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A Loglinear Tax and Transfer Function: Majority Voting and Optimal Rates

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Abstract

This paper explores the use of a loglinear tax and transfer function, displaying increasing marginal and average tax rates along with a means-tested transfer payment. The two parameters are a break-even income threshold, where the average tax rate is zero, and a tax parameter equivalent to the marginal tax rate at the break-even income level. When combined with Cobb-Douglas utility, the resulting labour supply is fixed and independent of the individual's wage rate. For an additive social welfare function involving the sum of logarithms of (indirect) utilities, a convenient expression is available for the optimal tax rate in a framework in which individuals differ only in the wage rate they face. It is shown that a unique optimal rate exists, depending on the preference for consumption and the inequality of wage rates. This coincides with the majority voting equilibrium rate. As with the linear tax function, higher inequality is associated with choice of a higher tax rate.

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1 Introduction

The simple linear tax function has long been a 'workhorse' of tax analysis. Its two parameters, the unconditional basic income received by everyone and the fixed marginal (and hence average) tax rate applied to non-transfer income, are easily interpreted and the linear form often makes analysis tractable. Furthermore a basic-income-flat-tax structure (BI-FT) has been advocated by some policy commentators. The linear structure gives rise to fairly simple labour supply functions, when combined with conventional utility functions such as Cobb-Douglas and CES. When used in models with a government budget constraint, the loss of a degree of freedom in policy choices means that only the marginal tax rate can be chosen independently, and hence the linear tax can be incorporated into (unidimensional) majority voting and optimal tax models, although strong assumptions are often required if explicit solutions are to be available.

The aim of this paper is to examine the use of a convenient alternative spcification – a loglinear relationship between net, or after-tax, income and gross income – which has received relatively little attention in the tax literature. This can also be useful when it is required to introduce a stylised income tax function into a wider model. For example, it can provide a smooth two-parameter approximation to a multi-step schedule having a range of marginal rates between income thresholds. The loglinear form has both increasing average tax rates with income, required for progressivity, and increasing marginal tax rates with income. Studies using this form purely as an income tax function include Edgeworth (1925), Dalton (1954), Creedy (1979), Atkinson (1983), Hersoug (1984), Waterson (1985), Creedy and McDonald (1992), Creedy and Gemmell (2006). However, a simple modification to the function allows it to cover means-tested transfer payments (negative income taxation) in addition to positive tax payments. Unlike the linear form, which has no means-testing and a flat tax, the loglinear tax has income-testing of transfers and an increasing marginal tax rate. Yet when combined with commonly used utility functions, it produces relatively simple labour supply behaviour. This form has been used, for example, by Benabou (1996) in the context of an overlapping generations model.

Section 2 describes the loglinear function and investigates the resulting government budget constraint. Section 3 then considers individual maximisation and labour supply behaviour using the simple Cobb-Douglas utility function. Majority voting is examined in Section 4. The standard optimal tax framework, involving maximisation of a utilitarian social welfare function specified in terms of individuals' utilities, is then examined in Section 5. Brief conclusions are in Section 6.

2 A Loglinear Tax Function

Subsection 2.1 describes the loglinear function and the interpretation of its two parameters. The resulting government budget constraint, which imposes a loss of a degree of freedom in policy choices, is derived in Subsection 2.2.

2.1 Progressive Taxes and Transfers

Define z_i and y_i respectively as the net income and gross income of individual i. Suppose taxes and transfers are described by a loglinear tax function, with parameters $\tau < 1$ and y_b, as follows:

$$z_i = y_i^{1-\tau} y_b^{\tau} \tag{1}$$

The parameter, y_b, is the break-even income level at which individuals neither pay tax nor receive transfers; that is, when $y_i = y_b$, then $z_i = y_i$.[1] Figure (1) demonstrates the nonlinear relationship between the disposable income and gross income. An individual with income above y_b pays tax and an individual below the break-even income receives a benefit. Transfers are effectively means-tested, falling to zero at y_b.

Consider the minimum level of net income, $z_{\min}$, which is obtained where gross income is equal to 1. Rewrite (1) in logarithm form as $\log(z_i) =$

[1] It would of course be possible to specify the function as $z_i = by_i^{\tau}$, where b represents the maximum transfer payment (available to those with $y_i = 1$). However, it is more convenient to have the break-even level more transparent, as in the specification above.

