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1 Francis Ysidro Edgeworth

Francis Ysidro Edgeworth (1845-1926) was a leading figure in the rapid development of economics during the last quarter of the 19th century and the first quarter of the 20th century. He held the Drummond Chair at Oxford from 1891 and was regarded as second only to the great Cambridge economist Alfred Marshall. He was a prolific and highly original author who, in a cosmopolitan age, had probably the widest correspondence with economists all over the world.[1] He was a man of enormously wide reading and considerable linguistic skills. He was the first editor of the *Economic Journal*, published by the newly formed Royal Economic Society. He was President of Section F of the British Association in 1889 and 1922. He achieved eminence as a statistician as well as an economist, becoming a Guy Medallist (Gold) of the Royal Statistical Society in 1907 and was President of the Society, 1912-14. Indeed, of about 170 papers, roughly three-quarters are concerned with statistical theory.[2]

His name is familiar to all economists, if only because of the 'Edgeworth box', one of the most widely used analytical devices in the subject. This diagrammatic tool was introduced by Edgeworth in 1881 in his first publication in economics, *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences*. This small book is remarkable for its highly original and far-reaching contributions to economics; indeed Marshall began his review with the statement that, 'This book shows clear signs of genius'. However, it was written in such a terse and unique style that it took many years before its contributions were fully appreciated, despite the fact that Edgeworth became one of the most prominent economists of his age. The title itself does not clearly signal a book on economics, and his use of sophisticated mathematics put it well beyond the reach of most of the economists of the period. The technical difficulty of much of his published output contributed to its slow assimilation into text books and he continues to remain relatively neglected in texts on the history of economic analysis.

Mathematical Psychics provides the key to all his later work and his lasting importance

[1] For a full-length treatment of Edgeworth's economics, see Creedy (1986), and for a biography, see Barbé (2010).

[2] His main contributions to statistics concern work on inference and the 'law of error', the correlation coefficient, transformations (what he called 'methods of translation'), and the 'Edgeworth expansion'. The latter, a series expansion which provides an alternative to the Pearson family of distributions, has been widely used (particularly since the work of Sargan, 1976) to improve on the central limit theorem in approximating sampling distributions. It has also been used to provide support for the bootstrap in providing an Edgeworth correction. His third and final book was *Metretike: or the Method of Measuring Probability and Utility* (1887). These contributions are not examined here; see Bowley (1928) and Stigler (1978). Edgeworth's work in probability and statistics has been collected by McCann (ed.) (1996).

to economics. He wrote extensively on a wide range of topics, but the central theme of Edgeworth's work is clear in his revealing statement, taken from his Presidential address to Section F of the Royal Society, that:

> It may be said that in pure economics there is only one fundamental theorem, but that is a very difficult one: the theory of bargain in a wide sense. (1925, ii, p.288)

Taking as his starting point Jevons's (1871, in 1957) basic analysis of exchange of two goods between two traders, Edgeworth supposed that the objective of each trader is to maximise utility, considered to be a general function of the quantities of the goods held and consumed after trade is concluded. The utility-maximising approach was immediately congenial to Edgeworth, who was steeped in Utilitiarian moral philosophy. He first concentrated on the nature of barter, instead of describing the characteristics of an equilibrium set of prices, that is, one which ensures that the individuals' responses are mutually consistent. If the traders in barter are allowed freely to vary the terms of provisional 'contracts', Edgeworth showed that there is a range of 'final settlements', from which no further 'recontracting' would take place. In a rectangular box where the base and height are determined by the initial stocks of the two goods, these final settlements define what Edgeworth called the 'contract curve'. These settlements are also efficient trades, in the sense that if a settlement is not on the contract curve, movement to it can make one person better off without the other being worse off: this original idea of efficiency later came to be called Pareto efficiency. Movement along the contract curve involves one trader becoming worse off while the other gains.

Edgeworth then defined indifference curves for a trader as showing combinations of amounts consumed for which utility is constant. Using several approaches, he demonstrated that the contract curve is the locus of points of tangency between traders' indifference curves, between limits given by their pre-trade curves (those going through the initial endowment point in the box). The existence of a range of final settlements has important implications. First, without introducing further structure to the barter framework, it is not possible to say what the implied rate of exchange is, given only information about preferences and endowments of individuals. It results in 'indeterminacy' whereby all that can be said is that the actual trade depends on the relative bargaining strength of the traders.

