

Does structure dominate regulation? The case of an input monopolist¹

Stephen P. King
Department of Economics
The University of Melbourne

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Abstract

This paper constructs a simple repeated game model to analyze how industry outcomes alter if a regulated input monopolist is allowed to integrate into the downstream retail market. Integration helps overcome double marginalization — a feature well known in the existing literature. Unlike existing static models, however, integration also makes tacit collusion more difficult in a repeated game framework. If the regulated input price exceeds marginal cost, an integrated monopolist has an incentive to increase retail sales as this raises upstream profits. It will be less willing to engage in any tacitly collusive conduct in the downstream market and it has a greater incentive to cheat on any collusive arrangement. We show that these effects may dominate input price regulation. A social planner may prefer the upstream monopoly to participate in the downstream market, even if integration leads to a higher regulated input price. The anti-competitive effects of the higher input price are more than offset by the pro-competitive effects of integration.

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1 Introduction

The analysis of vertical arrangements between firms has been an important part of recent competition policy. In utility industries, such as telecommunications and electricity, attention has focused on the interaction between potentially competitive downstream firms and an upstream monopoly that controls an essential input. Issues such as the optimal access price, vertical foreclosure and the incentive for the upstream firm to engage in anti-competitive conduct, have been explored by the literature.¹

Concern that an upstream monopoly will behave in an anti-competitive way if it is allowed to participate in a potentially competitive downstream market, has led to industry restructuring and regulatory intervention. For example, one of the factors leading to the break up of AT&T in the U.S. in the 1980s was a concern about anti-competitive behavior by an integrated, regulated telecommunications carrier.² More recently, the 1996 U.S. Telecommunications Act set out conditions that local exchange carriers had to satisfy before they could enter the competitive long-distance market. Anti-competitive concerns led the UK government to restrict entry by fixed carriers into mobile telephony (Geroski, *et. al.*, 1989), and to restructure the electricity industry before privatization.³ Similar restructuring to separate upstream monopoly and downstream firms has occurred in the electricity, gas and rail industries in Australia (King and Maddock, 1996).

¹Laffont and Tirole (2000) provide a survey of the literature on access pricing and vertical arrangements in telecommunications. Economides (1998) and Sibley and Weisman (1998) consider the incentive for an upstream monopoly to raise rivals' cost if it is integrated into the downstream sector. Rey and Tirole (1996) and McAfee and Schwartz (1994) consider the potential for foreclosure. Gans and De Fontenay consider the use of vertical integration to avoid hold-up.

²See Brennan (1987) and Noll and Owen (1989).

³Vickers and Yarrow (1988) and Armstrong, Cowan and Vickers (1994) provide surveys of British utility reforms.

Preventing an upstream monopoly from participating in downstream production may reduce the scope for some types of anti-competitive behavior. However, vertical separation is not costless. If there are economies of scope between upstream and downstream operations, forced vertical separation will tend to raise production costs. Separation may also alter a regulator's ability to set an efficient price for the input supplied by the monopoly. Gilbert and Riordan (1995) use an incentive regulation model to show that two vertically separated monopolies are more difficult to regulate than a single integrated monopoly. However, Vickers (1995) shows that integration can lead to a higher regulated input price when the downstream market is open to competitive production.

The consensus from the literature is that allowing an upstream monopoly that supplies an essential input, to participate in the downstream market, makes it more difficult to regulate the monopoly. In particular, following Vickers (1995), integration tends to raise the regulated access price. However, there may be offsetting benefits. Integration can reduce excessive downstream entry (Vickers 1995) or reduce excessive variety in a monopolistically competitive downstream market (Kuhn and Vives, 1999). As a result, there is an ambiguous trade-off between vertical separation, input pricing and social welfare.

One benefit of vertical integration focused on by existing studies is the moderation of double marginalization. In a one-shot model of competition, if the regulated input price exceeds marginal cost then downstream competitors face a distorted price and as a result set the retail price 'too high'. If the upstream monopoly also competes downstream then it faces the true marginal cost of the essential input and tends to price more aggressively. As a result, integration can lower the retail price.

Integration has an additional benefit that is not captured by the existing studies if downstream firms interact repeatedly over time. When the regulated input price exceeds marginal cost, the monopoly has an incentive to increase retail sales as this tends to raise upstream profits. If the monopoly is integrated into the downstream market, it can act aggressively to raise total sales. An integrated monopoly will be less willing to engage in any tacitly collusive conduct in the downstream market as this tends to reduce upstream profits. Further, the integrated firm has a greater incentive to cheat on any collusive arrangement than a non-integrated downstream competitor. The integrated firm finds the threat of a retail price war less of a deterrent than non-integrated firms. While the integrated monopoly loses retail profit during a price war, it gains wholesale sales and profit.

This paper constructs a simple repeated game model to capture this pro-competitive effect of integration. In general, competition will lead to a variety of potential equilibrium prices. We compare these equilibrium sets with and without integration for a variety of input prices, using different regulatory objectives. For example, if the objective is to maximize the minimum social welfare in equilibrium, then integration is preferred even if it involves a higher input price. We also consider the stability of any tacitly collusive equilibrium price and show that integration always makes the equilibrium less stable in the sense that it can only be supported with either a smaller number of downstream competitors or a higher discount factor. Finally, we consider particular subsets of equilibrium prices. For example, we consider the set of equilibrium prices that are dominant for the downstream firms, in the sense that no alternative equilibrium can make all firms better off. We show that with integration all dominant prices are strictly below the dominant price in the absence of integration. Again, this result is independent of the regulated

input price and remains valid even if integration raises this regulated price.

Overall, this paper shows that the case for allowing an upstream monopoly to participate in a downstream market is stronger than suggested by the existing literature. The potential for anti-competitive conduct, for example by reducing the quality of the input supplied to competitors, rises with integration. But integration also has significant competitive benefits. In particular, when downstream firms interact repeatedly, integration introduces an aggressive downstream competitor who has less interest in maintaining a tacitly collusive outcome. As a result, integration can lead to lower retail prices even when it involves a higher input price. In this sense, industry structure is more important for a competitive outcome than the specific regulated input price.

2 The model

Consider an industry with $N + 1$ retail firms and an upstream monopoly. We consider two cases. In the first case there is no vertical integration. The upstream monopoly is separate from the retail firms and each retail firm buys an essential input from the monopoly. In the second case, the upstream monopoly owns one of the retail firms. It supplies the input to that retail firm and also sells the input to the remaining N non-integrated retail firms.

The authorities regulate the price of the essential input at a per unit. For simplicity, we assume that the marginal cost of the input is constant and given by A . Production of the essential input might also involve a per-period fixed cost F , so if the upstream monopoly sells a total of Q units of the input in any period its profit is given by $[a - A]Q - F$. We will only consider situations where the authorities set an input price to at least cover marginal

cost so that $a \geq A$.⁴

Let n denote a generic retail firm. Each of these firms transforms the essential input into an identical final product using constant returns to scale technology. We normalize units so that one unit of the input is needed to produce one unit of the final product, where c denotes the cost per unit of transforming the input into final product. The retail firms compete in each period by simultaneously setting their individual prices. Competition continues for a potentially infinite number of periods and the common discount factor for the retail firms is denoted by δ . We assume that $\delta < 1$. In each period, all consumers simply buy from the cheapest retailer. If indifferent, consumers randomly choose between retailers, where ρ_n is the probability that each consumer will choose firm n . There are a large number of consumers so that ρ_n represents firm n 's market share.⁵

Demand for the final product is denoted by $Q(P)$, where P is the (lowest) retail price faced by consumers and Q is the total quantity of the retail product purchased. We assume that $Q(\cdot)$ is twice continuously differentiable with $Q'(\cdot) < 0$. For any price P , we denote the total profits that accrue to the retailers at this price in one period by $\pi(P)$. Each retailer's profit is denoted by π_n . The total profits that accrue to the retailers and to the upstream monopoly in one period equal $\pi(P) + [a - A]Q(P)$. This is denoted by $\Pi(P)$.

