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Choosing The Tax Rate in a Linear Income Tax Structure: An Introduction

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Abstract

This paper provides an introduction to modelling the choice of linear income tax rate in both majority voting and social welfare maximising contexts. Although the basic problem in each case – of finding the most preferred tax for the median voter and the welfare maximising tax for an independent judge or decision-maker – can be simply stated, it is usually not possible to obtain explicit solutions even for simple assumptions about preferences and population heterogeneity. The present paper instead gives special attention to a formulation of the required conditions in terms of easily interpreted magnitudes, the elasticity of average earnings with respect to the tax rate and a measure of inequality. The inequality measure takes the same basic form in each model (depending either on median earnings or a weighted average of earnings, where the weights depend on value judgements regarding inequality aversion. The approach enables the comparative static effects of a range of parameter changes to be considered. The results are reinforced using numerical examples based on the constant elasticity of substitution utility function.

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1 Introduction

The purpose of this paper is to provide an introduction to some of the problems involved in modelling the choice of tax rate in simple tax and transfer systems. The subject may be divided into two broad strands, positive and normative in nature. The positive line attempts to model policy choices and to investigate to what extent they can be explained as the outcome of voting behaviour. It combines modelling and empirical work. The normative line investigates the policy implications of adopting alternative value judgements. It typically uses modelling methods to examine choices implied by the maximisation of a social welfare function representing the views of a single decision-maker.[1]

Special attention is given below to the simplest possible progressive tax structure, one having an unconditional transfer payment (or 'basic income') and a proportional income tax (or 'flat tax').[2] The combined effect is to generate an increasing average tax rate over the whole range of income and therefore a high degree of progressivity, even though the system has a constant marginal tax rate (and thus a non-progressive rate scale). This system may be administered in several ways (depending for example on whether individuals simultaneously pay tax and receive a benefit, or whether the net tax, which may be positive or negative, is assessed for each person), giving rise to different descriptions. But the terms 'linear income tax' or 'BI/FT' seem to be the least ambiguous.

The linear tax involves just two tax policy parameters, the basic income and the tax rate. However, the government faces a budget constraint whereby the tax raised must be sufficient to finance the transfer payment and any other non-transfer expenditure. Hence, it is possible to choose the level of only one parameter independently. Having selected the tax rate, a value of the basic income is implied. This structure therefore appears to offer a very simple

[1] It is often suggested that determination of the optimal tax is an exercise in 'second best' policy, since a first-best tax is one imposed on the unobservable basic endowment of ability.

[2] The following discussion considers income units for which income is the only tax-relevant variable, so that no transfers for 'special needs' arise and income unit size and composition are irrelevant.

policy choice: only the single income tax rate needs to be set, assuming that the non-transfer expenditure needs have already been decided.

If individuals' labour supplies, and hence their incomes, are fixed independently of the tax system, consideration of this choice is also simple. For example, complete equality could be achieved by setting a tax rate of 1 and sharing all the revenue equally, after covering non-transfer needs.[3] If the tax were to be decided democratically using majority voting, the relatively rich would vote for the minimum tax consistent with raising non-transfer expenditure while the poor would vote for the maximum feasible tax rate of 1. But of course in practice labour supply incentives are important: they impose severe restrictions on the ability of governments to redistribute income. The question then arises of what insights can be obtained about the choice of tax rate, given explicit statements about the decision-making process and information about individuals' preferences and their productivities. As mentioned earlier, the decision may for example be made as a result of a democratic vote, or it may be imposed by a government (seen as a single decision-maker) holding a particular set of value judgements. The power to tax – to impose a non-voluntary imposition – is seen as a fundamental characteristic of governments.

It is simple to describe in principle the mechanics of modelling tax choices in the context of the linear income tax where just one policy dimension is involved and heterogeneity only of abilities, and thus wage rates, is assumed to exist. The indirect utility function of each individual is obtained (by substituting optimal values into the direct utility function) and by using the government's budget constraint (relating the basic income to the tax rate) these are expressed in terms of the tax rate only (along with preference parameters of course). Using the median voter theorem, the majority voting equilibrium is found by maximing the median voter's indirect utility with respect to the tax rate. To examine optimal policies the indirect utilities are substituted into a social welfare function, which is maximised with respect to the tax rate. Both are in principle unconstrained maximisation problems

[3]This assumes that the rich cannot vote for a poll tax (a negative basic income) to finance non-transfer expenditure.

involving one variable. However, the resulting first-order condition usually turns out to be highly nonlinear. It is very difficult to produce analytical solutions, even with extremely simple models.[4] The technical literature is therefore extensive and usually involves the use of numerical solution methods to examine a range of assumptions.

Conceptually, the problem can also be stated simply in terms of the tangency of indifference curves (either of the median voter or the social decision maker) with a government budget constraint. But it is difficult to take such purely diagrammatic insights much further. Nevertheless some useful insights can be obtained for the linear tax structure using some basic algebra and related diagrams, as shown below. It is possible to highlight the kind of information which would be required to determine the tax rate – either in positive or normative contexts – and about the potential limits to redistribution imposed by the existence of incentive effects. The emphasis here is on two summary measures, those of earnings inequality and the elasticity of average earnings with respect to the tax rate. The properties of tax choices expressed in terms of these concepts can guide intuition regarding comparative static parameter changes.

Section 2 begins by describing the linear tax structure and examining the form of the government's budget constraint. Individuals' preferences regarding the choice of tax rate, subject to that constraint, are considered in Section 3. Majority voting outcomes and optimal tax decisions are then examined in Sections 4 and 5 respectively. Some general properties of tax structures are then discussed briefly in Section 6.

[4] Hence numerical simulation methods are generally needed to examine particular model assumption. Special cases of utility functions giving rise to closed-form solutions for the optimal tax rate are given in Atkinson and Stiglitz (1980, pp. 408-9). Hindriks and Myles (2006, pp. 503-505, pp. 493-495) discuss a special case of majority voting, and two-person examples of the optimal tax rate structure. A further explicit solution to an optimal tax problem, assuming that income supply functions are linear in the wage rate, is given by Deaton (1983).

2 The Tax System

This section describes the simple linear tax model and considers the resulting constraints faced by individuals and the government.

2.1 Individuals

Suppose earnings from employment, y_i, for $i = 1, ..., n$ individuals, are the only form of income, other than transfer payments. The linear income tax involves a flat-rate tax at the rate, t, along with an unconditional (non means-tested) transfer payment of b per person. Hence the tax paid, $T(y_i)$, is:

$$T(y_i) = ty_i - b \tag{1}$$

and net income, z_i, is:

$$z_i = (1 - t) y_i + b \tag{2}$$

If $t > 0$ and $b > 0$ the tax is progressive.[5] The progressive nature of this type of structure can be seen from Figure 1, which shows an increasing average tax rate over the whole income range.

