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The Composition of Government Expenditure in an Overlapping Generations Model

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Abstract

This paper examines the choice of government expenditure on public goods and transfer payments (in the form of pension) under majority voting in an overlapping generations model, in which government expenditure is tax-financed on a pay-as-you-go (PAYG) basis. The condition required for majority support of the social contract involved in the PAYG scheme is established and shown to be independent of government expenditure, so that the choice of expenditure composition can be made conditional on acceptance of this social contract. The model yields a closed-form solution for the majority choice of the ratio of transfer payment to public goods, which depends negatively on the ratio of median to mean income, given parameters regarding preferences, tax, growth and interest rates. Informed by this result, a dataset for democratic countries is examined, suggesting that income inequalities play a minor role in accounting for the substantial variations in the compostion of government expenditure across democratic countries, while different preferences for public goods resulting from cultural differences may be an important determinant. Finally an alternative decision mechanism is also considered, in which a utilitarian government chooses expenditures to maximize a social welfare function. The solution is found to take a similar form to that of the majority voting context, except that a welfare-weighted average income replaces the median income.

JEL code: D72, H41, H53, H11

Keywords: Overlapping Generations, Majority Voting, Balanced Growth, Public Goods, Transfer Payments, Utilitarian Government

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1 Introduction

There are substantial variations among countries in their tax and expenditure policies, even among developed democracies sharing similar economic and political regimes.[1] For example, the US has relatively high income inequality and combines a low overall tax rate with a low ratio of expenditure on transfer payments to that on public goods. Scandinavian countries typically combine low inequality with high tax rates and substantially higher expenditure on transfer payments relative to public goods. British Commonwealth countries have higher income inequality than Scandinavian countries and yet devote much less to transfer payments compared with public goods. It therefore seems likely that some of the differences among countries can be attributed to cultural factors affecting attitudes towards income redistribution.

In order to attempt to understand the way in which preferences and economic conditions may combine to influence the composition of expenditure, this paper examines the choice of the division of government expenditure between public goods and a transfer payment. This choice is considered in the context of an overlapping generations model in which a pure public good and a transfer payment, in the form of a pension, are tax-financed on a pay-as-you-go (PAYG) basis. The decision clearly involves much more than simply the redistribution of income among members of the same cohort. The unconditional pension requires a decision regarding income shifting between periods within the life cycle as well as intra- and inter-generational redistribution.

The decision mechanism involves majority voting by members of each cohort regarding desired pension and public goods expenditure during the retirement period, on the understanding that during the working period each cohort finances the expenditure previously agreed by the preceding cohort and voters are aware of the nature of the government budget constraint. There is therefore a social contract in which each generation, in the retirement period, is able to benefit from the income and population growth of the following generation. The condition under which the social contract is supported by a majority of the members of any cohort involves extending the condition obtained by Aaron (1966) and Samuelson (1958). It is established and shown to be independent of individual preferences, tax rates, and choices of government expenditure, such that agreement can be obtained prior to voting over the expenditure composition. The majority voting equilibrium yields closed-form solutions for expenditures on transfer payments and public goods, and their ratio. This equilibrium exhibits balanced growth with pensions and public goods expenditure both

[1] See Tanzi (2000) for comparisons across countries over time. More details are given in Section 4 below.

growing with population and income growth, while their ratio remains constant. An alternative decision mechanism is also considered in which a utilitarian government maximises a discounted sum of all generations' lifetime utilities. It is shown that the government's choice of expenditure in each period takes a similar form to the majority voting outcome, except that the median income is replaced by a welfare-weighted average income measure.

There is a substantial literature on decisions involving government expenditure, most of which focuses on one type of government expenditure, either public goods expenditure or redistributive transfer payments.[2] Since the single type of government expenditure is financed by income taxation, the choice of government expenditure in these models is determined by the tax rate, which is often chosen by majority voting, stemming from the early work of Meltzer and Richard (1981).

Several recent studies consider more than one type of government expenditure. Bearse *et al.* (2001) study majority voting over a transfer payment and public education in a static framework. Hassler and *et al.* (2007) consider redistribution policy as well as provision of public goods financed by imposing a tax on the rich, which indicates the extent of redistribution. Creedy and Moslehi (2009) examine majority voting over government expenditure on transfer payments as well as public goods, within a static framework. The present paper follows this line of research, with a focus on the composition of government expenditure within an overlapping generations framework.[3]

Since the emphasis of the present study is on the composition of expenditure, rather than its total, the income tax rate is assumed, as in Bearse *et al.* (2001), to be exogenously fixed; it is not a decision variable. This assumption ensures that voting is over one dimension only – the pension (expenditure on the public good is then obtained from the government budget constraint). After pointing out that this is a common assumption, Tridimas (2001, p. 308) suggested that it 'is less restrictive than it first appears, since in practice governments are often constrained in the policy instruments that they may vary at anyone time'. In practice, tax and expenditure policies are usually debated separately and stronger constraints are usually imposed on changes in income tax rates. In the literature few voting models involve more than one dimension.[4] The voting problem considered here can be thought of as a

[2]For example, Tridimas (2001), Grossmann (2003) and Tridimas and Winer (2005) consider expenditure on public goods. Meltzer and Richard (1981), Grossman and Helpman(1998), Krusell and Rios-Rull (1999), Hassler *et al.* (2003), Hassler *et al.* (2005), Azzimonti *et al.* (2006) and Borck (2007) concentrate on transfer payments.

[3]Hassler *et al.* (2003, 2007), and Hassler *et al.* (2005) consider voting over taxes and transfers in two-period overlapping generations models in which individuals are ex ante identical but are subject to different probabilities of success regarding private investments.

[4]For example, Borge and Rattsø, J. (2004) use intermediate preferences and consider a multidimensional

second stage in a two-stage voting process in public choice, where in the first stage voters are concerned only with the tax rate, and in the second stage voting concerns composition of expenditure conditional on a given tax rate.[5]

In the present dynamic context, an explicit solution for the composition of government expenditure, the ratio of transfer payments to public goods expenditure, is derived. It depends negatively on the ratio of median to mean income, preference for public goods, and the discount rate, and positively on the tax rate, population and income growth rates. In particular, it is shown that more inequality (a lower ratio of median to mean income) gives rise to the transfer payment (here the pension) forming a larger proportion of total government expenditure, for given other parameters. This result is consistent with models examining majority voting over the tax rate, with an unconditional transfer payment, in which a uniform result is that more basic inequality leads to the choice of a more redistributive tax and transfer structure. For examples, see Meltzer and Richard (1981) and Krusell and Rios-Rull (1999).

However, as summarised in Lind (2005), empirical evidence concerning this relationship, based on cross-sectional data for a range of countries, has been found to be mixed. Lind (2005) and Alesina and Glaeser (2004) conjecture several reasons underlying this mixed result. In particular, Alesina and Glaeser (2004) emphasise the importance of unobserved cultural factors in shaping the redistribution policy. The present model suggests that in view of the wide range of factors affecting the composition of expenditure, it may indeed in practice be difficult to observe a simple relationship between redistribution and basic inequality in cross-country comparisons. On the other hand, the model also provides a way to examine the role of cultural factors in determing the expenditure patterns across democratic countries, since all determinants of expenditure composition identified in the model can be observed in the data except the preference parameter (utility weight attached to public goods) reflecting people's attitude toward public goods and redistribution expenditure.

Based on a dataset for a range of democratic countries, constructed by compiling data from *World Development Indicators* (2007), *World Income Inequality Database* (2007), and *Government Financial Statistics* (2008), the preference parameter is inferred for each country using the closed-form solution for expenditure composition. The results show that the U.S. has a much higher preference for public goods, compared with Commonwealth countries

policy with unidimensional conflict.

[5] On such two-stage voting in a public choice context, see McCaleb (1985) who emphasises uncertainties involved during the first stage.

and Scandinavian countries. This may explain why the U.S. has the lowest ratio of transfer payments to public goods although it has the highest degree of income inequality. Another finding is that the relationship between the expenditure composition and income inequality is quite flat over the most relevant range of income inequality, so the mixed empirical results regarding this relationship are not surprising.

The paper is arranged as follows. Section 2 describes the framework of analysis. In view of its central role, this section also examines the condition for majority support of the PAYG social contract. Section 3 characterizes the voting equilibrium. Closed-form solutions for expenditures on pensions and public goods and their ratio are obtained, and comparative static properties are examined and illustrated numerically. Section 4 presents some cross-country comparisons, informed by the model, on the composition of government expenditure among democracies. Section 5 examines a utilitarian government's choice of expenditure. Brief conclusions are in section 6.

2 The Economic Environment

This section begins in subsection 2.1 by describing the overlapping generations model with a public sector. In view of its central role, the condition under which a majority of each cohort supports the intergenerational social contract is established in subsection 2.2.

2.1 The Two-Period Framework

Each individual is assumed to live for two periods, a working and a retirement period, so that the economy is populated by two overlapping cohorts in any given period. Individuals have identical preferences but are heterogenous with respect to income endowments. A young individual i, born at time t, works in the first period and receives an exogenously fixed income, $y_{i,t}$.[6] Income is taxed at the rate τ, which is the same for all individuals and is assumed to be exogenously determined.

