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CRITICAL EVENTS

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ABSTRACT

This paper outlines the beginnings of a general theory of critical events. Four types are defined. Two are micro-events each affecting a small number of people and are shown to arise in everyday life. Two are macro-events that affect large numbers of people and are related to 'shocks' in economic theory. The paper makes some suggestions for a statistical theory of critical events, supported by well-known results from the theory of stochastic processes. Many examples are provided to illustrate the four suggested types of critical events. Finally, some strategies for coping with and/or anticipating critical events are briefly outlined.

Keywords: Critical events; Poisson processes; Phase transitions; Networks; Households.

[1] Victor Jennings was an Associate Professor in the Households Research Unit of the Department of Economics until his untimely death in April 2011; Bill Lloyd-Smith is a Research Associate and Duncan Ironmonger is an Associate Professor and Director of the Households Research Unit

Preface by Bill Lloyd-Smith

The original version of this paper was written by Victor E. Jennings in late 1991 as part of a more extensive work on households. He had intended that this work, together with necessary updates, should be published. Unfortunately, he died on 16 April 2011 before this work could be completed. Between 1991 and 2011, Victor had completed some other papers arising from work on household size distributions and projections for a large number of countries. This paper on critical events was discussed frequently with me and a number of novel ideas were discussed with a view to applying them to the study of critical events (also called 'shocks' in the economics literature) and the development of strategies to cope with them. These ideas were written down in various notes and emails, which are now included in the paper.

While the examples are somewhat 'dated', it is simple in principle to replace them with recent examples but similar conclusions would still hold. Also, it is simple in principle to provide examples from all over the world, but the main lessons given here still apply today.

I pause with a brief comment on 'shocks' that Duncan Ironmonger has recently shared with me. As is well known in economics, a shock is an unexpected or unpredictable event that affects an economy, either positively or negatively. A well known example is the oil supply embargo of 1973 which soon led to large rises in oil prices. More technically, it refers to an unpredictable change in exogenous factors which may have an impact on endogenous economic variables. The response of economic variables, such as output and employment, at the time of the shock and at subsequent times, is usually called an impulse response function. This term seems to come from signal processing in electrical engineering.

We pause to explain what 'events' are. Thus, we give a working definition of what constitutes a *critical event*. This notion is expanded to set out the beginnings of a theory of critical events. This will be useful for the management of critical events with the help of working rules. Following the Introduction in Cox and Isham (1980), we think in terms of point processes occurring in time. In this approach, 'events' are seen as taking place at discrete points of time, one after another. In a different approach favoured by Ironmonger (1989), 'events' have a start time and a completion time. This is sensible for activities that are regularly performed, such as eating a meal. In time use research, it is the natural way to proceed. Some events, such as a death in the family, do not have any time duration so we have to treat them differently. Other events have a dual aspect. For instance, a wedding ceremony only lasts for a few hours or a few days at the most. From the time use perspective, there is a starting time and a completion time for this august event. However, this event also marks the transition from the state 'not currently married' to the state 'married' with ensuing long-term consequences. The point process approach brings all these events under one heading for a general approach.

Not all events are equally important. An event which affects a single person, a household or a few related individuals will be considered to be a *micro-event*. Probably a large number of events will fall into this category. An event that affects whole countries or a large number of people, households and businesses in some area or region is called a *macro-event*. Some events might fall somewhere in between these two categories but it is expected that the vast majority of events can be assigned to one or the other category.

Shocks as defined above will be a subset of the macro-events. We will see that shocks broadly correspond to 'supercritical' events as Victor saw them. A supercritical event is generally cataclysmic, such as natural disasters and also includes a major breakdown in law and order. Major events such as the Depression of 1930, the oil embargo of 1973 and the global financial crisis of 2008 would come into this category as they quickly had far-reaching economic consequences.

Victor considered at great length events that affect the household. These are normally micro-events. These would be seen as major events to the householders who are directly affected but seldom as shocks to the economy as a whole. Perhaps these small-scale events could be termed 'micro-shocks' in direct analogy to the term 'shock' in economics. Some examples would include a death in the family or the loss of a job. Such events affect the household which experienced that event but the ripples from that event seldom, if ever, lead to major changes in the economy.

Micro-events are divided into Type 1 or Type 2, according as they arose from choices made by the individual (such as choosing a career or a spouse) or from events generally beyond a person's control, but still within a stream of events. Such events may include the acquisition of siblings, events that happen to relatives and medical problems. Some of these matters are grist for the demographer's mill.

Supercritical events are classified as Type 3 in our system of critical events and correspond to shocks to an economy.

A fourth type of critical event is called a 'phase change' (Type 4). A famous example in recent history is the collapse of the Berlin Wall in November 1989. There was no immediate cataclysmic change but it soon became clear that far-reaching changes would take place. This could also be regarded as a 'shock' in the economic sense since it led to the merger of two nations to form the unified state of Germany with attendant economic effects. Another term for this (Type 4) event can be 'phase transition' by analogy to physical changes in a substance such as water when it is heated to boiling point or made colder so that it freezes solid. The term 'shock' does not seem to be especially suitable since the changes need not be sudden.

Phase changes are a second type of macro-event that are distinct from the supercritical events.

There are numerous examples of critical events in this paper. They are intended to illustrate Victor's thinking. The purpose is that they should stimulate management to anticipate these events as part of life and hence to 'budget' for their expected occurrence, even though they are often unwelcome disruptions to daily routines and forward planning. Indeed, Victor intended that much of this paper should be suitable for a management readership.

Most examples given in the text come from Australian experience for obvious reasons but they can clearly be extended to the entire world.

Outline of paper

This paper was originally written as a chapter in a book that has not been published. There were six main sections to that chapter. Some additional material has been incorporated in this paper. It arises from numerous discussions, both verbal and by email that we had on many occasions.

The first section introduces the notion of a critical event and shows how a critical event can lead to constraints on or possible expansions of future activity. The idea of a supercritical event is introduced.

The second, third and fourth sections discuss various types of critical events. The four main types are defined above.

The fifth, sixth and seventh sections discuss, respectively, examples of personal, household and external sources of critical events in the household. In practice, personal sources of critical events often affect the entire household to which the individual belongs. Numerous statistics are provided as illustrations of the frequencies of the occurrence of critical events. Most examples are taken from the publications of the Australian Bureau of Statistics (ABS). Jennings also argues that there is a need for a statistical approach to the study of critical events, thereby providing a foundation for a sensible theory of critical events, even before they actually occur.

While these examples may resemble a catalogue of critical events, their real purpose is to illustrate the ubiquity and patterns that are commonly seen among critical events. These observations are a necessary preliminary to any attempt at a general approach to critical events.

The eighth section considers the important matter of how best to cope with critical events when they arise. Jennings stresses the need for built-in tolerances so that critical events can be averted. He draws on his engineering background to discuss such ideas as safety factors, fatigue, control and feedback. Codes of practice, rules and habits are other tools for avoiding undesirable events. Insurance and emergency reserves are also valuable for coping with adverse critical events.

The ninth section sums up and points out that critical events are part of life. Hence due allowance should be made in future planning and householding. The paper advocates the need for further study of these matters and the need to apply the findings of such studies.

The major purpose of these notes on critical events is to establish and promote the need to anticipate many of these critical events as part of regular planning and management in government, business and in our own lives and households.

Subsequently, Vic Jennings had many discussions with me about critical events and their statistical analysis. It is our task to present the results of this work, taking care to highlight his ideas throughout. However, we have amended the original text while striving to preserve the ideas originally presented by him along with the fruits of our frequent discussions. This, we have also added an outline of many subsequent discussions to this paper since they illustrate the later thoughts that Vic had put down in various emails and documents. In doing so, we are able to offer a few suggestions on the underlying patterns in critical events and how they can be adapted for practical action.

Because of Victor's untimely passing, the original structure of this paper has been somewhat amended with some editing at the suggestion of Margaret Jennings. Duncan Ironmonger has provided some valuable suggestions to improve this paper. Editorial notes have been added in the following text. They appear inside a box but can be read together with the main text or skipped in a first reading.

Bill Lloyd-Smith   January 2014

1 Introduction

People inhabit a world full of activity. Some of this activity generates events which have critical consequences for us. In this paper, we will demonstrate that critical events appear often in households. They arise from many sources, sometimes obscure, and should be treated as part of the mainstream of our lives, and not be regarded as isolated occurrences unique to one person. In this way people have a better chance of managing their effects and their frequency, both at the individual level and at the level of the society. In some countries the frequency of major critical events (Type 3 in our classification) may not be yearly or monthly but almost daily. One possible measure of the standard of living could be the inverse of the average frequency of major critical events in a society.

