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The Elasticity of Taxable Income and the Tax Revenue Elasticity

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Abstract

This paper examines the joint role of the elasticity of taxable income (the effect on taxable income of a tax rise) and the revenue elasticity (the effect on revenue of a change in taxable income) in influencing the revenue effects of tax rate changes. Traditionally, the revenue elasticity has been the central concept in examining fiscal drag, or obtaining local measures of tax progressivity. But it has an additional role in the context of the revenue effects of tax changes when incomes respond to rate changes. The elasticity of tax revenue with respect to a rate change is examined at both the individual and aggregate level. If there were no incentive effects, an equal proportional change in all marginal tax rates would produce the same proportional increase in total revenue – the elasticity is unity. This rapidly falls, at a linear rate, as the elasticity of taxable income increases. Illustrations are provided using the New Zealand income tax structures before and after the 2010 Budget, which reduced all rates while leaving income thresholds unchanged and, in particular, reduced the top marginal rate substantially.

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1 Introduction

In the analysis of income tax structures, two elasticities play an important role at individual and aggregate levels. First, the tax revenue elasticity – the elasticity of tax revenue with respect to a change in gross income – is the central concept in the literature on 'fiscal drag', which is concerned with the extent to which the non-indexation of tax thresholds leads to increasing average tax rates over time.[1] In this context, the change in gross income is considered to be exogenous and any consequent feedback disincentive effect on income arising from the change in the average tax rate is ignored. Indeed, at the individual level the literature concentrates on changes which do not involve a movement across tax thresholds, which would otherwise lead to a change in the marginal tax rate.[2]

Second, the elasticity of taxable income – the elasticity of taxable income with respect to a change in the marginal net-of-tax rate (one minus the marginal rate) – captures the net effect of all incentive effects associated with the marginal rate change. This approach to grouping all the various responses, such as labour supply, income shifting, under-reporting of income and so on, in a reduced-form specification has attracted much attention.[3] It avoids the considerable complexities of attempting to combine these effects into a structural model, as well as providing (under certain assumptions) a convenient method of measuring the marginal excess burden arising from tax changes. The elasticity can be influenced by policy changes concerning, for example, regulations regarding income shifting and the timing of income receipts and tax payments.

Hence, one elasticity concerns the way tax revenue changes in response to exogenous income changes while the other elasticity measures the extent to which income declared for tax purposes adjusts when the income tax rate varies.[4] The first elasticity

[1] See the survey in Creedy and Gemmell (2002). The revenue elasticity is also used in discussions of local measures of tax progressivity.

[2] In simulations generating aggregate elasticities from individual elasticities, care is also needed to avoid such movements because they can involve huge individual revenue elasticities for tiny changes in gross income. Labour supply incentive effects, in the context of the revenue elasticity with respect to wage rate changes, are examined by Creedy and Gemmell (2005).

[3] The seminal paper is Feldstein (1995). For reviews of evidence see Giertz (2007) and Saez, Slemrod and Giertz (2009), and for an introduction to the underlying analytics, see Creedy (2009).

[4] The tax rate may vary as a result of a deliberate policy change, or it may change as individuals move across income thresholds, particularly as a result of fiscal drag. As mentioned earlier, such

is concerned only with the nature of the income tax structure itself and, when considering aggregation over individuals, the form of the income distribution. The second elasticity is concerned with a wide range of behavioural adjustments associated with tax rate changes, captured in a single measure. Hence there is no direct connection between the two elasticity concepts. However, there is another associated elasticity in which the two elasticities have a role. The elasticity of tax revenue with respect to a change in the marginal tax rate is influenced, first, by the extent to which taxable income adjusts to the tax rate change and, second, by the way tax revenue adjusts to the taxable income change.

When discussing revenue changes resulting from marginal rate changes, the existing literature on the elasticity of taxable income has not generally identified a separate role for the revenue elasticity. Changes in total tax obtained from the top marginal rate in a multi-rate structure are examined in Saez *et al.* (2009), in the course of deriving the aggregate excess burden. But they do not consider revenue elasticities. It is shown below how the revenue elasticity has a clear role at the individual level in influencing the change in tax resulting from a rate change. In considering aggregate revenue over all individuals, changes are shown to depend on the revenue elasticity at the arithmetic mean income level within each tax bracket in a multi-rate income tax structure.

The aim of this paper is to explore the precise relationships among the three elasticities for the tax functioned mentioned above. Section 2 examines the the individual tax revenue elasticity, the individual elasticity of taxable income and presents the way in which the two elasticities combine to determine the elasticity of tax with respect to a change in the marginal tax rate. The relationships are examined for a completely general tax function. However, in view of the ubiquitous nature of the multi-step tax function, and the focus of the previous literature on the 'top rate', results are also given for this special cases. Section 3 looks at aggregation over individuals when a single marginal rate changes in a multi-rate tax structure. Brief conclusions are in Section 4.

transitions across thresholds are typically not considered in producing revenue elasticities.