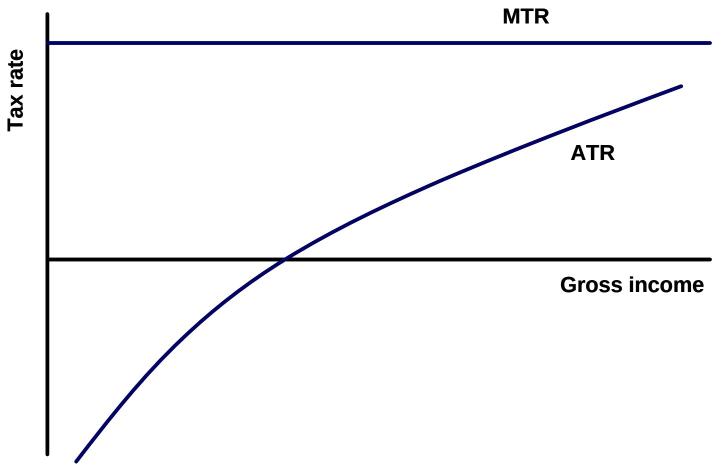

Figure 1: Average and Marginal Tax Rates in the Linear Income Tax Structure

[5] However, if $b < 0$ the transfer payment is instead a poll tax. This arises if the tax rate is not high enough to finance non-transfer expenditure.

2.2 The Government's Budget Constraint

In addition to financing transfer payments from the income tax, suppose the government needs to collects net revenue of R per person. In the following analysis it is assumed that any expenditure arising from this revenue does not affect individuals' utilities.[6] The government therefore faces a budget constraint of the form $nR + nb = t\sum_{i=1}^{n} y_i$, or:

$$R = t\overline{y} - b \tag{3}$$

Here $\overline{y}$ is arithmetic mean gross earnings, which depends on all the tax parameters as well as the distribution of wage rates and individuals' preferences. Holding R constant means that there is only one degree of freedom in the choice of t and b. The general form of the government's budget constraint, for fixed R, is shown in Figure 2. When $t = 0$, $b = -R$. So long as the tax rate exceeds $t_{\min}$, the linear tax structure is progressive.

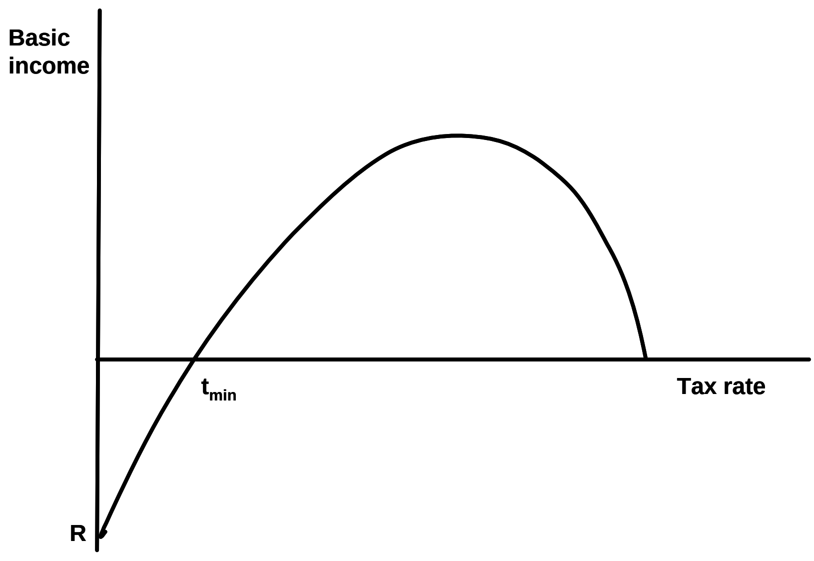

Figure 2: The Government Budget Constraint

The slope of the budget line, giving the variation in b as t varies, keeping R fixed, can be obtained by totally differentiating the budget constraint in

[6]The basic model can be extended to allow for the non-transfer expenditure to be used to finance public goods, introducing further modelling complexities.

(3) and setting $dR = 0$. Hence:

$$dR = \left(\overline{y} + t\frac{\partial \overline{y}}{\partial t}\right) dt + \left(t\frac{\partial \overline{y}}{\partial b} - 1\right) db \tag{4}$$

and:

$$\left.\frac{db}{dt}\right|_R = \frac{\overline{y} + t\frac{\partial \overline{y}}{\partial t}}{1 - t\frac{\partial \overline{y}}{\partial b}} \tag{5}$$

The variation in average gross earnings as tax parameters vary can be obtained by totally differentiating, so that:

$$d\overline{y} = \frac{\partial \overline{y}}{\partial t}dt + \frac{\partial \overline{y}}{\partial b}db \tag{6}$$

and:

$$\left.\frac{d\overline{y}}{dt}\right|_R = \frac{\partial \overline{y}}{\partial t} + \left(\frac{\partial \overline{y}}{\partial b}\right) \left.\frac{db}{dt}\right|_R \tag{7}$$

This result could easily be converted into an expression involving elasticities. Hence the elasticity of $\bar{y}$ with respect to t depends on the partial elasticities of $\bar{y}$ with respect to t and b, along with the elasticity of b with respect to t along the government budget constraint.

3 Individual Behaviour

The aim of this section is to characterise an individual's preferred position on the government's budget constraint relating b and t. First, individual utility maximisation involving consumption and labour supply choices is considered in order to generate the indirect utility function expressed in terms of the tax parameters. Then the tax preferences of individuals are examined, depending on their earnings. The results are entirely general in that they do not depend on the form of utility functions or the distribution of wage rates.

3.1 Utility Maximisation

Let c_i and ℓ_i denote the consumption and labour supply of individual i, who faces a pre-tax wage rate of w_i, for $i = 1, ..., n$. Gross earnings are thus $y_i = w_i \ell_i$. The price of consumption is normalised to 1 and as the model is static, so that no savings are made, consumption and net income are equal; hence $c_i = z_i$.

Each individual maximises $U(c_i, \ell_i)$ subject to the budget constraint:

$$c_i = (1-t)\, y_i + b \tag{8}$$

Suppose also that each individual is endowed with one unit of time, which may be divided between leisure and work. The effective price of leisure is the net wage. Full income, M_i, defined as the maximum income which can be obtained by devoting all available time to working, is thus:[7]

$$M_i = w_i\,(1-t) + b \tag{9}$$

Substituting the optimal values in the direct utility function gives the indirect utility function in terms of the net wage and full income, $V(w_i(1-t), M_i)$: for present purposes this can be written more succinctly as $V(t, b)$.