Critical events arise from a variety of sources, some of which we can control, some of which we cannot. Households with more than one person can expect a greater frequency of those events arising from personal sources such as illness. In this paper where we discuss the likelihood of events we prefer to argue in the negative, i.e. it is highly *unlikely* that at least one of a combination of such and such events will *not* occur in a given period. There are some quite awkward problems about applying risk analysis to individuals or to specific households, or even to groups. In an engineering context McDonnell has pointed out

> "One of the most important discontinuities in the perception of management of risk, particularly relevant, to ideas of acceptable risk, is that discontinuity called 'people changing their mind'."
>
> McDonnell (1991)

However, people can develop strategies to reduce risk, and to survive in this complex, indeterminate, diverse, and variable world.

Businesses, government authorities and other organisations also have to cope with critical events, which can seriously affect the work of those organisations. Thus, they may adopt rules intended to reduce the effects of excessive demands on their resources by running the organisation at somewhat less than full utilization. A simple example would be a large hospital. A commonly adopted rule is to run the hospital at approximately 85% capacity, i.e. 85% utilisation of its resources as averaged over a moderate timeframe, say a month. This rule actually arose from a study reported by Bagust et al (1999). While the soundness of this rule has attracted much strong criticism, especially from persons familiar with the underlying mathematics of queueing theory, this rule is simple to apply from an administrative viewpoint. No mathematical skill is needed by management in applying this rule but is the rule justifiable? Some discussion is available in Bain et al (2010) and in Goronescu et al (2002). In general, it is desirable to ensure that any proposed rule should be validated for a wide range of possible circumstances. It may be desirable to allow some flexibility in applying the rule in case unusual patterns occur.

1.1 Definition

Critical events upset the daily routine in the household, either temporarily or permanently. 'Critical' is defined to mean "Of the nature of, or constituting, a crisis; involving suspense as to the issue," - "decisive, crucial."[2] A physical analogy would be the case where or when a change of state or phase occurs. For instance, the boiling point of water defines a critical event, the conversion of liquid water into steam.

> Further explanation of the notion of critical events is given below. This arises from email discussion between BLS and VEJ in which a possible classification is provided. Several types will be distinguished. The first type of critical event (micro-event) consists of self-generated and household-generated events. These result from decisions made by the individual, e.g. choice of career, marriage partner, or choices arising in the household, e.g. births, divorce.

[2] Shorter Oxford English Dictionary, Third Edition (1956).

The second type of critical event is similar, except that they are encountered by the individual, not by choice or inclination. Thus, we acquire siblings, learn of a death of a friend or a divorce or acquire a serious medical condition. These are micro-events that change a person's daily life routine on a long-term basis.

The third type (macro-event) is considered to be "supercritical" because they affect a large number of people at once. These can arise from natural disasters and from the breakdown of law and order. Volcanic eruptions, tsunamis, earthquakes and wars all fall into this category. We can also include the Depression of 1930, the global financial crisis of 2008 and the incident of 11 September 2001.

The fourth type (macro-event) is akin to a phase transition. A good example is the fall of the Berlin Wall in 1989. It is difficult to predict these events as they reflect a change in the underlying state of the surroundings. This analogy comes from physics and engineering. A simple example in physics is the case of liquid water freezing solid at low temperatures and turning into steam at high temperatures.

Such events or happenings may, because of a coincidence of various circumstances, lead to a new path for the household which is different to the past and which does not allow us to return to the old way. For example a divorce usually leads in different directions; a major illness may lead to a new pattern of activities in a household.

1.2 Branching

This can be illustrated by a pipeline structure where one moves through time along a rubbery curved branching pipe from left to right and one's activities are constrained to lie along the surface of the pipe (perhaps a topological manifold) See below.

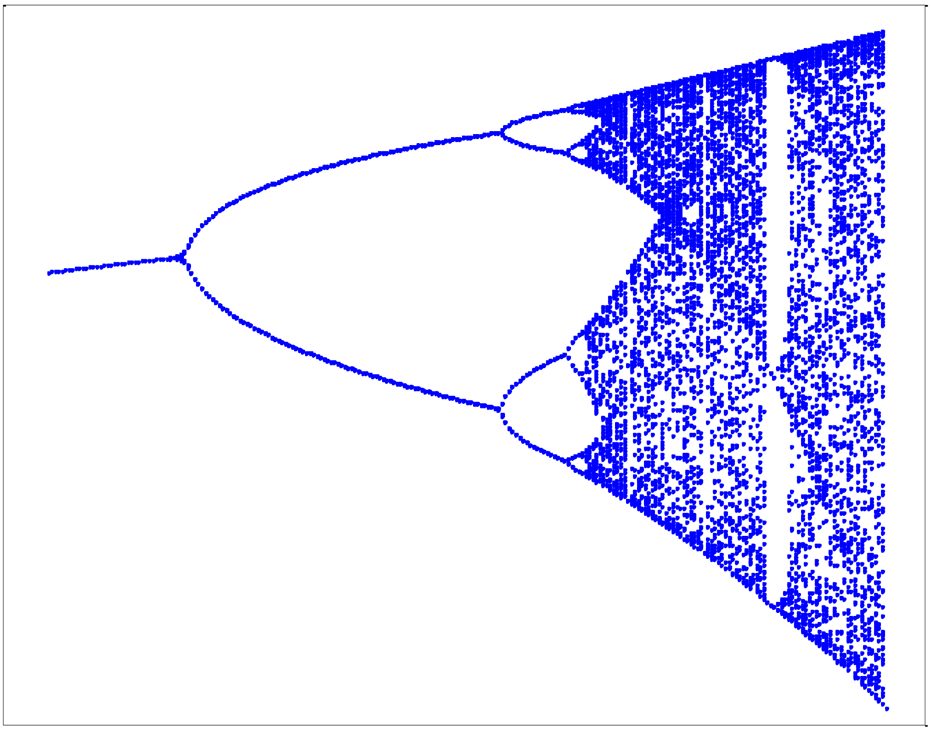

Figure 1: Branching

This example relies on a geometrical metaphor drawn from higher mathematics. In fact, we have used the logistic map $x_{n+1} = rx_n(1-x_n)$ with r in the range 2.7 to 4. A bifurcation occurs when $r=3$. More bifurcations occur as we increase r from 3 to 4. The next bifurcation occurs for $r=3.44949$ and this is followed by yet another one at $r=3.54409$. We have omitted the axes in the Matlab® run used to create the graph shown above. Admittedly, this is a simplification of reality but the basic idea is clear. As we progress through life, we are faced with decisions with consequences that are probably irrevocable. In this way, we encounter a situation demanding a decision between two or perhaps more directions for the future. This can happen many times so this is consistent with the bifurcation diagram shown above.

Another way to grasp this idea is to imagine an ant walking on a surface lying in ordinary three-dimensional space. The ant is 'intended' to follow a certain path. Initially, a small change in the ant's path can be easily diverted from the original path that had been 'planned'. If the ant continues to walk in a certain direction, the path may diverge further and further away from the original path and a change may no longer be easily carried out.

Examples of connected surfaces can be quite complicated. Simple examples include the ‘rubber ring’ (called a torus) and the hyperbolic paraboloid. Another example is the Moebius strip. Some surfaces can have a portion that branches well away from the rest of the surface so that different paths can lead to very different outcomes.

Some types of activity are not affected by the day to day fluctuations of external events and even events producing substantial change may not influence the central path. We need to define what is meant by the life course. The simplest answer is that a lifetime generally consists of several stages, starting with birth. This is followed by several stages. Initially there is the stage of childhood, followed by leaving home on reaching adulthood. A few years of life in the unmarried state is usually followed by marriage and children. This state persists for a long time until the last child moves out of the family household. The post-parenting stage commences, eventually leading to the death of one of the spouses and later the other one. However, if the life course leads to a fork in the path (a ‘bifurcation’), then we have to choose a new direction and therefore alternative plans may not be feasible.

