Financial Innovations and Endogenous Growth

Yuan K. Chou and Martin S. Chin
University of Melbourne

July 30, 2001

Abstract

This paper explores the channels through which innovations in the financial sector lead to economic growth. The channels identified are capital accumulation and technological innovation. The first is fulfilled by financial intermediaries which transform household savings into productive investment by firms, the second by venture capitalists which fund risky technological projects with high potential payoffs. The rate of financial innovation is determined by the amount of labor (or human capital) devoted to the sector as well as by spillovers from existing financial products. By embedding such a sector into the Romer (1990) - Jones (1995) and Lucas (1988) - Uzawa (1965) frameworks, it is shown that ultimately, financial innovations can only lead to long-run growth through its venture capital role. The transformative role of the financial sector only leads to temporary growth effects on the transitional path to the steady state.

Keywords: Economic Growth, Finance, Technological Change
JEL Codes: G20, O31, O33, O41

1 Introduction

Why does the financial sector matter to the real economy? The ever-rising number of graduates from top American and European universities being recruited by financial powerhouses and their handsome remuneration lend credence to the suggestion that the financial sector must be a highly valuable engine of growth in an advanced economy. Extensive media coverage of the activities of the finance industry seem to confirm its pre-eminence. Even in the other pillar of the New Economy, the real technological sector, financial firms in the guise of venture capitalists are seen as the key to inducing high-risk, potentially high-return ideas and innovations. However, the standard theoretical growth literature (including the New Growth Theory of the last fifteen years) notably excludes any meaningful role for the financial sector to influence long-run growth. Savings by households are automatically assumed to be transformed into productive investment by firms at every point in time by appealing to the "savings equal investment in equilibrium" argument. The vast majority of papers that have been written on the "finance-growth nexus" describe microeconomic models that detail how financial institutions alleviate borrowing constraints, perform risk management, acquire information to assist in resource allocation, monitor managers, mobilize savings and lead to rising specialization and efficiency in production. There is also an extensive literature on the empirical evidence linking development of the finance sector to economic growth [see, for example, Arestis and Demetriades (1997), Demetriades and Hussein (1996) and King and Levine (1993a)].

This paper aims to fill that important gap in the literature by explaining how a financial sector can be incorporated into an endogenous growth macroeconomic model such as those by Romer (1990) and Lucas (1988). Just as Romer constructs a dynamic equation describing the production of new designs or blueprints in the research and development sector, we develop a dynamic equation describing the production of financial innovations that continuously improves the efficiency of the intermediation process which transforms savings into investment and lubricates R&D activities in the real technological sector. In addition, the financial innovations sector may also be modelled in competition with a human capital producing sector for that scarce resource a là Lucas. We also explain the complicated ways in which households, financial innovators, financial intermediaries, R&D firms, intermediate and final goods producers interact and are intertwined in our model of the macroeconomy. Finally, we distinguish between the competitive, decentralized solution and that of a hypothetical social planner.

Levine (1997) lays out a sound theoretical approach to the study of the relationship between finance and growth. He argues that market frictions such as information and transaction costs motivate the emergence of financial markets and intermediaries, which serve multiple functions: facilitating the trading, hedging, diversifying, and pooling of risk; allocating resources; monitoring managers and exerting corporate control; mobilizing savings; and facilitating the exchange of goods and services. These financial functions in turn affect economic growth through the channels of capital accumulation and technological innovation. Figure 1, reproduced from Levine (1997), summarizes his theoretical approach. In the context of this approach, our paper may be seen as an elaboration on the *channels of growth*, while almost all of the existing literature relate to the *functions of financial systems*. For example, the impact of financial development on growth through its effect on borrowing constraints is studied by Bencivenga and Smith (1993), Japelli and Pagano (1994) and de Gregorio (1996). Bencivenga and Smith (1991) and Obstfeld (1994) study the impact of financial development on growth through its facilitation of risk management. Acemoglu and Zilibotti (1997) look at savings mobilization, while King and Levine (1993b) construct a model in which financial systems evaluate prospective entrepreneurs, mobilize savings to finance the most promising productivity-enhancing activities, and diversify the risks associated with these innovative activities, thereby improving the probability of successful innovation. Similarly, Saint-Paul (1992) looks at how capital markets facilitate the adoption of more specialized and productive technologies. Broadly speaking, our paper complements the existing literature by telling a macroeconomic story of how the production of financial innovations affect growth through capital accumulation and technological innovation, while the existing literature provide rich detailed examples of how financial markets and intermediaries fulfil their financial functions from a microeconomic perspective.

Figure 1: Levine's (1997) Theoretical Approach to Finance and Growth

The paper is organized as follows: the next section describes a basic growth model with a financial sector and derives its analytical solution. Section 3 delves into the details of a growth model with both endogenous technological progress and financial innovations, explaining the decentralized model in considerable detail, and explores the comparative statics of its solution. Section 4 examines a model with human capital and financial innovations and graphs the implications of its solutions, while Section 5 discusses the policy implications arising from these models. Section 6 concludes.

2 The Basic Model

To isolate the workings of our proposed financial sector, we first embed it in a standard no-frills growth model with intertemporal household optimization but without endogenous technological progress. We will see that, unsurprisingly, the model is incapable of generating endogenous growth in the steady-state. The efficiency and development of the financial sector generates non-zero growth in per-capita variables only on the transitional path to the steady state. Moreover, changes in the production function of financial innovations (or new financial products) generate only level but not growth effects.

The financial sector in this model comprises financial innovators and financial intermediaries. The former produce new financial "blueprints" (products and services) using labor that is diverted from the production of the final consumption good. These "blueprints" include innovations such as ATMs, phone and internet banking, derivatives of existing financial products (including new options), initial public offerings (IPOs) of companies and anything which enables funds to be channelled more effectively from savers (households) to borrowers (firms seeking to raise capital to finance the purchase of new plant and equipment). We denote the stock of financial products (ie. old financial innovations) as τ. Analogous to the Romer (1990) specification of the real R&D sector, the development of the financial sector is characterized by an ever-expanding variety of financial products. For simplicity, there is no "creative destruction" of existing financial products by successively superior products That is, there are no quality ladders in financial products. However, the existing stock of financial innovations/products affect the production of new financial ideas according to

$$\dot{\tau} = F(u_\tau L)^\lambda \tau^\phi,$$

where u_τ is the fraction of the labor force employed by the financial sector, and F, λ,and ϕ are constants.

The idea is that of a spillover effect from each financial innovation: financial innovators may build upon the ideas of other innovators to create a differentiated or improved financial product.

Financial intermediaries, on the other hand, are responsible for intermediating funds between borrowers and lenders. Borrowers are producers of the final consumption good while lenders are households with savings. The efficiency at which savings can be transformed into productive investment is specified to be dependent on the existing stock of financial innovations/products per adjusted capita (τ/L^κ, which we will label as ξ, $0 < \kappa < 1$), which proxies for the state of development and sophistication of the financial sector. The capital accumulation function hence looks like:

$$\dot{K}(t) = \frac{\tau(t)}{L(t)^{\kappa}} \left[AK(t)^{\alpha} \left(u_Y(t) L(t)\right)^{1-\alpha} - C(t) \right] - \delta K(t),$$

where K is the stock of capital, L is the number of workers, A is a (constant) technological parameter, and u_Y is the share of labor devoted to final goods production.[1] By including κ in our measure of transformative efficiency ξ, we are acknowledging that some financial innovations may be rivalrous (such as the creation of each new IPO, which may benefit from the knowledge gained from previous IPOs but nevertheless requires new labor to be expended in order to tailor it to the needs of individual firms) while others are not (such as a new financial instrument, which may in fact benefit from "thick market" effects as it becomes more widely traded). By restricting κ to lie strictly between 0 and 1, we are saying that in the *aggregate,* financial innovations or products are neither fully rivalrous nor fully non-rivalrous.[2]

In the steady state, τ/L^{κ} must be constant by definition. Therefore, if the labor force grows at the constant rate n, then the rate of financial innovations in the steady state must equal κn. Why must the number of financial products continually increase in the steady state even when all savings are completely transformed into investment? We argue that as the labor force or population increases, so does the volume of funds that have to be intermediated. Due to the rivalrous nature of some financial products and services, this rising volume results in congestion and decreased efficiency in the financial sector unless more financial products are devised to alleviate the strain on it. Loosely speaking, resources such as labor must continue to be directed to the financial sector as it services an expanding economy.

2.1 The Social Planner's Problem

We now proceed to lay out the social planner's problem and discuss the steady-state solutions of the model. The social planner seeks to maximize the representative consumer's stream of discounted utility assuming a Constant Relative Risk Aversion (CRRA) utility function:

$$\max_{c(t), u_Y(t)} U_0 = \int_0^{\infty} \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \tag{1}$$

[1] Pagano (1993) speciøes the saving-investment relationship as $\phi S = I$, where $1-\phi$ is the Æow of saving lost in the process of ønancial intermediation. This (exogenous, in his case) fraction goes to banks as the spread between lending and borrowing rates, and to securities brokers and dealers as commissions, fees and the like (pp. 614-615).

[2] If $\kappa = 1$, then all ønancial products are strictly rivalrous; if $\kappa = 0$, then all ønancial products are strictly non-rivalrous, so that the e ciency of ønancial intermediation is dependent only on the stock of ønancial products and independent of population size.

where $c \equiv C/L$, subject to

$$\dot{K}(t) = \frac{\tau(t)}{L(t)^{\kappa}} \left[AK(t)^{\alpha} (u_Y(t) L(t))^{1-\alpha} - C(t)\right] - \delta K(t), \quad (2)$$

$$\dot{\tau}(t) = F\left[(1-u_Y(t)) L(t)\right]^{\lambda} \tau(t)^{\phi}, \quad (3)$$

where

$$L(t) = L(0) e^{nt}. \quad (4)$$

Note that $\alpha \in (0,1)$, $u_Y(t) \in [0,1]$ $\forall t$ and $\{\theta, \rho, \delta, n\} > 0$.