In period t, a young individual, i, allocates disposable income between current consumption, $c_{1i,t}$, and savings, $s_{i,t}$. In the second period of life, the individual finances consumption of private goods, $c_{2i,t+1}$, using the unconditional and untaxed pension from the government, b_{t+1}, and the return on savings, $(1+r)s_{i,t}$, where r is the constant interest rate at which

[6] If the objective were to examine the choice of income tax rate, the assumption of exogenous incomes would make no sense, but the emphasis here is on the composition of expenditure. It is also not necessary to specify the precise form of the income distribution because, as shown below, only the ratio of median to mean income matters.

individuals can borrow or lend and, for simplicity, it is assumed that there is no tax on interest income. Government expenditures on pure public goods in t and $t+1$ are denoted as G_t and G_{t+1}. The price of the private consumption good is normalised to unity, so that c denotes private consumption expenditure.

The transfer payment is referred to here as a pension, since it is received in the second period of the life cycle.[7] However, it may be thought of more broadly as a standard type of income transfer since it augments the exogenously fixed income. Assuming perfect capital markets, some low-income people may wish to vote for a high value of b_{t+1}, part of which can be used to repay a loan in the first period while the remainder finances consumption in the second period. Conversely high-income individuals may prefer a low, or even zero, transfer while making positive savings during the working period.

In view of the inter-generational transfers in addition to the income shifting and intra-generational redistribution, it is desirable to allow for population and income growth. Suppose the average income of young individuals grows at a constant rate of ω over time, so that:

$$\frac{\bar{y}_{t+1}}{\bar{y}_t} = 1 + \omega. \tag{1}$$

Also, there is constant growth, at the rate n, in the population, so that:

$$\frac{N_{t+1}}{N_t} = 1 + n, \tag{2}$$

where N_t denotes the number of individuals born in period t. Here it is assumed that income growth involves an equal proportional change at all income levels and that population growth involves an equal proportional change in population frequencies at each income level, so that ω and n are independent.

2.2 Majority Support for The Social Contract

This section examines the condition required for the majority of each generation to be better off with the PAYG arrangement, thus ensuring the cooperation between generations. For any given value of expenditure on the public good and its associated tax rate, the question concerns the condition under which an individual (with given $y_{i,t}$) is better off with a transfer payment (and its associate higher tax rate) compared with a framework in which retirement income is provided by private savings alone, involving income shifting without the intra-

[7] The introduction of an additional transfer, received in the working period, would involve considerable problems arising from multidimensional voting.

and inter-generational redistribution of the transfers. Clearly, those who are most likely to prefer a private system are those with relatively high incomes.

A basic treatment of social insurance is given by Samuelson (1958) and Aaron (1966), who essentially establish the conditions under which the average consumption of each generation is higher with a PAYG system than with a fully funded scheme. They find that social insurance can increase average welfare 'if the sum of the rates of growth of population and real wages exceeds the rate of interest' (Aaron, 1966, p. 372). Under this condition, the PAYG system allows each generation to share the benefits arising from population and productivity growth, since later generations will be both larger and richer, so long as those gains are sufficient to outweigh the returns from private investment plans. The analysis, like the present context, is nevertheless confined to a partial equilibrium analysis, so no consideration is given to the possibility that different regimes imply different private returns to saving, arising because total savings in the two systems could be substantially different.[8]

The context here is similar to the basic Aaron-Samuelson setup, except that there is income heterogeneity among individuals of a generation and tax revenues also finance public goods expenditure (not transfer payment alone). In comparing the PAYG system and a fully funded scheme in the current context, it is assumed that the two systems provide the same expenditure on public goods in each period. Let a superscript, P, be used to denote values in the PAYG system with a basic pension (which may of course be augmented by private savings), while F indicates values in the scheme with private pension funds only; thus, $G_t^P = G_t^F = G_t$. The pension to be received in period $t+1$ under the PAYG system is denoted as b_{t+1}^P. In the private scheme the government finances only public goods, while in the PAYG system it finances public goods and the pension. The tax rates in the two systems must therefore differ; these are denoted as τ^F and τ^P respectively.

From the lifetime budget constraint, the present value of the ith individual's life time income under the PAYG system is given by $(1-\tau^P)y_{i,t} + b_{t+1}^P/(1+r)$. Consider next the privately funded system where there is no transfer payment and income tax finances only the provision of public goods. Individual i's lifetime income is simply $(1-\tau^F)y_{i,t}$. Since both systems provide the same amount of public good under the two systems, a sufficient

[8]The basic Aaron-Samuelson analysis also ignores the labour-leisure choice. Creedy and van de Ven (2000) extend the analysis by allowing for labour supply effects of taxation and for increasing longevity and find that the basic condition is not affected by an assumption that labour supply is endogenous. Walque (2005) uses non-cooperative game theory to examine an overlapping generations model with productivity and population growth. He assumes that some public transfer payment is proposed and voters decide to accept or reject it through their vote. The young form the majority at each time, since population growth is positive. He finds similar conclusions to Aaron (1966).

condition for utility to be higher in the PAYG system compared with private funding is that lifetime income is higher. This requires $(1-\tau^P)y_{i,t} + \frac{b^P_{t+1}}{1+r} > (1-\tau^F)y_{i,t}$, or equivalently:

$$(1+r) < \frac{b^P_{t+1}}{y_{i,t}(\tau^P - \tau^F)}. \tag{3}$$

The government budget constraint at time $t+1$ in the privately funded system is:

$$G_{t+1} = \tau^F N_{t+1}\bar{y}_{t+1}, \tag{4}$$

and in the PAYG system it is given by:

$$G_{t+1} + N_t b^P_{t+1} = \tau^P N_{t+1}\bar{y}_{t+1}. \tag{5}$$

Combining these two constraints gives the pension as:

$$b^P_{t+1} = (1+n)\bar{y}_{t+1}\left(\tau^P - \tau^F\right). \tag{6}$$

Substituting b^P_{t+1} from (6) into (3), and using $\bar{y}_{t+1} = \bar{y}_t\,(1+\omega)$ gives the following condition under which the ith individual is better-off in the PAYG system:

$$(1+n)\,(1+\omega)\,\frac{\bar{y}_t}{y_{i,t}} > (1+r). \tag{7}$$

This can be compared with the basic Aaron and Samuelson condition in a model without income heterogeneity and public goods, which requires $(1+n)\,(1+\omega) > (1+r)$.[9] From (7) the condition for the 'average' individual, for whom $y_{i,t} = \bar{y}_t$, is precisely the same as the Aaron-Samuelson condition. The higher-income individuals are more likely to prefer the privately funded scheme, which involves only income shifting rather than redistribution within and between generations.

An important feature of the condition in (7) is that it is independent of G, b and τ. In addition, it does not depend on the nature of preferences.[10] Hence agreement by a majority of each cohort for the use of a PAYG scheme and its associated social contract can be established prior to considerations regarding the choice of actual expenditure levels. If:

$$(1+n)\,(1+\omega)\,\frac{\bar{y}_t}{y_{m,t}} > (1+r), \tag{8}$$

the PAYG system gains majority support. For a positively skewed income distribution, median income, $y_{m,t}$, is less than the arithmetic mean income, $\bar{y}_t$: hence the greater the

[9] It is by ignoring cross-product terms that the condition is often stated as $r < n + \omega$.

[10] The condition can of course hold even if population growth is negative.

skewness, the more likely is the condition above to be satisfied for given values of the relevant rates. Those high-income individuals who would prefer only private funding are not allowed to 'contract out' of, or withdraw from, the social contract and the pension system. Without this compulsory element there would be an adverse selection problem arising from the gradual reduction in average income, of those remaining in the system, as the richest individuals gradually contract out.

3 The Voting Equilibrium

This section examines the majority voting equilibrium, assuming that the condition required for majority support of the PAYG system, established in (8) above, is satisfied. Voting takes place in each period and individuals vote only on the pension to be paid during the next period, for a given tax rate. Hence, there is no incentive for members of the old cohort to vote (as their preferences are entirely selfish). Those currently retired do not have a second vote over their pension (which they do not finance), since this has already been determined by their vote in the previous period.[11] The median member of the young cohort is the decisive voter. As part of the social contract, young individuals understand that they must finance the pension of those currently retired (and of course the majority of them recognise that they are better off by doing this). The resulting public good expenditure is determined from the government budget constraint. Individual preferences are examined in subsection 3.1. The voting outcome is derived in subsection 3.2, and subsection 3.3 shows that it is a balanced-growth equilibrium. Subsection 3.4 examines comparative static properties of the model, and numerical illustrations are reported in subseciton 3.5.

3.1 Individual Preferences

Each individual is assumed to have the following Cobb-Douglas lifetime direct utility function, expressed in logarithmic form:

$$U_{i,t} = \log c_{1i,t} + \gamma \log Q_{G,t} + \beta \left(\log c_{2i,t+1} + \gamma \log Q_{G,t+1} \right), \tag{9}$$

where $0 < \beta = \frac{1}{1+\rho} < 1$ is the discount factor and ρ is the time preference rate, and γ is the weight attached to consumption of public goods. Also, $Q_{G,t}$ and $Q_{G,t+1}$ are the quantities of the public good consumed by each person at time t and $t+1$.