Real life events can be considered as moving along a pipe-like *n*-dimensional manifold, with many branches. So what may have been a small change originally may over time lead to a different permanent path - people have moved out onto a new part of the manifold like an adjoining ‘pipe’. The placings of the pipe branches are not fixed but may vary over time, and they may change in size and orientation. A recession will tend to reduce the diameter of the pipe - we are more constrained. As people get older their investment of personal time in prior learning may predispose them to certain types of activity, the pipe gets smaller in diameter. But there are occasions where opportunities may expand with age; in that case one may regard the pipeline as increasing in diameter.

How can we move back to the main trunk pipeline? Well in many instances we may be unaware that we are on a branch until so many

commitments are made that it is difficult to regain the old course. Secondly we may just not want to change course. The marginal gain as we see it may not be worth the effort. It could be useful sometimes to broaden the horizons of people as to the consequences of their actions and this may lead them to re-evaluate the gains and losses of change.
The branching paths described above look a little like the paths in a bifurcation chart as described in Coveney (1990).

1.3 Lifestream

This approach is essentially statistical and probabilistic but has its roots in the theory of point processes as in Cox and Isham (1980). The simplest example is that of a Poisson process, which is well illustrated in counts of alpha particles from a radioactive source (radium, uranium) with a Geiger counter. A more realistic model will probably have extra features but the basic intention is the same. We hope to model the series of critical events via a statistical approach so that certain patterns can be predicted in advance with suitable error bounds.

These are Type 1 (chosen by the individual) and Type 2 (events beyond the individual's control that mainly affect one household).

An alternative approach is to consider a life stream of events which pass us by, sometimes buffeting us and occasionally critically so. With some 30,000 days in an average life of 82 years and many repeated events, there are many possibilities for critical events to occur. One may have forewarning of some critical events in which case we can take evasive action but it is almost impossible to avoid all critical events. If a critical event occurs, one hopes to have sufficient resources to cope. Sometimes the stream will exhibit more turbulence than at other times. We may be in turbulent times now.

Also, some events may be critical to one person or household and not another, even though they may be in similar circumstances.

The key points to come out of this discussion are:

- We live in a stream of events, some of which will be critical to us.
- We can anticipate some but not all of these events.
- Sudden changes in our situation can occur.
- We are not all equally able to deal with critical events.

1.4 Supercriticality and the population

This image comes from nuclear physics. The obvious example is of a quantity of fissile material such as uranium-235 or plutonium-239 in the state of supercriticality in which case a tremendous explosion can take place. This is a Type 3 critical event.

Supercritical events affect a very large number of people. Sometimes, they may have positive effects. For instance, the gold rush of 1851 in eastern Australia led to great population growth and development of the then colonies, later to federate in 1901. If events adversely affect sufficiently many households in a significant and deleterious manner, the whole fabric of society may be placed at risk. We often forget how fragile a society is since we take much for granted. A society depends very much for its existence on the level of expectation and trust between its members, and on habit and custom. We survive because our habitual ways of doing things usually serve us well. If these survival mechanisms break down, we seek ways to reassert ourselves in a way that ensures survival. If a society cannot deliver the basics of living to most of its members, it may be changed quite suddenly.

The raison d'être for studying supercritical events is clear. Over time, there may be a gradual or incremental increase in loads placed on all households and other related sections of society but these may not be noticed. Not all households are equally able to cope with such loads. If the total load is sufficient to increase the incidence of critical events in numerous households, we may finally reach a supercritical state in the community, and in this case a minor event may precipitate a major breakdown on a large number of households. This effect is made more likely now because of the day to day communication of events to most households so that a significant number may suddenly realise they are all in the same boat.

One way of considering these happenings is based on a theory of what is termed self-organised criticality which has been applied in some fields. This is clearly connected to *complex systems*, which already have an extensive literature. For a recent overview of complex systems in science and society see MacKay (2013).

Self-organised criticality as discussed below is probably an instance of Type 4 criticality. After all, a kind of order that emerges from a system thought to be disorganised is not really a catastrophe but a phase transition.

Self-organised criticality is discussed by Bak and Chen and the application of the theory to earthquakes, ecosystems and turbulence in fluids. To quote Bak and Chen (1991):

> "… We proposed the theory of self-organised criticality: many composite systems naturally evolve to a critical state in which a minor event starts a chain reaction that can affect any number of elements in the system. Although composite systems produce more minor events than catastrophes, chain reactions of all sizes are an integral part of the dynamics. According to the theory, the mechanism that leads to minor events is the same one that leads to major events. Furthermore, composite systems never really reach equilibrium but instead evolve from one metastable state to the next."
>
> "Self organised criticality is a holistic theory: the global features, such as the relative number of large and small events, do not depend on the microscopic mechanisms. Consequently, global features of the system cannot be understood by analyzing the parts separately. To our knowledge, self-organised criticality is the only model or mathematical description that has led to holistic theory for dynamic systems."

> "… If, on the other hand, the economy is a self-organised critical system more or less periodic large-scale fluctuations are to be expected even in the absence of any common jolts across sectors." [3]

The second, third and fourth Sections to this paper contain some notes towards a theory of critical events. Here, we describe the type of critical events that do occur in households drawing on data from the rest of this paper and other sources, particularly the Australian Bureau of Statistics. The State of Victoria relating to car accidents (seat belts) is one good example where a change in the law (a critical event) led to a significant reduction in the number of fatalities and severe injuries as people were required to wear seat belts.

Hence, it is important to consider critical events and their combined influence on millions of households. It is also important to consider critical events from the householder's point of view. In this way we are able to obtain a plausible idea as to the type of events that are likely to lead to criticality, and the way we may manage events. We first discuss personal sources, second household sources, third external sources, and finally discuss their management.

2 Point process models

A simple way to understand the probabilistic approach is as follows. On any one day, the probability p of a critical event is assumed to be very small. Different days are assumed to be independent of each other. Over all 365 days of a year, the probability of a critical event is no longer negligible. This is easy to see as the following argument shows. The probability for any one day of no critical event is $1-p$ so the probability of no critical event in one year is $(1-p)^{365}$. Hence the probability of a critical event in one year is $1-(1-p)^{365}$. For instance, if we take $p=0.01$, the probability of a critical event in one year is over 0.974. If we take $p=0.001$, the probability of a critical event in one

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[3] We are reminded of the consequences that follow from the Poisson distribution which applies generally to large populations. If we change the characteristics of the population, we change the frequency of events that are described e.g. vehicle accidents.

year is about 0.306. These examples show clearly that even seemingly unlikely events are likely to happen sooner or later. Hence, it is reasonable to anticipate the eventual occurrence of a seemingly unlikely event and hence to be prepared in case that event actually occurs.

A simple example of a probabilistic model that often featured in our thinking is the balls in cells model as expounded in the first two chapters of Feller (1968). Another account is available in Moran (1968). This is the problem of randomly allocating r balls among n cells. Assuming that the balls are distinguishable, the probability p_k of k balls being found in a given cell follows a binomial distribution. In fact, the required probability is of the form

$$p_k = \binom{r}{k} \frac{1}{n^k} \cdot \left(1 - \frac{1}{n}\right)^{r-k}$$

In the limiting case as $n \to \infty$ and $r \to \infty$ in such a way that $\lambda \to r/n$, it is well known that $p_k \to e^{-\lambda}\, \lambda^k / k!$, the Poisson distribution.

The Poisson distribution itself arises in a number of ways. We refer to Kingman (1993), Cox and Isham (1980), Papoulis (1984) and Karlin and Taylor (1975) for details. A well-known process that gives rise to the Poisson distribution is a Poisson process with intensity function $\lambda(t)$. In the simplest case, the intensity function is just a constant λ. The intensity function is defined as

$$\lambda(t) = \lim_{\delta \to 0+} \delta^{-1} \Pr\{N(t, t+\delta) > 0\}$$

Here, $N(u,v)$ is just the number of arrivals in the time interval (u,v). Also, the probability of at least one event in the time period $(t, t+h)$ is $\lambda + o(h), \quad h \downarrow 0$ and the probability of two events in $(t, t+h)$ is $o(h)$.

A Poisson arrival process model is perhaps the simplest model of this type. It is also attractive since it has a great deal of additional structure. This model is widely used in science and engineering. A well known instance is the work of A.K. Erlang in the modelling of telephone traffic. Telephone calls are assumed to arrive according to a Poisson process. In the simplest case, the duration of telephone calls (service time) is assumed to be exponential. Thus, Erlang obtained his well-known formula for the loss of a call for want of a free telephone line. This formula holds even for arbitrary service time distributions.