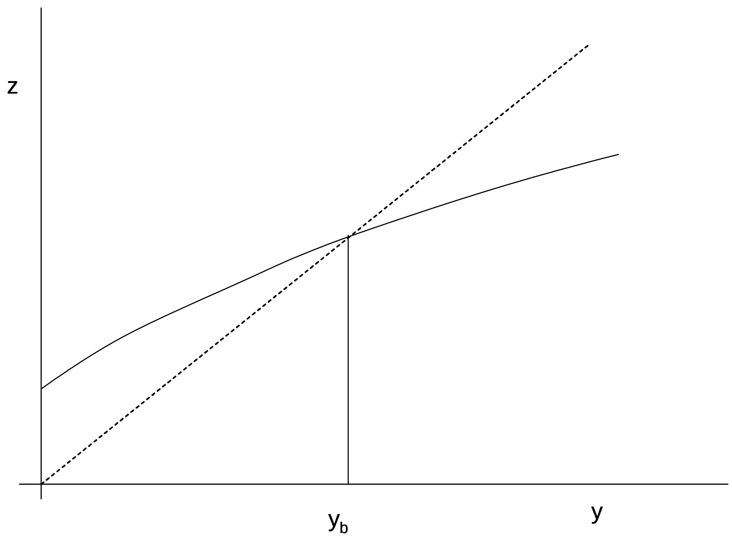

Figure 1: The Loglinear Tax Function

$(1-\tau)\log(y_i) + \tau\log(y_b)$. Hence:

$$\log(z_{\min}) = \log(z_i)\big|_{y_i=1} = \tau\log(y_b) \tag{2}$$

and minimum net income is equal to y_b^τ. This represents the maximum transfer payment available.

Unlike a standard piecewise linear tax function, where fixed marginal effective tax rates apply between specified income thresholds, the 'tax rate' facing any particular individual is not transparent. Let $T(y)$ denoted the tax paid by an individual with gross income, y, so that:

$$\begin{aligned} T(y_i) &= y - z \\ &= y - y^{1-\tau}y_b^\tau \end{aligned} \tag{3}$$

Therefore the marginal tax rate, MTR, and average tax rate, ATR, corresponding to income level, y, are respectively:

$$MTR = 1 - (1-\tau)\,y^{-\tau}y_b^\tau \tag{4}$$

$$ATR = 1 - y^{-\tau}y_b^\tau \tag{5}$$

The parameter, τ, is thus the marginal rate at the income level, $y = y_b$, at which the average rate is zero. A progressive tax system is defined as one in which the average tax rate increases when income increases, over the whole range of incomes. The average tax rate increases if the marginal tax rate exceeds the average tax rate, and applying this rule it can be shown that progression requires the tax parameter τ to be greater than zero, $\tau > 0$. The loglinear form may perhaps be regarded as providing a convenient smooth approximation, involving just two parameters, to a piecewise linear function having several marginal rates and income thresholds.

2.2 The Government Budget Constraint

In a pure transfer system within a single period framework with a pay-as-you-go system (which excludes borrowing or lending by the government), the government budget constraint is such that total net revenue is zero; that is, the total benefits paid to those below y_b are equal to the total tax revenue obtained from those with $y > y_b$. This means that it is not possible to set y_b and τ independently, as a degree of freedom in policy choices is lost. It is necessary to solve for y_b in terms of the parameter, τ.

At this point it is convenient to suppose there is a continuous distribution of income, with distribution function of gross income denoted by $F(y)$,with $0 < y < \infty$. Hence the government budget constraint can be written:

$$\int_0^{y_b} \left(y^{1-\tau} y_b^{\tau} - y\right) dF(y) = \int_{y_b}^{\infty} \left(y - y^{1-\tau} y_b^{\tau}\right) dF(y) \tag{6}$$

Therefore:

$$\int_0^{y_b} y^{1-\tau} y_b^{\tau} dF(y) + \int_{y_b}^{\infty} y^{1-\tau} y_b^{\tau} dF(y) = \int_0^{y_b} y dF(y) + \int_{y_b}^{\infty} y dF(y) \tag{7}$$

Hence:

$$y_b^{\tau} \int_0^{\infty} y^{1-\tau} dF(y) = \int_0^{\infty} y dF(y) \tag{8}$$

The right hand side of this constraint is clearly arithmetic mean gross income, $\bar{y}$.