[11] Hassler *et al.* (2007) use a similar assumption, that each generation votes once only, to find the political equilibrium in an overlapping generations context. In their study, agents vote on tax in each period and also decide how the revenue should be spent.

Normalising the price of the private consumption good to unity, the lifetime budget constraint of an individual is given by:

$$c_{1i,t} + \frac{c_{2i,t+1}}{(1+r)} = (1-\tau)y_{i,t} + \frac{b_{t+1}}{(1+r)} \equiv M_{i,t}. \tag{10}$$

This form allows for the fact that tax-financed public goods are non-excludable so that individuals are not charged at the point of consumption. To examine the voting outcome it is necessary to obtain each individual's indirect utility function, $V_{i,t}$, as follows.

The consumption plans, conditional on the values of public expenditure and the pension, are given, using the standard properties of Cobb-Douglas utility functions, as:

$$c_{1i,t} = \frac{M_{i,t}}{(1+\beta)}, \tag{11}$$

$$c_{2i,t+1} = \frac{\beta\,(1+r)\,M_{i,t}}{(1+\beta)}. \tag{12}$$

Hence planned private savings of the young individual, $s_{i,t}$, are:

$$s_{i,t} = \frac{\beta(1-\tau)y_{i,t}}{(1+\beta)} - \frac{b_{t+1}}{(1+r)\,(1+\beta)}. \tag{13}$$

From (13) an increase in the tax rate and the pension reduces savings, while an increase in the interest rate increases savings. Without the pension, the income and substitution effects of changes in the interest rates would offset each other such that savings would be independent of the interest rate.[12] Nevertheless, with the pension, the substitution effect outweighs the income effect such that an increase in the interest rate increases private savings. According to the savings function (13) the individual borrows if:

$$y_{i,t}\,(1-\tau) < b_{t+1}\left(\frac{1+\rho}{1+r}\right). \tag{14}$$

That is, borrowing takes place if disposable income is low in relation to the pension. Those low income individuals would borrow to finance their first period consumption and repay their debt with the pension received in their retirement period.

The indirect utility function, $V_{i,t}$, is obtained by substituting the optimal $c_{1i,t}$ and $c_{2i,t+1}$ into the direct utility function (9), whereby:

$$V_{i,t} = \log\left(\frac{M_{i,t}}{1+\beta}\right) + \beta\log\left(\frac{\beta\,(1+r)\,M_{i,t}}{(1+\beta)}\right) + \gamma\left(\log Q_{G,t} + \beta\log Q_{G,t+1}\right). \tag{15}$$

[12] This is a particular property of Cobb-Douglas utility functions.

However, the pension and public goods expenditure in each period are financed on a pay-as-you-go (PAYG) basis. Hence the values of $Q_{G,t}$ and $Q_{G,t+1}$ can be expressed in terms of b_t and b_{t+1} using the government budget constraint, which at time t is given by:

$$G_t + N_{t-1}b_t = \tau N_t \bar{y}_t, \tag{16}$$

where $G_t = pQ_{G,t}$, and p is the constant cost of producing one unit of the public good. Substituting into (15) gives indirect utility:

$$\begin{aligned} V_{i,t} &= \log\left(\frac{(1+r)(1-\tau)y_{i,t} + b_{t+1}}{(1+r)}\right) \\ &\quad +\beta \log\left(\beta\left((1+r)(1-\tau)y_{i,t} + b_{t+1}\right)\right) \\ &\quad +\gamma \log\left(\tau N_t \bar{y}_t - N_{t-1}b_t\right) + \beta\gamma \log\left(\tau N_{t+1}\bar{y}_{t+1} - N_t b_{t+1}\right) \\ &\quad -(1+\beta)\left\{\log(1+\beta) + \gamma \log p\right\}. \end{aligned} \tag{17}$$

3.2 Majority Voting

From (17), voting involves only one dimension, the value of b_{t+1}, because all other variables determining an individual's indirect utility are either predetermined or exogenously given. If the indirect utilities for all young individuals are single-peaked in b_{t+1}, the majority voting outcome is dominated by the median voter, who in the present context is the individual with median income, $y_{m,t}$.

Single-peakedness is guaranteed if the relationship between $V_{i,t}$ and b_{t+1} is strictly concave for all individuals, that is, if $\partial^2 V_{i,t}/\partial b_{t+1}^2 < 0$ for all i. This condition is confirmed by a differentiation of (17). Consequently, maximising the indirect utility function with respect to b_{t+1} gives the majority choice of pension at time $t+1$, $b_{m,t+1}$. The first-order condition is:

$$\frac{\partial V_{m,t}}{\partial b_{m,t+1}} = \frac{1+\beta}{(1+r)(1-\tau)y_{m,t} + b_{m,t+1}} - \frac{\beta\gamma N_t}{\tau N_{t+1}\bar{y}_{t+1} - N_t b_{m,t+1}} = 0, \tag{18}$$

and $b_{m,t+1}$ is solved as:

$$\begin{aligned} b_{m,t+1} &= \bar{y}_t\left(\frac{1+\beta}{1+\beta+\beta\gamma}\right) \\ &\quad \times\left\{(1+n)(1+\omega)\tau - \left(\frac{\beta\gamma}{1+\beta}\right)(1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}\right\}. \end{aligned} \tag{19}$$

The majority choice of public goods expenditure, $G_{m,t+1}$, can be solved by substituting $b_{m,t+1}$ into the government budget constraint at time $t+1$, giving:

$$\frac{G_{m,t+1}}{N_t} = \bar{y}_t\left(\frac{\beta\gamma}{1+\beta+\beta\gamma}\right)\left\{(1+n)(1+\omega)\tau + (1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}\right\}. \tag{20}$$

The focus here is on the ratio of the total expenditure on pensions to that on public goods, $R_{m,t+1}$, which is given by:

$$R_{m,t+1} = \frac{\frac{(1+\beta)}{\beta\gamma}(1+n)(1+\omega)\tau - (1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}}{(1+n)(1+\omega)\tau + (1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}}. \tag{21}$$

This result shows that $R_{m,t+1}$ depends, *inter alia*, on the ratio of median income to mean income at time t and parameters regarding population growth, income growth, the tax rate and preferences.[13] Note that the precise form of the income distribution is irrelevant; only the *ratio* of median to mean income matters. The growth rates of population and incomes, n and ω, appear in a symmetric fashion in (21); they both have the same effect on $R_{m,t+1}$.

Substituting (19) into (14) gives the criterion for borrowing by individual i, with the majority choice of pension, as:

$$\frac{y_{i,t}}{\bar{y}_t} < \frac{(1+\beta)(1+n)(1+\omega)\tau - \beta\gamma(1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}}{\beta(1+r)(1+\beta+\beta\gamma)(1-\tau)}. \tag{22}$$

This result shows that individual i borrows to finance first-period consumption if the ratio of i's income to average income is less than some critical value, which is determined by the ratio of median to mean income, among other parameters.

3.3 The Balanced Growth Path

In this subsection the voting equilibrium is shown to be a balanced-growth equilibrium in which all endogenous variables, including total expenditure on public goods and pensions, total consumption of young and old individuals, and total savings by young individuals, grow at the same rate.

First, the crucial ratio, $y_{m,t}/\bar{y}_t$, is unchanged with the population and income growth, according to the assumption that income growth involves an equal proportional change at all income levels and population growth involves an equal proportional change in population frequencies at each income level. This implies by equation (19) that the majority choice of pension per old individual, $b_{m,t+1}$, grows at the same rate as the average income of individuals, $\bar{y}_t$, and consequently total expenditure on the pension, $B_{m,t+1} \equiv N_t b_{m,t+1}$, grows at the same rate as the total income of individuals. That is, pensions per capita grows at rate ω and total pension expenditure grows at the rate, $n + \omega$.[14] Similarly, by

[13] The expenditure ratio does not depend on the constant cost of producing the public good, p. This property arises from the Cobb-Douglas form of utility functions.

[14] This ignores the term $n\omega$ in the expansion of $(1+\omega)(1+n)$.

equation (20) total expenditure on public goods, $G_{m,t}$, grows at the rate, $n+\omega$, and public goods expenditure per capita, $G_{m,t}/(N_{t-1}+N_t)$, grows at the rate, ω. As a result, the ratio of pension expenditure to public goods expenditure is a constant, as also confirmed by equation (21).

The period-t total consumption of young individuals, $N_t\sum_{i=1}^{N_t} c_{1i,t}$, total savings of young individuals, $N_t\sum_{i=1}^{N_t} s_{i,t}$, and total consumption of old individuals, $N_{t-1}\sum_{i=1}^{N_{t-1}} c_{2i,t}$, can be expressed by substituting $b_{m,t+1}$ into (11), (13) and (12). All these variables grow at the rate, $n+\omega$, and their per capita terms grow at the rate, ω.

Hence, the voting equilibrium is characterised by a balanced growth path, along which all aggregate endogenous variables grow at the same rate as aggregate income, and per capita variables grow at the same rate as average income. For simplicity incomes are assumed to be exogenously determined in the model. But it can be naturally incorporated into the model that income growth results from exogenous technological progress. Hence these implications regarding the balanced growth path are consistent with those of neoclassical growth models with exogenous PAYG social security, despite the fact that the social security is determined by majority choice in the present model.