Poisson processes have also been used in the social sciences, including demography, such as some recent work of Lloyd-Smith and Jennings (2010, 2011). In some cases, the Poisson process does not fit the data very well but it is still of value as a baseline for developing improved stochastic models. In this model, the events are considered to be purely 'random'. Not surprisingly as the name suggests, the number of events over a given time interval is found to satisfy a Poisson distribution.

In medicine, Poisson processes have been used in a mathematical model of epidemics in households. More details are in the references cited in Ball, Britton and Lyne (2004), especially Andersson and Britton (2000). The critical events are the occurrences of the disease of interest.

Poisson processes are discussed in such books as those by Kingman (1993), Cox and Isham (1980), Karlin and Taylor (1975). A key result in the study of Poisson processes is the following. We assume that events arrive at some rate λ events per time unit. This parameter λ is usually called the intensity function in the theory of point processes. In the simplest models, λ is constant and has to be estimated from the data. When the process has run for some time, a steady state is usually apparent. There is an extensive theory of Poisson processes that can be applied. Many of the main results can be found in the literature. It is known that the mean number of arrivals over a period of length t is given by λt with a standard deviation of $\sqrt{\lambda t}$. This result applies at once to critical events occurring at rate λ This result seems reasonable for events such as workplace absences in a large organisation, due to illness. It also allows one to write down a confidence interval (say 95%) for the anticipated range of critical events. Unexpected deviations can be detected in advance since there will be fewer or more events than usual over a shorter period of time so that corrective actions can be put into operation.

More generally, one may consider a point process model as defined in Cox and Isham (1980). The intensity function is still defined in the same way, namely as

$$\lambda(t) = \lim_{\delta \to 0+} \delta^{-1} \Pr\{N(t, t+\delta) > 0\}$$

As before, $N(u,v)$ is the number of arrivals in the time interval (u,v). In general, interarrival times may have an arbitrary distribution. In the special case of a Poisson process, interarrival times have an exponential distribution. However, it is often possible to simplify the model somewhat because of limit theorems on the superposition of large number of independent point processes, all having the same intensity function λ. From Section 4.5 of Cox and Isham (1980), we know that under broad conditions, the superposition of N processes approximate a Poisson process with arrival rate $N\lambda$. This allows one to approximate the point process with intensity λ by a Poisson arrival process with the same intensity λ. In practice, we often encounter large populations so we prefer to simplify the model by starting with a Poisson process model when possible.

Moreover, one can often take out insurance policies on a large variety of critical events. This is subject to a few caveats as spelled out in the fine print of the policy. In the spirit of Feller (1968), the events to be excluded from the policy are often of such magnitude that an attempt to pay out would bring financial ruin to the insurance company. Thus, nuclear accidents and wartime actions are normally excluded from insurance policies. Indeed, it is not enough to argue that the risk of a certain type of critical event has an exceedingly low probability of occurring. We also have to consider the cost of that critical event if it were ever to take place. This should be manifestly clear from Chapter X of Feller (1968) but it is not always realised in discussions of the odds of a disaster.

Feller (1968) points out that an insurance company has to be concerned minimizing the probability of ruin, due to a sudden increase in the number of insurance claims, due to a disaster such as the bushfires of Black Saturday in Victoria on February 2009. Hence, the company will be more concerned about the so-called ruin problem as explained in Chapters X and XIV of Feller (1968). After all, who could insure an insurance company against a catastrophic loss? The answer surely is that the company insures itself by refusing risks that would expose the company to a serious risk of ruin, such as wars and nuclear catastrophes.

In the same way, a bank can insure itself against ruin by not granting high-risk loans. The lure of high rates of interest payable on such loans may be tempting but if the borrower defaulted, the bank would have to bear the loss.

A useful result in the study of critical events is known as Little's Law, which has its origins in queueing theory. Thus, we imagine a series of critical events taking place at some given rate λ. For these events, some type of 'service' commences and after a certain period of time with average W, that service ceases. Then Little's Law asserts that the mean number of arrivals L in the system is given by $L = \lambda W$. This result is valid under very broad conditions. As a simple example, we suppose that $\lambda = 200{,}000$ births/year, $W = 80$ (life expectancy). Then Little's Law asserts that $L = 16{,}000{,}000$ persons.

A simple example is given by a stock-flow model of population change without immigration. Here, λ denotes the number of births per annum and W denotes the life expectancy at birth. Clearly, L is the mean number of persons in the population. Another example could relate to a certain medical condition in which the *incidence* of new sufferers from that condition and the mean time for which they have that medical condition can be multiplied together (using Little's Law) to estimate the number of persons (*prevalence*) in a population who have that condition. Little's Law also tells us how long staff are expected to remain with the organisation on average, given that we know the total number of employees and the mean number of new recruits per annum. (This assumes that the recruitment rate and length of employment there remain unchanged, or varies very slowly.)

Every household should expect a number of critical events to occur every year. However, it will be difficult to specify numerically reliable values of their frequency without the use of adequate records.

In general, these types of critical events may be analysed via standard statistical methods. Elementary approaches are available in Hoel (1984) or Fisher (1970). Sometimes, a more sophisticated probability model may be required such as those found in Karlin and Taylor (1975). Of course, more elementary probabilistic models as in Bailey (1964) or Ross (2000) may be considered for Government and business use that are not overly complex. We have to keep the models simple enough that they can be used with confidence by the people who requested them in the first place.

3 Supercritical events (Type 3)

Events outside of a person's usual way of life can also be critical. Examples could include Government decisions, epidemics, war, international trade decisions, natural disasters and so on. Many of these events may be of Type 3. In all of these circumstances, persons are compelled to change their behaviour. Yet, these incidents tend to be one-off and a statistical analysis is difficult to carry out for lack of sufficient data. These events are called *supercritical* because of their far-reaching effects.

Natural catastrophes include:

Hurricane/cyclone/typhoon (main peril is wind damage; some models also include storm surge); earthquake (main peril is ground shaking; some models can also include fire following earthquakes); tornado; volcanic eruption; flood; wind storm/hail; bushfire/wildfire; winter storm/ice storm; tsunami.

Human catastrophes include:

terrorist events; warfare; casualty/liability events;
economic collapse; displacement crises (refugees).

Here is one observation that may be useful. In many cases, a power law has been observed as for populations of cities and natural disasters such as earthquakes and volcanic eruptions. A power law is of the general form

$$\Pr(X > x) \propto x^{-\alpha} \quad x > 0$$

for some fixed $\alpha > 0$ where x is a measure of the 'size' of the event and X is a nonnegative random variable. Such models could be used for the purpose of modeling the probabilities in terms of the magnitude of these events and possibly the economic costs of a disaster (Type 3 critical event), such as tsunamis, volcanic eruptions, financial collapses and so on. We stress that such models may only be approximations to reality. Also, these models are unlikely to yield much information on when a disaster is likely to occur, apart from any statistical trends that have been gleaned from existing data.

There are on-going efforts to develop mathematical models for financial and geophysical disasters but these have to take into account the underlying causative factors, including some details of the underlying processes, and are beyond the scope of this paper. It would be even more difficult to develop a plausible theory for the occurrence of major decisions or disasters in national or international affairs or wars. Even so, it seems worthwhile to look for common features in such models in the hope that the development of these models would thereby be expedited. Advances in research on one type of catastrophe would then potentially be capable of extension to deal with another catastrophe.

4 Phase changes and networks (Type 4)

The last case for consideration is the phase transition (Type 4). The effects of a phase transition can be far-reaching as in the case of the collapse of the Berlin Wall in November 1989. Yet it is not easy to foresee events such as these, even though we may have a suspicion that a change is likely to occur sooner or later. In hindsight, we tend to say that it was expected but we seldom say in foresight that it will probably occur. In short, it is easy for us to be wise after the event but difficult to foresee it in advance.

One way to consider phase transitions is via a network model. Social networks are common examples that have been studied in numerous papers. In this case, it is known that a phase transition occurs when a substantial proportion of persons (at least a positive fraction) become linked via a social network. This is also called a 'giant component' in network theory as explained in Barrat et al (2008). An important historical example that Vic often mentioned was the case of the Berlin Wall which fell after a sufficient number of people in the former East Germany united in protest against the regime.