3.2 Individual Tax Preferences

Consider each individual's preferences over the tax rate rate. Non-transfer expenditure is assumed to be set exogenously, so the govenment's budget constraint implies a value of b for any t. Hence only the latter can be selected independently.[8] Diagrammatically, each individual's indirect utility function $V(t, b)$ defines a set of indifference curves in Figure 2. If the individual works, an increase in t must be compensated by an increase in the basic income, b, so indifference curves are upward sloping from left to right. An individual's preferred position on the government budget constraint is therefore characterised by a tangency between an indifference curve and the constraint. The slope of the budget constraint is given in equation (5). The slope of indifference curves is given in the usual way, by setting the total differential of V equal to zero, so that:

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial b}db \tag{10}$$

[7] The usual approach thus considers each individual as effectively obtaining full income and then purchasing consumption and leisure at prices 1 and $w_i(1-t)$ respectively.

[8] The choice must be over the tax rate, because there are two values of t corresponding to any given b, whereas choice of t gives rise to a unique b.

and when $dV = 0$:

$$\left.\frac{db}{dt}\right|_V = -\frac{\partial V/\partial t}{\partial V/\partial b} \tag{11}$$

At this point, it is useful to employ a standard result from duality theory. In general for an indirect utlity function of the form $V(p, m)$, for goods demanded, x_i, at prices, p_i, and a budget of m, Roy's Identity gives the Marshallian demands as $x_i = -(\partial V/dp_i)/(\partial V/dm)$. In the present context, this means that labour supply, ℓ_i, can be expressed as:

$$\begin{aligned}\ell_i &= \frac{\partial V/\partial(w_i(1-t))}{\partial V/\partial M_i} \\ &= -\frac{1}{w_i}\frac{\partial V/\partial t}{\partial V/\partial b} \end{aligned} \tag{12}$$

In the first line of this expression, the minus sign in the standard form of Roy's Identity has been deleted because the variable in question is the amount supplied, not demanded. Hence:[9]

$$y_i = -\frac{\partial V/\partial t}{\partial V/\partial b} \tag{13}$$

It should of course be remembered that y_i is not fixed, but depends on the nature of preferences as well as the tax parameters. In (13), $\partial V/\partial t < 0$ and $\partial V/\partial b > 0$, so that the right hand side is positive. Combining (13) and (11), the slope of an individual's indifference curve, relating to preferences regarding b and t, at any point is thus:[10]

$$\left.\frac{db}{dt}\right|_V = y_i \tag{14}$$

At the tangency position, equating the slope of the indifference curve with the slope of the government's budget constraint, the term $\left.\frac{db}{dt}\right|_R$ in (5)

[9] This insight is found in Tuomala (1985).

[10] This may be compared with the simple case where there are no labour supply incentive effects. In the fixed income case, individual net income, z_i, is given by $z_i = (1-t)y_i + b$. The individual is thus treated as selecting the tax structure which maximises net income. The indifference curves, from $dz_i = -y_i dt + db = 0$ are thus straight lines with $\left.\frac{db}{dt}\right|_{z_i} = y_i$. Furthermore, the government's budget constraint is a straight line with a slope equal to the (fixed) mean income of $\bar{y}$. Hence, as discussed in the introduction, all those with $y_i < \bar{y}$ prefer the maximum tax rate of unity.

is equated to $\frac{db}{dt}\big|_V = y_i$ from (14). The individual's preferred combination of tax parameters therefore satisfies:

$$y_i = \frac{\overline{y} + t\frac{\partial \overline{y}}{\partial t}}{1 - t\frac{\partial \overline{y}}{\partial b}} \tag{15}$$

which can be rearranged to produce the following form:

$$y_i - \overline{y} = t\left(\frac{\partial \overline{y}}{\partial t} + y_i \frac{\partial \overline{y}}{\partial b}\right) \tag{16}$$

Using equation (7),[11] the term in brackets on the right hand side of (16) can be replaced by $\frac{d\overline{y}}{dt}\big|_R$, so that:

$$y_i - \overline{y} = t \left.\frac{d\overline{y}}{dt}\right|_R \tag{17}$$

and dividing by $\overline{y}$ gives:

$$1 - \frac{y_i}{\overline{y}} = -\frac{t}{\overline{y}} \left.\frac{d\overline{y}}{dt}\right|_R = \left|\eta_{\overline{y},t}\right| \tag{18}$$

where $\left|\eta_{\overline{y},t}\right|$ represents the absolute value of the elasticity of average earnings with respect to the tax rate. This equality characterises individual i's preferred tax rate.

However, the expression in (18) applies only to individuals for whom $y_i < \bar{y}$, for which tangency solutions on the government's budget constraint apply. An individual with $y_i > \bar{y}$ has indifference curves (relating b and t) with a slope, y_i, that exceeds the slope of the government's budget constraints. Hence the optimal position for such an individual is a corner solution at $t = t_{\min}$.[12]

4 Majority Choice of Tax Rate

Having considered individuals' preferred tax rates, in terms of their preferred position on the government's budget line, this section examines majority voting over the tax rate. The context is therefore one of voting over

[11] And remembering that at the tangency, $\frac{db}{dt}\big|_R = \frac{db}{dt}\big|_V = y_i$.

[12] A regressive poll tax is ruled out by assumption.

a single-dimensional issue: voting simultaneously over more than one issue raises further complexities not examined here.[13] First, it is necessary to establish a feature of the model whereby individuals can be ranked in ascending order by their gross incomes in precisely the same way, independently of the tax rate. The majority outcome, corresponding to the preferences of the median voter, is then examined. Numerical examples are used to demonstrate comparative static properties of the result.

4.1 Individual Ranks

An important property of the present model is that the ranking of individuals by their gross earnings, which (as shown in the previous section) determines their preferred position on the government's budget constraint, and thus choice of tax rate, is not affected by the tax parameters. This condition of unchanged ranks is known as 'hierarchical adherence' or 'agent monotonicity'.