It is useful to define two reference prices. First, for any regulated input price a , let P^r refer to the monopoly price for a retailer with marginal cost $c + a$. In other words, $Q'(P^r)P^r + Q(P^r) - Q'(P^r)[c + a] = 0$. Second, let

⁴With a separate upstream monopoly, participation requires that the monopoly make non-negative profits in equilibrium. As a minimum, this requires that $a \geq A$.

⁵In other words, the retail firms play a standard infinitely repeated Bertrand competition game.

P^i refer to the monopoly price for an integrated upstream firm and retailer so that $Q'(P^i)P^i + Q(P^i) - Q'(P^i)[c + A] = 0$. The retail price P^i would maximize the joint profits of both the retailers and the upstream monopoly if it were set in every period. Given the input price, the retail price P^r would maximize the joint profits of the retail firms if it were set in every period. Because $a \geq A$, $P^r \geq P^i$ with equality only when $a = A$.⁶ We assume that the regulated input price is not so high that total industry profits Π would be maximized when the retailers sell at a loss. In other words, we assume that $a < P^i - c$.⁷

We concentrate on the stationary equilibria of the game between the retail firms and consider those equilibria that are supported by the ‘grim punishment’ strategy of reversion to the one-shot Nash equilibrium forever. In this model, such punishment involves selling at a price equal to retail marginal cost, $a + c$ in each period.

We make two further assumptions that simplify the analysis.

Assumption one: for a given regulated input price a , if there are two feasible stationary price equilibria that can be supported, but one equilibrium is preferred to the other by *all* participants in the market — the retail firms, the upstream monopoly and the consumers — in that one equilibrium involves a lower price but provides at least the same profits to each firm, then the firms do not play the non-preferred equilibrium.

Assumption two: For all P , $Q''(P) \leq 0$.

⁶Formally, if we consider the retail monopoly price P for any input price a ,

$$\frac{dP}{da} = \frac{Q'(P)}{Q''(P)[P - c - a] + 2Q'(P)}$$

But $Q''(P)[P - c - a] + 2Q'(P) < 0$ by the second order conditions for profit maximization, so that $(dP/da) > 0$.

⁷This assumption rules out certain trivial equilibria when one of the retail firms is owned by the upstream monopoly.

Under assumption one, if the upstream firm is not integrated into the retail market, it is trivial to show that we only need to focus on equilibria involving a price no greater than P^r .⁸ We show below that when the upstream monopoly is integrated with one of the retail firms, assumption one implies that we need only consider equilibria where the retail price is no greater than P^i .

Assumption two is a reasonably strong constraint on demand. It guarantees that the profit function for each firm is strictly concave, whether or not there is integration between a retail firm and the upstream monopoly. Because of this, assumption two allows us to avoid potential non-convexities that can arise when the number of retail firms change. The assumption is sufficient, but not necessary, for some of the results under integration.

2.1 Competition without integration

We first consider the situation when the retail firms are separate from the upstream monopoly. Because the input price is set by a regulator, the upstream monopoly cannot directly interact with the retail firms and has no active role in determining the market outcome.⁹ The potential equilibrium outcomes in this situation are well understood in the literature and are summarized by the following lemma.¹⁰

Lemma 1 *For a regulated input price a and $\delta \geq \frac{N}{N+1}$, any price $P \in [c + a, P^r]$ can be supported as a subgame perfect stationary equilibrium. For*

⁸Both the upstream firm and the consumers strictly prefer lower retail prices. By the definition of P^r any price greater than P^r results in less total profit for the retail firms than P^r . It is simple to show that if there is an equilibrium price greater than P^r then there exists an equilibrium among retail firms with price P^r that gives each retailer at least the same profit.

⁹In particular, the upstream monopoly has no strategies except to sell the amount of input that is demanded by the retailers.

¹⁰See for example Friedman (1971) and Tirole (1988).

$\delta < \frac{N}{N+1}$ the only stationary equilibria involve setting a price $P = c + a$.

All proofs are in the appendix.

From lemma 1 any retail price $P \in (c + a, P^r]$ can be supported as an equilibrium for $\delta \geq (N/N+1)$. But the supportable shares of the retail profits that accrue in equilibrium to each firm will vary as δ varies. Corollary 2 characterizes the equilibrium market (and hence profit) shares for the retail firms and follows directly from the proof of lemma 1.

Corollary 2 *Suppose $\delta \geq \frac{N}{N+1}$ and consider a retail price $P \in (c + a, P^r]$. P is supportable as a stationary equilibrium iff market shares are given by $\rho_n \in [1 - \delta, 1 - N(1 - \delta)]$ for all n .*

When $\delta \geq (N/N + 1)$ the equilibrium outcomes that maximize the joint profits of the retail firms involve setting a price of P^r . So when $\delta \geq (N/N + 1)$ the set of stationary equilibria that maximize the joint profits of the retailers is given by

$$D_r = \{\pi_n : \forall_n \quad \pi_n \in [(1 - \delta)\pi(P^r), 1 - N(1 - \delta)\pi(P^r)], \sum_n \pi_n = \pi(P^r)\}$$

From lemma 1, any other price that involves positive profits to the firms, $P \in (c + a, P^r)$ is only supportable as a stationary equilibrium price if δ is sufficiently large so that P^r is also sustainable as an equilibrium price. From corollary 2, the sustainable market shares do not depend on the equilibrium price for $P \in (c+a, P^r]$. So the set of equilibria D_r represents a ‘dominant’ set for the retail firms in the sense that for any stationary equilibrium involving a price $P \in (c + a, P^r)$, there exists an equilibrium in set D_r that provides at least as much profit to all retail firms and a strictly greater level of profit to at least one retail firm. If retail firms are able to co-ordinate on an equilibrium that is dominant in this sense, then they will play an equilibrium in the set D_r .

2.2 Competition with integration

We now consider the situation where one of the retail firms is integrated with the input monopoly. Without loss of generality we assume that retail firm $N + 1$ is vertically integrated. This firm receives profit from both its retail sales and from the sale of the essential input to its retail competitors. There are N non-integrated retailers. Again, the input price is set equal to a by a regulator and there is no ability for the integrated firm either to manipulate the price or quality of the essential input.¹¹

Integration allows the upstream monopoly to play an active role in the market outcome. While the monopoly must still supply whatever quantity of input is demanded by the (non-integrated) retailers, it can use its retail arm to influence the retail-level behavior. This is manifested in two ways. First, the integrated firm receives positive profits from selling the input. It makes profit $[a - A]Q(a + c)$ in the one-shot equilibrium. These wholesale profits will be lower whenever the retail price is higher than $(a + c)$. Because of this the wholesale profit affects the incentive for the integrated firm to maintain a high retail price. This in turn will affect the share of total profit that must accrue to each firm in a stationary equilibrium.