Other examples of phase transitions are less obvious. For instance, we consider the uptake of TV after the Melbourne Olympics of 1956 and more recently, the uptake of microwave ovens, PCs and mobile phones in recent years as discussed by Ironmonger et al (2002). Ironmonger (1972) has discussed the uptake of citrus fruits by British customers over 200 years ago. From the viewpoint of statistical methods, we might imagine a process over time in which the proportion of persons trying out the new product is initially very small. Over time, it increases and eventually becomes a majority of the population. This process has been modeled in terms of a logistic distribution. This is commonly done for the diffusion of new products and technologies. An alternative approach relies on the Gompertz curve.

In terms of networks, we may think of a collection of nodes that are joined by links that are assigned randomly according to a given model. We think of a link to mean that the two persons connected via this link both share a common outlook on some issue and know that the common outlook is held by both of them. If enough persons adopt this viewpoint, we may expect to obtain a 'large' connected component in that network where the proportion of nodes in that component is at least a positive fraction specified in advance of all nodes in the network (say at least one-tenth of all nodes, for example). Of course, statements such as these need to be made rigorous. This can be done via logistic regression models and the more sophisticated theory of Markov random fields.

At an elementary level, a different approach is demonstrated in Bak and Chen (1991) as cited earlier in this paper. More technical papers are found as reprints in Kadanoff (2000), especially at the end of Chapters 5 and 6 in that book. Two of them are written by Per Bak and colleagues. Some of them could be used as a source of ideas on research into phase transitions (Type 4 critical events).

The book by Barrat et al (2008) provides an introduction to random networks and dynamical process on such networks. The approach adopted in this book is influenced by the ideas of statistical physics, but the reader is not expected to have prior knowledge of that subject.

A simple example of a random graph model is the Erdős-Rényi model, in which E edges are chosen randomly from N vertices. This model is generally too simple for most real-world situations but it can serve as a prototype for better models. The Poisson distribution for the degree k of a node is readily obtained. (Here, the degree k of a given node is just the number of edges with that node as an endpoint. Thus, an isolated node has no edges going to that node so its degree is 0.)

More generally, we have the exponential random graph model, which includes the Erdős-Rényi model as a special case. It is usually assumed that the associated probabilities take an exponential form

$$P(\mathbf{X}) = \frac{\exp\left[\sum_i \theta_i z_i(\mathbf{X})\right]}{Z}$$

where $\mathbf{X}$ is the adjacency matrix with $X_{ij} = 1$ if (i, j) is an edge and $X_{ij} = 0$ otherwise, $z_i(\mathbf{X})$ are observed functions of the network, θ_i are parameters of the network and Z is a normalizing constant (called the partition function in information theory and statistical physics) so that we have a probability distribution. This topic is treated more fully in Barras et al (2008), Chapter 3. We are more concerned with the emergence of a phase transition as the probability *p* of a link existing is changed.

In general, for small *p* (say $p << 1$), we have only small clusters with high probability and for large *p* (say, with $1 - p << 1$), a single giant cluster is highly probable. At a certain critical value p_c, say, with $0 < p_c < 1$, there will be a phase transition. These ideas are developed further in Chapters 6 and 10 of Barras et al (2008) so the reader should consult that reference for a more formal description of phase transitions.

A useful example for understanding phase transitions in a model of a social network is summarised in Section 10.6 of Barras et al (2008) where a simple example in Ehrhardt et al (2006) is briefly discussed. The main parameters are, (1) the rate of creation of a new link, and (2) the rate of decay of a link. Here, 'creation' refers to the event of two agents coming to share the same opinion on a certain topic (a new link) and 'decay' refers to the event of that link ceasing to exist. As the creation rate relative to decay rate increases, the initial state of a large number of small clusters (each sharing the same opinion within that cluster) will exhibit a discontinuous change to a new state where one giant component with some small clusters are present. This is an informal description with technical details being available in Ehrhardt et al (2008) with Sections III and IV being especially useful.

The fall of the Berlin Wall is a plausible example of a phase transition where small groups who were critical of the existing regime gradually coalesced into a much larger group with negative views of the regime. Obviously, data on parameter values would be difficult to obtain for this example. Further, it is quite likely that the model of Ehrhardt et al (2006) need not fit well to reality, even if the necessary data could be obtained. Perhaps the real value of a phase transition model is to illustrate possible mechanisms by which the observed transition is likely to occur, along with credible results from such a model.

It is noted in Silver (2012) that even the experts failed to forecast the fall of the Berlin Wall. Political experts had difficulty in foreseeing the collapse of the USSR as Chapter 2 of Silver (2008) makes clear. However, the author does provide clues that could have been used to foresee a possible collapse. Indeed, Gorbachev opted to open up the media and the markets and a little extra freedom to the populace. Also, the weak state of the economy offered an opportunity for regime change as Silver states. Thus, it is a matter of putting these clues together to see that inference. Even so, it would seem difficult to accurately forecast the timing of such an event as the collapse of the Berlin Wall.

As far as predictions are concerned, some recent work was reported in *New Scientist* for 5 March 2011 on page 10. This reports some models developed by Yaneer Bar-Yam at the New England Complex Systems Institute in Cambridge, Massachusetts to predict when a dictatorship is ready to fall. The basic idea is to seek simple laws that describe a population's collective behaviour. The *New Scientist* report was clearly inspired by the so-called Arab Spring which began in late 2010. It is quite plausible that a similar model might have been useful to predict the fall of the Berlin Wall. However, good data is probably hard to find and the regimes concerned would probably hinder any attempts to gather such data. Bar-Yam believes that it may be enough to seek certain symptoms such as slower changes in the social system to even small changes.

A check of Bar-Yam's website reveals an online publication on the dynamics of complex systems. Suffice it to say that the reader has to work through a good deal of theory dealing with chaos, networks, fractals and ideas from statistical physics. With this remark, we now close with the observation that the successful modelling of transition phenomena is a highly nontrivial task. Householders and businesses can do little to avert these transitions except by watching for subtle changes in how their communities and governments work in order to detect possible clues of impending changes in the future. This may help householders and businesses to make contingency plans.

5 Personal sources of critical events (Types 1 and 2)

5.1 General

This Section provides some concrete examples of critical events at the personal level. Simple examples include births, deaths, marriages and accidents. These particular examples lend themselves well to statistical models. The Poisson distribution has been used as a basis for modeling the incidence of accidents in the literature. A well-known example occurs in Section 5.23 of Stuart and Ord (1987).

Critical events in this Section are generally of Types 1 and 2. An outline of the basic ideas is provided in Section 2.

Life evolution Some critical events are related to age, and changes in the composition of the household over time.

- Changes in the stages of life of individuals can impose changes on the household which has an effect on the balance of activities that go on in a household. Examples include pregnancy, birth of children, changes in schooling, retirement, people leaving the household, health, and finally death.
- Young people have more car accidents, probably because they are more active and take more risks.[4]
- As we get older some of our abilities become less well attuned to risky situations. Thus we find it increasingly difficult to negotiate a step and we may fall. The likelihood of back injuries increases markedly after the age of about 45 years is reached.
- The likelihood of major complications arising from an accident increases with age.
- Our general level of health changes with age.

Habits and customs We tend to repeat our life patterns every day. If our practices are prone to accident or disease, we should expect to

[4] See ABS (1989) Cat. No. 9405.0. Table 7.

sooner or later have an accident or disease, as surely as we expect a six if we toss a dice often enough.

• Some customs are related to our age. There are cultural differences between different generations as well as within a generation. To this extent we are all multi-cultural.

• Some people may have the inability to adjust to differences in customs and hence withdraw, or assume away differences. Ways to communicate across cultures, the willingness to live and let live, and to be open to other viewpoints, are particularly important in a multi-cultural society such as Australia.

• Personal habits may irritate others which can lead to conflict. However, people may be blissfully unaware that a habit underlies the conflict so a strong nudge may be needed as a reminder.

Personal behaviour

These are numerous and we will only give a few examples. A few examples are:

• Outstanding personal achievement or disaster.

• Decisions.

• Inconsistency of behaviour leading to misunderstanding by others.

• Personal relationships which may for various reasons sour over time.

• We may attempt to do more than we are capable of and strain a part of the body. The range of possible illnesses and injuries is very extensive.

5.2 Health and illness statistics

We include a few examples to illustrate the impact of illness and injury on the population. The economic costs are substantial. Illnesses are a rich source of critical events. We have related the data to households in Australia by taking averages but we do not consider specific types or regions of households. We use numbers of households at the 30th June of the year of interest as shown below:

TABLE 1: HOUSEHOLD NUMBER AND SIZE (est.)