It is possible to solve for y_b, given any value of τ, in terms of the ratio of the first moment about the origin to the $(1-\tau)$th moment about the origin. Here it is convenient to make an explicit assumption about the form of the distribution of income. Suppose gross income is lognormally distribution with mean μ_y and variance σ_y^2. Hence, $y \sim \Lambda(\mu_y, \sigma_y^2)$, and from the well-known properties of this distribution, the arithmetic mean income is equal to $\bar{y} = \exp\left(\mu_y + \sigma_y^2/2\right)$. Furthermore, $y^{1-\tau}$ is also distributed lognormally such that:

$$y^{1-\tau} \sim \Lambda\left((1-\tau)\,\mu_y, (1-\tau)^2\,\sigma_y^2\right) \tag{9}$$

Hence:

$$\textstyle\int_0^\infty y^{1-\tau} dF(y) = \exp\left((1-\tau)\,\mu_y + \frac{\sigma_y^2}{2}(1-\tau)^2\right) \tag{10}$$

Substituting in (8) gives:

$$y_b^\tau \exp\left((1-\tau)\,\mu_y + (1-\tau)^2\,\frac{\sigma_y^2}{2}\right) = \exp\left(\mu_y + \frac{\sigma_y^2}{2}\right) \tag{11}$$

Thus:

$$\begin{aligned} y_b^\tau &= \exp\left(\mu_y + \frac{\sigma_y^2}{2} - (1-\tau)\,\mu_y - (1-\tau)^2\,\frac{\sigma_y^2}{2}\right) \\ &= \exp\left(\tau\mu_y - \frac{\tau\sigma_y^2}{2}(\tau-2)\right) \end{aligned} \tag{12}$$

and:

$$\begin{aligned} y_b &= \exp\left(\mu_y + \frac{\sigma_y^2}{2} + \frac{\sigma_y^2}{2}(1-\tau)\right) \\ &= \bar{y}\exp\left(\frac{\sigma_y^2}{2}(1-\tau)\right) \end{aligned} \tag{13}$$

Therefore, the break-even level of income depends positively on average income and the variance of logarithms of the distribution of gross income and negatively on the tax parameter, τ.

If, instead of a pure transfer system, the tax structure has to raise net revenue of g per person in order to finance non-transfer expenditure. This means that g must be subtracted from the right hand side of (11), and the break-even income level becomes:

$$y_b = \left(1 - \frac{g}{\bar{y}}\right)^{1/\tau} \bar{y}\exp\left(\frac{\sigma_y^2}{2}(1-\tau)\right) \tag{14}$$

Hence if it is desired to extended the following analysis to cover the case where there is some non-transfer government expenditure per person, which does not enter individuals' utility functions, it is most convenient to express this expenditure as a fixed proportion of arithmetic mean income from employment.

3 Individual Maximization

This section examines utility maximisation, by examining optimal labour supply for Cobb-Douglas direct utility functions.[2] This allows the required form of indirect utility functions to be obtained.

3.1 Labour Supply and Earnings

Each individual is assumed to derive utility from consumption, c, which in this static framework is equal to net income, z, and leisure, h. The total time available for work and leisure is normalised to one unit, so that $h \leq 1$. Then the budget constraint facing the ith individual is:

$$c_i = \left(w_i \left(1 - h_i\right)\right)^{1-\tau} y_b^{\tau} \tag{15}$$

The direct utility function is assumed, for convenience, to be Cobb-Douglas, so that:

$$\begin{aligned} U_i &= c_i^{\alpha} h_i^{1-\alpha} \\ &= \left\{\left(w_i \left(1 - h_i\right)\right)^{1-\tau} y_b^{\tau}\right\}^{\alpha} h_i^{1-\alpha} \\ &= \left\{w_i^{\alpha(1-\tau)} y_b^{\alpha\tau}\right\} \left(1 - h_i\right)^{\alpha(1-\tau)} h_i^{1-\alpha} \end{aligned} \tag{16}$$

Differentiating with respect to h_i gives:

$$\frac{\partial U_i}{\partial h_i} = \left(w_i^{\alpha(1-\tau)} y_b^{\alpha\tau}\right) h_i^{-\alpha} \left(1 - h_i\right)^{\alpha(1-\tau)} \left(\left(1 - \alpha\right) - \alpha \left(1 - \tau\right) \frac{h_i}{1 - h_i}\right) \tag{17}$$

The first-order condition for utility maximisation, $\frac{\partial U_i}{\partial h_i} = 0$, therefore gives the interior solution for optimal leisure as:

$$h_i = \frac{1 - \alpha}{1 - \alpha\tau} \tag{18}$$

[2] The more general constant elasticity of substitution (CES) form gives rise to a non-linear equation for optimal hours of leisure.