3.4 Some Comparative Statics

This section presents some comparative static properties of the model. The aim is to examine how total expenditure on pensions, $B_{m,t+1}$, public goods, $G_{m,t+1}$, and their ratio, $R_{m,t+1}$, change in response to changes in parameters of the model. As shown in equations (19),(20) and (21), these relations are nonlinear. The signs of first and second derivatives of these variables with respect to each parameter are reported in Table 1.

Table 1: Comparative Statics of the Majority Choice of Expenditure on Pensions and Public Goods and Their Ratio

	$y_{m,t}/\bar{y}_t$	τ	γ	β	r	ω	n
First Derivative							
$B_{m,t+1}$	$-$	$+$	$-$	$-$	$-$	$+$	$+$
$G_{m,t+1}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$R_{m,t+1}$	$-$	$+$	$-$	$-$	$-$	$+$	$+$
Second Derivative							
$B_{m,t+1}$	0	0	$+$	$+$	0	0	$+$
$G_{m,t+1}$	0	0	$-$	$-$	0	0	$+$
$R_{m,t+1}$	$+$	$-$	$+$	$+$	$+$	$-$	$-$

A key determinant is $y_{m,t}/\bar{y}_t$, the ratio of median to average income. A rise in $y_{m,t}/\bar{y}_t$ represents a fall in inequality. Note that $B_{m,t+1}$ and $G_{m,t+1}$ are linearly decreasing and increasing in $y_{m,t}/\bar{y}_t$, respectively, suggesting that higher inequality causes voters to vote for higher pension expenditure and lower public goods expenditure. This implies that a rise in inequality would lead to a higher ratio of pension expenditure to public goods expenditure, which is confirmed by a negative first derivative of $R_{m,t+1}$ with respect to $y_{m,t}/\bar{y}_t$. The second derivative is positive, implying that the ratio of pension expenditure to public goods increases at an increasing rate as inequality rises.

It is also of interest to consider how the majority choice of total expenditure on pensions and public goods change with the tax rate. Clearly $B_{m,t+1}$ is linearly increasing in τ. However, $G_{m,t+1}$ is also linear in τ, but increases with τ only if $(1+n)(1+\omega)-(1+r)\frac{y_{m,t}}{\bar{y}_t} > 0$. This condition is the same as that derived above in (8) for majority support for the PAYG social security system. The majority choice of the composition of government expenditure, $R_{m,t+1}$ is increasing in τ unambiguously, but at a decreasing rate, if again the condition for majority support for the social contract is satisfied. These results suggest that an increase in the tax rate gives the government more income to spend on both types of expenditure, but the increase in pension expenditure is relatively higher than the increase in public goods expenditure.[15]

The comparative static results with respect to γ suggest that an increase in the preference for public goods unambiguously increases the total expenditure on public goods, but decreases total expenditure on pensions and the ratio of pension to public goods expenditure.

The results with respect to β suggest that an increase in the discount factor has a negative effect on the total expenditure on pensions and on the ratio of pensions to public goods, but increases the total expenditure on public goods. This result can be understood from a positive relationship between a young individual's savings and the discount factor; see equation (13). A higher discount factor, that is a higher weight on second-period utilities, leads to more private savings by individuals at young age, such that individuals tend to vote for a lower public-saving-pension, which results in a higher public goods expenditure and a lower ratio of pension to public goods expenditure.

An increase in the interest rate has similar effects to those of an increase in the discount factor. It increases public goods expenditure, but decreases expenditure on pensions and its ratio to public goods expenditure. This is clear from equation (8): an increase in the

[15] However, in a static model with transfers and a public good, Creedy and Moslehi (2009) find a concave relationship between the share of transfers and the tax rate arising from the adverse incentive effects of the tax and transfer system.

interest rate raises the return on private savings such that individuals are more likely to prefer the privately funded scheme; that is, individuals are more likely to vote for a lower pension.

Regarding parameters governing the growth of income and population, the results show that increases in ω and n increase total expenditure on pensions and on public goods, and also their ratio. With income growth or population growth, tax revenues of the government are increased such that the government is able to spend more on both types of expenditure. However, the increase in the expenditure on pensions is higher than the expenditure on public goods because a higher n and/or ω make individuals more likely to prefer the PAYG system, as shown in (7).

3.5 Numerical Illustrations

The comparative static analysis provides a general idea of whether the composition of government expenditure would increase or decrease following a change in a given parameter of the model. However, it does not show the sensitivity of the expenditure composition with respect to changes in parameters. This subsection reports some numerical examples to illustrate these properties, which may help identify some important factors underlying the observed variations in the composition of government expenditure across democratic countries.

To set a baseline value for each parameter, assume that the length of a time period in the model is 20 years. Using the sample of democratic countries discussed in Section 4 (and in Appendix A), the average annual growth rates of income and population, and the real interest rate are 0.024, 0.007, and 0.0052, giving $\omega = 0.73$, $n = 0.16$ and $r = 0.17$, respectively. Assuming that the time preference rate is equal to the interest rate, $\rho = r$, then $\beta = 0.97$.[16] The baseline value for the tax rate is $\tau = 0.4$, the average income tax rate across the democratic countries. Given the baseline values for other parameters and an average median to mean income ratio of $y_m/\bar{y} = 0.85$, γ is chosen to match the average expenditure composition for the sample ($R = 1.59$), which gives $\gamma = 0.65$ (see Section 4 for details).

Figure 1 illustrates variations in the relationship between $R_{m,t+1}$ and $y_{m,t}/\bar{y}_t$, for given values of the other parameters. The parameter variations considered are 15 and 30 per cent changes around the baseline values. As expected, increasing $y_{m,t}/\bar{y}_t$ reduces $R_{m,t+1}$ at a

[16] These are obtained respectively using $(1+0.024)^{20}-1$, $(1+0.007)^{20}-1$ and $(1+0.0052)^{20}-1$, which assumes a 20 year period. Also, the value of time preferences rate, β, obtained from $1/(1+0.0052)^{20}$.

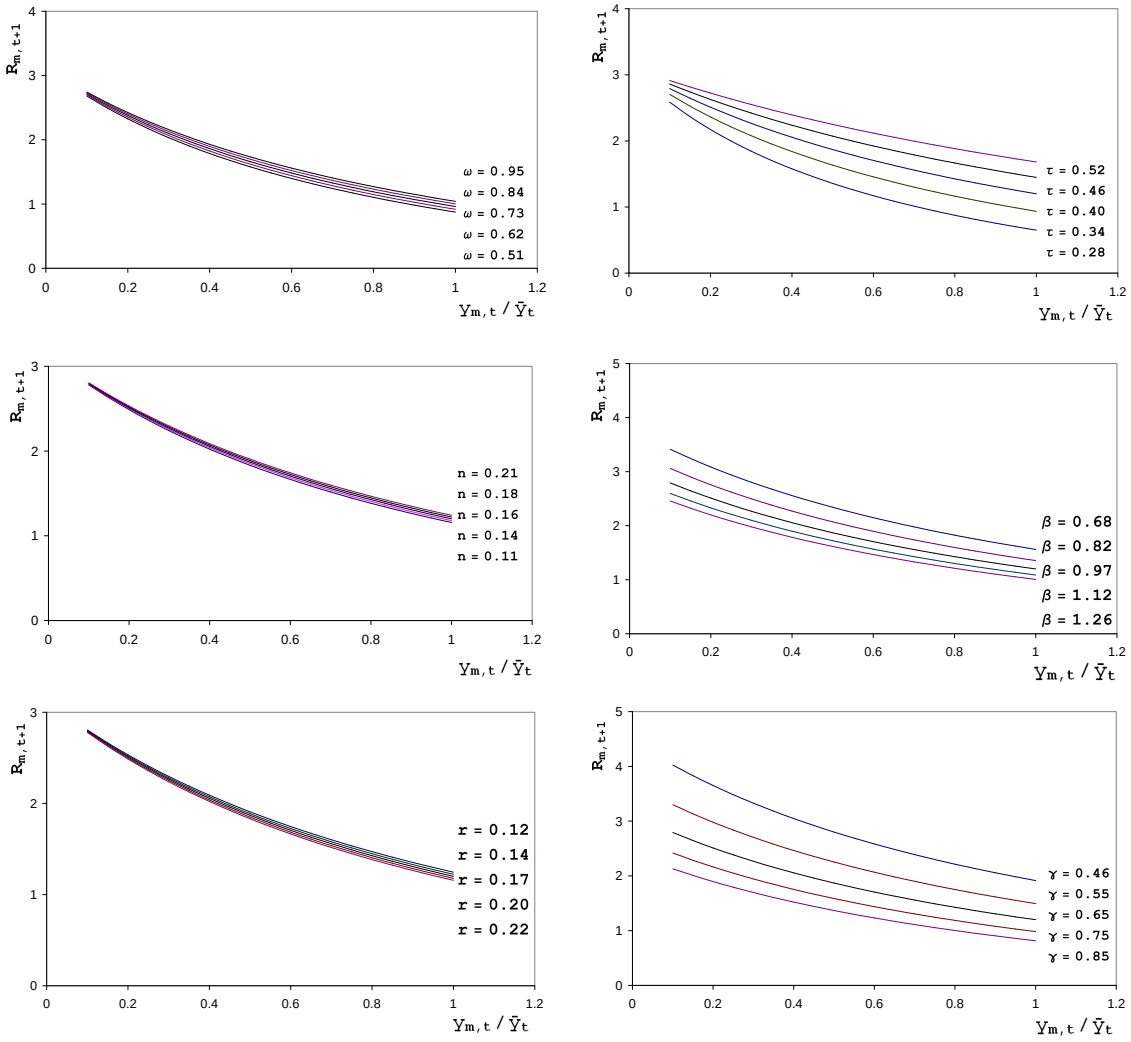

Figure 1: Variation in Expenditure Ratio for Alternative Parameters

decreasing rate in all figures, demonstrating that lower inequality is consistently associated with a lower ratio of pension expenditure to public goods expenditure. However, the relationship between R and $y_m/\bar{y}$ is also quite flat around empirically relevant values.[17] It is therefore not surprising that empirical studies have generally failed to find a strong, or even negative, relationship. The diagrams show that the majority choice of R is quite sensitive to variations in the tax rate, discount factor and the utility weight attached to public goods, and less sensitive to changes in the interest rate and growth rates of population and income. This indicates the potential importance of preferences over public goods, as summarized by γ, in shaping the composition of expenditure.

