Measuring the Natural Output Gap using Actual and Expected Output Data*

by

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Abstract

An output gap measure is suggested based on a multivariate Beveridge-Nelson decomposition of output using a vector-autoregressive model that includes data on actual output and on expected output obtained from surveys. The gap is estimated using an integrated approach to identifying permanent and different types of transitory shocks to output. The gap has a statistical basis but is provided economic meaning by relating it to natural output in DSGE models. The approach is applied to quarterly US data over 1970q1-2007q4. Estimated gaps have sensible statistical properties and perform well in explaining inflation in estimates of New Keynesian Phillips curves.

Keywords: Trend Output, Natural Output Level, Output Gap, Beveridge-Nelson Decomposition, Survey-based Expectations, New Keynesian Phillips Curve.

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1 Introduction

This paper offers a novel approach to measuring the natural level of output, and associated gap measure, based on a time series analysis of actual output data and direct measures of expected output obtained from surveys. The approach is in the spirit of Blanchard and Quah (1989) [BQ] in that it recognises that output is reasonably characterised as a unit root process and uses the single stochastic trend that drives actual and expected output to identify the permanent shocks to the series. BQ interpret these as permanent 'supply' shocks but note also that the trend derived from the permanent shocks alone will not adequately represent the trend in a standard business cycle decomposition. They note that this trend needs to accommodate the fluctuations in output caused by short-term, transitory supply shocks as well as permanent ones. Failure to incorporate the short-lived effects into the trend means the associated gap measure will over-react to changes in output and the size and timing of corresponding price pressures will be misjudged. Policy based on the gap will also over-react to output change and cause unnecessary policy-induced volatility. In this paper, we use the direct measures of output expectations to identify and distinguish the permanent shocks from different types of transitory shocks in an integrated approach to measuring the gap. The idea is that survey respondents state explicitly which parts of output movements they believe will be short-lived in describing their expectations on future output levels. Their effect can then be excluded from the gap measure so that it better reflects price pressure and is relevant in policy-formation.

The proposed gap measure is based on a vector-autoregressive model of the actual and expected data and has a statistical basis independent of any behavioural macroeconomic model. However, the proposed measure can be readily interpreted in terms of the economically-meaningful natural output concept. To see this, we note that the measure is underpinned by the familiar Beveridge-Nelson (1981) [BN] trend which we believe has a reasonable economic interpretation as the "steady-state" output level at the heart of many recent behavioural models: see, for example, Woodford's (2003) text and recent DSGE models concerned with measuring the output gap in Smets and Wouters (2003), Andres *et al.* (2007), Basistha and Nelson (2007), Edge *et al.* (2008), Justiniano and Primiceri (2008) and Coenen *et al.* (2009). In these models, a macroeconomic framework is derived based on optimising behaviour on the part of households and firms operating in an economy with differentiated goods and monopolistic competition. The models are subject to various types of shock, including productivity shocks that have a permanent effect on output, 'efficient' shocks that have a transitory effect on output which should be accommodated by welfare-maximising monetary authorities (associated with transitory technological disturbances, for example), and 'inefficient' shocks that also have a transitory effect on output but which the monetary authorities would wish to counteract (monetary shocks, for example). The steady-state output concept in these models describes the level achieved in the absence of stochastic variation and abstracting from the dynamic effects of real rigidities (from habits or investment adjustment costs, for example) and nominal rigidities (from Calvo pricing, sticky wages and so on) found in the model. In its turn, the BN trend can be interpreted as comprising the current observed value of output plus all forecastable future changes in the output series, abstracting from the dynamics of the path taken to obtain this level. It is the infinite-horizon forecast of the output level that will be achieved when all of the adjustments to the current and historical disturbances have been worked through, reflecting the complete effect of permanent shocks and entirely eliminating the effect of transitory shocks therefore. While the BN trend is a purely statistical concept, then, its forward-looking nature means it matches closely with the steady-state concept in many behavioural models.

The steady-state output concept is important in macroeconomics but is distinct from the natural level of output that is of interest to price-setters and policy-makers. This point is also made clearly in Woodford and others' structural models. The natural output concept accommodates the influence of the 'efficient' shocks which would cause output to deviate from steady-state even in the presence of flexible prices. These include shocks associated with stochastic variability in individuals' preferences, transitory technological disturbances, and temporarily high or low levels of government purchases relative to the spending plans, for example. Woodford shows that it is the gap between output and this natural level of output that influences real marginal costs, and hence price-setting decisions, and that this is also the gap concept that should enter into monetary policy decisions if they are to have a micro-founded welfare basis. In the context of these DSGE models, a measure of natural output requires the effects of the efficient transitory disturbances to be identified and separated from the inefficient transitory effects of changes in monetary policy regime, policy control errors and markup shocks and from the permanent innovations in output arising from technological progress.[1]

In this paper, we argue that the efficient transitory disturbances have, by their nature, a more time-limited impact than the inefficient changes in policy regime, policy control errors or markup shocks. This is not to say that the effects of the efficient disturbances disappear quickly since there are likely to be mechanisms that propagate the effects of shocks over time which are common to all shocks. But we argue that the *impact effects* of the efficient disturbances will be short-lived. This

[1]Many papers side-step the permenant-transitory decomposition by 'detrending' the data, often using a Hodrick-Prescott filter, before identifying different types of business cycle shock. This approach will generate its own difficulties in time series analysis, as emphasised in Harvey and Jaeger (1993) for example.

means we can use survey respondents' statements on expected future output levels to distinguish the time-limited efficient disturbances from the longer-lived monetary and markup disturbances. The argument is that agents are aware of the efficient transitory disturbances impacting on today's output and purge the series of these transitory elements in their responses to surveys asking what output levels they believe will be achieved in the future. Having identified the efficient disturbances, we can add their influence on output levels back to the steady-state level measured by the BN trend to obtain a natural output level and corresponding gap that properly reflects price pressures and that can be used in policy decisions.

The layout of the remainder of the paper is as follows. Section 2 describes the modelling framework. It defines the BN trend measures in a multivariate framework and considers these in the context of a vector-autoregressive (VAR) model which accommodates the time series properties of actual output and direct measures of output expectations. The relationships between the theoretical output concepts introduced in the DSGE model and the statistical concepts embodied in the VAR (including the presence of cointegrating relations, the BN decomposition and infinite horizon forecasts) are also described in Section 2. Section 3 describes the application of the methods to quarterly US data over the period 1970q1-2007q4. A VAR is estimated based on data on actual and expected output, inflation and interest rates and the corresponding natural output gap measure is calculated. The properties of the gap measure are discussed and compared to those of other popular gap measures. The performance of the gap measure in explaining US inflation is also explored in the context of various estimated versions of the New Keynesian Phillips curve in Section 4. Section 5 concludes.

2 Measuring BN Trends and the Steady-State and Natural Levels of Output

2.1 A VAR Model of Actual and Expected Output

It is straightforward to describe a statistical model of the joint determination of actual output and direct measures of expected future output using a VAR framework if we assume that actual output is first-difference stationary, and that expectational errors are stationary. The first of these assumptions is supported by considerable empirical evidence, and the latter assumption is consistent with a wide variety of hypotheses on the expectations formation process, including the Rational Expectations hypothesis (REH). Under these assumptions, and if direct measures are available for upto two periods ahead, for example, then we can write a statistical model for the series in a variety of different ways. For example, given the assumptions, actual and expected output growth are stationary and have the following fundamental Wold representation:[2]

$$\begin{bmatrix} y_t - y_{t-1} \\ {}_t y^e_{t+1} - y_t \\ {}_t y^e_{t+2} - {}_t y^e_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} + \mathbf{A}(\mathbf{L}) \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{2.1}$$

where (the logarithm of) actual output at time t is denoted by y_t and the direct measure of (the logarithm of) the expectation of output at time $t+h$, formed by agents on the basis of information available to them at time t, is denoted by ${}_t y^e_{t+h}$, for $h = 1, 2$. Here, α_h is mean expected output growth in $t+h$ for $h = 0, 1, 2$, $\mathbf{A}(\mathbf{L})=\sum_{j=0}^{\infty} \mathbf{A}_j(L)$, the $\{\mathbf{A}_j\}$ are 3×3 matrices of parameters and L is the lag-

[2]Expected growth in output at time $t+1$, $y^e_{t+1} - y_t$, is stationary as it can be decomposed into actual output growth $(y_{t+1} - y_t)$ and expectational error $(y^e_{t+1} - y_{t+1})$, both of which are stationary by assumption.