Consequently leisure, and thus labour supply, depends on only the preference parameter, α, and the tax parameter, τ, and is independent of the wage rate, w_i. In a population in which heterogeneity is reflected only in variations among individuals in the wage rate, then all individuals work the same number of hours. This contrast strongly with the corresponding result for the linear tax function. Labour supply is always positive, under the assumption made above that $\tau < 1$. Hence, again unlike the case of the linear tax function, corner solutions with $h_i = 0$ and their associated complications do not arise.

Gross income is thus simply proportional to the wage rate for each individual, with:

$$y_i = w_i (1 - h_i) = w_i \frac{\alpha (1 - \tau)}{1 - \alpha\tau} \tag{19}$$

3.2 The Indirect Utility Function

Indirect utility, V_i, is obtained by substituting optimal leisure into the direct utility function:

$$V_i = \left\{ w_i^{\alpha(1-\tau)} y_b^{\alpha\tau} \right\} \left(\frac{\alpha (1 - \tau)}{1 - \alpha\tau} \right)^{\alpha(1-\tau)} \left(\frac{1 - \alpha}{1 - \alpha\tau} \right)^{1-\alpha} \tag{20}$$

Furthermore, substituting for y_b from the government budget constraint gives, for the term in curly brackets in (20):[3]

$$w_i^{\alpha(1-\tau)} y_b^{\alpha\tau} = w_i^{\alpha(1-\tau)} \bar{y}^{\alpha\tau} \left\{ \exp\left(\frac{\sigma_y^2}{2} (1 - \tau) \right) \right\}^{\alpha\tau} \tag{21}$$

It is also necessary to consider the arithmetic mean gross income, $\bar{y}$, since it is also a function of the tax parameters and the wage rate distribution. Suppose the wage rate distribution is lognormally distributed as $\Lambda(\mu_w, \sigma_w^2)$. Then from (19), the wage rate and gross earnings distributions are related using:

$$\mu_y = \mu_w + \log \frac{\alpha (1 - \tau)}{1 - \alpha\tau} \tag{22}$$

[3] In the case mentioned above where a fixed proportion $\delta = g/\bar{y}$ of revenue is raised for non-transfer purposes, the right hand side of (21) is simply multiplied by $(1 - \delta)^{\alpha}$. Holding δ fixed instead of g means that this adjustment is independent of the tax rate.

and:

$$\sigma_y^2 = \sigma_w^2 \tag{23}$$

Hence, $\bar{y} = \frac{\alpha(1-\tau)}{1-\alpha\tau}\bar{w}$, in (21) is equal to:

$$\bar{y} = \exp\left[\mu_w + \frac{\sigma_w^2}{2} + \log\frac{\alpha(1-\tau)}{1-\alpha\tau}\right] \tag{24}$$

which uses the standard property of the lognormal distribution that, for example, $\bar{w} = \exp\left(\mu_w + \frac{\sigma_w^2}{2}\right)$. As expected, an increase in the tax rate reduces the average income, since:

$$\frac{\partial\bar{y}}{\partial\tau} = -\frac{\bar{w}\alpha(1-\alpha)}{(1-\alpha\tau)^2} < 0 \tag{25}$$

and an increase in the (common) preference for consumption raises $\bar{y}$. In addition:

$$\frac{\partial^2\bar{y}}{\partial\alpha\partial\tau} = -\frac{\bar{w}(1+\alpha^2\tau-2\alpha)}{(1-\alpha\tau)^3} \tag{26}$$

which may be positive or negative. For the lognormal distribution, median income is less than the average income and from (13) it is clear that that average income is less than break-even income level. This relationship between median, mean and break-even levels of income holds for all feasible levels of the tax parameter.