2 Relationships Among Elasticities

This section demonstrates, at the individual level, how the revenue elasticity and the elasticity of taxable income combine to generate the elasticity of tax with respect to the marginal rate. Subsections 2.1 and 2.2 are for a general tax function, but subsection 2.3 explores the special case of the ubiquitous multi-step tax function. In these sections, the distinction between gross income and taxable income is ignored, though this distinction is likely to be important for countries with extensive income tax deductions.[5] Subsection 2.4 therefore extends the results to deal with the case where there are endogenous, income-related deductions.

2.1 The Revenue Elasticity

In the literature on the tax revenue elasticity, concentration is on the effects of changes in taxation resulting from exogenous changes in taxable income, *with the tax rates and thresholds held constant*.[6] Let $T(y)$ denote the tax paid by an individual with income of y, and let τ denote the marginal tax rate facing the individual. Totally differentiating $T(y)$ gives:

$$dT = \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial \tau}d\tau \tag{1}$$

This assumes that any deductions and tax thresholds in the tax structure remain unchanged. The existence of separate deductions, which may depend on y, are discussed below. Using the general notation, $\eta_{b,a} = \frac{a}{b}\frac{db}{da}$, to denote a 'total' elasticity and letting $\eta'_{b,a} = \frac{a}{b}\frac{\partial b}{\partial a}$, so that a prime indicates that the elasticity is partial, rearrangement of (1) gives:

$$\eta_{T,y} = \eta'_{T,y} + \eta'_{T,\tau}\eta_{\tau,y} \tag{2}$$

However, it can be seen that the partial elasticity $\eta'_{T,\tau} = 1$, so that:

$$\eta_{T,y} = \eta'_{T,y} + \eta_{\tau,y} \tag{3}$$

[5] For discussion of the empirical importance of income-related deductions in personal income tax regimes in OECD countries, see Caminada and Goudswaard (1996) and Wagstaff and van Doorslaer (2001). For the US, Feldstein (1999, p. 675) estimated that total income tax deductions in 1993 amounted to about 60% of estimated taxable income.

[6] The restriction to exogenous income changes is easily controlled in considering individual elasticity values but of course the nature of the overall distribution of income, which is needed to obtain aggregate values, may well be influenced by the incentive effects of the consequent tax changes.

In obtaining expressions for the revenue elasticity, it is usual in the literature to assume that the exogenous change in income does not cause the individual to move into a higher tax bracket. Such a movement, where the tax function involves discrete changes in marginal rates, gives rise to a large jump in the elasticity, and this can – when carrying out appropriately tax-share weighted aggregation – distort the aggregate elasticity. Hence $\eta_{\tau,y}$ is considered to be zero, and it may be said that the literature concentrates on $\eta'_{T,y}$.

It may be tempting here to rewrite $\eta_{\tau,y}$ as $1/\eta_{y,\tau}$ and think of the latter as reflecting an incentive effect of a change in the marginal tax rate. However, this is not legitimate: for example the assumption that the individual does not move into a higher tax bracket when income rises is not consistent with an infinitely large response of income to a change in the tax rate.

2.2 The Elasticity of Taxable Income

The individual elasticity of taxable income, $\eta_{y,1-\tau}$, measures the behavioural response of taxable income to a change in the marginal net-of-tax rate, $1-\tau$, facing the individual. This is positive, because increases in the net-of-tax rate are expected to lead to increases in y. Expressed instead as the elasticity of taxable income with respect to the *tax rate*, this simply becomes:

$$\eta_{y,1-\tau} = -\left(\frac{1-\tau}{\tau}\right)\eta_{y,\tau} \ , \tag{4}$$

which is negatively signed. Consider a change in the individual's tax liability resulting from an exogenous increase in the marginal tax rate. From (1), dividing by $d\tau$ gives:

$$\frac{dT}{d\tau} = \frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial y}\frac{dy}{d\tau} \tag{5}$$

The first term may be said to reflect a pure 'tax rate' effect of a rate change, with unchanged incomes, while the second term reflects the net 'tax base' effect resulting from the incentive effects on taxable income combined with the revenue consequences of that income change. When discussing the effect on total revenue of a change in the top income tax rate, Saez *et al.* (2009) refer to these as the 'mechanical' and 'behavioural' effects respectively.[7] In terms of elasticities, equation (5) becomes:

$$\eta_{T,\tau} = \eta'_{T,\tau} + \eta'_{T,y}\eta_{y,\tau} \tag{6}$$

Furthermore, using (4), along with the fact, mentioned above, that $\eta'_{T,\tau} = 1$, it can be seen that:

$$\eta_{T,\tau} = 1 - \left(\frac{\tau}{1-\tau}\right)\eta'_{T,y}\eta_{y,1-\tau} \tag{7}$$