2.1.1 Model Set-Up

The Hamiltonian is

$$\begin{aligned} \mathsf{H} \equiv{}& \frac{c^{1-\theta}-1}{1-\theta} e^{-\rho t} + \nu \left[\frac{\tau}{L^{\kappa}} \left(AK^{\alpha} u_Y^{1-\alpha} L^{1-\alpha} - C\right) - \delta K\right] \\ & + \mu F (1-u_Y)^{\lambda} L^{\lambda} \tau^{\phi}, \end{aligned} \quad (5)$$

where the control variables are c and u_y, the state variables are K and τ, and ν and μ are the costate variables associated with K and τ respectively. The first-order conditions for the control variables are

$$\frac{\partial \mathsf{H}}{\partial C} = c^{-\theta} e^{-\rho t} - \nu \tau = 0, \quad (6)$$

$$\frac{\partial \mathsf{H}}{\partial u_y} = \nu \tau A K^{\alpha} (1-\alpha) u_Y^{-\alpha} L^{1-\alpha-\kappa} - \mu F \lambda (1-u_Y)^{\lambda-1} L^{\lambda} \tau^{\phi} = 0. \quad (7)$$

The first-order conditions for the state variables are

$$\frac{\dot{K}}{K} = \frac{\tau}{L^{\kappa}} \left(AK^{\alpha-1} u_Y^{1-\alpha} L^{1-\alpha} - \frac{C}{K}\right) - \delta, \quad (8)$$

$$\frac{\dot{\tau}}{\tau} = F (1-u_Y)^{\lambda} L^{\lambda} \tau^{\phi-1}. \quad (9)$$

The first-order conditions for the costate variables are

$$\dot{\nu} = -\frac{\partial \mathsf{H}}{\partial K} = -\nu \left(\tau A \alpha K^{\alpha-1} u_Y^{1-\alpha} L^{1-\alpha-\kappa} - \delta\right), \quad (10)$$

$$\dot{\mu} = -\frac{\partial \mathsf{H}}{\partial \tau} = -\nu \left(AK^{\alpha} u_Y^{1-\alpha} L^{1-\alpha-\kappa} - \frac{C}{L^{\kappa}}\right) \quad (11)$$

$$-\mu F (1-u_Y)^{\lambda} L^{\lambda} \phi \tau^{\phi-1}. \quad (12)$$

Finally, the transversality conditions are

$$\lim_{t\to\infty} \nu(t) K(t) = 0, \quad (13)$$

$$\lim_{t\to\infty} \mu(t) \tau(t) = 0. \quad (14)$$

2.1.2 Variables in the Steady State

To arrive at the steady-state solutions, we first define the following three variables $k \equiv K/L$, $\chi \equiv C/K$ and $\xi \equiv \tau/L^{\kappa}$. In the steady state, we require the output-capital ratio given by

$$\frac{Y}{K} = Ak^{\alpha-1}u_Y^{1-\alpha}, \tag{15}$$

to remain constant. This implies, from equation (15), that

$$\frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} = (\alpha - 1)\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{u}_Y}{u_Y} = 0. \tag{16}$$

We also require that $\dot{u}_Y/u_Y = 0$ in the steady state. Hence, $\dot{k}/k = 0$ as well in order to satisfy equation (16), so the growth rate of output per capita y/y is zero. Furthermore, it is assumed that $\dot{\chi}/\chi = \dot{\xi}/\xi = 0$ in the steady state. Since L grows at the exogenous rate n according to equation (4), these assumptions imply that, to have a balanced growth path, we must have $\dot{Y}/Y = \dot{K}/K = \dot{C}/C = n$ and $\dot{\tau}/\tau = \kappa n$ in the steady state. From $\dot{\tau} = Fu_\tau^{\lambda}L^{\lambda}\tau^{\phi}$, we have $\gamma_\tau \equiv \dot{\tau}/\tau = Fu_\tau^{\lambda}L^{\lambda}\tau^{\phi-1}$. Taking logarithms of the latter equation and differentiating both sides with respect to time provide us the solution to the steady-state growth rate of τ, $\gamma_\tau^* = \lambda n/(1-\phi)$. Since $\dot{\tau}/\tau = \kappa n$, the solution implies that $\kappa = \lambda/(1-\phi)$. The steady-state growth rate of n is comparable to the Cass (1965) - Koopmans (1965) formulation of the Solow (1956) - Swan (1956) model without technological progress. In their model, the absence of technological progress eventually results in aggregate output growing at rate n since there are no increases in productivity to offset the diminishing marginal product of physical capital. Here, output grows at rate n in the steady state since there are limits to the efficiency of financial innovations in transforming the flow of savings to new physical capital.

The model is solved in terms of the four unknowns k, χ, ξ and u_Y. The four equations needed to pin down the solutions to the four unknowns are given by $\dot{k}/k = 0$, $\dot{\chi}/\chi = 0$, $\dot{\xi}/\xi = 0$ and $\dot{u}_Y/u_Y = 0$. These four conditions lead to the following equations respectively:

$$\xi Ak^{\alpha-1}u_Y^{1-\alpha} - \xi\chi = n + \delta, \tag{17}$$

$$\xi A\alpha k^{\alpha-1}u_Y^{1-\alpha} = \rho + n + \delta, \tag{18}$$

$$F(1-u_Y)^{\lambda}\xi^{\phi-1} = \frac{\lambda n}{1-\phi}, \tag{19}$$

$$\frac{\lambda^2 n}{(1-\alpha)(1-\phi)}\frac{u_Y}{1-u_Y}\left[1 - \left(\xi^{-1}A^{-1}k^{1-\alpha}u_Y^{\alpha-1}\right)\xi\chi\right] = \xi A\alpha k^{\alpha-1}u_Y^{1-\alpha} - \delta - (1-\lambda)n. \tag{20}$$

2.1.3 Analytical Solutions to the Model

Using equations (17) to (20), we obtain the following solutions for u_Y, u_τ, ξ, χ and k:

$$u_\tau^* = \frac{\Gamma}{\Gamma + \Phi}, \tag{21}$$

where $\Gamma \equiv \alpha\lambda\gamma_\tau^*(n+\delta)$, $\Phi \equiv (1-\alpha)(\rho+\lambda n)(\rho+n+\delta)$, $\gamma_\tau^* = \lambda n/(1-\phi)$,

$$\begin{aligned} u_Y^* &= 1 - u_\tau^* \\ &= \frac{\Phi}{\Gamma+\Phi}, \end{aligned} \tag{22}$$

$$\xi^* = \left[\frac{F u_\tau^{*\lambda}}{\gamma_\tau^*}\right]^{\frac{1}{1-\phi}} \tag{23}$$

$$= \left[\frac{F}{\gamma_\tau^*}\left(\frac{\Gamma}{\Gamma+\Phi}\right)^{\lambda}\right]^{\frac{1}{1-\phi}}, \tag{24}$$

$$\begin{aligned} \chi^* &= \frac{\rho + (1-\alpha)(n+\delta)}{\alpha\xi^*} \\ &= \frac{\rho + (1-\alpha)(n+\delta)}{\alpha}\left[\frac{\gamma_\tau^*}{F}\left(\frac{\Gamma+\Phi}{\Gamma}\right)^{\lambda}\right]^{\frac{1}{1-\phi}}, \end{aligned} \tag{25}$$

$$\begin{aligned} k^* &= \left(\frac{\xi^* A\alpha}{\rho+n+\delta}\right)^{\frac{1}{1-\alpha}} u_Y^* \\ &= \left[\frac{A\alpha}{\rho+n+\delta}\left[\frac{F}{\gamma_\tau^*}\left(\frac{\Gamma}{\Gamma+\Phi}\right)^{\lambda}\right]^{\frac{1}{1-\phi}}\right]^{\frac{1}{1-\alpha}} \frac{\Phi}{\Gamma+\Phi}. \end{aligned} \tag{26}$$

2.2 Implications of the Model

Proposition 1 *The financial innovations sector has no influence on the steady-state growth rate of the economy.*

In the steady state, the variables Y, K, and C all grow at the rate n, the population growth rate, in order to achieve a balanced growth path, while τ grows at rate κn. In spite of its role in transforming funds into productive investments, the financial innovations sector does not alter the balanced growth path requirement at all.

We relegate the proofs of the following propositions to the Appendix.

Proposition 2 *The steady-state proportion of labor employed in the financial innovations sector, u_τ^*, is lower in the decentralized economy than in the social planner's case.*

The divergence arises because the social planner internalizes the spillover effects of existing financial products on financial innovations.

We now discuss the implications of the model with regard to the steady-state proportion of labor employed in the financial innovations sector, u_τ^*. We specifically investigate the impact on u_τ^* of a change in the following parameters: (i) the spillover parameter in the financial innovations sector, ϕ; (ii) the rate of time preference, ρ; and (iii) the degree of risk aversion, θ.

Proposition 3 *An increase in the financial innovations spillover effect, ϕ, increases the steady-state proportion of labor employed in the financial innovations sector, u_τ^*.*

An increase in ϕ raises the marginal product of labor of financial innovators. The share of labor in the financial innovations sector must thus rise so that the wage in this sector once again equals that of the final goods sector in the new equilibrium.

Proposition 4 *An increase in the rate of time preference (or households' discount factor), ρ, decreases the steady-state proportion of labor employed in the financial innovations sector, u_τ^*.*

As households become more impatient, they care more for current consumption then future consumption. Hence, more labor is devoted to the final goods sector to produce the final consumption good, and correspondingly less labor is devoted to the financial innovations sector..

Proposition 5 *The degree of risk aversion, θ, does not affect the steady-state proportion of labor employed in the financial innovations sector, u_τ^*.*

2.3 Transitional Dynamics

To discuss the properties of the model away from the steady state, we need to reduce the dimensionality of the problem by assuming that the share of labor in the financial innovations sector, u_τ, and the physical investment rate, s_K, are constant and exogenous. The model then reduces to

$$\begin{aligned} Y &= AK^\alpha(1-u_\tau)^{1-\alpha}L^{1-\alpha} \\ \dot{K} &= \xi s_K Y - \delta K \\ \dot{\tau} &= F u_\tau^\lambda L^\lambda \tau^\phi. \end{aligned}$$

In the steady state, $\dot{k} = \dot{\xi} = 0$. The $\dot{k} = 0$ and $\dot{\xi} = 0$ schedules are given by

$$
\begin{aligned}
k^* &= \left[\frac{\xi^* s_K A}{n+\delta}\right]^{\frac{1}{1-\alpha}} (1 - u_\tau) \\
\xi^* &= \left[\frac{F u_\tau^\lambda (1-\phi)}{\lambda n}\right]^{\frac{1}{1-\phi}}
\end{aligned}
$$

and are plotted in the phase diagram below:

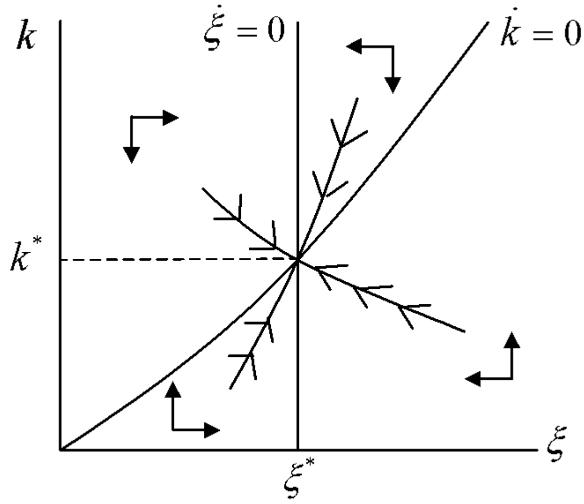

Figure 2: Transitional Dynamics of the Basic Model

Suppose the productivity parameter in the production function for financial innovations, F, increases, possibly due to efficiency-promoting deregulation of the financial sector The rise in F shifts the $\dot{\xi} = 0$ schedule to the right but leaves the $\dot{k} = 0$ schedule unchanged. >From the diagram below, we see that both k and ξ must rise smoothly along the saddle path to their new levels. The increase in F has no effect on the long-run growth rates of k and y (which still remain at zero because there is no technological progress), but it has temporary growth effects in the transition to the new steady state at higher levels of k^* and y^*.

