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# A PEDAGOGICAL TOOL FOR ILLUSTRATING THE REAL IMPACT OF THE FINANCIAL SECTOR

by

Yuan K. Chou

Department of Economics
The University of Melbourne
Melbourne Victoria 3010
Australia.

# A Pedagogical Tool for Illustrating the Real Impact of the Financial Sector

Yuan K. Chou<sup>1</sup>

Department of Economics, University of Melbourne

#### **Abstract**

We devise a simple way of incorporating the financial sector into a growth model that is useful pedagogically. Financial innovation raises the efficiency of financial intermediation, which facilitates capital accumulation. The model may be extended to include real R&D as a symbiotic source of endogenous growth.

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JEL keywords: Financial innovations, economic growth

#### 1. Introduction

Students in introductory and intermediate-level undergraduate economics courses are often puzzled about the role of the financial sector in the functioning and performance of the real economy. While they are exposed to constant media coverage of financial firms and their activities, discussions about the financial sector are often cursory or non-existent in most standard macroeconomic texts. On the topic of economic growth, real R&D activities that produce technological innovations are often touted as the primary engine of growth in advanced economies. Since financial firms attract many of the brightest college and business school graduates, the financial sector may appear to affect growth adversely by draining precious talent and human capital away from the science and engineering fields.

In this paper, we construct an extension of the classic Ramsey-Cass-Koopmans model to show in a succinct fashion why the financial sector matters for the real economy. The financial sector in our model comprises financial innovators and financial intermediaries. Financial innovations, designed by those talented college and MBA graduates, increase the efficiency of financial intermediation by improving the match between the needs of savers and borrowers, which in turn facilitates capital accumulation and increases future output. As our model of finance and growth differs from previous models by focusing on innovative financial activities, we begin with a short discussion on financial innovations.

Financial innovations may be motivated by the need to hedge some new economic risk, or by new regulation, a change in fiscal and monetary policies, demand for intertemporal or spatial wealth transfers, the need to lower transactions costs, or the desire to reduce agency costs due to asymmetric information. Financial innovation may involve inventing a brand new class of products, the modification of existing products, or the combination of the characteristics of several existing products ("spectrum filling"), which moves the financial system closer to the Arrow-Debreu ideal where all transactors can ensure themselves delivery of goods and services in all states of nature.

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<sup>&</sup>lt;sup>1</sup> Address: Department of Economics, University of Melbourne, VIC 3010, Australia. Phone: +61 3 8344 5287 Fax: +61 3 8344 6899 Email: ychou@unimelb.edu.au. The author would like to thank Martin Chin for his research assistance.

Financial innovations differ from real technological innovations in that they are not protected by patents. However, financial innovators gain first-mover advantages such as acquiring reputation and expertise in pricing and exploiting the properties of the product. [Tufano (1989), Herrera and Schroth (2001)]. Financial innovations may be classified according to their risk-transferring, liquidity-enhancing, or equity-generating abilities. Financial innovations may aid real innovative activities (such as venture capital), assist corporate expansion (bonds), facilitate profit-taking and the spreading of risk (mutual funds, CDs, derivatives, hedge funds), and help refinance obligations or mobilize assets (swaps, mergers, futures).

### 2. Modelling the Financial Sector: The Key Equations

#### 2.1 Financial Innovators

Financial innovators produce new financial products and services using labor (and the embodied human capital) that is diverted from the production of the final consumption good. The stock of financial products (that is, old financial innovations) is denoted as  $\tau$ .

The development of the financial sector is characterized by an ever-expanding variety of financial products. The existing stock of financial products affects the production of new financial ideas according to

$$\dot{\tau} = F \left( u_{\tau} L \right)^{\lambda} \tau^{\phi} \,, \tag{1}$$

where  $\tau$  denotes the quantity of financial innovations per unit time,  $u_{\tau}$  is the fraction of the labor force employed by the financial sector, L is the aggregate stock of labor which grows at rate n, F is a productivity parameter,  $\lambda \in (0,1)$  is an elasticity parameter, and  $\phi \in (0,1)$  measures the extent of positive spillovers from existing financial products (the "spectrum filling effect" discussed previously). The production function for financial innovations exhibits diminishing marginal returns with respect to labor: as more and more individuals are engaged in designing new financial products, the probability of inefficient replication rises.

