Zehnwirth Ben - 18

ISBN 0 7325 0587 9

THE UNIVERSITY OF MELBOURNE

OUTSTANDING CLAIM LIABILITIES: ARE THEY PREDICTABLE?

by

Ben Zehnwirth
The University of Melbourne

RESEARCH PAPER NUMBER 18

February 1995

Centre for Actuarial Studies
Department of Economics
The University of Melbourne
Parkville, Victoria, 3052
Australia.

OUTSTANDING CLAIMS LIABILITIES: ARE THEY PREDICTABLE?

TABLE OF CONTENTS

OUTSTANDING CLAIMS LIABILITIES: ARE THEY PREDICTABLE?

Inaugural Professional Lecture
The University of Melbourne

Ben Zehnwirth

GIO Professorial Fellow in Insurance

ACCEPTANCE AND ACKNOWLEDGMENTS

I am proud and honoured to have been appointed to the position of GIO Professorial Fellow in (General) Insurance at The University of Melbourne. Since, General Insurance is a relatively new practice area for actuaries, the creation of the Professorial Fellowship represents a milestone in the advancement of actuarial education, research and practice, both nationally and internationally. In recognition of the obligations incumbent upon me by the appointment, I regard it a duty, privilege and challenge to enhance The University's and the Centre for Actuarial Studies' already eminent reputations by advancing (General) Insurance actuarial education, research and practice with great vigour.

During my actuarial career of the last fifteen years, I have had the good fortune of enjoying the acquaintance and friendship of many remarkable actuaries to whom I owe thanks for intellectual stimulation and encouragement, and moral support. In the early stages, it was Bob Buchanan, at that time chief actuary of the GIO of NSW, who influenced and inspired me to strike out in a new direction. Buchanan was one of the first actuaries to recognise that General Insurance companies would benefit from the practical applications of modern statistical technology. I greatly benefited from Buchanan's informal and personal approach to discussing General Insurance problems.

I believe that the collegeality and mutual support that a small but growing group of actuaries extend to one another has already resulted in some significant advancements in the area of estimation of outstanding claim liabilities. These advancements, I hope, will lead to actuaries playing a dominant role in the application of statistical techniques to General Insurance matters.

Finally, I would like to thank Owen Roach, Executive Director of GIO Insurance Limited for his dedication and service to the actuarial profession. I believe the funding of the Professorship by GIO Australia is a way for actuaries to strengthen the academic base of their work, and is part of securing a strong actuarial education process for the future.

DEFINITIONS

1. An insurer's **outstanding claim liabilities** at a given date are the amounts which it is liable to pay, after that date, for claims which arose on or before that date.

2. An outstanding claim **provision** or **claims reserve** or **loss reserve** or just **reserve** is an amount set aside in the insurer's accounts, to provide for outstanding claim liabilities.

3. A **long tail** class of business is one where the insurer's liability does not cease at the expiry of the risk period, as there are delays in reporting and subsequent settlements of claims.

4. **Loss reserving** is the term used to denote the actuarial process of the estimation of the outstanding claim liabilities.

5. **Balance sheet**

   Assets = <u>Liabilities</u> + Shareholders' Equity.

   An insurer's total assets equals the sum of its liabilities plus its shareholders' funds. For an insurer writing long tail lines, a very large component of the <u>Liabilities</u> is represented by the provisions (loss reserves) for outstanding claims.

6. **Incurred Claims**

   In the underwriting statement,

   Incurred Claims = Paid Losses + End of year Loss Reserve
- Beginning of year Loss Reserve

   The Incurred Claims are the total claim costs incurred in the underwriting year. The Paid Losses represent the total payments made in the underwriting year, in respect of all claims incurred in the current and all prior underwriting years.

ABSTRACT

This paper describes a (probabilistic) statistical MODELLING FRAMEWORK for conducting loss reserving analysis. The modelling framework affords many advantages including extraction of maximum information, simplicity, formal testing of assumptions and, most importantly, the quantification of loss reserve variability (or uncertainty). The last advantage is of paramount importance for premium rating purposes, assessment of risk based capital, testing of solvency and valuation of the company.

Four real life portfolios are analysed using the modelling framework in order to demonstrate its power and flexibility, and moreover dispel many pervasive myths surrounding loss reserving. These include concepts such as estimation of inflation, sources and extraction of pertinent information, stability, predictability, actuarial judgment and incorporation of business knowledge. Much of the paper is iconoclastic. This is because standard actuarial techniques for loss reserving are very ad hoc, have a "cookbook" experimental flavour, do not address the critical issue of variability (uncertainty) and assumptions are neither stated nor tested. It's times to declare "The Emperor has no clothes".

Significant findings for the new CTP (Compulsory Third Party Personal Injury) Scheme in NSW are presented. It is demonstrated that the current industry pure premium (as advised by some consultants) is substantially too low. In my view, major upgrades in premiums, and claims provisions in the accounts of insurers writing CTP in NSW, are prudent.

The paper is long and contains many technical details, so a special self-contained Executive Summary is included for insurance management. The Summary also provides a "road map" of what's to come in the paper.

Finally, the views expressed in this paper are entirely my own (except for quotes) and if you agree with all of them, then only one person is doing the thinking.

0.0 EXECUTIVE SUMMARY

When I deliver a public lecture I usually start out communicating the fact that I have both a statistical designation and an actuarial designation. One nice feature is that I have a repertoire of at least two jokes.

Statistical joke

A statistician is someone who puts their head beside a heater and their feet in a bucket of ice water and declares "On the average I am fine".

Actuarial joke

There are <u>three</u> types of actuaries. Those who can count and those who can't.

Both jokes have immediate applications to actuarial solutions of General Insurance problems.

Statistics is much more than just calculating an average. Statistics is the study and measurement of variability or uncertainty. For many processes, the probability of observing an outcome equal to the average or even close to the average is very small, if not zero.

The primary purpose of statistical endeavour is inferring about realisable values not observed, based on values that were observed. That is, predicting or forecasting uncertain outcomes (events) in terms of probability distributions.

If we roll a symmetric die numbered 1 to 6 (many times), the average (mean) is 3.5, but it is never observed. The average conveys very little information, if any. The **probability (statistical)** distribution of the outcomes is much more informative. In this case, each outcome occurs with probability 1/6. The probability distribution measures the variability or uncertainty associated with outcomes of die rolling. This type of variability is called **process variability**. If the die is mutilated and accordingly biased, we would <u>estimate</u> the **probability distribution** of future outcomes (unobserved) based on a sample of (past) observed outcomes. The uncertainty as a result of not knowing the probability distribution of outcomes perfectly (we estimate it) is termed **estimation** error.

This paper addresses the critical issue of assessing the variability (uncertainty or predictability) of outstanding claim liabilities. It is shown how to infer from the past experience (data) and other sources of information, the probability distribution of the outstanding claim liabilities.

Turning to the actuarial joke, many practising General Insurance actuaries substitute judgment for straight forward arithmetic (or modern statistics) when in fact they are capable of counting.

Before we launch into a summary discussion of the technical ideas contained in the paper, we present some background material on the actuarial profession, General Insurance, academia and the critical importance of loss reserving to an insurer writing long tail business.

0.1 The Actuarial Profession, General Insurance (GI) and Academia

The actuarial profession is small and is dwarfed by thousands in accounting, engineering, etc. But, like many other professions, it is undergoing a revolution, as a result of a rapidly changing financial environment, the advent of high speed computers and the rapid convergence of related fields.

Traditional actuarial practice areas include life assurance, pension insurance and investment. General Insurance (GI), on the other hand, is a relatively new field for (Aussie) actuaries and has only been an examinable subject by the Institute of Actuaries (London) since the late 1970's. In Australia, actuaries do not have a statutory role in GI, as they do in life assurance. Increasing involvement of actuaries in GI is critical for the future growth and success of the profession outside its traditional roles.

The underlying GI process is much more variable than life assurance and pension insurance, and so deterministic actuarial techniques based on the "Law of Averages", formed and honed in these fields, offer little in GI. The GI paradigm is stochastic (probabilistic), where the assessment of variability or uncertainty is critical to the solutions of most problems.

A number of celebrated actuaries including William Sutton (1888), then President of the Institute of Actuaries (London) and Arthur Bailey (1942), an American GI actuary, recognised that the GI field would benefit from the "practical applications of the doctrine of probabilities". It appears, unfortunately, that their views have been ignored by most practising GI actuaries.

The increasing computer power available to the actuary today and recently developed statistical technology make the use of probabilistic (stochastic) techniques relatively simple. The actuarial profession can no longer rely on the deterministic approach to future planning in GI.

If the actuarial profession embraces stochastic techniques, then it is likely to be ideally equipped to solve GI problems. It is only by delivering quality GI education to actuarial students and actuaries, and by fostering collaborative research relationships between industry and academia, that actuaries may again play a dominant role in the application of statistics to financial matters, and will have increasing involvement in GI.

0.2 Outstanding Claim Liabilities and Financial Accounts

Prediction of outstanding claim liabilities is a major issue in any assessment of the financial condition of an insurer writing long tail lines, whether performed in supervision as part of solvency testing, or in company management. Since solvency is a probabilistic concept, its assessment necessarily includes in part the quantification of loss reserve variability or uncertainty.

For a 'long tail' line of business there are delays between the time period for which insurance protection is afforded (risk period) under the policy, and the actual claim payments. Accordingly, the insurer may take many years to discharge its obligations assumed under the policy.

An insurer's outstanding claim liabilities at a given date are the amounts which it is liable to pay, after that date, for claims which arose on or before that date.

In order to understand the necessity for the estimation of outstanding claim liabilities, it is helpful to have a conceptual understanding of the basic accounting principles applicable to GI companies.

GI companies use the accrual basis of accounting which recognises revenue when it is earned, not when it is received. Costs are likewise, recognised as expenses in the same period as the revenues giving rise to these costs. This results in financial statements that more appropriately match costs with appropriate revenues.

At the end of an accounting year the insurer sets aside a **provision** (equivalently a **loss reserve**), in the accounts to provide for the outstanding claim liabilities. The total outstanding claim liabilities is uncertain or variable. It has a probability distribution, and it is incumbent on the actuary to compute or estimate the distribution. The distribution is estimated from past experience and any other sources of pertinent information. This is a critical point, as the actual financial provision in the accounts has a direct impact on shareholder's equity (and therefore solvency), and also on underwriting profit. The reliability and usefulness of both the balance sheet and underwriting statement, are dependent on the 'accuracy' and 'probabilistic' interpretation of the **provision** and **incurred losses** shown in the insurers accounts. Management must recognise and accept variability (uncertainty) and should not only be concerned with the final figures, which it reports in its company accounts.

0.3 Statistical Modelling Framework

The paper introduces and describes a unified statistical approach to loss (claims) reserving with its principal advantages and benefits. At the core, is the paradigm shift, from the non-statistical actuarial techniques to the statistical actuarial techniques.

It is always difficult to predict the future.

Forecasting

Indeed it (forecasting) has been likened to driving a car blindfolded while following directions given by a person looking out the back window. Nevertheless, if this is the best we could do, it is important that it should be done properly, with the appreciation of the potential errors involved. In this way it should at least be possible to negotiate straight stretches of road without a major disaster.

Andrew C. Harvey [9]

(The person in the back seat does not have supernatural powers).

In the loss reserving context, the 'straight stretches' are the stable trends in the (incremental) payments. If the trends have been stable in past years, we are confident (but not absolutely sure) in supposing the same trends in the future. This almost perfect analogy will be used throughout the paper.

The mechanisms by which claim sizes, frequencies and delays are generated are invariably complex. When a model is constructed, it is not intended to be an accurate description of every aspect of the claims processes. The aim is to simplify the underlying processes in such a way that the essential features are brought out. According to Milton Friedman [7]: *'A hypothesis is important if it 'explains' much by little...'*. Similar views are expressed by Popper [13]; *'Simple statements... are to be prized more highly than less simple ones because they tell us more; because their empirical content is greater, and because they are better testable.'*

From the statistical point of view, the key feature of a simple model is that it contains a small number of parameters. This is known as the principle of parsimony. Moreover, a simple model is testable. There is no need to model every basic element of the claims process. Instead, we construct a simple model that identifies the trends and deviations (random fluctuations) about the trends in the (aggregate) payments.

It is useful to think of data (measurements) as comprising two components: a **signal** or a **message** which is distorted by a second component, termed **noise**. The **signal** is regarded as deterministic and the **noise** as random. Therefore, a mathematical model of the data combining both signal and noise is stochastic (probabilistic) and is called a statistical model.

Another way of thinking of a statistical model is to consider the signal component as a mathematical description of the main features of the data, and the noise component as all those characteristics not 'explained' by the signal component.

Typically the mathematical description of the signal involves several unknown constants, termed **parameters**.

In the loss reserving context the signal itself has three components of interest, viz., the trends in the three directions, **development year, accident year** and **payment/calendar year** of a "loss development array" described in Section 1.2. For each direction there are trend parameters. The fourth component is the noise, equivalently, the random fluctuations or deviations about the trends. The random fluctuation component is just as important as the three trend components and is necessarily an integral part of the model. The data or transform thereof are decomposed thus:

DATA = TRENDS + RANDOM FLUCTUATIONS

The concept of **trends** and **random fluctuations** about trends is over two hundred years old. These concepts have been widely used in analysing (and forecasting) univariate time series such as sales, stock market prices, interest rates, consumption, energy and so on.

The principal aim of analysing a loss development array is to estimate the trends in the past, especially in the payment/calendar year direction, and determine the random fluctuations about the trends. In this way it can be best judged which assumptions should be used for future trends (and random fluctuations). The probability distributions of the random fluctuations are also computed.

IF THE TRENDS IN THE DATA ARE STABLE THEN THE (OPTIMAL) MODEL WILL VALIDATE WELL AND BE STABLE. If the trends in the data are unstable then the decision about future trends is no longer straight forward. Instability in trends with little random variation about the trends makes data less predictable than stable trends with much random fluctuation. See Sections 4.2 and 4.3 for real life examples.

The 'best' identified model contains assumptions (equivalently, information). All the assumptions must be tested to ensure they are supported by the data (experience).

As we proceed through the model identification strategy we are extracting information (about trends and stability thereof and the amount of random variation) and we 'hope' that the 'best' identified model tells us that the calendar year trend in the data is stable (especially more recently). If trends are not stable then we may not necessarily use the optimal statistical model for forecasting. See Section 5.0.

None of the numerous models contained in the MODELLING FRAMEWORK actually represent explicitly the underlying claims generating processes. The multitude of variables involved in generating the claims are invariably complex. What we attempt to achieve is the identification of a parsimonious model in terms of the simple components of interest for which all the assumptions inherent in the (probabilistic) model are supported by the data. It is subsequently argued that the experience (data) can be regarded as a sample (path) from the identified probabilistic model. The multitude of variables that are the determinants of the claims processes are proxied by the TRENDS and the (residual) variance of the RANDOM FLUCTUATIONS.

The principal advantage of an explicit statistical model is that it makes the assumptions clear. Other advantages include improved accuracy and quantification of variability required for assessment of risk based capital and testing of solvency.

0.4 Loss Reserving Myths and Uncertainty

We debunk a number of loss reserving myths by employing simulation studies and a number of real life examples. In spite of the critical importance of loss reserving in insurance, the statistical foundations are not well understood by many insurance experts. Misstatements and fallacies regarding loss reserving and related concepts are pervasive, ranging from insurance courses in the classroom to insurance cases in the courtroom.

<u>**Myth 1**</u>

If we know the (exact) probabilistic model including the values of the parameters generating the paid losses, there is no variability or uncertainty.

<u>**Reality 1**</u>

There is variability or uncertainty even when we know the exact probabilistic model generating the losses. (Recall the die rolling example). This variability (uncertainty) is called **process variability** (uncertainty). See Section 3.1.

Myth 2

Variability or uncertainty is inversely proportional to the size of the insurer's exposure base.

Reality 2

There is no relationship. It is only the process uncertainty (noise) that may reduce with increasing exposure. In Section 4.2 we analyse an experience of a large U.S. insurer with a large exposure base where the paid loss experience has a major shift in trend, and accordingly there is much uncertainty about the future. By contrast, in Section 4.3, we consider a company with a relatively small exposure base where the paid losses fluctuate widely, BUT, the trend is relatively stable and so the future experience is not as uncertain.

Myth 3

Large fluctuations in paid losses implies instability in trends and so the future experience is very uncertain.

Reality 3

Large fluctuations may be due to the "random" component, equivalently, the "noise", not an instability in trends. See Section 4.3 for a real life example where the paid losses fluctuate widely but due to stability in trends, the model estimated three years earlier would have "predicted" the last three years experience and would have yielded the same estimates statistically of the outstanding claim liabilities, as the model estimated at valuation date.

Myth 4

Escalation in payments is due to "claims closing faster", and so less will be paid later.

Reality 4

This is one of the "great lies" in loss reserving. Some insurance practitioners have used this argument to explain the rapid escalation in the claims experience for the new CTP NSW Scheme. See Section 6.0 for a description of the "relationship" between aggregate payments and closure rates in the CTP NSW industry experience, and Section 4.3 for the "relationship" in the (individual) AMP General Insurance CTP (NSW) experience.

Charles McLenahan, a distinguished U.S. GI actuary in referring to Myth 4 remarked:

> *"If I had a nickel for every time I heard this as an explanation for increasing loss development factors, I wouldn't have nearly enough to cover the reserve deficiency of the company which believes it. In twenty-five years, the only situation in which I have ever witnessed a material speedup of claims closure was a company in liquidation. The teller of this untruth is usually armed with various recently-instituted changes in claims handling policies and procedures which account for the change."*

Predictability is intimately related to the concept of uncertainty which abounds in everyday business life. The various components or sources of uncertainty or variability are discussed in Section 5.0.

Uncertainty (variability) is modelled in terms of probability (statistical) distributions. There are four principal sources of uncertainty that are interrelated. These sources of uncertainty determine the predictability of the outstanding claim liabilities.

1. Noise or random variation

   This is called **process uncertainty** or **process risk**. It represents the inherent variability in the process. We have no control over it and cannot reduce it.

2. Estimation error

A statistical model contains parameters that are estimated from data. Due to sampling variation (noise) the parameters are not known exactly.

3. Trend stability or lack thereof

Based on Andrew Harvey's car example in Section 0.3, uncertainty about future trends is related to stability or lack thereof of past trends.

Assumptions about future trends are based on identification of past trends and other sources of information, including business knowledge. See Section 5.0.

This type of uncertainty is commonly referred to as **risk parameter** uncertainty and is intimately related to the next type of uncertainty.

4. The future ain't what it used to be

This source of uncertainty may be difficult to measure statistically, but that does not mean we should ignore it. The future may be very different to the past. A pricing actuary working in 1975 would have had no way of predicting the explosion in pollution liability claims that would have occurred in the 1980's, in respect of claims incurred in 1975. However, had he been aware that there is a strong probability that the environment, legal and economic, may change, judgemental changes to the parameters of the model, could have been made.

The quantification of the first three sources of uncertainty are dependent on the information extracted from the historical experience.

0.5 CTP NSW Industry Experience

The NSW Government established a new Scheme, which commenced in July 1989, for compensating people injured as a result of the fault of others in motor vehicle accidents. Compulsory Third Party Personal Injury

Insurance (or CTP) provides funds to compensate people injured in motor vehicle accidents.