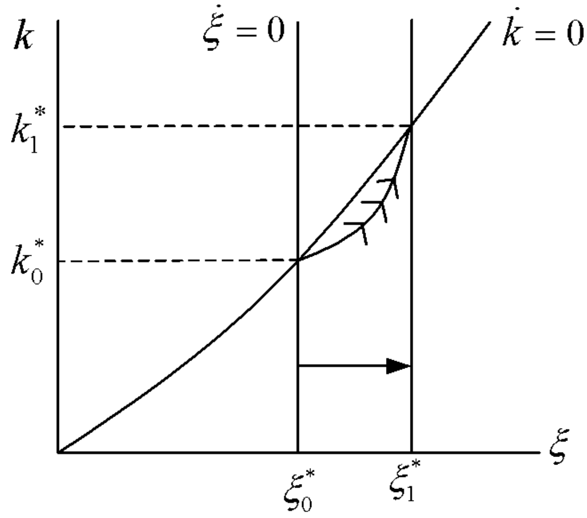

Figure 3: Transitional Dynamics for an Increase in F

3 Financial Innovations and Endogenous Technological Progress

We now examine a full-blown growth model with a financial sector akin to that in the previous section as well as endogenous technological progress in the mold of Romer (1990) and Jones (1995). In this class of models, technological progress is characterized by an increasing variety of intermediate goods used in the production of the final consumption good. Unlike the "creative destruction" models of Aghion and Howitt (1992) and Grossman and Helpman (1991c), the producers of these intermediate goods never lose the monopoly rights to their production nor are they ever superseded by new producers. The blueprints for new intermediate goods are in turn created by a real research and development sector which draws labor away from final goods production.

We allow for the stock of financial innovations, τ, to influence the rate at which new designs for intermediate goods are produced in the R&D sector. Implicitly, we are using the stock of financial innovations as a proxy for the stage of development of an economy's financial sector: a more sophisticated financial sector is associated with a higher innovation rate. This formulation attempts to capture the role that venture capitalists play in encouraging high-risk R&D activities with potentially large technological payoffs. We argue that these venture capital firms are only ubiquitous in economies with highly-developed and sophisticated financial sectors.

As before, the financial sector consists of financial innovators who create new financial products, and financial intermediaries who use the existing stock of financial products to intermediate funds between households and firms by transforming the savings of the former into productive investment by the latter. Unlike the model discussed in the previous section, however, now financial intermediaries (or more accurately, their venture capitalist arms) also extract rents from the real R&D sector for identifying and financing high-risk research projects with potentially huge future pay-offs. For tractability's sake, we do not differentiate between financial innovations which improve the efficiency of the intermediation process and those which make the financing of ever-riskier projects possible.

In the rest of this section, we first present the decentralized, competitive model, followed by a discussion of the characteristics and implications of the planner's solution. The decentralized model will explain how the different actors (households, final goods firms, intermediate goods producers, real R&D firms, financial intermediaries and financial innovators) and constituent components of the model function and interact. A flowchart of the model is illustrated in Figure 2.

3.1 The Decentralized Model

As in Jones (1995), the final goods sector produces the consumption good Y using labor $u_Y L$ and a collection of intermediate inputs x, taking the available variety of intermediate inputs A as given:

$$Y = (u_Y L)^{1-\alpha} \int_0^A x(i)^\alpha \, di. \tag{27}$$

This specification of the production function characterizes technological change as increasing variety, as in Dixit and Stiglitz (1977). Inventions are basically the discovery of new varieties of producer durables that provide alternative methods of producing the final consumer good.

A representative producer of final goods solves the following profit maximization problem

$$\max_{u_Y, x(i)} \pi_Y = (u_Y L)^{1-\alpha} \int_0^A x(i)^\alpha \, di - w_Y u_Y L - \int_0^A p(x(i)) \, x(i) \, di, \tag{28}$$

where w_Y is the prevailing wage in the final goods sector and $p(x(i))$ is the price of intermediate good i. The price of the final good is normalized to unity. The first-order conditions dictate that

$$w_Y = (1-\alpha) \frac{Y}{A u_Y L}, \tag{29}$$

and

$$p(x(i)) = \alpha u_Y^{1-\alpha} L^{1-\alpha} x(i)^{\alpha-1} \quad \forall i. \tag{30}$$

The intermediate sector comprises an infinite number of firms on the interval $[0, A]$ that have purchased a design from the real R&D sector, now acting as monopolists in the production of their specific variety. Following Romer (1990) and Jones (1995), each firm rents capital at rate r_K and, using the previously purchased design, effortlessly transforms each unit of capital into a single unit of the intermediate input. (For simplicity, producer durables are transformed costlessly back into capital at the end of the period and no depreciation takes place.) Each intermediate firm therefore solves the following problem period-by-period:

$$\max_x \pi_x = p(x)\,x - r_K x. \tag{31}$$

Being monopolists, they see the downward-sloping demand curve for their producer durables generated in the final goods sector. This results in a standard monopoly problem with constant marginal cost and constant elasticity of demand, giving rise to the following solutions:

$$\bar{p}(i) = \bar{p} = \frac{r_K}{\alpha} \quad \forall i, \tag{32}$$

$$\bar{x}(i) = \bar{x} = \left[\frac{\alpha\,(u_Y L)^{1-\alpha}}{\bar{p}}\right]^{\frac{1}{1-\alpha}} \quad \forall i, \tag{33}$$

and

$$\pi_{x(i)} = \bar{\pi}_x = (1-\alpha)\,\bar{p}\bar{x} = \alpha(1-\alpha)\frac{Y}{A} \quad \forall i. \tag{34}$$

Each intermediate firm thus sets the same price and sells the same quantity of its produced durable. Moreover, since

$$K = \int_0^A \bar{x}\,di = A\bar{x}, \tag{35}$$

we can rewrite the aggregate final goods production function as

$$Y = K^{\alpha}\,(A u_Y L)^{1-\alpha}. \tag{36}$$

Next, we examine the production of new designs in the real R&D sector. Here, the rate of innovation is governed by the following production function

$$\begin{aligned} \dot{A} &= \widetilde{B}\,[(1-u_Y-u_\tau)\,L]^{\eta}\,\tau^{\beta}, && (37)\\ \widetilde{B} &\equiv BA^{\psi}, && (38) \end{aligned}$$

where $(1-u_Y-u_\tau)$ is the share of labor devoted to the production of new technical designs. In the decentralized model, R&D firms do not take into account spillovers from existing designs, A^{ψ}, so they regard $\widetilde{B}$ as exogenously given. As argued previously, a more sophisticated financial sector (with a greater stock of financial innovations, τ) is associated with a higher innovation rate.

Each R&D firm derives revenue from the sale of blueprints to intermediate goods producers, $P_A \dot{A}$, and incurs costs $w_A (1 - u_Y - u_\tau) L$ from labor hired, and $R_\tau \tau$ from services rendered by financial intermediaries. Its profits are therefore

$$\pi_A = P_A \dot{A} - w_A (1 - u_Y - u_\tau) L - R_\tau \tau, \tag{39}$$

where L and τ are both compensated according to their marginal productivities in R&D production:

$$w_A = P_A \tilde{B} \eta \left[(1 - u_Y - u_\tau) L \right]^{\eta - 1} \tau^{\beta}, \tag{40}$$

$$R_\tau = P_A \tilde{B} \left[(1 - u_Y - u_\tau) L \right]^{\eta} \beta \tau^{\beta - 1}, \tag{41}$$

where w_A is the prevailing wage in the real R&D sector, R_τ is the "rental rate" of τ charged by financial intermediaries, and P_A is the price of each new technical design.

In our model, the financial sector is composed of financial innovators and financial intermediaries-*cum*-venture capitalists. The former are responsible for producing financial innovations, τ, which then determines the degree of sophistication of the financial sector, proxied by ξ (equal to the ratio τ / L^{κ}, or the number of financial innovations per adjusted capita). A greater value of ξ allows more efficient intermediation between lenders (households) and borrowers (intermediate goods producers), resulting in a higher percentage of savings being transformed into useful capital. In addition, a greater value of ξ also raises the rate at which new R&D designs are produced, as explained previously.

Financial innovators are monopolists who make extra-normal profits by producing new financial products, using raw labor as input, according to the production function

$$\dot{\tau} = \widetilde{F} (u_\tau L)^{\lambda}, \tag{42}$$

where

$$\widetilde{F} \equiv F \tau^{\phi}.$$

As in the real R&D sector, financial innovators do not internalize the spillover effect from the existing stock of financial products. They therefore treat $\widetilde{F}$ as exogenously given.

The profit of a representative financial innovator, to be maximized by its choice of u_τ, is

$$\pi_\tau = P_\tau \dot{\tau} - w_\tau u_\tau L, \tag{43}$$

where P_τ is the price of each financial innovation. Since ξ is constant in the steady state (and specifically equals 1), $\dot{\tau}/\tau = \kappa n$ in the steady state. With these substitutions, the first order condition dictates that

$$\hat{p}_\tau \equiv \frac{P_\tau}{AL^{1-\kappa}} = \frac{\hat{w}_\tau u_\tau}{\xi \lambda \gamma_\tau^*}, \tag{44}$$

where n is the population growth, $\hat{w}_\tau = w_\tau / A$, $\xi \equiv \tau / L^\kappa$ and $\gamma_\tau^* = \lambda n / (1-\phi)$. From this equation, we see that the price of each financial innovation is simply a constant mark up on the marginal factor cost of labor in the financial innovations sector.

Downstream in the financial sector, financial intermediaries purchase innovations from financial innovators (which, in the real world, are probably sister divisions in the same financial firms) and use them in transforming savings into productive investment as well as in the funding of real R&D activities. The financial intermediaries derive their income from: (a) charging the R&D firms the rate R_τ to finance their production of new designs; and (b) by charging firms in the (real) intermediate sector a higher interest rate (r_K) for renting capital than it pays out to households for their savings (r_V). The interest rate differential, $(r_K - r_V)$, may be thought of as the commission charged for intermediating funds. For simplicity, we assume that financial intermediation requires no labor input. Financial intermediaries make zero profits as this sector is assumed to be perfectly competitive.