This result links the two relevant elasticities. The total response of tax revenue to a change in the marginal tax rate is one, minus $\tau/(1-\tau)$ multiplied by the product of the revenue elasticity and the elasticity of taxable income. The latter elasticity governs the way income changes when the marginal tax rate varies, while the first elasticity reflect the consequent change in revenue as a result of that income change. If there is no incentive effect, the total change $\eta_{T,\tau}$ is thus equal to the partial change $\eta'_{T,\tau} = 1$.

Tax paid by the individual increases, when the marginal rate increases, only if:

$$\frac{1-\tau}{\tau} > \eta'_{T,y}\eta_{y,1-\tau} \tag{8}$$

Or, in terms of $\eta_{y,\tau}$, tax increases if:

$$\left|\eta_{y,\tau}\right| < \frac{1}{\eta'_{T,y}} \tag{9}$$

In any progressive income tax structure, the partial elasticity, $\eta'_{T,y}$, exceeds 1, unless (as shown in the following section) allowances vary sufficiently with income. Hence revenue increases only if the absolute value of the elasticity, $\eta_{y,\tau}$, is sufficiently small.

2.3 The Multi-Step Tax Function

Consider the case of the multi-step tax function, which is defined by a set of income thresholds, a_k, for $k = 1, ..., K$, and marginal income tax rates, τ_k, applying in tax brackets, that is between adjacent thresholds a_k and a_{k+1}. The funcrion can be written as:[8]

$$\begin{aligned} T(y) &= \tau_1(y-a_1) & & a_1 < y \leq a_2 \\ &= \tau_1(a_2-a_1) + \tau_2(y-a_2) & & a_2 < y \leq a_3 \end{aligned} \tag{10}$$

[7] Thus, their 'behavioural effect' combines the revenue elasticity and elasticity of taxable income effects. Saez *et al.* (2009, p. 5) do not discuss the separate role of the revenue elasticity in this context. Discussion of the rate and base effects is often discussed in the context of a simple proportional tax structure, with constant average and marginal rate, t, where the revenue elasticity is everywhere unity. Thus if $\bar{y}$ is arithmetic mean income, $\frac{dT}{dt} = \bar{y} + \frac{td\bar{y}}{dt}$ and in terms of elasticities, $\eta_{T,t} = 1 + \eta_{\bar{y},t}$.

[8] This is examined in more detail in Creedy and Gemmell (2006).

and so on. If y falls into the kth tax bracket, so that $a_k < y \leq a_{k+1}$, $T(y)$ can be written for $k \geq 2$ as:

$$T(y) = \tau_k (y - a_k) + \sum_{j=1}^{k-1} \tau_j (a_{j+1} - a_j) \tag{11}$$

This expression for $T(y)$ can be rewritten as:

$$T(y) = \tau_k (y - a_k^*) \tag{12}$$

where:

$$a_k^* = \frac{1}{\tau_k} \sum_{j=1}^{k} a_j (\tau_j - \tau_{j-1}) \tag{13}$$

and $\tau_0 = 0$. Thus the tax function facing any individual taxpayer in the kth bracket is equivalent to a tax function with a single marginal tax rate, τ_k, applied to income measured in excess of a single threshold, a_k^*. Therefore, unlike a_j, a_k^* differs across individuals depending on the marginal income tax bracket into which they fall. For this structure, and supposing that the income thresholds, a_k, remain fixed, the revenue elasticity is:

$$\begin{aligned} \eta'_{T,y} &= \frac{y}{y - a_k^*} \\ &= 1 + \frac{a_k^*}{y - a_k^*} \end{aligned} \tag{14}$$

and the individual partial elasticity must exceed unity.

For this tax function, appropriate substitution gives the result that the elasticity of revenue with respect to the marginal rate faced by an individual in the kth tax bracket is given by:

$$\eta_{T,\tau} = 1 - \left(\frac{y}{y - a_k^*}\right)\left(\frac{\tau_k}{1 - \tau_k}\right)\eta_{y,1-\tau} \tag{15}$$

2.4 Endogenous Allowances

Suppose that, instead of having only fixed income thresholds, there is a range of deductions which are income related. In the case of the multi-rate tax schedule discussed in the previous subsection, the deductions can be integrated into the income thresholds, a_k, which are then considered to adjust when income varies.