Year	Households	Households size	Population

	No. millions	persons per household	No. millions
1981	4.67	2.99	14.927
1986	5.19	2.88	16.0184
1987	5.30	2.87	16.2639
1988	5.40	2.85	16.5322
1989	5.51	2.84	16.8144
1990	5.62	2.83	17.0651
1991	5.72	2.82	17.284

The estimated growth in numbers of households was about 105,000 households per year for this period.

Personal health statistics: The determination of the range and distribution of health factors as well as risk factors has become a major field of study in epidemiology.

Health surveys Most of the health surveys referred to below are really disease and accident surveys since these are more easily measurable. It should be noted that in the literature the word 'health' is often used when 'disease' is meant. A health survey is often an illness survey. Quoting from the 1989-90 National Health Survey[5]

• "Of the Australian population aged 18 years or more, nearly 80 per cent felt that they were in good or excellent health. Only 5 per cent felt they were in poor health."
• "Despite this, two out of three persons reported suffering a long-term medical condition ..."
• "About one in seven persons had been admitted to hospital during the twelve months prior to interview."
• "Almost one fifth of the population had consulted a doctor in the two weeks prior to interview."
• "Seventy per cent of people reported using medications during the previous two weeks."

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[5] The ABS 1989-90 National Health Survey[5] Preliminary Estimates (4361.0) May 1991.

• “Almost one in ten employed persons took one day or more away from work due to illness or injury in the previous two weeks., and the same proportion of students had one day or more away from school, university, etc …”

• “One third of both male and females reported no exercise.”

ABS (1991a, b); 4664.0

The conclusion from these observations is that health difficulties are widespread in households. This is confirmed by earlier data:

"The 1977-78 Health Survey reports **three quarters** of older Australians (aged 65 years and over) had experienced recent illness compared with **two-thirds** of younger Australians."

See Kendig & McCallum (1986)

“The 1983 Australian Health Survey (AHS) provided some indicators of the health status of the non-institutionalised population." "The survey was conducted throughout 1983 and obtained information on illnesses and injuries experienced during the two weeks prior to interview." "The survey found that children, particularly 5 to 14 year olds, appeared to be healthier than most adults. Even so,

About half of all children,
55 per cent of children under 5 years of age
46 per cent of 5 to 14 year olds, has
experienced one or more illnesses or injuries
in the previous two weeks.

most of these children (70 per cent) had only one, around 22 per cent had two while 8 percent had three or more conditions. The common cold was the most frequently reported ailment, followed by dental problems. Injuries comprised the third largest group of conditions experienced by 5 to 14 year olds and the fifth largest, after eczema/dermatitis and cough/sore throat, for children under 5 years.”

See ABS (1989a) *Australia’s Children* 1989; 4119.0,23

It is likely that a family with children will experience several illnesses or accidents per year. This result is consistent with the results of the Australian Health Survey referred to above. It is noted from the survey that children are healthier than adults. The incidence of illness and probably incidence of accidents is greater for adults.

Births In Australia there were 246,193 live births in the year ended 31 December 1988. See ABS(1989a) .

4.6 births per 100 households per year.

(Resident population at 30th June 1988 was estimated to be 16.54 million, see ABS (1991c) Year Book Australia 1991). Therefore there were close to 14.9 births per 1000 persons.)

Children About 2.178 million families (which we take to be equivalent to family households in the ABS sense) had dependent children in the 1986 Census. Thus we find about

42 households with children out of every 100 households

See ABS (1988c), *Australian families and households. Census 86,* 2506.0, 21. Children are a source of many critical events.

Deaths In Australia in the 1988 calendar year there were 119,866 deaths or about 7.3 deaths per 1000 persons. This rate is about

2.2 deaths per 100 households per year

See ABS Australian Demographic Statistics June quarter 1989; 3101.0, Table 15.

Psychological disruptions These are described in much of the psychiatric and psychological literature. Old memories intrude on the present in quite bizarre ways. These can lead to self-generated critical events although the worst fears are often never realised. Anecdotal evidence indicates a substantial impact on households from psychological disruption. The absence of quantitative measurements in this area may lead us to underestimate its importance.

Other psychological disruptions may arise from various sources of stress, such as a divorce, a disabling accident, loss of job and so on. More critical events may flow from such sources.

Traffic injury accidents There are about 25,000 injury accidents from traffic per year, and no doubt many more from non-reported accidents. This is a rate of roughly

0.5 per 100 households per year

See ABS (1989b), 1987 Road Traffic Accidents Involving Casualties Australia; 9405.0, Table 1.

6 Household sources of critical events

Some critical events can be viewed as taking place at a household level. For instance, marriages are not just personal events since they affect the families of the parties concerned and often lead to the creation of new households. Standard statistical models can often be considered for this type of event (normally Type 1 or 2). An outline of these models is sketched in Section 2.

It is clear from previous remarks in this paper that many personal sources of critical events are also household sources of critical events.

6.1 Household as a composite

The likelihood of critical events in a given time period in households of size > 1 is greater than for any one of its members. For instance, suppose the probability of a person not having an illness in a given time period is ½. In a three person household, the probability of no-one having an illness in that time period is ½ times ½ times ½, which is $\frac{1}{8}$, assuming independence. (This hypothesis is questionable.) In reality, the distribution of illness is uneven across age and we expect older households to have a greater prevalence of chronic illness than others. Infectious disease tends to be more common in family households with children. Larger households are more likely to experience critical events in the household in a given time period.

6.2 Relationships reaching critical levels

General There are many relationships that can generate critical events from time to time. Examples include a new family member, death of spouse or close family member or close friend, births, marriage, divorce, arguments, marital reconciliation, marital separation, sexual difficulties, son or daughter leaving home or trouble with in-laws.

- Interaction between householders is frequent and there is often the possibility of amplification of a mistake, particularly if both are tired. One mistake leads to an over-reaction by a partner which leads to further increasing tension and possibly leading to a change of state.
- Role overload where one is expected to do more than is possible for one person, either at home or at work. Public figures may have role overload because their public relations people have built up the character of the person to be more than life-size.

Marriages In Australia there were 116,816 marriages in the 1988 calendar year. That is about

2 'newly-married' households per 100 households per year.

See ABS (1989c) Australian Demographic Statistics June Quarter 1989; 3101.1, Table 17.

Divorces In Australia there were 41,007 divorces in the 1988 calendar year. i.e. about

0.8 divorces per 100 households per year.

See ABS (1989c) Australian Demographic Statistics June Quarter 1989; 3101.1, Table 18.

About one in six children can expect to experience the divorce of their parents before their sixteenth birthday, see MacDonald, P. (1988)

Change of residence Of the 17.5% of people who moved between 1985 and 1986, the great majority (88%) stayed in the same State. That is, of those who moved, $0.88 \times 17.5\% = 15.4\%$ stayed in the same State or 15.4 persons out of each 100 persons or out of 34.7 households.

There are two household-person changes per move, one out of a dwelling and one into a dwelling. This may be somewhat modified by migration and new housing effects although we will regard such effects here as being secondary. For young households, rates of change are much greater. Over the period between the censuses of 1981 and 1986, 43% of people moved, of whom 37.4% stayed in the same State; Table 1.9 of ABS (1988a).

7 Outside sources of critical events

There are many outside sources of events which can be critical to householders so we only consider a few examples.

7.1 Environment

General We may not be as well attuned to the world as we may think. An understanding of critical events in the natural environment may help us to appreciate our own limits in the world.

The Darling River algae bloom of 1991 provided a very good example of a critical event at a regional level which affects all households in the region as well as all natural systems. In addition this provides lessons in many other fields as well. Allen Creer, Senior Research Scientist, The Australian Museum wrote a letter (1991) to *The Australian Financial Review* about the Darling River.

Darling River takes the wraps off reality

SIR, One good thing to come out of the blue-green algae bloom along the Darling River is that it provides an excellent example of what the collapse of an ecosystem looks like.

We've all read about them, and now we've got one in our own backyard.

Have a close look at it.

All the elements are there: long years of slow but steady degradation of the natural infrastructure and systems (vegetation and soils, and the water and nutrient cycles); every human user in for as much as he can get in the absence of an overall management plan; derision of early warnings based on experience and knowledge of the way the natural system operates; and then a small perturbation (this year's dry spell) to push the whole thing into collapse over the space of just a few days, leaving us stunned before pictures of fish gasping for oxygen, incontinent animals in their death throes, warnings to humans to not even touch the water let alone drink it … and the inevitable search for scapegoats (forgetting Pogo's discovery that "I have met the enemy and it is us!")