#### 2.2 Financial Intermediaries

Financial intermediaries are responsible for intermediating funds between borrowers (producers of the final consumption good) and lenders (households). Unlike conventional growth models, not all household savings will automatically be transformed into funds that are utilizable by firms for investment in new plant and machinery. In particular, some risk-averse savers will continue to hold liquid but unproductive assets unless offered a sufficient variety of financial products. The efficiency of intermediation is measured by

$$\xi \equiv \tau / L^{\kappa} \,, \tag{2}$$

where  $\xi \in (0,1)$ , and  $\kappa \in (0,1)$  is a parameter that measures the degree of rivalry in  $\tau$ .  $\xi$  may be interpreted as the state of development or sophistication of the financial sector.

Why does the efficiency of intermediation diminish as L increases? As the labor force increases, so does the volume and complexity of funds that have to be intermediated. A larger population may exhibit more diverse preferences for the risk, maturity and other characteristics of financial instruments. For some financial products and services (such as branch banking), the increase in population creates congestion that can only be relieved by financial innovations (like phone and Internet banking). By restricting  $\kappa$  to lie strictly between 0 and 1, financial innovations in the aggregate are neither fully rivalrous ( $\kappa = 1$ ) nor completely non-rivalrous ( $\kappa = 1$ ) nor completely non-rivalrous ( $\kappa = 1$ )

= 0). Our model allows  $\kappa$  to be infinitely small but not zero. The capital accumulation process therefore takes the form:

$$\dot{K} = \xi(Y - C) - \delta K \,, \tag{3}$$

where K denotes the stock of capital, C is the level of aggregate consumption, and  $\delta$  is the rate at which capital depreciates.

The aggregate production function for the final good, *Y*, is of the Cobb-Douglas form:

$$Y = AK^{\alpha} \left( uL \right)^{1-\alpha},\tag{4}$$

where A denotes the exogenous level of technology,  $u_Y$  the fraction of the labor force employed by the final goods sector, and  $\alpha \in (0,1)$  is capital's share of income from final goods production.

In the steady state,  $\xi$  must be constant by definition (and bounded from above at one in a closed economy). Therefore, if the labor force grows at the constant rate n, then the rate of financial innovations in the steady state must equal  $\kappa n$ .

#### 3. Microeconomics of the Model

Tufano (1989)'s empirical findings on the pricing behavior of financial innovators are consistent with the hypothesis of competitive innovation: investment banks that create new products do not charge higher prices in the brief period of monopoly before imitative products appear. The profit of a price-taking financial innovator, to be maximized by its choice of  $\tau$ , is

$$\pi_{\tau} = P_{\tau} \dot{\tau} - w_{\tau} u_{\tau} L \,, \tag{5}$$

where  $P_{\tau}$  is the price of each financial innovation. The first order condition implies that

$$P_{\tau} = \frac{W_{\tau}}{\lambda \, \tilde{F}^{1/\lambda}} \, \dot{\tau}^{(1-\lambda)/\lambda} \,. \tag{6}$$

This equation may be interpreted as the optimal pricing schedule for  $\tau$ .

Downstream in the financial sector, financial intermediaries purchase innovations from financial innovators (which, in the real world, are probably sister divisions of the same financial firms) and use them in transforming savings into productive investment. As the focus of our model is on financial innovations, we model the financial intermediaries very simply. They are passive, price-taking entities engaged in perfect competition who derive their income from charging firms a higher interest rate,  $r_K$ , for renting capital than it pays out to households for their savings,  $r_V$ . The interest rate differential,  $r_K - r_V$ , may be thought of as the commission charged for intermediating funds. For simplicity, we assume that financial intermediation requires no labor input.