Consider first the revenue elasticity. In this case, again assuming that the individual does not cross into a higher-rate tax bracket when income increases, it can be shown that the total revenue elasticity is given by:

$$\eta_{T,y} = \eta'_{T,y} + \eta'_{T,a^*}\eta_{a^*,y} \tag{16}$$

where $\eta_{a^*,y}$ measures the extent to which the 'effective threshold', a^*, varies when income changes (a k subscript is suppressed here for convenience). In the case of the multi-rate schedule, the elasticity of tax with respect to the effective threshold, η'_{T,a^*}, is given by:

$$\eta'_{T,a^*} = -\frac{a^*}{y - a^*} \tag{17}$$

Hence:

$$\eta_{T,y} = \eta'_{T,y}\left(1 - \frac{a^*}{y}\eta_{a^*,y}\right) \tag{18}$$

Next, consider the elasticity of taxable income. With endogenous deductions, a modification must also be made to the expression given above for $\eta_{T,\tau}$, such that:

$$\begin{aligned}\eta_{T,\tau} &= \eta'_{T,\tau} + \eta'_{T,y}\eta_{y,\tau} + \eta'_{T,a^*}\eta_{a^*,\tau} \\ &= 1 + \left(\eta'_{T,y} + \eta'_{T,a^*}\eta_{a^*,y}\right)\eta_{y,\tau}\end{aligned} \tag{19}$$

and using (4):

$$\eta_{T,\tau} = 1 - \left(\frac{\tau}{1-\tau}\right)\left(\eta'_{T,y} + \eta'_{T,a^*}\eta_{a^*,y}\right)\eta_{y,1-\tau} \tag{20}$$

Hence, the total effect on revenue as a result of a change in the marginal tax rate is modified by the endogenous effect of an income change on the deductions from gross income. Indeed, to the extent that an increase in the marginal tax rate reduces declared income, it also reduces the level of deductions claimed, so that the fall in tax revenue is not as large as it otherwise would be if deductions were fixed.

For the multi-rate schedule, appropriate substitution gives (again neglecting the k subscript), for the individual elasticity:

$$\eta_{T,\tau} = 1 - \left(\frac{y}{y-a^*}\right)\left(\frac{\tau}{1-\tau}\right)\left(1 - \frac{a^*}{y}\eta_{a^*,y}\right)\eta_{y,1-\tau} \tag{21}$$

This can alternatively be expressed in terms of responses of 'taxable income', $y_T = y - a^*$, as follows:

$$\begin{aligned}
\eta_{T,\tau} &= 1 - \left(\frac{\tau}{1-\tau}\right)\left(\frac{y}{y-a^*}\right)\left(\frac{d(y-a^*)}{dy}\right)\eta_{y,1-\tau} \\
&= 1 - \left(\frac{\tau}{1-\tau}\right)\eta_{y_T,y}\eta_{y,1-\tau} \\
&= 1 - \left(\frac{\tau}{1-\tau}\right)\eta_{y_T,1-\tau} \qquad (22)
\end{aligned}$$

Hence the elasticity of interest, $\eta_{T,\tau}$ can be expressed simply in terms of the marginal tax rate, τ, and the responsiveness of *taxable* income, y_T, to the net-of-tax rate, $1-\tau$.

3 Aggregate Revenue

For tax policy purposes attention is often devoted to aggregate revenue and its variation as component tax rates are changed. This section therefore examines aggregation over individuals. Emphasis is on the effect on total income tax revenue of a change in a single tax rate, and the effect of a simultaneous similar change in all rates. In order to obtain clear results, attention is restricted to the case of the multi-rate tax function. It is assumed that all individuals face the same income thresholds, so that endogenous allowances are not considered here. Aggregate elasticities are derived in subsection 3.1 and, in subsection 3.2, these are compared with an earlier result produced by Saez *et al.* (2009). The potential orders of magnitude involved are then examined in subsection 3.3.

When dealing with population aggregates it is necessary to distinguish various tax and revenue terms, for both clarity and succinctness. In the previous section, the tax liability facing an individual with an income of y has been denoted by $T(y)$. In the multi-tax form, if y is in the kth tax bracket a distinction can be made between $T(y) = \tau_k (y - a_k^*)$ and the tax paid by the individual at the marginal rate, τ_k, thereby ignoring tax paid on income falling into lower thresholds.

For aggregate revenue amounts defined over populations, or population sub-groups, R is used. Thus, in this section R represents aggregate revenue, while R_k refers to the aggregate revenue obtained from all individuals whose incomes fall in the kth tax bracket: that is, R_k is the aggregate over individuals in the kth bracket of $\tau_k (y - a_k^*)$

values. Let $R_{(k)}$ denote the aggregate amount raised only at the rate k from individuals who fall into the kth bracket: that is, $R_{(k)}$ is the sum over individuals in the kth bracket of $\tau_k (y - a_k)$ values. Furthermore, $R_{(k)}^{+}$ refers to the aggregate revenue obtained at the kth rate from individuals whose incomes fall into higher tax brackets: that is, $R_{(k)}^{+}$ is the number of all individuals in higher tax brackets multiplied by $\tau_k (a_{k+1} - a_k)$.