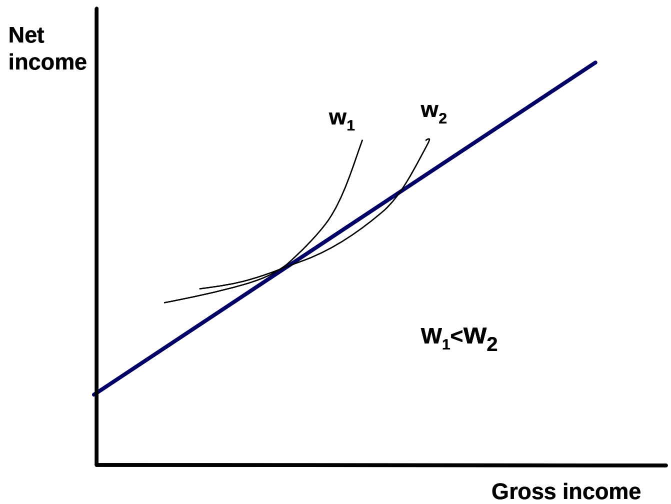

Figure 3: Hierarchical Adherence with Single Crossing Indifference Curves

This property is guaranteed if a 'single crossing' condition applies to individuals' indifference curves defined in terms of gross and net income.[14] This

[13] On difficulties raised by multidimentional voting, see Muller (2003, pp. 87-92).
[14] See Ganns and Smart (1996).

is illustrated in Figure 3, which has gross earnings on the horizontal axis and net income on the vertical axis. The tax function, with a slope of $1 - t$, is shown as a straight line and the preferred position of person 1 is shown as the tangency of indifference curve w_1 with the tax function. This determines labour supply (and hence gross earnings) and consumption (net income). An individual with a higher wage, w_2, must have an indifference curve that is flatter than than associated with w_1 (at the tangency position) because any given increase in gross earnings can be obtained by working a smaller number of extra hours. Hence a given increase in gross earnings (a reduction in leisure) does not need to be compensated by as high an increase in net income as for the person with the lower wage rate. The person with the higher wage rate therefore reaches a tangency to the right of that of the person with the lower wage. This must apply for all linear tax functions, whatever their slope. Those with relatively high w necessarily have relatively high y.

4.2 The Median Voter

Consider the majority choice of the tax rate, for given R. It is well known that single-peaked preferences guarantee a voting equilibrium in which the median voter dominates. The median voter unambiguously holds the balance of power, and coalitions would not be formed between those on either side of the median. In the present framework, single-peakedness arises if the relationship between V and t is concave, so that $d^2V/dt^2 > 0$. With positive marginal utility and a concave relationship between b and t, single-peakedness is guaranteed if the individual always works, that is if w_i exceeds the minimum wage, w_L, necessary to avoid the corner solution where $\ell_i = 0$. Hence if all $w_i > w_L$, combined with the fact that the ranks of individuals are independent of the tax system, the median voter can unambiguously be identified with the median wage, w_m, and hence the median earnings, y_m.

If there are individuals who do not work, that is if $w_i < w_L$ for some i, their preferences are not single peaked. Remembering that w_L depends on the tax rate, such an individual may work when t is very low and suffer from increases in t. However, once the tax rate is sufficiently high for them to stop working altogether, they prefer t to rise further because they desire only the highest possible transfer payment and are not themselves directly affected by further increases in t. Roberts (1977) showed that there is nevertheless a majority voting equilibrium, so long as the ranking of individuals are independent of the tax system. This condition has already been established above.[15]

Having demonstrated that the median voter theorem applies in this case, it can be seen from (18) that substitution of y_m for y_i shows that the majority choice satisfies the condition:

$$1 - \frac{y_m}{\overline{y}} = \left| \eta_{\overline{y},t} \right| \tag{19}$$

The left hand side of this expression can be interpreted as a measure of the inequality of earnings: it is the proportional difference between the median and the arithmetic mean gross earnings.[16] While this condition is simply stated and has terms with convenient interpretations, it does not of course provide a 'closed form' solution for the majority choice of tax rate. The simplicity of this condition disguises considerable complexity, since $\bar{y}$ and $\eta_{\overline{y},t}$ depend on the complete wage rate distribution and the tax parameters.[17]

4.3 Illustrative Examples

It is useful to consider how the terms in (19) are likely to vary with t. This can be achieved using numerical examples based on specific assumptions regarding the wage rate distribution and utility functions. Suppose that individuals have constant elasticity of substitution (CES) utility functions, which can be written as:

$$U = \left\{ \alpha c^{-\rho} + (1-\alpha)(1-\ell)^{-\rho} \right\}^{-1/\rho} \tag{20}$$

[15] The existence of a majority voting equilibrium in the case of a tax-free threshold is demonstrated by Creedy and Francois (1993), who also show that in a multi-period context hierachical adherence is not sufficient to guarantee an equilibrium.

[16] It is assumed that the wage rate distribution is positively skewed, so that the median wage is less than the arithmetic mean.

[17] Furthermore, an attempt to solve for the choice of t analytically would proceed directly from the first-order condition. An example using Cobb-Douglas utility functions is given the Appendix.

where the endowment of time is normalised to unity, so that $1-\ell$ is leisure, and the elasticity of substitution, θ, is equal to $1/(1+\rho)$. Defining Ψ as:

$$\Psi = \left\{ \alpha^{\theta} + w \left(\frac{1-\alpha}{w(1-t)} \right)^{\theta} \right\}^{-1} \tag{21}$$

It can be shown that interior solutions for labour supply and consumption are:

$$\ell = 1 - \Psi M \left(\frac{1-\alpha}{w(1-t)} \right)^{\theta} \tag{22}$$

and:[18]

$$c = \Psi M \alpha^{\theta} \tag{23}$$

Where, as above, M is full income given by $w_i(1-t)+b$. Equation (22) applies only for wage rates in excess of a minimum, w_L, for which $\ell = 1$. Suppose that wage rates are lognormally distributed as $\Lambda(w|\,\mu, \sigma^2)$. A simulated population of individuals' wages can therefore be obtained by selecting random values from this lognormal distribution. For given preference parameters (common to all individuals) it is possible to find the value of b which satisfies the government's budget constraint, for given values of t and R, using a process of trial and error.[19]

Figure 4 shows the variation in arithmetic mean gross income, $\bar{y}$, with the marginal tax rate, t, for three different values of non-transfer expenditure, R. These results are for a wage rate distribution, $\Lambda(w|\,10, 0.5)$, and $\alpha = 0.98$ with $\theta = 0.7$.[20] Clearly, average income decreases as the tax rate increases. The profile for the smaller level of non-transfer expenditure is higher because, for a given tax rate, the transfer payment must be smaller so that labour supply is higher. The corresponding government budget constraints showing

[18] The expenditure function, expressed in terms of full income, is found to be $E(w,U) = U\Psi^{-1}\alpha^{\sigma/\rho}\left\{1 + w^{1-\sigma}\left(\frac{1-\alpha}{\alpha}\right)^{\sigma}\right\}^{\sigma/(1-\sigma)}$.

[19] This involves using a trial value of b and solving each individual's labour supply decision. The resulting total values of tax revenue and expenditure can be checked. If revenue exceeds expenditure, the value of b is increased and the process is repeated until convergence is reached. For further discussion of such iterative methods, see Creedy (1996).

[20] On the choice of parameter values in the CES case, see Creedy (1996, pp. 136-139). For example, the choice cannot be made independently of the units of measurement of wages.