4 Cross-Country Comparisons

Informed by the model, this section examines patterns of government expenditure for a cross-sectional sample of democratic countries.[18] The aim is to examine whether there are systematic variations in expenditure composition across different groups of democratic countries and, if so, identify some important factors that may help explain such variations.

It is very difficult indeed to obtain detailed comparable information on government expenditure patterns for a wide range of countries. Appendix A describes the construction of a dataset for a range of democratic countries, along with a number of non-democratic countries for which data can be obtained. Brief summary information is reported here. A major difficulty in the present context is that inadequate information about expenditure components is available. Ideally, separate details regarding pension or superannuation expenditure are required, but only aggregate expenditure on transfer payments can be obtained: the use of the aggregates effectively requires an assumption that the composition of transfers is similar across countries. Furthermore, not all expenditure on health and education involves a pure public good. In the absence of clear information on the public-private mix, calculations are reported for three assumptions: the term R_F denotes the ratio of total transfer expenditure to that on public goods, where the latter includes a proportion, F, of that on health and education.[19]

[17] In the following section it is seen that $y_m/\bar{y}$ varies from about 0.75 to 0.90 for the democratic countries examined.

[18] Comparisons of expenditure patterns over time for a single country are extremely difficult to make, given the paucity of data, and would anyway be unlikely to reveal any significant influence of the term $y_m/\bar{y}$, in view of its relative stability over time. A more rigourous regression-based examination is also not realistic given the small size of the current data set.

[19] For example, $R_{1/4}$ refers to the ratio of total transfer expenditure to public goods expenditure, where public goods expenditure includes only 1/4 of health and education expenditure.

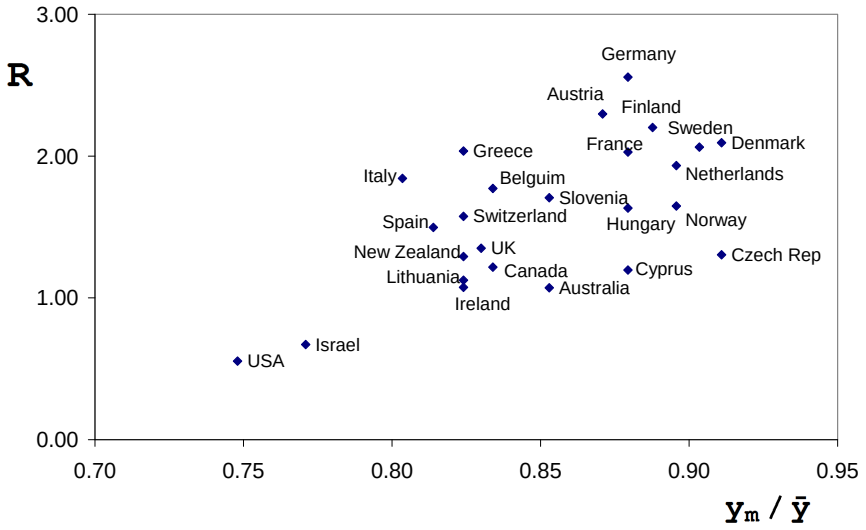

Figure 2: Expenditure Shares and Income Ratios for a Sample of Democratic Countries

Despite these severe data problems, some indication of variations among democratic countries is shown in Figure 2. This shows the partial relationship between the ratio of transfer expenditure to public goods expenditure ($R_{1/2}$) and the ratio of median to arithmetic mean income, using average values over the period $2000 - 06$. Care must be taken in interpreting this diagram, given differences in the other relevant variables (including the tax rate) among countries. Nevertheless, there is clearly no systematic negative relationship between R and $y_m/\bar{y}$ across all these countries.[20] For example, the U.S. has both the lowest value of R and $y_m/\bar{y}$ (and has the lowest tax rate), while the Scandinavian countries have much higher $y_m/\bar{y}$ and are more closely grouped with relatively high values of R. Similarly, the Commonwealth countries of U.K., Australia, Canada and New Zealand are more closely grouped, showing a roughly negative relationship between R and $y_m/\bar{y}$.

The expenditure patterns discussed above suggest that cultural differences may play a substantial role. A society which favours income equality tends to support redistributive government policies and, as a result, relatively less government expenditure is devoted to public goods. In the model, the weight attached to public goods consumption in the utility function, γ, could be regarded as an indicator of the cultural factors that may influence the distribution of government expenditure. Despite the fact that γ is not directly observable,

[20] Nevertheless, Lind (2005) discussed various complexities not treated by the present model, such as the existence of multiple social contracts, prospect of upward income mobility, multi-dimensional policies, race and redistribution versus social insurance.

it can be inferred for each country using the analytical result in equation (21), given data on all other relevant variables for each country: population growth rate n, income growth rate ω, tax rate τ, median to mean income ratio $y_m/\bar{y}$, real interest rate r, and discount factor β (assuming that $\rho = r$ such that $\beta = \frac{1}{1+r}$). Since equation (21) is derived conditional on (8), the majority support condition for the PAYG system, it is interesting to see whether this condition holds in the data. We find that (8) holds in all democratic countries in our sample. More details for the data on these variables are given in Appendix A.

Table 2: Average (and coefficient of variation) of Variables for Different Country Groups

Country	$\frac{y_m}{\bar{y}}$	τ	$R_{1/4}$	$R_{2/4}$	$R_{3/4}$	$\gamma_{1/4}$	$\gamma_{2/4}$	$\gamma_{3/4}$
All democracies	0.85	0.40	2.27	1.59	1.23	0.49	0.65	0.78
	(0.05)	(0.17)	(0.34)	(0.32)	(0.31)	(0.33)	(0.29)	(0.27)
Scandinavian	0.90	0.50	2.99	2.00	1.51	0.40	0.56	0.70
	(0.01)	(0.08)	(0.13)	(0.12)	(0.12)	(0.10)	(0.08)	(0.07)
Others	0.84	0.40	2.14	1.51	1.17	0.51	0.66	0.79
	(0.05)	(0.15)	(0.35)	(0.34)	(0.33)	(0.34)	(0.30)	(0.28)
Developing	0.87	0.38	1.88	1.35	1.06	0.62	0.80	0.97
	(0.05)	(0.20)	(0.20)	(0.19)	(0.19)	(0.16)	(0.16)	(0.17)
Developed	0.85	0.42	2.33	1.62	1.25	0.48	0.62	0.75
	(0.05)	(0.16)	(0.34)	(0.32)	(0.32)	(0.34)	(0.29)	(0.27)
Commonwealth	0.84	0.38	1.71	1.23	0.95	0.51	0.67	0.80
	(0.01)	(0.09)	(0.09)	(0.10)	(0.11)	(0.02)	(0.02)	(0.04)
Others	0.85	0.42	2.37	1.66	1.28	0.49	0.64	0.77
	(0.06)	(0.17)	(0.34)	(0.32)	(0.31)	(0.36)	(0.31)	(0.29)
United States	0.75	0.30	0.75	0.55	0.44	0.78	0.92	1.03
Non_Democratic	0.77	0.32	1.32	0.99	0.8			
	(0.15)	(0.46)	(0.57)	(0.55)	(0.53)			

Note: R_F denotes the ratio of total transfer expenditure to that on public goods, where the latter includes a proportion, F, of that on health and education. γ_F is the corresponding calculated preferences parameter for public goods

Table 2 reports a summary of the data, in the form of means and coefficients of variation of relevant variables, for several groups of countries: information for each country is reported in Table 3 Appendix A. In particular, the calculated values for the preference parameter, γ, are reported in the last three columns. The democratic countries are divided into various groups, as suggested by the scatter diagram of Figure 2, within which important cultural characteristics may be thought to be similar. The Scandinavian countries includes Denmark, Finland, Norway and Sweden. Developing economies includes Czech Republic, Hungary, and

Lithuania.[21]

The U.S. has a much higher γ, indicating a much lower preference for redistributive transfer payments, compared with Scandinavian and Commonwealth countries. This may explain why the U.S. has the lowest ratio of transfer payment to public goods expenditure despite having the highest income inequality among democracies. In contrast, the Scandinavian countries have the lowest average γ among the democratic country groups, contributing to the high ratio of transfer payments to public goods despite the relatively low degree of inequality (a value of $y_m/\bar{y}$ closer to unity). The commonwealth countries have a γ that is comparable to the average of all democracies, but the coefficient of variation is relatively small compared with other groups, demonstrating a similar preference over income redistribution among these countries. There is an implied higher preference for redistribution (lower γ) in developed democratic countries, consistent with the relatively higher proportion of transfer payment in those countries although they have a degree of inequality similar to that of developing democratic countries.