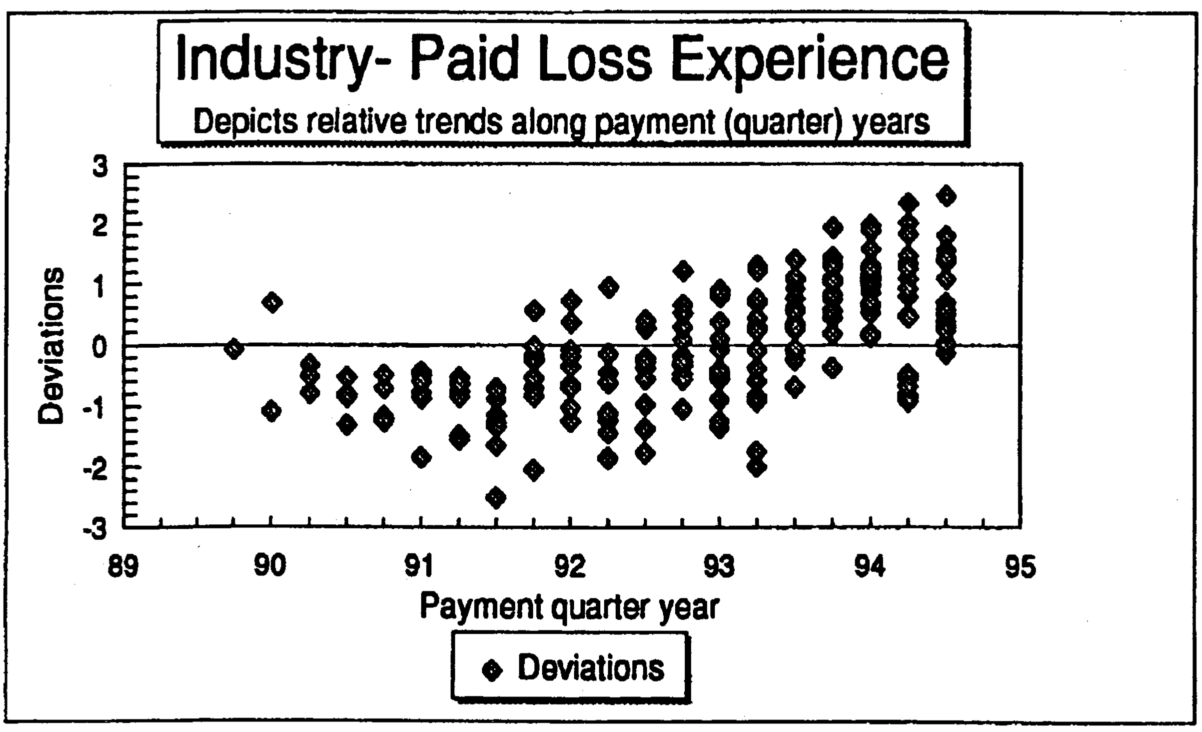
The Scheme includes the following features:

- Each registered owner of a vehicle in NSW is required to insure, with an insurer licensed under The Motor Accidents Act 1988.

- Insurers are licensed under the Act and must file with the Motor Accident Authority, at least once a year, a full set of premiums it proposes to charge for third party policies.

Applying the statistical modelling framework described in the paper, we have determined an alarming claims escalation (deterioration) in the paid losses that has existed for over three years.

The graph below displays the trends in the paid losses across the payment quarter years from the September quarter of 1989 (3-89) to the June quarter of 1994 (2-94). We observe a favourable experience (zero trend) from 3-89 to 2-91. Thereafter, there is a distinct positive trend estimated as 8.42% ± 1.38% per quarter year. That is, 40% p.a.!



The alarming trend has existed for over three years, so that, if the 'optimal' statistical model is estimated at payment quarter year end 3-92 (omitting the 4-92 to 2-94 paid losses), for example, it "predicts" the paid losses for the payment quarter years 4-92 to 2-94 and yields statistically the same outstanding paid losses, as the model estimated at payment quarter year end 2-94! See Section 6.5 for detailed analysis.

The alarming deterioration in claims experience has resulted in substantial underwriting losses for NSW based CTP business, in the current and recent financial years. Some industry practitioners have attributed the deterioration to "claims closing faster". That this contention (Myth 4) is not supported by the experience is demonstrated in Section 6.4.

If we apply the Harvey [9] motor car analogy, even though we have identified an inordinate high inflation rate in the CTP experience, we are not guaranteed it will continue. Indeed, given that the principal reason for the inflation, is a continued trend towards litigation as an avenue by claimants for higher award payments, one would expect and hope to reach 'saturation' in the near future. A downward adjustment to the future trend ought to be made to reflect the fact that the legal environment will stabilise.

In any event, the trends arising from the deterioration in claim costs have only been marginally factored into the current pure premium (before allowing for expenses, profit margins and other contingencies) of $190 (per vehicle). Assuming a discount rate of 8% - 10% p.a., it could only be substantiated by a less than 4% p.a. combined AWE + Superimposed inflation in the future.

In my view, unless there are major upgrades to the industry pure premium and to the provisions carried by companies writing this line, then losses will continue and are indeed likely to increase.

The companies that are inadequately reserved are attracting taxes on profits that are very unlikely to emerge.

Section 6.0 of the paper presents more detailed analysis of CTP NSW.

0.6 Organisation of the paper

The problem of estimation of outstanding claim liabilities including accounting concepts is discussed in Section 1.0. Probabilistic concepts and statistical models in general are presented in Section 2.0.

The probabilistic modelling framework is introduced in Section 3.0 with a few simple examples using simulated data. The difference between process uncertainty and estimation error is illustrated. Model validation, stability and updating are also discussed.

In Section 4.0 we describe a number of loss reserving myths that are related to the measurement of uncertainty. The myths are debunked using real life examples. Formulation of assumptions about the future based on extraction of information from the historical experience, business knowledge and judgment are discussed in Section 5.0.

In Section 6.0 we use the statistical modelling framework to analyse the NSW based CTP experience, in order to illustrate the power and flexibility of the statistical methodology. We also show that the CTP experience is subject to an alarming claims escalation rate.

1.0 THE PROBLEM

One of the major challenges to the General Insurance (GI) actuary is the estimation of the necessary financial provisions for the unpaid outstanding claim liabilities of an insurer to claimants. The determination of the provisions is essential to the long term management of a GI company. Accurate assessment is required for solvency considerations, as well as premium setting.

1.1 Outstanding claims

A GI policy is a short term contract, usually one year. However, the insurer's liability may not necessarily cease at the expiry of the (one year) risk period.

For a 'long tail' line of business there are delays between the time period for which insurance protection is afforded (risk period) under the policy, and the actual claim payments. Accordingly, the insurer may take many years to discharge its obligations assumed under the policy.

An insurer's outstanding claim liabilities at a given date are the amounts which it is liable to pay, after that date, for claims which arose on or before that date.

We define the 'accident year' as the 'year of origin' in which the incident leading to a claim occurred. The year in which a payment is made is referred to as 'payment year' and the difference between 'payment year' and 'accident year' is referred to as the 'development year'.

Each 'accident year' gives rise to a stream of payments in emerging years.

1.2 Triangulation (Loss Development Array)

The claim experience of an insurer in respect of a particular class of business can be summarised in a run-off triangle exemplified below. 'Year of origin' is the year in which the incident leading to a claim occurs.

Incremental Paid Losses ($000)

Year of Origin	Development year (Delay) 0	1	2	3	4
1990	580	1079	131	80	25
1991	494	993	118	91	
1992	551	1060	129		
1993	648	1312			
1994	746				

The diagonals in the array represent the payment years. For example, in respect of claims originating in 1992, payments totalling $129,000 were made in 1994 (development year 2).

Run-off triangles for other (aggregate) data types including number of claims notified, number of claims closed and case estimates can also be created.

The objective is to complete the rectangle in order to compute the total ultimate incurred cost for each year of origin (accident year).

1.3 Accounting Concepts

In order to understand the necessity for the estimation of outstanding claim liabilities, it is helpful to have a conceptual understanding of the basic accounting principles applicable to insurers.

The accounting process produces two important statements, the balance sheet and the income statement, that document the financial position and performance of a firm respectively. The reliability and usefulness of both these statements are dependent on the accuracy and interpretation of the provisions (for outstanding claim liabilities) shown in the insurer's accounts.

1.4 Accrual basis of accounting

The accrual basis of accounting recognises revenue as it is earned. Likewise, costs are reported as expenses in the same period as revenues giving rise to these costs are recognised. This results in an income statement that more appropriately matches costs with appropriate revenues.

1.5 Provision

An outstanding claim provision is an amount set aside in the insurer's accounts, to provide for outstanding claim liabilities.

1.6 The Balance Sheet and Underwriting Statement

The balance sheet reports on the financial position of the firm at a specific point in time. It shows the levels of assets and liabilities, and the status of the shareholders' equity, or surplus, for the insurer.

Assets = Liabilities + Shareholders' Equity

The liabilities include the outstanding claims liabilities defined in Section 1.1. Through common usage the term "loss reserve" or "claims reserve" has come to denote the GI company's provision in the balance sheet for its outstanding claims liability.

In the Underwriting statement

Incurred Claims = Paid Losses + End of year Loss Reserve
- Beginning of year Loss Reserve

The Incurred Claims are the total claim costs incurred in the underwriting year. The Paid Losses represent the total payments made in the underwriting year, in respects of claims incurred in the current and all prior underwriting years.

1.7 Solvency and Income

Any change in the (financial) provisions in the accounts have a direct impact on Shareholders' Equity and accordingly solvency, and Incurred Losses and accordingly income.

1.8 Loss (claims) reserving

Loss or claims reserving is the process of estimating the amount of the company's outstanding claim liabilities.

2.0 LOSS (CLAIMS) RESERVING METHODS

The basic goal of this paper is to introduce and describe a unified statistical approach to loss (claims) reserving with its principal advantages and benefits. At the core, is the paradigm shift, from the non-statistical standard actuarial techniques to the statistical actuarial techniques.

In spite of the critical importance of loss reserving in insurance, the statistical foundations are not well understood by many insurance experts. Misstatements and fallacies regarding loss reserving and related concepts are pervasive, ranging from insurance courses in the classroom to insurance cases in the courtroom.

Paradigm Shift on the Port Bow

The following true story of a naval cammander's brush with a new reality occurred some years ago during US navy practice manoeuvres. The ship in question was steaming just after dark in heavy fog when a light was reported by a lookout.

The captain ordered his signalman to flash the message 'We are on a collision course. Advise you change course 20 degrees'. The reply came back through the fog, 'Advise you change course'.

The next signal said 'I am a captain. Change course 20 degrees'. The reply was 'I am a seaman, 2nd class. You had better change course'.

The captain, now infuriated, sent back, 'I'm a battleship. Change course 20 degrees'. The reply? 'I'm a lighthouse'.

Stephen R. Covey [5]

The statistical approach to loss reserving requires a totally different perception and/or framework. It requires a paradigm shift.

Celebrated actuaries have suggested "statistical thinking" as the principal approach to solving GI problems.

In his Presidential address (Institute of Actuaries, London) in 1888, William Sutton expressed the wish that insurance offices other than life offices should benefit from the practical application of the doctrine of probabilities.

More recently, the celebrated American actuary Arthur Bailey, in spite of his mathematical cum statistical brilliance, had a way of presenting ideas so lucidly that even lay people could get his message. For example, in his 1942 paper, *"Sampling Theory in Casualty Insurance"*, he said:

"Thus the losses paid by an insurer never actually reflect the hazard covered, but are always an isolated sample of all possible amounts of losses which might have been incurred. It is this condition, of never being able to determine, even from hindsight, what the exact value of the inherent coverage was, that has brought the actuary into being."

2.1 Claims Processes

The mechanisms by which claim severities, frequencies and delays are generated are invariably complex. When a model is constructed, it is not intended to be an accurate description of every aspect of the claims processes. The aim is to simplify the underlying processes in such a way that the essential features are brought out. According to Milton Friedman [8]: *'A hypothesis is important if it 'explains' much by little....'.* Similar views are expressed by Popper [14]: *'Simple statements... are to be prized more highly than less simple ones because they tell us more; because their empirical content is greater, and because they are better testable'.*

From the statistical point of view, the key feature of a simple model is that it contains a small number of parameters. This is known as the principle of <u>parsimony</u>. Moreover, a simple model is <u>testable</u>.

The purpose of constructing a statistical model is to systematically account for as much of the variation in the observations with as few parameters as possible.

The "essential features" of the data in the loss reserving context are the trends and the random fluctuations about the trends. We decompose the data thus:

Log 'payments' = Trends + Random Fluctuations

Another way of thinking of this statistical model is to regard the Trends as a mathematical description of the main features of the data and the Random Fluctuations (or error or noise component) as all of those characteristics not 'explained' by the Trends. All the complex mechanisms involved in generating the data are implicitly included in the model as creating the Trends plus the residual variance in the Random Fluctuations.

The final identified model that 'explains' the data does not represent explicitly the underlying claims generating process. The model has probabilistic properties for which the data may be regarded as a sample (path) from it. Another classical modelling example in insurance where the same kind of modelling concepts are used is when we fit a Pareto distribution, say, to loss sizes. We do not assume that the Pareto distribution represents the underlying generating process. Whatever is driving the claims is very complex and depends on many factors. All we are saying is that our experience (sample) can be regarded as a random sample from the estimated Pareto distribution. The estimated Pareto distribution describes the variability in the loss sizes.

2.2 Statistics, Statistical Models and Forecasting

The best way to suppose what may come, is to remember what is past.

George Savile, Marquis of Halifax.

In this section we discuss a number of fundamental statistical forecasting concepts including which salient features of the data should be used to *"remember what is past"*.

Statistics can be defined as the art and science of gathering, analysing and making inferences from data. Statistics is the study and modelling of variability and uncertainty.

The basic principles of statistical inference necessarily involve probabilities. Indeed, statistics is primarily concerned with the application of probability theory to data. The statistical approach to modelling is based on the construction or estimation of a probabilistic model. The model does not necessarily represent the underlying generating process of the losses. Whatever generates the losses is complex and depends on a myriad of factors. Instead, the statistical model is simple, and defines the probabilistic mechanisms (or laws) which are regarded as being capable of having produced the data (observations). If the model were to generate several sets of data (or observations), each data set would be different but they would all obey the same probabilistic laws.

Forecasting

Indeed it (forecasting) has been likened to driving a car blindfolded while following directions given by a person looking out the back window. Nevertheless, if this is the best we could do, it is important that it should be done properly, with the appreciation of the potential errors involved. In this way it should at least be possible to negotiate straight stretches of road without a major disaster.

Andrew C. Harvey [10]

In the loss reserving context the 'straight stretches' are the stable trends in the (incremental) payments. If the trends have been stable in past years, we are confident (but not absolutely sure) in supposing the same trends in the future. This almost perfect analogy will be used throughout the paper.

Predictability, as will become apparent, is intimately related to stability of trends.

3.0 PROBABILISTIC MODELS

We use probabilistic models to extract information from data. Based on:

- information extracted from the incremental paid losses development array (triangle);
- information extracted from other data types;

and

- business knowledge,

the actuary determines the most appropriate assumptions about the future. Information extracted from the loss development arrays will necessarily involve (i) validation analysis, (ii) stability analysis, (iii) sensitivity analysis and (iv) 'what if?' analysis.

3.1 Example of A Statistical Model Generating A Loss Development Array (Triangle)

We describe a (simple) probabilistic model representing the generation of incremental paid losses in a loss development array (triangle).

Consider first, only one accident year, and denote by p(d) the incremental paid loss in respect of development year d.

d	:	0	1	2	......	s
p(d)	:	p(0)	p(1)	p(2)	......	p(s)

Assume p(d) is generated by the trend curve $\exp(\alpha + \gamma d)$, an exponential curve. So,

$$\begin{aligned} y(d) &= \ln p(d) \\ &= \alpha + \gamma d. \end{aligned}$$

That is, y(d), the logarithm of p(d) is generated by the constant trend line $\alpha + \gamma d$.

The parameter α (alpha) represents the intercept whereas the parameter γ (gamma) represents the slope or trend.

Note that logarithms are like percentages and are used to measure trends.

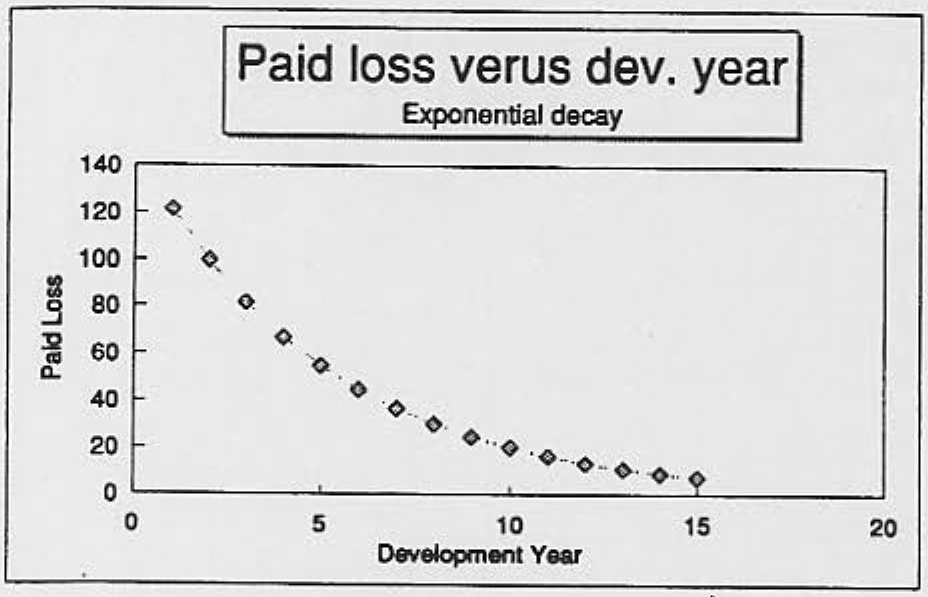


Figure 3.1.1

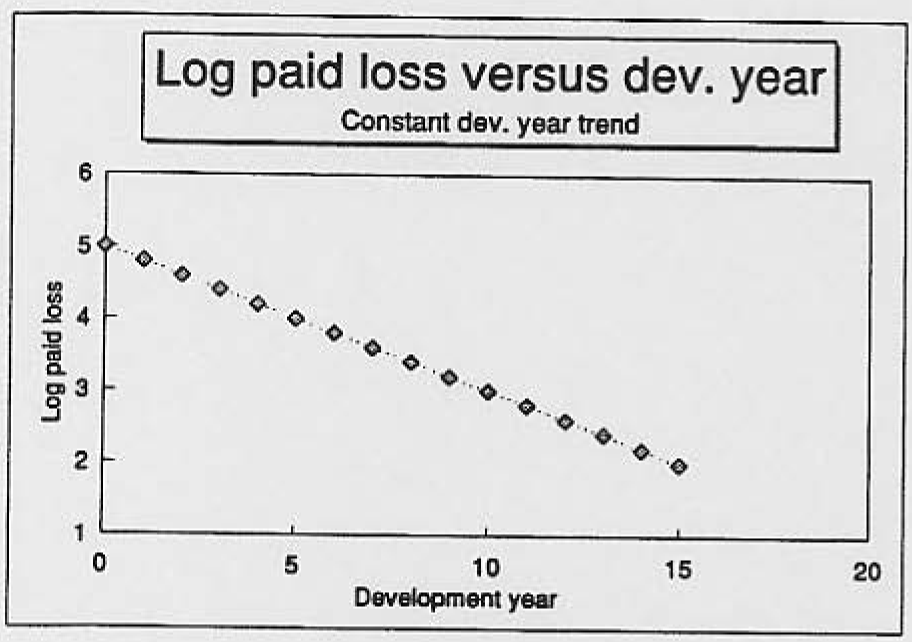


Figure 3.1.2

Figure 3.1.1 depicts the exponential curve $\exp(\alpha+\gamma d)$ and Figure 3.1.2 the corresponding logarithm.