operator. Actual output growth at time t and the growth in output expected to occur in times $t+1$ and $t+2$, based on information at time t, are determined and published in surveys at time t and driven by disturbances ε_{0t}, ε_{1t} and ε_{2t} respectively. The ε_{0t} represents "news on output growth in time t becoming available at time t", while ε_{ht} is "news on output growth expected in time $t+h$ becoming available at time t" for $h = 1, 2$.[3]

As is shown in detail in the Appendix, the model in (2.1) can be written as a VAR in actual and expected output growth assuming that the lag polynomial $\mathbf{A}(\mathbf{L})$ is invertible or as a cointegrating VAR describing $\Delta \mathbf{z}_t$ where $\mathbf{z}_t = (y_t,\ {}_t y^e_{t+1},\ {}_t y^e_{t+2})'$:

$$\Delta \mathbf{z}_t = \mathbf{a} + \Pi \mathbf{z}_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \mathbf{u}_t \ , \tag{2.2}$$

and the error terms of $\mathbf{u}_t$ are interpreted as "news on the successive output levels" with $\mathbf{u}_t = (\varepsilon_{0t}, \eta_{1t}, \eta_{2t})' = (\varepsilon_{0t},\ (\varepsilon_{0t} + \varepsilon_{1t}),\ (\varepsilon_{0t} + \varepsilon_{1t} + \varepsilon_{2t}))'$. Both the VAR in actual and expected output growth and the cointegrating VAR are straightforward to estimate. The model can also be written, through recursive substitution of (2.2), as the moving average representation

$$\Delta \mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t. \tag{2.3}$$

The parameters in Π, Γ_j and $\mathbf{C}(L)$ are functions of the parameters of the model in (2.1) and the assumptions underlying (2.1) translate into restrictions on the parameters of the cointegrating VAR and the moving average representation. Specifically, Π

[3]It is worth emphasising that all the terms on the left-hand-side of (2.1), other than y_{t-1}, are dated at t and that, for example, ${}_t y^e_{t+1} - y_t$ is a "quasi difference" since ${}_t y^e_{t+1} - y_t \neq \Delta\ {}_t y^e_{t+1}$ $(= {}_t y^e_{t+1} - {}_{t-1} y^e_t)$

and $\mathbf{C}(1) = \sum_{i=0}^{\infty} \mathbf{C}_i$ take the forms

$$\Pi = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{C}(1) = \begin{bmatrix} k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \end{bmatrix} \tag{2.4}$$

for scalars k_{ij}, $(i = 1, 2, 3,\ j = 1, 2)$, k_4 , k_5 and k_6. All of these forms will provide an equivalent statistical characterisation of the data. They capture the potentially complicated dynamic interactions between the actual and expected output series but are restricted to reflect the underlying stationarity assumptions that ensure the series, while each growing according to a unit root process, are tied together over the long run.

2.2 Multivariate BN Trends

The BN trend of a variable is defined as the infinite horizon forecast obtained having abstracted from deterministic growth. For a $n \times 1$ vector process $\mathbf{z}_t$, the BN trends $\overline{\mathbf{z}}_t$ are defined by

$$\overline{\mathbf{z}}_t = \lim_{h \longrightarrow \infty} E[\mathbf{z}_{t+h} \mid I_t] - \mathbf{g}h \tag{2.5}$$

where $E[.|I_t]$ represents the expectation based on information available at time t, I_t, and $\mathbf{g}$, the element of deterministic growth, is a vector of constants. As Garratt *et al.* (2006) point out, any arbitrary partitioning of $\mathbf{z}_t$ into permanent and transitory components, $\mathbf{z}_t = \mathbf{z}_t^P + \mathbf{z}_t^T$ will have the property that the infinite horizon forecast of the transitory component is zero while the infinite horizon forecast of any permanent component converges on the BN trend; i.e.

$$\lim_{h \longrightarrow \infty} E[\mathbf{z}_{t+h}^T \mid I_t] = 0 \quad \text{and} \quad \lim_{h \longrightarrow \infty} E[\mathbf{z}_{t+h}^P \mid I_t] = \overline{\mathbf{z}}_t. \tag{2.6}$$

The various alternative measures of trends and cycles provided in the literature effectively represent alternative methods of characterising the dynamic path of the permanent component to the BN steady state therefore.[4]

In the multivariate moving average representation of (2.3), the BN trend can be expressed as

$$\Delta \overline{\mathbf{z}}_t = \mathbf{g} + \mathbf{C}(1)\mathbf{u}_t \tag{2.7}$$

so the trends are correlated random walks with the change in the trends reflecting the accumulated future effects of the system shock $\mathbf{u}_t$. Given the structure of the $\mathbf{C}(1)$ in (2.4) imposed by the initial stationarity assumptions on output growth and expectational errors, (2.7) shows the steady-state value of all three series in $\mathbf{z}_t$ is the same, denoted $\overline{y}_t$, and this is driven by the stochastic term $q_t = k_4\varepsilon_t + k_5\eta_{1t} + k_6\eta_{2t}$.

2.3 A Measure of the Natural Level of Output

We argued in the introduction that the BN trend is readily interpreted in terms of the steady-state output concept elaborated in Woodford's and others' structural models. But we noted also that the output concept of interest to price-setters and monetary authorities is the natural level incorporating the effects of any efficient transitory disturbances that cause output to fluctuate around the steady-state. To identify the effects of efficient disturbances, our approach assumes that the impact effects of these are known to be relatively short-lived compared to those of inefficient monetary or markup disturbances. The assumption is justified by the nature of the different types of shock. Efficient disturbances relate to the impatience of individuals to consume,

[4] Alternative approaches to characterising trends and cycles based on the BN decomposition include, for example, Blanchard and Quah (1989), King *et al.* (1991), Crowder *et al.* (1999), Gonzalo and Granger (1993), Garratt *et al.* (2006) and Dees *et al.* (2009).

or fluctuations in workers' attitudes to leisure, or changes in government expenditure patterns within spending plans, say. Inefficient disturbances include changes in monetary policy regime, as policy shifts from targeting a monetary aggregate to the exchange rate or inflation, say, as well as changes in a government's policy stance on inflation, or shifts in firms' or workers' market power. Our identifying assumption is that these shifts in policy or in market structure or in bargaining power have a more prolonged impact effect than the efficient disturbances. There will be mechanisms that cause the effects of all shocks to be propagated over time: time-to-build investment, costs of adjustment in hiring and firing, and so on. But we can take the common propagation mechanisms into account, and the survey data on expected future output can be used to identify the separate types of shock if the efficient disturbances have a shorter impact effect than the inefficient ones.[5]

In practice, survey data is often available for many quarters ahead (and indeed our empirical work below uses output expectations upto a year ahead). But for the purpose of exposition, we consider here the simple three variable system of (2.1), where we have direct measures of expected output at $t+2$. Here we can distinguish between three staged, time-limited transitory shocks. These are: a shock that has a direct effect on output on impact only, s_{0t}; a shock that effects output directly for one further period only, s_{1t}; and a shock that effects output for at least two periods and possibly more, s_{2t}. The effects of all three are transitory in the sense they have no effect on output in the long run although all three could effect output over a prolonged period through unspecified propagation mechanisms. News arriving at time t on output at t can be decomposed into the separate elements relating to the

[5] Thapar (2008) also makes use of direct measures of expectations and timing assumptions to identify economically-meaningful shocks. But Thapar's approach is different, assuming a Choleski ordering on the variables in his system and rationality to identify monetary policy shocks.

permanent supply shock and the three staged transitory shocks:

$$\varepsilon_{0t} = \beta_0 q_t + \lambda_{02} s_{2t} + \lambda_{01} s_{1t} + s_{0t}.$$

News arriving at t on expected output in $t+1$ reflects the effects of ε_{0t} as they are propagated over time plus the continued direct effects of q_t, s_{2t} and s_{1t}:

$$\eta_{1t} = \rho_{10}\varepsilon_{0t} + \beta_1 q_t + \lambda_{12} s_{2t} + s_{1t}.$$

News on expected output at $t+2$ reflects the further propagation of earlier news plus the remaining direct effects of q_t and s_{2t}:

$$\eta_{2t} = \rho_{20}\varepsilon_{0t} + \rho_{21}\eta_{1t} + \beta_2 q_t + s_{2t}.$$

Assuming the staged structural shocks are independent of each other, the β, λ, and ρ coefficients can be estimated through simple regressions using the residuals from the estimated VECM model explaining $\Delta \mathbf{z}_t$, (2.2), to observe η_{2t}, η_{1t}, ε_{0t} and $q_t = k_4\varepsilon_{0t} + k_5\eta_{1t} + k_6\eta_{2t}$. The s_{2t}, s_{1t} and s_{0t} are obtained as the residuals from these subsidiary regressions (estimated in the reverse order to the way they are presented above).