3.1 Changes in Aggregate Revenue

First, it is useful to clarify the general relationship between the elasticity of aggregate revenue with respect to a single marginal rate change, and the elasticity with respect to changes in all rates. Suppose all marginal tax rates change, but income thresholds remain fixed. Totally differentiating R gives:

$$dR = \sum_{k=1}^{K} \frac{\partial R}{\partial \tau_k} d\tau_k \tag{23}$$

Hence if all rates change by the same proportion, $\partial \tau_k / \tau_k = d\tau / \tau$ for all k and:

$$\eta_{R,\tau} \equiv \frac{\tau}{R} \frac{dR}{d\tau} = \sum_{k=1}^{K} \frac{\tau_k}{R} \frac{\partial R}{\partial \tau_k} = \sum_{k=1}^{K} \eta'_{R,\tau_k} \tag{24}$$

Thus the elasticity of aggregate revenue with respect to a simultaneous equal proportional change in all tax rates is the sum of the separate elasticities, η'_{R,τ_k}, over all $k = 1, ..., K$.

In the multi-step tax function with K brackets, suppose P_k people are in each bracket, for $k = 1, ..., K$, and the arithmetic mean income in each bracket is $\bar{y}_k$. Then aggregate revenue is:

$$\begin{aligned} R &= \tau_1 (\bar{y}_1 - a_1) P_1 \\ &\quad + \{\tau_2 (\bar{y}_2 - a_2) + \tau_1 (a_2 - a_1)\} P_2 \\ &\quad + \{\tau_3 (\bar{y}_3 - a_3) + \tau_1 (a_2 - a_1) + \tau_2 (a_3 - a_2)\} P_3 \\ &\quad + \text{etc} \end{aligned} \tag{25}$$

Let $P_k^{+} \equiv \sum_{j=k+1}^{K} P_j$ denote the number of people above the kth tax bracket. For the top marginal rate, where $k = K$, clearly $P_K^{+} = 0$. Thus aggregate revenue can be written more succinctly as:

$$R = \sum_{k=1}^{K} \tau_k \left(\bar{y}_k - a_k\right) P_k + \sum_{k=1}^{K-1} \tau_k \left(a_{k+1} - a_k\right) P_k^+ \tag{26}$$

Consider next the response of aggregate revenue to a change in the kth marginal tax rate. This has two basic components. First, there is the direct effect of the change in the kth tax rate on tax from that bracket alone. From previous sections above, this is made up of the behavioural effect of the tax rate change on the incomes of those in the kth bracket, along with the revenue elasticity effect (which is not a reflection of behaviour but depends on the tax structure). Second, there is an indirect effect on individuals in higher tax brackets, as a result of the term $\tau_k \left(a_{k+1} - a_k\right)$. Assume first that there are no behavioural responses. It can be seen that, letting $\eta'_{R,\tau_k} \equiv \frac{\tau_k}{R} \frac{\partial R}{\partial \tau_k}$:

$$\eta'_{R,\tau_1} = \frac{\tau_1}{R} \left\{ \left(\bar{y}_1 - a_1\right) P_1 + \left(a_2 - a_1\right) P_1^+ \right\} \tag{27}$$

$$\eta'_{R,\tau_2} = \frac{\tau_2}{R} \left\{ \left(\bar{y}_2 - a_2\right) P_2 + \left(a_3 - a_2\right) P_2^+ \right\} \tag{28}$$

and so on. Hence in general:

$$\eta'_{R,\tau_k} = \frac{\tau_k}{R} \left\{ \left(\bar{y}_k - a_k\right) P_k + \left(a_{k+1} - a_k\right) P_k^+ \right\} \tag{29}$$

For this 'no behavioural response' case, these elasticities sum to:

$$\eta_{R,\tau} = \sum_{k=1}^{K} \eta'_{R,\tau_k} = 1 \tag{30}$$

and the elasticity of total revenue with respect to an equal proportional change in all rates, in (24), is unity. Any behavioural response clearly reduces the elasticity below 1, as shown below.