Hitherto, we have assumed that each y(d) value sits on the straight line $\alpha + \gamma d$.

Suppose that in actual fact the observations y(d) fluctuate about the line $\alpha + \gamma d$, such that positive fluctuations (deviations) are as likely as negative fluctuations (deviations). Indeed, the deviations of y(d) about $\alpha + \gamma d$ can be described by the symmetric bell-shaped normal distribution. That is, the deviations or fluctuations follow a particular type of probabilistic law, depicted in Figure 3.1.3.

The symmetric bell shaped curve about the trend line represents the (relative) frequency of the deviations.

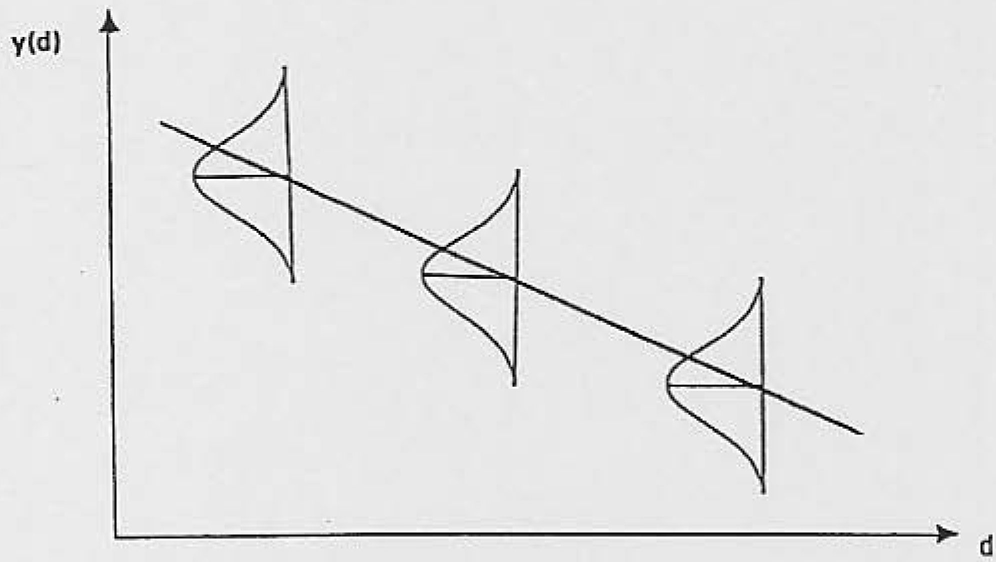


Figure 3.1.3

The model can now be written

$$y(d) = \alpha + \gamma d + \epsilon(d)\,, \tag{3.1.1}$$

where the "error" or "deviation" $\epsilon(d)$ of y(d) from the straight line $\alpha + \gamma d$ is a random selection from a normal distribution with mean zero (average deviation = 0) and variance σ^2, say. (The Greek letter, σ, denotes the standard deviation of the "deviations").

If we estimate (or fit) the above model to one accident year's observed experience we are not only estimating the straight line $\alpha + \gamma d$ but also estimating the normal distribution of the deviations of the observed from the fitted line. The estimated normal distribution with means lying on the estimated line define the probabilistic mechanisms which are regarded as being capable of having produced the observations.

Note that the model assumes that the mean effective yearly trend (on the $ scale) between any two development years is constant and equal to $\exp(\gamma) - 1$. The mean continuous trend (like force of mortality, or force of inflation) is represented by the parameter γ. We call γ a development factor on a log scale.

Suppose now that this constant development year trend model (on a log scale) applies to every accident year in the triangle with the same parameters α, γ and σ^2 for each accident year.

So what does this model containing only two mean parameters α and γ, and one variance parameter σ^2, assume about the observations in a triangle?

Succinctly, it assumes complete homogeneity of accident years with a constant trend along development years. More specifically,

(A1): The mean trend between any two development years is constant and is the same for each accident year. Accident years are homogeneous in respect of development year trends.

(A2): Accident years are homogeneous in respect of mean level. The same parameter α applies to each accident year.

(A3): The deviations of the (log) observations from the trend line follow a normal distribution with mean zero and constant variance σ^2.

N.B. The distribution of the deviations, equivalently, the random fluctuations, about the trend is an integral part of the model.

The model decomposes the (log) observations into trend plus deviations or random fluctuations.

DATA = TREND(S) + DEVIATIONS (RANDOM FLUCTUATIONS)

The above model contains very few assumptions (parameters). It is only useful for projections if all the assumptions contained in the model are supported by the data.

Violations of Assumptions

1. A constant trend along development years

   The violation of this assumption can be detected diagnostically by fitting (estimating) a constant trend to development years and examining the graph of observed deviations (residuals) versus development years for any residual trends.

   For example, the display of observed deviations versus development years in Figure 3.1.4 below exhibits non-randomness. Therefore, the trend along development years is not constant. Indeed, there appear to be four distinct trends.

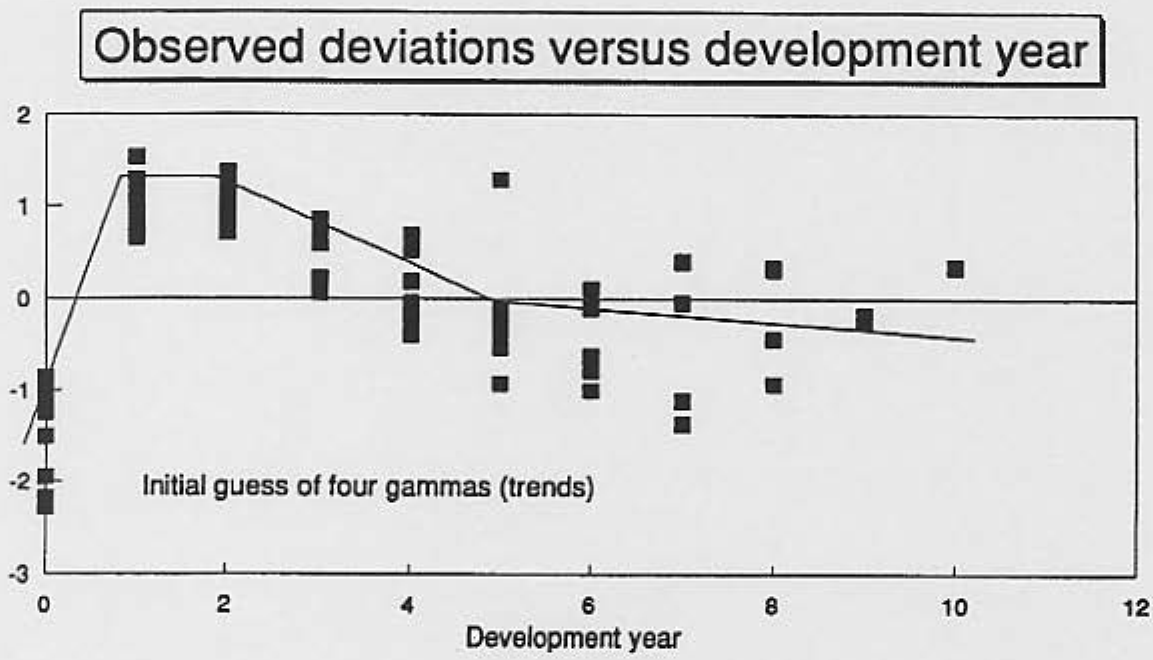


Figure 3.1.4

2. Accident years are homogeneous in respect of levels

The violation of this assumption can be detected diagnostically by examining the graph of observed deviations (residuals) versus accident years for systematic patterns.

For example, the display of observed deviations versus accident years in Figure 3.1.5 below indicates a systematic pattern (trend).

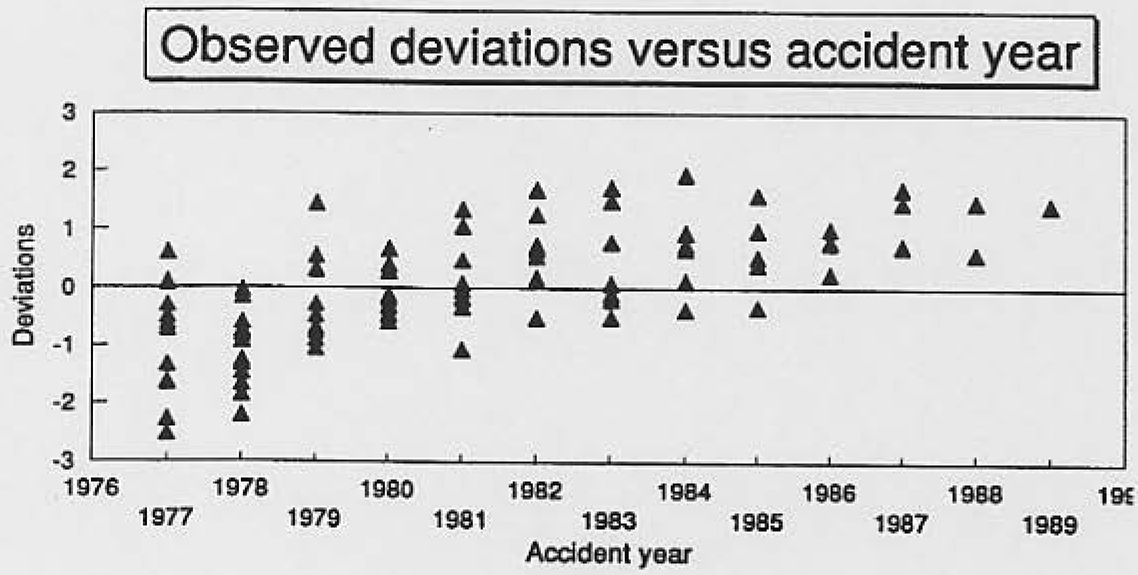


Figure 3.1.5

3. Payment/Calendar year trend is zero

The most important dimension or direction in the triangle is the payment/ calendar year direction, equivalently, the diagonals in the triangle. Model (3.1.1) assumes that the trend between any two contiguous payment/calendar years is zero. The violation of this assumption can be detected diagnostically by examining the graph of observed deviations versus the payment years.

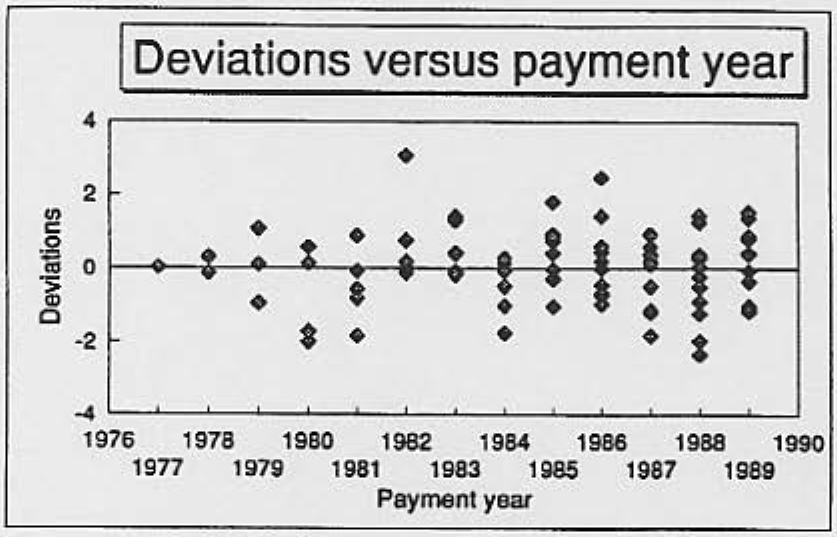


Figure 3.1.6

For example, the observed deviations in Figure 3.1.6 indicate diagnostically a zero trend whereas the observed deviations in Figure 3.1.7 indicate diagnostically a positive constant trend.

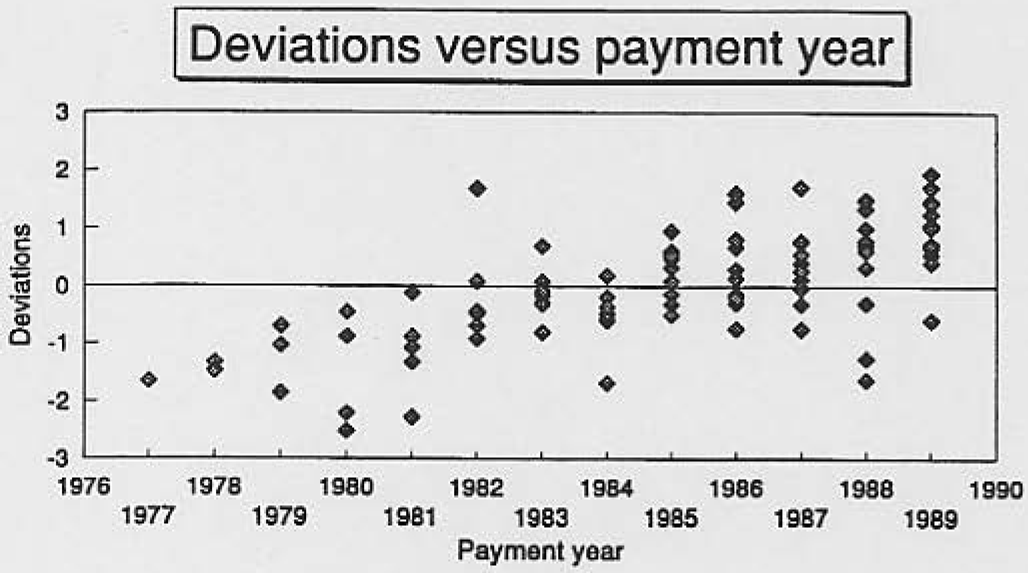


Figure 3.1.7

The most important assumption is concerned with the distribution of the deviations. The distribution of the deviations is assumed to be normal with mean zero and constant variance. This assumption must also be tested.

3.2 A Model with Three Inflation Parameters

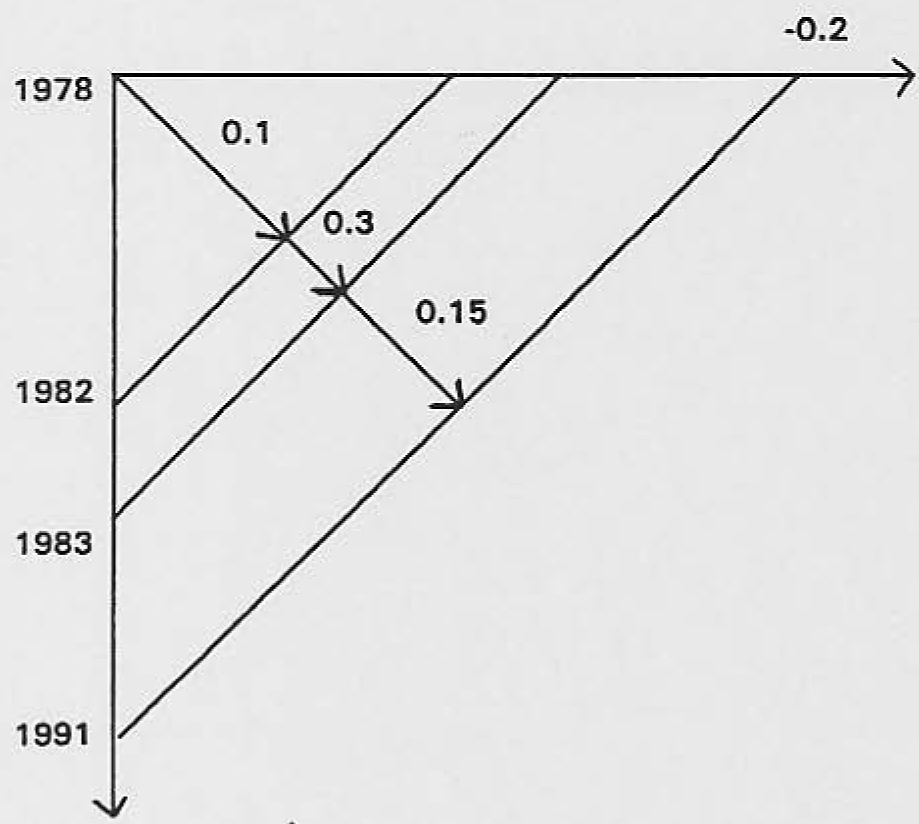
The data in Appendix A1 to Appendix A9 are generated as follows.

First, we create payments based on the formula:

$$p(w,d) = \exp(\alpha - 0.2*d).$$

That is, each accident year w is generated by the same exponential curve with γ (gamma) or decay factor equal to -0.2. The Greek letter α (alpha) represents the intercept, level or (log) "exposure". See Appendix A1 for a display of the data.

<u>TRENDS</u>



On a log scale we introduce payment/calendar year trends thus: 10% trend from 1978-82, 30% trend from 1982-83 and 15% trend from 1983-91. The logarithms of the payments with these trends are given in Appendix A2.

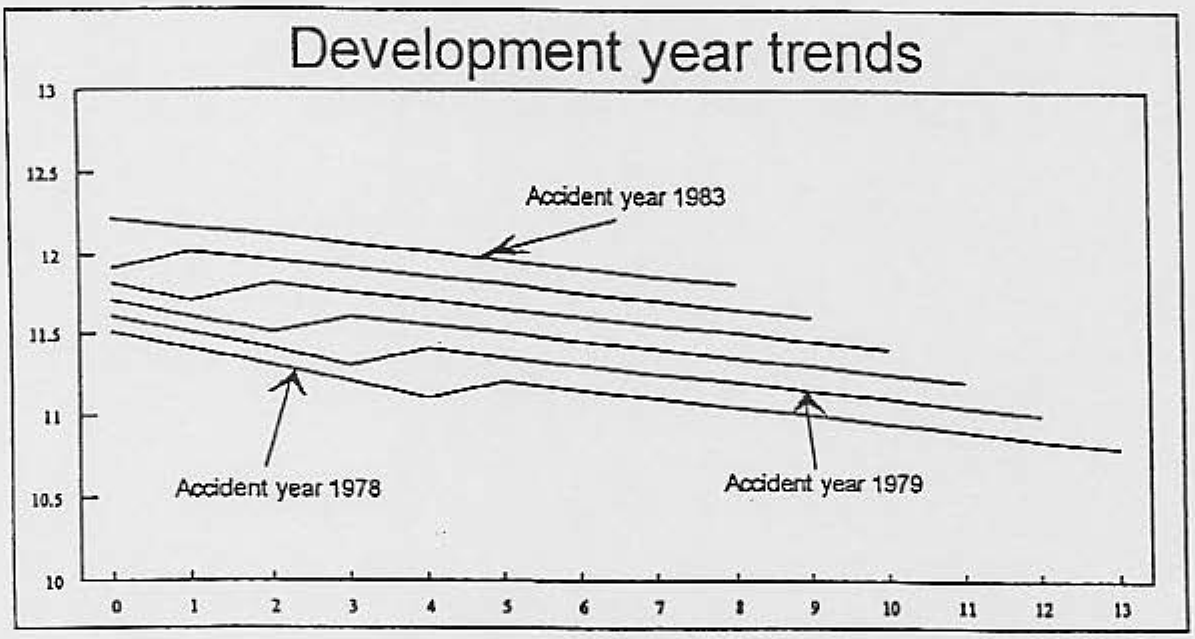


Figure 3.2.1

Figure 3.2.1 displays the graph of the log paid losses versus development year for the first six accident years. (The log paid losses are presented in Appendix A2).