4 Majority Voting

This section examines the majority voting equilibrium choice of tax parameter, τ. First, it can be shown that the indirect utility function is concave in τ, so that preferences are single-peaked.[4] Hence the median voter theorem can be invoked and it is only necessary to examine the preferred value of the individual with median w, denoted w_m. It is useful to write the median voter's indirect utility function in (20), after substituting for y_b and $\bar{y}$, using (13) and (24). Furthermore, for the lognormal distribution, the logarithm of

[4] This contrasts with the case of the linear tax function, where preferences can be double-peaked. However, as Roberts (1977) showed, a majority-voting equilibrium exists if there is hierarchical adherence whereby the ranking of individuals does not depend on the tax rate.

median income is equal to the mean of logarithms, so that $\exp(\mu_w) = w_m$. It is also most convenient to take logarithms, since the value of τ which maximise indirect utility is not affected by monotonic transformation. Then $\log V_m$ becomes:

$$\begin{aligned}\log V_m &= \alpha \log w_m + \alpha \log \alpha + (1-\alpha)\log(1-\alpha) \\ &\quad + \alpha\tau\sigma_w^2 - \frac{\alpha\tau^2\sigma_w^2}{2} + \alpha\log(1-\tau) - \log(1-\alpha\tau) \qquad (27)\end{aligned}$$

Differentiating gives:

$$\frac{dV_m}{d\tau} = \alpha(1-\tau)\sigma_w^2 - \frac{\alpha\tau(1-\alpha)}{(1-\tau)(1-\alpha\tau)} \qquad (28)$$

Hence, the median's choice of tax parameter, τ_m, is given by the solution to:

$$\sigma_w^2 = \frac{\tau_m(1-\alpha)}{(1-\tau_m)^2(1-\alpha\tau_m)} \qquad (29)$$

At first sight, a difference between this result for the loglinear tax function and the standard linear function is that the majority-voting equilibrium does not appear to depend on the median wage in relation to the arithmetic mean wage. However, this is subsumed in the variance of logarithms of the wage rate. Rearrangement of $\bar{w} = \exp\left(\mu_w + \frac{\sigma_w^2}{2}\right)$, and using $\mu_w = \log w_m$, gives:

$$\sigma_w^2 = 2\log\frac{\bar{w}}{w_m} \qquad (30)$$

and (29) could easily be written in terms of $\frac{\bar{w}}{w_m}$. A higher variance of logarithms of wages implies greater skewness of the wage rate distribution and a larger distance between the median and arithmetic mean wage rates.

It would be possible to rewrite (29) as a cubic equation in τ_m, suggesting the possibility of three roots. However, Writing the right hand side of (29) as $f(\tau) = \tau(1-\alpha)/\left\{(1-\tau)^2(1-\alpha\tau)\right\}$, it can easily be seen that $f(0) = 0$ and $f(1) = \infty$, with $df(\tau)/d\tau > 0$, and $d^2f(\tau)/d\tau^2 > 0$. Thus $f(\tau)$ strictly increases with τ when $0 < \tau < 1$, is convex, and approaches an asymptote at $\tau = 1$. Importantly, this implies that there is a unique solution to (29). The comparative static properties of the model, involving the effects on the majority choice of τ of variations in α and σ_w^2, can be investigated using implicit differentiation, where (29) is written as $F(\tau_m, \alpha, \sigma_w^2) = 0$. It can be shown that:

$$\frac{d\tau_m}{d\alpha} = -\frac{F_a}{F_\tau} > 0 \tag{31}$$

and:

$$\frac{d\tau_m}{d\sigma_w^2} = -\frac{F_{\sigma_w^2}}{F_\tau} > 0 \tag{32}$$

The clear result that the tax parameter increases as basic wage rate inequality increases is also obtained for the linear income tax structure. Furthermore, the linear tax combined with homogeneous preferences also produces the result that the median voter's choice of tax rate increases as the preference for consumption (net income) increases. Essentially, this arises in the linear tax case because a higher preference for consumption implies higher labour supplies and hence a higher arithmetic mean taxable income and unconditional transfer: redistribution benefits the median voter, who is below the arithmetic mean, and so a higher tax rate is unambiguously chosen.[5] However, with the present loglinear tax function, the intuition is less obvious. The median voter's utility must increase as a result of a joint increase in α and τ, which satisfies the government budget constraint. Hence the median voter's choice must satisfy:

$$\frac{dV_m}{d\tau} = \frac{\partial V_m}{\partial \tau} + \frac{\partial V_m}{\partial \alpha}\frac{d\alpha}{d\tau} = 0 \tag{33}$$

rather than simply setting the right hand side of (28) equal to zero. Hence, of concern is whether:

$$\left.\frac{d\tau}{d\alpha}\right|_{V_m} = -\frac{\partial V_m/\partial \alpha}{\partial V_m/\partial \tau} > 0 \tag{34}$$