Second, with integration the joint profit maximizing price for all retail firms is lower and equal to P^i . Because integration internalizes the spill over between retail pricing and upstream sales, integration creates an incentive for firms to co-ordinate on a lower price.

We start by noting that if there is no gap between the regulated input price and the marginal cost of the input, then integration is irrelevant.

¹¹This enables us to ignore various potential anti-competitive behaviours by the integrated firm. For example, see Economides (1998).

Comment 3 *If the input price a is set equal to marginal cost A then integration has no affect on the set of stationary equilibria.*

Comment 3 follows from the absence of wholesale profits when $a = A$. In this situation, integration has no affect on any firm's profits and cannot affect the set of stationary equilibria. For this reason, we concentrate on situations where $a > A$ to show how integration alters the potential equilibrium outcomes for the industry.

If $a > A$ then integration will change the conditions for equilibrium. For any price $P \in (a + cP^i]$ these conditions are given by

$$\begin{aligned} \frac{1}{1-\delta}(\pi_{N+1}(P) + [a - A]Q(P)) &\geq \pi(P) + (a - A)Q(P) \\ &+ \frac{\delta}{1-\delta}[a - A]Q(a + c) \end{aligned} \quad (1)$$

and

$$\forall_{n \in \{1, \dots, N\}} \quad \frac{1}{1-\delta}\pi_n(P) \geq \pi(P) \quad (2)$$

Inequality (1) requires that the integrated firm prefers to set the equilibrium price rather than deviate, seize the entire retail profit for one period, and then face the 'grim punishment' forever. Inequality (2) is the equivalent condition for each of the non-integrated firms. If the equilibrium price is $P > P^i$ then any deviation by the integrated firm will involve setting the price P^i for one period. Thus, P^i replaces P on the right-hand-side of (1). If $P > P^r$ then any deviation by a non-integrated firm will involve setting the price P^r for one period. Thus, P^r replaces P on the right-hand-side of (2).

To allow comparison with the non-integrated situation, we consider retail prices up to and including P^r . Lemma 4, however, shows that for any equilibrium with a retail price that exceeds P^i , there is an equilibrium with

a retail price equal to P^i that provides each firm with at least the same profits. Such an equilibrium will clearly also be preferred by consumers, so any equilibrium with a price greater than P^i violates assumption 1. Thus, from lemma 4, we only need to consider equilibria with a price no greater than P^i .

Lemma 4 *Consider any stationary equilibrium with $P \in (P^i, P^r]$. There exists a stationary equilibrium with $P = P^i$ that gives at least the same profit to each firm and is strictly preferred by consumers.*

It remains to show if and when P^i or any other price can be sustained as a stationary equilibrium. Lemma 5 examines this. For any price $P \in (a+c, P^i]$ this lemma characterizes the minimum value of the discount factor, δ , that is needed to sustain the price as a subgame perfect stationary equilibrium. We denote this minimum value of the discount factor for a retail price P and an input price a by $\tilde{\delta}(P, a)$.

Lemma 5 *Given the input price a , a price $P \in (c+a, P^i]$ can be supported as a subgame perfect stationary equilibrium iff $\delta \geq \tilde{\delta}(P, a)$ where*

$$\tilde{\delta}(P, a) = \frac{NQ(P)[P - a - c]}{(N + 1)Q(P)[P - a - c] - [a - A][Q(a + c) - Q(P)]}$$

It is easy to confirm that when $a > A$ then $\tilde{\delta}(P, a) \in (\frac{N}{N+1}, 1)$ for all $P \in (a+c, P^i]$. First, note that the denominator of $\tilde{\delta}(P, a)$ is equivalent to $N\pi(P) + \Pi(P) - [a - A]Q(a + c)$. But this exceeds $N\pi(P)$ for $P \in (a+c, P^i]$, so that $\tilde{\delta}(P, a) < 1$. Also note that $[a - A][Q(a + c) - Q(P)] > 0$ when $a > A$ so that $\tilde{\delta}(P, a) > \frac{N}{N+1}$. Further, by LeHopital's rule,

$$\lim_{P \rightarrow a+c} \tilde{\delta}(P, a) = \frac{N\pi'(a+c)}{(N+1)\pi'(a+c) + [a-A]Q'(a+c)} > \frac{N}{N+1}$$

However, when $a = A$, $\tilde{\delta}(P, A) = \frac{N}{N+1}$ for all P .

In general, $\tilde{\delta}(P, a)$ need not be monotonic in P . Lemma 6 shows that $\tilde{\delta}(P, a)$ will be increasing either in the neighborhood of P^i or, under assumption two. It also shows that $\tilde{\delta}(P, a)$ is increasing in both the number of non-integrated retailers and in the regulated input price.

Lemma 6 (i) $\tilde{\delta}(P, a)$ is increasing in P at $P = P^i$, (ii) $\tilde{\delta}(P, a)$ is increasing in N for all $P > a + c$, (iii) $\tilde{\delta}(P, a)$ is increasing in a for all $P > a + c$ and (iv) under assumption two, $\tilde{\delta}(P, a)$ is increasing in P for $P \in (c + a, P^i]$.

Finally, we can consider the set of stationary equilibrium outcomes if the firms only play equilibria that are not dominated from their perspective. Assume that the discount factor is sufficiently high to support all prices $P \in (a + c, P^i]$ as equilibria. In other words, $\delta > \tilde{\delta}(P, a)$ for all $P \in (a + c, P^i]$. Under assumption two, it is sufficient that $\delta > \tilde{\delta}(P^i, a)$.

Unlike the non-integrated equilibria presented in section 2.1, the non-dominated equilibria under integration can involve a range of prices. In particular, the best non-dominated equilibrium for a subset of firms will depend on both the number of firms in the subset and the composition of the subset.

First suppose that there is a subset of firms composed of \mathcal{N} non-integrated retailers. If $\mathcal{N} < N$ then the non-dominated equilibrium that maximizes the joint profits of the subset will involve a price strictly below P^i . This contrasts with the situation where all firms are non-integrated and non-dominated equilibria only involved a price equal to P^r . The difference arises because the incentive for the integrated firm to deviate from an equilibrium tends to decrease as the retail price decreases. The optimal equilibrium for the subset is found by maximizing the subset's profits subject to guaranteeing no deviation by the integrated firm and other non-integrated firms. As the

retail price falls below P^i , there are less retail profits for the subset of firms to seize but this is more than offset by the reduced need to share retail profits with the integrated firm.

Let $P_r^*(\mathcal{N})$ be the price associated with the best non-dominated equilibrium for a subset involving \mathcal{N} non-integrated firms. Lemma 7 characterizes $P_r^*(\mathcal{N})$ and shows that $P_r^*(\mathcal{N}) \in (c + a, P^i]$ with $P_r^*(\mathcal{N}) < P^i$ if $\mathcal{N} < N$.

