De Vylder Approximations to the Moments and Distribution of the Time to Ruin

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Abstract

De Vylder (1978) proposed a method of approximating the probability of ultimate ruin in the classical risk model. In this paper we show that his ideas can be extended to approximate the moments and distribution of the time to ruin.

1 Introduction

In the classical risk model the insurer's surplus process $\{U(t)\}_{t\geq 0}$ is given by

$$U(t) = u + ct - S(t)$$

where u is the insurer's initial surplus, c is the insurer's premium income per unit time, which is assumed to be received continuously, and $\{S(t)\}_{t\geq 0}$ is the aggregate claims process. This latter process is a compound Poisson process, and we denote the Poisson parameter by λ and let individual claims have distribution function P and moments m_k, $k = 1, 2, 3, \ldots$.

The random variable T denotes the time to ruin so that

$$T = \inf\{t \mid U(t) < 0\}$$

with the understanding that $T = \infty$ if $U(t) \geq 0$ for all $t > 0$. The probability of ultimate ruin is denoted by $\psi(u)$ and given by

$$\psi(u) = \Pr(T < \infty)$$

and we assume that $c = (1 + \theta)\lambda m_1$ where $\theta > 0$ is the premium loading factor, so that $\psi(u) < 1$ for all $u \geq 0$.

In the special case when $P(x) = 1 - \exp\{-\alpha x\}$, $x \geq 0$, it is well known that

$$\psi(u) = \frac{\lambda}{\alpha c} \exp\{-(\alpha - \lambda/c)\, u\}\,.$$

See, for example, Gerber (1979). Indeed, for this individual claim amount distribution many explicit results are known for ruin related quantities. In particular, Drekic and Willmot (2003) derive results for the moments of T_c where $T_c = T \mid T < \infty$, giving explicit solutions for the mean, variance and coefficient of skewness of T_c which are very easy to compute. Further, Dickson et al (2003) show that the density of T_c is

$$g(t) = \alpha c e^{-\lambda u/c-(\lambda+c\alpha)t} \left\{ I_0\left(\sqrt{4\alpha\lambda t(ct+u)}\right) - \frac{ct}{ct+u} I_2\left(\sqrt{4\alpha\lambda t(ct+u)}\right) \right\}$$

where

$$I_v(t) = \sum_{n=0}^{\infty} \frac{(t/2)^{2n+v}}{n!(n+v)!}$$

is the modified Bessel function of order v. It is a straightforward exercise to compute this density with software such as Mathematica which contains the modified Bessel function as a supplied function.

However, for many individual claim amount distributions explicit solutions for ψ and for the moments and density of T_c do not exist. Lin and Willmot (2000) present a recursive scheme from which explicit solutions for the moments of T_c can be found when an explicit solution for ψ exists, and Drekic et al (2004) provide an efficient means of implementing this scheme with Mathematica.

De Vylder (1978) proposed a simple approximation to ψ by approximating the surplus process $\{U(t)\}_{t\geq 0}$ by a process $\{\tilde{U}(t)\}_{t\geq 0}$ given by

$$\tilde{U}(t) = u + \tilde{c}t - \tilde{S}(t),$$

where the aggregate claims process $\{\tilde{S}(t)\}_{t\geq 0}$ is a compound Poisson process with Poisson parameter $\tilde{\lambda}$ and individual claim amount distribution $\tilde{P}(x) = 1 - \exp\{-\tilde{\alpha}x\}$, $x \geq 0$. The parameters of the approximating process are chosen by matching the first three moments of $U(t)$ and $\tilde{U}(t)$, leading to

$$\tilde{\alpha} = \frac{3m_2}{m_3}, \qquad \tilde{\lambda} = \frac{9\lambda m_2^3}{2m_3^2},$$

and $\tilde{c} = c - \lambda m_1 + \tilde{\lambda}/\tilde{\alpha}$. De Vylder's approximation to $\psi(u)$ is

$$\frac{\tilde{\lambda}}{\tilde{\alpha}\tilde{c}} \exp\left\{-\left(\tilde{\alpha} - \tilde{\lambda}/\tilde{c}\right) u\right\},$$

and in his paper De Vylder shows that the approximation works well in cases when the moment generating function of the individual claim amount distribution exists, but does not work as well when this moment generating function does not exist.