The relationships between the VECM residuals in $\mathbf{u}_t$ and the structural shocks $\mathbf{w}_t = (s_{0t}, s_{1t}, s_{2t})'$ are summarised by

$$\begin{bmatrix} 1-\beta_0 k_4 & -\beta_0 k_5 & -\beta_0 k_6 \\ -\rho_{10}-\beta_1 k_4 & 1-\beta_1 k_5 & -\beta_1 k_6 \\ -\rho_{20}-\beta_2 k_4 & -\rho_{21}-\beta_2 k_5 & 1-\beta_2 k_6 \end{bmatrix} \begin{bmatrix} \varepsilon_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix} = \begin{bmatrix} 1 & \lambda_{01} & \lambda_{02} \\ 0 & 1 & \lambda_{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{0t} \\ s_{1t} \\ s_{2t} \end{bmatrix};$$

that is

$$\mathbf{u}_t = \mathbf{Q}\mathbf{w}_t,$$

where $\mathbf{Q} = \begin{bmatrix} 1-\beta_0 k_4 & -\beta_0 k_5 & -\beta_0 k_6 \\ -\rho_{10}-\beta_1 k_4 & 1-\beta_1 k_5 & -\beta_1 k_6 \\ -\rho_{20}-\beta_2 k_4 & -\rho_{21}-\beta_2 k_5 & 1-\beta_2 k_6 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \lambda_{01} & \lambda_{02} \\ 0 & 1 & \lambda_{12} \\ 0 & 0 & 1 \end{bmatrix}.$

Hence, we can rewrite (2.3) as

$$\begin{aligned}\Delta \mathbf{z}_t &= \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t \\ &= \mathbf{g} + \mathbf{C}(L)\mathbf{Q}\mathbf{Q}^{-1}\mathbf{u}_t \\ &= \mathbf{g} + \widetilde{\mathbf{C}}(L)\mathbf{w}_t \end{aligned} \tag{2.8}$$

where $\widetilde{\mathbf{C}}(L) = \mathbf{C}(L)\mathbf{Q}$. This is an alternative MA representation for $\Delta \mathbf{z}_t$ in which the shocks have a structural interpretation.

Clearly, $\widetilde{\mathbf{C}}(1)\mathbf{w}_t = \mathbf{C}(1)\mathbf{Q}\mathbf{w}_t = \mathbf{C}(1)\mathbf{u}_t$ so that the output series are, of course, driven by the same single stochastic shock, q_t, in the long run and the steady-state measure provided by the BN trend remains unchanged. Our identifying assumptions assume that the natural level of output deviates from the steady-state because of the influence of the short-lived transitory disturbances only. Using (2.3) and (2.7), the deviation of output from its long-run level can be written as

$$\mathbf{z}_t - \overline{\mathbf{z}}_t = \mathbf{C}^*(L)\mathbf{u}_t = \widetilde{\mathbf{C}}^*(L)\mathbf{w}_t$$

where $\mathbf{C}^*(L) = \sum_{j=0}^{\infty} \mathbf{C}_j^* L^j$, $\mathbf{C}_j^* = -\sum_{i=j+1}^{\infty} \mathbf{C}_i$, and $\widetilde{\mathbf{C}}^*(L) = \mathbf{C}^*(L)\mathbf{Q}$. If we assume that the natural level should accommodate the influence of the first, very-short-lived staged disturbance, then the natural level of output can be defined by

$$\widetilde{y}_t = \overline{y}_t + \widetilde{C}_{11}^*(L)s_{0t} \tag{2.9}$$

where the $\widetilde{C}_{ij}^*$ are the i, j^{th} elements of $\widetilde{\mathbf{C}}^*$, while we have

$$\widetilde{y}_t = \overline{y}_t + \widetilde{C}_{11}^*(L)s_{0t} + \widetilde{C}_{12}^*(L)s_{1t}$$

if we treat the first two short-lived disturbances as efficient disturbances. The natural output gap, defined by the difference between the actual and natural levels of output, $\widetilde{x}_t = y_t - \widetilde{y}_t$, will not be affected by the permanent or the short-lived transitory disturbances. The steady-state gap measure, $\overline{x}_t = y_t - \overline{y}_t$, will incorporate the effect of the short-lived transitory disturbances.[6]

3 Estimating Steady-State and Natural Output Gap Measures for the US

This section provides estimates of the steady-state and natural output gap measures defined above based on US data over the period 1970q1-2007q4. In order to capture the macroeconomic dynamics as fully as possible, the model on which we base our estimates makes use of inflation and interest rate series as well as data on actual output and on expected future output at the one-, two-, three- and four-period ahead horizons. Hence we have $\mathbf{z}_t = (y_t, {}_ty^e_{t+1}, {}_ty^e_{t+2}, {}_ty^e_{t+3}, {}_ty^e_{t+4}, p_t, r_t)$ where y_t is (the logarithm of) US real GDP and ${}_ty^e_{t+h}$, $h = 1, .., 4$ are the corresponding direct measures of expected future output obtained form the Survey of Professional Forecasters. Prices p_t and the short term interest rate, r_t are measured by the GDP deflator and the 3-month Treasury Bill rate respectively. A full description of the data, their sources and the transformations used are provided in the Data Appendix.

[6] It is worth noting that the BN trend and the natural output level are both expressed in terms of currently observable data and are readily obtained on the basis of the estimated parameter values and residuals from (2.2). This is an important feature for any trend that is to be used in real-time decision-making; see papers by Orphanides (2001), Orphanides and van Norden (2002) and Garratt *et al.* (2008), for example. We abstract from these real-time issues in the empirical work of this paper, however, so that we can compare our proposed measures with others found in the literature based on final vintage data.

3.1 Model Specification and Estimation

The empirical counterpart of the VECM model in equation (2.2) was estimated for the seven variables in $\mathbf{z}_t$ with a lag order of two. The underlying assumptions that actual and expected outputs are difference-stationary but (pairwise) cointegrated with vector $(1, -1)'$ were tested and shown to hold. Prices were also found to be difference-stationary. The interest rate was found to be stationary in levels but this feature can be readily accommodated into the cointegrating VAR framework of (2.2), treating the single variable r_{t-1} as a fifth 'artificial' cointegrating combination of variables.[7]

The model underlying our US output gap measures is simple in form but is complex in the sense that each of the equations of the system explaining the seven terms in $\Delta\mathbf{z}_t$ includes two lags of all seven variables plus feedback from the five cointegrating vectors plus intercepts; a total of 140 parameters are estimated in total. The estimated model is able to capture very sophisticated dynamic interactions, then, and in the event we find large and statistically significant feedbacks captured both among the actual and expected future output measures and between output, prices and interest rates.[8] In order to illustrate the properties of the estimated system, Table 1 reports the estimated (loading) coefficients on the long-run terms along with the diagnostics for each of the seven equations in our VECM system. These estimated coefficients give

[7]Details of the tests on the order of integration for the variables and those for the choice of lag order in the VAR are available from the authors on request.

[8]The model at (2.1), and the equivalent forms in (5.11)-(5.15) of the Appendix, are quite general and have no implications for the expectations formation process. However, the assumption that expectations are formed rationally can be accommodated in the model through the imposition of restrictions that ensure $y_t =_{t-1} y_t^* + \varepsilon_{0t}$ and $_ty_{t+1} =_{t-1} y_{t+1} + \eta_{1t}$. In the context of the larger 7-variable system estimated here, the rational expectations hypothesis implies 57 restrictions on the VAR. The likelihood ratio test statistic takes a value of 290.53. This provides strong evidence to reject the rationality assumption when compared to a χ^2_{57} distribution.

a sense of the complexity of the underlying dynamics and the statistical significance of the four equilibrating terms in the individual equations. The diagnostic statistics show that the equations fit the data well and that there are no serious problems of serial correlation, non-normality and heteroskedasticity in the residuals.[9]

Figure 1 illustrates the dynamic properties of the system as they relate to actual and forecast values of the output series, plotting the forecast growth rates of actual output and the direct measure of four-period ahead expected output, y_{T+h} and ${}_{T}y_{T+4+h}$ for $h = 1, ..., 28$, at the end of the sample, $T = 2007q4$. The plot shows the characteristic smoothness of the four-period ahead expectation series relative to the actual series over the seven years prior to the end of the sample and then shows the gradual convergence of the forecasts of the actual and expected series to close to zero by the end of 2010. The fact that the series converge is, of course, a property of the model that assumes stationarity in the expectational errors. But the rate of convergence is a property of the estimated model dynamics and Figure 1 suggests that the "infinite-horizon" steady-state output level is obtained over a three- or four-year time frame.