And lest you think this is a one-off ecosystem collapse, let me remind you of a thing called the ozone hole …

Creer, A. (1991)

We do not know whether this process is reversible or not. Perhaps the then current heavy rains may have had the desired effect. We can think of many processes that are slowly changing and which can after some time show noticeable signs of failure.

7.2 Other systemic effects

Leaving the environment for other critical events, one may consider possible examples of degeneration in the education system, average work hours per month and an infrastructure that cannot respond quickly to a crisis. Adverse consequences are shown in high youth unemployment, breakdowns in transport, a flight of investment out of manufacturing or commercial building and central city pollution.

Cumulative effects We convert a great deal of food and liquid over a life time. If our filters, especially the liver and kidneys, deteriorate, the concentration of harmful substances may become high enough to cause a general breakdown in health.

Biological Epidemics. In recent times, AIDS, hepatitis B and C and also various mutations of influenza have become widespread. This is leading many people to review the idea of complete freedom of travel in order to ensure that public health is not jeopardised. Fast travel may indicate a need for stricter rules of personal behaviour to limit transmission of disease.

Social rules and schedules The total demands of social rules and schedules made on the household or person may exceed the capacity to cope. Sometimes a society or an individual or government creates events in order to induce change. Major events such as the Olympic Games are said to produce such changes. Whether or not they do so is another matter. In an unfamiliar country, a simple but highly dangerous mistake is to be unaware of the local road rules.

Large computer systems Most households are dependent upon large computer systems run by banks, Medicare, the Taxation Office, social security and so on. In reality, a system that works adequately for a limited series of tasks should only have a loose connection to another system with a different set of tasks. This avoids the problem of two systems becoming dependent on one another with greater complexity. For instance, audit trail information should only be loosely linked to management data so that the variable nature of the data does not overload the system.

7.3 Law

Far-reaching influence Many decisions have legal consequences, such as marriage, purchases, contracts, partnerships, employment contracts, accidents. Some of these can turn out to be disastrous if incorrectly dealt with at the appropriate time legally. There are books providing guidance as to when to seek legal advice. See for example Bowen (1990). Very common problems include drawing up Wills, and resolving Partnership difficulties.

Crime According to a survey of the Australian population conducted by the Australian Bureau of Statistics during 1983, 9% of households (462,700) experienced an illegal entry and/or household theft in the previous 12 months

9 illegal entries per 100 households per year

See ABS (1983) Victims of Crime Australia 1983, 4506.0.1, p.1. Numerous homes are now left unattended by adults during the day, because of the increase in paid work for all adults. This provides a relatively easy target for thieves and other predators. Theft thus becomes a source of critical events in the household, and security becomes necessary to reduce the frequency of theft.

7.4 Related to outside work

Paid work Many critical events can occur in the workplace such as difficulties with the boss or colleagues. Bullying can be a problem. Loss of a job or job changes can have far-reaching consequences. Some critical events can be positive, for instance, a pay rise or a promotion.

Retirement It can take an individual at least two years to adjust. This would then involve about 100,000 persons per year and over two years 200,000 persons. See ABS (1989c), *Australian Demographic Statistics June Quarter 1989*; 3101.1, Table 13. On 30th June 1988 the estimated numbers were 359,938 males and 369,404 females in the age category 60-64 years. If we assume that half the females and all the males have been in paid work and we take a one year group, we arrive at an approximate figure of 108,000. We then multiply by two to get the effect of the two year changeover.

Four retirements per 100 households per year.

Starting up and closing down small businesses The average number of business (small) operations in Australia during the 1986-87 financial year was 580,900. See ABS (1988b) *Small Business in Australia 1983-84 to 1986-87*; 1321.0. That was about one small business to 9 households. Assuming a five year life for these businesses on average (probably the right order of magnitude) and about the same number of start-ups, this implies about 230,000 close-downs and start-ups each year. See Stutchbury, M. (1990). This leads to approximately

4 small business changes per 100 households per year

7.5 Money

Financial problems commonly lead to critical events, due to problems in meeting financial obligations. A sharp drop in income can lead to difficulties. Various studies have been done by marriage guidance organizations on the influence of money on marriage problems. Inflation creates extra worry since one is continually concerned with turning money into something more inflation proof. It distorts economic decisions away from the common good towards personal survival.

A sudden increase in wealth, such as winning a lottery or receiving a large inheritance can change a person's lifestyle. Sometimes, this can lead to negative results, apparently due to the person not being able to manage the new-found wealth well. Even so, critical events such as these would mostly be seen as positive

Major change in the financial world With the much more flexible relationships between the world currencies, countries such as Australia which are small economically in a world sense are not insulated from world events. Indeed, the effects of world events may be amplified. A recession in such circumstances may alter confidence about the future even more than in larger countries and lead to increased saving, or lower employment prospects, or lower household income. Governments need a fallback position on how to ensure the survival of most households in these turbulent circumstances. It is not sufficient to think in broad economic terms since there are substantial underlying variation, orders of magnitude problems, spatial distribution problems and so on. We can expect one major currency upset every two to four years in Australia

Mortgages A large number of households are under mortgage. 1,604,300 households have mortgages or about:

30 mortgages per 100 households

See Table 3.14 in ABS (1988c) *Australian Families and Households, Census 86* (2506.0).
A large change in the cost of money for housing finance (e.g. 25% increase in interest rates) can affect a household because it rapidly alters costs while incomes cannot be changed much in the short term. A significant drop in interest rates may be seen by an investor as highly undesirable but highly desirable for borrowers!

7.6 International War, floods, bushfires, earthquake, climatic events, international trade problems are a few that come to mind. Cyclones occur regularly in Northern Australia and can produce short term but serious problems for some households. Many people may not anticipate the possibility of a war. Clearly these happen fairly often in certain parts of the world. The world is far from stable politically and a war can upset many households even indirectly.

8 Managing criticality

One definition of management that is relevant here is: 'knowingly applying methods to our activities in order to achieve a desired result.' Consider four aspects which help to control the flow of critical events: Avoidance, coping with critical events, decision making and recovery.

8.1 Avoidance

Engineering practice provides a helpful guide to some techniques for dealing with criticality in the household and elsewhere. There is a long history of designing and constructing physical structures to cope with the variability of the real world, of risk, probability and critical events, and to design to accommodate extremes or at least to take account of the cost of loss due to failure. Avoidance of criticality is a central theme of engineering.

Tolerances If there are a number of parts in a machine with lack of 'play' between them, the machine will seize up. Engineers aim to create conditions where inherent fluctuations in one part are not passed on to another part, thus avoiding serial effects. Tolerances are designed into interacting parts, thus creating some degree of independence between them. This idea of tolerances can be applied to households.

Hazards are avoided by maintaining a distance between ourselves and the hazardous object. This is the case when driving a car. It is surprising how rare major accidents are, given the high frequency of potential hazards. Products and services are often allowed to have some degree of ‘play’ or slack between differing parts or functions. Homes are generally designed with sufficient space in various rooms to allow for free movement.

Slack In a chain of critical events as in a critical path schedule, it may be useful to build in slack, points of zero activity but taking time. This ensures that time slippage in some early items does not create problems with later items in the process.

Safety factors Safety factors are important in engineering practice. This allows for conditions which are more severe than those assumed in the design. This ensures that excess loading is very unlikely to create a failure in a part or structure.

Fatigue It may be found that a load X can be carried by a structure now but that continuous repetition of such loads will lead to failure - this is called fatigue in engineering. This failure can be avoided by a more sophisticated safety factor which takes into account the repetition of load as well as the size of the load. Clearly there are analogous circumstances in human life at the individual level, at the household level and at the level of society as a whole. Fatigue (due to stress) may be a source of critical events.

Control and feedback Information from current activity of a machine is used to ensure that if the machine is running too fast, power is reduced and if the machine is running too slowly then more power is provided. In engineering, there is the associated idea of ‘hunting’. This is an unwanted amplification where a machine or other system drifts further away from equilibrium until a failure occurs. Control systems using negative feedback dampen oscillations in the system so that failures do not occur. Similarly household members need to be able to communicate with each other so that conflicts do not escalate to serious disputes.