Given that explicit solutions for the moments and density of T_c exist when the individual claim amount distribution is exponential, it is natural to ask whether De Vylder's idea can be extended to approximate these quantities when the individual claim amount distribution is not exponential. In other words, can we approximate the moments and density of T_c by the moments and density of the time to ruin, given that ruin occurs, in De Vylder's approximating process $\{\tilde{U}(t)\}_{t\geq 0}$? In this paper we explore this question by considering moments of T_c in Section 2 and the density of T_c in Section 3. Our examples illustrate that by using De Vylder's approximating surplus process we can obtain good approximations to both the moments and density of T_c in the same circumstances that De Vylder's approach provides good approximations to ψ.

Although methods exist to calculate and approximate the moments and density of T_c (see Dickson and Waters (2002)), the more accurate methods are computationally intensive. De Vylder approximations on the other hand can be calculated extremely quickly, making them an attractive alternative approach.

2 Moments of T_c

In this section we illustrate De Vylder approximations to the moments of T_c. For our first two examples, we consider individual claim amount distributions used in De Vylder's (1978) paper, while our final two examples are for Pareto and lognormal individual claim amount distributions. Without loss of generality, we can set $\lambda = m_1 = 1$ so that $c = 1 + \theta$, and this has been done in each of the examples in this section, and in Section 3.

Example 2.1 *Let the individual claim amount distribution be a mixture of exponentials with distribution function*

$$P(x) = 1 - ae^{-\alpha_1 x} - (1-a)e^{-\alpha_2 x}$$

for $x \geq 0$, *where*

$$\alpha_1 = a/(1-a), \quad \alpha_2 = 1/\alpha_1 \quad \text{and} \quad a = \frac{1}{2} + \left[\frac{1}{4} - \frac{2}{7+s^2}\right]^{1/2}.$$

Then $m_1 = 1$, $m_2 = 1 + s^2$ *and* $m_3 = 1.5s^4 + 6s^2 - 1.5$. *This distribution was discussed by Bohman (1977), and we refer to it subsequently as*

Bohman's distribution. As the distribution is just a mixture of two exponentials, an explicit solution for ψ exists (see, for example, Gerber et al (1987)), and hence we can calculate moments of T_c using the algorithms of Drekic et al (2004). In the tables that follow, we have used three values of s which allows us to study the effect of increasing the variance of the individual claim amount distribution on the approximations. For $s = \sqrt{2}$ ($m_2 = 3$), Tables 2.1 and 2.2 show exact and approximate values of the mean, variance and coefficient of skewness of T_c when $\theta = 10\%$ and 25% respectively. Similarly Tables 2.3 and 2.4 display the corresponding values when $\theta = 10\%$ and 25% respectively with $s = 5$ ($m_2 = 26$), while Tables 2.5 and 2.6 are for $\theta = 10\%$ and 25% respectively with $s = 6.496012$ ($m_2 = 43.1982$). We remark that this final choice of s gives the same variance as the individual claim amount distribution considered in the next example.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	15.00	18.33	69.46	76.39	13.74	12.49
20	196.66	196.55	256.75	256.59	3.72	3.72
40	374.58	374.77	354.85	354.73	2.69	2.69
60	552.50	552.99	431.17	431.09	2.21	2.22
80	730.42	731.21	495.89	495.82	1.93	1.93
100	908.34	909.42	553.09	553.03	1.73	1.73

Table 2.1: Mean, standard deviation and coefficient of skewness of T_c, Bohman claims, $s = \sqrt{2}$, and $\theta = 10\%$.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	6.00	7.33	18.44	20.14	8.97	8.19
20	68.46	68.61	65.24	65.16	2.54	2.55
40	129.23	129.89	89.96	89.92	1.85	1.85
60	190.01	191.16	109.23	109.21	1.52	1.52
80	250.78	252.44	125.57	125.56	1.32	1.32
100	311.56	313.72	140.02	140.02	1.19	1.19

Table 2.2: Mean, standard deviation and coefficient of skewness of T_c, Bohman claims, $s = \sqrt{2}$, and $\theta = 25\%$.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	130.00	139.23	597.75	617.56	13.74	13.30
20	319.90	319.88	949.43	949.41	8.65	8.65
40	500.55	500.53	1192.23	1192.22	6.89	6.89
60	681.20	681.19	1393.35	1393.34	5.90	5.90
80	861.85	861.84	1568.89	1568.88	5.24	5.24
100	1042.50	1042.49	1726.68	1726.67	4.76	4.76
200	1945.75	1945.75	2362.50	2362.50	3.48	3.48
300	2849.01	2849.01	2860.32	2860.31	2.87	2.87