In the case where there are behavioural effects of marginal rate changes, it is convenient to assume that all those in a given bracket have the same elasticity, η_{y,τ_k}. In this case, it can be shown that an appropriate adjustment to the average income level within the tax bracket gives:

$$\eta'_{R,\tau_k} = \frac{\tau_k}{R} \left[\left\{ \bar{y}_k \left(1 + \eta_{y,\tau_k}\right) - a_k \right\} P_k + \left(a_{k+1} - a_k\right) P_k^+ \right] \tag{31}$$

The expression in (31), while quite straightforward, does not bring out the separate elements influencing η'_{R,τ_k} in a transparent way. First, rewrite this as:

$$\eta'_{R,\tau_k} = \frac{\tau_k P_k}{R} \left[(\bar{y}_k - a_k) - \bar{y}_k \left(\frac{\tau_k}{1 - \tau_k} \right) \eta_{y,1-\tau_k} + (a_{k+1} - a_k) \frac{P_k^+}{P_k} \right] \tag{32}$$

Then multiplying and dividing by $(\bar{y}_k - a_k^*)$ gives:

$$\eta'_{R,\tau_k} = \frac{R_k}{R} \left[\frac{R_{(k)} + R_{(k)}^+}{R_k} - \left(\frac{\bar{y}_k}{\bar{y}_k - a_k^*} \right) \left(\frac{\tau_k}{1 - \tau_k} \right) \eta_{y,1-\tau_k} \right] \tag{33}$$

In this expression $R_{(k)} + R_{(k)}^+ \neq R_k$.

From equation (14), $\bar{y}_k / (\bar{y}_k - a_k^*)$ is the revenue elasticity at arithmetic mean income in the kth bracket. This expression therefore shows how the elasticity, η'_{R,τ_k}, depends on the elasticity of taxable income of those in the kth tax bracket, $\eta_{y,1-\tau_k}$, along with the revenue elasticity at $\bar{y}_k$, and various tax-share terms. Furthermore, it can be shown that $\eta'_{R,\tau_k} > 0$ if:

$$\eta_{y,1-\tau_k} < \left(\frac{1 - \tau_k}{\tau_k} \right) \left[1 - a_k \left(1 - \frac{P_k^+}{P_k} \right) + a_{k+1} \left(\frac{P_k^+}{P_k} \right) \right] \frac{1}{\bar{y}_k} \tag{34}$$

For the top bracket, the final term within square brackets in equation (34) is zero and the elasticity is positive if:

$$\eta_{y,1-\tau_K} < \left(\frac{1 - \tau_K}{\tau_K} \right) \left(\frac{\bar{y}_K - a_K}{\bar{y}_K} \right) \tag{35}$$

and although the first term in brackets exceeds 1 as long as the tax rate, τ_K, is less than 0.5, the second term in brackets is likely to be well below 1. Hence the elasticity of taxable income must be relatively low for a tax rate increase to increase aggregate revenue.

3.2 Comparison with Earlier Results

The above result for any tax rate in a multi-rate structure may be compared with that given by Saez *et al.* (2009, p. 5, equation 4) for the top marginal rate. They consider changes in taxation paid at the top rate only. When converted to the present notation and written in elasticity form, their result thus refers not to actual tax paid but to the elasticity of tax paid at the rate τ_K, which can be defined as $\eta'_{R_{(K)},\tau_K}$. Hence:

$$\eta'_{R_{(K)},\tau_K} = \left[1 - \left(\frac{\bar{y}_K}{\bar{y}_K - a_K}\right)\left(\frac{\tau_K}{1-\tau_K}\right)\eta_{y,1-\tau_K}\right] \tag{36}$$

Saez *et al.* (2009) discuss the term $\bar{y}_K/(\bar{y}_K - a_K)$, which is constant if the income distribution above the top threshold follows the Pareto form. Their expression therefore does not indicate the separate role for the revenue elasticity at $\bar{y}_K$. Furthermore, their 'behavioural response' actually includes both the behavioural response and the revenue elasticity effect, which depends on the full tax structure as well as average income above the top threshold. It is useful to convert (36) into an expression which does separate the these two elasticity effects, by writing:

$$\eta'_{R_{(K)},\tau_K} = \frac{R_K}{R_{(K)}}\left[\frac{R_{(K)}}{R_K} - \left(\frac{\bar{y}_K}{\bar{y}_K - a_K^*}\right)\left(\frac{\tau_K}{1-\tau_K}\right)\eta_{y,1-\tau_K}\right] \tag{37}$$

and remembering that $\bar{y}_K/(\bar{y}_K - a_K^*)$ is the revenue elasticity at $\bar{y}_K$.