These results are consistent with Alesina and Glaeser (2004), who focus on different attitudes toward redistribution in Europe and the U.S. to explain the observed considerable differences between U.S. and Europe in welfare expenditure, tax policy and regulation. They argue that these differences can not be explained by the economic theory that higher inequality leads to higher redistribution because they also document a higher income inequality across many demographic groups in the U.S. compared with Europe. They summarise the evidence from a *World Values Survey* which reveals different attitudes toward redistribution in the U.S. and Europe as follows: 'Americans believe that they live in a mobile society in which individual effort can lift people up the social ladder. Likewise, the European welfare states are supported by European beliefs that the poor are unfortunate and would be stuck in their poverty without government intervention' (Alesina and Glaeser, 2004, p.220).[22]

Some summary statistics for a small sample of non-democratic countries are also reported in Table 2. Since the majority voting model is not suitable for these countries, γ is not computed. It is clear that compared with democracies, non-democratic countries on average have higher income inequality, a lower tax rate, and a lower ratio of transfer payments to public goods expenditure.

[21] These belong to the group of middle-income economies in the World development Indicator (WDI) classification which contains countries in which GNI per capita in 2006 was between \$876 and \$10,725.

[22] They highlight other issues such as political institutions, racial and ethnic fractionalisation.

5 Choice of a Utilitarian Government

Finally, it is of interest to compare the majority voting outcome with the choice of a utilitarian government acting to maximize a multi-period social welfare function defined in terms of individual utilities. This section examines expenditure compositions under this alternative political regime. The utilitarian welfare function can be defined as the following discounted sum of all generations' utilities:

$$SW = \sum_{t=-1}^{\infty} \delta^t W_t, \tag{23}$$

where δ is the discount factor of the government, which could be different from private individuals' discount factor, β. The social welfare associated with generation t, W_t, is defined as a function of utilities of all individuals born in period t:

$$W_t = W\left(V_{1,t}, ..., V_{i,t}, ...V_{N_t,t}\right), \tag{24}$$

where $V_{i,t}$ is the lifetime indirect utility of individual i as defined in (15), and the function W is time-invariant, individualistic and Paretean. To ensure the government's problem is well defined, it is also assumed that W is additively separable, and strictly increasing and (weakly) concave in each of its argument.[23]

The utilitarian government, recognizing that individuals' consumption and saving decisions are conditional on its expenditure choices, chooses a sequence of expenditures on public goods and pensions, $\{b_{t+1}, G_{t+1}\}_{t=0}^{\infty}$ (b_0 and G_0 are given), to maximize the social welfare function, subject to pay-as-you-go financing in each period. That is, the government faces a budget constraint in each period as defined in (16). The government is assumed to have full commitment over its policy decisions.[24]

The optimising conditions for choice of b_{t+1}, for $t = 0, 1, \ldots$, are given by:

$$\sum_{t=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \left(\frac{\partial V_{i,t}}{\partial b_{t+1}} + \frac{\partial V_{i,t}}{\partial G_{t+1}} \frac{dG_{t+1}}{db_{t+1}} \right) = 0. \tag{25}$$

Notice that the imposition of pay-as-you-go financing as well as the two-period overlapping generations nature of the model ensures that maximisation of the social welfare function in (23) is equivalent to the maximisation of W_t in each t.

[23] A simple form of W_t which is adopted by many studies, for example Ghiglino (2000), defines W_t as the weighted sum of lifetime utilities of individuals in cohort t.

[24] This assumption is made simply to avoid any time consistency issues. In fact, the social welfare function defined here is consistent with requirements for time-consistent policy. See Calvo and Obstfeld (1988) and Ambler (2000) for further discussion on this issue in overlapping generations models.

Define $v_{i,t} = \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial b_{t+1}}$ as the welfare weight attached to an increase in i's income. The derivative of W_t with respect to G_{t+1} can thus be written as:

$$\sum_{i=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial G_{t+1}} = \sum_{i=1}^{N_t} v_{i,t} \left(\frac{\partial V_{i,t}/\partial G_{t+1}}{\partial V_{i,t}/\partial b_{t+1}} \right). \tag{26}$$

From the government budget constraint, (16):

$$\frac{dG_{t+1}}{db_{t+1}} = -N_t. \tag{27}$$

Hence (25) can be rewritten for each t as:

$$\frac{1}{N_t} = \sum_{i=1}^{N_t} \tilde{v}_{i,t} \left(\frac{\partial V_{i,t}/\partial G_{t+1}}{\partial V_{i,t}/\partial b_{t+1}} \right), \tag{28}$$

where:

$$\tilde{v}_{i,t} = \frac{v_{i,t}}{\sum_{i=1}^{N_t} v_{i,t}} \tag{29}$$

and $\sum_{i=1}^{N_t} \tilde{v}_{i,t} = 1$. From equation (15), and using $G_{t+1} = pQ_{G,t+1}$:

$$\frac{\partial V_{i,t}}{\partial G_{t+1}} = \frac{\beta\gamma}{G_{t+1}}, \tag{30}$$

$$\frac{\partial V_{i,t}}{\partial b_{t+1}} = \frac{1+\beta}{(1+r)(1-\tau)y_{i,t} + b_{t+1}}. \tag{31}$$

Substituting into (28) and writing $\tilde{y}_t = \sum_i^{N_t} \tilde{v}_{i,t} y_{i,t}$ gives:

$$\frac{1}{N_t} = \frac{\beta\gamma \left\{ (1+r)(1-\tau)\tilde{y}_t + b_{t+1} \right\}}{(1+\beta)\, G_{t+1}}. \tag{32}$$

The term $\tilde{y}_t$ is a weighted average of individual incomes with weights, $\tilde{v}_{i,t}$. Substituting the government budget constraint into (32) gives the optimal b_{t+1} chosen by the government:

$$\begin{aligned} b_{W,t+1} \;=\; & \frac{\bar{y}_t\,(1+\beta)}{(1+\beta+\beta\gamma)} \\ & \times \left\{ (1+n)\,(1+\omega)\tau - \frac{\beta\gamma}{(1+\beta)}(1+r)(1-\tau)\frac{\tilde{y}_t}{\bar{y}_t} \right\}. \end{aligned} \tag{33}$$

Comparing this result with the median voter's choice of $b_{m,t+1}$ in (19) shows that the two expressions are identical except for the fact that the majority choice depends on $y_{m,t}/\bar{y}_t$ whereas maximization of a social welfare function depends on $\tilde{y}_t/\bar{y}_t$. The same feature must of course apply to expenditure on the public goods and the expenditure ratio.

The expression for $\tilde{y}_t$ conceals considerable complexity. Strictly, (33) is not a closed-form solution because $\tilde{y}_t$ actually depends on the optimal value of b_{t+1}. However, it is shown in Appendix B that if the distribution of income is lognormal, b is relatively small and the welfare function takes the additive iso-elastic form, the following approximation holds:

$$\frac{\tilde{y}_t}{\bar{y}_t} \approx \left(\frac{y_{m,t}}{\bar{y}_t}\right)^{\varepsilon}, \tag{34}$$

where ε is a measure of relative inequality aversion of the government. The optimal expenditure levels and their ratio can thus be expressed in terms of $y_{m,t}/\bar{y}_t$, just as in the majority voting framework, except that there is an additional degree of nonlinearity in the expressions, involving the term ε. By substituting (34) into (33) it can be shown that, as expected, an increase in ε, that is a higher degree of inequality aversion, is associated with an increase in the optimal ratio of the transfer payment to per capita expenditure on the public goods.[25] Only a government having $\varepsilon = 1$ would evaluate the majority voting outcome as approximately optimal.[26]

6 Conclusions

This paper examined the composition of government expenditure under majority voting in a two-period overlapping generations model. The main focus was on public goods and a transfer payment, in the form of a pension, which are financed by income tax on a pay-as-you-go basis. The condition under which there is majority support for the social contract involved in a pay-as-you-go financing structure was established, providing an extension to the familiar Aaron-Samuelson condition. This condition was found to be independent of the tax rate and expenditure levels, so that it is possible to consider the choice of expenditure conditional on the social contract being supported.

Under the majority voting mechanism, selfish individuals vote in the first period on their pensions to be received in the next period. Therefore, voting is over one dimension and expenditure on public goods is obtained from the government's budget constraint. Preferences are single-peaked and the decisive voter is the young individual with the median

[25] The relationships between optimal pension, public goods and their ratio and $y_{m,t}/\bar{y}_t$ are different from those with majority voting. First, although total expenditure on pension, $B_{W,t+1}$, and public goods, $G_{W,t+1}$, are decreasing and increasing in $y_{m,t}/\bar{y}_t$ respectively, this relationship is not linear. If $\varepsilon < 1$ the second derivative of $B_{W,t+1}$ and $G_{W,t+1}$ with respect to $y_{m,t}/\bar{y}_t$ is positive and negative respectively. The relationship between the optimal ratio and $y_{m,t}/\bar{y}_t$ is negative, but the second derivative is undetermined.