In each period, the representative financial intermediary ensures that revenues received from the real intermediate sector and R&D firms equal the cost of acquiring deposits from households and purchasing new products from financial innovators:

$$r_K K + R_\tau \tau = r_V K + P_\tau \dot{\tau}. \tag{45}$$

Finally, to close the model, we examine the consumption decision of households. As usual, we assume that this decision may be characterized by a representative consumer maximizing an additively separable utility function subject to a dynamic budget constraint. We use a conventional CRRA utility function and assume that households are ultimate owners of all capital and shareholders of final goods firms, real intermediate firms, R&D firms, financial intermediaries and financial innovators. The optimization problem is thus:

$$\max_{c, u_Y, u_\tau} \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{46}$$

subject to

$$\begin{aligned} \dot{V} &= r_V K + w_Y u_Y L + w_\tau u_\tau L + w_A u_A L \\ &\quad + A\pi_x + \pi_\tau + \pi_A - P_A \dot{A} - C, \qquad (47) \\ \dot{K} &= \xi \dot{V}, \qquad (48) \end{aligned}$$

where $\dot{V}$ represents the flow of households' stock of assets (i.e. saving), π_x, π_τ and π_A are the monopolistic profits from the real intermediate sector, the financial innovators and the R&D sector respectively. The monopolistic pofits of financial innovators, π_τ, equal to revenue $P_\tau \dot{\tau}$, less labor costs $w_\tau u_\tau L$, is paid out to households who are alsoshareholders of these firms. Unlike Jones (1995), the real R&D sector is also allowed to generate monopolistic profits which again accrue ultimately to households. In equilibrium, wages are equal across all labor markets, i.e. $w_Y = w_\tau = w_A = \bar{w}$. These conditions together with equation (45) yield the following households budget constraint

$$\begin{aligned}\dot{K} &= \xi \left(r_K K + \bar{w} u_Y L + \bar{w} u_A L + R_\tau \tau \right. \\ &\qquad \left. + A \bar{\pi}_x + \pi_A - P_A \dot{A} - C \right). \end{aligned} \tag{49}$$

We can show that the prices of R&D blueprints and financial innovations are determined by the following arbitrage equations respectively:

$$\xi r_K = \frac{\bar{\pi}_x}{P_A} + \frac{\dot{P}_A}{P_A} \tag{50}$$

$$\xi r_K = \frac{\dot{V}}{P_\tau \tau} + \frac{\dot{P}_\tau}{P_\tau} \tag{51}$$

Equation (50) states that the opportunity cost to an intermediate producer of investing in a R&D blueprint, $\xi r_K P_A$, must equal the flow of profits that it generates, $\bar{\pi}_x$, and its associated capital gain, $\dot{P}_A$. Equation (51) similarly indicates that the opportunity cost to a financial intermediary of purchasing a financial innovation, $\xi r_K P_\tau$, must be equal to the average flow of savings intermediated by a unit of financial product, $\dot{V}/\tau$, and the associated capital gain, $\dot{P}_\tau$.

The solutions for the steady-state levels of u_A and u_τ, the shares of labor devoted to the real R&D sector and the financial innovations sector respectively, are shown in the Appendix. Using numerical simulations, we can demonstrate that their steady-state levels are lower in the decentralized model compared to their counterparts in the social planner's solution. The sources of divergence are the externalities arising from existing R&D designs and financial products (which are only internalized by the social planner), as well as the monopoly power of intermediate good producers (which is eliminated by the social planner).

Figure 4: Flowchart of the Economy

3.2 The Social Planner's Problem

The representative consumer seeks to

$$\max_{c(t),u_Y(t),u_\tau(t)} U_0 = \int_0^\infty \frac{c(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{52}$$

subject to

$$\dot{K}(t) = \frac{\tau(t)}{L(t)}\left[K(t)^\alpha (A(t) u_Y(t) L(t))^{1-\alpha} - C(t)\right] - \delta K(t), \tag{53}$$

$$\dot{\tau}(t) = F[u_\tau(t) L(t)]^\lambda \tau(t)^\phi, \tag{54}$$

$$\dot{A}(t) = B[(1-u_Y(t)-u_\tau(t)) L(t)]^\eta \tau(t)^\beta A(t)^\psi, \tag{55}$$

where

$$L(t) = L(0) e^{nt}. \tag{56}$$

Note that $\alpha \in (0,1)$, $\{u_Y(t), u_\tau(t)\} \in [0,1]$ $\forall t$ and $\{\theta, \rho, \delta, n\} > 0$. Furthermore, $\{\lambda, \eta\} \in (0,1]$ and $\{\phi, \psi, \beta\} \in [0,1]$. To arrive at the steady-state solutions, we define the following three variables $\hat{k} \equiv K/AL$, $\chi \equiv C/K$ and $\xi \equiv \tau/L^\kappa$.

As detailed in Appendix C, the growth rates of technological and financial innovations are

$$\gamma_A^* = \frac{[(1-\phi)\,\eta + \lambda\beta]\,n}{(1-\phi)\,(1-\psi)}, \tag{57}$$

$$\gamma_\tau^* = \frac{\lambda n}{1-\phi}. \tag{58}$$

We note two salient features of the solution for the steady-state growth rate of the economy. The first is that $\psi < 1$, $\therefore\ \psi \in [0,1)$. In contrast to Romer's (1990) model where ψ is arbitrarily assigned the value of unity, our model indicates that it must be strictly less than that. Jones (1995) has argued that empirical investigations of time series data on various research and development variables suggest that $\psi \neq 1$.

The second feature is that the financial sector now has a direct impact on the growth rate of technology and thus output as well[3]. The growth rate of Y, given by $\gamma_A + n$, is a monotonically increasing function of the four elasticity parameters λ, ϕ, η, β and ψ which affect the production of new technologies in the research and development sector. Therefore any policy that raises these elasticity parameters will lead to a higher rate of economic growth. Specifically, these policies should be targeted at the researchers employed in R&D sector (to influence η), easing their access to the stock of knowledge embodied in existing inventions (to influence ψ), and at the projects undertaken by venture capitalists in encouraging high-risk R&D activities (to influence β).

The model is solved in terms of the five unknowns $\hat{k}$, χ, ξ, u_Y and u_τ.

3.2.1 Solutions

Using $\dot{\hat{k}}/\hat{k} = 0$, $\dot{\chi}/\chi = 0$, $\dot{\xi}/\xi = 0$, $\dot{u}_Y/u_Y = 0$ and $\dot{u}_\tau/u_\tau = 0$, we obtain the following solutions for u_τ (the share of labor devoted to the financial innovations sector), u_Y (the share of labor devoted to the final goods sector), u_A (the share of labor devoted to the real technological sector), ξ (the efficiency of financial intermediation, equal to τ/L^κ), χ (the consumption-capital ratio, or C/K) and $\hat{k}$ (the capital stock per effective unit of labor):

$$u_\tau^* = \frac{\Gamma}{\Gamma + \Phi}, \tag{59}$$

[3] This stands in contrast to our basic model where the same sector only leads to level but not growth e ects. (In that model, spillovers from existing ønancial products to ønancial innovations, measured by ϕ, had no impact on the steady-state growth rate.)

where $\Gamma \equiv \Gamma_1 \Gamma_2 + \Gamma_3$ and

$$\begin{aligned}
\Gamma_1 &\equiv \frac{\alpha\left(n+\delta+\gamma_A^*\right)}{(1-\alpha)\left(\rho+n+\delta+\theta\gamma_A^*\right)}, \\
\Gamma_2 &\equiv \frac{\rho+(\theta-\psi)\gamma_A^*}{\rho+(\theta+\eta-\psi)\gamma_A^*}, \\
\Gamma_3 &\equiv \frac{\beta\gamma_A^*}{\rho+(\theta+\eta-\psi)\gamma_A^*}, \\
\Phi &\equiv \frac{\rho+\lambda n+(\theta-1)\gamma_A^*}{\lambda\gamma_\tau^*},
\end{aligned}$$

$$\begin{aligned}
u_Y^* &= \Gamma_2\left(1-u_\tau^*\right) \\
&= \Gamma_2 \frac{\Phi}{\Gamma+\Phi},
\end{aligned} \tag{60}$$

$$\begin{aligned}
u_A^* &= \left(1-\Gamma_2\right)\left(1-u_\tau^*\right) \\
&= \left(1-\Gamma_2\right)\frac{\Phi}{\Gamma+\Phi},
\end{aligned} \tag{61}$$

$$\begin{aligned}
\xi^* &= \left[\frac{F u_\tau^{*\lambda}}{\gamma_\tau^*}\right]^{\frac{1}{1-\phi}} \\
&= \left[\frac{F}{\gamma_\tau^*}\left(\frac{\Gamma}{\Gamma+\Phi}\right)^\lambda\right]^{\frac{1}{1-\phi}},
\end{aligned} \tag{62}$$

$$\begin{aligned}
\chi^* &= \frac{\rho+(1-\alpha)(n+\delta)+(\theta-\alpha)\gamma_A^*}{\alpha\xi^*} \\
&= \frac{\rho+(1-\alpha)(n+\delta)+(\theta-\alpha)\gamma_A^*}{\alpha}\left[\frac{\gamma_\tau^*}{F}\left(\frac{\Gamma+\Phi}{\Gamma}\right)^\lambda\right]^{\frac{1}{1-\phi}},
\end{aligned} \tag{63}$$

$$\begin{aligned}
\hat{k}^* &= \left[\frac{\alpha\xi^*}{\rho+n+\delta+\theta\gamma_A^*}\right]^{\frac{1}{1-\alpha}} u_Y^* \\
&= \left\{\frac{\alpha}{\rho+n+\delta+\theta\gamma_A^*}\left[\frac{F}{\gamma_\tau^*}\left(\frac{\Gamma}{\Gamma+\Phi}\right)^\lambda\right]^{\frac{1}{1-\phi}}\right\}^{\frac{1}{1-\alpha}} \Gamma_2 \frac{\Phi}{\Gamma+\Phi}.
\end{aligned} \tag{64}$$