From the general result above, the value of η'_{R,τ_K} is given by:

$$\eta'_{R,\tau_K} = \frac{R_K}{R}\left[\frac{R_{(K)}}{R_K} - \left(\frac{\bar{y}_K}{\bar{y}_K - a_K^*}\right)\left(\frac{\tau_K}{1-\tau_K}\right)\eta_{y,1-\tau_K}\right] \tag{38}$$

For comparison with (37), it is necessary to use the general relationship between $\eta'_{R_{(K)},\tau_K}$ and η'_{R,τ_K}. For the top rate, this takes the simple form:

$$\eta'_{R_{(K)},\tau_K} = \left(\frac{R}{R_{(K)}}\right)\eta'_{R,\tau_K} \tag{39}$$

Thus multiplication of (38) by $R/R_{(K)}$ gives the rearranged form of the Saez *et al.* result in (37). Hence, as expected, the Saez result is a special case of the more general result derived above. But instead of focussing on a term such as $\bar{y}_K/(\bar{y}_K - a_K)$, which depends purely on the form of the distribution of income, the present formulation emphasises the *joint* role of the elasticity of taxable income and the appropriate revenue elasticity (at the income level, $\bar{y}_K$), which depends on the nature of the tax function (the lower rates and thresholds, not simply the top threshold) as well as the income distribution (which, together with a_K, affects $\bar{y}_K$).

3.3 Illustrative Examples

In order to provide an illustration of potential orders of magnitude, it is useful to consider the change to the income tax structure in New Zealand, made in the 2010 Budget. For simplicity, the following calculations are made on the assumption that the distribution of taxable income follows the lognormal distribution with mean and variance of logarithms of income of 10.0 and 0.7 respectively. These parameter values imply an arithmetic mean income of \$31,257. These assumptions are clearly only approximate (and in New Zealand the distribution of taxable income has a lower mode associated with taxable benefits), but serve to indicate the nature of the relationships involved and the sensitivity to variations in the elasticity of taxable income.

Table 1: The New Zealand Income Tax Structure Before and After the 2010 Budget

k	τ_k	a_k	a_k^*	$\bar{y}_k$	P_k/N	R_k/P_k	R_k/R	$\eta_{T(\bar{y}_k),y}$
Tax structure pre-2010 Budget								
1	0.125	1	1.00	8935.33	0.29	1116.79	0.051	1.000
2	0.210	14000	5667.26	26745.01	0.53	4426.33	0.367	1.269
3	0.330	48000	21060.99	57367.53	0.09	11981.16	0.173	1.580
4	0.380	70000	27500.33	109607.80	0.08	31200.85	0.408	1.335
Tax structure post-2010 Budget								
1	0.105	1	1.00	8935.33	0.29	938.10	0.051	1.000
2	0.175	14000	5600.60	26745.01	0.53	3700.27	0.360	1.265
3	0.300	48000	23267.02	57367.53	0.09	10230.15	0.174	1.682
4	0.330	70000	27515.47	109607.80	0.08	27090.48	0.415	1.335

Table 1 provides summary information about the pre- and post-2010 Budget tax structures, for the assumed income distribution. In the column headed P_k/N, N represents the total number of individuals, so that the values show the proportion of people in the respective tax bracket. In the Budget, all the income thresholds were left unchanged, but the marginal tax rates were reduced, in particular the top marginal rate. Given the relatively low value of the income threshold above which the top rate applies, this tax bracket contributes a higher proportion of total income tax revenue than the other brackets, even though it contains less than ten per cent of taxpayers. This compares with the second tax bracket which contains over half of all taxpayers. The final column of the table reports the revenue elasticity, $\eta_{T,y}$, in each tax bracket, evaluated at arithmetic mean income within the bracket. For each tax structure, this elasticity