On the argument that such higgling is widespread, Edgeworth stated in his unique style that, 'The whole creation groans and yearns, desiderating a principle of arbitration, and

end of strifes' (1881, p.51). His next argument involved two steps. First, he showed that the Utilitarian principle of maximising total utility places individuals on the contract curve, because the mathematical conditions are equivalent to the tangency of indifference curves. Indeed, if it is possible to make someone better off without someone being worse off, total utility cannot be a maximum and individuals cannot be on the contract curve. While this may seem a small step, to Edgeworth it was of great significance:

> It is a circumstance of momentous interest that one of the in general indefinitely numerous settlements between contractors is the utilitarian arrangement ... the contract tending to the greatest possible total utility of the contractors. (1881, p.53)

However, he recognized that this result is not sufficient to justify the use of Utilitarianism as a principle of arbitration; it is only a necessary condition. Edgeworth's justification for Utilitarianism as a principle of justice, comparing points along the contract curve, was as follows:

> Now these positions lie in a reverse order of desirability for each party; and it may seem to each that as he cannot have his own way, in the absence of any definite principle of selection, he has about as good a chance of one of the arrangements as another ... both parties may agree to commute their chance of any of the arrangements for ... the utilitarian arrangement. (1881, p.55)

The important point to stress about this statement is that Edgeworth clearly considered willingness to accept the Utilitarian arbitration in terms of choice under uncertainty. His argument is that the contractors, faced with uncertainty about their prospects but viewing alternatives along the contract curve as equally likely, would choose to accept an arrangement along Utilitarian lines. Thus a crucial component of this argument is the use of equal *a priori* probabilities, something that was later important to Edgeworth in his statistical work. In taking this second step Edgeworth believed that he had provided an answer to an age old question, stating, 'by what mechanism the force of self-love can be applied so as to support the structure of utilitarian politics, neither Helvetius, nor Bentham, nor any deductive egoist has made clear' (1881, p.128).

The importance to him of this new justification of Utilitarianism cannot be exaggerated. Indeed the whole of *Mathematical Psychics* seems to be imbued with a feeling of excitement generated by his discovery of this justification based on a 'social contract'. This provided

the crucial link between 'impure' and 'pure' utilitarianism in a more satisfactory way than his earlier appeal to evolutionary forces, made in his book on *New and Old Methods of Ethics*, written in 1877, before turning to economics.

The nature of price-taking behaviour – involving an equimarginal principle whereby the ratio of prices must be equal, for both traders, to the ratio of their marginal utilities for each of the relevant goods, had been explored with great originality by Jevons with his 'equations of exchange'. Edgeworth made important extensions to this analysis, as well as providing his succinct diagrammatical synthesis (which included showing, in 1881, p. 113, how Marshall's 'offer curves' can be derived from indifference curves). He showed how his box diagram can be used to illustrate a price-taking equilibrium. This arises where one or more of the mutual tangency positions of indifference curves along the contract curve also corresponds to tangency with a straight line going through the endowment point. This line represents a common budget constraint for the choices of the individuals, whereby the slope represents the exchange ratio and hence the relative price. In equilibrium, individuals acting in isolation and taking prices as given (in contrast to those engaged in barter) have mutually consistent demands and supplies. A price-taking equilibrium, as such a tangency position, must therefore correspond to a point on the contract curve.

Edgeworth was thus able to clarify the sense in which a price-taking (often called competitive) equilibrium is 'optimal', fully recognising that it is just one of many Pareto optimal points. This gives rise to what is now referred to as the 'First Fundamental Theorem' of welfare economics – that a price-taking equilibrium is Pareto efficient. The use of price-taking also provides a considerable reduction in the amount of information required by traders compared with barter. Individuals only need to know the equilibrium prices, whereas in barter they have to learn a considerable amount of information about other individuals' preferences and endowments. Of course, this merely describes the properties of an equilibrium and does not, as Edgeworth was fully aware, explain how it may be achieved in practice. However, he later showed that a sequence of price adjustments, where trading – at the minimum of demand and supply – takes place at disequilibrium prices, leads to a point on the contract curve although precisely where is indeterminate.

