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Corporation Tax Asymmetries: Effective Tax Rates and Profit Shifting

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Abstract

This paper examines the way in which the asymmetric treatment of losses within corporate tax codes can be expected to affect behavioural responses to changes in tax rates. The paper introduces the concept of an equivalent tax function, raising the same present value of tax payments as the actual function, in which the effective rate on losses in any period, and thus the degree of asymmetry, is explicit. The influence on the elasticity of tax revenue with respect to the tax rate of this effective rate is then examined, where 'loss-shifting' occurs. Results suggest that estimates of the behavioural effect of changes in tax rates on tax revenues can be expected in general to be smaller in regimes that involve greater asymmetries in the tax treatement of losses. As losses vary over the economic cycle, asymmetric treatment also generates effects on tax revenues that are asymmetric (non-linear) between above-trend and below-trend parts of the cycle.

*The views, opinions, findings and conclusions or recommendations expressed in this paper are strictly those of the authors. They do not necessarily reflect the views of the New Zealand Treasury.

1 Introduction

The corporation tax treatment of losses typically involves an asymmetry in the tax function in that losses do not give rise to a tax rebate equivalent to the tax on positive profits. Often losses can only be used concurrently if they can offset positive profits from other sources within the corporation, or across members of a group of 'associated' corporations defined for tax purposes.[1] Alternatively they can be carried forward or back to be used as profit offsets in future or previous periods.[2] The overall effect of these restrictions is that some losses become 'stranded', even if temporarily, such that the effective 'rate of rebate' on losses is less than the rate of tax on profits. These sources of asymmetry apply, for example, to the UK and US corporation tax regimes, leading Cooper and Knittel (2007, p. 651) to conclude for the US that, 'many tax losses are used with a substantial delay' so that 'certain firms and industries suffer a significant penalty from the partial loss refund regime due to the erosion in the real value of their loss refund'. They also report (2007, p. 651) that up to 50 per cent of corporate losses remain unused after 10 years, and that around 25-30 per cent of losses are never used.[3] For the UK, Klemm and McCrea (2002) report that Inland Revenue estimates suggest that in the UK a higher proportion of about 80 per cent of losses arising in 2000-01 were used as tax offsets in that same

[1] The terms 'corporation', 'company' and 'firm' are used interchangeably below. Donnelly and Young (2002, pp. 446-449) review the tax treatment of groups of companies in 30 OECD countries, of which two thirds allow some form of group relief.

[2] There are sometimes restrictions on flexibility when losses are used in this way. For example, in the UK's 'schedular' system some loss pools can be used only as offsets against the same source, whereas current losses can be used contemporaneously against profits from different sources. For multinational corporations, losses in overseas subsidiaries cannot be offset against profits arising in the UK.

[3] See also Office of Tax Policy (2007), which also compares policies in G-7 countries, and suggests that the asymmetry reduces the automatic stabilization properties of taxes and encourages uneconomic mergers, as well as affecting investment incentives. For some deductions against profits, such as allowances for investment expenditures, the effective rebate rate can exceed the rate of tax on profits if, for example, the capital allowance regime is designed to be especially generous, perhaps to encourage investment. The present paper is concerned only with loss asymmetries and hence ignores this possibility but the analysis below carries over in a straightforward fashion to the case where the asymmetry involves a more, rather than less, generous tax deduction.

year.[4] This asymmetry aquires particular importance in view of the large size of losses, amounting to about £80bn in the UK in 2000-1 and $418bn in the US in 2002.

The effect on investment behaviour of this asymmetry has been investigated by Auerbach (1986), Devereux (1989), Altshuler and Auerbach (1990), and more recently by Edgerton (2007). The importance of asymmetry for companies' average tax rates was stressed by Auerbach (2007), who showed that, *ceteris paribus*, restrictions on loss use have significantly increased US corporations' implicit average tax rates above statutory rates, especially during cyclical downturns. He also argued that, although US corporations have generally been making greater losses since the early 1980s, they have been using them less to offset their corporate tax liabilities, so raising their effective average tax rates. This led Auerbach to cast doubt on arguments that US corporates are increasingly avoiding tax by engaging in international profit-shifting.

This paper provides an analysis of how the asymmetric treatment of losses can be expected to affect companies' behavioural responses to changes in tax rates, as measured by the elasticity of tax revenue with respect to the tax rate. It introduces the concept of an equivalent tax function, raising the same present value of tax payments as the actual function, but in which the effective rate on losses in any period, and thus the degree of asymmetry, is explicit. The influence of this effective rate on the variation in the elasticity of tax revenue with respect to the corporate tax rate over the business cycle is then examined.[5]

This tax revenue elasticity has been the focus of a number of empirical studies of corporate profit-shifting, such as Demirgüç-Kunt and Huizinga (2001), Bartelsman and Beetsma (2003) and Huizinga and Laeven (2007).[6] Responses to corporation tax changes can take two forms. First, there are

[4] See Devereux (1989) for earlier evidence on the extent of losses in the UK.

[5] Cooper and Knittel (2006, p. 663) concluded that, 'Future work should examine how the time delay in loss utilization manifests itself in marginal tax rate discrepancies between industries and over time'.

[6] In some case these authors report semi-elasticities of tax revenue, or taxable profits, with respect to the tax rate. For recent evidence relating to Europe, see Dischinger (2007).

real responses, whereby activities are transferred to other tax jurisdictions. The second form of response involves income-shifting in which the location of economic activity is unchanged but the extent to which profits and deductions are declared in the home country changes, for example through the use of transfer pricing. The present paper does not separate these responses but shows that overall behavioural responses of tax revenue to tax rate changes depend crucially on the extent of the tax function asymmetry. Since losses can be expected to vary over the economic cycle, it is shown that the asymmetric treatment of losses generates effects on tax revenue elasticities that are asymmetric between above-trend and below-trend parts of the cycle.