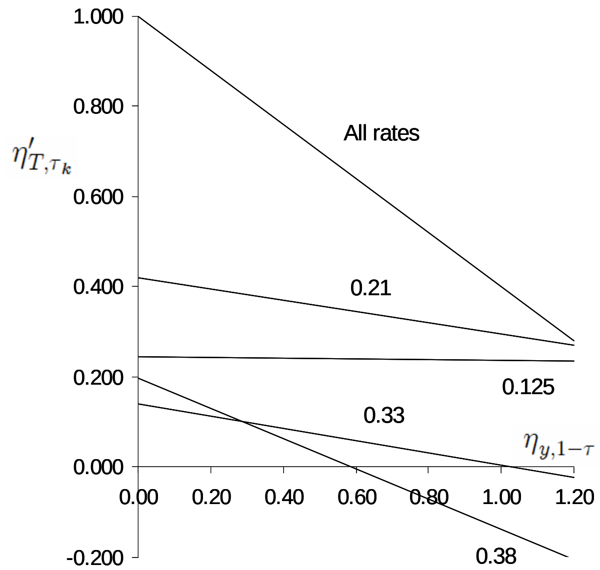

Figure 1: Elasticity of Total Tax Revenue wrt Tax Rates: Pre-Budget Changes

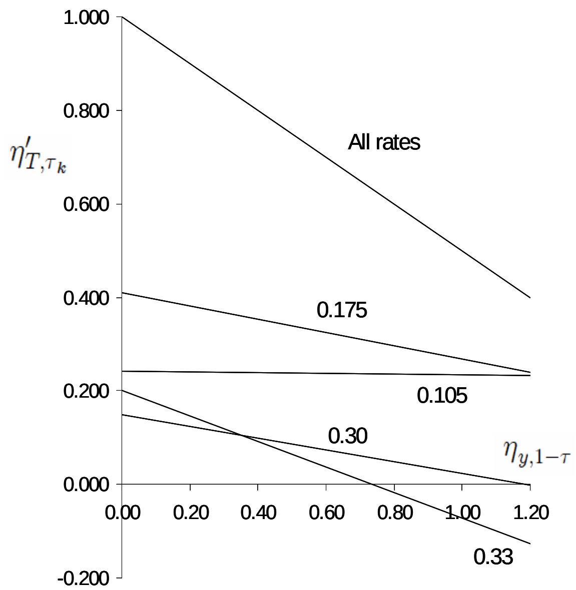

Figure 2: Elasticity of Total Tax Revenue wrt Tax Rates: Post-Budget Changes

is highest in the third tax bracket because the value of $\bar{y}_3$ is relatively closer to the effective income threshold, a_3^* than for the other brackets. For those in the first tax bracket, the average and marginal tax rates are equal, so that the revenue elasticity is unity. The Budget change in marginal rates has little effect on the revenue elasticities.

Figures 1 and 2 show the variations in the elasticity, η'_{R,τ_k}, for each tax bracket, as the elasticity of taxable income, $\eta_{y,1-\tau}$, increases. As demonstrated above, the value of each η'_{R,τ_k} falls linearly with $\eta_{y,1-\tau}$, but the rate of decrease is noticeably less in the post-Budget structure. In each case the elasticity, η'_{R,τ_k}, for the lowest income tax bracket remains approximately constant. Although the elasticity $\eta_{T(\bar{y}),y}$ is highest in the third tax bracket, the value of η'_{R,τ_k} falls faster in the top marginal rate bracket. This is because the value of $\tau/(1-\tau)$ is higher for the higher marginal tax rate, along with the fact that the top-rate bracket contributes a much higher proportion of aggregate tax revenue.

Some evidence regarding the elasticity of taxable income of New Zealand taxpayers is reported in Claus *et al.* (2010). They found that for those in the lower tax brackets, the estimated elasticities were very small, but for the top marginal tax rate the responses were substantial. For top-rate taxpayers, the values were mainly in the range 0.5 to 1.2.

4 Conclusions

This paper has examined the joint role of the elasticity of taxable income (which refers to the effect on taxable income of a tax rise) and the revenue elasticity (which reflects the effect on revenue of a change in taxable income) in influencing the revenue effects of tax rate changes. Traditionally, the revenue elasticity has been the central concept in examining fiscal drag, or obtaining local measures of tax progressivity. But it has an additional role in the context of the revenue effects of tax changes when incomes respond to rate changes. This separate effect has not previously been discussed explicitly. The elasticity of tax revenue with respect to a rate change was examined at both the individual and aggregate level.

When a single marginal tax rate in a multi-rate income tax structure is changed, those in the relevant tax bracket adjust their incomes in accordance with the elasticity of taxable income, and this affects the tax paid via the revenue elasticity. There is also a revenue effect on those individuals who are in higher tax brackets, since marginal rate changes in lower tax brackets imply a change in their effective income threshold. But there are no incentive effects on higher-rate taxpayers because only their average tax rate changes. If there were no incentive effects, an equal proportional change in all marginal tax rates would produce the same proportional increase in total revenue – the elasticity is unity. This rapidly falls, at a linear rate, as the elasticity of taxable income increases.

Illustrations were provided using the New Zealand income tax structures before and after the 2010 Budget, which reduced all rates while leaving income thresholds unchanged and, in particular, reduced the top marginal rate substantially. The elasticity of total tax revenue with respect to a single tax rate was found to be particularly sensitive to the elasticity of taxable income in the top tax bracket. In the pre-Budget structure, an elasticity of taxable income in excess of 0.6 was found to produce a negative tax response to an increase in the top marginal rate of 0.38. When this rate is lower, as in the post-Budget structure, the elasticity of taxable income needs to be over 0.7 before tax revenue is expected to fall in response to an increase in the marginal rate. These values must be considered as purely illustrative, since they are based on an approximation to the income distribution, rather than using the precise proportions of people and income within each tax bracket. However, estimates of the elasticity of taxable income in the top tax bracket in New Zealand are in the range (with some estimates in excess of 1) where tax revenue may fall. These results therefore suggest that further detailed empirical investigation is warranted.

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