In each period, the financial intermediary ensures that interest revenues received from firms equal the costs of acquiring deposits from households and purchasing new financial products from financial innovators:

$$r_{\scriptscriptstyle K}K = r_{\scriptscriptstyle V}K + P_{\scriptscriptstyle \tau}\dot{\tau} \ . \tag{7}$$

We can show that this results in an arbitrage equation governing the evolution of  $P_{\tau}$ :

$$\xi r_K = \frac{\dot{V}}{P_\tau \tau} + \frac{\dot{P}_\tau}{P_\tau} \,. \tag{8}$$

In the final goods sector, firms maximize profits by choosing the optimal account of labor and capital at each point in time:

$$\max_{u_Y \in K} Y - w_Y u_Y L - r_K K . (9)$$

The first order conditions require

$$w_{Y} = (1 - \alpha)Y/(u_{Y}L),$$
 (10)

$$r_K = \alpha Y / K - \delta . \tag{11}$$

Finally, to close the model, we examine the consumption decision of households. The representative household maximizes a discounted stream of lifetime utility subject to an intertemporal budget constraint where instantaneous utility is of the CRRA form:

$$\max_{C,u_Y} \int_0^\infty \frac{C^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt$$

subject to

$$\begin{split} \dot{V} &= r_V K + w_Y u_Y L + w_\tau u_\tau L + \pi_\tau - C ,\\ \dot{K} &= \xi \dot{V} ,\\ 1 &= u_Y + u_\tau , \end{split} \tag{12}$$

where  $\dot{V}$  represents the flow of households' stock of assets (that is, savings). We assume that households are ultimate owners of all capital and shareholders of final goods firms and financial innovators. In equilibrium, wages are equal across all labor markets, that is  $w_{\gamma} = w_{\tau}$ . Substituting this and equation (7) into the constraint results in the Hamiltonian:

$$\mathbf{H} = \frac{C^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \upsilon \xi \left( r_K K + \overline{w} u_Y L - C \right) + \mu \tilde{F} u_\tau^{\lambda} L^{\lambda}$$
(13)

where v and  $\mu$  are co-state variables associated with the state variables K and  $\tau$  respectively. The control variables are c and  $u_v$ . A flowchart of the economy is shown in Fig.1.

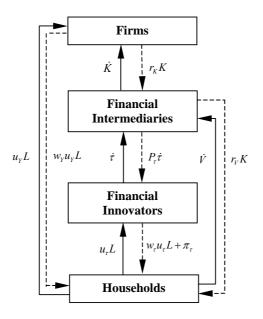


Fig 1. Flowchart of the model

## 4. Equilibrium, Solutions and Results

We define the balanced growth path or steady state of the model as one in which all variables grow at constant rates. As there is no technological progress, aggregate output, aggregate capital and aggregate consumption grow at the same rate as the labor force in the steady state. Defining  $k \equiv K/L$ ,  $c \equiv C/L$  and  $y \equiv Y/L$ , we can show that the steady state solutions to the model are, in sequential fashion:

$$u_{\tau}^* = \frac{\Gamma}{\Gamma + \Phi} = 1 - u_{\gamma}^*, \tag{14}$$

$$\xi^* = \left(\frac{Fu_{\tau}^{*\lambda}}{\kappa n}\right)^{\frac{1}{1-\phi}},\tag{15}$$

$$k^* = \left(\frac{\xi^* A \alpha}{\rho + \delta}\right)^{\frac{1}{1 - \phi}} u_{\gamma}^* \tag{16}$$

$$c^* = \frac{\rho + \delta - \alpha(n+\delta)}{\alpha} \frac{k^*}{\xi^*},\tag{17}$$

where  $\Gamma \equiv \alpha \lambda \kappa (n+\delta)$  and  $\Phi \equiv (1-\alpha)(\rho+\delta)[\rho-(1-\kappa)n]$ . These solutions imply that:

- 1. An increase in the financial sector spillover effect,  $\phi$ , increases the steady-state proportion of labor employed in the financial sector,  $u_{\tau}^*$ . That is,  $\partial u_{\tau}^* / \partial \phi > 0$ .
- 2. An increase in the rate of time preference,  $\rho$ , and the degree of risk aversion,  $\theta$ , decreases  $u_{\tau}^*$ . That is,  $\partial u_{\tau}^*/\partial \rho < 0$  and  $\partial u_{\tau}^*/\partial \theta < 0$ .
- 3. The steady state growth rate of the economy is independent of the characteristics of the financial sector. In the steady state,  $\dot{Y}/Y = \dot{K}/K = \dot{C}/C = n$  and  $\dot{y}/y = \dot{k}/k = \dot{c}/c = 0$ .