Section 2 considers the tax treatment of losses in the context of a single corporation and provides a precise measure of the asymmetry involved. While concentration on a single firm is somewhat artificial in this context, it avoids the unnecessary complications arising from modelling a distribution of firms' profits and losses, and focuses on the key asymmetry that applies to individual corporations.[7] Section 3 demonstrates the role of this tax asymmetry in the analysis of the behavioural responsiveness of revenue to tax rate changes. Section 4 illustrates the magnitude of the elasticity of tax revenue with respect to the tax rate during periods of above-trend and below-trend growth, comparing symmetric and asymmetric loss treatments. It is shown that the asymmetric treatment of losses reduces size of the behavioural component of the tax revenue elasticity compared with their symmetric treatment. It also gives rise to asymmetric behaviour of the elasticity over the business cycle which becomes more pronounced as company profits (net of losses) move further below trend, but not when they move further above trend. Conclusions are in Section 5.

[7] For an extensive microsimulation analyis of the effects on potential behavioural responses to corporation tax rate changes in the UK, see Creedy and Gemmell (2007). This allows for the tax minimising use of losses within corporations and between them, that is within groups defined for tax purposes.

2 Taxes and Loss Asymmetries

This section provides a framework for examining the value of losses as profit offsets in the context of a single corporation with more than one profit source. Subsection 2.1 considers the relationship between losses and deductions, and subsection 2.2 provides some numerical illustrations. The approach involves defining the value of losses as tax deductions in present value terms under the alternatives of symmetric and asymmetric treatment. This allows an effective tax rate on losses to be derived.

2.1 Losses and Deductions

Consider a firm obtaining profits from several sources. Some of these generate positive profits in time period j with a total of P_j. Other sources produce total losses during j of L_j. To focus on the asymmetric treatment of losses, assume that losses are the only deductions allowable against profits in determining corporate tax liabilities. The typical tax function can be described as having a constant marginal tax rate, t, applied to taxable profits.[8] The *actual* tax liability that a corporation faces in period j, T_j^A, is given by the function:

$$T_j^A = t\left(P_j - D_j\right) \tag{1}$$

where D_j is the value of loss deductions offset in period j, such that:

$$D_j = \min(P_j, L_j + L_j^P) \tag{2}$$

where L_j^P represents the 'loss pool' carried over from the previous period, consisting of unused earlier losses.[9] Any losses in excess of P_j are then carried forwards (or backwards) to be offset in future (or previous) periods, depending on the time profile of past and future profits and losses.

The restriction in (2) produces the fundamental corporation tax asymmetry. Whereas the tax liability for each additional unit of profit generated

[8] In the UK there is a lower rate band and other bands where the marginal rate varies, but the vast majority of tax is paid at the single higher rate. On the details of the UK tax function, see Creedy and Gemmell (2008).

[9] In practice some companies may not claim all the tax loss deductions to which they are entitled. This complication is ignored here.

in period j is t, the negative tax liability on each extra unit of losses generated in j is not $-t$. Rather, it depends on the future or past periods, if any, for which tax code restrictions allow these losses to be used as tax offsets. For this reason it is important to compare the tax liabilities associated with profits and losses generated in a given period in present value terms. For example, if limited future profits mean that some period j losses are never used as offsets for tax purposes then the tax (or rebate) rate on those losses is effectively zero. On the other hand, if all period j losses can be used to reduce future, but not current, tax liabilities, they have a tax liability in present value terms that is greater than $-t$ (that is, an effective rebate rate of less than t). The precise magnitude depends on the relevant time profile of loss offsets and the associated discount rate. The relevant tax rate on losses therefore lies between $-t$ and 0.

The effective tax rate applied to losses at any time is therefore not transparent from (1), as the present value of losses as tax offsets does not appear in the tax function. However, it is the effective rate that is required in order to examine the incentives facing firms regarding their real and profit-shifting responses to tax changes. Any current loss involves a time stream of tax offsets which needs to be captured in some way. The following analysis produces an appropriate framework. First, it is convenient to ignore the 'carry back' of losses, as this does not affect the argument. Also uncertainty regarding future profits and losses is ignored: the fundamental issue concerns the nature of the time profiles rather than uncertainty.

The objective is to construct an alternative tax function which collects the same present value of tax revenue from the firm as the actual tax function, over the period during which the losses are able to be used as offsets. This time period is not necessarily fixed, though a time limit may be specified in the tax code.[10] The 'parameters' of the function, expressed in terms of a single-period tax schedule, need to encapsulate the use of the losses in such a way that allows subsequent analysis to trace how the effective tax rate varies for different assumptions about the time profile of profits and the nature of

[10] Donnelly and Young (2002, p. 448) summarise the time restrictions, along with provisions for group allowances, in OECD countries.

the asymmetric treatment of losses.

Define q_j as the proportion of losses arising in period j which are used as a deduction in period j. Furthermore, define q_{j+k}, as the proportion of the j losses used as tax offsets in period $j+k$. These proportions are determined by the firm's profits and losses in period j and future profits and losses over periods $k = 1, 2, ...$ and so on. For example, if the firm expects a profit in $j+k$ of P_{j+k}, and it expects to be able to use some of j's losses (depending on their earlier use) to offset part of those profits, say an amount equal to $P^*_{j+k} < P_{j+k}$, then $q_{j+k} = P^*_{j+k}/L_j$.[11]

Let s_j denote the present value at j, of period j's losses as tax off-sets, as a proportion of their nominal value L_j, over the relevant period. Letting r denote the discount rate, s_j is given by:

$$s_j = q_j + \frac{q_{j+1}}{1+r} + \frac{q_{j+2}}{(1+r)^2} + ... \tag{3}$$

where summation is over the relevant time horizon, say K.[12] Thus:

$$s_j = \sum_{k=0}^{K} \frac{q_{j+k}}{(1+r)^k} \tag{4}$$

where $0 < s_j < 1$. The first term q_j $(k = 0)$, is simply, as defined above, the proportion of losses arising in j that are used in period j. Each subsequent term, for periods $j+k$, where $k = 1, ..., K$, reflects the suitably discounted proportions of period j losses which are used in periods $j+k$.