Observe how payment/calendar year trends project onto development years and accident years. Each of the first six accident years has a different run-off development.

Consider the first accident year 1978. The 10% calendar year trend projects onto the development year, so that the resultant trend from development year 0 to development year 4 is -0.2 (the gamma) + .1 (the payment year trend) = -.1. The 30% trend between calendar years 1982 and 1983 also projects onto the development year so that the trend between development year 4 and 5 is -0.2+0.3=+0.1. Thereafter the trend is -.2+.15=-.05. Since .15 is larger than .1, the resultant decay in the tail is less rapid (-.05>-.1).

Consider the next accident year 1979. First up to development year 3, this accident year is 10% higher than the previous one since the 10% calendar year trend also projects onto the accident years. The 10% upward trend is one development year earlier than in previous accident year since the 30% trend is a calendar year change.

So, changing payment/calendar year trends can cause some interesting development year patterns. The run-off pattern is different for each accident year. The calendar year trends cannot be determined by the link ratios (age-to-age development factors) displayed in Appendix A4.

The patterns became much more complicated in the presence of random fluctuations superimposed on the trends.

The model describing the data we have constructed can be represented pictorially thus:

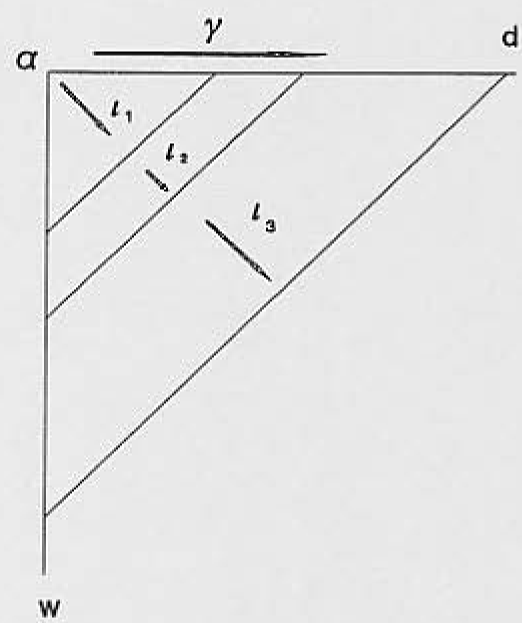


Figure 3.2.2

where γ=-0.2, ι_1=0.1, ι_2=0.3 and ι_3=0.15. The Greek letter ι (iota) represents a trend or inflation along payment/calendar years.

Writing the equations explicitly is not necessary. Indeed, it is too complicated.

We note that the <u>resultant</u> trend (age-to-age development factor) between development years j-1 and j is the (base) development factor γ between the two development years plus the payment year trend ι (iota) between the two corresponding payment years.

We now introduce random fluctuations or deviations from trends.

To all the log "payments" in the triangle we add random numbers from a normal distribution with mean zero. Equivalently, to the trends depicted in Figure 3.2.1, we add random numbers from a normal distribution displayed in Appendix A5. The sum of trends (Appendix A2) plus random fluctuations (Appendix A5) is displayed in Appendix A6.

The graph of the first six accident years of the data in Appendix A6 is given in the Figure 3.2.3 below.

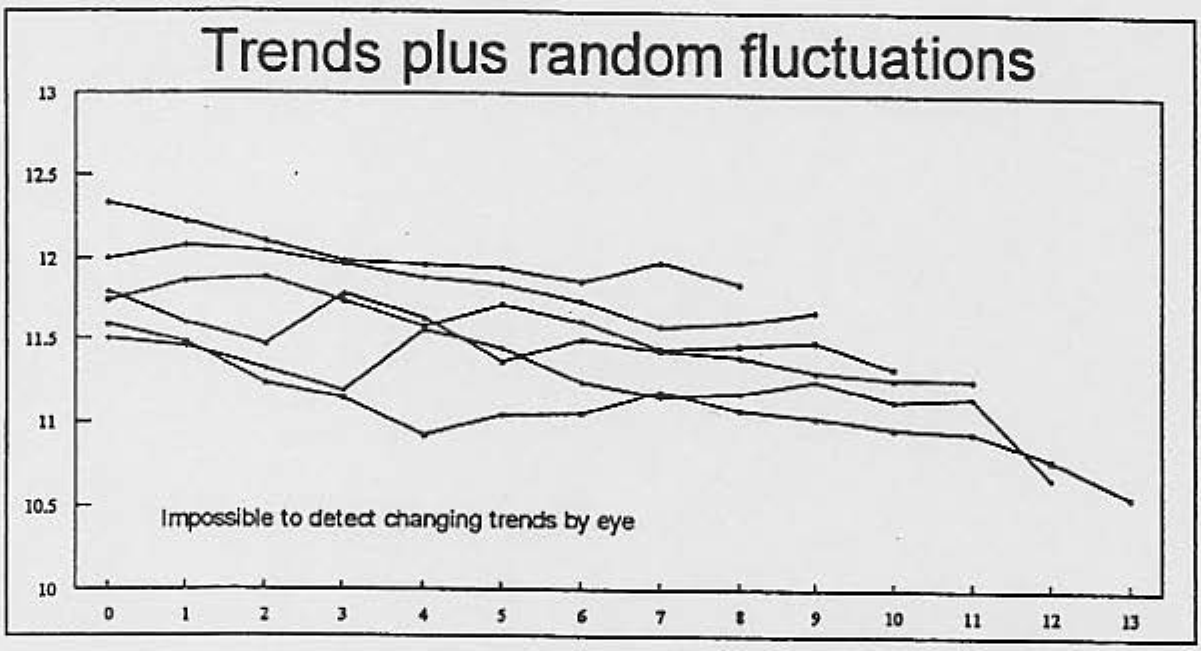


Figure 3.2.3

NOTE that it is impossible to determine the trends and/or changes in trends by eye or from the age-to-age link ratios of the cumulative payments (Appendix A9). See Appendices A7 - A9. THE TRENDS CAN ONLY BE DETERMINED BY USING REGRESSION.

The incremental paid losses we have generated in Appendix A7 were generated by five trend parameters (α , γ , ι_1 , ι_2 , ι_3) and one variance (noise, randomness) parameter σ^2 .

Since the incremental paid losses possess a stable trend (15%) along the payment years from 1983 to 1991 we would expect that the estimated model will validate well and be stable. See Section 3.3. Basically, when we look out the back window we determine that the road has been straight for quite a long distance.

3.3 Model Validation and Stability

Suppose we generate a sample triangle using model (3.3.1) with selected values of the three parameters $\alpha=10$, $\gamma=-0.3$ and $\sigma^2=0.4$.

When we use the generated observations in the triangle to estimate the parameters α, γ and σ^2 , our estimates, due to sampling variation, will <u>not</u> be identical to the actual selected values of the parameters.

What should we expect if we re-estimate the model from a sub-sample of the triangle, say the triangle excluding the last three diagonals?

First, we would expect the estimates of the three parameters to be stable. That is, the estimates based on the sub-sample should not be statistically different to those based on the whole triangle. This is because when we look out the back window of our car (along the diagonals) we determine that the road has been straight (stable (zero) trend) for many years.

Secondly, we would expect the estimated model based on the sub-sample to accurately forecast the observations in the last three diagonals. When we use the estimated model for forecasting we are projecting not only the (average) trends for the future but most importantly the distribution of the deviations of the observations from the trends. We would expect that the observed deviations of the actual observations from the forecast trends to be governed by the probabilistic mechanisms of the forecast distributions. Moreover, the completion of the rectangle should be statistically non-different to basing our projections on the estimated model from the whole triangle.

The following table displays results of estimating the model $\alpha + \gamma d + \epsilon$ from a sample triangle.

TABLE 3.3.1

Payment yrs in Estimation	Estimate of gamma %	Forecast
1978-1994	-28.67±1.26	299,660±35,487
1978-1993	-28.58±1.46	303,980±37,885
1978-1992	-28.65±1.66	302,601±38,843
1978-1991	-29.26±1.95	304,711±42,149
1978-1990	-29.40±2.28	296,650±43,625

The true model for which $\alpha = 10$, $\gamma = -0.3$ and $\sigma^2 = 0.4$ yields a (true) mean reserve forecast of 284,125 and a (true) standard deviation of 30,970. The standard deviation of 30,970 is referred to as process uncertainty. So,

EVEN IF WE KNOW ALL THE PARAMETERS OF THE TRUE MODEL, THERE IS STILL UNCERTAINTY OR VARIABILITY THIS IS CALLED PROCESS UNCERTAINTY

We now give a summary of the analysis of the incremental paid losses array generated by the probabilistic model of Section 3.2. The model contains a stable payment year trend of 15% since 1983.

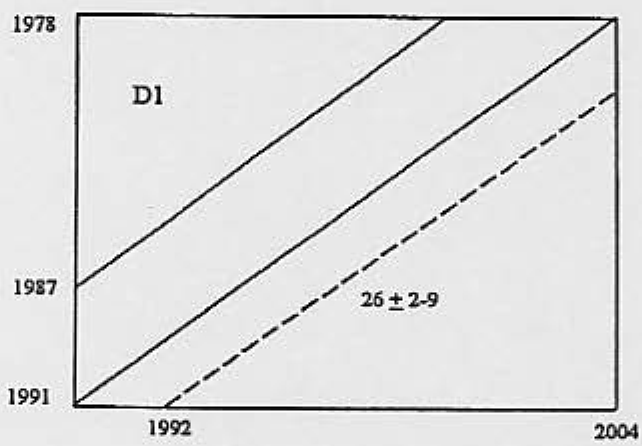
The model has four parameters, one α = 11.51293, one γ = -0.2, and three inflation parameters ι_1 = 0.1 for payment years 1978-82, ι_2 = 0.3 for payment years 1982-83 and ι_3 = 0.15 for payment years 1983-91. The incremental paid losses array appears in Appendix A7.

The following table gives estimates of parameters and outstanding total payments (i) using the whole triangle, (ii) using the triangle up to year end 1990, (iii) using the triangle up to year end 1989, (iv) using the triangle up to year end 1988 and finally (v) using the triangle up to year end 1987. In case (v) we do not use over 50% of the data points.

TABLE 3.3.2

Payment yrs in Estimation	Estimate of gamma (in tail) %	Estimate of iota (since 1983) %	Forecast $M
(i) 1978-91	-20.62±0.33	14.46±0.46	23±0.9
(ii) 1978-90	-20.75±0.36	15.27±0.51	25±1.2
(iii) 1978-89	-20.86±0.42	15.12±0.64	25±1.5
(iv) 1978-88	-21.19±0.45	15.75±0.75	26±2.0
(v) 1978-87	-21.31±0.55	15.63±1.03	26±2.9

Case (v)



D1 represents data points in the 1978-1987 payment years.

When we use the data set D1 to estimate the model parameters and then project payments for payment/calendar years 1992-2004, we obtain the same answer (26±2.9) statistically as when we use the whole triangle (23±0.9), that is data points from 1978-1991. Moreover, at year end 1987 the estimated model accurately forecasts the distribution of the deviations of the observed values from projected trends for payment/calendar years 1988-1991.

The identified model informs us that there is a stable trend in the data since 1983.

3.4 Model Maintenance and Updating

Once a model has been identified up to year end 1993, say, one year later at year end 1994, there is <u>no</u> need to analyse the history again. A number of post-sample predictive tests are conducted and the model is subsequently updated.

Consider the analysis in Section 3.3 of the data generated in Section 3.2.

At year end 1991 the model is stored. Next year, 1992, on receipt of the 1992 experience the same model is restored and zero weight assigned to the 1992 experience, in order to determine whether the estimated model at year end 1991 forecast the 1992 experience, and if not why not? Which assumption is the culprit if the answer is in the negative? Is it, for instance, that inflation between 1991 and 1992 is not $14.46\% \pm 0.46\%$. The post-sample predictive testing is a type of forward validation analysis.

3.5 Family of Models

Hitherto, we have assumed a constant trend γ (gamma) across development years. This is often not the case. It is usually the case that development year trends change in the early development years and become constant somewhere in the tail. Hence, we need the flexibility to determine the trends between every two contiguous development years and whether they change significantly. Accordingly, the modelling framework allows for a γ_j parameter between development years j-1 and j.

Similarly, the modelling framework allows for an inflation parameter ι_t (iota) between payment year t-1 and payment year t and a level parameter α_w for accident year w. Each parameter is readily interpretable.

Development factors

γ_j - trend between development years j-1 and j represents the development factor (on a log scale) between development years j-1 and j.

Inflations

ι_t - trend between payment/calendar years t-1 and t represents the inflation (superimposed inflation) between payment/calendar years t-1 and t. If the data are adjusted by some kind of CPI index then the trend represents superimposed inflation, otherwise the sum of the two effects: economic and social.

Exposures

α_w - level for accident year w represents the log "exposure".

The model of Section 3.2 has one γ parameter (constant base trend along development years), one α parameter (one constant 'exposure') and three iota parameters (inflation parameters). It also has a parameter σ^2 that represents the variance of deviations about the trends. So, the paid losses of Section 3.2 were only created by six parameters.

4.0 LOSS RESERVING MYTHS AND UNCERTAINTY

In the present paper we debunk some persistent loss reserving myths including a number of misconceptions concerning uncertainty. We use real life data to demonstrate our assertions.

4.1 Uncertainty

It is part of the actuary's task to respond to uncertainty, both as a technical matter and in the presentation of results.

There are a number of components of this uncertainty.

(i) **Process uncertainty (Noise or random variation)**

Even if we know the (exact) probabilistic model including the values of the parameters generating the paid losses, there is variability or uncertainty. See model (3.3.1) and the discussion in Section 3.3 for an example of process uncertainty. (If a coin, that is unbiased, is to be tossed 100 times, we know the probabilistic model generating the number of heads but we do not know how many heads we will observe).

(ii) **Estimation error or uncertainty**

The parameters of a probabilistic model are estimated from the historical experience. Accordingly, there is uncertainty associated with the true values of the parameters. See Section 3.3.

(iii) **Trend stability or lack thereof**

Based on Andrew Harvey's car example in Section 0.3, uncertainty about future trends is related to stability or lack thereof of past trends.

Assumptions about future trends are based on identification of past trends and other sources of information, including business knowledge. See Section 5.0.

This type of uncertainty is commonly referred to as **risk parameter** uncertainty and is intimately related to the next type of uncertainty.

(iv) **The future ain't what it used to be**

This source of uncertainty may be difficult to measure statistically, but that does not mean we should ignore it. The future may be very different to the past. A pricing actuary working in 1975 would have had no way of predicting the explosion in pollution liability claims that would have occurred in the 1980's, in respect of claims incurred in 1975. However, had he been aware that there is a strong probability that the environment, legal and economic, may change, judgemental changes to the parameters of the model, could have been made.

An actuary working in 1975 would have had no way of predicting the explosion in pollution liability that would have occurred in the 1980's (in respect of claims incurred in 1975).

4.2 Myth 1

Smooth data and/or smooth age-to-age link ratios means stability of trend in (incremental) payments.

The principal objectives of the analysis of the real life data in this section are to demonstrate that:

1. Age-to-age link ratios (or development factors) based on the cumulative paid losses give no indication about the trends and random fluctuations in the (incremental) payments.

2. Smooth data may have major shifts in payment year trends.

3. A large company's run-off payments are not necessarily stable in respect of payment year trends, even though the payments may be extremely smooth (with very little random fluctuations about the trends). So, even a large company with a large exposure base can have significant problems.

The data (save a multiplicate factor in order to preserve confidentiality) come from a large insurer and are given in Appendix C1. Accident year exposures, (from memory), represent earned premium (relativities). As we shall see in the next section, the exposures are not that important.

The age-to-age link ratios presented in Appendix C2 are relatively smooth. For the early development years they tend to decrease slightly in the middle accident years and then increase in the latter payment years.

The graph below is that of the residuals of the (statistical) chain ladder model. The model adjusts the data for the average trends between every two contiguous development years and every two contiguous accident years. We use the model as a powerful diagnostic tool to determine the relative payment/calendar year trends. Note a major shift in trend around 1984-1985. The trend changes quite alarmingly from 8% to 16%.

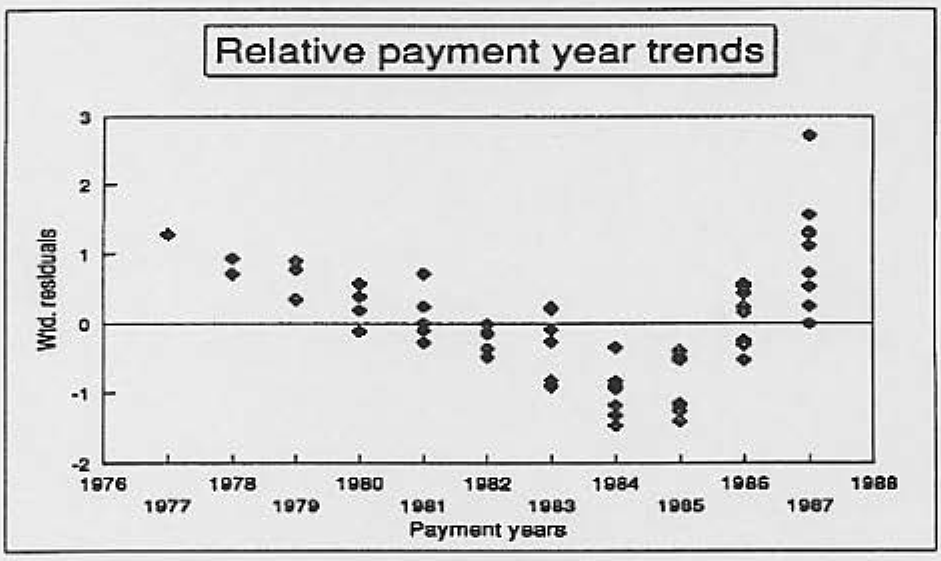


Figure 4.2.1

4.3 Myth 2

Rough data and/or age-to-age link ratios means instability in trend in incremental payments.

In the present sub-section we present some of the results of the analysis of a real (incremental) paid loss development array which we name GLD1. The exposure base is very small relative to example in Section 4.2.

The paid losses, presented in Appendix E1, are rough (i.e., have much variation), have rough link ratios or age-to-age development factors (Appendix E3), YET the payment/calendar year trend is essentially stable. Had we estimated the optimal model at year end 1989, we would have forecast accurately the trends and the distribution of deviations about the trends for the years 1990 to 1992, and moreover, the estimate of the outstanding liabilities beyond 1992 would be statistically the same as estimating the model to all the years at year end 1992.

So, loss (claims) reserve myth number two of the next Section is debunked, namely, **rough paid losses and/or rough link ratios (age-to-age development factors) imply payment year trend instability in the paid losses.**

It turns out that the identified optimal model has one (constant) trend parameter along the payment/calendar years. The estimate of this (average) inflation parameter is $12.77\% \pm 3.93\%$.

A graph of the deviations (residuals) of the observed from the fitted trends versus payment years is presented in Figure 4.3.1 below.