Lemma 7 *Suppose $\delta > \tilde{\delta}(P^i, a)$ and consider any subset of non-integrated firms with \mathcal{N} members. The equilibrium that maximizes the joint profit of this subset involves a price $P_r^*(\mathcal{N}) \in (a + c, P^i]$ where $P_r^*(\mathcal{N})$ solves*

$$\pi'(P)[1 - (N + 1 - \mathcal{N})(1 - \delta)] + \delta[a - A]Q'(P) = 0 \quad (3)$$

Further, if $\mathcal{N} < N$ then $P_r^(\mathcal{N}) < P^i$. But $P_r^*(N) = P^i$*

As the relevant set of non-integrated firms increases in size, the best non-dominated equilibrium for this set involves an increasingly higher price. To see this, note that by totally differentiating (3)

$$\frac{dP_r^*(\mathcal{N})}{d\mathcal{N}} = \frac{-(1 - \delta)\pi'(P_r^*)}{\pi''(P_r^*)[1 - (N + 1 - \mathcal{N})(1 - \delta)] + \delta[a - A]Q''(P_r^*)}$$

which is greater than zero under assumption two.

Similarly,

$$\frac{dP_r^*(\mathcal{N})}{da} = \frac{-(N - \mathcal{N})Q'(P_r^*)}{\pi''(P_r^*)[1 - (N + 1 - \mathcal{N})(1 - \delta)] + \delta[a - A]Q''(P_r^*)}$$

so that $P_r^*(\mathcal{N})$ is decreasing in a for $N > \mathcal{N}$ and is constant in a when $N = \mathcal{N}$.¹²

Now, suppose that there is a subset of firms including the integrated firm and \mathcal{N} non-integrated retailers. Let $P_i^*(\mathcal{N})$ be the price associated with

¹²Remembering that $\tilde{\delta}(P, a)$ depends upon a so that any change in a must continue to satisfy the assumption that δ is above $\tilde{\delta}(P^i, a)$.

the best non-dominated equilibrium for this subset of firms, where $\mathcal{N} \in \{0, \dots, N\}$. Lemma 8 characterizes $P_i^*(\mathcal{N})$.

Lemma 8 *Suppose $\delta > \tilde{\delta}(P^i, a)$ and consider any subset of firms including the integrated firm and \mathcal{N} non-integrated firms. The equilibrium that maximizes the joint profit of this subset involves a price $P_i^*(\mathcal{N}) \in (a+c, P^i]$ where $P_i^*(\mathcal{N})$ solves*

$$\pi'(P)[1 - (N - \mathcal{N})(1 - \delta)] + [a - A]Q'(P) = 0 \quad (4)$$

Further, if $\mathcal{N} < N$ and $a + c < P^i$ then $P_i^(\mathcal{N}) < P^i$. But $P_i^*(N) = P^i$*

Again, it is easy to see that as the number of non-integrated firms in the relevant subset increases, the best non-dominated equilibrium for the subset involves an increasingly higher price. Also

$$\frac{dP_i^*(\mathcal{N})}{da} = \frac{-[(N - \mathcal{N})(1 - \delta)]Q'(P_r^*)}{\pi''(P_i^*)(1 - (N - \mathcal{N})(1 - \delta)) + [a - A]Q''(P_i^*)}$$

so that $P_i^*(\mathcal{N})$ is decreasing in a when $N > \mathcal{N}$ and is constant in a when $N = \mathcal{N}$.¹³

We can compare the prices associated with particular non-dominated equilibria. In particular, consider $P_r^*(1)$ and $P_i^*(0)$. From (13) we know that

$$\pi'(P_r^*(1))[1 - N(1 - \delta)] + \delta[a - A]Q'(P_r^*(1)) = 0 \quad (5)$$

From (15) and assumption 2, we know that if $P > P_i^*(0)$ then

$$\pi'(P)[1 - N(1 - \delta)] + [a - A]Q'(P) < 0 \quad (6)$$

Substituting $P_r^*(1)$ into the left-hand-side of (6) and using equation (5) gives $(1 - \delta)[a - A]Q'(P_r^*(1))$. But this is less than zero so that $P_r^*(1) > P_i^*(0)$.

¹³Remembering that we have assumed that $\delta > \tilde{\delta}(P^i, a)$ and this constraint must continue to hold as a alters.

In other words, the best non-dominated equilibrium for the integrated firm involves a strictly lower price than the best non-dominated equilibrium for a single non-integrated firm.

Lemma 7 and 8 can be used to characterize the set of non-dominated equilibria when one retail firm is integrated with the network owner. For example, if $N = 1$ and $\delta > \tilde{\delta}(P^i)$, the set of non-dominated stationary equilibria involve per period profits given by

$$D_i = \{\pi_1, \Pi_2 : \Pi_2 \in [(1-\delta)\Pi(P^i) + \delta[a-A]Q(a+c), \delta\pi(P_n^*(0)) + [a-A]Q(P_n^*(0))],$$

$$\pi_1 \in [(1-\delta)\pi(P_n^*(0)), \delta\pi(P^i) - \delta[a-A][Q(a+c) - Q(P^i)]],$$

$$\pi_1 + \Pi_2 = \pi(P) + [a-A]Q(P) \text{ where } P \in [P_n^*(0), P^i]\}$$

3 Does structure dominate input price regulation?

The analysis in section 2 allows us to compare market outcomes either with or without vertical integration for a wide range of input prices. In particular, we can compare outcomes involving both different regulated input prices and different industry structures.

As noted in the introduction, a regulated input price can differ depending on the industry structure. A social planner, when considering the desirable industry structure will need to consider how structure interacts with the regulated input price. Further, for any given input price, a range of final product prices can arise as equilibria. Hence, the social planner will need to compare between sets of potential final product prices to determine the preferred industry structure.

In this section, we compare equilibrium outcomes when the regulated input price differs depending on industry structure. We show how, for a

variety of reasonable objectives and beliefs of the social planner, an integrated industry structure is preferred *even if this structure necessarily involves a higher regulated input price*. In other words, integration will tend to result in lower final product prices even if it leads to a higher input price. In this sense, industry structure is more important to the social planner than the specific input price regulation.

To consider the social planner's preferred industry structure, we first need to consider the set of possible retail price equilibria under different structures and for different input prices. Let $\mathcal{S}_r(\delta, a_r)$ be the set of sustainable equilibrium retail prices when the upstream monopoly is independent of all retailers and the regulated input price is a_r . $\mathcal{S}_i(\delta, a_i)$ is the equivalent set when the upstream monopoly is integrated with one retailer and the regulated input price is a_i . Similarly, let $\mathcal{D}_r(\delta, a_r)$ and $\mathcal{D}_i(\delta, a_i)$ represent the sets of non-dominated equilibrium prices without and with integration respectively. Again, a_r refers to the regulated input price in the absence of integration and a_i is the regulated input price with integration.

From lemma 1, under assumption one, if $\delta > \frac{N}{N+1}$ then $\mathcal{S}_r = \{P : c + a_r \leq P \leq P^r(a_r)\}$. From lemma 5, under assumption one and two, if $\delta \geq \tilde{\delta}(P^i, a_i)$, then $\mathcal{S}_i = \{P : c + a_i \leq P \leq P^i\}$. As (i) $P^i \leq P^r(a_r)$ with equality only if $a_r = A$ and (ii) $\tilde{\delta}(P^i, a_i) \geq \frac{N}{N+1}$ for all a_i , corollary 9 immediately follows.