The corporation's tax liability in period j, T_j, can therefore be defined using an alternative function given by:

$$T_j = t\,(P_j - s_j L_j) \tag{5}$$

where t is the proportional tax rate as defined in the actual tax function, and there is no restriction preventing a rebate in j if $s_j L_j > P_j$. It can be shown

[11] It is appropriate here to assume that earlier losses, if any exist, are used before current losses. In practice, as mentioned above, loss pools may be less flexible than current lossses: see Creedy and Gemmell (2007).

[12] As mentioned earlier, the time horizon itself depends both on the tax code and the firm's time profile of profits and losses. See Devereux (1989, p.105) for a comparable expression for the effective tax rate associated with capital allowances.

that the present value of tax payments is the same under (5) as with the actual tax function. The term s_j may be referred to as the loss 'deductions rate'.[13] The company's effective tax rate on losses in j is therefore $-s_j t$. In the case of *symmetric* tax treatment of profits and losses, $s_j = 1$. The asymmetry is demonstrated in Figure 1 where the tax function in (5), T_j, is the solid line. The symmetric case is represented by a linear function, such that tax and rebate schedules are mirror images. The actual tax function (1) follows the horizontal axis until $P_j > D_j$, after which it slopes upwards at the rate, t.

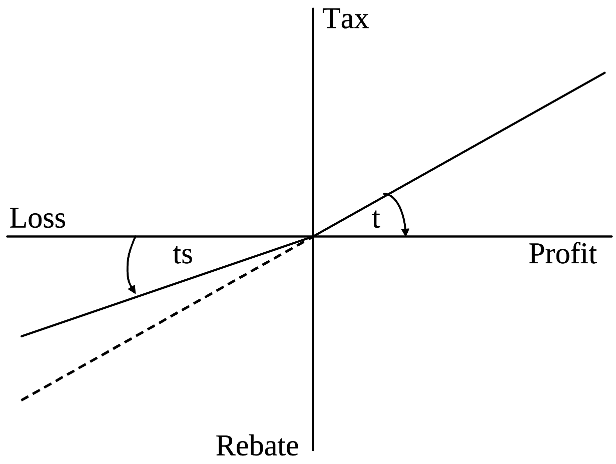

Figure 1: The Asymmetric Treatment of Losses

The value of s_j can be related to Auerbach's (2007) comparison of the statutory tax rate, t, and implicit average tax rate, τ, faced by companies. Define the implicit average tax rate in period j as $\tau_j = T_j/(P_j - L_j)$. It is then readily shown from (5) that:

$$\frac{\tau_j}{t} = \frac{P_j - s_j L_j}{P_j - L_j} \tag{6}$$

Furthermore, defining the ratio of net profit to positive profit as $\theta_j = (P_j -$

[13] By defining the deductions value of losses in this way, the average and marginal 'rates of deduction', s, are assumed to be the same. This could be relaxed (for example, by defining $D = s_0 + sL$, where D is the deductions value of losses) but the simpler form here is sufficient to demonstrate the impact of the asymmetries of interest.

$L_j)/P_j$, for $P_j > 0$, this becomes:

$$\frac{\tau_j}{t} = \frac{1 - s_j(1-\theta_j)}{\theta_j} \tag{7}$$

When the function is symmetric $s_j = 1$ and $\tau_j = t$. The result in (7) demonstrates the combined impact on the ratio of τ_j to t of s_j and the economic cycle, since $1 - \theta_j$ captures the ratio of company losses to profits. In particular, τ_j/t is expected to rise during cyclical downturns, as observed by Auerbach for the US. An additional effect arising when there is a positive correlation between s_j and θ_j, is discussed in section 3.

2.2 Numerical Illustrations

Consider a simple example where a firm undertakes only one type of activity over several periods. Hence if $P_j > 0$ it must be true that $L_j = 0$, and conversely if $L_j > 0$ then $P_j = 0$. Let the tax code dictate that any losses which cannot be used immediately can only be carried forward and used against future profits within the firm. This and the following example assume that losses cannot be carried back to earlier periods, or across firms.

Assume further that if losses are not used as tax off-sets within three years, they will not be usable at all, so that $K = 3$. This helps to illustrate stranded losses. The firm's actual profit in period j and its expected profits and losses, for $k = 1, ..., 3$, are shown in Table 1. There is a loss in only period j and positive profits in $j + k$, so in this example concern is simply with the value of s_j (all $s_{j+k} = 0$).

Table 1: Example 1

Period	j	$j+1$	$j+2$	$j+3$
Profits/losses	-500	100	100	100
Losses used	0	100	100	100
Losses available	500	400	300	200

The example shows that a loss of \$500 in period j is used to offset profits of \$100 in each of three subsequent years. Given the time limitation, 200 of period j's lossses are 'stranded'. Hence $q_j = 0$ and $q_{j+k} = 100/500 = 1/5$

for $k = 1, ..., 3$. Using a discount rate of 10 per cent, s_j is calculated, by substituting in (3), as:

$$\begin{aligned} s_j &= 0 + \left(\frac{1}{5(1.1)}\right) + \left(\frac{1}{5(1.1)^2}\right) + \left(\frac{1}{5(1.1)^3}\right) \\ &= 0 + 0.182 + 0.165 + 0.150 \\ &= 0.497 \end{aligned} \tag{8}$$

Here $s_j < 1$ because of the fact that, by assumption, losses in period j cannot be used beyond $j + 3$ and because their use in future profit-making periods must be discounted to produce the present value. It can be seen that the present value of tax liabilities for the four years under the equivalent function in (5), T_j, is:

$$\begin{aligned} T_j &= t\left\{-500(0.497) + 100/1.1 + 100/1.1^2 + 100/1.1^3\right\} \\ &= 0 \end{aligned} \tag{9}$$

which is the same as the actual tax payments, T_j^A, of zero under an asymmetric structure, allowing losses to be carried forward for only three periods.

Consider a second example, in which there are two profit schedules, A and B. Source A makes positive profits in each period while source B makes losses in each period. These B losses are allowed to be carried forward and offset against A profits. The firm is assumed to use the earliest available vintage of losses first. Rows 2 and 3 of Table 2 show the stream of A profits and B losses over 5 periods. Row 4 shows the losses used within each period, with those brought forward from previous periods. Losses carried forward to the next period are shown in row 5. The time profiles of profits and losses are such that there are no losses in period 5; hence $s_5 = 0$.