In view of the nonlinearity of the first-order condition, reliance must be placed on the implicit differentiation above.

The nature of the solution can be illustrated further using Figure **??**. The vertical axis shows how the right hand side of (39), $f(\tau)$, varies with τ, for

[5] For the linear structure, Holder (2008) has shown that if the preference for leisure varies among individuals (such that hierarchical adherence exists), a greater deviation between the median voter's preference for leisure and the average preference parameter is important. A higher median preference compared with the arithmetic mean is associated with a higher chosen tax rate. In that case, the median voter benefits by being able to have relatively more leisure and a higher degree of redistribution.

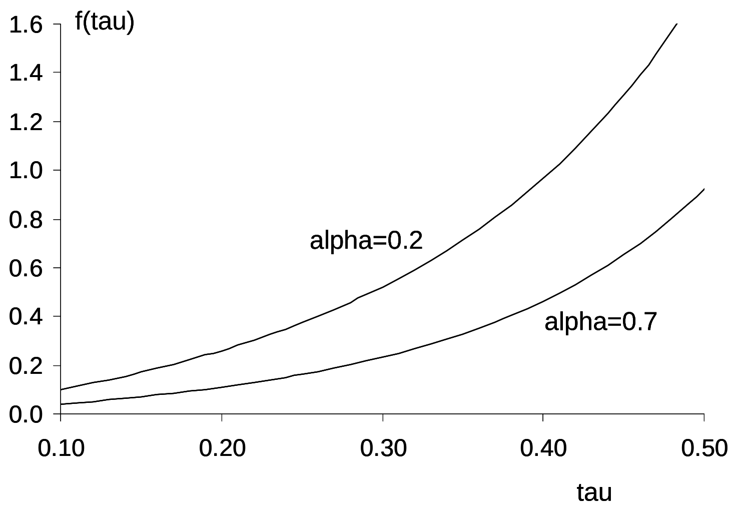

Figure 2: Variation in $f(\tau)$ with τ for Alternative α

two different values of α. It is again clear from these shedules in Figure **??** that the median voter's choice of τ has one feasible solution. For any given value of wage inequality, σ_w^2, the value of τ_m is thus easily obtained from the diagram. For example, if $\sigma_w^2 = 0.5$, τ_m is equal to 0.29 for $\alpha = 0.2$ and is 0.41 for $\alpha = 0.7$. Furthermore, the diagram illustrates the comparative static result reported above that the choice of tax parameter increases as the inequality of the wage rate distribution increases, and as the weight attached to consumption in the (common) utility function increases.

5 A Social Welfare Function

Consider the choice of tax rate by an independent judge or policy maker, whose value judgements can be described by an individualistic, additive and Paretean welfare, or evaluation, function, W, satisfying the principle of transfers. The welfare function is expressed in terms of individuals' (indirect) utilities, and thus described as 'welfarist'. Aversion to inequality on the part of the judge is reflected in the degree of concavity of the weighting function,

$H(V_i)$, with:[6]

$$W = \frac{1}{N}\sum_{i=1}^{N} H(V_i) \tag{35}$$

In the literature on optimal linear taxation, $H(V_i)$ is usually specified as taking a constant relative inequality aversion form, $H(V_i) = V_i^{1-\varepsilon}/(1-\varepsilon)$, with ε measuring the degree of aversion. Even in the simplest of frameworks, where individuals are assumed to have identical utility functions and therefore differ only in the value of w_i, it is well known that a closed-form solution for the optimal linear tax cannot be obtained unless special (quasi-linear) utility functions are used, or various approximations are made; see Creedy (2009).