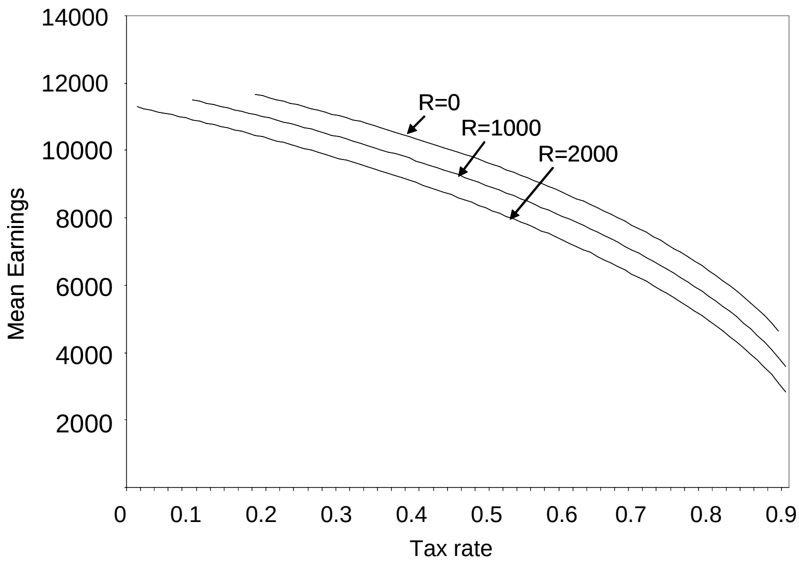

Figure 4: Mean Earnings

the variation in b as t increases are shown in Figure 5, again for three values of non-transfer expenditure. When the tax rate increases from low levels, the 'tax rate effect' on total revenue, and hence b, is greater than the 'tax base effect' arising from the adverse incentive effects. However, after a certain point the reduction in the tax base outweighs the effect of the increasing tax rate, and the tranfer payment falls as t rises.

The profiles of $1 - \frac{y_m}{\bar{y}}$ and $\left|\eta_{\bar{y},t}\right|$, for the three R values, are illustrated in Figure 6 for variations in the tax rate, t. Both profiles are upward sloping as both the absolute elasticity of $\bar{y}$ with respect to t, and the inequality of gross earnings, in this case expressed in the form, $1 - \frac{y_m}{\bar{y}}$, rise with t. However, the (absolute) elasticity rises faster than inequality and there is consequently a single equilibrium position. The majority choice of tax rate can thus be read from the appropriate point of intersection of the two profiles. Increasing R shifts the profile of $1 - \frac{y_m}{\bar{y}}$ and that of $\left|\eta_{\bar{y},t}\right|$ downwards. In the examples shown here, the shift in the latter profile is relatively larger. Hence the median voter's choice of t increases.

The effect of an increase in inequality can be examined by varying the

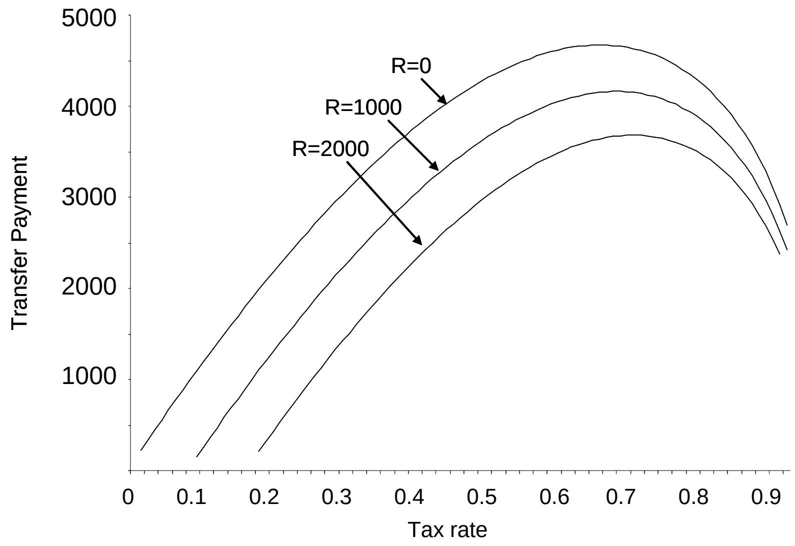

Figure 5: Government Budget Constraint

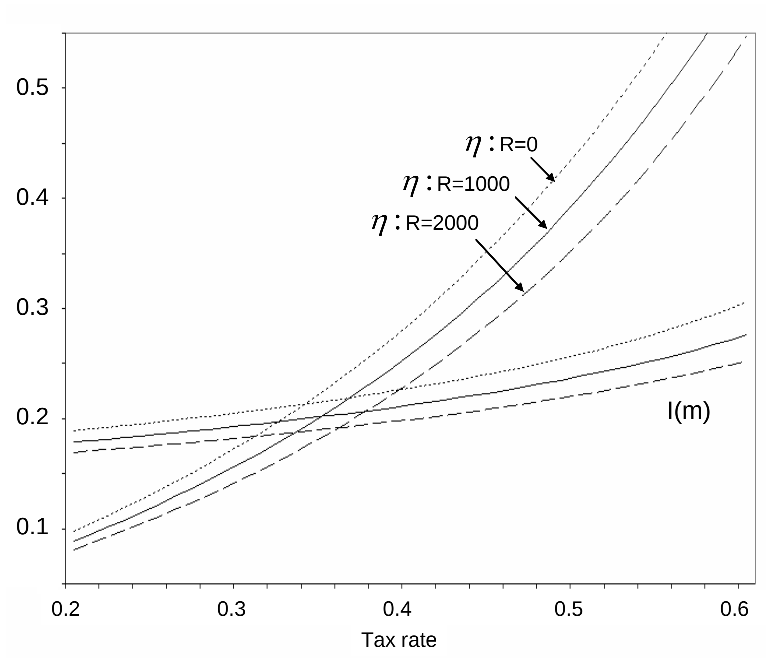

Figure 6: Median Voter Outcomes

assumed value of the variance of logarithms of w. A higher variance implies a lower value of $w_m/\bar{w}$ and thus of $y_m/\bar{y}$, for any given tax rate.[21] An increase in wage rate inequality resulting from an increase in σ^2 has the effect of shifting the profile of $\left|\eta_{\overline{y},t}\right|$ downwards slightly. That is, it produces a small reduction in the absolute elasticity for any given tax rate. However, it clearly produces an upward shift in the profile of $1-\frac{y_m}{\overline{y}}$, since $y_m/\bar{y}$ is lower for any given tax rate. Both of these effects imply an unequivocal increase in the majority voting equilibrium tax rate. The increase is smaller, the smaller is the effect on $\left|\eta_{\overline{y},t}\right|$. This suggests that higher inequality (associated with the higher skewness of the distribution of earnings) produces a majority vote for a higher tax rate, and thus a more redistributive (and progressive) tax structure.[22]

5 Optimal Taxation

This section turns to the normative analysis of tax rate choices, where emphasis is on examination of the implications of adopting particular value judgements, summarised by a social welfare function. It is shown in subsection 5.1 that the condition satisfied by the optimal linear tax is very similar to that obtained for the majority voting outcome. The difference is that the inequality measure on the left hand side of $1-\frac{y_m}{\overline{y}}=\left|\eta_{\overline{y},t}\right|$ is replaced by one in which the median voter's earnings are replaced by a weighted average of earnings. Illustrative examples are then presented in subsection 5.2.