3.2 Co-movements and Business Cycle Properties of Alternative Gap Measures

The measures of the steady-state output gap ($\overline{x}_t$) and natural output gap ($\widetilde{x}_t$) obtained using the model described above are plotted in Figure 2 and summary statistics provided in Table 2. The measures are based, respectively, on the trends defined in (2.7) and (2.9) updated to reflect the dimensions of $\mathbf{z}_t$ in the empirical application. Hence, the steady-state output trend is the multivariate BN trend obtained as the

[9]More complete details of the model, including the associated impulse responses describing the system dynamics, are available from the authors on request.

infinite-horizon forecast of output from the seven equation cointegrating VAR model. In deriving the natural output trend in this larger system, there are two permanent shocks (one to output, q_t^y, and one to prices, q_t^p) and there are five 'staged' transitory shocks to output, s_{it} $i = 0, .., 4$. The news arriving at time t about output at t, found from the residual from the output equation in the VAR, is decomposed to show the influence of the permanent shocks, the five staged transitory shocks to output and the shock to interest rates:

$$\varepsilon_{0t} = \beta_0^y q_t^y + \beta_0^p q_t^p + \gamma_0 \varepsilon_{rt} + \sum_{j=1}^{4} \lambda_{0j} s_{jt} + s_{0t},$$

where ε_{rt} is the residual from the interest rate equation in the VAR. News arriving at t on expected output in $t+i$, for $i = 1, .., 4$, is decomposed as:

$$\eta_{it} = \rho_{i0}\varepsilon_{0t} + \sum_{j=1}^{i-1} \rho_{ij}\eta_{jt} + \beta_4^y q_t^y + \beta_4^p q_t^p + \gamma_4 \varepsilon_{rt} + \sum_{j=1}^{4-i} \lambda_{i.i+j} s_{i+j,t} + s_{it}, \qquad i = 1, .., 4,$$

following the identifying structure in the earlier illustrative example. The natural gap measure shown in Figure 2 and described in the subsequent discussion incorporates the effects of all 5 of the short-lived disturbances into the trend; i.e. we assume that the shocks having an impact effect of upto one year are included in the trend and excluded from the natural gap. Very similar results are obtained for the natural gap measures that include at least s_{0t}, s_{1t} and.s_{2t}.[10] The plot shows a strong similarity between the natural and steady-state gap measures, with a contemporaneous correlation of 0.61. However, there are periods when the two diverge by upto 3% and the two gap measures actually have different signs in about a third of the sample observations. The BN trend clearly represents a key element of the natural output level therefore but the effects of the short-lived transitory disturbances are not trivial.

[10]The correlations between the natural gap measure based on 5 short-lived disturbances and the measures obtained including 4, 3, 2 or 1 short-lived disturbances are, respectively, 0.99, 0.95, 0.88 and 0.83.

Table 2 and Figures 3 and 4 compare the natural and steady state gap measures with four other regularly-used gap measures: a gap based on marginal costs, x_t^{MC}; the measure produced by the Congressional Budget Office (CBO), x_t^{CBO}; the gap obtained using a simple linear trend, x_t^{LT}; and a gap obtained applying the HP smoother to the output series, x_t^{HP}. The marginal cost measure is advocated by Gali and Gertler (1999) [GG], Gali, Gertler and Lopez-Salido (2001, 2005) [GGL] and others and, as explained in the Data Appendix, is given by the (logarithm of demeaned) average unit labour costs.[11] The CBO series is the Office's 2007q4 estimate of the maximum level of sustainable output achievable in each period based around a neoclassical production function and calculated levels of factor inputs (see CBO, 2001, for detail of the estimation methods employed). The gap based on the linear and HP trends are standard detrended measures found in the literature (the latter calculated using a smoothing parameter of 1600).

The summary statistics of Table 2 show that, in terms of the standard deviation and minimum and maximum values of the series, the size of the natural output gap is broadly in line with the alternatives found in the literature, with output lying in the range $\pm 3.8\%$ of trend and with mean about zero and standard deviation of 1.4%. The plots show relatively persistent dynamics in the natural gap and the steady-state gap, with first-order autocorrelation coefficients of 0.71 and 0.87, broadly in line with the corresponding autocorrelations for the other gaps in Table 2. This is an interesting finding that contrasts with gap estimates based on BN trends obtained in univariate

[11] GG note that, under certain conditions on the form of nominal rigidities and the nature of capital accumulation, there is a proportional relationship between the natural output gap measure derived in a micro-founded DSGE model and the deviation of marginal cost from its steady-state. Although this measure is not directly observable either, GG use theory-based restrictions to propose the demeaned unit labour cost series as an alternative means of measuring the gap and show that this performs well in estimates of the New Keynesian Phllips curve.

exercises. These typically find that much of the variation in output is variation in trend and that the gap is small and noisy. For example, the gap based on the BN trend obtained from a univariate AR(1) model of Δy_t for our sample has a standard deviation of 0.36% and a first order autocorrelation of 0.31 (See Morley *et al.* (2003) for further discussion). This feature of the gap measures in Figure 2 is retained even in experiments where the inflation and interest rate variables are dropped from the analysis and the model concentrates on the various output measures only; it is the interaction of the actual and expected output series in the estimated model that underlies the finding, not the relationship with the other variables.

The table shows there is a strong consensus in the size and timing of the cycles based on the natural output trend and on the marginal cost measure. The correlation between these is 33% and there is agreement on the sign of the gap on 63% of occasions, both statistically significant. There is also considerable similarity among the three cycles based on the statistical 'smoothing' algorithms underlying the linear trend, CBO and HP definitions of trend. The correlations between these three are typically around 0.8, the agreement on the sign of these gap is in the region of 75%, and all the values are statistically significant. The correlations and levels of agreement between these three and the $\widetilde{x}_t$,x_t^{MC} pair are much lower and statistically insignificant so that the $\widetilde{x}_t$,x_t^{MC} pair appear qualitatively different to the group of three smoothed series. Interestingly, the steady-state gap $\overline{x}_t$ appears to share features of both the $\widetilde{x}_t$,x_t^{MC} pair and the group of smoothed series. The differences between the sets of gap measures are illustrated in Figures 3a,b which plot, in turn, the $\widetilde{x}_t$ and x_t^{MC} series against x_t^{LT} (chosen as a representative of the three smoothed cycles) and the smoothed series together.

These similarities are perhaps even more striking in the dynamic cross-correlations provided in Figure 4 which show a statistically-significant positive correlation between

$\widetilde{x}_t$ and x_{t+k}^{MC} at all horizons $k = -8, .., 8$ and peaking close to $k = 0$; there is clearly a strong synchronisation of the cycles captured by these measures. This contrasts with the cross-correlations between x_{t+k}^{MC} and $\overline{x}_t$ and, even more starkly, x_{t+k}^{MC} and x_t^{LT}. In the case of the steady-state gap measure, it is clear that this measure "over-reacts" to a shock to output, compared to the natural gap measure, since it does not take into account the part associated with short-lived transitory effects. While both $\overline{x}_t$ and $\widetilde{x}_t$ return to zero over time, the over-reaction means that the steady-state gap appears to lead the natural gap and the correlogram peaks at $t + 4$. A similar argument explains the apparent lead of the gaps based on the smoothed series (x_t^{LT}, x_t^{CBO}, x_t^{HP}) where the excessively smooth trend translates to an apparent over-reaction in the gap following a shock to output. Here though, the smoothing across observations before and after the shock also means the effects of a shock to output are anticipated in the measured trend so that there is a negative correlation between the x_{t+k}^{MC} and the x_t^{LT}, x_t^{CBO}, x_t^{HP} for $k < 0$.

In summary, then, the proposed natural gap measure has reasonable statistical properties comparable to those of many gap measure found in the literature. The natural output gap's time series properties are quite distinct from those of the other statistically-based series but appears closely aligned with the marginal cost gap measure. This provides some support for the view that the natural output gap measure has the structural interpretation we suggested in the previous section as well as having a clear statistical basis.

4 Using the Natural Output Gap in a New Keynesian Phillips Curve

A further means of judging the properties of the suggested natural output gap measure is to investigate its usefulness in the analysis of inflation, π_t. Figure 4 shows the dynamic cross-correlations between the gap measure and inflation over the sample period, along with corresponding plots for the marginal cost and linear trend gaps. This shows that the natural output gap measure is highly positively correlated with inflation with correlation coefficients in excess of 0.3 found between $\widetilde{x}_t$ and π_{t+s}, $s = -7, .., 7$ and peaking at $k = 0$. A similar pattern is found for the marginal cost gap. There is a very strong synchronisation between these gap measures and inflation, therefore, and they will serve as very good indicators of contemporaneous inflationary pressures for use in policy-making.