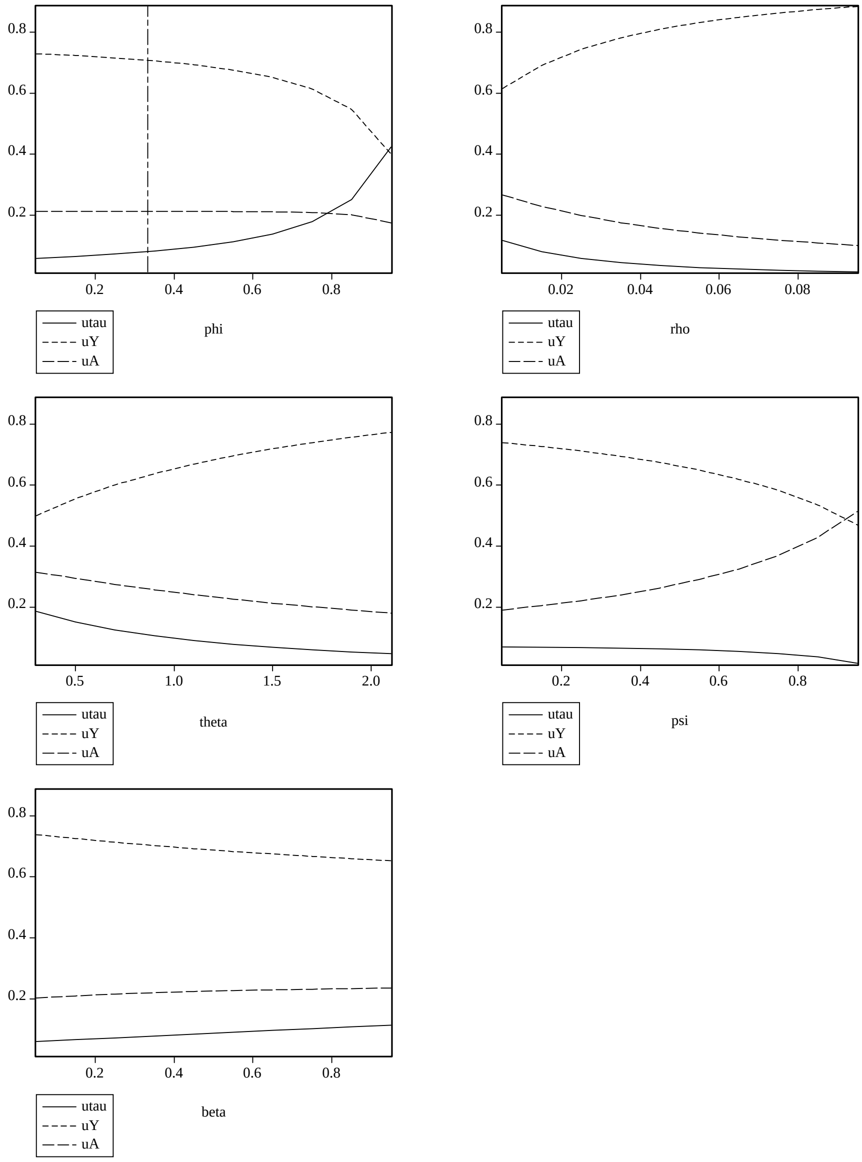

Figure 5: Simulated Comparative Statics

3.2.2 Model Implications

Due to the complexity of the analytical solutions, we utilize simulation techniques to investigate the comparative statics of the model. Specifically, we analyze the impact of a change in ϕ, ρ, θ, ψ and β on the three shares of labor u_{τ}^{*}, u_{Y}^{*} and u_{A}^{*}. The comparative statics are performed with respect to a particular parameter holding the other parameters constant. They should be interpreted relative to the base model with the following set of baseline values:

α	δ	ρ	θ	n	λ	ϕ	η	ψ	β
$\frac{1}{3}$	0.0	0.02	1.5	0.02	$\frac{2}{3}$	0.2	$\frac{2}{3}$	0.2	0.2

Figure 5 presents the impact on u_τ^*, u_Y^* and u_A^* over a range of values for each of the five parameters. The results of our comparative statics exercise are summarized in the following table (the arrows indicate the direction of change in the variables on the first column given an increase in the value of a parameter around the neighborhood of its baseline value):

	ϕ	ρ	θ	ψ	β
u_τ^*	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
u_Y^*	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
u_A^*	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$

We find that a rise in ϕ decreases u_τ^* but increases u_Y^* and u_A^*: since the stock of financial products grow at the fixed rate κn in the steady state, a rise in ϕ means that financial innovators get a bigger "kick" from existing financial products, so that less labor is required to produce the pre-determined number of financial innovations. This frees up labor to be channelled to the final goods sector and the real R&D sector. A rise in β increases the importance of τ on the production of new R&D designs (the ultimate source of growth in this model) as well as raises the marginal product of labor in that sector. The former creates a higher demand for financial products which leads to a rise in u_τ^* while the latter induces a rise in u_A^* in order to bring wages across the sectors back to equilibrium again. Similarly, an increase in ψ raises the productivity of labor in the R&D sector thereby inducing u_A^* to rise at the expense of u_Y^* and u_τ^* to equalise wages.

The impact of ρ and θ on the shares of labor in the three sectors are similar. An increase in either ρ or θ increases u_Y^* but decreases u_τ^* and u_A^*. The explanation is straightforward: an increase in either ρ or θ indicates a rise in households' preference for current consumption. Consequently, more labor is devoted to the production of the final consumption good at the expense of the other two sectors.

4 Human Capital and Financial Innovations

We now insert our formulation of the financial sector into the well-known Lucas (1988) model of human capital, which is based on earlier work by Uzawa (1965). The idea here is that the production of financial innovations is costly not only in terms of foregone production of the final consumption good, but also because it draws human capital away from the generation of *new* human capital. In the real world, for example, the aggressive recruiting of the best graduates from each cohort by financial powerhouses prevents such talent from being channelled into academia and the teaching profession.

In this model, we examine the optimum allocation of human capital between the final goods, financial and human capital sectors and observe how this varies according to the elasticities and productivity parameters of the various inputs in these sectors.

The representative consumer's problem in this model is

$$\max_{C(t),u_Y(t),u_\tau(t)} U_0 = \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{65}$$

subject to

$$\begin{aligned}
\dot{K}(t) &= \frac{\tau(t)}{H(t)^\kappa}\left[AK(t)^\alpha (u_Y(t)H(t))^{1-\alpha} - C(t)\right] - \delta K(t), \quad (66)\\
\dot{\tau}(t) &= F\left[u_\tau(t)H(t)\right]^\lambda \tau(t)^\phi, \quad (67)\\
\dot{H}(t) &= D\left[1-u_Y(t)-u_\tau(t)\right]H(t) - \delta H(t). \quad (68)
\end{aligned}$$

Note that $\alpha \in (0,1)$, $\{u_Y(t), u_\tau(t)\} \in [0,1]$ $\forall t$, $\{\theta,\rho,\delta,n\} > 0$, $\lambda \in (0,1]$ and $\phi \in [0,1]$.

To arrive at the steady-state solutions, we first define the following three variables $\omega \equiv K/H$, $\chi \equiv C/K$ and $\zeta \equiv \tau/H^\kappa$. In the steady state, we require the output-capital ratio given by

$$\frac{Y}{K} = A\omega^{\alpha-1}u_Y^{1-\alpha}, \tag{69}$$

to remain constant. This implies, from equation (69), that

$$\frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} = (\alpha-1)\frac{\dot{\omega}}{\omega} + (1-\alpha)\frac{\dot{u}_Y}{u_Y} = 0. \tag{70}$$

We also require that $\dot{u}_Y/u_Y = 0$ in the steady state. Hence, $\dot{\omega}/\omega = 0$ in order to satisfy equation (70). This then suggests that $\dot{K}/K = \dot{H}/H$ in the steady state. Furthermore, it is assumed that $\dot{\chi}/\chi = \dot{\zeta}/\zeta = 0$ in the steady state. Hence, these assumptions imply that to have a balanced growth path, we must have $\dot{Y}/Y = \dot{K}/K = \dot{C}/C = \gamma_H$ and $\dot{\tau}/\tau = \kappa\gamma_H$ in the steady state, where $\gamma_H \equiv \dot{H}/H$ and $\kappa = \lambda/(1-\phi)$. The growth rate is now determined endogenously instead of being equal to some exogenous population growth rate, as was the case in the first two models.

The model is solved in terms of the five unknowns ω, χ, ζ, u_Y and u_τ. The five equations needed to pin down the solutions to these variables are given by $\dot{\omega}/\omega = 0$, $\dot{\chi}/\chi = 0$, $\dot{\zeta}/\zeta = 0$, $\dot{u}_Y/u_Y = 0$ and $\dot{u}_\tau/u_\tau = 0$. These five conditions lead to the following equations respectively:

$$\begin{aligned}
\zeta A\omega^{\alpha-1}u_Y^{1-\alpha}-\zeta\chi &= \gamma_H^*+\delta, \qquad (71)\\
\zeta A\alpha\omega^{\alpha-1}u_Y^{1-\alpha} &= \rho+\theta\gamma_H^*+\delta,\\
Fu_\tau^\lambda\zeta^{\phi-1} &= \frac{\lambda\gamma_H^*}{1-\phi}, \qquad (72)
\end{aligned}$$

$$\begin{aligned}
&\zeta A\alpha\omega^{\alpha-1}u_Y^{1-\alpha}-\gamma_H^*-\delta=\\
&Du_\tau+\frac{Du_Y}{1-\alpha}\left[(1-\alpha-\kappa)+\kappa\left(\zeta^{-1}A^{-1}\omega^{1-\alpha}u_Y^{\alpha-1}\right)\zeta\chi\right], \qquad (73)
\end{aligned}$$

$$\begin{aligned}
&\frac{\lambda\gamma_\tau^*}{1-\alpha}\frac{u_Y}{u_\tau}\left[1-\left(\zeta^{-1}A^{-1}\omega^{1-\alpha}u_Y^{\alpha-1}\right)\zeta\chi\right]-\lambda\gamma_H^*=\\
&Du_\tau+\frac{Du_Y}{1-\alpha}\left[(1-\alpha-\kappa)+\kappa\left(\zeta^{-1}A^{-1}\omega^{1-\alpha}u_Y^{\alpha-1}\right)\zeta\chi\right]. \qquad (74)
\end{aligned}$$

Using equations (71) to (74), we obtain the following solutions for u_τ, u_Y, u_H, ζ, χ and ω:

$$\begin{aligned}
u_\tau^* &= \frac{\lambda\gamma_H^*}{D}\frac{\Phi}{\Gamma-\Phi}, \qquad (75)\\
u_Y^* &= \frac{\rho+(\theta-1)\gamma_H^*}{D}\frac{\Gamma}{\Gamma-\Phi}, \qquad (76)\\
u_H^* &= \frac{\left[D-\rho-(\theta-1)\gamma_H^*\right]\Gamma-\left[D+\lambda\gamma_H^*\right]\Phi}{D(\Gamma-\Phi)}, \qquad (77)\\
\zeta^* &= \left[\frac{F}{\gamma_\tau^*}\left(\frac{\lambda\gamma_H^*}{D}\frac{\Phi}{\Gamma-\Phi}\right)^\lambda\right]^{\frac{1}{1-\phi}} \qquad (78)\\
\chi^* &= \frac{\rho+(\theta-\alpha)\gamma_H^*+(1-\alpha)\delta}{\alpha}\\
&\quad\times\left[\frac{\gamma_\tau^*}{F}\left(\frac{D}{\lambda\gamma_H^*}\frac{\Gamma-\Phi}{\Phi}\right)^\lambda\right]^{\frac{1}{1-\phi}}, \qquad (79)\\
\omega^* &= \left\{\frac{A\alpha}{\rho+\theta\gamma_H^*+\delta}\left[\frac{F}{\gamma_\tau^*}\left(\frac{\lambda\gamma_H^*}{D}\frac{\Phi}{\Gamma-\Phi}\right)^\lambda\right]^{\frac{1}{1-\phi}}\right\}^{\frac{1}{1-\alpha}}\\
&\quad\times\frac{\rho+(\theta-1)\gamma_H^*}{D}\frac{\Gamma}{\Gamma-\Phi}, \qquad (80)
\end{aligned}$$

where $\gamma_\tau^*=\kappa\gamma_H^*$, γ_H^* is the root of

$$Du_H^*-\delta=\gamma_H^*, \qquad (81)$$

and

$$\Gamma \equiv (1-\alpha)(\rho+\theta\gamma_H^*+\delta)[\rho+(\theta+\lambda-1)\gamma_H^*],$$
$$\Phi \equiv \kappa\alpha(\gamma_H^*+\delta)[\rho+(\theta-1)\gamma_H^*].$$