Regular maintenance All machines wear out, but their life can be prolonged with regular maintenance. Different parts need different maintenance schedules. There are obvious parallels with health. For example, about one third of the male population and about a quarter of the female population were overweight in 1991, presumably due to the body not being able to efficiently metabolise the food ingested.[6]

Repeated events Repetition is considered in two ways. First, in a large population of households, many similar types of activity may occur nearly concurrently. Second people tend to repeat their daily routine throughout their lives. Thus, a few occurrences of a critical event should be expected on probabilistic grounds in a large population, e.g. the Poisson distribution, laws of large numbers. Also, infrequent unwanted critical events in large populations of events may be sufficiently important to warrant redesign or change of practices, e.g. motor car accidents or accidents in homes, or quality control applications.[7] It has been found that, for example, accident frequency can be reduced by modifying the population or its behaviour; in the case of motor vehicle accidents, roads, drivers and vehicles can be modified. This involved the construction of divided roads, the design of more reliable and safer vehicles, wearing safety belts and regulating drivers to have greater driving restraint and skill.
Accidents in or near the household are especially likely to occur when carrying out repeated events, due to the person relaxing their guard in a familiar situation. Much can be done to reduce their incidence by adopting safety features in housing design and in household maintenance.

Resources for anticipation of critical events There are periods in the history of a household where there are likely to be a greater number of critical events per month than at other times. Recognition of the nature of critical events and when they are likely to occur can help in their anticipation, avoidance or moderation. Also, each individual will find that at a particular stage of life what may be critical to one is not necessarily so to another and vice versa.

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[6] See Chart 33 Australian Bureau of Statistics (1991) Catalogue No. 4364.0

[7]The distribution of accidents can often be described by a Poisson distribution. An example is found in Sec. 5.23 of Stuart and Ord (1987).

Rules, codes of practice, routines and habits are obvious tools for reducing the incidence of negative critical events which are likely to arise out of repeated actions. This is particularly so if these rules are aligned to skills. Meta-rules such as general ways of behaving towards others can be beneficial and save a great deal of time; they help us to order specific rules into groups. For example "Assume firstly that people are good". If there is proof otherwise then apply exceptional methods. See Jennings (1989). Such a rule leads to great economies of time and effort. It is suspected that in recent times, regulations and rules are being introduced at an excessive rate because people are assumed to be potential criminals.

Conflict settlement Conflict is very frequent in interactive groups such as the household, and hence can be a source of critical events. The observance of rules can help to reduce or avoid conflict. An example of an approach from Argyle and Henderson (1985) is given showing eight conflict-regulating rules. One example will suffice to give the general idea. The various rules shown are designed to avoid conflict by regulating behaviour so that conflicts do not occur.

> "We list some of the most important ones below, together with the relationship to which they apply."
>
> 1. Should respect the other's privacy (all relationships) *Separate spaces.*
> 2. Should not disclose confidences (all relationships) *Otherwise cuts out feedback.*
> 3. Should not criticise the other person in public (most relationships) *Otherwise undermines social support structures.*
> 4. Should be faithful (dating, cohabiting, and husband-wife) *Maintains stable relationships.*
> 5. Should be punctual/prompt in keeping appointments (dating, repairman, student) *Simplifies planning for others.*
> 6. Should not be overprotective/over possessive (sibling, parent) *Allows people to develop their own sense of control and independence.*
> 7. Should not encroach shared boundaries (neighbour) *Personal space limitations.*

8. Should accept fair share of the work load (work colleague). *Social support.*

Argyle and Henderson (1985) , p.307

Simon has some pertinent comments to make on conflict.

"Choices bearing mixed consequences and choices where all alternatives have unpleasant consequences represent conflict situations for the subject, and behaviour in a conflict situation is qualitatively different from behaviour when conflict is absent. For example, one kind of behaviour has no place in utility theory, but it is frequently observed in conflict situations - this is the subject's refusal to accept alternatives as given and his search for new alternatives. Where the conflict is stronger, various kinds of neurotic behaviour come into evidence - fixation, regression, aggression, withdrawal, and so on.

"The empirical evidence on choice in conflict situations challenge the validity of the utility function as a basis for conceptualizing decision making behaviour. Any microeconomic description of behaviour that purports to fit empirical data will have to deal with this problem. The study of choice under conflict may prove to be even more fruitful for reexamining the utility concept than the experiments on choice under uncertainty that were mentioned earlier."

Simon (1963), p.694

Insurance cover This needs to be adequate to cover the full range of possible accidents and the full extent of likely losses. There is a tendency to underestimate the current cost of accidents and hence under-insure. An insurance policy cannot be taken out for excluded events such as arson, war and highly dangerous activities.

8.2 Coping with critical events

There are numerous techniques for managing in critical situations - crisis management has quite a substantial literature.

Calling on reserves People carry within them reserves of nutrients and water so that they can go for hours without food or water.

Households need to have stocks of various items to last for a few days. Households also need to have some source of credit or savings to cover them for loss of income for a period. Above all human beings have reserves of skill and habit, a stock of knowledge of our own, and we can call on the collective memory of the human race.

From the theory of point processes as expounded by Cox and Isham (1980), the timing of critical events (at least those of Types 1 or 2) will generally be 'spread out' over a substantial interval of time. Thus, it will seldom happen that a large number of critical events will occur in a small timeframe. Consequently, it is possible to reduce the average reserve per household. Thus, governments raise taxes to cover the unemployed and other needy people. Charities raise funds in order to provide for the needy. Friends and relatives can help in times of difficulty. The increase in single person households can increase the need for emergency relief systems since outside help will be obviously necessary if a serious critical event occurs.

Dampening of behaviour It is useful to be aware that behaviour at critical times is exceptional and should not be regarded as normal. We should not act like an out-of-control machine. Various techniques can be adopted to stop escalation of a dispute, an apology may be all that is needed if we do not permanently hurt others. Different people have different flash points, and will vary over time as to the level at which the flash occurs.

Change of behaviour By definition a critical event leads to a change of state, either temporarily or permanently. Habits need to be modified to conform to the new state. For example, the number of persons in the household may change or various personal attributes of one or more individuals may change significantly. These changes can be done, possibly with some effort, if it is recognised as being necessary but we are often a slave to our habits. The consequences of a change of state should be considered in our future planning and management.

Decision making Some personal decisions can turn out to be critical. We cannot avoid them even if they are made for us. Choice of home location, decisions about major expenditures, marriage and divorce, choice of job, choice of the method of bringing up children. Some of these may be irrevocable directions for future activities. Such decisions may be time consuming and weighty.

8.3 Grouping of sources of critical events

We have implied grouping in our previous discussions but it is useful to make explicit some of the main forms that grouping can take.
Mathematical form The best known mathematical form for rare events is perhaps the Poisson distribution where the frequencies of rare events, such as critical events, are described. In order to alter the frequency of critical events such as road accidents, we have to alter the population or the associated parameters. More details are given in Sections 2, 3 and 4.

Household grouping Some critical events do not merely affect the individual but are best considered at the household level. If one member of a household contracts a contagious illness, other members of that household may catch it too. Medical treatment and some other matters, such as stress and caring issues, could perhaps be best given within the closest socially interactive group, the household.

It seems that there has been a social dogma extant in recent years that has downplayed the importance of household and individual behavioural practices as major sources of critical events. Service groups concerned with reducing the incidence of critical events cannot ignore the importance of small social groups where intense, frequent communication occurs, in producing change. This includes close friends, and family, and households, and small groups at work. Groups such as 'Alcoholics Anonymous' and many therapy groups have been using such techniques for many years. But the importance of the household is still not publicised adequately.

9 Conclusions

Avoiding adverse critical events is likely to be practical. Coping with stressful situations is sometimes regarded as a sign of weakness, but is probably an optimal way of operating in many circumstances – sufficient effort for the purpose. Stopping to review the way critical events group together can lead to efficient management practices.

In this paper, we only give a broad outline of a possible theoretical approach. It relies on stochastic processes and network models as a source of possible models. It is probably more important at this stage to get the correct order of magnitude for the occurrence of critical events, rather than strive for a high degree of exactness or precision. These events occur at various points in time so that we can apply results from the theory of random processes, especially point processes.

There are many sources of critical events and their frequency is not low. People seek to maximize the impact of positive critical events and try to minimise the adverse effects of negative critical events. This paper advocates the need for further study of critical events and the need to apply the findings to the regular planning and management in government, business and in our own lives and households.

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