To discuss the properties of the model away from the steady state, we need to reduce the dimensionality of the model by assuming that the share of labor in the financial innovations sector,  $\overline{u}$ , and the physical investment rate,  $\overline{s}$ , are constant and exogenous. The resulting  $\dot{k}=0$  and  $\dot{\xi}=0$  schedules are given by

$$k^* = \left(\frac{\xi^* \overline{s} A}{n+\delta}\right)^{\frac{1}{1-\alpha}} \left(1 - \overline{u}\right),\tag{18}$$

$$\xi^* = \left(\frac{F\overline{u}^{\lambda} \left(1 - \phi\right)}{\lambda n}\right)^{\frac{1}{1 - \alpha}}.$$
 (19)

Figure 2 illustrates the dynamics of the model in  $k-\xi$  space and the impact of a rise in the productivity of financial innovators due, for example, to financial liberalization. Steady state intermediation efficiency,  $\xi^*$ , and capital per worker,  $k^*$ , are both permanently higher after the shock.

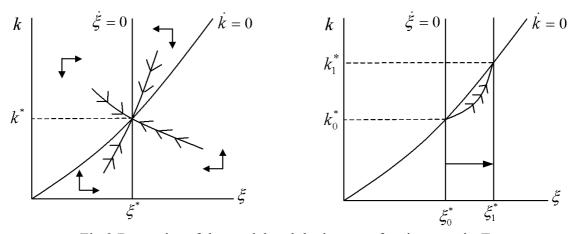


Fig.2 Dynamics of the model and the impact of an increase in F.

#### 5. Extensions

#### 5.1 Financial Innovations and Technological Innovations

We can incorporate endogenous technological progress into the model by adding an R&D equation, such as that in Jones (1995) but including a spillover effect for  $\tau$ :

$$\dot{A} = B(u_{\scriptscriptstyle A}L)^{\eta} \tau^{\beta} A^{\psi}, \tag{19}$$

where  $\dot{A}$  denotes the quantity of financial innovations per unit time,  $u_A$  is the fraction of the labor force engaged in R&D, B is a productivity parameter,  $\psi \in (0,1)$  measures the extent of spillovers from previous R&D activities, while  $\beta \in (0,1)$  measures the impact of financial development on R&D productivity. The idea is that some financial innovations, such as convertible loan notes and redeemable convertible preference stock, are used by venture capitalists to fund risky R&D projects with potentially high payoffs and may therefore raise the rate of technological innovation. We can show that the growth rate of the economy is now given by:

$$\gamma_A^* = \left(\eta + \frac{\beta \lambda}{1 - \phi}\right) n / (1 - \psi).$$

This growth rate depends in part on the parameters characterizing the financial sector, and neatly demonstrates the inter-dependence between financial and technological innovations in generating long-run growth.

### 6. Conclusion

In this paper, we demonstrated a simple and succinct way of incorporating the financial sector into an economic growth model. This is particularly useful as a pedagogical tool for explaining the real macroeconomic impact of the financial sector in undergraduate economics courses.<sup>2</sup> In our model, the financial sector produces financial innovations, which raises the efficiency of financial intermediation, thereby facilitating capital accumulation. We derived the steady state solutions of the model, explained its implications, and studied its transitional dynamics. Finally, we also showed how the model may be extended to include endogenous technological progress.

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 $<sup>^2</sup>$  For teaching purposes, it is easier to assume a fixed saving rate (s) as in the Solow (1956) model. Then the key equations are:  $Y = AK^\alpha (u_\gamma L)^{1-\alpha}$ ,  $\dot{K} = \xi(sY) - \delta K$ , and  $\dot{\tau} = F \left(u_\tau L\right)^\lambda \tau^\phi$  where  $\xi \equiv \tau / L^\kappa$ . For introductory courses, a description of the model and a graphical illustration like fig.1 would suffice.

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