Table 2.3: Mean, standard deviation and coefficient of skewness of T_c, Bohman claims, $s = 5$, and $\theta = 10\%$.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	52.00	55.69	157.23	162.08	8.96	8.69
20	118.81	118.80	244.04	244.03	5.79	5.79
40	181.91	181.90	304.69	304.68	4.64	4.64
60	245.01	245.00	355.12	355.12	3.98	3.98
80	308.11	308.11	399.24	399.24	3.54	3.54
100	371.21	371.21	438.95	438.94	3.22	3.22
200	686.72	686.73	599.23	599.22	2.36	2.36
300	1002.23	1002.25	724.89	724.89	1.95	1.95

Table 2.4: Mean, standard deviation and coefficient of skewness of T_c, Bohman claims, $s = 5$, and $\theta = 25\%$.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	215.99	225.53	991.87	1012.47	13.74	13.46
20	406.62	406.62	1374.44	1374.43	9.92	9.92
40	587.72	587.71	1659.23	1659.23	8.22	8.22
60	768.81	768.80	1901.84	1901.84	7.17	7.17
80	949.90	949.89	2116.83	2116.83	6.44	6.44
100	1130.99	1130.98	2311.91	2311.91	5.90	5.90
300	2941.90	2941.90	3739.59	3739.59	3.65	3.65
500	4752.81	4752.81	4756.49	4756.49	2.87	2.87

Table 2.5: Mean, standard deviation and coefficient of skewness of T_c, Bohman claims, $s = 6.496012$, and $\theta = 10\%$.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	86.40	90.21	260.46	265.50	8.96	8.79
20	153.65	153.65	354.66	354.66	6.59	6.59
40	217.09	217.09	425.53	425.53	5.50	5.50
60	280.53	280.53	486.18	486.18	4.82	4.82
80	343.97	343.97	540.06	540.06	4.34	4.34
100	407.41	407.41	589.03	589.03	3.98	3.98
300	1041.80	1041.81	948.63	948.63	2.47	2.47
500	1676.20	1676.21	1205.33	1205.33	1.94	1.94

Table 2.6: Mean, standard deviation and coefficient of skewness of T_c, Bohman claims, $s = 6.496012$, and $\theta = 25\%$.

We see from Tables 2.1–2.6 that the approximations are generally excellent and that the pattern is the same for each value of s suggesting that the variance of the individual claim amount distribution has little effect on the quality of the approximation. Further, the pattern is the same for each value of θ. We see that when $u = 0$ the approximations are poorer than when $u > 0$, which is as expected since De Vylder's approximation to $\psi(0)$ is poor. See De Vylder (1978) for details.

Example 2.2 *Now let the individual claim distribution be another mixture of exponential distributions, this time with distribution function*

$$P(x) = 1 - 0.0039790e^{-0.014631x} - 0.1078392e^{-0.190206x} - 0.8881815e^{-5.514588x}$$

for $x \geq 0$. *Then* $m_1 = 1$, $m_2 = 43.1982$ *and* $m_3 = 7717.23$. *This distribution was discussed by Wikstad (1971), and we refer to it subsequently as Wikstad's distribution. As in Example 2.1, Tables 2.7 and 2.8 show exact and approximate values of the mean, variance and coefficient of skewness of* T_c *when* $\theta = 10\%$ *and* 25% *respectively. We see that the approximations are good, but not as accurate as in the previous example, and that the percentage errors of the approximations are smaller when* $\theta = 10\%$.

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	215.99	595.49	1069.41	1710.85	13.79	8.58
20	704.09	752.27	1891.72	1947.05	7.78	7.54
40	932.67	909.04	2179.02	2157.53	6.75	6.81
60	1103.98	1065.82	2382.43	2349.24	6.18	6.26
80	1262.31	1222.60	2559.77	2526.44	5.75	5.82
100	1417.82	1379.37	2723.65	2692.00	5.41	5.47
200	2191.50	2163.26	3425.37	3400.98	4.30	4.33
300	2964.97	2947.14	4005.89	3985.78	3.68	3.70
400	3738.47	3731.02	4512.29	4495.14	3.27	3.28
500	4512.00	4514.90	4967.27	4952.38	2.97	2.98

Table 2.7: Mean, standard deviation and coefficient of skewness of T_c, Wikstad claims, $\theta = 10\%$.