In contrast, the smoothed linear trend gap x_t^{LT} is positively correlated only with future inflation at $t+s$, $s = 2, 3, 4, ..$, and negatively correlated with lagged inflation. As GGL point out, the patterns found for the gap measures based on the linear trend are inconsistent with the forward-looking behaviour underlying the New Keynesian Phillips curve relationships in the DSGE literature. These accommodate the idea that nominal rigidities arise because wages and prices are reset only periodically and, recognising this, firms and households make current decisions based on what is likely to happen between now and the next opportunity to change wages and prices. The pattern in the linear trend gap has the gap leading inflation and this is inconsistent with this type of forward-looking behaviour.

The point is illustrated again in the results of Table 3 which reports on the estimation of some "hybrid" New Keynesian Phillips Curves of the form considered in GGL:

$$\pi_t = \lambda x_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} + \epsilon_t, \tag{4.10}$$

estimated using four alternative gap measures, $x_t = x_t^{LT}$, x_t^{MC}, $\overline{x}_t$ or $\widetilde{x}_t$, and subject to the restrictions

$$\lambda = (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}, \qquad \gamma_f = \beta\theta\phi^{-1}, \qquad \gamma_b = \omega\theta^{-1}, \qquad \text{and } \phi = \theta + \omega[1-\theta(1-\beta)],$$

where, in the underlying theoretical formulation based on Calvo pricing, θ represents the degree of price stickiness (proportion of firms who do not re-set prices in each period), ω represents a measure of backwardness (the proportion of firms using a backward-looking rule of thumb in price-setting) and β is a discount factor. This hybrid formulation has the advantage of being able to capture both the forward-looking behaviour of the type suggested in the DSGE literature and any inertia-based backward-looking behaviour.

The measure of inflation used in the empirical work is the change in (the logarithm of) the GDP deflator and the period of estimation is 1970q3-2004q4. The table reports the outcome of four different specifications estimated using each of the four alternative gap measures. The first 'baseline' specification estimates (4.10) using a GMM estimator using as instruments four lags of inflation, two lags of detrended output, marginal costs and wage inflation, matching the instrument set used in GGL. The alternative 'closed form' specification follows the suggestion in Rudd and Whelan (2005) and writes inflation in terms of its discounted sum of current and expected future values of the gap, but still takes into account the cross-parameter restrictions implied by (4.10). We solve forward for upto twelve quarters in this case (and it is for this reason that the sample period ends in 2004q4 in this exercise). In both the baseline and closed form versions, we also estimate the relationship with and without imposing the restriction $\gamma_b + \gamma_f = 1$.

The four columns of the table show first how poorly the smoothed linear trend gap performs in the Phillips curve relations. The coefficient on the gap term is not statistically significant in any of the equations presented and is wrongly-signed in three out of the four. The marginal cost, natural and steady-state gap measures are much more successful in explaining inflation having positive coefficients on the gap in all but two cases (namely the unrestricted baseline for $\widetilde{y}_t$ and $\overline{y}_t$) and these are statistically significant for both the marginal cost and natural gaps in the restricted baseline models and the unrestricted closed form models. All three of these gaps provide similar conclusions on the balance between backward- and forward-looking influences on inflation too, being broadly in the ratio 4:6 across the various specifications using either gap measure. In short, then, the natural gap measure has good explanatory power in the hybrid Phillips curve relationships explaining inflation, providing estimates broadly in line with those of GG and GGL and those obtained here using the marginal cost-based measure.

5 Conclusions

The natural output gap measure suggested in this paper has a straightforward statistical basis. It is simple to calculate as it can be obtained using (actual and expected) output data alone: the steady-state stochastic trend obtained applying the BN decomposition to the output series is supplemented by the effects of the disturbances that survey respondents say will be short-lived and transitory. However, the trend that is obtained is readily interpreted in terms of the 'natural' level of output described in many behavioural models. The steady-state output level in DSGE models is readily conceived in terms of the BN trend, and the short-lived transitory disturbances can be identified with 'efficient' shocks given the different likely durations of the efficient and inefficient disturbances in these models. The analysis of the US data showed that the gap based on the BN trend derived from a multivariate system including the actual and expected series has sensible properties (compared to that using actual data only). But the gap is quite distinct from the natural gap obtained by eliminating the effects of the short-lived shocks from the steady-state gap following our suggested approach. The natural gap measure is highly synchronised with measured marginal cost movements, supporting the economic interpretation of our measure as being the 'natural gap'. It is also highly synchronised with inflation and performs well in New

Keynesian Phillips curves. The natural gap measure therefore provides an indicator of price pressure which has a straightforward economic interpretation , which can be estimated easily and which can be readily applied in policy.

Appendix: Alternative Statistical Representations for Actual and Expected Output

The general model in (2.1) gives the Wold representation for actual and expected growth driven by $\mathbf{v}_t = (\varepsilon_{0t}, \varepsilon_{1t}, \varepsilon_{2t})'$, a vector of mean zero, stationary innovations, with non-singular covariance matrix $\Psi = (\psi_{jk})$, $j, k = 1, 2, 3$. This model can be expressed in a variety of alternative ways. For example, assume $\mathbf{A}^{-1}(\mathbf{L})$ can be approximated by the lag polynomial $\mathbf{A}^{-1}(\mathbf{L}) = \mathbf{B}_0 + \mathbf{B}_1\mathbf{L} + .. + \mathbf{B}_{p-1}\mathbf{L}^{p-1}$, where $\mathbf{B}_0 = \mathbf{I}_2$. In this case, (2.1) can be rewritten to obtain the AR representation

$$\begin{bmatrix} y_t - y_{t-1} \\ {}_t y^e_{t+1} - y_t \\ {}_t y^e_{t+2} - {}_t y^e_{t+1} \end{bmatrix} = \mathbf{B} - \mathbf{B}_1 \begin{bmatrix} y_{t-1} - y_{t-2} \\ {}_{t-1} y^e_{t} - y_{t-1} \\ {}_{t-1} y^e_{t+1} - {}_{t-1} y^e_{t} \end{bmatrix} - ...$$

$$...-\mathbf{B}_{p-1} \begin{bmatrix} y_{t-p+1} - y_{t-p} \\ {}_{t-p+1} y^e_{t-p+2} - y_{t-p+1} \\ {}_{t-p+1} y^e_{t-p+3} - {}_{t-p+1} y^e_{t-p+2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (5.11)$$

where $\mathbf{B} = \mathbf{A}^{-1}(\mathbf{1})\boldsymbol{\alpha}$ and hence

$$\begin{bmatrix} y_t \\ {}_t y^e_{t+1} \\ {}_t y^e_{t+2} \end{bmatrix} = \mathbf{a} + \Phi_1 \begin{bmatrix} y_{t-1} \\ {}_{t-1} y^e_{t} \\ {}_{t-1} y^e_{t+1} \end{bmatrix} + \Phi_2 \begin{bmatrix} y_{t-2} \\ {}_{t-2} y^e_{t-1} \\ {}_{t-2} y^e_{t} \end{bmatrix} + ... \quad (5.12)$$

$$... + \Phi_p \begin{bmatrix} y_{t-p} \\ {}_{t-p} y^e_{t-p+1} \\ {}_{t-p} y^e_{t-p+2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix},$$

where $\mathbf{a} = \mathbf{M}_0^{-1}\mathbf{B}$, $\Phi_j = \mathbf{M}_0^{-1}\mathbf{M}_j$, $j = 1, \ldots\ldots, p$, and

$$\mathbf{M}_0 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \ \mathbf{M}_p = \mathbf{B}_{p-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\text{and } \mathbf{M}_j = \mathbf{B}_{j-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \mathbf{B}_j \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

for $j = 1, \ldots, p-1$. The error terms $\mathbf{u}_t = (\varepsilon_{0t}, \eta_{1t}, \eta_{2t})'$ are defined by

$$\begin{bmatrix} \varepsilon_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix} = \mathbf{M}_0^{-1} \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} = \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{0t} + \varepsilon_{1t} \\ \varepsilon_{0t} + \varepsilon_{1t} + \varepsilon_{2t} \end{bmatrix},$$

and the covariance matrix of the $\mathbf{u}_t$ is denoted $\Omega = (\sigma_{jk})$, $j, k = 1, 2, 3$. Note that ε_{0t} has the interpretation of "news on output level in time t becoming available at time t", which is equivalent to news on output growth given that y_{t-1} is known, while η_{ht} is the "news on the level of output expected in time $t+h$ becoming available at time t". The latter incorporates the news on output levels at time t and the news on growth expected to be experienced over the coming period ($\eta_{ht} = \varepsilon_{0t} + \sum_{j=1}^{h} \varepsilon_{jt}$).