Unlike the earlier simple case, this example shows values of s_j for $j = 1, ..., 5$. In each case, the profiles of profits and losses are such that losses in j are exhausted in $j + 1$, so only two terms are needed in obtaining the sum in equation (4), and no losses are stranded. In each of periods 1 to 4 some losses are carried forward to be used in the next period, so they are discounted at $r = 0.1$. The values carried forward are shown in bold. Consider the case of

Table 2: Example 2

1	Period (j)	1	2	3	4	5
2	A profits	20	20	10	10	10
3	B losses	25	20	15	10	0
4	B losses used	20	$\mathbf{5}+15$	$\mathbf{5}+5$	**10**	**10**
5	B losses c/f	5	5	10	10	0
6	s_j	0.982	0.977	0.939	0.909	0

$j = 1$, where $q_j = 20/25$ and $q_{j+1} = 5/25$. Substitution in (3) gives:

$$s_1 = \frac{20}{25} + \left(\frac{5}{25}\right)\left(\frac{1}{1.1}\right) = 0.982 \tag{10}$$

The values of s_j for subsequent periods are shown in the last row of the table. It can be seen that s_j falls from period 1 and is zero in period 5 when losses are zero. The declining value of s_j reflects the fact that in each successive period an increasing fraction of each period's losses are carried forward before they are used as deductions: these are: 5/25; 5/20; 10/15; and 10/10. Hence a greater fraction of losses are subject to discounting. In this second example, based on the actual tax function the firm pays no tax in each of the five periods. Again, this is the same as the present value of tax liabilities in the equivalent function (5) for the five periods, since:

$$\begin{aligned} \frac{T_j}{t} &= \{20 - 25\,(0.982)\} + \frac{\{20 - 20\,(0.977))\}}{1.1} \\ &\quad + \frac{\{10 - 15\,(0.939)\}}{1.1^2} + \frac{\{10 - 10\,(0.909)\}}{1.1^3} + \frac{10}{1.1^4} \\ &= 0 \end{aligned} \tag{11}$$

As mentioned above, these examples exclude loss carry back. If this option were available the firm could effectively offset previous profits immediately (that is, in period j), such that if x is the value of previous profits available to be offset, q_j becomes x/L_j, subject to $x \leqslant L_j$.[14]

Although the precise value of s_j has been seen to depend on the time profile of the firm's profits and losses, the specification of the equivalent tax

[14]There is no need to discount losses carried back since the tax code implies that period j losses are used concurrently, although in accounting terms they are offsetting previous profits.

function using this concept of asymmetry has the advantage that it allows valuable insights to be obtained into the determinants, and possible pattern over the business cycle, of the elasticity of revenue with respect to the tax rate, without having to specify the full time profile of profits.[15] This is explored in the following sections.

3 Behavioural Responses to Tax Changes

The question of interest here is how the asymmetry in tax loss treatment described in the previous section, and summarised by s_j, affects the behavioural response of tax revenue to changes in the corporate tax rate. The analysis in this section considers an individual firm, though this does not preclude the possibility that the losses available to that firm as tax offsets are generated by a partner firm, to the extent that the tax code permits such loss-sharing. The basic elasticity describing the behavioural responses to a tax rate change is discussed first in subsection 3.1 and the effects of changes in net profits over the business cycle are examined in subsection 3.2.

3.1 Profit Shifting and the Tax Rate

Consider a taxpaying firm for whom $P_j > s_j L_j$. From equation (5), defining $P_j^T = P_j - s_j L_j$ as taxable profit, so that $T_j = tP_j^T$, the effect on T_j of a change in the tax rate, t, is (omitting time subscripts for convenience):

$$\frac{dT}{dt} = P^T + t\frac{dP^T}{dt} \tag{12}$$

where dP^T/dt measures the extent of the corporation's response to the tax rate change. This includes any induced 'real' changes in profit levels arising from a change in location and profit shifts into or out of the tax jurisdiction. Dividing both sides of (12) by P^T and using the fact that $\frac{dT}{P^T dt} = \frac{tdT}{Tdt} = \eta_{T,t}$:

$$\eta_{T,t} = 1 + \eta_{P^T,t} \tag{13}$$

[15] However, the distribution of effective rates across firms could be obtained using the simulation model discussed in Creedy and Gemmell (2007). Effective rates were obtained, using simulation methods, by Myers and Majd (1986).

This uses the general notation $\eta_{x,y}$ to denote the elasticity of x with respect to y. The term $\eta_{P^T,t} \leq 0$ is the elasticity of taxable profits with respect to the tax rate, capturing possible behavioural responses. This elasticity is closely related to the Feldstein (1995, 1999) elasticity of taxable income with respect to the net-of-tax, or retention, rate, $\eta_{P^T,1-t}$, such that:

$$\eta_{P^T,t} = -\left(\frac{t}{1-t}\right)\eta_{P^T,1-t} \tag{14}$$

Taxable profit, P_j^T, changes if either gross profits, P_j, or losses, L_j, alter in response to tax rate changes. It is useful for present purposes to define net profits as $N_j = P_j - L_j$ and work with the ratio of net profits to gross profits, θ_j, defined above. This allows the effects of the economic cycle to be examined by specifying systematic changes in θ_j over time. For $P_j > 0$:

$$\theta_j = N_j/P_j \tag{15}$$

Losses can therefore be written as $L_j = (1-\theta_j)\,P_j$. Substituting for L_j in the equivalent tax function of equation (5) allows the tax liability to be re-written as:

$$T_j = t\left\{1 - s_j(1-\theta_j)\right\}P_j \tag{16}$$

Clearly, $\theta_j \leq 1$, but if $L_j > P_j$, θ_j can be negative. When $\theta_j < 0$, it is nevertheless possible to have $s_j(1-\theta_j) < 1$ if s_j is sufficiently low, so that a positive tax payment is required in the equivalent function in period j even if losses exceed positive profits in the corporation.