Example 2.3 *We now consider the situation when the individual claim amount distribution is Pareto(4,3), so that* $P(x) = 1-(3/(3+x))^4$ *for* $x \geq 0$. *For this distribution, an explicit solution for* ψ *does not exist, but numerical values of* ψ *can be calculated. (See, for example, Dickson and Waters (1991).) The moments of* T_c *can be calculated approximately, using the method described in Dickson and Waters (2002). However, as only the first three moments of this Pareto distribution exist, only the first two moments of* T_c *exist – see Delbaen (1988). Tables 2.9 and 2.10 show values of the mean and standard deviation of* T_c *for* $\theta = 10\%$ *and* 25% *respectively. In these tables, we present 'calculated' rather than exact values for comparison with approximations based on De Vylder's method, and these values come from Table 4.3 of*

	Mean		St. Dev.		Coeff. of Skewness	
u	Exact	Approx.	Exact	Approx.	Exact	Approx.
0	86.40	238.20	305.63	470.50	9.12	5.80
20	277.53	285.55	525.40	525.99	5.24	5.21
40	354.09	332.91	594.02	576.16	4.65	4.77
60	405.10	380.27	641.50	622.30	4.31	4.42
80	451.23	427.63	683.34	665.24	4.06	4.15
100	496.48	474.99	722.37	705.58	3.84	3.91
200	721.86	711.78	891.92	879.95	3.13	3.15
300	947.21	948.57	1034.01	1025.07	2.70	2.71
400	1172.58	1185.36	1158.76	1152.06	2.42	2.41
500	1397.98	1422.15	1271.24	1266.38	2.21	2.19

Table 2.8: Mean, standard deviation and coefficient of skewness of T_c, Wikstad claims, $\theta = 25\%$.

Dickson and Waters (2002). We see from Table 2.9 that the approximations are reasonable when $\theta = 10\%$, although not nearly as good as in the previous two examples. However, when $\theta = 25\%$ the approximations are poor, particularly for large values of u. As De Vylder approximations to ψ are generally not particularly good when the individual claim amount distribution is Pareto, the quality of the approximations in Tables 2.9 and 2.10 is not surprising.

	Mean		St. Dev.	
u	Calculated	Approx.	Calculated	Approx.
0	15.00	30.00	71.94	99.50
20	203.77	196.67	271.39	264.39
40	372.13	363.33	373.14	360.42
60	531.90	530.00	456.49	435.78
80	681.88	696.67	535.33	499.90

Table 2.9: Mean and standard deviation of T_c, Pareto claims, $\theta = 10\%$.

Example 2.4 *As our final example, we consider a lognormal individual claim amount distribution with parameters μ and σ, so that*

$$P(x) = \Phi\left(\frac{\log x - \mu}{\sigma}\right)$$

for $x > 0$, where Φ denotes the standard normal distribution function. Setting $\mu = -(\log 3)/2$ and $\sigma = \sqrt{\log 3}$ we obtain $m_1 = 1$, $m_2 = 3$ and $m_3 = 27$,

u	Mean		St. Dev.	
	Calculated	Approx.	Calculated	Approx.
0	6.00	12.00	19.90	26.83
20	70.49	65.33	75.50	67.53
40	119.00	118.67	113.74	91.65
60	155.88	172.00	164.94	110.63
80	186.27	225.33	233.05	126.81

Table 2.10: Mean and standard deviation of T_c, Pareto claims, $\theta = 25\%$.

which are identical to the first three moments of the Pareto(4,3) distribution, resulting in the same De Vylder approximations as in Example 2.3. As in that example, an explicit solution for ψ does not exist, but we can calculate approximate values of the moments of T_c. Tables 2.11 and 2.12 display the calculated and approximate values of the mean, variance and coefficient of skewness of T_c for $\theta = 10\%$ and 25% respectively, and we observe a similar pattern in these tables to that in the previous example.

u	Mean		St. Dev.		Coeff. of Skewness	
	Calculated	Approx.	Calculated	Approx.	Calculated	Approx.
0	15.00	30.00	71.94	99.50	14.00	9.92
20	203.35	196.67	269.83	264.39	3.73	3.74
40	367.48	363.33	367.15	360.42	2.74	2.75
60	523.57	530.00	443.50	435.78	2.27	2.27
80	672.09	696.67	509.28	499.90	1.98	1.98

Table 2.11: Mean, standard deviation and coefficient of skewness of T_c, log-normal claims, $\theta = 10\%$.

The tables suggest that the quality of the approximation depends on the form of the individual claim amount distribution, rather than on a distribution's moments. The findings in these examples are very much in line with the findings in De Vylder's (1978) paper regarding approximations to ψ.