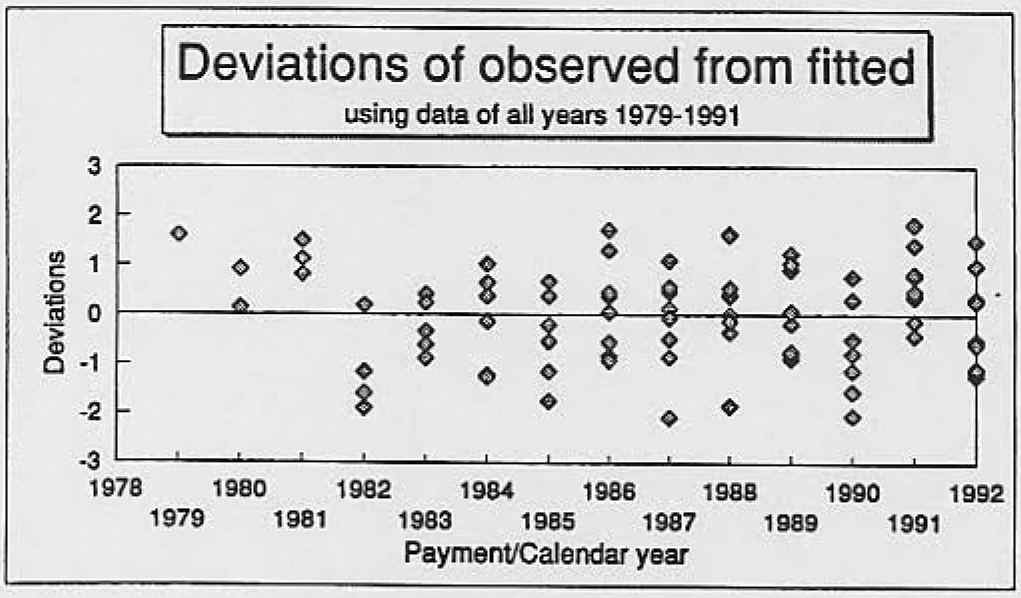


Figure 4.3.1

We observe some oscillations in the deviations about the zero line in the years 1989 to 1992. The deviations for the year 1992 appear almost symmetric about the zero line, whereas the deviations for 1991 are mostly positive, the deviations for 1990 mostly negative and those for 1989 almost symmetric.

So, when we remove the payments in 1992 from the estimation of the model we may expect the estimate of average trend (inflation) to increase slightly. Removal of 1991 and 1992 is expected to decrease the average trend and removal of 1990, 1991 and 1992, is not easy to call but we do not expect the estimate to change significantly.

We now present the validation and stability analysis results.

TABLE 4.3.1

	Years in Estimation	Inflation estimate (%)	Trend estimate along develop yrs. 4-8 (%)
(i)	1979-1992	12.77±3.93	-59.53±10.42
(ii)	1979-1991	13.83±4.28	-63.52±11.26
(iii)	1979-1990	11.68±4.54	-65.78±12.19
(iv)	1979-1989	14.18±4.63	-65.14±13.09

TABLE 4.3.2

	Years in Estimation	Estimate of resultant dev. yr. trend 4-8 (%)	Forecast (mean±s.e.) $M
(i)	1979-1992	-46.76± 9.59	202±53
(ii)	1979-1991	-49.69±10.57	212±62
(iii)	1979-1990	-54.10±11.76	164±52
(iv)	1979-1989	-50.97±12.87	222±82

Note that both the inflation and development year parameters are statistically stable. In case (iii), since we assume a much lower mean trend for the future, the mean forecast drops considerably. This essentially results from excluding one high year 1991 from the estimation and including one low year 1990 in the estimation.

So, what assumption do we invoke for the trends along the future payment/calendar years? We determined the past trend to be essentially stable. Three years earlier, at year end 1989, had we assumed a similar trend, 14.18%±4.63%, we would have forecast accurately the experience of 1990, 1991 and 1992 and our estimate of the outstanding liability beyond 1992 would not have been statistically different.

It is therefore reasonable to assume for the future 12.77%±3.93%, as in case (i). That is, actual trend in the future is a random value from a normal distribution with mean (average) 12.77% and standard deviation 3.93%. This implies that the mean payment in the future is not derived by just using a trend of 12.77%. Inflating the payments by 12.77% will yield the median payment not the mean payment. The mean payment is obtained by using inflation of 12.77% + ½ x $(3.93\%)^2$ = 12.85%.

Consider case (iv) where the model is estimated at year end 1989. Figure 4.3.2 below displays the deviations of observed from the fitted trends for the years 1979-1989 and the deviations (prediction errors) of the observations from the predicted trends for years 1990 to 1992. Note that the deviations from fitted for the years 1990 to 1992 in Figure 0.7.1 are very similar to deviations (prediction errors) from predicted for the years 1990 to 1992 in Figure 4.3.2.

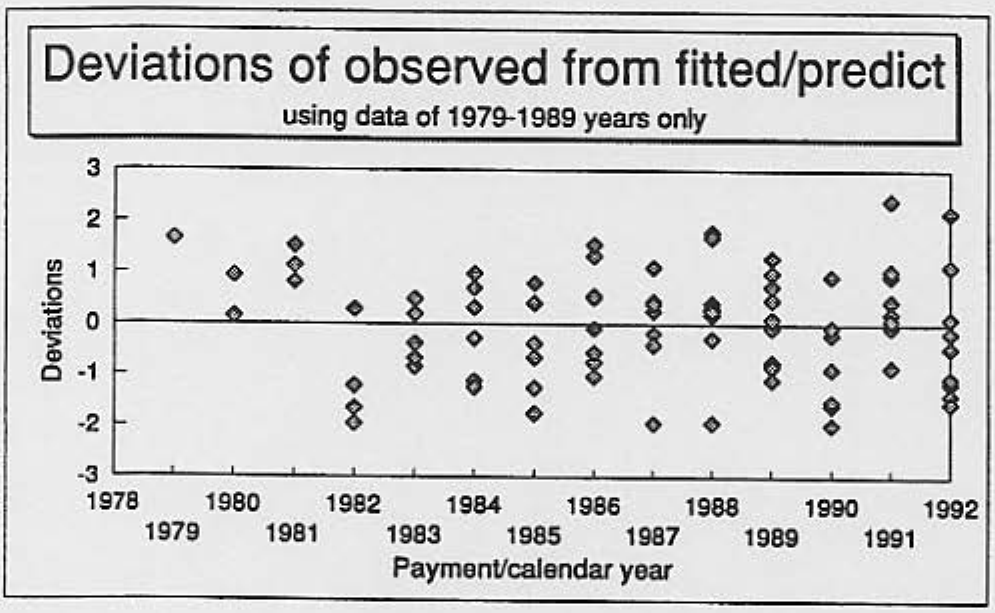


Figure 4.3.2

The model as at year end 1989 also predicts normal distributions for the deviations of the (log) observations from predicted trends for the years 1990-1992. Figure 0.7.3 below displays a normal probability plot of the deviations (prediction errors) for the years 1990-1992, as at year end 1989.

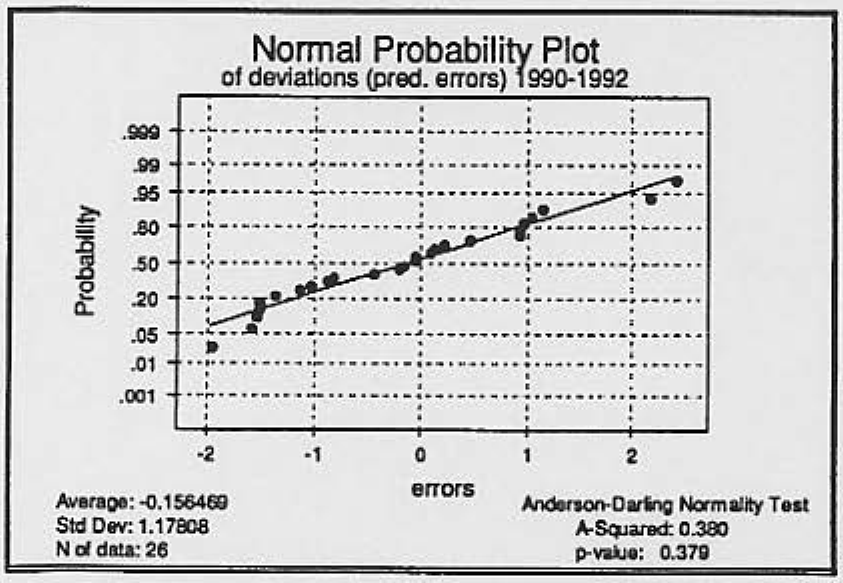


Figure 4.3.3

So there is considerable evidence that the predicted deviations for the years 1990-1992 are values selected at random from the predicted normal distribution.

Most importantly, all our calculations about the future are <u>conditional</u> on our assumptions for the future remaining true. For example, the assumption regarding future payment/calendar year inflation is 12.77%±3.93%. If, in some years in the future, inflation turns out to be 28%, say, then our assumption is violated and our subsequent calculations do not apply. This is analogous to seeing an almost straight road out the back window, but the road may not remain as straight in the future.

4.4 **Myth 3**

Increase in incremental payments is associated with increase in speed of closures (finalisations) of claims.

Often the contrary of the above statement is true. Figure 4.4.1 below displays the residuals of the statistical chain ladder applied to the incremental paid losses of CTP NSW for AMP General Insurance. Note there is a favourable trend in the early years and thereafter a relatively stable trend. In 1987 the payments are relatively lower.

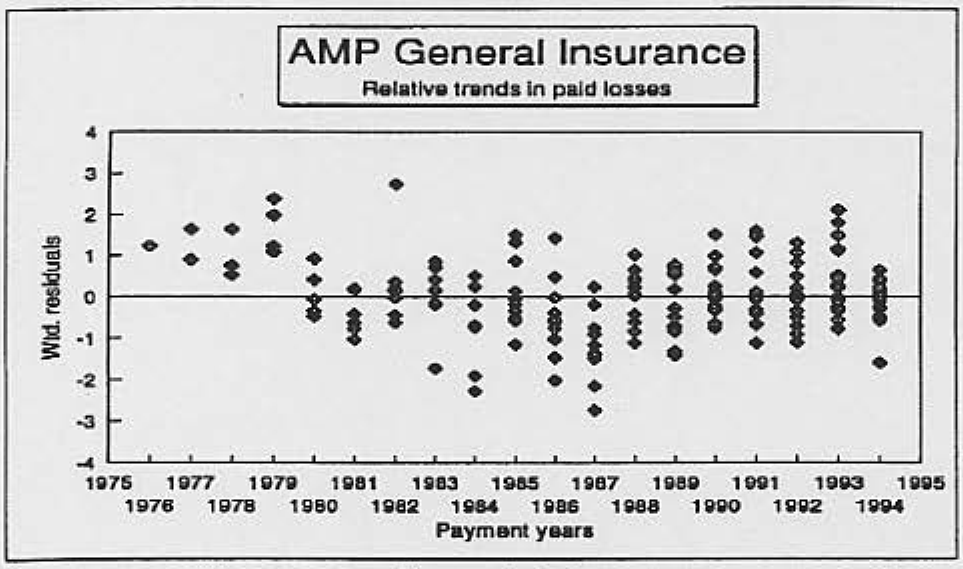


Figure 4.4.1

Figure 4.4.2 displays the relative payment/calendar year trends for the closed claim counts.

Note:

(i) Higher closures do not imply higher payments.

(ii) Closed claim counts are less stable than the paid losses in terms of trends.

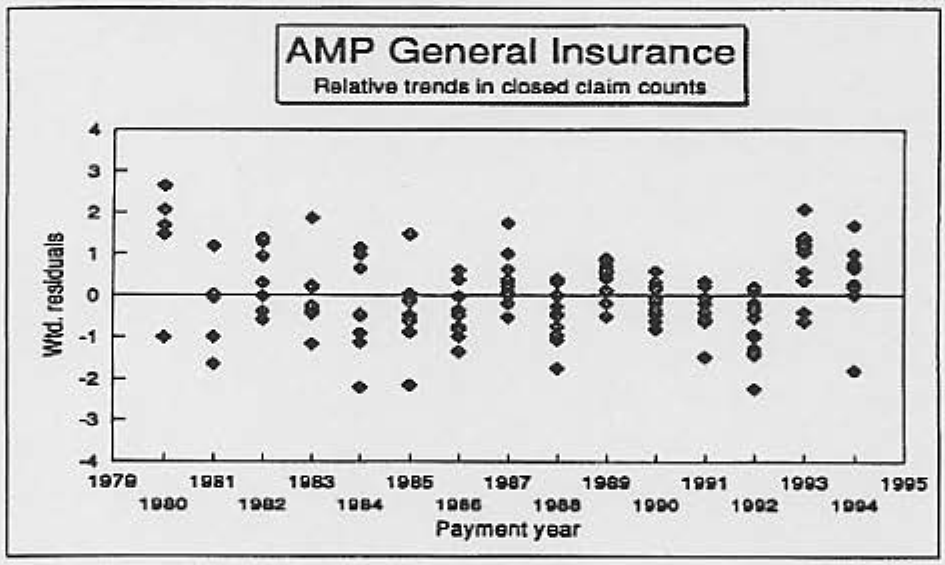


Figure 4.4.2

4.5 Myth 4

Closed (finalised) claim counts have more stable trends than incremental payments and are so better predictable.

Displays 4.4.1 and 4.4.2 demonstrate that the incremental paid losses have more stable trends than the closed claim counts and so are better predictable. Indeed, according to the author's expeience, it is rare that closed claim counts have stable payment/calendar year trends.

5.0 STABILITY, ASSUMPTIONS ABOUT FUTURE JENSEN'S INEQUALITY AND PREDICTABILITY

In this section we discuss related issues of trend stability, assumptions about the future and Jensens' inequality.

5.1 Stability

Returning to our example of Section 4.3, we ask the question whether at year end 1989 our completion of the rectangle should be materially different from completion at year end 1992. The answer, as was demonstrated, is in the negative since trends, especially in the payment year direction are stable. (Applying the Harvey motor car analogy, the road was essentially straight).

We illustrate with another four examples. (There are numerous others that occur in practice.)

Example 1: Suppose payment year trends (after adjusting for trends in the other two directions) are as depicted in Figure 5.0.1 below. The trend is stable and suppose its estimate is 10% $\pm$ 2%. How do we know that the trend is stable? Well, as we remove the more recent payment years from the estimation, the estimates of trends do not change (significantly). For example, after removing 1990 and 1991, the estimate of trend is 9.5% $\pm$ 2.1%, say. Alternatively, we could estimate a new trend parameter from 1989-1991 and examine whether the trend has changed significantly.

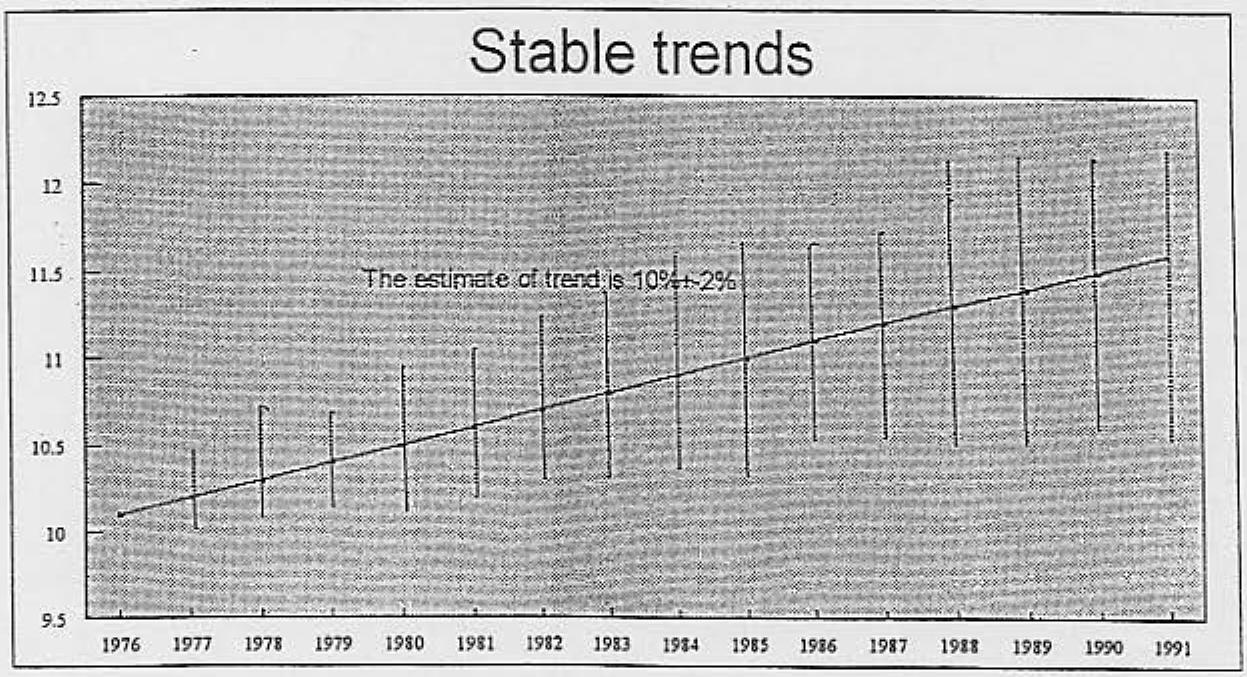


Figure 5.0.1

Typically, if the payment/calendar year trend is stable, the model will also validate well. Here our estimates of outstanding payments do not change significantly as we omit recent years.

Example 2: Consider the payment year trends depicted in Figure 5.0.2 below.

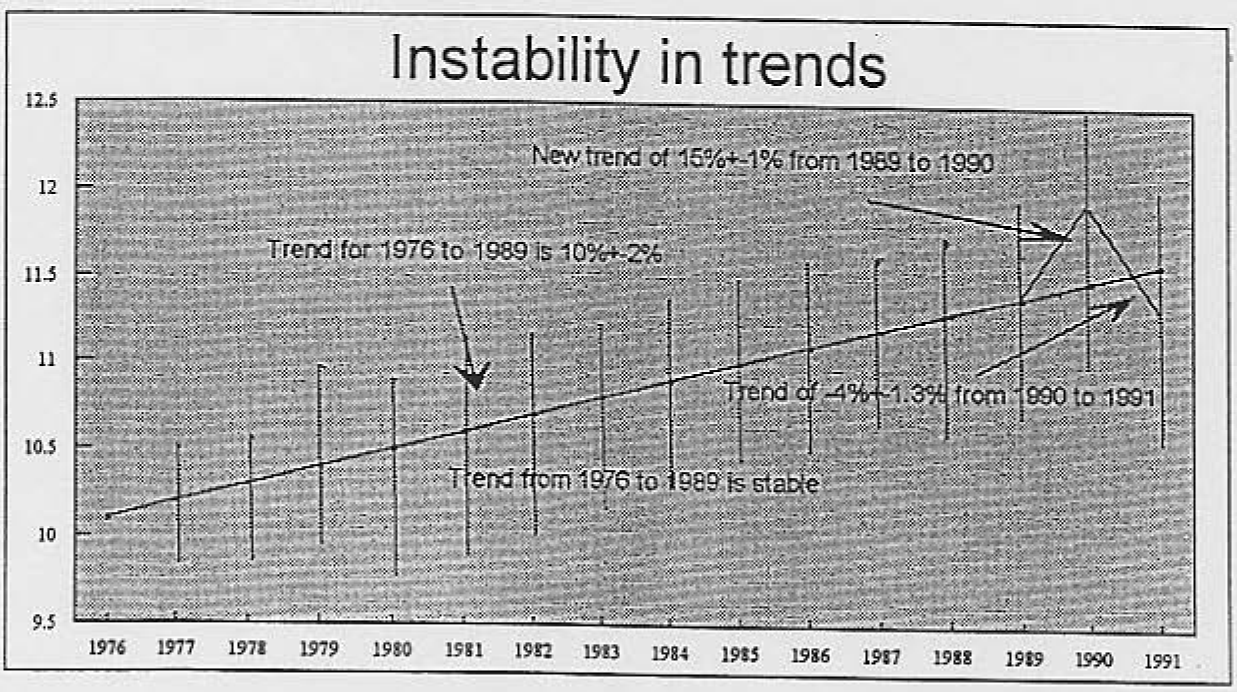


Figure 5.0.2

The trend in the years 1976 to 1989 is relatively stable. Its estimate is 10% ± 2%, say. However, the trend from 1989 to 1990 is higher at 15% (± 1%) and from 1990 to 1991 it is -4% (± 1.3%), say. This information is extracted from the "optimal" statistical model. The shifts in trends is a property of the data (determined through the model). A question now emerges as to which trend assumption do we make for the future, first in the absence of any other information. It would be foolhardy to assume the estimate between the last two years of -4% ± 1.3%. The most reasonable assumption (for the future) is a mean trend of 10% with a standard deviation of 2%, that which was estimated for the years 1976-1989.