[26] This is subject to the qualification that the approximation examined in the Appendix applies to a particular cardinalisation of utility, using the multiplicative form of the Cobb-Douglas function.

income. It was found that the majority choice of the ratio of expenditure on pensions to public goods falls at a decreasing rate when the ratio of median to mean income increases. It therefore has in common with those models which focus on the choice of tax rate the property that a higher degree of basic inequality is associated with a more redistributive structure.

However, the numerical results based on a calibrated version of the model suggest that, over realistic values, differences in the ratio of transfer payments to public goods expenditure arising from differences in income inequality are likely to be small. Furthermore, the ratio is relatively sensitive to preference parameters (the time preference rate and weight attached to public goods in utility functions). This suggests that in a cross-sectional comparison of democratic countries, a simple relationship between redistribution and basic inequality is unlikely to be observed.

Despite the considerable data problems, an empirical examination for a sample of democratic countries illustrate that there are systematic differences in expenditure patterns across different cultural groups of countries. Using the analytical result from the model, the preference parameter (utility weight attached to public goods) is inferred for each country, and the results suggest that different attitudes toward redistribution resulting from cultural differences may play an important role in explaining these differences in expenditure patterns among democratic countries.

An alternative approach considered the choice of a utilitarian government maximising a multi-period social welfare function, subject to the government's pay-as-you-go budget constraint. The expressions for the choice of expenditure levels and their ratio were found to take similar forms to those in the majority voting context, except that a welfare-weighted average of each generation's income replaces its median income.

The modelling framework, involving two overlapping generations and fixed incomes, is in some ways extremely simple. Nevertheless, consideration of the choice of tax-financed public good expenditure and the level of a pension is substantially complicated by the fact that the pension involves a combination of income-shifting between phases of the life cycle (in addition to that provided by private savings) with both inter-generational and intra-generational transfers. The latter arises because the basic pension is unrelated to income whereas the tax is proportional to income. The former arises from the pay-as-you-go feature of financing whereby each generation can benefit from productivity and thus income growth accruing to the following generation. It is suggested that the approach therefore offers useful insights into the relevant relationships involved.

Appendix A: Further Empirical Evidence

This Appendix describes the construction of the dataset used in Section 4. The first question concerns the countries to be included in the group of democratic countries. The *Polity IV* (2007) dataset provides an index of democracy for all countries, which varies between 0 and 10, with 10 representing the highest level of democracy. Countries with an index of 9 and 10 since the year 2000 are classified in the group of democratic countries.[27] The rest are classified in the group of nondemocratic countries.[28]

The ratio of median to average income is a key variable. In the absence of direct information about medians, this ratio was obtained from reported Gini measures by assuming that the distribution of income is lognormal. Data on Gini inequality measures were obtained from *World Development Indicators* (2007) and *World Income Inequality Database* (2007).[29] Suppose income follows the lognormal distribution with mean and variance of logarithms of income being μ and σ^2 respectively, then the ratio of median to average income is equal to $e^{-\sigma^2/2}$, which depends only on the variance of logarithms of income. This variance can be obtained from the Gini index using $\sigma = \sqrt{2}\Phi^{-1}(\frac{Gini+1}{2})$, where Φ^{-1} is the inverse function of the standard normal cumulative distribution.[30] Since for most of the countries the Gini index is available only for selected years, the average of available years between 2000 and 2006 was used to calculate the ratio of median to mean income. Consequently, averages of growth of GDP per capita, population growth, the real interest rate, government expenditures and revenues were computed over the same period.

The growth rate of income and population as well as the real interest rate were obtained from the average annual growth rates of GDP per capita and population, and the average annual real deposit interest rate between 2000 and 2006 for each country. The various rates were adjusted to correspond to the OLG framework used here. Suppose the average annual

[27] The resulting democratic countries are: Australia, Austria, Belgium, Canada, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Lithuania, Netherlands, New Zealand, Norway, Slovenia, Spain, Sweden, Switzerland, United Kingdom, and United States. This sample does not contain all countries with index of 9 and 10. Some countries such as Costa Rica, Jamaica, Japan, Mauritius, Papua New Guinea, Portugal, Trinidad and Tobago and Uruguay are excluded because the data are not available for these countries.

[28] This sample of 22 non-democratic countries does not contain all those with an index of democracy below 9. Most countries are excluded because the data on the composition of general government expenditure were not available. The resulting non-democratic countries are: Albania, Belarus, Bolivia, Bulgaria, El Salvador, Estonia, Georgia, Iran, Kazakhstan, Kuwait, Kyrgyz Republic, Latvia, Lesotho, Moldova, Mongolia, Poland, Romania, Russia, Singapore, Slovak Republic, South Africa, Ukraine.

[29] Unfortunately, the income concept varies across countries. In some countries gross income is considered and in others it is net income.

[30] On the lognormal properties used here, see Aitchison and Brown (1957).

growth of GDP per capita between 2000 and 20006 is x percent, and the length of a time period in this overlapping generation model is 20 years, then the growth rate in income is obtained as $(1+x)^{20}-1$. The same approach was used for population growth and the interest rate for each country. Also, by assuming that the time preference rate is equal to the real interest rate, $\rho = r$, then the discount factor, β, is obtained from $1/(1+r)^{20}$. The source for the data for growth in income and population and real interest rate is WDI (2007).[31]

Data on government expenditures were obtained from *Government Financial Statistics*, GFS, (2008). A difficulty arises with the measurement of public goods and expenditure on pensions. For instance, there is no characteristic between pure public goods and private goods. Also, the type of transfer payment system is different across countries.

Expenditure on public goods was obtained as the sum of expenditure by general government on defence, public order and safety, environmental protection, health and education. In the data expenditure on health and education contains publicly provided private goods, rather than public goods. Therefore different proportions of these two expenditures (a quarter, half and three quarters) were considered as public goods. Expenditure on pensions is not available separately in *Government Financial Statistics* (2008), and thus has to be measured by aggregate data on 'social protection' at the general government level, which includes expenditure on sickness and disability, old age, survivors, family and children, unemployment, housing, social exclusion and R&D social protection. Aggregate tax rates were obtained as general government's revenue divided by GDP. The source for the data on revenue and GDP are *Government Financial Statistics* (2008) and *World Development Indicators* (2007) respectively. The revenue includes taxes and social contributions, where the group of taxes includes: tax on income, profit, capital gain; payroll and workforce; properties; good and services; international trade and transactions and 'other taxes'.

Summary information for all democratic countries is reported in Table 3.

Appendix B: Comparing Income Ratios

In section 5, it was found that a welfare-weighted average income, $\tilde{y}_t$, plays a crucial role in determining the optimal composition of expenditure in each period. This measure is highly complex, even for simple social welfare functions. However, it is useful to consider

[31] Data for interest rates in U.S., U.K. and Austria were obtained respectively from websites of Federal Reserve Bank of USA, Bank of England and Central Bank of Austria.

Table 3: Summary Information for All Democracies

Country	$\frac{y_m}{\bar{y}}$	τ	$R_{1/4}$	$R_{2/4}$	$R_{3/4}$	$\gamma_{1/4}$	$\gamma_{2/4}$	$\gamma_{3/4}$
Australia	0.85	0.33	1.53	1.07	0.82	0.51	0.66	0.79
Austria	0.87	0.47	3.55	2.30	1.70	0.30	0.44	0.56
Belgium	0.83	0.50	2.61	1.77	1.34	0.43	0.60	0.75
Canada	0.83	0.37	1.80	1.22	0.92	0.51	0.69	0.84
Cyprus	0.88	0.42	1.57	1.20	0.97	0.64	0.78	0.91
Czech Republic	0.91	0.38	1.76	1.30	1.04	0.56	0.71	0.85
Denmark	0.91	0.53	3.23	2.09	1.55	0.37	0.53	0.68
Finland	0.89	0.46	3.28	2.20	1.66	0.36	0.51	0.64
France	0.88	0.48	2.94	2.03	1.55	0.37	0.51	0.63
Germany	0.88	0.42	3.69	2.56	1.96	0.25	0.33	0.41
Greece	0.82	0.33	2.56	2.04	1.69	0.39	0.48	0.56
Hungary	0.88	0.46	2.29	1.63	1.27	0.55	0.74	0.90
Ireland	0.82	0.36	1.62	1.07	0.80	0.66	0.95	1.22
Israel	0.77	0.37	0.82	0.67	0.57	0.94	1.05	1.14
Italy	0.80	0.45	2.59	1.84	1.43	0.38	0.51	0.62
Lithuania	0.82	0.30	1.58	1.12	0.87	0.73	0.95	1.15
Netherlands	0.90	0.43	2.64	1.93	1.52	0.35	0.45	0.54
New Zealand	0.82	0.39	1.93	1.29	0.97	0.50	0.68	0.82
Norway	0.90	0.47	2.45	1.65	1.24	0.44	0.60	0.74
Slovenia	0.85	0.47	2.49	1.71	1.30	0.50	0.70	0.87
Spain	0.81	0.40	2.08	1.50	1.17	0.51	0.67	0.82
Sweden	0.90	0.54	3.01	2.06	1.57	0.42	0.60	0.75
Switzerland	0.82	0.31	2.22	1.57	1.22	0.31	0.41	0.49
United Kingdom	0.83	0.41	1.81	1.35	1.07	0.52	0.65	0.77
United States	0.75	0.30	0.75	0.55	0.44	0.78	0.92	1.03