3 The density of T_c

In this section we consider De Vylder approximations to the density of T_c for the same individual claim amount distributions as in the previous section. In

	Mean		St. Dev.		Coeff. of Skewness	
u	Calculated	Approx.	Calculated	Approx.	Calculated	Approx.
0	6.00	12.00	19.90	26.83	9.76	6.62
10	41.81	38.67	54.60	51.38	3.62	3.49
20	69.25	65.33	72.39	67.53	2.80	2.66
30	93.02	92.00	86.58	80.50	2.42	2.23
40	113.45	118.67	99.08	91.65	2.22	1.96

Table 2.12: Mean, standard deviation and coefficient of skewness of T_c, lognormal claims, $\theta = 25\%$.

each of our examples we choose a value for the initial surplus u which yields a value of $\psi(u)$ less than 5% as we consider such values to be of practical interest, and we compare the approximating density with a density which we refer to as the exact density. In each example this density has been calculated by the method referred to as 'Algorithms' by Dickson and Waters (2002, Section 5.1).

Example 3.1 *In this example we consider Bohman's individual claim amount distribution, again allowing the parameter s to vary to consider the effect of the individual claim amount distribution's variance. Figure 3.1 shows the exact and approximate densities of T_c for $s = \sqrt{2}$, $u = 60$ and $\theta = 10\%$ (giving $\psi(60) = 0.0253$), Figure 3.2 shows the exact and approximate densities of T_c for $s = 5$, $u = 200$ and $\theta = 25\%$ (giving $\psi(200) = 0.0380$), and Figure 3.3 shows the exact and approximate densities of T_c for $s = 6.496012$, $u = 350$ and $\theta = 25\%$ (giving $\psi(350) = 0.0319$). In each of these three figures, two densities are plotted, but the approximation is so good in each case that it is very difficult to distinguish between them. As in Example 2.1, we observe that a change in the parameter s, and hence in the variance of the individual claim amount distribution, has little effect on the quality of the approximation.*

Example 3.2 *We now consider the density of T_c when the individual claim amount distribution is Wikstad's, with $u = 400$ and $\theta = 25\%$ (giving $\psi(400) = 0.0388$). The exact and approximate densities are plotted in Figure 3.4, and although the approximation is not as good as in the previous example, the approximate density is very close to the exact one.*

Example 3.3 *Here we consider the case when the individual claim amount distribution is Pareto(4,3), and we set $u = 80$ and $\theta = 10\%$ (giving $\psi(80) = 0.010$). The exact and approximate density functions are plotted in Figure 3.5. This example was studied by Dickson and Waters (2002) and a comparison with their Figure 3 shows that the De Vylder approximation compares*

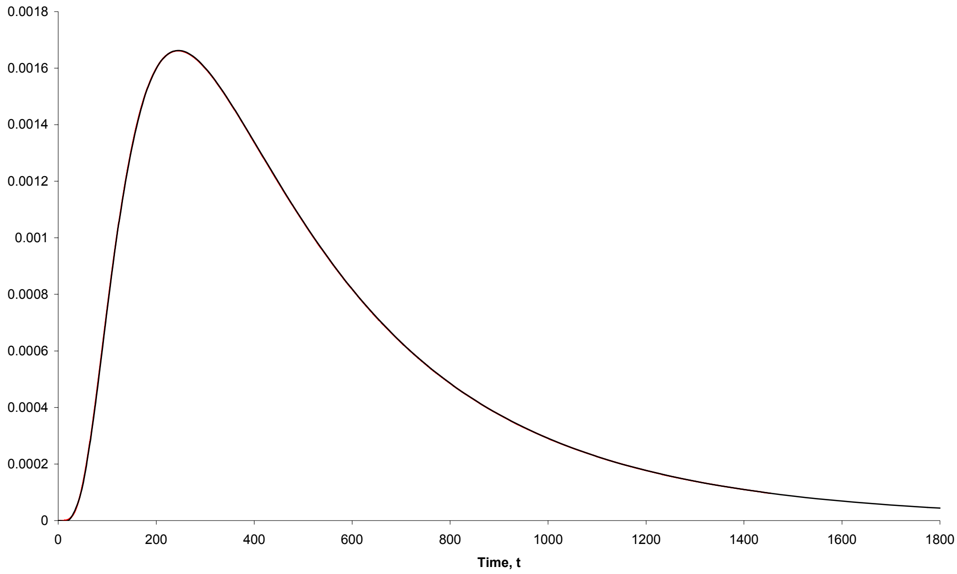

Figure 3.1: Exact and approximate densities of T_c, Bohman claims with $s = \sqrt{2}$, $u = 60$ and $\theta = 10\%$.