Expression (5.12) can be written

$$\mathbf{z}_t = \mathbf{g} + \Phi_1 \mathbf{z}_{t-1} + \Phi_2 \mathbf{z}_{t-2} + \ldots + \Phi_p \mathbf{z}_{t-p} + \mathbf{u}_t \tag{5.13}$$

where $\mathbf{z}_t = (y_t,\ {}_ty^e_{t+1},\ {}_ty^e_{t+2})'$ and this can also provide the VECM representation

$$\Delta \mathbf{z}_t = \mathbf{a} + \Pi \mathbf{z}_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \mathbf{u}_t, \tag{5.14}$$

where $\Phi_1 = \mathbf{I}_2 + \Pi + \Gamma_1$, $\Phi_i = \Gamma_i - \Gamma_{i-1}$, $i = 2, 3, .., p-1$, and $\Phi_p = -\Gamma_{p-1}$. Given the form of the Φ_i described in (5.12), it is easily shown that Π takes the form

$$\Pi = \begin{bmatrix} k_{11}+k_{12} & -k_{11} & -k_{12} \\ k_{21}+k_{22} & -k_{21} & -k_{22} \\ k_{31}+k_{32} & -k_{31} & -k_{32} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix},$$

where k_{ij}, $i = 1, 2, 3$ $j = 1, 2$ are scalars dependent on the elements of the $\mathbf{B}_j$, $j = 0, 1, .., p-1$. The form of the cointegrating vector captures the fact that actual and expected output cannot diverge indefinitely by assumption and is incorporated through the inclusion of the disequilibrium terms $y_{t-1} - {}_{t-1}y_t^e$ and $y_{t-1} - {}_{t-1}y_{t+1}^e$ in each of the system's equations in (5.14).

Alternatively, through recursive substitution of (5.13), we can obtain the moving-average form given by

$$\Delta \mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t, \tag{5.15}$$

where $\mathbf{C}(L) = \sum_{j=0}^{\infty} \mathbf{C}_j L^j$, $\mathbf{C}_0 = \mathbf{I}$, $\mathbf{C}_1 = \Phi_1 - \mathbf{I}_n$, and $\mathbf{C}_i = \sum_{j=1}^{p} \Phi_j \mathbf{C}_{i-j}$. The presence of the cointegrating relationships between the y_t , ${}_{t-1}y_t^e$ and ${}_{t-1}y_{t+1}^e$ imposes restrictions on the parameters of $\mathbf{C}(L)$; namely, $\boldsymbol{\beta}'\mathbf{C}(1)=0$, as shown in Engle and Granger (1987). Given the form of $\boldsymbol{\beta}'$ in (5.14), $\mathbf{C}(1)$ takes the form

$$\mathbf{C}(1) = \begin{bmatrix} k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \end{bmatrix} \tag{5.16}$$

for scalars k_4 , k_5 and k_6. Hence, the BN trend defined by (2.7) shows the steady-state value of all three series in $\mathbf{z}_t$ is the same and driven by the stochastic trend $k_4\varepsilon_{0t} + k_5\eta_{1t} + k_6\eta_{2t}$.

Data Appendix

The sources and transformations for the data are as follows:

y_t : the natural logarithm of US real GDP. Source: St Louis Federal Reserve Economic Database [FRED].

p_t : the natural logarithm of the US GDP Price Deflator. Source: FRED.

${}_t y^e_{t+h}$, $h = 1, 2, 3$ and 4 : the natural logarithm of expected h quarter ahead US real GDP reported at time t. The series used in the estimation is defined as ${}_t y^e_{t+h} = g^h_t + y_t$ where g^h_t is expected output growth reported in the SPF at t, based on expected output in $t + h$ relative to the real-time "nowcast" of current output, ${}_t y^{SPF}_{t+h} - {}_t y^{SPF}_t$. Source: Survey of Professional Forecasters.

r_t : the annualised US three month treasury bill rate, averaged over the three months in each quarter, expressed as a quarterly rate: $r_t = 1/4 \times \ln[1 + (R_t/100)]$, where R_t is the annualised rate. Source: FRED.

π_t : US GDP price deflator inflation, defined as: $400 * (p_t/p_{t-1})$

mc_t : marginal cost or real (demeaned) unit labour cost, defined as $mc_t = ulc_t + 4.596299$, where 4.596299 is the average unit labor cost (ulc_t) for the sample period 1970q3-2004q4 and $ulc_t = \ln(comnfb_t/ophnfb_t) - \ln(pnfb_t)$ where

$comnfb_t$: non-farm business sector compensation per hour. Source: US Department of Labour, Bureau of Labour Statistics

$ophnfb_t$: non-farm business sector output per hour of all persons. Source: FRED

$pnfb_t$: implicit prices deflator in the non-farm business sector. Source: FRED.

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Figure 1: Growth Forecasts of y_{T+h} and ${}_{T}y_{T+4+h}$ for $h = 1, ..., 28$