Note that γ_H^* is the value of $\dot{H}/H$ in the steady state which yields the steady-state growth rate of the economy. It is apparent from equation (81) that $\gamma_H^* = f(\alpha,\delta,\rho,\theta,\phi,D)$. Although it is impossible to obtain γ_H^* in analytic form, we use numerical techniques for its solution and perform comparative statics numerically using the following set of baseline values for the parameters:

α	δ	ρ	θ	λ	ϕ	A	D
$\frac{1}{3}$	0.05	0.02	1.5	$\frac{2}{3}$	0.2	1	0.15

The simulation results are presented in the figures below. Figure 6 presents the graphs for the shares of human capital. The results for γ_H^* are also presented in Figure 7 to help analyze the size of the impact.

The set of baseline values yield a steady-state growth rate, γ_H^*, of 4.17% approximately. The matrix below provides an overview of the direction of change in γ_H^* as well as the three shares of human capital u_τ^*, u_Y^* and u_H^*, given an increase in α, δ, ρ, θ, ϕ and D around the neighborhood of their baselines values.

	α	δ	ρ	θ	ϕ	D
γ_H^*	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
u_τ^*	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
u_Y^*	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
u_H^*	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

An increase in α raises the marginal product of human capital in the final goods sector. Since wages must equate across all markets for human capital in equilibrium according to its marginal product, the share of human capital in the final goods sector must rise relative to the share in the other sectors. Our results show that the rise comes at the expense of a fall in u_H^* thus leading to a fall in γ_H^*. An increase in δ *ceteris paribus* decreases γ_H^* since it affects directly the accumulation of human capital. However, given that both physical and human capital accumulation depend on the stock of human capital, more human capital is needed in the human capital sector to counteract the higher rate of depreciation, i.e. u_H^* increases. This has the opposite effect of raising γ_H^*. Our simulations indicate that the former effect dominates the latter.

An increase in either ρ or θ decreases γ_H^*. Both have the effect of favoring current consumption *vis-a-vis* future consumption, leading to more human capital being channelled to the final goods sector at the expense of the other two sectors. Naturally, the fall in u_H^* then leads to a fall in γ_H^*. An increase in ϕ increases the marginal product of human capital in the financial innovations sector. In order to bring the marginal products back to equilibrium across the sectors, the share of human capital in the financial innovations sector has to rise relative to the share in the other sectors. Our results indicate that u_H^* falls while u_τ^* and u_Y^* increase. The fall in u_H^* hence causes γ_H^* to fall.

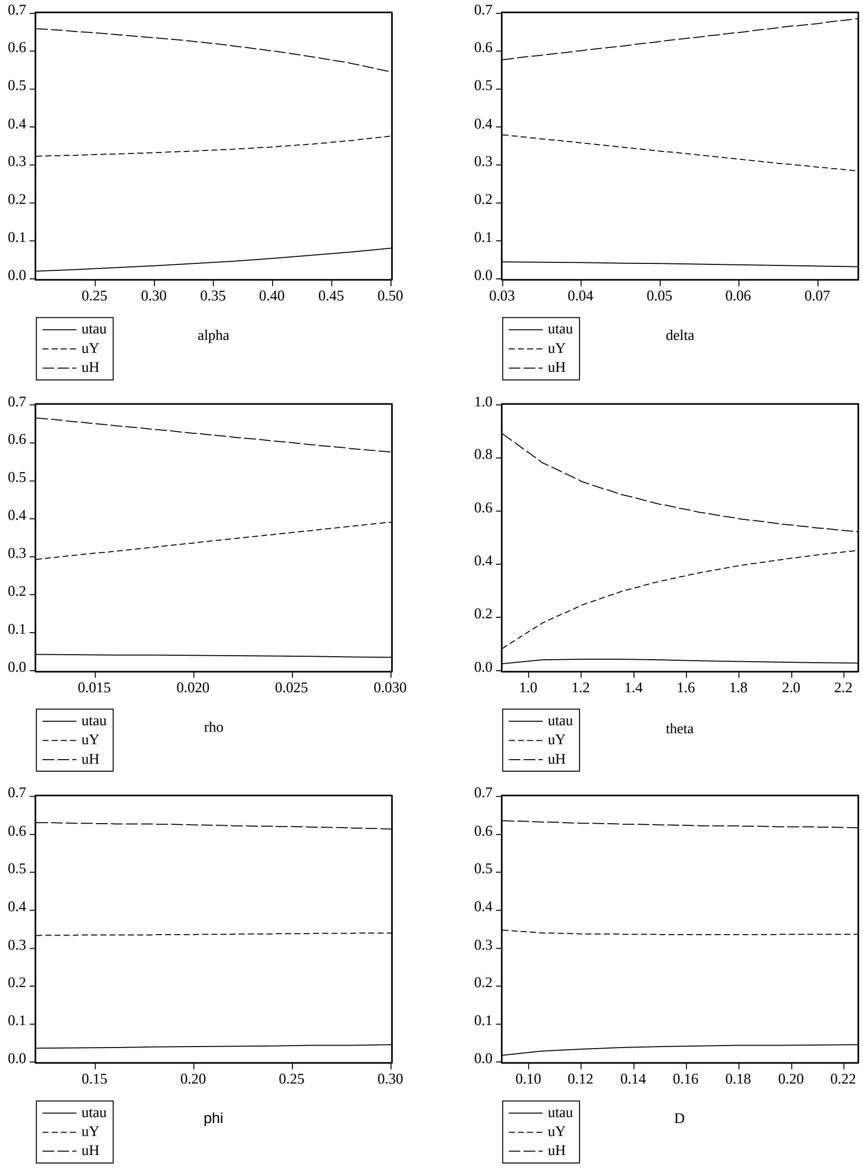

Figure 6: Simulated Comparative Statics

Finally, an increase in the level of productivity in the education sector, D, has a direct effect of raising the rate of human capital accumulation and thus γ_H^*. On the other hand, our simulations indicate a rise in D also channels more human capital to the financial innovations sector at the expense of the other two, which has the opposite effect on γ_H^*. It appears that the former effect dominates the latter in our simulations.

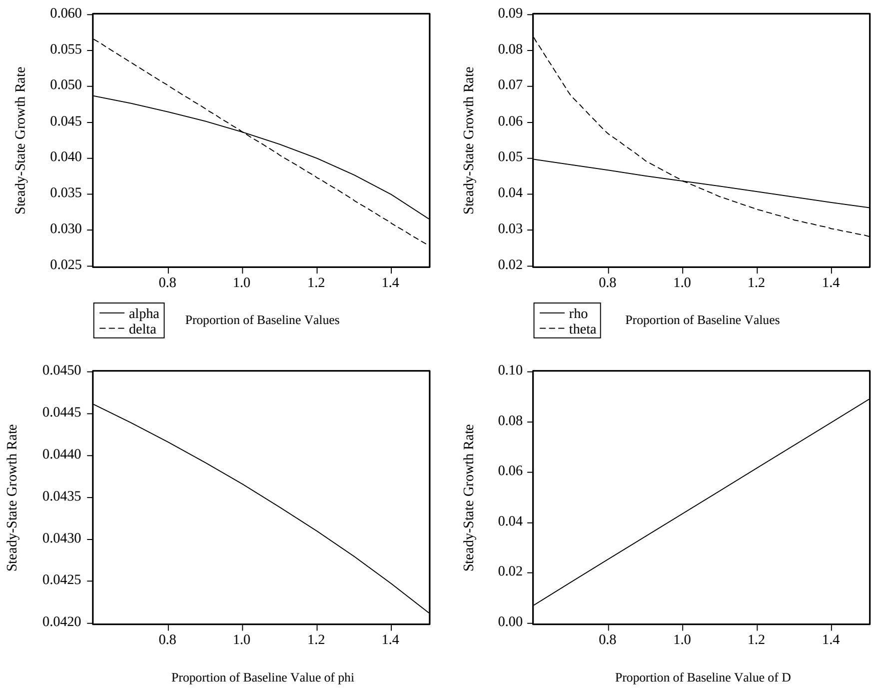

Figure 7: Impact of Model Parameters on the Steady-State Growth Rate

5 Policy Implications

Our model with technological progress suggests that government subsidies for financial innovations may raise the steady-state level of capital and output per capita through its effect on the rate of technological innovations. With these subsidies, the financial sector develops more rapidly (its maturity being measured by the stock of financial products) and assists the real R&D sector more capably through its venture capital role. However, as none of the parameters in the financial innovations equation affect the steady-state growth rate of the economy, subsidies have level but not long-run growth effects.

Deregulation of the financial sector may lead to increased productivity of financial innovators (captured in our model by a rise in F), which raises the steady-state per-capita capital stock and output but not their growth rates. (We can show that when F is too low, the economy may never achieve a 100 per cent transformation of savings into investment, i.e. $\xi < 1$ in the steady state.) Similarly, opening the financial sector of a less developed economy to leading-edge financial firms from advanced countries will enable a transfer of financial expertise from the more advanced country to the less developed one, allowing the latter to raise its F parameter and thereby attain its steady-state sooner while achieving a higher level of GDP per capita. This effect is not to be confused with the issue of increasing capital flows between countries.

The results from our model with endogenous human capital accumulation suggest that government policies which affect the productivity of the education sector raises the long-run growth rate of the economy. However, varying the exponent on the spillover effect of existing financial products on financial innovations has no impact on the steady-state growth rate in our model with endogenous technological progress. So perhaps we can argue that a government intent on generating high long-run growth should focus its attention and direct its subsidies more towards the educational sector rather than the technological or financial sectors.

6 Conclusion

In this paper, we set out to investigate how the extraordinary expansion in the variety of financial products and the increasing sophistication of the financial sector lead to rising affluence in the context of an endogenous growth model. The channels we explored are capital accumulation and technological innovation.