Corollary 9 *Suppose $\delta > \tilde{\delta}(P^i, a_i)$. Then under assumptions one and two:*

1. $\mathcal{S}_r(\delta, a_r) = \mathcal{S}_i(\delta, a_i)$ iff $a_r = a_i = A$;
2. if $a_i \geq a_r > A$ then $\mathcal{S}_i(\delta, a_i)$ is a strict subset of $\mathcal{S}_r(\delta, a_r)$, and the maximal element $\mathcal{S}_r(\delta, a_r)$ is strictly greater than the maximal element of $\mathcal{S}_i(\delta, a_i)$; and

3. if $A \leq a_i < a_r$ then $\mathcal{S}_i(\delta, a_i)$ is the union of two sets \mathcal{S}_i^0 and \mathcal{S}_i^1 where
- (i) all elements of \mathcal{S}_i^0 are less than the smallest element of $\mathcal{S}_r(\delta, a_r)$ and
 - (ii) \mathcal{S}_i^1 is a strict subset of $\mathcal{S}_r(\delta, a_r)$, and the maximal element $\mathcal{S}_r(\delta, a_r)$ is strictly greater than the maximal element of \mathcal{S}_i^1

This corollary shows that when firms are relatively patient the two equilibrium price sets only coincide if the regulated input price is set at exactly marginal cost regardless of industry structure. If however the social planner is unsure of the exact marginal input cost or is constrained to set $a > A$ to guarantee firm viability, then either a_i and/or a_r is likely to be set above A . In this case, the sets of potential equilibrium prices will differ under different industry structures.

The social planner will generally be unsure of the exact equilibrium that the firms will play, but it may have beliefs over the sets of equilibrium prices. Assume that the social planner is interested in maximizing social surplus or consumers' welfare, so that it prefers a lower equilibrium price. We say that one pair of industry structure and input price is *socially preferred* to another if it supports equilibria with lower prices. Formally:

Definition: A set of equilibrium prices $S_1(\delta, a_1)$ under industry structure 1 and with input price a_1 is *socially preferred* to another set $S_2(\delta, a_2)$ if (i) $p_1 \in S_1(\delta, a_1)$, $p_1 \notin S_2(\delta, a_2)$ implies that $p_1 < \min_p\{p : p \in S_2(\delta, a_2)\}$ and (ii) $p_2 \in S_2(\delta, a_2)$, $p_2 \notin S_1(\delta, a_1)$ implies that $p_2 > \max_p\{p : p \in S_1(\delta, a_1)\}$.

If one set of equilibrium prices is socially preferred to another, then this means that where the two sets do not coincide, the preferred set always has lower prices and the other set always has higher prices. The socially preferred set will lead to greater expected social welfare or consumers' surplus for a variety of beliefs over the equilibrium sets. In particular, for any beliefs that

coincide over prices that are in both sets, the socially preferred set will never lead to lower expected social surplus or consumers' surplus. In this sense, an industry structure and input price that involves a socially preferred set of equilibrium prices is a desirable outcome for the social planner.

Result 10 *If $A \leq a_i \leq a_r$ then a vertically integrated industry structure leads to a set of equilibrium prices that is socially preferred to the set of equilibrium prices under a vertically separated industry structure.*

Result 10 states that if vertical integration does not result in a higher regulated input price, then integration will be preferred by the social planner (in the sense that it leads to a socially preferred set of equilibrium prices). The result follows directly from corollary 9.

If $a_i > a_r > A$ then neither $\mathcal{S}_r(\delta, a_r)$ nor $\mathcal{S}_i(\delta, a_i)$ is socially preferred. But these are exactly the situations that have raised concern in the literature — where integration raises the regulated input price. In such a situation there are a number of ways that the planner can determine the optimal combination of input price and industry structure. One reasonable approach is that the planner might wish to maximize the minimum level of consumer surplus or social surplus. If the social planner follows this rule then result 11 follows immediately from corollary 9.

Result 11 *Suppose that $\delta > \tilde{\delta}(P^i, a_i)$ and $a_r > A$. If the social planner wants to maximize the minimum possible equilibrium consumer or social surplus, the planner will strictly prefer an industry structure where the input monopoly owns a retail firm rather than an industry structure without any vertical integration, regardless of the actual input prices.*

Result 11 is very strong. If the social planner has a 'maximin' objective then it will prefer to have an integrated industry structure regardless of the

regulated input prices. In this sense, the industry structure dominates the input price regulation. For example, suppose integration leads to a larger gap between the true marginal input cost and the regulated input price. Despite this, the social planner will still prefer an integrated industry structure to one where the input monopoly is separate from the retail firms.

The social planner, of course, might use an alternative approach. For example, the planner wish to maximize expected social surplus, given its beliefs over the equilibrium prices that might arise under different specifications of industry structure and input price. In the absence of further information the planner might have uniform beliefs over the prices in $\mathcal{S}_r(\delta, a_r)$ and $\mathcal{S}_i(\delta, a_i)$. If $a_r > A$ then the planner will strictly prefer integration if $A \leq a_i \leq a_r$ as $\mathcal{S}_i(\delta, a_i)$ is socially preferred to $\mathcal{S}_i(\delta, a_i)$. Further, as the set $\mathcal{S}_i(\delta, a_i)$ is continuous in a_i , there exist values $\varepsilon > 0$ such that if $a_i < a_r + \varepsilon$, the expected social surplus with integration exceeds that without integration. Result 12 summarizes this conclusion.

Result 12 *Suppose that $\delta > \tilde{\delta}(P, \bar{a})$ and given the industry structure, the social planner has a uniform prior belief over all potential equilibrium prices. Then, there exists values $\varepsilon > 0$ such that if $A < a_i \leq a_r \leq \bar{a}$ or if $A < a_r < a_i$ but $a_i < a_r + \varepsilon \leq \bar{a}$ then the planner prefers an integrated industry structure.*

Result 12 is not as strong as result 11. The planner might prefer a separated industry structure, but only if the input price under integration is sufficiently large compared with the input price under separate ownership.

While results 11 and 12 consider all potential equilibrium prices, it might be reasonable for the social planner to focus on the equilibria that are undominated from the perspective of the firms. Result 13 follows directly from lemma 7 and 8 and the fact that $\mathcal{D}_r(\delta, a_r) = P^r(a_r)$ when $\delta > (N/N + 1)$.

Result 13 *Suppose that $\delta > \tilde{\delta}(P^i, a_i)$ and $a_r > A$. If the social planner believes that the firms will only play undominated equilibria and wants to maximize expected social surplus, then the regulator always strictly prefers an integrated industry structure regardless of the actual access prices.*

This result holds regardless of the beliefs that the social planner might have over the set of undominated equilibria under an integrated industry structure. The planner always finds integration preferable because all elements of the set of undominated equilibrium prices under integration, $\mathcal{D}_i(\delta, a_i)$ are less than P^r regardless of the value of a_i when $a_r > A$. In other words, even if the input price is significantly higher under an integrated market structure, the social planner will still prefer integration unless they know that the input price can be set at exactly marginal cost in the absence of integration.

Result 13 is partially driven by double marginalization. When the input price is set above marginal cost and there is no vertical integration, the joint profit maximizing retail price must exceed the integrated monopoly price. However, the implications of result 13 go further than this. Suppose, for example, that under an integrated market structure, the integrated firm will be a natural price leader. From lemma 8, the best non-dominated price for the integrated firm will be below the integrated monopoly price whenever the input price exceeds marginal cost. Because the integrated firm cares about wholesale profits it is prepared to trade-off some retail profits for increased wholesale profits. As a price leader, the integrated firm will tend to set a price that is both below the price that non-integrated firms would set and below the integrated monopoly price.