However, s_j is a function of current and expected future profits and losses, and hence $s_j = s_j(\theta_j)$, with $ds_j/d\theta_j > 0$.[16] That is, a higher θ_j, implying lower reported losses for a given level of profits, raises the prospect that a given $ of loss is used, or used sooner, to offset profits. In addition, to the extent that there is a behavioural response of declared losses to the tax rate, $\theta_j = \theta_j(t)$, where it is expected that $d\theta_j/dt_j < 0$ if increased tax rates encourage lower declared profits, or higher declared losses.

[16] Strictly, allowing for future profits and losses, $s_j = s_j(\theta_{j+k})$, $k = 0, ..., K$. To keep the analytics simple, the additional complications arising from the effects on s_j of future values of θ are ignored below.

3.2 Tax and Net Profit Changes

In order to focus on the asymmetric impact of losses, the following analysis treats profits as fixed, that is, independent of the tax rate.[17] Thus, emphasis is on the response of losses to tax rate change. In general, and omitting j subscripts for convenience, totally differentiating $T = T(t, \theta, s)$ and remembering that $s = s(\theta)$, gives:

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial \theta}\frac{d\theta}{dt} + \frac{\partial T}{\partial s}\frac{\partial s}{\partial \theta}\frac{d\theta}{dt} \tag{17}$$

Multiplying both sides by t/T, and using the fact that the partial elasticity, $\frac{t\partial T}{T\partial t}$, is equal to 1 for a proportional tax structure, allows the elasticity, $\eta_{T,t}$, to be written as:

$$\eta_{T,t} = 1 + \eta_{T,\theta}\eta_{\theta,t} \tag{18}$$

where $\eta_{T,\theta}$ is:

$$\eta_{T,\theta} = \eta'_{T,\theta} + \eta'_{T,s}\eta'_{s,\theta} \tag{19}$$

The elasticities on the right-hand-side of (19), indicated by a prime, are partial elasticities associated with the partial derivatives in (17). The elasticity $\eta_{\theta,t}$ captures behavioural responses to tax rate changes associated with changes in losses, with an expected sign of $\eta_{\theta,t} < 0$. Thus an increase in the tax rate is expected to encourage a decrease in θ, reflecting an increase in losses declared for tax. This can arise either because real losses rise or losses generated elsewhere are shifted into the tax jurisdiction.

Other sign expectations are: $\eta'_{T,\theta} > 0$; $\eta'_{s,\theta} > 0$; and $\eta'_{T,s} < 0$. That is, a greater ratio of net-to-gross profits, θ, raises tax revenues directly and also raises s, since there are fewer losses competing to be offset against each unit of profit. However, the greater deductability of losses arising from a larger value of s reduces tax revenue.

Equation (19) demonstrates that any behavioural effect, captured by $\eta_{\theta,t}$, is transmitted into an effect on the tax revenue elasticity, $\eta_{T,t}$, via $\eta_{T,\theta}$, which in turn is determined by the sizes of $\eta'_{T,\theta}$ and $\eta'_{T,s}\eta'_{s,\theta}$. Any asymmetric effect

[17] Equivalent results in which either losses or θ is held fixed, while profits vary, can also be examined.

on $\eta_{T,t}$ of losses must therefore arise via the direct effect, $\eta'_{T,\theta}$, and the indirect effect, $\eta'_{T,s}\eta'_{s,\theta}$, on tax revenues from changes in θ.

Expressions for the two partial elasticities $\eta'_{T,\theta}$ and $\eta'_{T,s}$ in terms of s and θ can be obtained by differentiating (16), whereby:

$$\eta'_{T,\theta} = \frac{s\theta}{1 - s(1-\theta)} \tag{20}$$

and:

$$\eta'_{T,s} = \frac{-s(1-\theta)}{1 - s(1-\theta)} \tag{21}$$

If $s = 1$, so that the tax function is symmetric, (20) shows that $\eta'_{T,\theta} = 1$ and, since it must also be true in this case that $\eta'_{s,\theta} = 0$, (18) gives the simple result that:

$$\eta_{T,t} = 1 + \eta_{\theta,t} \tag{22}$$

Furthermore, if $\theta = 1$, so that the firm makes no losses from any source, a change whereby some losses are made (a fall in θ) implies $\eta'_{T,\theta} = s$.

Substituting (20) and (21) into (18) gives:[18]

$$\eta_{T,t} = 1 + \left[\frac{s\left\{\theta - (1-\theta)\eta'_{s,\theta}\right\}}{1 - s(1-\theta)}\right]\eta_{\theta,t} \tag{23}$$

The component elasticities in (20) and (21) capture the impact of any asymmetries. They are respectively positive and negative for $0 < \theta < 1$, but the signs of $\eta'_{T,\theta}$ and $\eta'_{T,s}$ can be reversed when $\theta < 0$ (when losses exceed profits) depending on the values of θ and s. Thus, the value of $\eta_{T,t}$ in (23) depends, *inter alia*, on the extent of tax loss asymmetries, s, the economic cycle as captured by the ratio of net-to-gross profits, θ, together with the impact on s of changes in θ, $\eta'_{s,\theta}$.

4 Asymmetry over the Business Cycle

This section examines the way in which the elasticity of tax revenue with respect to the tax rate is likely to vary over the business cycle, and in particular

[18] Furthermore, by comparison with (13), the second term in (23) is also $\eta_{PT,t}$.

how this depends on the degree of asymmetry, as measured by s. It proceeds by considering the cyclical pattern of a corporation's profits and losses as being represented as increases or decreases in θ around a trend value. The absolute size of the trend value of θ, which is likely to vary across firms, is not itself important. What matters is the behaviour of the component partial elasticities $\eta'_{T,\theta}$, $\eta'_{T,s}$, and $\eta'_{s,\theta}$ as θ moves upwards towards $\theta = 1$ and and downwards, including $\theta \leqslant 0$ (above-trend and below-trend respectively). As a point of reference, the examples below initially consider the case where the behavioural response, $\eta_{\theta,t}$, is fixed; that is, it does not vary as θ varies over the cycle. This allows variations in the elasticity $\eta_{T,t}$ to be related directly to variations in the total elasticity, $\eta_{T,\theta}$, and hence to θ for alternative s, for specific assumptions about the partial elasticity $\eta'_{s,\theta}$.