In the present context the derivation of the optimal loglinear tax rate becomes intractable except for the case where $\varepsilon = 1$, whereby $H(V_i) = \log V_i$. Hence, this section concentrates on the optimal value of the tax parameter, τ, for a utilitarian social evaluation function of the form:[7]

$$W = \frac{1}{N}\sum_{i=1}^{N} \log V_i \tag{36}$$

Substituting V_i from (20) into (36), and remembering that $\sigma_y^2 = \sigma_w^2$, gives for a pure-transfer system:

$$\begin{aligned} W &= \log\left[\left(\frac{\alpha(1-\tau)}{1-\alpha\tau}\right)^{\alpha(1-\tau)}\left(\frac{1-\alpha}{1-\alpha\tau}\right)^{1-\alpha}\left\{\exp\left(\frac{\sigma_w^2}{2}(1-\tau)\right)\right\}^{\alpha\tau}\right] \\ &\quad +\alpha(1-\tau)\frac{1}{N}\sum_{i=1}^{N}\log w_i + \alpha\tau\bar{y} \end{aligned} \tag{37}$$

[6] For convenience the discrete summation over $i = 1, ..., N$ people has been used here, whereas in specifying the wage rate distribution above, a continuous form was used. This merely simplifies the notation.

[7] Mention should also be made of the fact that the optimal rate is not invariant with respect to monotonic transformations of utility. That is, the cardinalisation of utlity functions – a fundamental requirement if interpersonal comparisons are to be made – matters. Indeed the form in (36), with unit inquality aversion, in combination with (20), is equivalent to the optimal rate with zero inequality version and a cardinalisation of utility given by taking logarithms of the indirect utility function.

with $\bar{y}$ given by (24). Using $\frac{1}{N}\sum_{i=1}^{N}\log w_i = \mu_w$, this can be written as:

$$\begin{aligned} W &= \alpha(1-\tau)\log\alpha(1-\tau) - (1-\alpha\tau)\log(1-\alpha\tau) \\ &\quad + (1-\alpha)\log(1-\alpha) + \frac{\alpha\sigma_w^2}{2}\tau(1-\tau) \\ &\quad + \alpha\mu_w + \alpha\tau\left\{\frac{\sigma_w^2}{2} + \log\frac{\alpha(1-\tau)}{1-\alpha\tau}\right\} \end{aligned} \tag{38}$$

The first two lines of (38) correspond to the first line of (37), while the last line of (38) corresponds to the second line of (37). Differentiating with respect to τ, setting $dW/d\tau = 0$ and rearranging eventually gives the optimal tax parameter, τ_{SWF}, as the root of the following nonlinear equation:

$$\sigma_w^2 = \frac{\tau_{SWF}(1-\alpha)}{(1-\tau_{SWF})^2(1-\alpha\tau_{SWF})} \tag{39}$$

Comparison with (29) shows that the optimal choice, with $\varepsilon = 1$, is exactly the same as the majority-voting equilibrium, $\tau_m = \tau_{SWF}$, in this case where wage rates are lognormally distributed and preferences follow the Cobb-Douglas form.

6 Conclusions

This paper has explored the use of a loglinear tax and transfer function as an alternative to the linear form that has received so much attention in the public finance literature. The loglinear function displays increasing marginal and average tax rates along with a means-tested transfer payment. It consists of two parameters, a break-even income threshold, where the average tax rate is zero, and a tax parameter. The latter is the marginal tax rate at the gross income level for which the average tax rate is zero.

When combined with Cobb-Douglas utility, the resulting labour supply is particularly simple, implying a fixed positive labour supply that is independent of the individual's wage rate. It was found that preferences regarding the tax parameter are single-peaked, so that the median-voter theorem can be applied. For common preferences and a lognormal wage rate distribution, the majority choice of tax parameter was found to be the unique root of a nonlinear equation involving the variance of logarithms of wages and the exponent on net income (consumption) in the utility function. As with the linear tax function, higher basic inequality is associated with a higher choice of tax parameter. Considering the optimal choice of tax parameter by an independent judge who maximises an additive social welfare function involving the sum of logarithms of (indirect) utilities, the same expression is obtained as with the median voter's choice. Hence the use of constant relative inequality aversion of unity coincides, which implies a relatively high trade-off between equity and efficiency, coincides with the majority choice, even though the median voter is entirely selfish and has no concern for inequality.

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