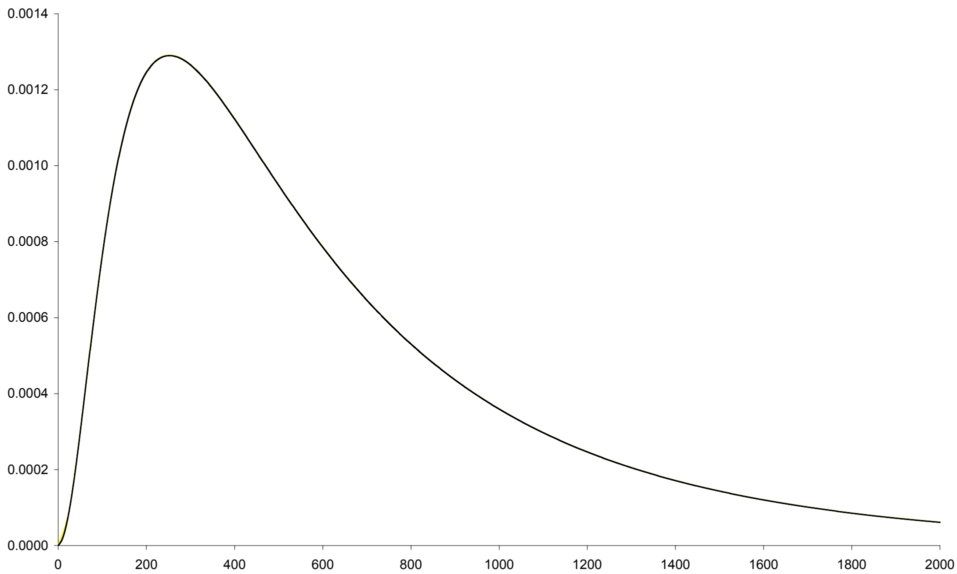

Figure 3.2: Exact and approximate densities of T_c, Bohman claims with $s = 5$, $u = 200$ and $\theta = 25\%$.

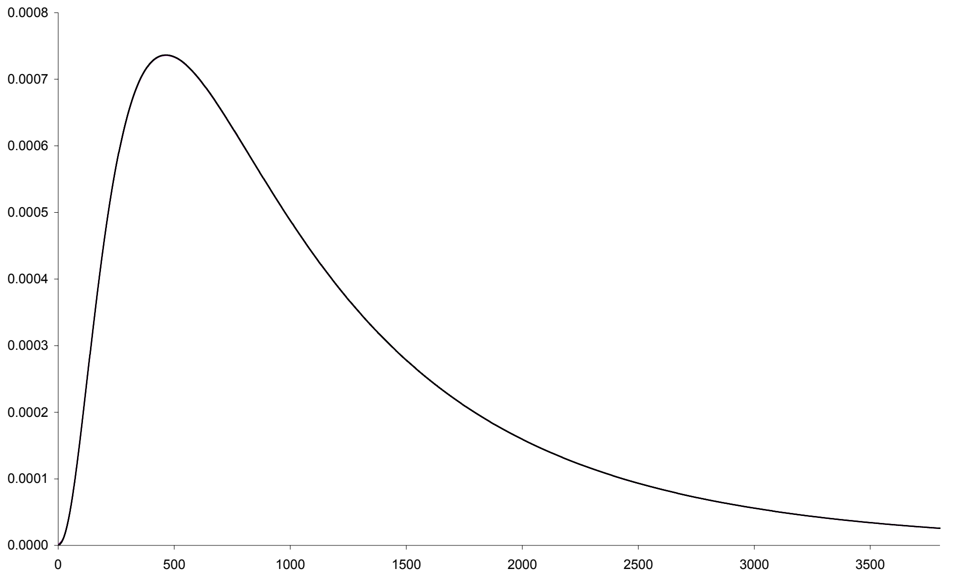

Figure 3.3: Exact and approximate densities of T_c, Bohman claims with $s = 6.496012$, $u = 350$ and $\theta = 25\%$.