5.1 A Social Welfare Function

Suppose the aim of the government is to maximise a social welfare function of the form $W=W\left(V_1,...,V_n\right)$. This function of individual (indirect) utilities can be regarded, since each V_i depends on the tax parameters, as a function

[21] It can be shown that in a lognormal distribution with variance of logarithms equal to σ^2, the ratio $w_m/\bar{w}=\exp\left(-\sigma^2/2\right)$.

[22] Empirical studies of the relationship between inequality and redistributive taxation have found mixed results; see, for example, Borck (2007). This simple majority voting model abstracts from numerous practical factors (the role of parties, political influence, voter participation, uncertainty and so on) as well as dynamic considerations.

of t and b. It therefore gives a set of 'social indifference curves' in Figure 2. The choice of optimal tax rate is thus represented as a tangency of a social indifference curve with the government budget constraint.[23] The slope of an indifference curve is given by:

$$\left.\frac{db}{dt}\right|_W = -\frac{\partial W/\partial t}{\partial W/\partial b} \tag{24}$$

where:

$$\frac{\partial W}{\partial t} = \sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial t} \tag{25}$$

and:

$$\frac{\partial W}{\partial b} = \sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b} \tag{26}$$

The term $\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b} = v_i$ is the 'welfare weight' attached by the social welfare function to an addition to person i's income (from an increase in the basic income). Furthermore, using (13):

$$\sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial t} = \sum_{i=1}^{n} \left(\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b}\right)\left(\frac{\partial V_i/\partial t}{\partial V_i/\partial b}\right) = -\sum_{i=1}^{n} v_i y_i \tag{27}$$

Thus:

$$\left.\frac{db}{dt}\right|_W = \sum_{i=1}^{n} \left(\frac{v_i}{\sum_{i=1}^{n} v_i}\right) y_i \tag{28}$$

The right hand side of this expression is a weighted average of the y_is, which can be denoted $\widetilde{y}$. Hence equating the slopes of the social indifference curve and the government budget constraint gives:

$$\widetilde{y} = \frac{\overline{y} + t\frac{\partial \overline{y}}{\partial t}}{1 - t\frac{\partial \overline{y}}{\partial b}} \tag{29}$$

[23] An alternative route to the following result, followed by Tuamala (1985) involves using the Lagrangian, $L = W + \lambda\left(t\overline{y} - b - R\right)$. The first-order conditions for maximisation, $\frac{\partial L}{\partial t} = 0$ and $\frac{\partial L}{\partial b} = 0$, give, in addition to the budget constraint, the two equations $\sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial t} = -\lambda\left(\overline{y} + t\frac{\partial \overline{y}}{\partial t}\right)$ and $\sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b} = \lambda\left(1 - t\frac{\partial \overline{y}}{\partial b}\right)$. Dividing these two conditions therefore reproduces the result below.

The optimal tax rate therefore satisfies the same kind of condition as the majority voting outcome, but with median earnings replaced by $\widetilde{y}$, so that:

$$1 - \frac{\widetilde{y}}{\overline{y}} = \left|\eta_{\overline{y},t}\right| \tag{30}$$

The left hand side of this expression, as with (19), defines an inequality measure, this time involving the proportional difference between the weighted average earnings and the unweighted average earnings, where the weights depend on the form of the social welfare function. The optimal tax is that rate which satisfies equation (30).[24]

Since the majority choice and the social welfare function maxima are both described in terms of tangency solutions of a form of indifference curve along the government's budget constraint, it is perhaps not surprising that they can be expressed in fundamentally the same way. The only difference relates to the inequality measure used. Hence, if in general a measure of the inequality of earnings, taking the form of a proportional difference between average earnings and some other measure of location, is denoted by $I(y)$, the desired tax rate is the root of:

$$I(y) = \left|\eta_{\overline{y},t}\right| \tag{31}$$

Again, this does not provide a closed-form solution, and of course any solution must also satisfy $t > t_{\min}$ if the tax system is to be progressive. That is, if the tax does not raise sufficient revenue to finance the non-transfer expenditure, R, the shortfall must be made up by imposing a poll tax (a negative b) on each individual. But, as with majority voting, the expression in (30) provides a condition in terms of easily interpreted concepts.

5.2 Illustrative Examples

In section 4.2, profiles of $\left|\eta_{\overline{y},t}\right|$ were shown, on the assumption that utility functions display constant elasticity of substitution. Examination of optimal

[24]This is the result given by Tuomala (1985). However, in the accompanying diagram, he draws a downward sloping curve of $1 - \frac{\widetilde{y}}{\overline{y}}$ against t. It is shown below that both the right and left hand sides of (30) slope upwards.

tax choices therefore requires corresponding values of the inequality measure, $I(y) = 1 - \frac{\tilde{y}}{\overline{y}}$ to be produced, where $\tilde{y}$ depends on the welfare weights, v_i. Although it is possible to examine the implications of using any type of social welfare function, a convenient approach for present purposes is to specify welfare in terms of each individual's income, rather than utility.[25] Using a constant relative inequality aversion form of the welfare function, the contribution to social welfare of the ith individual is $y_i^{1-\varepsilon}/(1-\varepsilon)$, where ε is the constant relative inequality aversion coefficient. The term $\tilde{y}$ can thus be replaced by the equally distributed equivalent level of income, y_e, defined as the level of income which, if obtained by everyone, produces the same social welfare as the actual distribution. Hence, the term $I(y)$ can be replaced by Atkinson's inequality measure, A, since it is the proportional difference between arithmetic mean income and the equally distributed equivalent level and:

$$A = \frac{\overline{y} - y_e}{\overline{y}} \tag{32}$$

where:

$$y_e = \left\{ \frac{1}{n} \sum_{i=1}^{n} y_i^{1-\varepsilon} \right\}^{1/(1-\varepsilon)} \tag{33}$$

Figure 7 illustrates the variation in the inequality of net income and gross earnings as the tax rate increases, for a given value of $\varepsilon = 0.2$. Inequality of gross earnings increase as t rises, whereas the inequality of net income falls steadily. The relatively small decrease in the inequality of net income over a wide range of t indicates the severe constraint on the government's ability to redistribute income imposed by adverse incentive effects. Figure 8 illustrates the relevant profiles for $R = 1000$ and $\varepsilon = 0.8$, for two values of the variance of logarithms of wage rates. In this case the profile of $\left|\eta_{\overline{y},t}\right|$ is virtually unchanged, so the upward movement in the inequality profile clearly produces a higher choice of optimal tax rate. Hence, higher inequality is again associated with a more redistributive tax structure. Similarly, a higher value of the inequality aversion coefficient shifts only the profile of $1-\tilde{y}/\overline{y}$ upwards, implying a higher optimal tax rate.