Edgeworth then returned to the indeterminacy in barter, asking whether this indeterminacy results from the absence of competition in the simple two-person market. Edgeworth quickly moved on to examine the implications of introducing further pairs of traders. The analysis of barter with numerous traders again involves Edgeworth's stylized description of the recontracting process of barter mentioned above. With more traders, the importance

of the recontracting process, apart from allowing the dissemination of information, lies in the fact that it makes it possible to analyse the use of collusion among some of the traders. Individuals are allowed to form coalitions in order to improve bargaining strength. Recontracting enables the coalitions to be broken up by outsiders who may attract members of a group away with more favourable terms of exchange.

The analysis of many traders, where coalitions can be temporarily formed and broken up by the offer of improved terms from other traders, would appear to present formidable difficulties. Yet Edgeworth rapidly demonstrated, again using his famous box diagram, that the introduction of further similar pairs of traders gradually reduces the range of indeterminacy; that is, the length of the contract curve shrinks. With a sufficiently large number of traders, the range of indeterminacy shrinks to the finite number of price-taking equilibria. Barter thus replicates price-taking behaviour. Given that coalitions among traders are allowed in the recontracting process, a price-taking equilibrium cannot be 'blocked' by a coalition of traders. In this sense the competitive equilibrium is robust.

The argument that a complex process of bargaining among a large number of individuals produces a result which is identical to a price-taking equilibrium is an important result that is far from intuitively obvious. The recontracting process can be said to represent a competitive process, and the contract curve shrinks essentially because of the competition between suppliers of the same good, although it is carried out in a barter framework in which explicit prices are not used. The price-taking equilibrium, in contrast, does not actually involve a competitive process. Individuals simply believe that they must take market prices as given and outside their control. They respond to those prices without any reference to other individuals. But the result is that the price-taking equilibrium looks just like a situation in which all activity is perfectly co-ordinated.

Great stress was placed by Edgeworth on comparison with Lagrange's 'Principle of least action' in examining the overall effects produced by the interactions among many particles. The connection with Edgeworth's analysis of competition, involving interaction among a large number of competitors to produce a determinate rate of exchange, is clear. The fact that in the natural sciences so much could be derived from a single principle was important for both Jevons and Edgeworth. But Edgeworth took this to its ultimate limit in arguing that the comparable single principle in social sciences, that of maximum utility, would produce results of comparable value. Referring to Laplace's massive work, *Mécanique Celeste*, he suggested that:

> 'Mécanique Sociale' may one day take her place along with 'Mécanique Celeste',

> throned each upon the double-sided height of one maximum principle, the supreme pinnacle of moral as of physical science . . . the movements of each soul, whether selfishly isolated or linked sympathetically, may continually be realising the maximum energy of pleasure, the Divine love of the universe. (1881, p.12)

A strong belief in the value of mathematical analysis in economics, even where the precise numerical form of the relevant relationships cannot be known, imbues all of Edgeworth's work. When this is combined with his strong adherence to Utilitarianism, it is not difficult to see how Edgeworth was excited to be showing not only why this principle may be accepted in the form of a 'social contract', but how the actions of many utility maximising individuals in a market can lead to a determinate solution. Thus, while the comparison with Laplace may seem fanciful to some readers, it was far from fanciful to Edgeworth. These elements provide the 'plan' with which virtually all his work in economics could be viewed. It is no wonder that Alexander Pope's statement, in his *Essay on Man*, that it presents 'A mighty maze, but not without a plan' was borrowed by Edgeworth to describe the competitive barter process. It also nicely fits Edgeworth's own oevre. Although he went on to write on a wide range of economic topics, and to make original contributions to mathematical statistics which alone would guarantee a lasting reputation, an appreciation of the preoccupations leading towards, and nature of, this first work is important in placing everything else in perspective.[3]

It is clear from even a small sample of Edgeworth's work that the writer brings to it not just a deep and fertile originality, but also a vast range of knowledge covering natural sciences and literature. His writing is highly allusive and contains quotations from Greek and Latin classics as well as well a range of English poets. It displays a sharp wit of a kind found in no other writing in the subject, and continues to repay repeated reading.

[3]His economic papers are collected in Edgeworth (1925).