Suppose we also have access to another data type, the number of closed claims development array. We find utilising the modelling framework that the additional 5% above the 10% between 1989 and 1990 can be explained by a corresponding increase in speed of closures of claims and the -15% from 1990 to 1991 below the 10% from 1976-1989 can be explained by a corresponding decrease in the speed of closures of claims. What assumption about future trends in payments should we adopt then? I would still recommend 10% ± 2% for the future. That's a decision based on my judgement and experience. The instability in trends in the last few years means that the model will not validate well. At year end 1990, we would not have forecast the distributions for 1991, for example.

Example 3: It is possible to have a transient change in trend. Consider Figure 5.0.3. The business has been moving along 10% ± 2% but between the last two calendar years 1990 and 1991 the trend increases to 20% ± 3%. What do we assume for the future? Well, that depends on the explanation for the increase in trend. Suppose its a "transient" change that can be explained by a new level of benefits that apply retrospectively. Then it is reasonable to assume 10% ± 2% for the future. Suppose instead that subsequent to analysis of claims closed triangle, the trend change is explained by increase in severities. That's a problem, because this means that it is now more likely that the new trend will continue.

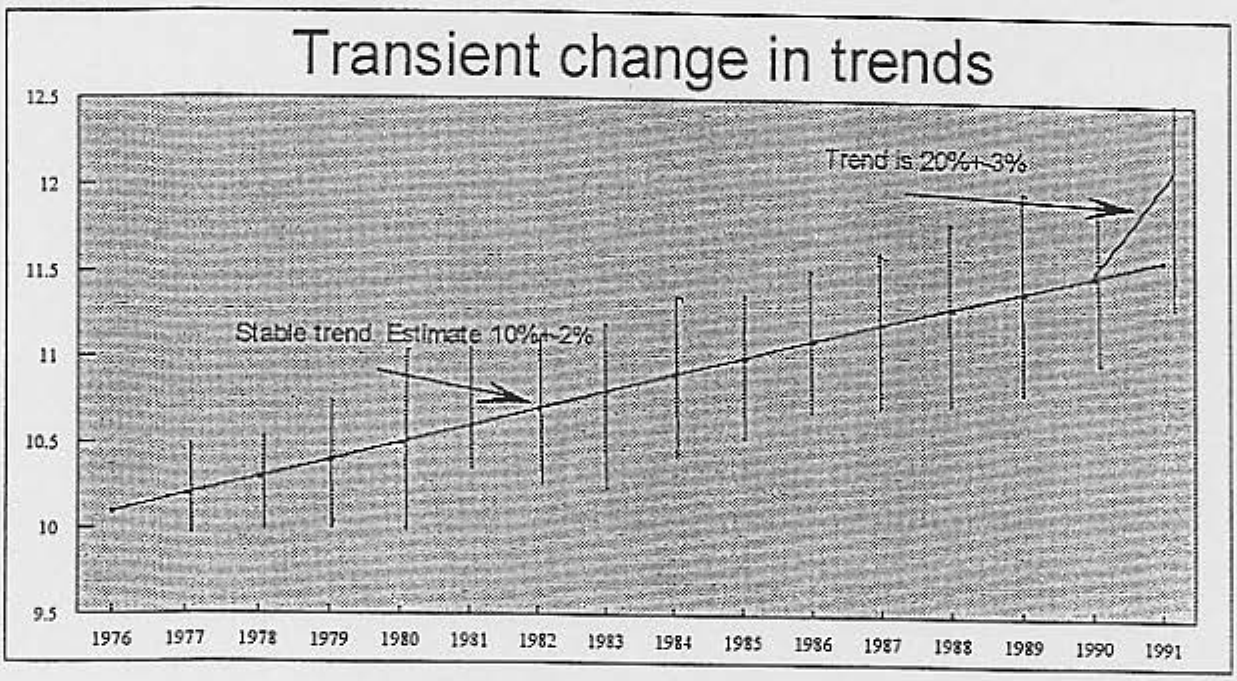


Figure 5.0.3

So the decision making process about the future becomes more complicated when trends are unstable. We are talking about trends in the (incremental) payments not age-to-age link ratios.

The last example illustrates an 'unpredictable' loss development array.

Example 4: The payment year trends are depicted in Figure 5.0.4 below.

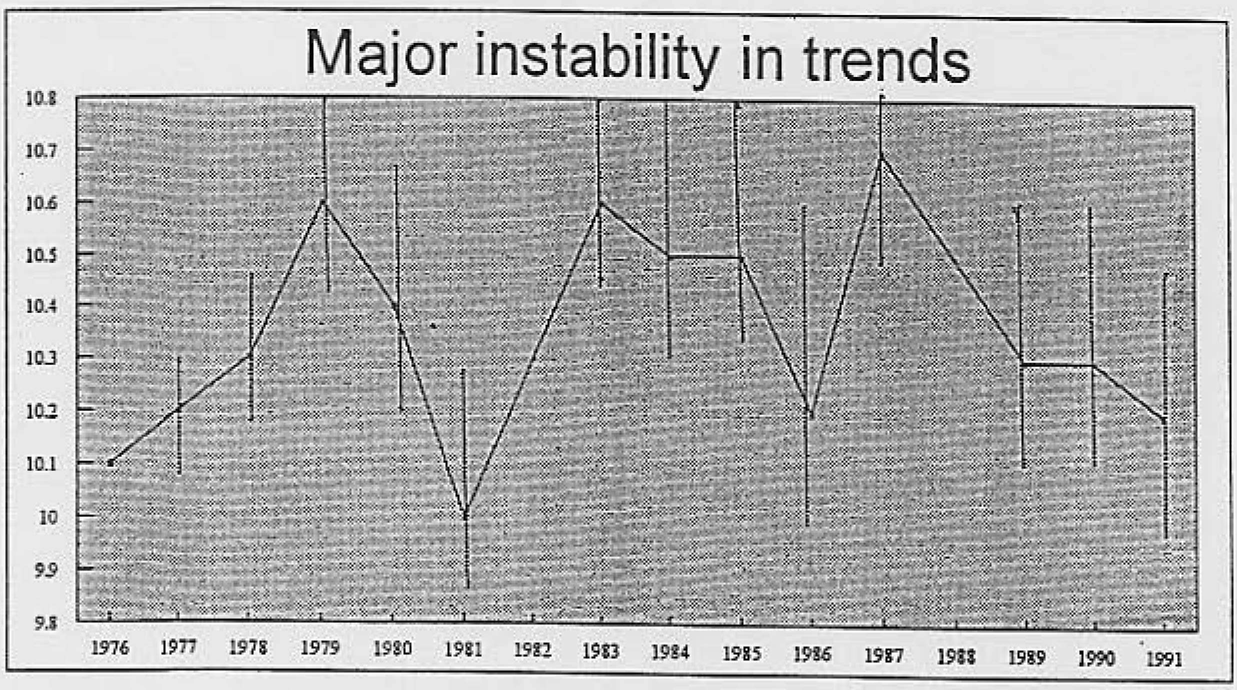


Figure 5.0.4

Note the instability in the trends. At year end 1989, would anyone be able to predict a flat trend for the next year and a downward trend for the following year?

Here, maybe, one could calculate $\hat{\imath}$, a weighted average of trends estimated in the past with a weighted variance $\hat{\sigma}^2$ and assume for the future a mean trend of $\hat{\imath}$ with standard deviation of trend $\hat{\sigma}$. Since $\hat{\sigma}$ will be relatively large, mean forecasts will be well above the median forecasts and the standard deviation of the distributions will be relatively large.

It is instructive to relate the foregoing discussion with the quote from Harvey [9] given at the end of Section 2.2.

5.2 Assumptions about the Future and Jensen's Inequality

We demonstrated in Section 5.1 that if payment/calendar year trend has been stable, then the assumption about the future trend is relatively straightforward, although we may not be absolutely sure how long the trend may continue. It may be helpful to determine what factors are driving the trend, in order to make appropriate assumptions about the future. This determination may be based on analysing data types other than the paid losses and knowledge of the business.

If on the other hand, payment/calendar year trend has been unstable, as in examples 2 and 3 of Section 5.1, the nature of the instability, analysis of other data types and business knowledge would be critical in formulating assumptions about the future. Of course, business knowledge should be combined with the objective facts in formulating sound judgment.

Application of Jensen's inequality tells us that the mean payment using a variable inflation rate is higher than if we just inflate by the mean inflation. Indeed, ignoring this result is dangerous, especially, if the standard error of inflation is large.

6.0 CTP NSW AGGREGATE EXPERIENCE

6.1 Background

The NSW Government established a new Scheme, which commenced in July 1989, for compensating people injured as a result of the fault of others in motor vehicle accidents. Compulsory Third Party Personal Injury Insurance (or CTP) provides funds to compensate people injured in motor vehicle accidents.

The Scheme includes the following features:

- Each registered owner of a vehicle in NSW is required to insure, with an insurer licensed under The Motor Accidents Act 1988.
- Insurers are licensed under the Act and must file with the Motor Accident Authority, at least once a year, a full set of premiums it proposes to charge for third party policies.

The Scheme has only five years experience in its current form, and accordingly the principal loss development arrays analysed have a quarterly sampling period.

6.2 Relevant Findings Based on Probabilistic Models

The following findings are relevant:

1. The incremental paid (loss) experience has a high inflation (combined AWE + superimposed) rate of 8.42% $\pm$ 1.38% (continuous per quarter) since payment quarter year 2-91. The inflation has been relatively stable, save for some seasonality.
2. The high inflation rate is <u>not</u> explained by the speed of closures of claims, although there has been a relatively small increase in closure rates especially in the early development quarter years.
3. There is a small increase in the number of claims notified, but only for the early development quarter years.

4. The high trends (inflation) in the case estimates and incurred losses parallel those in the paid losses.

5. There is some evidence that the tail in the incremental paid loss experience is beginning to decay.

6. Projections of payments outstanding based on quarterly incremental paid losses data are statistically the same as projections based on yearly incremental paid losses data.

6.3 Inflation in the Paid Loss Experience

The graph presented below (Figure 6.3.1) depicts the relative payment quarter year (diagonal) trends after adjusting the paid loss data only in the development quarter year direction.

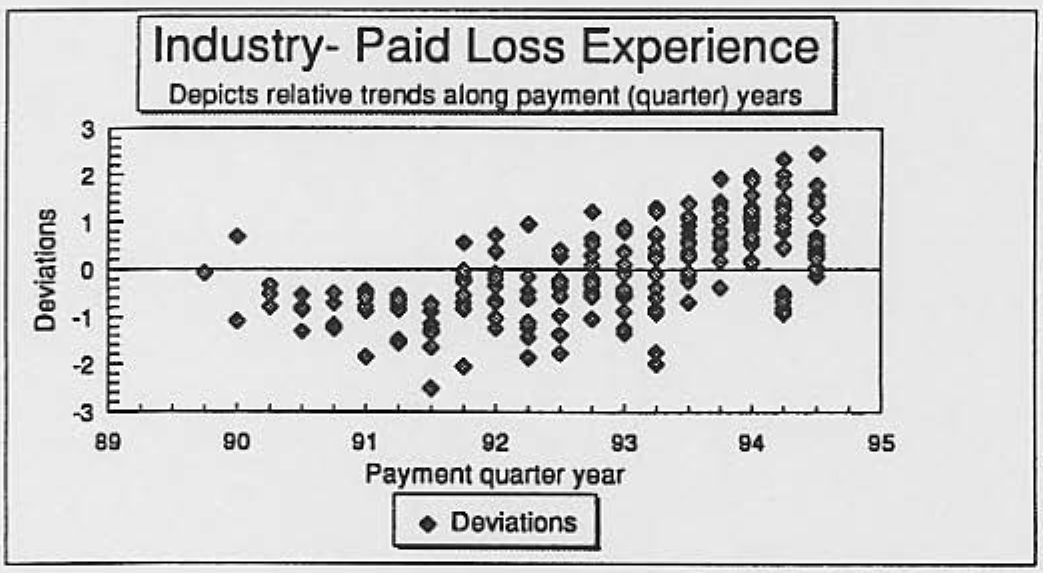


Figure 6.3.1

From payment quarter 3-89 to 2-91 (first two years) there is a relatively favourable experience.

Thereafter, the trend is relatively stable but very steep, indicating a <u>high</u> level of inflation.

6.4 Inflation in the Paid Losses and Claim Settlement Rates

Much of the high trend in the paid loss experience is not explained by the trends (speed of closure) in the closed claim count experience. Figure 6.3.2 below depicts the trends along the payment quarter years (diagonals) in the closed claim count experience.

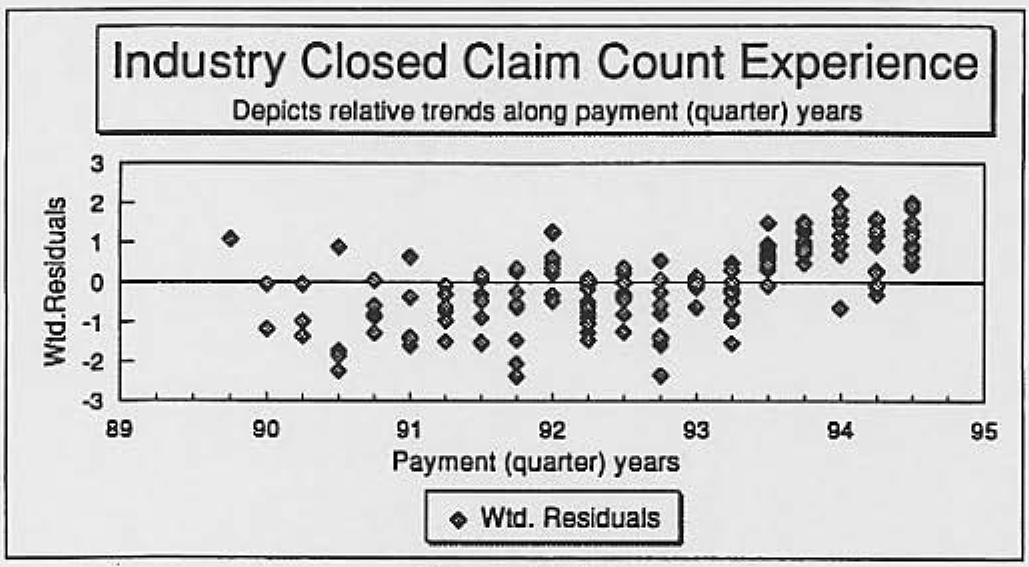


Figure 6.4.1

Note positive trend in closed claims only from payment quarter 1-93 to 4-93, which also partly explains the higher payments in those quarter years. However, the higher payments in other years including 2-94 are not explained by the speed of closures of claims. The trends in the closed claim counts are not as high as those in the paid losses and are mainly in the early development quarter years where payments are low.

Accordingly we now present two displays showing the relative payment quarter year trends in the paid losses and the closed claim counts beyond development quarter 4, respectively.

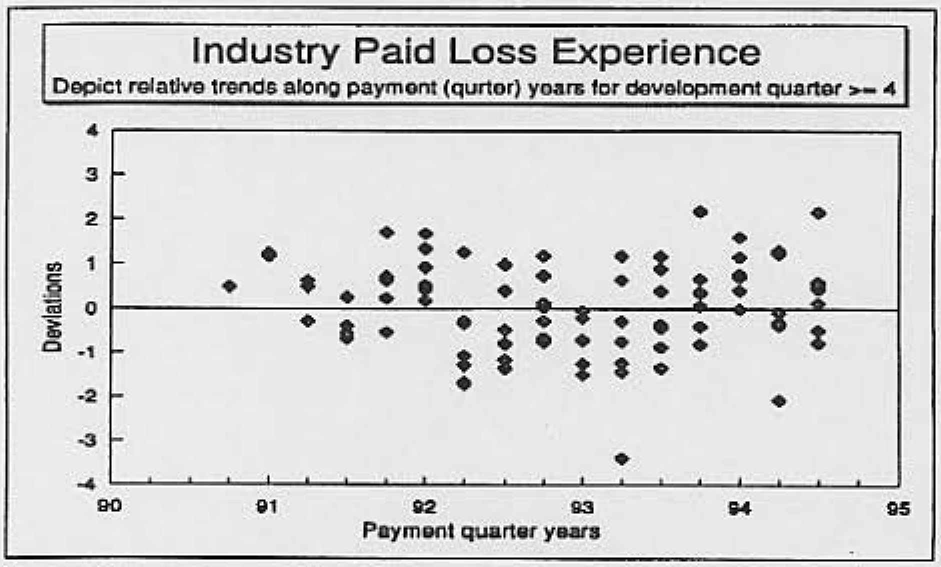


Figure 6.4.2

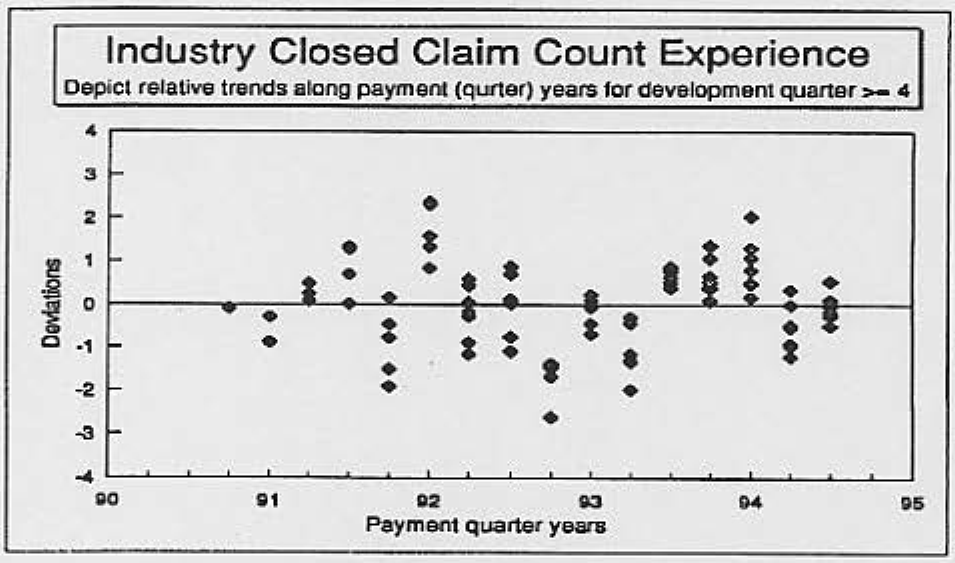


Figure 6.4.3

We observe:

- Paid losses experience has more stable trend and accordingly is better predictable.