So far we have considered the social planner's preference over industry structure when evaluating the set of possible equilibrium prices when δ is

sufficiently large. Alternatively, suppose the planner is uncertain about the level of ‘patience’ of the firms in the industry, or about the potential number of firms that might enter the industry. In these circumstances, the planner might be concerned about the stability of any equilibrium price above $c + a$ under either industry structure. If a price above $c + a$ is less stable under one structure, in the sense that it will only be sustainable as an equilibrium if either firms are more patient or if fewer firms compete in the retail market, then the social planner might prefer the structure that lowers stability.

For any given values of a_r and a_i and for any specific price P that exceeds both $c + a_r$ and $c + a_i$ but is no greater than P^i , consider the set of discount factors such that this price is sustainable as an equilibrium under either structure. Denote this set by $\Delta_r(P, a_r)$ if the input monopoly is vertically separated from the retailers and by $\Delta_i(P, a_i)$ if one of the retailers is owned by the input monopoly. Corollary 14 follows from lemma 1, 5 and 6.

Corollary 14 *Suppose $a_i > A$ and consider any price that exceeds both $c + a_i$ and $c + a_r$ but is no greater than P^i . Then $\Delta_i(P, a_i)$ is a strict subset of $\Delta_r(P, a_r)$. Further, any δ that is an element of $\Delta_r(P, a_r)$ but not an element of $\Delta_i(P, a_i)$ is strictly less than all elements of $\Delta_i(P, a_i)$.*

This corollary shows how any price that is a potential equilibrium under both industry structures will always be ‘less stable’ if the upstream firm is integrated into the downstream market. This is formalized by the following result.

Result 15 *Suppose $a_i > A$ and consider any price P that exceeds both $c + a_i$ and $c + a_r$ but is no greater than P^i . Further, suppose the social planner has beliefs over the set of potential δ and wishes to choose the industry structure where P is less likely to be an equilibrium. Then for all beliefs, the integrated*

structure is at least as desirable to the social planner as the non-integrated structure. Further, for some beliefs over δ the social planner strictly prefers the integrated structure.

Similarly, suppose the social planner is uncertain about the exact number of firms that will participate in the retail market, but believes that the same number of firms will participate under either structure. Given the discount factor δ we can consider the stability of a potential equilibrium price P by looking at the minimum number of firms that need to engage in retail competition to ensure that the price is not sustainable. Denote this critical minimum number of firms without integration by N_r . From lemma 1, $N_r = \frac{\delta}{1-\delta}$ for any relevant P . Similarly, if we denote the critical number of firms in the absence of integration by N_i , from lemma 5, $N_i < \frac{\delta}{1-\delta}$ for any relevant P . In this sense, integrated equilibria are less stable when there is uncertainty about the level of entry.

4 Conclusion

In this paper we have considered the consequences of alternative industry structures when a monopoly supplies an essential input to a potentially competitive downstream market. Our analysis suggests that there are a variety of circumstances when a social planner will prefer an integrated industry structure.

Our results are clearly at odds with many regulatory pre-conceptions. In part this reflects the tendency in the literature to focus on the actual input price. Integration may tend to raise the regulated input price and in a one-shot model of competition this increased input price usually translates into a higher retail price. The concerns about the input price are well founded.

But the analysis above shows that these concerns might be overwhelmed by the effect that integration has on lowering retail prices. Integration makes one retailer ‘care’ about wholesale profits and provides an impetus to lower prices.

The results in this paper also indicate the circumstances when integration might be undesirable. In particular, if it is likely that $A \leq a_r < a_i$ and firms will tend to behave highly competitively in the retail market regardless of structure, then having a non-integrated structure with a lower input price will improve social surplus. If integration makes it more likely that the regulated input price will be set high then integration is undesirable if retail competition is likely to be strong. Similarly, if integration is likely to make entry into retail competition less likely, possibly because an integrated competitor can more effectively pursue anti-competitive conduct than a non-integrated firm, then integration might be undesirable. Of course, for separate ownership to be preferred, the anti-competitive potential under integration must outweigh the innate tendency to lower prices that holds under integration.

Appendix

Proof of Lemma 1: Consider any $P \in (c + a, P^r]$. This price will be supportable as a stationary equilibrium with market shares ρ_n for all $n = 1, \dots, N + 1$ iff

$$\forall_n \quad \frac{\rho_n}{1 - \delta} \pi(P) \geq \pi(P) \quad (7)$$

Simplifying, this implies that P is supportable as a stationary equilibrium price iff

$$\forall_n \quad \rho_n \in [1 - \delta, 1 - N(1 - \delta)] \quad (8)$$

Note that (8) does not depend on the value of P .

For $\delta \geq \frac{N}{N+1}$, there always exist a set of market shares $\{\rho_n\}$ such that, for all n , $\rho_n \in [0, 1]$ and $\sum_n \rho_n = 1$ with (8) is satisfied. To see this, let $\rho_1 = 1 - N(1 - \delta)$ and $\rho_n = 1 - \delta$ for all $n \neq 1$. By construction, these market shares satisfy (8) and sum to unity. Further, as $\delta \geq \frac{N}{N+1}$, $\rho_n \in [0, \frac{1}{N+1}]$ for all $n \neq 1$ and $\rho_1 \in [\frac{1}{N+1}, 1]$. So, $\rho_n \in [0, 1]$ for all n .

In contrast, for $\delta < \frac{N}{N+1}$ there never exists a set of market shares such that (8) is satisfied. To see this, from (8)

$$\sum_n \rho_n \geq (N+1)(1-\delta) > (N+1) \left(1 - \frac{N}{N+1}\right) = 1$$

so that any market shares that satisfy (8) must sum to greater than unity. ■

Proof of lemma 4: For $\hat{P} \in (P^i, P^r]$ to be an equilibrium, it must simultaneously satisfy (1) and (2) where P^i replaces P on the right-hand-side of (1).

Consider a putative equilibrium with price P^i and market shares such that $\rho_n \pi(P^i) = \pi_n(\hat{P})$ for $n = 1, \dots, N$. As $\Pi(P^i) > \Pi(\hat{P})$ by definition of P^i , $\rho_{N+1}(P^i) + [a - A]Q(P^i)$ must exceed $\pi_{N+1}(\hat{P}) + [a - A]Q(\hat{P})$ because the profits of all other firms has been held constant. Further, a price of P^i is strictly preferred by consumers to a price $\hat{P} > P^i$. Thus if the putative equilibrium exists, it is Pareto preferred to the equilibrium with price \hat{P} . It remains to prove that this putative equilibrium exists.