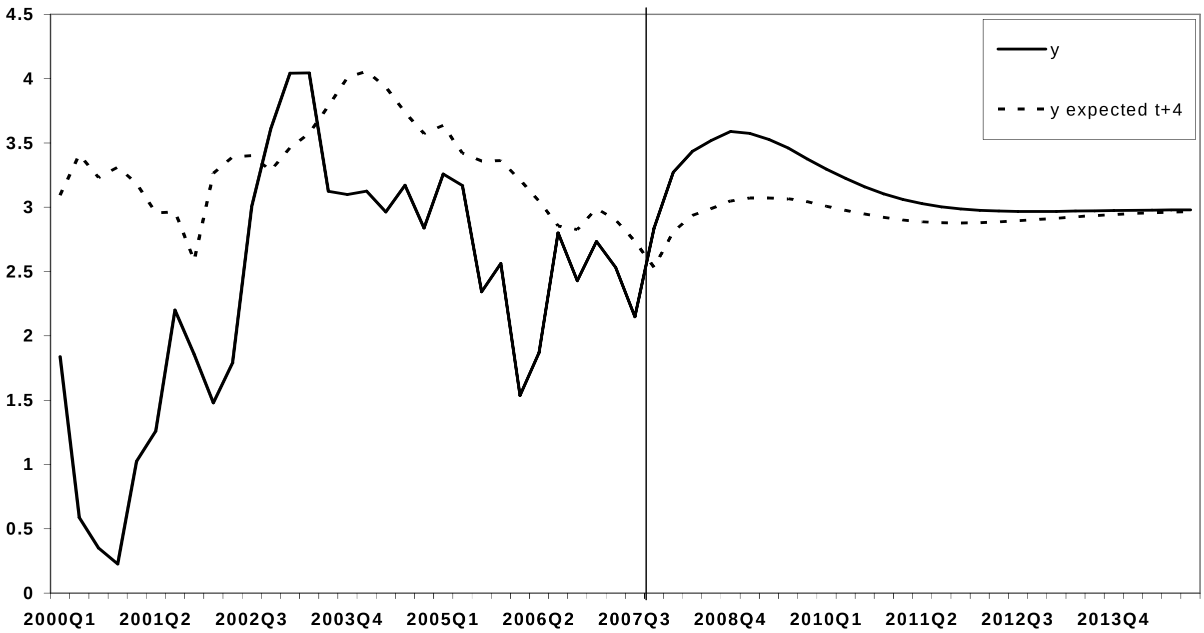

Figure 2: Natural and Steady State Output Gaps

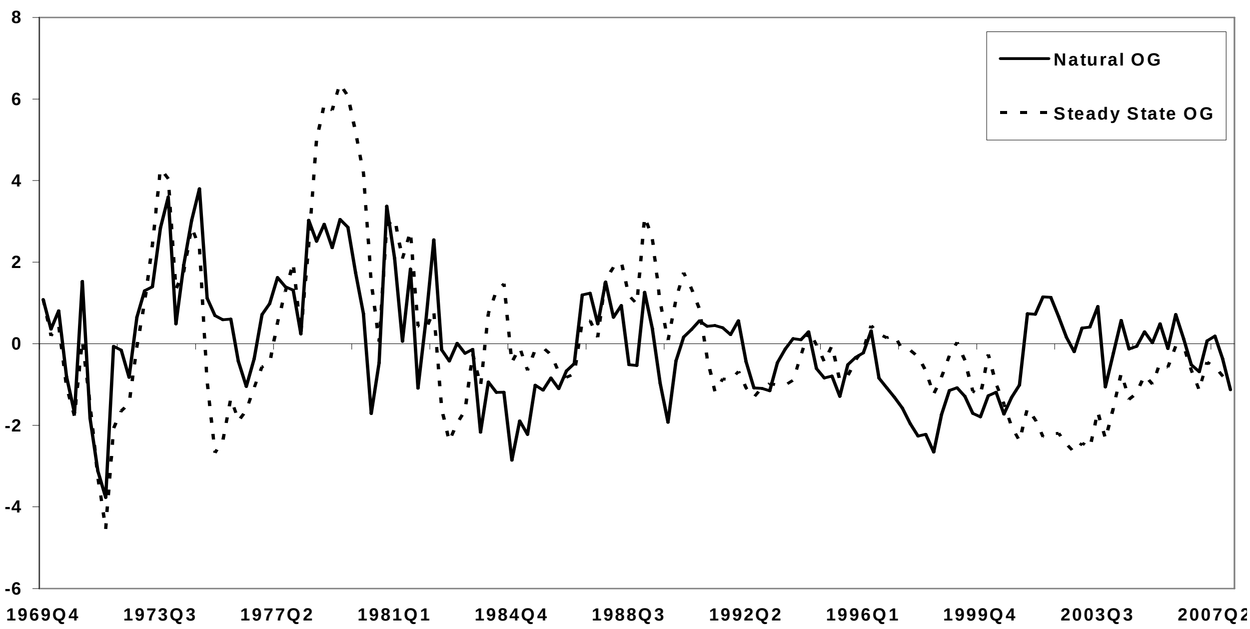

Figure 3a Natural OG Compared to Marginal Cost and Linear Trend Measures

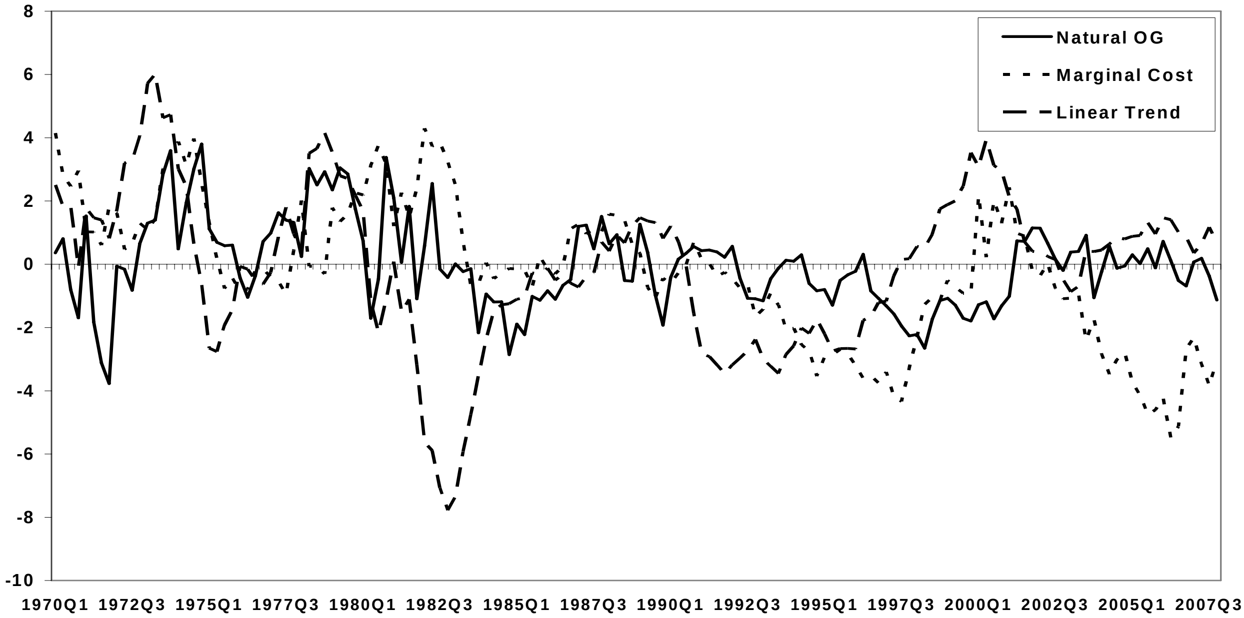

Figure 3b: Linear Trend, CBO and Hodrick-Prescott Output Gaps

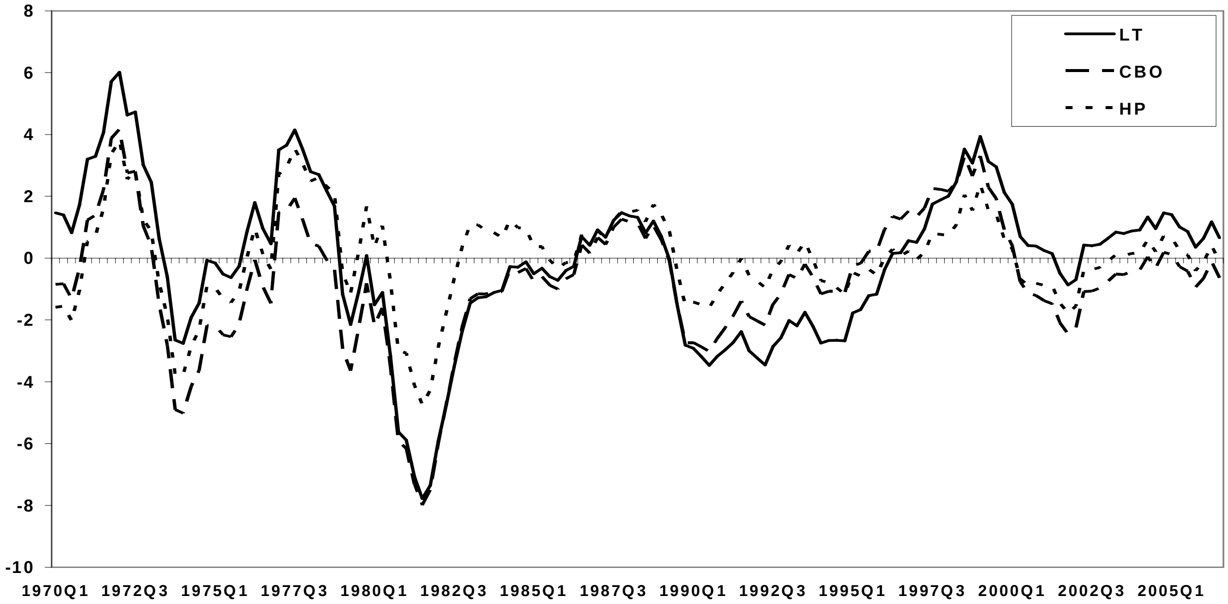

Figure 4: Dynamic Cross-Correlations.