As a preliminary exercise, Figure 2 plots $\eta_{T,\theta}$ against θ for alternative values of s, for the case where $\eta'_{s,\theta} = 0$ in (23). In this extreme case, $\eta_{T,\theta} = \eta'_{T,\theta}$ and equation (20) can be used. Consider the range $\theta > 0$ for which net profits are positive. It has been shown that $\eta_{T,\theta} = 1$ for all θ in the symmetric case of $s = 1$: this is the top horizontal line in the figure. For $s < 1$, $\eta_{T,\theta}$ clearly increases as θ increases but the profile becomes flatter for relatively higher values of θ. When $\theta = 0$, the value of $\eta_{T,\theta}$ is zero for all values of s, and Figure 2 shows that the further is s below 1 (the greater the asymmetry in the tax code), the smaller is the value of $\eta_{T,\theta}$: the profiles pivot around the origin and at the same time become flatter for lower s.

Given the general result in (18) that $\eta_{T,t} = 1 + \eta_{T,\theta}\eta_{\theta,t}$, these results show that, for a given (negative) behavioural response, $\eta_{\theta,t}$, greater loss asymmetry (lower s) results in smaller tax responsiveness; that is $\eta_{T,\theta}$ is lower and so $\eta_{T,t}$ is closer to 1. This effect is more pronounced the smaller is θ. Figure 2, also shows that for lower values of $\theta > 0$, the profiles of $\eta_{T,\theta}$ are closer together; hence the effect on $\eta_{T,\theta}$ of differences in s diminishes as θ tends to zero. As a result, the precise degree of asymmetry – as measured by s – becomes less relevant in determining the revenue elasticity.

This latter point is important for estimates of behavioural responses in below-trend and above-trend situations. For the sake of illustration, assume that a trend value of θ is around 0.6. As the firm's performance moves

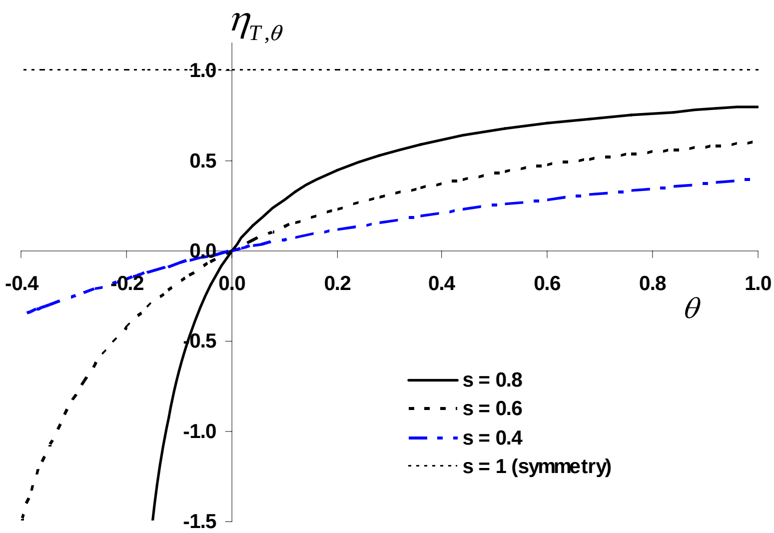

Figure 2: The Impact of Tax Loss Asymmetries

above-trend (towards $\theta = 1$) this has little *additional* impact on the revenue elasticity $\eta_{T,\theta}$. However, as the firm's performance moves below trend (towards or below $\theta = 0$), the impact on $\eta_{T,\theta}$ becomes magnified, given the concavity of the profiles, which must all go through the origin. Thus, moving into recession has an asymmetric effect on $\eta_{T,\theta}$ compared with movements into boom periods for a given value of s. In other words, $\eta_{T,\theta}$ declines by more when moving into recession than it increases when moving into a boom. As a result, for a given behavioural response, $\eta_{\theta,t}$, the same asymmetric effects will be observed for $\eta_{T,t}$, as is clear from (23). Perhaps counter-intuitively, Figure 2 also suggests that this asymmetry, or nonlinearity, between above-trend and below-trend changes in $\eta_{T,\theta}$, is *less* when the asymmetry in the treatment of losses is greater (smaller s). This reflects the fact that $\eta_{T,\theta}$ is reduced so much at all values of θ when s is low (see, for example, the profile for $s = 0.4$ in Figure 2), that the impact of *differences in* θ becomes of limited importance.

Figure 2 also shows values of $\eta_{T,\theta}$ for negative values of θ; that is, when losses exceed profits. Negative θ values can be associated with a tax liability when s is sufficiently small such that $P > sL$, even though $P < L$. The Figure shows that the asymmetry between below-trend and above-trend effects is magnified further when $\theta < 0$ is considered. However, larger values of s (smaller asymmetries in loss treatment) are associated with larger deviations from $\eta_{T,\theta} = 1$ (the symmetric case), and $\eta_{T,\theta}$ also becomes negative. These negative values of $\eta_{T,\theta}$ arise from the fact that when $\theta < 0$ the firm has a positive tax liability associated with its negative net profits. Reducing the size of s, *ceteris paribus* increases the range of negative net profits over which the firm has a positive tax liability, so that there is a less pronounced effect on $\eta_{T,\theta}$ (it is closer to zero) at a given negative value of θ.