- The trends in the paid losses are <u>not</u> positively correlated with the trends in the closed claim counts.

The latter observation means that Taylor's See Saw hypothesis is valid: "The faster claims are closed the less is paid per claim". As a result of the validity of this hypothesis we expect the first observation to be true.

6.5 Validation Analysis and Stability of Inflation

We examine the behaviour of the estimated model as we remove payment quarter years (diagonals) from the estimation process. The important question is whether the model estimated one to seven payment quarter years ago would have predicted the most recent experience.

If we estimate the best model a number of times, each time removing more of the recent experience, the following results are obtained.

Table 6.5.1

Projections to quarter 20			
Quarter Years used in estimation	Trend Estimate Since 1-92 (Inflation) %	Mean Forecast $M	Standard Error $M
3-89 to 2-94	8.42 ± 1.38	2,963	225
3-89 to 1-94	9.01 ± 1.59	2,931	282
3-89 to 4-93	10.25 ± 1.56	3,459	393
3-89 to 3-93	9.94 ± 1.82	3,236	461
3-89 to 2-93	8.24 ± 2.09	2,654	457
3-89 to 1-93	6.60 ± 2.33	2,356	490
3-89 to 4-92	7.70 ± 2.49	2,675	686
3-89 to 3-92	6.68 ± 2.77	2,804	945

Changes in mean projections as years are removed from the estimation process are closely reflected by the change in assumption as to future "inflation". The assumed future payment quarter year inflation is based on that estimated since payment quarter year 1-92.

Note from Table 6.5.1 above, that had the best model been estimated at payment (quarter) year end 3-92, the estimated total outstanding of $2,804M ± $945M is statistically not different to $2,963M ± $225M, that obtained by the model using the experience to payment quarter year end 2-94. Indeed, the two answers are remarkably close especially that $2,804M ± $945M is obtained after removing 56% of the most recent experience.

We now explore whether the model estimated at quarter year end 3-92 "predicts" the subsequent paid losses in payment quarter years of 4-92 to 2-94.

Figure 6.5.1 displays the deviations based on the model estimated at payment quarter year end 3-92. For payment quarter years 3-89 to 3-92, the deviations are the observed values minus the fitted values, whereas for payment quarter years 4-92 to 2-94 (last seven payment quarter years!) the deviations represent the prediction errors. It is remarkable how the prediction errors for the last seven payment quarter years are centred around zero.

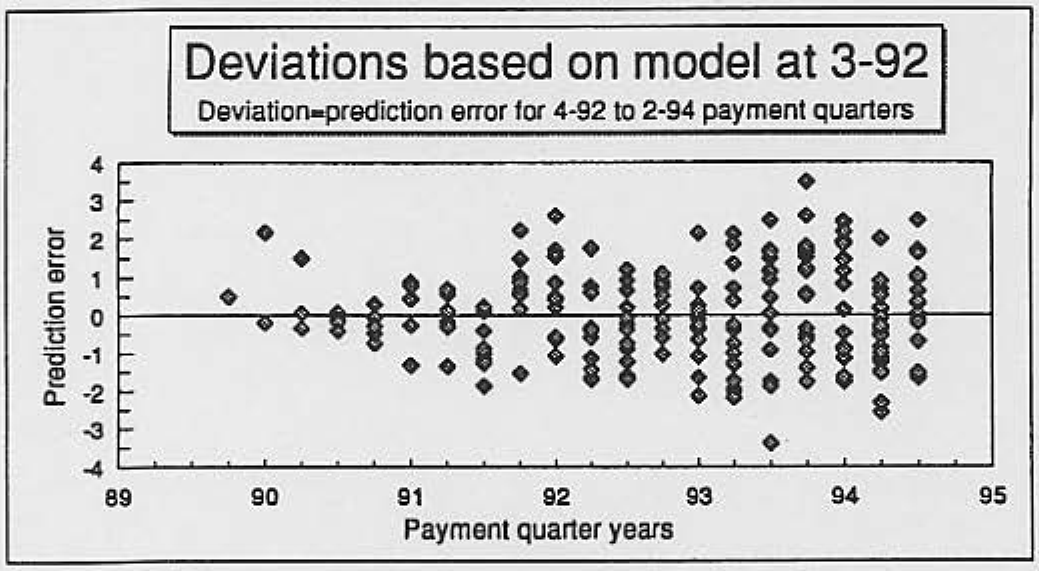


Figure 6.5.1

Figure 6.5.2 presents the prediction errors for payment quarter years 4-92 to 2-94 based on the model estimated at year end 3-92. Note the slight curvature due to higher payments in 3-93 and 4-93 and lower payments in 1-94.

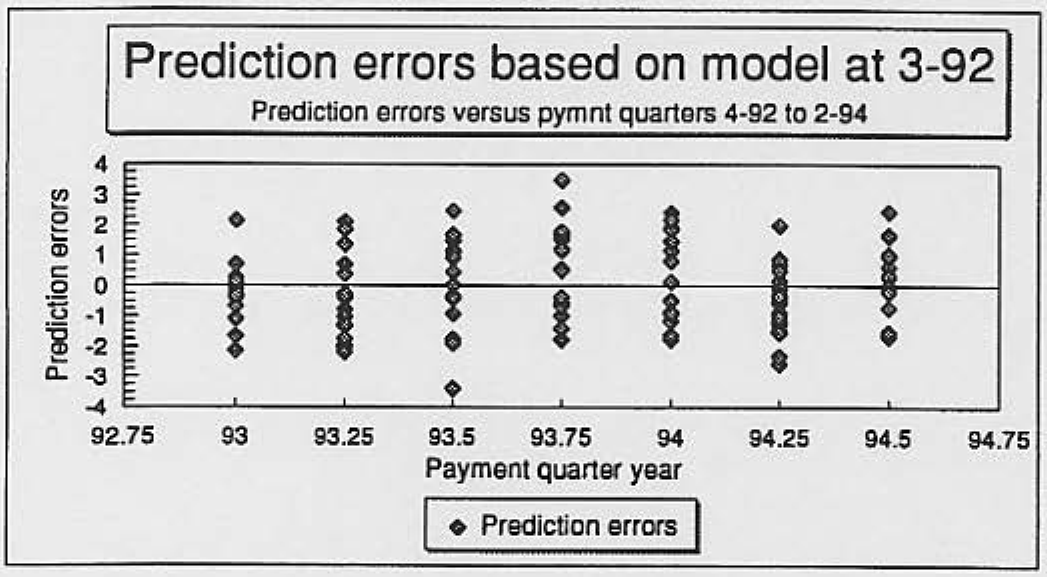


Figure 6.5.2

Finally, the normality plot for the prediction errors of observations in payment quarters 4-92 to 2-94, based on model estimated at payment quarter year 3-92 is displayed below and one can see it is in good shape.

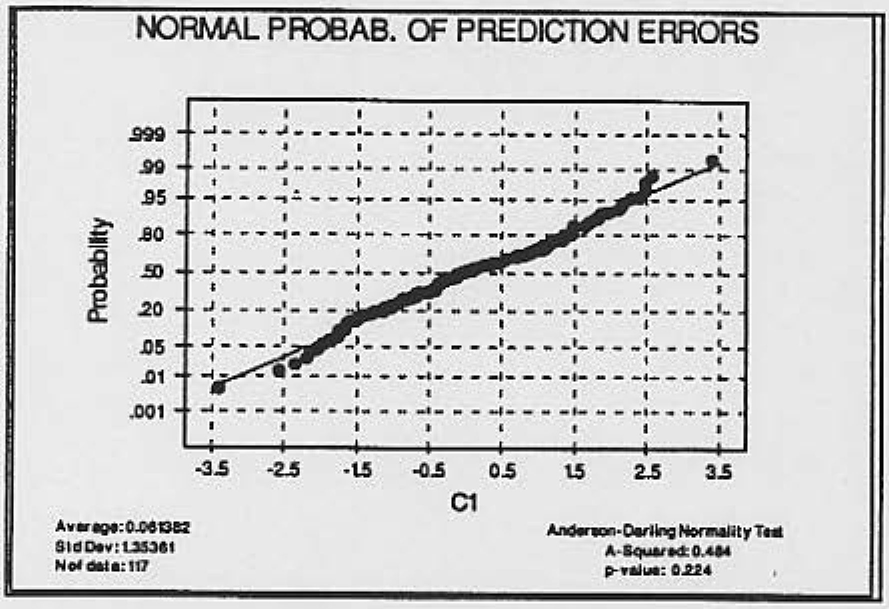


Figure 6.5.3

By way of summary:

- The paid loss experience contains a high inflation rate not explained by the speed of closures of claims.
- The high inflation rate has existed since 2-92 and has been essentially stable, save for some seasonality.
- The model estimated at payment quarter year end 3-92, predicts accurately the next seven quarter payment years experience and yields the same outstanding claims estimate statistically as the model estimated at quarter year end 2-94.

6.6 Premium and Future Inflation

If we apply the Harvey [9] motor car analogy, even though we have identified an inordinate high inflation rate in the five year CTP experience, we are not guaranteed it will continue. Indeed, given that the principal reason for the inflation is a continued trend towards litigation as an avenue by claimants for higher award payments one would expect and hope to reach 'saturation' in the near future.

In any event, the current pure premium of approximately $190 (per vehicle), as advised by a number of actuarial consultants is substantially too low. Assuming a discount rate of 10% - 11% p.a., it could only be substantiated by a less than 4% combined AWE + Superimposed inflation in the future, and moreover we would have to ignore Jensen's inequality.

The principal reason for the high inflation seems to be a continued trend towards litigation as an avenue by claimants for higher award payments, although some practitioners are arguing that the inflation is principally driven by increase in speed of closures of claims.

7.0 CONCLUSIONS

We have argued that the four components of interest regarding a loss development array are the trends in the three directions and the distributions of the deviations (random fluctuations) about the trends.

A MODELLING FRAMEWORK was introduced where each model contained therein possesses the four components of interest. The modelling approach offers the actuary a way of fitting (estimating) distributions to the cells in a loss development array and predicting (forecasting) distributions for future years that affords numerous advantages including:

- simplicity;
- clarity of assumptions;
- testing of assumptions;
- assessment of loss reserve variability;
- asset/liability matching;
- model maintenance and updating.

We showed how the identified optimal statistical model for the (incremental) payments conveys information about the loss experience to date. In applying the model to predicting distributions of future payments the actuary may (need to) adjust some of the parameters to reflect knowledge about the business and to incorporate his view of the future. View of the future may be based on analysis of other data types, especially if there are instabilities in the trends in the paid losses in the recent payment years.

A prediction interval computed from the forecast distributions is <u>conditional</u> on the assumptions made about the future remaining true.

In passing we have debunked a number of pervasive loss reserving perceptions concerning data types, age-to-age link ratios, stability, forecasting and regression.

Methods based on age-to-age link ratios do not (and cannot) separate trends from random fluctuations and moreover do not satisfy the <u>basic fundamental</u> property of additivity of trends.

REFERENCES

[1] Abraham, B. and Ledholter, J. (1983). Statistical Methods for Forecasting, John Wiley, New York.

[2] Bailey R.A., "Sampling theory in casualty insurance", PCAS, 1942, pp.29-50.

[3] Catterjee, S. and Price, B. (1977). Regression Analysis by Example. John Wiley, New York.

[4] Christofidies, S. (1980). "Regression models based on log-Incremental Payments". Institute of Actuaries Loss Reserving Manual, Volume 2., Chapter 2.

[5] Covey, S.R. (1989). The 7 Habits of Highly Effective People. Fireside, Simon & Schuster.

[6] Foundations of Casualty Actuarial Science, 1989, Casualty Actuarial Science, New York.

[7] Fisher, W.H. and Lange, J.T. (1973). "Loss reserve testing: a report year approach", PCAS, 60, pp.189-207.

[8] Friedman, M. (1953). "The methodology of positive economics", in Essays in Positive Economics, University of Chicago Press, pp.3-43.

[9] Gauss, K.F. (1809). Theoria Motus corpurum coelstium, Werke 7, Hamburg.

[10] Harvey, A.C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press, New York.

[11] Laurenzano, V. (1992). "Balancing capital with risks", Best's Review, June, 1992.

[12] Loss Reserving Manual, 1990, Institute of Actuaries, London.

[13] Muth, J.F. (1960). "Optimal properties of exponentially weighted forecasts". J. Amer, Statist. Assoc., 55, 299-306.

[14] Popper, K.F. (1959). The Logic of Scientific Discovery. Hutchinson.

[15] Zehnwirth, B. (1985). "Linear Filtering and Recursive Credibility Estimation", ASTIN Bulletin, Vol. 15, No. 1, 19-35.

Appendix A1 (Lotus Worksheet M3INF)

Model is p = exp(alpha-.2d) with no error or randomness
alpha = 11.51293

Year/delay

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080	9072	7427
1979	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080	9072	
1980	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080		
1981	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534			
1982	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530				
1983	100000	81873	67032	54881	44933	36788	30119	24660	20190					
1984	100000	81873	67032	54881	44933	36788	30119	24660						
1985	100000	81873	67032	54881	44933	36788	30119							
1986	100000	81873	67032	54881	44933	36788								
1987	100000	81873	67032	54881	44933									
1988	100000	81873	67032	54881										
1989	100000	81873	67032											
1990	100000	81873												
1991	100000													

Appendix A2 (Lotus Worksheet M3INF)

y=log(p) plus .1 inf. from 1978-82, .3 inf. from 1982-83 and .15 inf. from 1983-91

Year\delay

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	11.5129	11.4129	11.3129	11.2129	11.1129	11.2129	11.1629	11.1129	11.0629	11.0129	10.9629	10.9129	10.8629	10.8129
1979	11.6129	11.5129	11.4129	11.3129	11.4129	11.3629	11.3129	11.2629	11.2129	11.1629	11.1129	11.0629	11.0129	
1980	11.7129	11.6129	11.5129	11.6129	11.5629	11.5129	11.4629	11.4129	11.3629	11.3129	11.2629	11.2129		
1981	11.8129	11.7129	11.8129	11.7629	11.7129	11.6629	11.6129	11.5629	11.5129	11.4629	11.4129			
1982	11.9129	12.0129	11.9629	11.9129	11.8629	11.8129	11.7629	11.7129	11.6629	11.6129				
1983	12.2129	12.1629	12.1129	12.0629	12.0129	11.9629	11.9129	11.8629	11.8129					
1984	12.3629	12.3129	12.2629	12.2129	12.1629	12.1129	12.0629	12.0129						
1985	12.5129	12.4629	12.4129	12.3629	12.3129	12.2629	12.2129							
1986	12.6629	12.6129	12.5629	12.5129	12.4629	12.4129								
1987	12.8129	12.7629	12.7129	12.6629	12.6129									
1988	12.9629	12.9129	12.8629	12.8129										
1989	13.1129	13.0629	13.0129											
1990	13.2629	13.2129												
1991	13.4129													

Appendix A3 (Lotus Worksheet M3INF)

Cumulative data (on a $ scale) derived from Appendix A2

100000	190484	272357	346439	413471	487552	558021	625053	688816	749469	807164	862045	914250	963908
110517	210517	301001	382874	473358	559428	641302	719182	793263	863732	930764	994527	055180	
122140	232657	332657	443174	548302	648302	743425	833908	919979	001852	1079732	1153814		
134986	257126	392112	520515	642655	758838	869355	974482	1074482	1169605	1260089			
149182	314055	470886	620068	761975	896961	1025363	1147504	1263687	1374204				
201375	392929	575141	748467	913339	1070170	1219352	1361259	1496245					
233965	456519	668219	869594	1061148	1243360	1416685	1581557						
271828	530399	776359	1010324	1232878	1444578	1645954							
315819	616236	902001	1173829	1432400	1678360								
366930	715964	1047976	1363795	1664212									
426311	831831	1217574	1584504										
495303	966450	1414619											
575460	1122855												
668589													

Appendix A4 (Lotus Worksheet M3INF)

Age-to- age link ratios of the cumulative losses of Appendix A3

1.904837	1.429816	1.272002	1.193488	1.179170	1.144535	1.120124	1.102011	1.088054	1.076981	1.067992	1.060558	1.054316
1.904837	1.429816	1.272002	1.236327	1.181830	1.146351	1.121440	1.103008	1.088834	1.077607	1.068505	1.060986	
1.904837	1.429816	1.332224	1.237213	1.182381	1.146726	1.121712	1.103213	1.088994	1.077736	1.068611		
1.904837	1.524979	1.327463	1.234652	1.180786	1.145639	1.120925	1.102618	1.088529	1.077362			
2.105170	1.499375	1.316812	1.228856	1.177152	1.143152	1.119119	1.101248	1.087456				
1.951229	1.463726	1.301361	1.220279	1.171712	1.139400	1.116378	1.099162					
1.951229	1.463726	1.301361	1.220279	1.171712	1.139400	1.116378						
1.951229	1.463726	1.301361	1.220279	1.171712	1.139400							
1.951229	1.463726	1.301361	1.220279	1.171712								
1.951229	1.463726	1.301361	1.220279									
1.951229	1.463726	1.301361										
1.951229	1.463726											
1.951229												

Appendix A5 (Lotus Worksheet M3IR5)

Random error random from Normal with mean 0

Year\delay

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	0.083	0.075	-0.076	-0.065	-0.188	-0.164	-0.101	0.078	0.021	0.029	0.005	0.03	-0.073	-0.241
1979	-0.113	-0.049	-0.086	-0.123	0.148	0.09	-0.06	-0.099	-0.032	0.096	0.028	0.1	-0.331	
1980	0.086	-0.007	-0.037	0.17	0.071	-0.138	0.047	0.022	0.036	0.003	0.004	0.058		
1981	-0.071	0.147	0.067	-0.028	-0.132	0.049	0	-0.117	-0.042	0.026	-0.078			
1982	0.081	0.059	0.073	0.048	0.025	0.029	-0.023	-0.133	-0.044	0.066				
1983	0.117	0.059	-0.017	-0.081	-0.051	-0.024	-0.048	0.124	0.033					
1984	-0.024	-0.026	0.134	0.214	0.071	0.193	-0.022	0.012						
1985	0.022	0.015	0.076	-0.028	-0.004	0.155	0.032							
1986	-0.043	0.181	0.184	-0.192	-0.16	-0.048								
1987	0.07	0.106	0.144	0.032	-0.102									
1988	0.056	-0.195	0.032	0.041										
1989	0.145	0.187	-0.159											
1990	0.001	-0.153												
1991	-0.142													

Appendix A6 (Lotus Worksheet M3IR5)