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2 Philip Henry Wicksteed

Philip Henry Wicksteed (1844-1927) was an English economist who was also a Unitarian theologian (succeeding James Martineau at the Little Portland Street Chapel in London in 1974, and resigning in 1897), translator and classicist (with a particular interest in Dante) and literary critic.[4] Turning to economics after reading Henry George's (1879) *Progress and Poverty*, for many years he gave University of London Extension Lectures on economics (as part of an adult education programme). Robbins (1933, p. v) makes the point that 'there can be few men who have so successfully combined such a wide range of intellectual pursuits with such conspicuous excellence in each of them'. The greatest influence on his economics was Jevons's *Theory of Political Economy*, and he can be described, with Edgeworth, as a disciple of Jevons and a careful exponent of the subjectivist approach in which cost is interpreted in terms of foregone alternatives rather than as a 'real cost'. Robbins (1931, p. 229) describes how Wicksteed's copy of the second edition of Jevons's *Theory*, purchased in 1882, is covered with marginal annotations.

Wicksteed's first publication in economics was his 1884 criticism of Marx, the first along Jevonian lines by a British economist.[5] He published his first economics book, *The Alphabet of Economic Science* in 1888. This is primarily a pedagogic work expounding the utility theory of value, with a long introductory section on basic calculus. In this, he is responsible for introducing the term 'marginal utility' as an improvement on Jevons's 'final utility'. This was followed in 1894 by the celebrated *Essay on the Co-ordination of the Laws of Distribution* in which, in contrast to the earlier work, he states in the preface that, 'I address myself only to experts', although at the same time 'without any claim to originality' (1894, p. 3). Although the main elements of the marginal productivity theory of distribution, according to which (using modern terminology) factors receive their marginal revenue product (marginal physical product multiplied by marginal revenue, which in a competitive goods market is equal to price or average revenue), had been proposed by a number of authors, Wicksteed is famous for his original argument that the total remuneration of all factors will precisely exhaust total revenue. Hence there is no 'residual' available for distribution (in contrast to the classical approach in which rent is regarded as a residual). This led to the famous review by Alfred Flux (a Cambridge Senior Wrangler in mathematics in 1887 who, as a student of St. John's College, came into contact with Alfred Marshall). Flux made the important point that Wicksteed had implicitly assumed constant returns to scale, or linear

[4] Wicksteed's life is described by Herford (1931).

[5] This led to a debate with George Bernard Shaw.

homogeneous, production functions and, in addition, that the result is immediately given by the application of Euler's Theorem for homogeneous functions. For $f(x_1, ..., x_n)$ homogeneous of degree k, then Euler's Theorem states that $\sum_{i=1}^{n} x_i \frac{\partial f(x_1,...,x_n)}{\partial x_i} = kf(x_1, ..., x_n)$. Hence for $k = 1$ and perfectly competitive markets, the 'product exhaustion' result follows. Flux suggested that, 'there seems no need for delaying to prove a relation so well known as this, as Mr. Wicksteed does'. It may have been 'well known' to mathematicians, but Wicksteed was not a trained mathematician; indeed Herford (1931, p. 200) mentions that he had been taking lessons in calculus from John Bridge, a mathematics tutor at University College London.

The assumption of linear homogeneity, along with the continuous substitutability of factors in production, was subsequently strongly criticised by Pareto, Barone and a bad-tempered Walras who unfairly accused Wicksteed of plagiarism.[6] Furthermore, it elicited the following comment by Edgeworth, made in his unique style: 'There is a magnificence in this generalization which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage; and rational conduct is a homogeneous function, adds the modern savant'.[7] As a result of these attacks, Wicksteed himself became somewhat dissatisfied with his argument, though not of course with the main points of the marginal productivity theory.

In 1910 Wicksteed published his massive *Common Sense of Political Economy*, described by Robbins (1931, p. 235) as 'the most exhaustive non-mathematical exposition of the technical and philosophical complications of the so-called "marginal" theory of pure economics, which has appeared in any language'. Again, Wicksteed makes no claims of originality, but this book does contain a strong criticism of the partial equilibrium analysis of supply and demand, suggesting that it does not pay adequate attention to the role of stocks of goods. He went so far as to describe the standard diagrammatic analysis as 'profoundly misleading' (1910, in 1933, II, p. 785) and actually stated that there is 'no such thing' as a supply curve. He argued that although it is useful to separate the supply and demand sides of the market in considering the process of adjustment by which an equilibrium may be reached, this separation is of dubious value in examining the determinants of that price. He suggested that:

> the cross curves of demand and supply, so often employed by economists, are really no more than two sections of the true collective curve of demand, separated out from each other, and read, for convenience, in reverse directions.