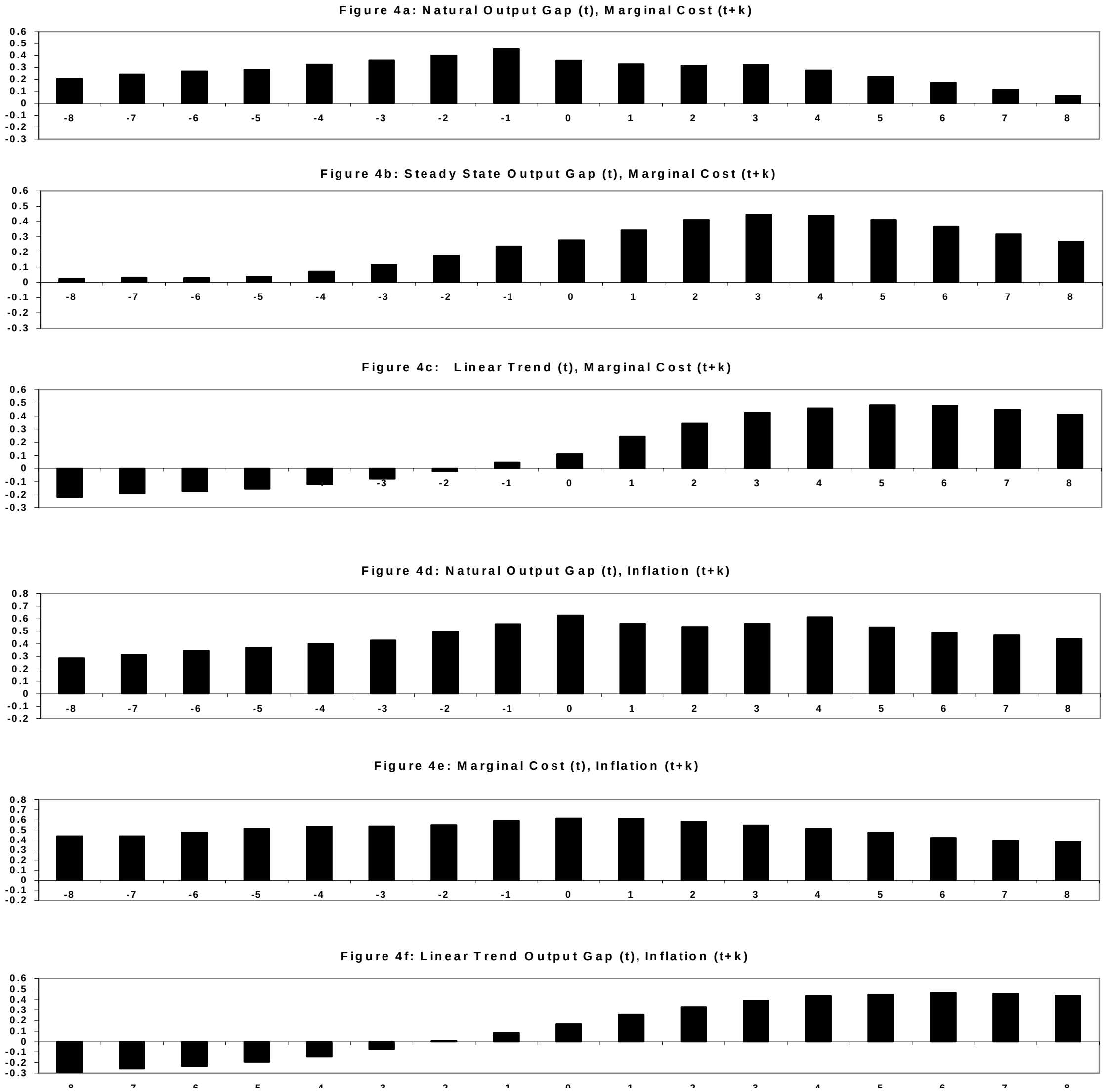

Table 1: VECM Long Run Terms and Diagnostics

Equation	Δy_t	$\Delta_t y^e_{t+1}$	$\Delta_t y^e_{t+2}$	$\Delta_t y^e_{t+3}$	$\Delta_t y^e_{t+4}$	Δp_t	Δr_t
$\widehat{\xi}_{1,t-1}$	-1.38 (1.24)	3.03* (1.35)	2.59 (1.41)	2.80* (1.44)	2.37 (1.45)	-1.02* (0.52)	0.24 (0.28)
$\widehat{\xi}_{2,t-1}$	-6.25* (1.77)	-7.16* (1.94)	-6.28* (2.02)	-7.27* (2.06)	-6.40* (2.08)	0.94 (0.74)	-0.18 (0.40)
$\widehat{\xi}_{3,t-1}$	5.26* (1.86)	5.41* (2.03)	5.39* (2.11)	6.80* (2.15)	5.68* (2.18)	0.40 (0.77)	-0.29 (0.41)
$\widehat{\xi}_{4,t-1}$	-1.22 (0.83)	-1.30 (0.91)	-1.65 (0.94)	-2.25* (0.96)	-1.67* (0.97)	-0.34 (0.35)	0.31* (0.19)
$\overline{R}^2$	.293	.274	.227	.201	.194	.782	.945
$\hat{\sigma}$	.007	.008	.008	.008	.008	.003	.002
$\chi^2_{SC}[4]$	{.749}	{.658}	{.552}	{.413}	{.288}	{.000}	{.001}
$\chi^2_H[23]$	{.002}	{.003}	{.007}	{.009}	{.002}	{.033}	{.000}
JB_N	{.179}	{.427}	{.249}	{.254}	{.363}	{.054}	{.000}

Notes: The five long-run terms are given by:

$$\begin{aligned}
\widehat{\xi}_{1,t} &= y_t - {}_t y^e_{t+1} + 0.0066,\\
\widehat{\xi}_{2,t} &= y_t - {}_t y^e_{t+2} + 0.0137,\\
\widehat{\xi}_{3,t} &= y_t - {}_t y^e_{t+3} + 0.0214,\\
\widehat{\xi}_{4,t} &= y_t - {}_t y^e_{t+4} + 0.0291.
\end{aligned}$$

Standard errors are given in parenthesis. "*" indicates significance at the 5% level and the remaining diagnostics are p-values denoted {.}. $\overline{R}^2$ is the squared multiple correlation coefficient, $\widehat{\sigma}$ the standard error of the regression, χ^2_{LM} is a chi-squared test statistic (with 4 d.f.) for serial correlation (SC), χ^2_H the Breusch-Pagan chi-squared test statistic for heteroscedasticity (H) and JB_N Jarque-Bera test for normality (N).

Table 2: Output Gap Measures: 1971q2 – 2007q4

	x_t^{MC}	$\widetilde{x}_t$	$\overline{x}_t$	x_t^{LT}	x_t^{CBO}	x_t^{HP}
Mean	-0.34	-0.01	0.00	-0.12	-0.76	0.01
SD	2.16	1.40	1.93	2.48	2.14	1.53
Min	-5.48	-3.77	-4.54	-7.78	-7.99	-4.75
Max	4.30	3.81	6.37	6.01	4.17	3.80
AR1	0.93	0.71	0.87	0.95	0.93	0.87
x_t^{MC}	1	0.33**	0.30**	0.02	-0.19	-0.03
$\widetilde{x}_t$	*62.5%***	1	0.61**	0.18	-0.04	0.16
$\overline{x}_t$	*70.0%***	*68.0%***	1	0.34**	0.32**	0.62**
x_t^{LT}	*55.8%*	*56.5%*	*54.4%*	1	0.88**	0.79**
x_t^{CBO}	*59.2%*	*50.3%*	*66.0%***	*76.2%***	1	0.86**
x_t^{HP}	*51.7%*	*48.3%*	*62.6%***	*74.1%***	*76.2%***	1

Notes: The output gaps measures are: the natural $(\widetilde{x}_t)$, steady state $(\overline{x}_t)$, marginal cost (x_t^{MC}), linear trend (x_t^{LT}), Congressional Budget Office (x_t^{CBO}), and Hoderick-Prescott (x_t^{HP}) output gaps. Summary statistics in the upper panel refer to the mean, standard deviation, minimum and maximum values, and the first-order serial correlation coefficient respectively. Figures in the lower panel refer to correlation coefficients across the gap measures and, in italics, the percentage of the sample for which there is agreement that the output gap is positive or negative. "**" indicates significance at the 1% level

Table 3: Estimates of the New Keynesian Phillips Curve

	Linear Trend $[x_t^{LT}]$			Marginal Cost $[x_t^{MC}]$		
	γ_b	γ_f	λ	γ_b	γ_f	λ
Baseline GMM	0.216 (.091)	0.467 (.081)	−0.022 (0.029)	0.330 (.132)	0.591	0.075 (.096)
$\gamma_b + \gamma_f = 1$	0.221 (.120)	0.779	0.039 (.039)	0.351 (.102)	0.649	0.094 (.044)
Closed form GMM	0.267 (.088)	0.742 (.092)	−0.019 (.013)	0.418 (.028)	0.563 (.029)	0.029 (.011)
$\gamma_b + \gamma_f = 1$	0.345 (.056)	0.655	−0.010 (.009)	0.412 (.028)	0.588	0.009 (.005)

	Steady State $[\overline{x}_t]$			Natural $[\widetilde{x}_t]$		
	γ_b	γ_f	λ	γ_b	γ_f	λ
Baseline GMM	.252 (.078)	0.456 (.098)	−0.008 (.046)	.268 (.085)	0.435 (.147)	−0.008 (.046)
$\gamma_b + \gamma_f = 1$	0.300 (.094)	0.700	0.071 (.038)	0.344 (.091)	0.656	0.088 (.037)
Closed form GMM	0.452 (.028)	0.531 (.030)	0.036 (.023)	0.420 (.027)	0.561 (.030)	0.071 (.044)
$\gamma_b + \gamma_f = 1$	0.437 (.027)	0.563	0.002 (.009)	0.379 (.027)	0.621	0.016 (.011)

Notes: The results relate to estimates of the New Keynesian Phillips Curve $\pi_t = \lambda x_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} + \epsilon_t$, estimated using four alternative gap measures and subject to the restrictions that $\lambda = (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}$, $\gamma_f = \beta\theta\phi^{-1}$, $\gamma_b = \omega\theta^{-1}$, and $\phi = \theta + \omega[1-\theta(1-\beta)]$; see text for details.