We developed a formulation of the financial sector which was then embedded in three growth models, including one in which technological progress is modelled endogenously as an expansion in the variety of intermediate goods and another in which a broad definition of capital used in the production of final goods includes both physical and human capital. Our financial sector comprises financial innovators and financial intermediaries. Financial innovators utilize labor (or human capital) and the existing catalog of financial products to develop new financial products and services. Financial intermediaries then purchase these innovations to improve their efficiency in transforming household savings into productive investment by firms. In the model with endogenous technological progress, we also allowed for spillovers from financial innovations into the production of new designs in the real R&D sector.

By solving for the steady-state values of the variables of interest and analyzing the resulting comparative statics, we showed that financial innovations ultimately affect the long-run growth rate only through the channel of technological innovation The rise in transformative efficiency of savings into new capital through the adoption of financial innovations slows and eventually comes to a halt in the steady state, so that an increase in the marginal productivity of the financial sector leads to growth effects on the transitional path to the steady state but only level effects in the long run. We then discussed the policy implications arising from these results.

Extensions to be explored and future research plans include opening the economy to allow for capital inflows and outflows, as well as formulating a model with financial innovation, endogenous technological progress *and* endogenous human capital accumulation.

References

Acemoglu, D., and F. Zilibotti (1997): "Was Prometheus Unbound by Chance? Risk, Diversification and Growth," *Journal of Political Economy*, 105, 709–751.

Aghion, P., and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60, 323–351.

Arestis, P., and P. O. Demetriades (1997): "Financial Development and Economic Growth: Assessing the Evidence," *Economic Journal*, 107, 783–799.

Barro, R., and X. Sala-I-Martin (1995): *Economic Growth*. McGraw-Hill, New York.

Bencivenga, V. R., and B. D. Smith (1991): "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*, 58, 195–209.

——— (1993): "Some Consequences of Credit Rationing in an Endogenous Growth Model," *Journal of Economic Dynamics and Control*, 17, 97–122.

Bencivenga, V. R., B. D. Smith, and R. M. Starr (1995): "Transaction Costs, Technological Choice and Endogenous Growth," *Journal of Economic Theory*, 67, 153–177.

——— (1996): "Equity Markets, Transactions Costs and Capital Accumulation: An Illustration," *World Bank Economic Review*, 10, 241–265.

Berthelmy, J. C., and A. Varoudakis (1996): "Economic Growth, Convergence Clubs and the Role of Financial Development," *Oxford Economic Papers*, 48, 300–328.

Betts, C., and J. Bhattacharya (1998): "Unemployment, Credit Rationing and Capital Accumulation: A Tale of Two Frictions," *Economic Theory*, 12(3), 489–518.

Blackburn, K., and V. T. Y. Hung (1998): "A Theory of Growth, Financial Development and Trade," *Economica*, 65, 107–124.

Bose, N., and R. Cothren (1996): "Equilibrium Loan Contracts and Endogenous Growth in the Presence of Asymmetric Information," *Journal of Monetary Economics*, 38, 363–376.

Boyd, J., and B. D. Smith (1996): "The Coevolution of the Real and Financial Sector in the Growth Process," *World Bank Economic Review*, 10, 371–396.

Buiter, W. H., and K. M. Kletzer (1995): "Capital Mobility, Fiscal Policy and Growth under Self-Financing of Human Capital Formation," *Canadian Journal of Economics*, 28, S163–S194.

Cass, D. (1965): "Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, 32, 233–240.

Chiang, A. C. (1992): *Elements of Dynamic Optimization*. McGraw-Hill, New York.

de Gregorio, J. (1996): "Borrowing Constraints, Human Capital Accumulation and Growth," *Journal of Monetary Economics*, 37, 49–71.

de la Fuente, A., and J. M. Marin (1996): "Innovation, Bank Monitoring and Endogenous Financial Development," *Journal of Monetary Economics*, 38, 269–301.

Demetriades, P. O., and K. A. Hussein (1996): "Does Financial Development Cause Economic Growth? Time-Series Evidence from 16 Countries," *Journal of Development Economics*, 51, 387–411.

Devereux, M. B., and G. W. Smith (1994): "International Risk Sharing and Economic Growth," *International Economic Review*, 35, 535–550.

Dixit, A., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297–308.

Gertler, M. (1988): "Financial Structure and Aggregate Economic Activity: An Overview," *Journal of Money, Credit and Banking*, 20, 559–588.

Greenwood, J., and B. Jovanovic (1990): "Financial Development, Growth and the Distribution of Income," *Journal of Political Economy*, 98, 1076–1107.

Greenwood, J., and B. D. Smith (1997): "Financial Markets in Development and the Development of Financial Markets," *Journal of Economic Dynamics and Control*, 21, 145–181.

Grossman, G. M., and E. Helpman (1991a): *Innovation and Growth in the Global Economy*. MIT Press, Cambridge, Massachussetts.

——— (1991b): "Quality Ladders and Product Cycles," *Quarterly Journal of Economics*, 106, 557–586.

——— (1991c): "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, 58, 43–61.

Japelli, T., and M. Pagano (1994): "Saving, Growth and Liquidity Constraints," *Quarterly Journal of Economics*, 108, 83–109.

Jones, C. I. (1995): "Research and Development Based Models of Economic Growth," *Journal of Political Economy*, 103, 759–784.

King, R. G., and R. Levine (1993a): "Finance and Growth: Schumpeter Might Be Right," *Quarterly Journal of Economics*, 108, 717–737.

——— (1993b): "Finance, Entrepreneurship and Growth: Theory and Evidence," *Journal of Monetary Economics*, 32, 513–542.

Koopmans, T. C. (1965): "On the Concept of Optimal Economic Growth," in *The Economic Approach to Development Planning*. North-Holland, Amsterdam.

Levine, R. (1997): "Financial Development and Economic Growth: Views and Agenda," *Journal of Economic Literature*, 35, 688–726.

Levine, R., and S. Zevos (1998): "Stock Markets, Banks and Economic Growth," *American Economic Review*, 88, 537–558.

Lucas, Robert E., J. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3–42.

Ma, C. H., and B. D. Smith (1996): "Credit Market Imperfections and Economic Development: Theory and Evidence," *Journal of Development Economics*, 48, 351–387.

Obstfeld, M. (1994): "Risk-Taking, Global Diversification and Growth," *American Economic Review*, 84, 1310–1329.

Pagano, M. (1993): "Financial Markets and Growth: An Overview," *European Economic Review*, 37, 613–622.

Romer, P. M. (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98, S71–S102.

Saint-Paul, G. (1992): "Technological Choice, Financial Markets and Economic Development," *European Economic Review*, 36, 763–781.

Shi, S. (1996): "Asymmetric Information, Credit Rationing and Economic Growth," *Canadian Journal of Economics*, 29, 665–687.

Solow, R. M. (1956): "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70, 65–94.

Swan, T. W. (1956): "Economic Growth and Capital Accumulation," *Economic Record*, 32, 334–361.

Tsiddon, D. (1992): "A Moral Hazard Trap to Growth," *International Economic Review*, 33, 299–321.

Uzawa, H. (1965): "Optimum Technical Change in an Aggregative Model of Economic Growth," *International Economic Review*, 6, 12–31.

A Mathematical Proofs

Proof for Proposition 2. The representative consumer in the decentralized economy seeks to

$$\max_{c,u_Y} U_0 = \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta} e^{-\rho t} dt,$$

subject to

$$\begin{aligned} \dot{V} &= r_V K + w_Y u_Y L + w_\tau u_\tau L + \pi_\tau - C, \\ \dot{K} &= \xi \dot{V}, \\ \dot{\tau} &= \tilde{F} \left(u_\tau L\right)^\lambda, \end{aligned}$$

where $\dot{V}$ is the flow of savings accumulated by households, r_V is the rate of interest paid by financial intermediaries to households on the stock of savings that has been successfully transformed i.e. K, π_τ is the monopolistic profits earned by the producers of financial products, τ, and $\tilde{F} \equiv F\tau^\phi$. The equation for $\dot{\tau}$ differs from that in the social planner's case because the financial innovators do not internalize the spillovers from existing financial products. The monopolistic profits, equal to revenue $P_\tau \dot{\tau}$ less labor costs $w_\tau u_\tau L$, accrue to households who also own the firms in the financial innovations sector. In every period, financial intermediaries earn $r_K K$ from their loans to firms which is just enough to cover their payments, $r_V K$ and $P_\tau \dot{\tau}$, to households (for their deposits) and the financial innovations sector (for the financial products) respectively. In equilibrium, wages are equal across all labor markets, i.e. $w_Y = w_\tau = \bar{w}$. These conditions together yield the following households budget constraint

$$\dot{K} = \xi \left(r_K K + \bar{w} u_Y L - C\right).$$

Solving the Hamiltonian then yields

$$u_\tau^* = \frac{\Gamma}{\Gamma + \Phi},$$

where $\Gamma \equiv \alpha\lambda n\gamma_\tau^*$, $\Phi \equiv (1-\alpha)(\rho+n)[\rho+\gamma_\tau^*]$ and $\gamma_\tau^* = \lambda n/(1-\phi)$. Expressing the ratio of the two shares of labor, $(u_\tau^*/u_Y^*)^{DC}$, as a function of $(u_\tau^*/u_Y^*)^{SP}$, where DC and SP denote the decentralized economy and the social planner respectively, we have

$$\left(\frac{u_\tau^*}{u_Y^*}\right)^{DC} = \frac{(1-\phi)(\rho+\lambda n)}{(1-\phi)\rho+\lambda n}\left(\frac{u_\tau^*}{u_Y^*}\right)^{SP},$$

which shows that $u_\tau^{*DC} < u_\tau^{*SP}$ as long as $\phi > 0$. The divergence between the two increases as ϕ increases. ■

Proof for Proposition 3. The partial total derivative of u_τ^* with respect to ϕ is

$$\begin{aligned}\frac{\S u_\tau^*}{\S\phi} &= \frac{\partial u_\tau^*}{\partial\Gamma}\frac{\partial\Gamma}{\partial\phi} + \frac{\partial u_\tau^*}{\partial\Phi}\frac{\partial\Phi}{\partial\phi} \\ &= \frac{\Gamma\Phi}{(\Gamma+\Phi)^2(1-\phi)} > 0.\end{aligned}$$

Note that $\frac{\partial\Gamma}{\partial\phi} = 0$. ■

Proof for Proposition 4. The partial total derivative of u_τ^* with respect to ρ is

$$\begin{aligned}\frac{\S u_Y^*}{\S\rho} &= \frac{\partial u_\tau^*}{\partial\Gamma}\frac{\partial\Gamma}{\partial\rho} + \frac{\partial u_\tau^*}{\partial\Phi}\frac{\partial\Phi}{\partial\rho} \\ &= -\frac{\Gamma\Phi}{(\Gamma+\Phi)^2}\frac{2\rho+(1+\lambda)n+\delta}{(\rho+\lambda n)(\rho+n+\delta)} < 0,\end{aligned}$$