[25]The conditions under which this is consistent are discussed by Banks *et al.* (1996).

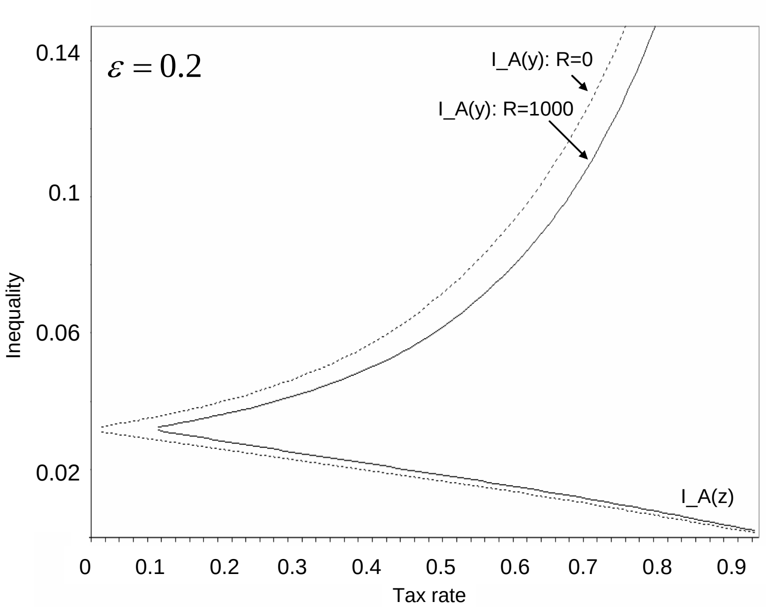

Figure 7: Inequality of Net Income and Gross Earnings

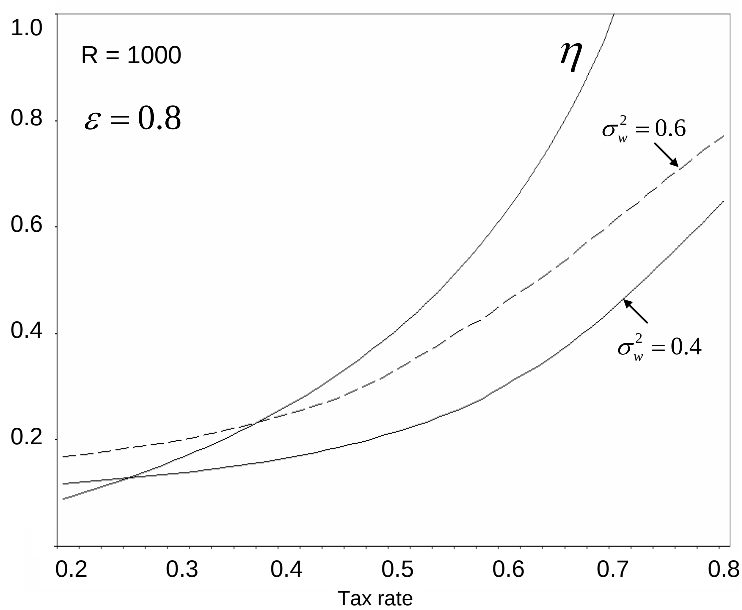

Figure 8: Change in Inequality of Wages

The effect of differences in the elasticity of substitution are less clear cut in this framework because it depends on the non-transfer revenue needed. If there is a pure transfers system, with $R = 0$, the optimal tax rate falls as the elasticity of substitution increases. However, if $R > 0$ the optimal rate falls as the elasticity of substitution, θ, increases from low values. But at higher values of θ, further increases actually lead to an increase in the optimal tax rate. This arises because the minimum tax rate needed to finance the non-transfer expendure is higher for higher values of θ.

6 Optimal Tax Structures

The above analysis has concentrated on the choice of tax rate in the simplest possible redistributive tax and transfer system, the linear income tax. A more general, and much more difficult, question concerns the nature of the optimal tax structure itself.[26] It is not clear that a linear structure, involving a constant marginal tax rate, is optimal, and it is likely that the optimal structure depends on the nature of the social welfare function examined as well as the wage rate distribution and the preference parameters. However, emphasis on the linear tax arises not only because of its simplicity, but also because numerical analyses have suggested that, for a range of assumptions, the optimal structure is approximately linear.

The general treatment of optimal tax structures yields few clear results, yet perhaps ironically the most unambiguous results can easily be established diagrammatically. First consider Figure 9 which shows a tax function in a diagram with net income on the vertical and gross income on the horizontal axis. This displays a range AB where the marginal tax rate (equal to 1 minus the slope of the tax schedule) is greater than 100 per cent. It is clear that this range is irrelevant, since indifference curves relating net income and gross earnings are upward sloping and convex: an increase in gross earnings involves an increase in hours worked, which must be compensated by an increase in net income (consumption). An example of a preferred position,

[26] This is in fact a problem in the calculus of variations, and was examined in detail by Mirrlees (1971).

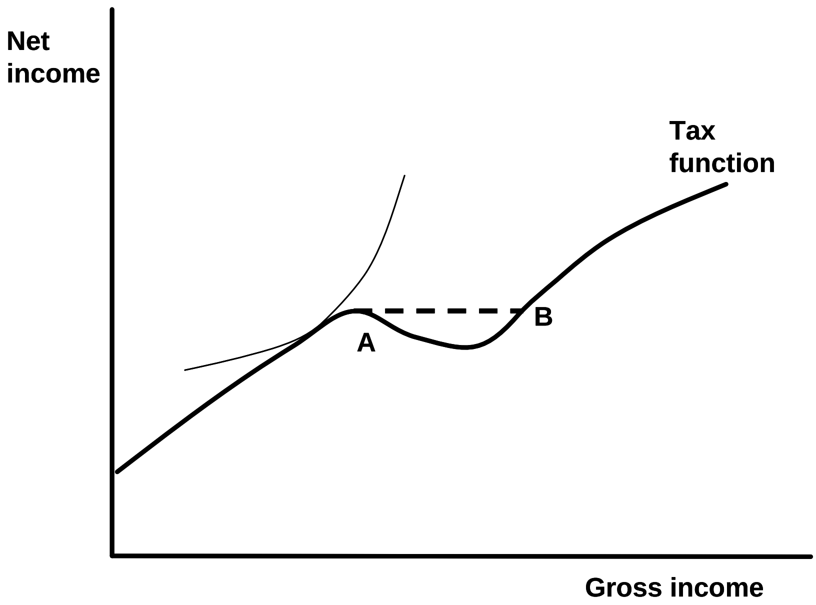

Figure 9: Maximum Tax Rate

for a particular wage rate, is shown as the tangency between the tax function and the indifference curve. Hence AB can be replaced by a marginal tax rate of unity, showing that the maximum rate is 1.