Note: R_F denotes the ratio of total transfer expenditure to that on public goods, where the latter includes a proportion, F, of that on health and education.
γ_F is the corresponding calculated preferences parameter for public goods.

an approximation, for the ubiquitous case where social welfare within each generation takes the iso-elastic form:

$$W_t = \begin{cases} \frac{1}{1-\varepsilon} \sum_{i=1}^{N} V_{i,t}^{1-\varepsilon} & \varepsilon \neq 1, \varepsilon > 0 \\ \sum_{i=1}^{N} \log V_{i,t} & \varepsilon = 1 \end{cases} . \tag{B.1}$$

In considering optimal choices, a decision must also be made regarding the cardinalisation of individuals' utility functions. Monotonic transformations do not of course affect individuals' plans, and hence the majority voting outcome. However, choices of the government are affected by such transformations.[32] Here the cardinalisation examined is the multiplicative form of the Cobb-Douglas utility function, $U_{i,t} = c_{1i,t} c_{2i,t+1}^{\beta} Q_{G,t}^{\gamma} Q_{G,t+1}^{\beta\gamma}$. Consequently, indirect utility, from (17), can be rewritten as:

$$V_{i,t} = \frac{\left((1-\tau) y_{i,t} + \frac{b_{t+1}}{(1+r)}\right)^{(1+\beta)}}{(1+\beta)^{(1+\beta)}} \left(\beta(1+r)\right)^{\beta} Q_{G,t}^{\gamma} Q_{G,t+1}^{\beta\gamma}. \tag{B.2}$$

From (B.1), $\partial W_t / \partial V_{i,t} = V_{i,t}^{-\varepsilon}$, which holds for all $\varepsilon > 0$. And using $\partial V_{i,t} / \partial b_{t+1}$ from (B.2), the welfare weights, $v_{i,t} = \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial b_{t+1}}$, are:

$$v_{i,t} = \frac{(1+\beta)\left(\frac{\beta^{\beta}(1+r)^{\beta} Q_{G,t}^{\gamma} Q_{G,t+1}^{\beta\gamma}}{(1+\beta)^{(\beta+1)}}\right)^{1-\varepsilon}}{(1+r)} \left((1-\tau) y_{i,t} + \frac{b_{t+1}}{(1+r)}\right)^{\beta - \varepsilon(1+\beta)} . \tag{B.3}$$

Suppose b_{t+1} is small relative to $y_{i,t}$. In this case an approximation, denoted $\tilde{y}_{A,t}$, for $\tilde{y}_t = \sum y_{i,t} \left(v_{i,t} / \sum v_{i,t}\right)$, is obtained as:

$$\tilde{y}_{A,t} = \frac{\frac{1}{N_t} \sum y_{i,t}^{(1+\theta)}}{\frac{1}{N_t} \sum y_{i,t}^{\theta}}, \tag{B.4}$$

with $\theta = \beta - \varepsilon(1+\beta)$. Thus $\tilde{y}_{A,t}$ is the ratio of two fractional moments. Suppose further that $y_{i,t}$ is lognormally distributed as $\Lambda(y_{i,t} | \, \mu_t, \sigma_t^2)$, with mean and variance of logarithms of μ_t and σ_t^2 respectively. Using the properties of the lognormal moment generating function, it can be found that:[33]

$$\tilde{y}_{A,t} = \exp\left(\mu_t + (1-\varepsilon)\frac{\sigma_t^2}{2}\right) \exp\left((2\beta(1-\varepsilon) - \varepsilon)\frac{\sigma_t^2}{2}\right). \tag{B.5}$$

[32] For example, suppose $\varepsilon = 1$, so that $W_t = \sum_i \log V_{i,t}$. The optimal choice is therefore the same as that obtained using $\varepsilon = 0$ with an alternative cardinalisation using the logarithm of utility.

[33] On the lognormal distribution, see Aitchison and Brown (1957).

The final term in this expression is close to, but less than, unity. This is because β is less than one. Also, reasonable values of ε are small and ≤ 1.[34] However, the use of the assumption that b_{t+1} can be neglected in (B.3) actually attaches too much weight to the lower incomes, and thus imparts a downward bias to the approximation. One approach is thus to 'correct' for this downward bias by excluding the final term in (B.5). This gives:

$$\tilde{y}_{A,t} = \exp\left(\mu_t + (1-\varepsilon)\frac{\sigma_t^2}{2}\right). \tag{B.6}$$

A feature of this result in (B.6) is that $\tilde{y}_{A,t}$ is closely related to Atkinson's measure of the inequality of income. Following Atkinson (1970), let $y_{ede,t}$ denote the 'equally distributed equivalent' income, representing the equal income which gives the same social welfare for cohort t as would be achieved by the current heterogeneous income distribution, using a welfare function of the form $\frac{1}{1-\varepsilon}\sum_{i=1}^{N} y_{i,t}^{1-\varepsilon}$. This is the same as the above but with $V_{i,t}$ replaced by $y_{i,t}$. Thus:

$$y_{ede,t} = \left(\frac{1}{N_t}\sum_{i=1}^{N_t} y_{i,t}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{B.7}$$

Again using lognormal properties, the term $y_{ede,t}^{1-\varepsilon}$ has mean and variance of logarithms respectively of $(1-\varepsilon)\mu_t$ and $(1-\varepsilon)^2\sigma_t^2$, so that:

$$y_{ede,t} = \exp\left((1-\varepsilon)\mu_t + (1-\varepsilon)^2\frac{\sigma_t^2}{2}\right)^{\frac{1}{1-\varepsilon}} = \exp\left(\mu_t + (1-\varepsilon)\frac{\sigma_t^2}{2}\right). \tag{B.8}$$

Consequently, the ratio of the equally distributed equivalent to the arithmetic mean income, $\bar{y} = e^{\mu_t + \sigma_t^2/2}$, is:

$$\frac{y_{ede,t}}{\bar{y}_t} = \frac{\exp\left(\mu_t + (1-\varepsilon)\frac{\sigma_t^2}{2}\right)}{\exp\left(\mu_t + \frac{\sigma_t^2}{2}\right)} = \left(\exp\left(-\frac{\sigma_t^2}{2}\right)\right)^{\varepsilon}. \tag{B.9}$$

Furthermore, as $y_{m,t} = e^{\mu_t}$:

$$\frac{y_m}{\bar{y}_t} = \exp(-\frac{\sigma_t^2}{2}), \tag{B.10}$$

giving:

$$\frac{y_{ede,t}}{\bar{y}_t} = \left(\frac{y_{m,t}}{\bar{y}_t}\right)^{\varepsilon}. \tag{B.11}$$

[34] In the cross-sectional inequality context, questionnaire studies involving consideration of the 'leaky bucket' experiment found values for respondents which averaged around 0.2; see Amiel *et al.* (2001).

If $\tilde{y}_t$ is approximated using $\tilde{y}_{A,t} = y_{ede,t}$, (B.11) gives the required relationship between the two income ratios reported in Section 5 above.

It is important to test the value of the above approximation. Hence values of the expenditure components using the approximation $\tilde{y}_A = y_{ede}$ were compared with those obtained using a simulated population of size 15000 drawn at random from a lognormal distribution with $\mu = 9.0$ and $\sigma^2 = 0.5$.[35] Using the simulated distribution, a range of values of b_{t+1} were investigated. For each b_{t+1} the government budget constraint was used to obtain G_{t+1} and the resulting values were used to calculate each individual's level of utility. These were then used to obtain social welfare, using the iso-elastic function with a specified inequality aversion parameter, ε. Finally, given a large number of W_t measures, the maximum was determined, giving the optimal composition. In order to compare welfare values, the expenditure values obtained from the approximation was used with the simulated population. Table 4 gives the results for a range of inequality aversion, ε, with other baseline parameters.

Table 4: Composition of Government Expenditure: Alternative Solutions

	Approximation			Simulation			
ε	b_{t+1}	$\frac{G_{t+1}}{(N_t+N_{t+1})}$	R_{t+1}	b_{t+1}	$\frac{G_{t+1}}{(N_t+N_{t+1})}$	R_{t+1}	$\%\Delta W_t$
0.8	3087	1532	0.863	3460	1372	0.925	-0.011
0.5	2907	1609	0.774	3452	1376	0.930	-0.028
0.2	2713	1692	0.687	3442	1380	0.936	-0.043

These results show that the approximation does indeed give values of expenditure levels and ratios which are reasonably close to those obtained using a large simulated population. The percentage difference of the social welfare function using the approximation from that obtained by simulation, $\%\Delta W$, is in each case found to be small, at less than half a percentage point. This reflects the relative flatness of the profile relating W_t to b_{t+1} (for given parameter values) as well as the closeness of the approximation.

[35] This implies that $\bar{y} = 10405$ and $y_m/\bar{y} = 0.78$.

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