First it is necessary to shows that sufficient retail profits exist at P^i to allow $\rho_n \pi(P^i) = \pi_n(\hat{P})$ for all $n = 1, \dots, N$. Note that as price \hat{P} is an equilibrium, from (1) with P on the right-hand-side replaced by P^i , we know that

$$\begin{aligned} \sum_{n=1}^N \pi_n(\hat{P}) &\leq \pi(\hat{P}) - (1-\delta)\pi(P^i) - (1-\delta)[a-A]Q(P^i) \\ &\quad + [a-A]Q(\hat{P}) - \delta[a-A]Q(a+c) \end{aligned}$$

Thus

$$\begin{aligned} \sum_{n=1}^N \pi_n(\hat{P}) - \pi(P^i) &\leq [\Pi(\hat{P}) - \Pi(P^i)] - (1 - \delta)\pi(P^i) \\ &\quad - \delta[a - A][Q(a + c) - Q(P^i)] < 0 \end{aligned}$$

so that $\pi(P^i) > \sum_{n=1}^N \pi_n(\hat{P})$ and there exists a set of market shares $\rho_n \in [0, 1]$, $\sum_{n=1}^N \rho_n < 1$ such that $\rho_n \pi(P^i) = \pi_n(\hat{P})$. We denote these market shares by ρ_n^* for $n = 1, \dots, N$.

Second, it is necessary to show that the strategies of both firms setting a retail price equal to P^i with market shares $\rho_n = \rho_n^*$ and $\rho_{N+1} = 1 - \sum_n \rho_n^*$ form an equilibrium. As \hat{P} is an equilibrium, from (2) we know that $\pi_n(\hat{P}) \geq (1 - \delta)\pi(\hat{P})$ for all n . But, $\rho_n^* \pi(P^i) = \pi_n(\hat{P})$ by construction and $\pi(\hat{P}) > \pi(P^i)$ for $\hat{P} \in (P^i, P^r]$. Hence $\rho_n^* \pi(P^i) > (1 - \delta)\pi(P^i)$ for all n and no non-integrated firm, $n = 1, \dots, N$ will deviate. Similarly, by (1) with P^i replacing P on the right-hand-side,

$$\pi_{N+1}(\hat{P}) + [a - A]Q(\hat{P}) \geq (1 - \delta)\Pi(P^i) + \delta[a - A]Q(a + c)$$

But, as $\Pi(P^i) > \Pi(\hat{P})$ and $\rho_n^* \pi(P^i) = \pi_n(\hat{P})$ for all $n = 1, \dots, N$ so that $(1 - \sum_n \rho_n^*)\pi(P^i) + [a - A]Q(P^i) > \pi_{N+1}(\hat{P}) + [a - A]Q(\hat{P})$. Hence,

$$(1 - \sum_n \rho_n^*)\pi(P^i) + [a - A]Q(P^i) > (1 - \delta)\Pi(P^i) + \delta[a - A]Q(a + c)$$

and the integrated firm $N + 1$ will not deviate. As no firm will deviate, the putative equilibrium is an actual equilibrium, and this equilibrium Pareto dominates the original equilibrium at price \hat{P} . ■

Proof of lemma 5: We can rewrite (1) and (2) respectively as

$$\pi_{N+1}(P) \geq \pi(P) - \delta(\pi(P) - [a - A][Q(a + c) - Q(P)]) \quad (9)$$

$$\forall_{n \in \{1, \dots, N\}} \quad \pi_n(P) \geq \pi(P) - \delta\pi(P) \quad (10)$$

But $\pi(P) = \pi_{N+1}(P) + \sum_n \pi_n(P)$ so by (9) and (10)

$$\pi(P) \geq \pi(P) - \delta(\pi(P) - [a - A][Q(a + c) - Q(P)]) + N\pi(P) - \delta N\pi(P)$$

Simplifying, this means that P is sustainable as an equilibrium iff

$$\delta \geq \frac{N\pi(P)}{(N + 1)\pi(P) - [a - A][Q(a + c) - Q(P)]} \quad (11)$$

Substitution shows that (11) is identical to $\delta \geq \tilde{\delta}(P, a)$. ■

Proof of lemma 6: For (i), note that $\tilde{\delta}(P, a)$ can be written as

$$\tilde{\delta}(P, a) = \frac{N\pi(P)}{N\pi(P) + \Pi(P) - \Pi(a + c)}$$

Taking the derivative of this expression, the sign of $(\partial\tilde{\delta}(P, a)/\partial P)$ will be the same as the sign of $N\pi'(P)[\Pi(P) - \Pi(a + c)] - N\Pi'(P)\pi(P)$. But, $[\Pi(P^i) - \Pi(a + c)] > 0$ and $\Pi'(P^i) = 0$ while $\pi'(P^i) > 0$ so that $(\partial\tilde{\delta}(P^i, a)/\partial P) > 0$.

For (ii), from the equation for $\tilde{\delta}(P, a)$ given in lemma 5,

$$\begin{aligned} \frac{\partial\tilde{\delta}(P, a)}{\partial N} &= \frac{\pi(P)[N\pi(P) + \Pi(P) - \Pi(a + c)] - N\pi(P)\Pi(P)}{((N + 1)Q(P)[P - a - c] - [a - A][Q(a + c) - Q(P)])^2} \\ &= \frac{\Pi(P) - \Pi(a + c)}{((N + 1)Q(P)[P - a - c] - [a - A][Q(a + c) - Q(P)])^2} \\ &> 0 \text{ for all } P > a + c \end{aligned}$$

For (iii), from the equation for $\tilde{\delta}(P, a)$ given in lemma 5,

$$\begin{aligned} \frac{\partial\tilde{\delta}(P, a)}{\partial a} &= \frac{[Q(a + c) - Q(P)][P - a - c]NQ(P)}{((N + 1)Q(P)[P - a - c] - [a - A][Q(a + c) - Q(P)])^2} \\ &> 0 \text{ for all } P > a + c \end{aligned}$$

For (iv), taking the derivative of $\tilde{\delta}(P, a)$ with regards to P , it is easy to confirm that if for all $P > a + c$

$$-\pi(P)(a - A)Q'(P) - (\pi'(P)(a - A)[Q(a + c) - Q(P)]) < 0 \quad (12)$$

then $(\partial\tilde{\delta}(P, a)/\partial P) > 0$. Substitution and simplification shows that (12) is equivalent to

$$\forall_{P>a+c} \quad Q(a+c)Q'(P)[P-a-c] + [Q(a+c) - Q(P)]Q(P) < 0$$

To see that this always holds under assumption two, let $z(P) = Q(a+c)Q'(P)[P-a-c] + [Q(a+c) - Q(P)]Q(P)$. Then, $z(a+c) = 0$ and $z'(P) = Q(a+c)Q''(P)(P-a-c) + 2Q'(P)[Q(a+c) - Q(P)] < 0$ if $Q''(P) \leq 0$. So if $Q''(P) \leq 0$ then $z(P) < 0$ for all $P > a+c$. ■

Proof of lemma 7: Let \mathcal{N} refer to the subset of firms as well as the number of firms in the subset and \mathcal{N}^c refer to the set of all non-integrated firms not in the set \mathcal{N} . By (1) and (2) the profit maximizing equilibrium price for a subset of \mathcal{N} non-integrated firms is given by the solution to

$$\max_P \sum_{n \in \mathcal{N}} \pi_n(P)$$

subject to

$$\pi_{N+1}(P) \geq (1-\delta)\pi(P) + \delta[a-A][Q(a+c) - Q(P)]$$

and

$$\forall_{n \in \mathcal{N}^c} \quad \pi_n(P) \geq (1-\delta)\pi(P)$$

By substitution and differentiation, the solution to this problem is given by $P_r^*(\mathcal{N})$ such that

$$\pi'(P_r^*(\mathcal{N}))[1 - (N+1-\mathcal{N})(1-\delta)] + \delta[a-A]Q'(P_r^*(\mathcal{N})) = 0 \quad (13)$$

But this is identical to (3). Further, it is easy to confirm that the second order conditions for this solution are satisfied under assumption 2.