Note that $\frac{\partial\Gamma}{\partial\rho} = 0$. ■

Proof for Proposition 5. The partial total derivative of u_τ^* with respect to θ is

$$\frac{\S u_Y^*}{\S\theta} = 0.$$

■

B The Decentralized Economy in the Model with Technological Progress

The Hamiltonian is

$$\begin{aligned}\mathsf{H} \equiv\ & \frac{c^{1-\theta}-1}{1-\theta}e^{-\rho t} + \nu\xi\Big(r_K K + \bar{w}u_Y L + \bar{w}u_A L + R_\tau\tau \\ & + A\bar{\pi}_x + \pi_A - P_A\tilde{B}u_A^\eta L^\eta\tau^\beta - C\Big) \\ & + \mu\tilde{F}u_\tau^\lambda L^\lambda + \upsilon\tilde{B}u_A^\eta L^\eta\tau^\beta.\end{aligned}$$

Solving the Hamiltonian yields

$$u_\tau^* = \frac{\Gamma}{\Gamma + \Phi},$$

where $\Gamma \equiv \Gamma_1\Gamma_2 + \Gamma_3$ and

$$\begin{aligned}
\Gamma_1 &\equiv \frac{\alpha^2 \left(n + \gamma_A^*\right)}{(1-\alpha)\left(\rho + n + \theta\gamma_A^*\right)}, \\
\Gamma_2 &\equiv \frac{\rho + \theta\gamma_A^*}{\rho + (\theta + \alpha\eta)\,\gamma_A^*}, \\
\Gamma_3 &\equiv \frac{\alpha\beta\gamma_A^*}{\rho + (\theta + \alpha\eta)\,\gamma_A^*}, \\
\Phi &\equiv \frac{\rho + \lambda n + (\theta - 1)\,\gamma_A^*}{\lambda\gamma_\tau^*},
\end{aligned}$$

$$\begin{aligned}
u_Y^* &= \Gamma_2 \left(1 - u_\tau^*\right), \\
u_A^* &= \left(1 - \Gamma_2\right)\left(1 - u_\tau^*\right).
\end{aligned}$$

Note that depreciation is dropped here for simplicity.

C The Social Planner's Problem in the Model with Technological Progress

The Hamiltonian is:

$$\begin{aligned}
\mathsf{H} \equiv{}& \frac{c^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \nu \left[\frac{\tau}{L^\kappa}\left(K^\alpha A^{1-\alpha} u_Y^{1-\alpha} L^{1-\alpha} - C\right) - \delta K\right] \quad (82) \\
&+ \mu F u_\tau^\lambda L^\lambda \tau^\phi + \upsilon B \left(1 - u_Y - u_\tau\right)^\eta L^\eta \tau^\beta A^\psi,
\end{aligned}$$

where the control variables are c, u_Y and u_τ, the state variables are K, τ and A, and ν, μ and υ are the costate variables associated with K, τ and A respectively. The first-order conditions are:

$$\begin{aligned}
\frac{\partial \mathsf{H}}{\partial C} &= c^{-\theta} e^{-\rho t} - \nu\tau = 0, && (83) \\
\frac{\partial \mathsf{H}}{\partial u_y} &= \nu\tau K^\alpha A^{1-\alpha} (1-\alpha)\, u_Y^{-\alpha} L^{1-\alpha-\kappa} - \upsilon B \eta \left(1 - u_Y - u_\tau\right)^{\eta - 1} L^\eta \tau^\beta A^\psi \\
&= 0, && (84) \\
\frac{\partial \mathsf{H}}{\partial u_\tau} &= \mu F \lambda u_\tau^{\lambda - 1} L^\lambda \tau^\phi - \upsilon B \eta \left(1 - u_Y - u_\tau\right)^{\eta - 1} L^\eta \tau^\beta A^\psi = 0. && (85)
\end{aligned}$$

$$\dot{\nu} = -\frac{\partial \mathsf{H}}{\partial K} = -\nu\left(\tau\alpha K^{\alpha-1}A^{1-\alpha}u_Y^{1-\alpha}L^{1-\alpha-\kappa}-\delta\right), \tag{86}$$

$$\begin{aligned}\dot{\mu} &= -\frac{\partial \mathsf{H}}{\partial \tau} = -\nu\left(K^{\alpha}A^{1-\alpha}u_Y^{1-\alpha}L^{1-\alpha-\kappa}-\frac{C}{L^{\kappa}}\right)-\mu F u_\tau^{\lambda}L^{\lambda}\phi\tau^{\phi-1} \\ &\quad -\upsilon B\left(1-u_Y-u_\tau\right)^{\eta}L^{\eta}\beta\tau^{\beta-1}A^{\psi},\end{aligned} \tag{87}$$

$$\begin{aligned}\dot{\upsilon} &= -\frac{\partial \mathsf{H}}{\partial A} = -\nu\tau K^{\alpha}\left(1-\alpha\right)A^{-\alpha}u_Y^{1-\alpha}L^{1-\alpha-\kappa} \\ &\quad -\upsilon B\left(1-u_Y-u_\tau\right)^{\eta}L^{\eta}\tau^{\beta}\psi A^{\psi-1}.\end{aligned} \tag{88}$$

In addition, the growth rates of the state variables are given by

$$\frac{\dot{K}}{K} = \frac{\tau}{L^{\kappa}}\left(K^{\alpha-1}A^{1-\alpha}u_Y^{1-\alpha}L^{1-\alpha}-\frac{C}{K}\right)-\delta, \tag{89}$$

$$\frac{\dot{\tau}}{\tau} = F u_\tau^{\lambda}L^{\lambda}\tau^{\phi-1}, \tag{90}$$

$$\frac{\dot{A}}{A} = B\left(1-u_Y-u_\tau\right)^{\eta}L^{\eta}\tau^{\beta}A^{\psi-1}. \tag{91}$$

and the transversality conditions are

$$\lim_{t\to\infty}\nu\left(t\right)K\left(t\right) = 0, \tag{92}$$

$$\lim_{t\to\infty}\mu\left(t\right)\tau\left(t\right) = 0, \tag{93}$$

$$\lim_{t\to\infty}\upsilon\left(t\right)A\left(t\right) = 0. \tag{94}$$

To arrive at the steady-state solutions, we first define the following three variables $\hat{k} \equiv K/AL$, $\chi \equiv C/K$ and $\xi \equiv \tau/L^{\kappa}$. In the steady state, we require the output-capital ratio given by

$$\frac{Y}{K} = k^{\alpha-1}A^{1-\alpha}u_Y^{1-\alpha}, \tag{95}$$

to remain constant. This implies, from equation (**??**), that

$$\frac{\dot{Y}}{Y}-\frac{\dot{K}}{K} = \left(\alpha-1\right)\frac{\dot{k}}{k}+\left(1-\alpha\right)\frac{\dot{A}}{A}+\left(1-\alpha\right)\frac{\dot{u}_y}{u_y} = 0. \tag{96}$$

In addition, $\dot{u}_Y/u_Y = 0$ in the steady state. Hence, $\dot{k}/k = \dot{A}/A$ in order to satisfy equation (96). This then suggests that $\dot{K}/K = \dot{A}/A + n$, given equation (56), in the steady state. Furthermore, it is assumed that $\dot{\chi}/\chi = \dot{\xi}/\xi = 0$ in the steady state. Hence these assumptions imply that a balanced growth path requires $\dot{Y}/Y = \dot{K}/K = \dot{C}/C = \dot{A}/A + n$ and $\dot{\tau}/\tau = \kappa n$ in the steady state.

With endogenous technological progress embedded in the model, the growth rate of output is now augmented by the growth rate of technology. Again, this result is comparable to that of the Cass-Koopman's model with technological progress. The presence of technological progress offsets the diminishing marginal product of physical capital, thereby continually raising the productivity level of labor.

To solve for $\dot{A}/A$, we impose the conditions that $\dot{A}/A$ and $\dot{\tau}/\tau$ are constant in the steady state. Let $\gamma_A \equiv \dot{A}/A$ and $\gamma_\tau \equiv \dot{\tau}/\tau$. These conditions imply that

$$\frac{\dot{\gamma}_A}{\gamma_A} = -\eta \frac{\dot{u}_Y + \dot{u}_\tau}{1 - u_Y - u_\tau} + \eta n + (\psi - 1)\gamma_A + \beta\gamma_\tau = 0, \tag{97}$$

$$\frac{\dot{\gamma}_\tau}{\gamma_\tau} = \lambda \frac{\dot{u}_\tau}{u_\tau} + \lambda n + (\phi - 1)\gamma_\tau = 0. \tag{98}$$

Since $\dot{u}_Y = \dot{u}_\tau = 0$ in the steady state, solving equations (97) and (98) for γ_A and γ_τ thus yields

$$\gamma_A^* = \frac{[(1-\phi)\eta + \lambda\beta]\, n}{(1-\phi)(1-\psi)}, \tag{99}$$

$$\gamma_\tau^* = \frac{\lambda n}{1-\phi}. \tag{100}$$

D Mathematical Notation

C = consumption
ρ = subjective discount rate
θ = coefficient of risk-aversion in the utility function
δ = rate of depreciation
t = time
K = physical capital
L = labor
n = rate of growth of the labor force
u_Y = share of labor (or human capital) devoted to production of final consumption good
u_τ = share of labor (or human capital) devoted to production of financial innovations
u_A = share of labor devoted to R&D of new technological designs
u_H = share of labor devoted to production of human capital
H = stock of human capital
τ = stock of financial innovations
$\xi \equiv \tau/L^\kappa$ = efficiency of intermediation between savings and investment
$\varsigma \equiv \tau/H^\kappa$ = number of financial innovations per adjusted unit of human capital
$\chi \equiv C/K$ = consumption-capital ratio
$k \equiv K/L$ = capital-labor ratio
$\widehat{k} \equiv K/AL$ = technology-augmented capital-labor ratio
$\omega \equiv K/H$ = physical to human capital ratio

γ_τ^* = steady-state growth rate of the stock of financial innovations
γ_A^* = steady-state growth rate of the number of intermediate goods
γ_H^* = steady-state growth rate of human capital
i = index of intermediate goods
A = number of intermediate goods
x = quantity of any intermediate
w_j = wage rate in sector j
r_V = interest rate on transformed savings earned by households
r_K = interest rate paid by financial intermediaries by borrowers (firms)
$p(x_i)$ = price of intermediate good i
π_x = profits earned by a producer of an intermediate good
π_τ = profits earned by a financial innovator
α = capital's share of income generated in final goods production
λ = elasticity of financial innovation production with respect to labor
ϕ = elasticity of financial innovation production with respect to the existing stock of financial products
κ = a measure of the average degree of rivalry in financial products
η = elasticity of R&D production with respect to labor
ψ = elasticity of R&D production with respect to the existing stock of R&D designs
β = elasticity of R&D production with respect to the stock of financial innovations