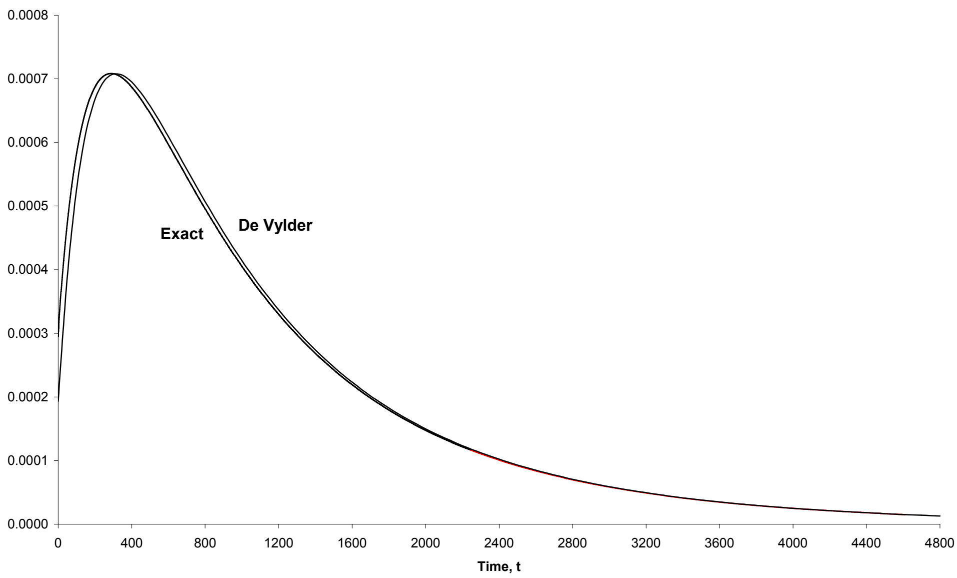

Figure 3.4: Exact and approximate densities of T_c, Wikstad claims, $u = 400$ and $\theta = 25\%$.

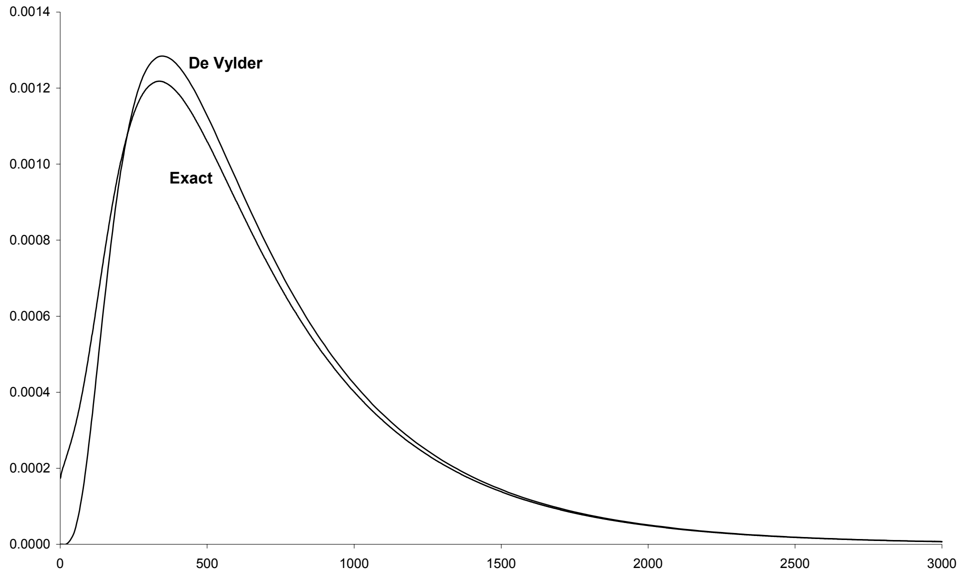

Figure 3.5: Exact and approximate densities of T_c, Pareto claims, $u = 80$ and $\theta = 10\%$.

very favourably with the best approximation given by Dickson and Waters (their translated gamma method), but has the significant advantage of being much more efficient from a computational point of view. It is clear from Figure 3.5 that the approximation is not as good as in Examples 3.1 and 3.2, although the is good approximation in the tail.

Example 3.4 *As our final example, we consider the situation when the individual claim amount distribution is the lognormal distribution from Example 2.4, and we set $u = 80$ and $\theta = 10\%$ (giving $\psi(80) = 0.0104$). The exact and approximate densities are plotted in Figure 3.6, and we can see that this figure has very similar features to Figure 3.5.*

The pattern of approximations in this section is the same as in the previous section. In the cases when the moment generating function of the individual claim amount distribution exists, the approximations to the density of T_c are excellent, but when it does not exist, the approximations are not as good, but are reasonable in each density's tail.