To see that $P_r^*(\mathcal{N}) < P^i$ when $\mathcal{N} < N$ but $P_r^*(N) = P^i$ note that we can rewrite (13) as

$$\delta\Pi'(P) - (N-\mathcal{N})(1-\delta)\pi'(P) = 0 \quad (14)$$

When $\mathcal{N} = N$, (14) becomes $\delta\Pi'(P) = 0$ which has a unique solution at $P = P^i$. When $\mathcal{N} < N$, $(N - \mathcal{N})(1 - \delta)\pi'(P) > 0$ at all $P < P^r$ so that $\Pi'(P_r^*(\mathcal{N})) > 0$ and $P_r^*(\mathcal{N}) < P^i$.

To see that $P_r^*(\mathcal{N}) > c + a$ for all \mathcal{N} , note that if $P_r^*(\mathcal{N}) \leq c + a$ then $\sum_{n \in \mathcal{N}} \pi_n(P_r^*) \leq 0$. But this clearly cannot be the optimal price as $\sum_{n \in \mathcal{N}} \pi_n(P^i) > 0$ for $a + c < P^i$. ■

Proof of lemma 8: Let \mathcal{N} refer to the subset of firms as well as the number of non-integrated firms in the subset and \mathcal{N}^c refer to the set of all non-integrated firms not in the set \mathcal{N} . By (1) and (2) the profit maximizing equilibrium price for a subset of firms including the integrated firm and \mathcal{N} non-integrated firms is given by the solution to

$$\max_P [a - A]Q(P) + \sum_{n \in \mathcal{N}} \pi_n(P)$$

subject to

$$\forall_{n \in \mathcal{N}^c} \pi_n(P) \geq (1 - \delta)\pi(P) \text{ and } \forall_n \pi_n(P) \geq 0$$

By substitution and differentiation, the solution to this problem is given by the price P such that

$$\pi'(P)[1 - (N - \mathcal{N})(1 - \delta)] + [a - A]Q'(P) = 0 \quad (15)$$

if this is greater than $a + c$. But this is identical to (4). Further, it is easy to confirm that the second order conditions for this solution are satisfied under assumption two. So long as the solution to (4) is no less than $a + c$ then this price is $P_i^*(\mathcal{N})$.

To see that $P_i^*(\mathcal{N}) < P^i$ when $\mathcal{N} < N$ but $P_i^*(N) = P^i$ note that we can rewrite (15) as

$$\Pi'(P) - (N - \mathcal{N})(1 - \delta)\pi'(P) = 0 \quad (16)$$

When $\mathcal{N} = N$, (16) becomes $\Pi'(P) = 0$ which has a unique solution at $P = P^i$. When $\mathcal{N} < N$, $(N - \mathcal{N})(1 - \delta)\pi'(P) > 0$ at all $P < P^i$ so that $\Pi'(P_i^*(\mathcal{N})) > 0$ and $P_i^*(\mathcal{N}) < P^i$.

To see that the solution to (4) is strictly greater than $c + a$ for all \mathcal{N} , note that the left hand side of (16) is increasing in δ for any $P \leq P^i$. Also, note that by LeHopital's rule, $\tilde{\delta}(a + c, a) = \frac{N\pi'(a+c)}{N\pi'(a+c) + \Pi'(a+c)}$. Under assumption 2, $\tilde{\delta}(a + c, a) < \tilde{\delta}(P^i, a)$ and by assumption, $\delta > \tilde{\delta}(P^i, a)$. So if the left hand side of (16) exceeds zero for $\delta = \tilde{\delta}(a + c, a)$ and $P = a + c$, it follows that $P_i^*(\mathcal{N}) > a + c$. But this is easily confirmed. Substitution of $\tilde{\delta}(a + c, a)$ and $P = a + c$ into the left-hand-side of (16) yields

$$\frac{(\Pi'(a + c))^2 + \mathcal{N}\pi'(a + c)\Pi'(a + c)}{N\pi'(a + c) + \Pi'(a + c)} > 0$$

Finally, it is necessary to confirm that the price $P_i^*(\mathcal{N})$ with the relevant division of profits is sustainable as an equilibrium. But this directly follows from the assumption that $\delta > \tilde{\delta}(P^i, a)$ so that all prices $P > a + c$ are sustainable as equilibria, and from construction as all firms not in set \mathcal{N} are held to their minimum profit levels that guarantee that they will not deviate. Thus, there is some division of profits among the firms in set \mathcal{N} that ensures that the price $P_i^*(\mathcal{N})$ is an equilibrium. ■

References

- Armstrong, M., Cowan, S. and Vickers, J. (1994) *Regulatory reform: economic analysis and the British experience*, MIT Press, Cambridge, MA.
- Brennan, T. (1987) "Why regulated firms should be kept out of unregulated markets: understanding divestiture in United States v. AT& T", *Antitrust Bulletin*, 32, 741-93.

- Economides, N. (1998) “The incentive for non-price discrimination by an input monopolist”, *International-Journal-of-Industrial-Organization*; 16, 271-84.
- Friedman, J. (1971) “A non-cooperative equilibrium for supergames”, *Review of Economic Studies*, 38, 1-12.
- Gans, J. and De Fontenay, C. (2000) “Extending market power through vertical integration”, mimeo, University of Melbourne, Australia.
- Geroski, P., Thompson, D. and Toker, S. (1989) “Vertical separation and price discrimination: cellular phones in the UK”, *Fiscal Studies*, 10, 83-103.
- Gilbert, R. and Riordan, M. (1995) “Regulating complementary products: a comparative institutional analysis”, *Rand-Journal-of-Economics*, 26, 243-56.
- King, S. and Maddock, R. (1996) *Unlocking the infrastructure: the reform of public utilities in Australia*, Allen and Unwin, Sydney.
- Kuhn, K. and Vives, X. (1999) “Excess entry, vertical integration and welfare”, *RAND Journal of Economics*, 30, 575-603.
- Laffont, J. and Tirole, J. (2000) *Competition in telecommunications*, MIT Press, Cambridge, MA.
- McAfee, R.P. and Schwartz, M. (1994) “Opportunism in multilateral vertical contracting: nondiscrimination, exclusivity, and uniformity”, *American Economic Review*, 84, 210-30.

- Noll, R. and Owen, B. (1989) "The anticompetitive uses of regulation: United States v. AT& T" in J. Kwoka and L. White (eds) *The antitrust revolution*, Scott, Foresman and Company, Glenview, Illinois.
- Rey, P. and Tirole, J. , (1996) "A primer on foreclosure", mimeo, IDEI, France
- Sibley, D. and Weisman, D. (1998) "Raising rivals' costs: the entry of an upstream monopolist into downstream markets", *Information economics and policy*, 10, 451-470.
- Tirole, J. (1988) *The Theory of Industrial Organization*, MIT Press, Cambridge, MA.
- Vickers, J. (1995) "Competition and regulation in vertically related markets", *Review of Economic Studies*, 62, 1-17.
- Vickers, J. and Yarrow, G. (1988) *Privatization, an economic analysis*, MIT Press, Cambridge, MA.