[6] For further discussion of Wicksteed's contribution, see Stigler (1941, pp. 38-60).

[7] Edgeworth's 'ancient sage' is Aristotle, who in his *Nicomachean Ethics* argued that justice requires equality in all directions.

> These cross curves, then, as usually presented, confuse the methods by which the equilibrating price is arrived at with the conditions that determine what it is. (1933, II, pp. 797-8)

The situation he had in mind, along with other major neoclassical economists, was not of firms producing only for sale but of exchange. He envisaged the standard exchange situation in which individuals hold stocks of a good which are brought to a market. His examples include the results of a harvest, or the 'catch' of a fishing fleet (1933, II, p. 787). Those who hold stocks also consume the good and therefore have a demand for it comparable with that of individuals who do not hold stocks. This context is thus the same as that of Jevons and Walras, Mill and Marshall (when considering international trade) and Edgeworth, yet Wicksteed took a quite independent position: See Wicksteed (1933, I, p. 229-34; 1933, II, pp. 772-96; 1933, II, pp. 797-800; 1933, II, pp. 822-6). He referred to the conventional supply curve as a 'reverse demand curve' and argued:

> I say it boldly and baldly: there is no such thing ... what is usually called the supply curve is in reality the demand curve of those who possess the commodity.' (1933, II, p. 785)

Wicksteed's preferred diagram showed a curve relating the price to the total demand of possessor and non-possessors (on the horizontal axis). The equilibrium price is then obtained by the intersection of this total demand curve with a vertical line drawn from the total stock of the good available. He went on to suggest that:

> a change in its initial distribution (if the collective curve is unaffected, while the component or intersecting curves change) will have no effect on the market, or equilibrating price itself, which will come out exactly the same. (1933, II, pp. 785-6)

However, this result was simply assumed by Wicksteed, who failed to recognise that the basic assumption, that a change in the allocation of stocks does not affect the total demand curve, requires very special conditions and generally will not hold: for details see Creedy (1991). In criticising the partial equilibrium demand and supply analysis, Wicksteed simply replaced it with another partial equilibrium approach, instead of using the exchange context of Jevons, Walras and Edgeworth which in fact made explicit the stocks of goods held by traders in exchange.

Wicksteed's strong rejection of the supply curve is of course associated with his 'Austrian' view that all productive resources are ultimately fixed in 'supply' and that cost must be seen in terms of opportunity cost; see, for example, Hutchison (1953, p. 104). Again, this is entirely consistent with an emphasis on exchange. As stressed by Fraser (1937, p. 104), the view of cost in terms of foregone alternatives is 'merely the extension of the exchange relationship to the whole range of economic life', which was of course the agenda set out by Jevons. This makes Wicksteed's approach rather curious.

His interest in Jevons also led Wicksteed to critise Jevons's discussion of the famous King-Davenant law of demand, where Jevons provided a functional form which he 'fitted' to the basic data which were presented by Davenant in tabular form. Wicksteed actually recognised that a third degree polynomial fits the data points exactly, and gave the parameters, adding that it 'can hardly fail to stimulate curiosity as to the origin of this most interesting estimate, and the grounds on which it was formed' (1933, II, p. 738). He acknowledged the help of Bridge in finding the polynomial, using the 'method of differences'. Yet it is interesting that neither Wicksteed nor Jevons recognised that Whewell had earlier given the precise form of the polynomial, and yet Jevons explicitly referred to Whewell; for details see Creedy (1986).

Wicksteed's reputation stands because of his serious and extended analyses of fundamental theoretical questions. Although these analyses attracted the attention of a only a small number of his contemporaries, they were without question the leaders of the economics profession. At a time when economics was becoming dominated by established academics, this was a remarkable achievement by one who was clearly an 'outsider'. More general readers are indeed likely to find his expansive style difficult to penetrate, yet it is likely that those concerned with basic questions will continue to find much interest and food for thought in his works.

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