There is a further result which is perhaps surprising and which states that the marginal tax rate on the highest income should be zero. In Figure 10 consider the tax function AB, where the person with the highest wage rate reaches a tangency position at C. If the tax function is changed to ACD, where CD is parallel to the 45 degree line, the individual then faces a zero marginal rate on any extra income earned. This induces a movement to a new tangency on a higher indifference curve. The total tax revenue is unchanged and no one else is affected: hence there is a Pareto improvement and the non-zero top marginal rate cannot have been optimal. However, this result is of no practical relevance as there is no way to determine just where the rate should become zero.

Instead of considering the optimal form of the tax function in general, one approach has been to consider piecewise-linear tax functions. This allow for consideration of the question of whether marginal tax rates should be

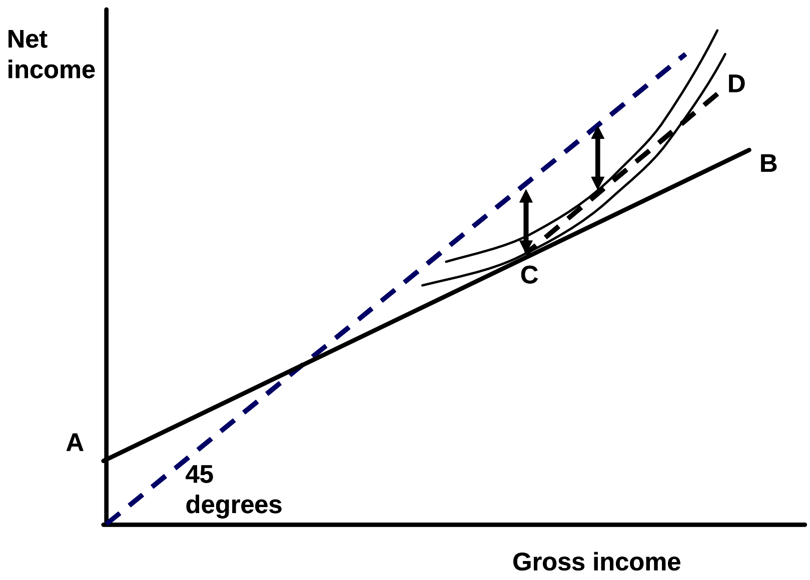

Figure 10: Tax Rate on Top Income

higher for those with relatively low earnings. Such higher marginal rates arise from the means-testing of transfer payments, whereby benefits are reduced as earnings rise. The resulting non-convexity of budget sets facing individuals can give rise to complex labour supply behaviour. Means-testing is preferred by those who advocate 'target efficiency' as the criterion by which schemes should be judged. However, numerical analyses show that in a very wide range of situations, a social welfare function is increased by a shift towards a flatter rate schedule.[27]

7 Conclusions

This paper has considered the choice of linear income tax rate in both majority voting and social welfare maximising contexts. Although the basic problem in each case – of finding the most preferred tax for the median voter and the welfare maximising tax for an independent judge or decision-maker

[27] See, for examples, simulation results reported in Creedy (1998). However, assumptions leading to relatively higher taxes on the poor and the rich are given by Diamond (1998). See also Atkinson (1995).

– can be simply stated, it is usually not possible to obtain explicit solutions even for simple assumptions about preferences and population heterogeneity. The present paper has instead given special attention to a formulation of the required conditions in terms of easily interpreted magnitudes, the elasticity of average earnings with respect to the tax rate and a measure of inequality. The inequality measure takes the same basic form in each model. It is either the proportional difference between the median and arithmetic mean earnings (in the voting model) or the proportional difference between a weighted average of earnings and arithmetic mean earnings (for maximisation of a social welfare function), where the weights depend on value judgements regarding inequality aversion. The approach enables the comparative static effects of a range of parameter changes to be considered. The results were reinforced using numerical examples based on the constant elasticity of substitution utility function.

Appendix: Majority Voting with Cobb-Douglas Preferences

Consider the case where all utility functions take the Cobb-Douglas form (omitting subscript i), $U = c^{\alpha} h^{1-\alpha}$, where $\ell = 1 - h$, so that with full income of $M = w(1-t) + b$, the demands are $c = \alpha M$ and $h = (1-\alpha) M / \{w(1-t)\}$. Earnings are thus equal to $y = \alpha w - (1-\alpha) b / (1-t)$, and if all individuals work, arithmetic mean earnings are therefore:

$$\bar{y} = \alpha \bar{w} - \frac{(1-\alpha) b}{1-t} \tag{1}$$

and using the government budget constraint, $b = t\bar{y} - R$, the transfer payment can be expressed as:

$$b = \frac{\alpha t \bar{w} - R}{1 + (1-\alpha)\left(\frac{t}{1-t}\right)} \tag{2}$$

Substituting c, h into U and using (2), indirect utility can be written, using $k = \alpha^{\alpha} (1-\alpha)^{1-\alpha}$, as:

$$V = k \frac{M}{\{w(1-t)\}^{1-\alpha}} \tag{3}$$

with:

$$\begin{aligned} M &= w(1-t)\left[1 + \frac{\bar{w}}{w}\left\{\frac{\alpha t - R/\bar{w}}{1 - \alpha t}\right\}\right] \\ &= w(1-t) A \end{aligned} \tag{4}$$

where A denote the term in square brackets in the preceding expression. Then $V = k\{w(1-t)\}^{\alpha} A$ and where the median wage is w_m, the majority choice is the solution to $dV/dt = 0$. This gives the condition:

$$\frac{\alpha A}{1-t} = \frac{dA}{dt} = \left(\frac{\bar{w}}{w_m}\right) \frac{\alpha(1 - R/\bar{w})}{(1-\alpha t)^2} \tag{5}$$

Rearrangement of (5) gives the majority choice as the appropriate root of the following quadratic:

$$at^2 + bt + c = 0 \tag{6}$$

where $a = \alpha^2 (1 - \bar{w}/w_m)$, $c = 1 - R/\bar{w}$ and:

$$b = \frac{\bar{w}}{w_m}\left\{(1-\alpha)\left(1 - \frac{R}{\bar{w}}\right) + 2\alpha\right\} - 2\alpha \tag{7}$$

Hence even in what might appear to be a very simple model, no convenient expression for the majority choice of tax rate is available, although numerical examples can easily be obtained using these results.

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