The above results assume somewhat unrealistically that $\eta'_{s,\theta} = 0$. When $\eta'_{s,\theta} \neq 0$, then s, θ, and $\eta'_{s,\theta}$ cannot be set independently. It is also not approporiate to assume that $\eta'_{s,\theta}$ is constant, since the implied relationship between logarithms of s and θ is incompatible with $\theta < 0$. Instead, assumed that $ds/d\theta$ is constant. Thus, rearranging the term in square brackets in (23) gives:

$$\eta_{T,\theta} = \left[\frac{\theta\left\{s - (1-\theta)ds/d\theta\right\}}{1 - s(1-\theta)}\right] \tag{24}$$

The importance of the additional effect on $\eta_{T,\theta}$ of allowing for the effect on s of changes in θ can be seen in Figure 3 which shows the relation between $\eta_{T,\theta}$ and θ, using (24) and setting $ds/d\theta = 0.1$. To assist comparisons the maximum value of s, at $\theta = 1$ is set at $s = 0.8$, the equivalent fixed case also shown in the figure. This value of $ds/d\theta$ means that reductions in θ (increases in losses) modestly lower the firm's deductions rate, s, such that the present value of those losses, as tax deductions, falls.[19] This shows that the concavity (to the x-axis) of the relationship between $\eta_{T,\theta}$ and θ when s is fixed, is reduced. This reflects the fact that as θ falls due to rising losses, fewer of those losses can be used to reduce tax liability when s is variable compared to s fixed. As a result the highly non-linear effect on $\eta_{T,\theta}$ generated by decreases in θ when s is fixed is ameliorated by an opposing effect due to decreased s.[20]

[19] For example, with $ds/d\theta = 0.1$, and $s = 0.8$ at $\theta = 1$, s falls to 0.7 at $\theta = 0$.

[20] It can be shown, for this variable s case, that if the maximum s (at $\theta = 1$) is lower, the concavity of the relationship between θ and $\eta_{T,\theta}$ can even be reversed. For example, at

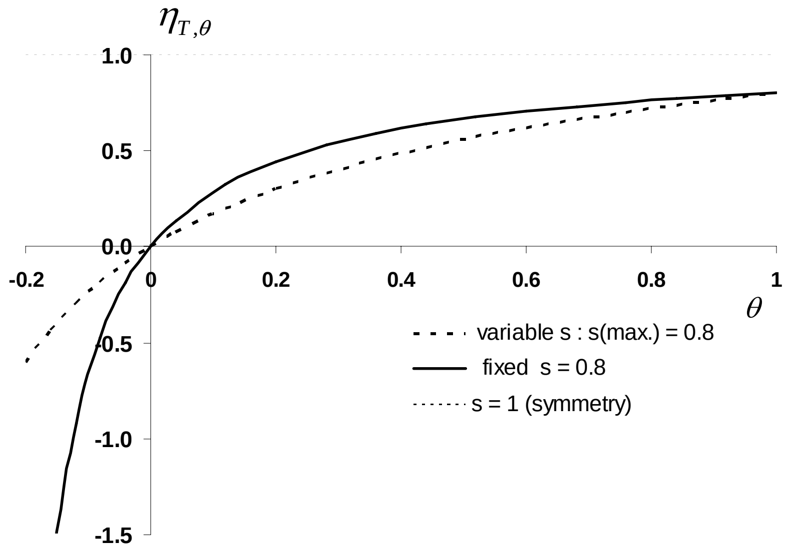

Figure 3: Loss symmetries with $ds/d\theta > 0$

The intuition here is as follows. A tax-induced fall in θ may arise because losses are shifted into the tax jurisdiction when t rises. This has a direct effect of reducing tax revenue because loss deductions rise. However, the indirect effect is to reduce s which reduces loss deductions and raises tax revenue. Hence the negative behavioural effect on tax revenues of a rise in the tax rate due to the increase in losses declared for tax is compensated by a tendency for asymmetric loss restrictions to bind more tightly, which boosts tax revenue, *ceteris paribus*. The negative, non-linear 'tax base' effect is therefore compensated by an endogenous change in the effective rebate rate, $s_j t$.

The tax revenue elasticity, $\eta_{T,t}$, is shown in Figure 4, for a behavioural response of $\eta_{\theta,t} = -0.8$. The symmetric ($s = 1$), and three asymmetric, cases are illustrated: $ds/d\theta = 0$, and $ds/d\theta = 0.1$, for $s^{\max} = 0.8$ and 1. This shows that in the case of symmetric treatment of losses, the elasticity, $\eta_{T,t}$,

maximum $s = 0.4$, and other parameters held at their values in Figure 3, the relationship becomes approximately linear. Of course, when there are no current losses, such that $\theta = 1$, a value of s close to, or equal to, 1 is to be expected (depending on past loss availability).

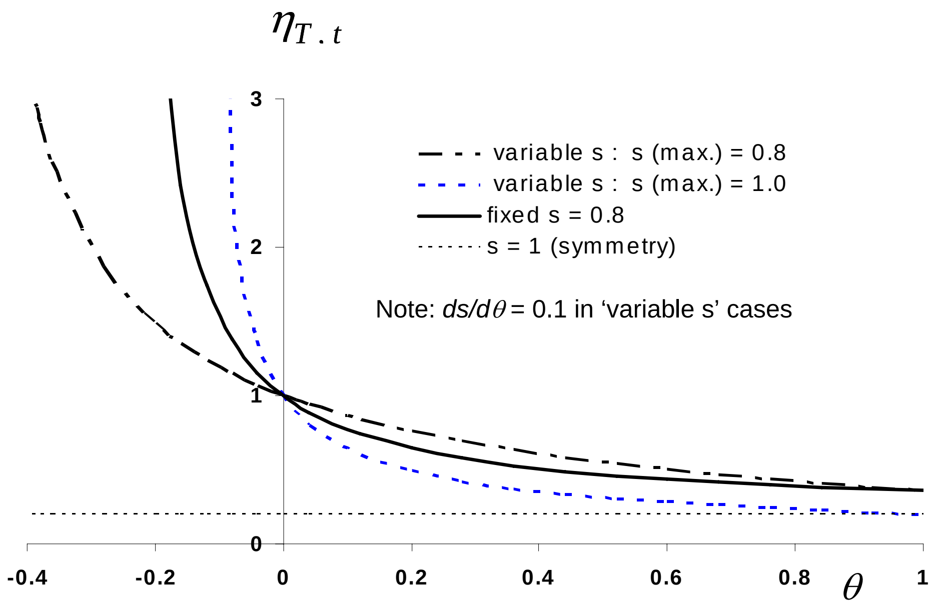

Figure 4: The Elasticity $\eta_{T,t}$; with $\eta_{\theta,t} = -0.8$

is flat at $1 + \eta_{T,\theta}\eta_{\theta,t} = 1 - 0.8 = 0.2$. However, the asymmetric treatment of losses causes the elasticity to exceed 0.2 for all values of θ and to fall sharply as θ increases. As with $\eta_{T,\theta}$, the $\eta_{T,t}$ elasticity falls more steeply as θ rises, at low values of θ (that is, in more severe recessions), when the asymmetric treatment of losses is less (that is, s closer to unity). However, the convexity (to the x-axis) of this relationship is reduced when $ds/d\theta \neq 0$, and with $s^{\max}$ set at the equivalent fixed value of 0.8. Raising $s^{\max}$ to 1.0 has the opposite effect of increasing the degree of non-linearlity.