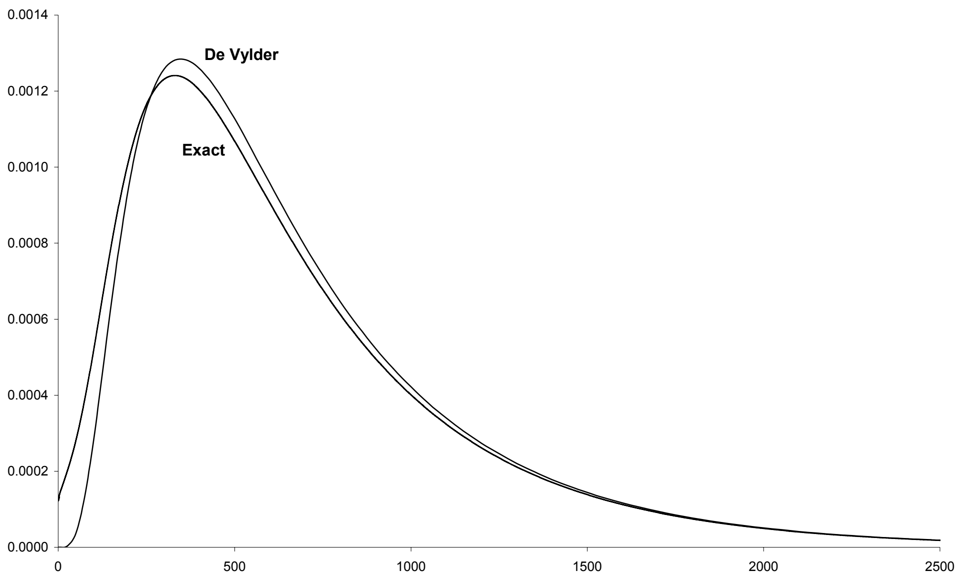

Figure 3.6: Exact and approximate densities of T_c, lognormal claims, $u = 80$ and $\theta = 10\%$.

4 Concluding remarks

The examples in the previous two sections suggest that De Vylder approximations to the moments and density of T_c are good in the same circumstances that De Vylder's original approximation to ψ is good. Considering that we are approximating more complex quantities, it is perhaps surprising that we obtain such good approximations. De Vylder's original idea is attractive because of its simplicity, and its application appears to be far more widespread than he originally suggested. We have restricted our focus in Section 3 to combinations of u and θ which give a ruin probability in a range of practical interest, and would suggest that the approximation is most applicable in this circumstance.

References

[1] De Vylder, F. (1978) *A practical solution to the problem of ultimate ruin probability.* Scandinavian Actuarial Journal, 114-119.

[2] Delbaen, F. (1988) *A remark on the moments of ruin time in classical risk theory.* Insurance: Mathematics & Economics 9, 121-126.

[3] Dickson, D.C.M. and Waters, H.R. (1991) *Recursive calculation of survival probabilities.* ASTIN Bulletin 21, 199-221.

[4] Dickson, D.C.M. and Waters, H.R. (2002) *The distribution of the time to ruin in the classical risk model.* ASTIN Bulletin 32, 299-313.

[5] Dickson, D.C.M., Hughes, B.D. and Zhang, L. (2003) *The density of the time to ruin for a Sparre Andersen process with Erlang arrivals and exponential claims.* Centre for Actuarial Studies Research Paper Series No. 111, University of Melbourne.

[6] Drekic, S. and Willmot, G.E. (2003) *On the density and moments of the time to ruin with exponential claims.* ASTIN Bulletin 33, 11-21.

[7] Drekic, S., Stafford, J.E. and Willmot, G.E. (2004) *Symbolic calculation of the moments of the time to ruin.* Insurance: Mathematics & Economics 34, 109-120.

[8] Gerber, H.U. (1979) An Introduction to Mathematical Risk Theory. *S.S. Huebner Foundation, Philadelphia, PA.*

[9] Gerber, H.U., Goovaerts, M.J. and Kaas, R. (1987) *On the probability and severity of ruin.* ASTIN Bulletin 17, 151-163.

[10] Lin, X. and Willmot, G.E. (2000) *The moments of the time to ruin, the surplus before ruin, and the deficit at ruin.* Insurance: Mathematics & Economics 27, 19-44.

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