Consider the extreme right hand side where $\theta = 1$, which implies that $\eta_{T,\theta} = \eta'_{T,\theta} = s$ and $\eta_{T,t} = 1 + s\eta_{\theta,t}$: hence the revenue elasticity is, as expected, closer to the symmetric case the greater is s. However, the profiles intersect at $\theta = 0$; $\eta_{T,\theta} = 1$, as shown in the figure. This can be seen from (24) to result from the fact that $\eta_{T,\theta} = 0$ if $\theta = 0$. Hence $\eta_{T,t} = 1 + \eta_{T,\theta}\eta_{\theta,t} = 1$ in all cases all the profiles shown in Figure 4 must intersect when $\eta_{T,t} = 1$. The revenue elasticity can thus exceed 1, to an extent depending on s, where losses are relatively high (that is, when θ is negative), the magnitude depending on the partial elasticity $\eta'_{s,\theta} > 0$. This reflects the fact that at low θ and/or s, the revenue-enhancing effect of tighter loss restrictions is sufficient to outweigh the revenue-depleting effect of increased losses ('real' or shifted into the tax jurisdiction) in response to the tax rate rise.

These results suggest that the elasticity of tax revenues with respect to changes in tax rates, $\eta_{T,t}$, (a variable commonly estimated in the empirical profit-shifting literature) can be expected in general to be higher (indicating a lower behavioural response) in regimes that involve greater asymmetries in the tax treatement of losses. This holds in both below-trend and above-trend situations. However, during recessions, when there are large losses, estimates of behavioural responses are likely to be especially sensitive to the precise combination of relative loss sizes, θ, and the degree of asymmetry, s.

The results obtained above have taken the value of the behavioural elasticity, $\eta_{\theta,t}$, as being constant for different values of θ. However, it may be that, faced with increasing difficulty of gaining tax deductions from their losses when θ is small, firms tend to increase their behavioural responsiveness to compensate; that is $\eta_{\theta,t}$ becomes more negative when $\eta_{T,\theta}$ is low during recessions. In the limit this could have the effect of keeping the term $\eta_{T,\theta}\eta_{\theta,t}$ in (19) constant over the cycle. The ability of firms to do this is likely to depend on the ease with which firms can adjust the amount of losses that they shift into or out of the tax jurisdiction in response to tax rate changes. To the extent that such adjustments can be made, the nonlinear profiles in Figures 2 to 4 provide measures of the size of adjustments that would be required to achieve 'full compensation' for the differential impact of loss asymmetries over the cycle. The evidence of Auerbach (2007), that there are sizeable cyclical fluctuations in US corporations' effective average tax rates, suggests that US firms are unable or unwilling to shift profits and losses sufficiently to prevent the cyclical impact of tax loss asymmetries from raising their effective tax rates during downturns.

5 Conclusions

Losses tend to be treated less generously than profits in the corporate tax codes of many countries. This typically means that, in present value terms, losses generate a lower tax rebate than the positive tax levied on equivalent sized profits. This asymmetry has two opposing effects on corporate tax liabilities when corporate losses increase. On the one hand increased losses give rise to a larger 'base' for tax rebates. On the other hand, the asymmetries of the tax code bind more tightly when losses are larger, which has the effect of pushing up the effective average tax rate. This paper has considered the relevance of this phenomenon for estimates of companies' behavioural responses to changes in corporate tax rates. These responses involve shifting profits and losses into or out of the tax jurisdiction.

In order to make the effective tax rate applied to losses more transparent, an equivalent tax function facing a firm was specified having the same present value of tax revenue as the actual function but allowing for a (partial) rebate in each period, where appropriate. This function involved a measure of asymmetry equal to the present value of the period's losses as tax off-sets (as a ratio of their nominal value).

It was shown that the response of tax revenue to a change in the tax rate, $\eta_{T,t}$, can be decomposed into the response, $\eta_{\theta,t}$, of the ratio of net-to-gross profits, θ, to the tax rate, t (a behavioural response), and the response of tax revenues to a change in the ratio of net-to-gross profits. The latter arises both directly, via a partial elasticity $\eta'_{T,\theta}$, and indirectly via changes in the impact of loss asymmetries, measured by the product of the partial elasticities, $\eta'_{T,s}$ and $\eta'_{s,\theta}$, which are determined by the tax code and firms' profit and loss situations. These latter responses 'translate' behavioural responses into tax revenue changes.

Asymmetries in the way losses are treated in the tax code reduce tax revenue responses to tax rate changes by reducing the sum of the direct and indirect effects, $\eta_{T,\theta} = \eta'_{T,\theta} + \eta'_{T,s}\eta'_{s,\theta}$, below unity (the value when there are no asymmetries). The size of the reduction increases as the asymmetry increases. This effect was found to be non-linear with respect to the economic cycle.

It is disproportionately strong during recessionary periods (when losses are relatively large) but is dispoportionately weak during boom periods (when losses are small). These disproportionate recessionary effects are larger the *smaller* is the asymmetry.

Therefore behavioural responses of tax revenue to tax rate changes are likely to be smaller (that is, $\eta_{T,t}$ deviates less below unity) for tax regimes which impose greater constraints on loss use within the tax code and when these responses are measured during periods when corporate losses are abnormally high. However, when losses are abnormally low, *ceteris paribus* larger behavioural responses are likely to be observed.

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