Sum of data in Appendices A2 and A5 to produce trends + randomness

Year\delay

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	11.5959	11.4879	11.2369	11.1479	10.9249	11.0489	11.0619	11.1909	11.0839	11.0419	10.9679	10.9429	10.7899	10.5719
1979	11.4999	11.4639	11.3269	11.1899	11.5609	11.4529	11.2529	11.1639	11.1809	11.2589	11.1409	11.1629	10.6819	
1980	11.7989	11.6059	11.4759	11.7829	11.6339	11.3749	11.5099	11.4349	11.3989	11.3159	11.2669	11.2709		
1981	11.7419	11.8599	11.8799	11.7349	11.5809	11.7119	11.6129	11.4459	11.4709	11.4889	11.3349			
1982	11.9939	12.0719	12.0359	11.9609	11.8879	11.8419	11.7399	11.5799	11.6189	11.6789				
1983	12.3299	12.2219	12.0959	11.9819	11.9619	11.9389	11.8649	11.9869	11.8459					
1984	12.3389	12.2869	12.3969	12.4269	12.2339	12.3059	12.0409	12.0249						
1985	12.5349	12.4779	12.4889	12.3349	12.3089	12.4179	12.2449							
1986	12.6199	12.7939	12.7469	12.3209	12.3029	12.3649								
1987	12.8829	12.8689	12.8569	12.6949	12.5109									
1988	13.0189	12.7179	12.8949	12.8539										
1989	13.2579	13.2499	12.8539											
1990	13.2639	13.0599												
1991	13.2709													

Appendix A7 (Lotus Worksheet M3IR5)

Incremental paids derived from Appendix A6

1978	108651	97529	75879	69418	55542	62875	63697	72468	65114	62436	57983	56551	48528	39023
1979	98706	95216	83025	72396	104914	94174	77103	70538	71747	77567	68934	70467	43560	
1980	133106	109743	96365	130993	112860	87108	99698	92494	89224	82117	78190	78504		
1981	125731	141478	144336	124854	107034	122015	110514	93517	95885	97626	83692			
1982	161765	174888	168704	156514	145495	138954	125480	106927	111179	118054				
1983	226364	203191	179136	159835	156670	153108	142187	160637	139511					
1984	228411	216837	242050	249422	205644	220996	169549	166858						
1985	277868	262472	265375	227499	221660	247187	207918							
1986	302519	360015	343485	224336	220334	234427								
1987	393525	386054	383425	326081	271278									
1988	450855	333667	398276	382277										
1989	572576	568013	382277											
1990	576021	469724												
1991	580068													

<u>Appendix A8</u> (Lotus Worksheet M3IR5)

Cumulative paids from Appendix A7

1978	108651	206180	282059	351477	407019	469894	533591	606059	671173	733609	791592	848143	896671	935694
1979	98706	193922	2769473	49343	454257	548431	625534	696072	767819	845386	914320	984787	1028347	
1980	133106	242849	339214	470207	583067	670175	769873	862367	951591	1033708	1111898	1190402		
1981	125731	267209	411545	536399	643433	765448	875962	969479	1065364	1162990	1246682			
1982	161765	336653	505357	661871	807366	946320	1071800	1178727	1289906	1407960				
1983	226364	429555	608691	768526	925196	1078304	1220491	1381128	1520639					
1984	228411	445248	687298	936720	1142364	1363360	1532909	1699767						
1985	277868	540340	805715	1033214	1254874	1502061	1709979							
1986	302519	662534	1006019	1230355	1450689	1685116								
1987	393525	781579	1165004	1491085	1762363									
1988	450855	784522	1182798	1565075										
1989	572576	1140589	1522866											
1990	576021	1045745												
1991	580068													

Appendix A9 (Lotus Worksheet M3IR5)

Age-to-age factors (link ratios) of the cumulative payments

1978	1.897635	1.368023	1.246111	1.158024	1.154476	1.135556	1.135811	1.107438	1.093025	1.079038	1.071439	1.057216	1.043519
1979	1.964642	1.428136	1.261407	1.300318	1.207314	1.140588	1.112764	1.103074	1.101022	1.081541	1.077070	1.044232	
1980	1.824478	1.396810	1.386166	1.240021	1.149396	1.148764	1.120141	1.103464	1.086294	1.075640	1.070603		
1981	2.125243	1.540161	1.303378	1.199541	1.189631	1.144378	1.106759	1.098903	1.091636	1.071962			
1982	2.081123	1.501121	1.309709	1.219823	1.172107	1.132597	1.099763	1.094321	1.091521				
1983	1.897629	1.417026	1.262588	1.203857	1.165487	1.131861	1.131616	1.101012					
1984	1.949328	1.543629	1.362902	1.219536	1.193454	1.124361	1.108850						
1985	1.944592	1.491125	1.282356	1.214534	1.196981	1.138421							
1986	2.190057	1.518441	1.222993	1.179081	1.161597								
1987	1.986097	1.490577	1.279896	1.181933									
1988	1.740076	1.507667	1.323197										
1989	1.992030	1.335157											
1990	1.815463												
1991													

One cannot determine changing calendar year trends from the age-to-age link ratios.

APPENDIX C1

	DELAY										
ACC. YEAR	0	1	2	3	4	5	6	7	8	9	10
1977	153638	188412	134534	87456	60348	42404	31238	21252	16622	14440	12200
1978	178536	226412	158894	104686	71448	47990	35576	24818	22662	18000	
1979	210172	259168	188388	123074	83380	56086	38496	33768	27400		
1980	11448	253482	183370	131040	78994	60232	45568	38000			
1981	219810	266304	194650	120098	87582	62750	51000				
1982	205654	252746	177506	12952	96786	82400					
1983	197716	255408	194648	142328	105600						
1984	239784	329242	264802	190400							
1985	326304	471744	375400								
1986	420778	590400									
1987	496200										

ACCI EXPOSURES

YEAR	
1977	2.20
1978	2.40
1979	2.20
1980	2.00
1981	1.90
1982	1.60
1983	1.60
1984	1.80
1985	2.20
1986	2.50
1987	2.60

APPENDIX C2

AGE LINK RATIOS OF CUMULATIVE PAYMENTS

DELAYS

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/10
1977	2.226337	1.393316	1.183505	1.106992	1.067912	1.046848	1.030445	1.023109	1.019622	1.016259
1978	2.268158	1.392381	1.185665	1.106873	1.064853	1.045149	1.030135	1.026712	1.020665	
1979	2.233123	1.401389	1.187119	1.106787	1.064900	1.041831	1.035220	1.027606		
1980	2.198791	1.394403	1.202128	1.101360	1.070173	1.049607	1.039413			
1981	2.211519	1.400420	1.176416	1.109359	1.070629	1.053616				
1982	2.228986	1.387229	1.203681	1.126446	1.095567					
1983	1.291792	1.429568	1.219719	1.133653						
1984	2.373077	1.465360	1.228344							
1985	2.445719	1.470397								
1986	2.403115									

APPENDIX E1 (Project GLD1, Incremental paid losses)

DEVELOPMENT YEAR

ACCI. YEAR	0	1	2	3	4	5	6	7	8
1979	2737400	1818600	1902200	838900	1134000	200600	748000	319600	9300
1980	989500	3285100	464700	413900	348400	12900	127900	268200	145100
1981	2442600	263300	1007100	1530500	1418700	943700	225600	2056500	53800
1982	364500	1710700	2168400	296300	853300	13200	331700	258200	532000
1983	884800	3607600	757900	3630900	3519900	3317200	1856200	28700	615100
1984	1791600	1982000	4784100	2089900	968400	1266400	111700	759300	1068600
1985	1631600	2348500	3247100	2160700	1555700	272800	711300	112200	
1986	2024600	3231300	4406200	4349300	1662500	2647200	622400		
1987	3704500	5610500	6892300	3529400	11132200	6099200			
1988	5272000	7300600	4549400	4573100	2873500				
1989	3643600	6201300	10902500	5080700					
1990	4902900	7321000	7763000						
1991	10358000	10870200							
1992	6435800								

ACCI YR	RELATIVE EXPOSURES
1979	2.75
1980	2.75
1981	2.75
1982	3.00
1983	3.00
1984	3.50
1985	3.50
1986	3.50
1987	4.00
1988	4.00
1989	4.00
1990	5.50
1991	5.50
1992	5.50

APPENDIX E2 (Project GLD1, Payment per claim "incurred")

DEVELOPMENT YEAR

ACCI YEAR	0	1	2	3	4	5	6	7	8
1979	995418	661309	691709	305054	412363	72945	272000	116218	3381
1980	359818	1194581	168981	150509	126690	4690	46509	97527	52763
1981	888218	95745	366218	556545	515890	343163	82036	747818	19563
1982	121500	570233	722800	98766	284433	4400	110566	86066	177333
1983	294933	1202533	252633	1210300	1173300	1105733	618733	9566	205033
1984	511885	566285	1366885	597114	276685	361828	31914	216942	305314
1985	466171	671000	927742	617342	444485	77942	203228	32057	
1986	578457	923228	1258914	1242657	475000	756342	177828		
1987	926125	1402625	1723075	882350	2783050	1524800			
1988	1318000	1825150	1137350	1143275	718375				
1989	910900	1550325	2725625	1270175					
1990	891436	1331090	1411454						
1991	1883272	1976400							
1992	1170145								

APPENDIX E3 (Project GLD1, Age-to-Age link ratios for the cumulative paid losses)

DEVELOPMENT YEAR

ACCI YEAR	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8
1979	1.664353	1.417515	1.129897	1.155404	1.023793	1.086657	1.034074	1.000959
1980	4.319960	1.108712	1.087334	1.067608	1.002345	1.023193	1.047533	1.024549
1981	1.107795	1.372187	1.412200	1.270564	1.141650	1.029661	1.262593	1.005441
1982	5.693278	2.044911	1.069823	1.187956	1.002448	1.061353	1.044997	1.088721
1983	5.077306	1.168707	1.691560	1.396332	1.267492	1.118092	1.001633	1.034943
1984	2.106274	2.267781	1.244213	1.090950	1.109022	1.008671	1.058434	1.077697
1985	2.439385	1.815834	1.298968	1.165713	1.024928	1.063416	1.009407	
1986	2.596019	1.838334	1.450140	1.118653	1.168892	1.033972		
1987	2.514509	1.739914	1.217766	1.564036	1.197584			
1988	2.384788	1.361850	1.267089	1.132449				
1989	2.701971	2.107426	1.244884					
1990	2.493198	1.635067						
1991	2.049450							

RESEARCH PAPER SERIES

No.	Date	Subject	Author
1	MAR 93	AUSTRALIAN SUPERANNUATION : THE FACTS, THE FICTION, THE FUTURE	David M Knox
2	APR 93	AN EXPONENTIAL BOUND FOR RUIN PROBABILITIES	David C M Dickson
3	APR 93	SOME COMMENTS ON THE COMPOUND BINOMIAL MODEL	David C M Dickson
4	AUG 93	RUIN PROBLEMS AND DUAL EVENTS	David CM Dickson Alfredo D Egidio dos Reis
5	SEP 93	CONTEMPORARY ISSUES IN AUSTRALIAN SUPERANNUATION - A CONFERENCE SUMMARY	David M Knox John Piggott
6	SEP 93	AN ANALYSIS OF THE EQUITY INVESTMENTS OF AUSTRALIAN SUPERANNUATION FUNDS	David M Knox
7	OCT 93	A CRITIQUE OF DEFINED CONTRIBUTION USING A SIMULATION APPROACH	David M Knox
8	JAN 94	REINSURANCE AND RUIN	David C M Dickson Howard R Waters
9	MAR 94	LIFETIME INCOME, TAXATION, EXPENDITURE AND SUPERANNUATION (LITES): A LIFE-CYCLE SIMULATION MODEL	Margaret E Atkinson John Creedy David M Knox
10	FEB 94	SUPERANNUATION FUNDS AND THE PROVISION OF DEVELOPMENT/VENTURE CAPITAL: THE PERFECT MATCH? YES OR NO	David M Knox
11	JUNE 94	RUIN PROBLEMS: SIMULATION OR CALCULATION?	David C M Dickson Howard R Waters
12	JUNE 94	THE RELATIONSHIP BETWEEN THE AGE PENSION AND SUPERANNUATION BENEFITS, PARTICULARLY FOR WOMEN	David M Knox
13	JUNE 94	THE COST AND EQUITY IMPLICATIONS OF THE INSTITUTE OF ACTUARIES OF AUSTRALIA PROPOSED RETIREMENT INCOMES STRATEGY	Margaret E Atkinson John Creedy David M Knox Chris Haberecht
14	SEPT 94	PROBLEMS AND PROSPECTS FOR THE LIFE INSURANCE AND PENSIONS SECTOR IN INDONESIA	Catherine Prime David M Knox

15	OCT 94	PRESENT PROBLEMS AND PROSPECTIVE PRESSURES IN AUSTRALIA'S SUPERANNUATION SYSTEM	David M Knox
16	DEC 94	PLANNING RETIREMENT INCOME IN AUSTRALIA: ROUTES THROUGH THE MAZE	Margaret E Atkinson John Creedy David M Knox
17	JAN 95	ON THE DISTRIBUTION OF THE DURATION OF NEGATIVE SURPLUS	David C M Dickson Alfredo D Egidio dos Reis
18	FEB 95	OUTSTANDING CLAIM LIABILITIES: ARE THEY PREDICTABLE?	Ben Zehnwirth
19	MAY 95	SOME STABLE ALGORITHMS IN RUIN THEORY AND THEIR APPLICATIONS	David C M Dickson Alfredo D Egidio dos Reis Howard R Waters
20	JUN 95	SOME FINANCIAL CONSEQUENCES OF THE SIZE OF AUSTRALIA'S SUPERANNUATION INDUSTRY IN THE NEXT THREE DECADES	David M Knox
21	JUN 95	MODELLING OPTIMAL RETIREMENT IN DECISIONS IN AUSTRALIA	Margaret E Atkinson John Creedy
22	JUN 95	AN EQUITY ANALYSIS OF SOME RADICAL SUGGESTIONS FOR AUSTRALIA'S RETIREMENT INCOME SYSTEM	Margaret E Atkinson John Creedy David M Knox
23	SEP 95	EARLY RETIREMENT AND THE OPTIMAL RETIREMENT AGE	Angela Ryan
24	OCT 95	APPROXIMATE CALCULATION OF MOMENTS OF RUIN RELATED DISTRIBUTIONS	David C M Dickson
25	DEC 95	CONTEMPORARY ISSUES IN THE ONGOING REFORM OF THE AUSTRALIAN RETIREMENT INCOME SYSTEM	David M Knox
26	FEB 96	THE CHOICE OF EARLY RETIREMENT AGE AND THE AUSTRALIAN SUPERANNUATION SYSTEM	Margaret E Atkinson John Creedy
27	FEB 96	PREDICTIVE AGGREGATE CLAIMS DISTRIBUTIONS	David C M Dickson Ben Zehnwirth
28	FEB 96	THE AUSTRALIAN GOVERNMENT SUPERANNUATION CO-CONTRIBUTIONS: ANALYSIS AND COMPARISON	Margaret E Atkinson
29	MAR 96	A SURVEY OF VALUATION ASSUMPTIONS AND FUNDING METHODS USED BY AUSTRALIAN ACTUARIES IN DEFINED BENEFIT SUPERANNUATION FUND VALUATIONS	Des Welch Shauna Ferris
30	MAR 96	THE EFFECT OF INTEREST ON NEGATIVE SURPLUS	David C M Dickson Alfred D Egídio dos Reis

31	MAR 1996	RESERVING CONSECUTIVE LAYERS OF INWARDS EXCESS-OFF-LOSS REINSURANCE	Greg Taylor
32	AUG 1996	EFFECTIVE AND ETHICAL INSTITUTIONAL INVESTMENT	Anthony Asher
33	AUG 1996	STOCHASTIC INVESTMENT MODELS: UNIT ROOTS, COINTEGRATION, STATE SPACE AND GARCH MODELS FOR AUSTRALIA	Michael Sherris Leanna Tedesco Ben Zehnwirth
34	AUG 1996	THREE POWERFUL DIAGNOSTIC MODELS FOR LOSS RESERVING	Ben Zehnwirth
35	SEPT 1996	KALMAN FILTERS WITH APPLICATIONS TO LOSS RESERVING	Ben Zehnwirth
36	OCT 1996	RELATIVE REINSURANCE RETENTION LEVELS	David C M Dickson Howard R Waters
37	OCT 1996	SMOOTHNESS CRITERIA FOR MULTI-DIMENSIONAL WHITTAKER GRADUATION	Greg Taylor
38	OCT 1996	GEOGRAPHIC PREMIUM RATING BY WHITTAKER SPATIAL SMOOTHING	Greg Taylor
39	OCT 1996	RISK, CAPITAL AND PROFIT IN INSURANCE	Greg Taylor
40	OCT 1996	SETTING A BONUS-MALUS SCALE IN THE PRESENCE OF OTHER RATING FACTORS	Greg Taylor
41	NOV 1996	CALCULATIONS AND DIAGNOSTICS FOR LINK RATION TECHNIQUES	Ben Zehnwirth Glen Barnett
42	DEC 1996	VIDEO-CONFERENCING IN ACTUARIAL STUDIES – A THREE YEAR CASE STUDY	David M Knox
43	DEC 1996	ALTERNATIVE RETIREMENT INCOME ARRANGEMENTS AND LIFETIME INCOME INEQUALITY: LESSONS FROM AUSTRALIA	Margaret E Atkinson John Creedy David M Knox
44	JAN 1997	AN ANALYSIS OF PENSIONER MORTALITY BY PRE-RETIREMENT INCOME	David M Knox Andrew Tomlin
45	JUL 1997	TECHNICAL ASPECTS OF DOMESTIC LINES PRICING	Greg Taylor
46	AUG 1997	RUIN PROBABILITIES WITH COMPOUNDING ASSETS	David C M Dickson Howard R Waters
47	NOV 1997	ON NUMERICAL EVALUATION OF FINITE TIME RUIN PROBABILITIES	David C M Dickson
48	NOV 1997	ON THE MOMENTS OF RUIN AND RECOVERY TIMES	Alfredo G Egídio dos Reis
49	JAN 1998	A DECOMPOSITION OF ACTUARIAL SURPLUS AND APPLICATIONS	Daniel Dufresne
50	JAN 1998	PARTICIPATION PROFILES OF AUSTRALIAN WOMEN	M. E. Atkinson Roslyn Cornish

51	MAR 1998	PRICING THE STOCHASTIC VOLATILITY PUT OPTION OF BANKS' CREDIT LINE COMMITMENTS	J.P. Chateau Daniel Dufresne
52	MAR 1998	ON ROBUST ESTIMATION IN BÜHLMANN STRAUB'S CREDIBILITY MODEL	José Garrido Georgios Pitselis
53	MAR 1998	AN ANALYSIS OF THE EQUITY IMPLICATIONS OF RECENT TAXATION CHANGES TO AUSTRALIAN SUPERANNUATION	David M Knox M. E. Atkinson Susan Donath
54	APR 1998	TAX REFORM AND SUPERANNUATION – AN OPPORTUNITY TO BE GRASPED.	David M Knox
55	APR 1998	SUPER BENEFITS? ESTIMATES OF THE RETIREMENT INCOMES THAT AUSTRALIAN WOMEN WILL RECEIVE FROM SUPERANNUATION	Susan Donath
56	APR 1998	A UNIFIED APPROACH TO THE STUDY OF TAIL PROBABILITIES OF COMPOUND DISTRIBUTIONS	Jun Cai José Garrido
57	MAY 1998	THE DE PRIL TRANSFORM OF A COMPOUND R_k DISTRIBUTION	Bjørn Sundt Okechukwu Ekuma
58	MAY 1998	ON MULTIVARIATE PANJER RECURSIONS	Bjørn Sundt
59	MAY 1998	THE MULTIVARIATE DE PRIL TRANSFORM	Bjørn Sundt
60	JUNE 1998	ON ERROR BOUNDS FOR MULTIVARIATE DISTRIBUTIONS	Bjørn Sundt
61	JUNE 1998	THE EQUITY IMPLICATIONS OF CHANGING THE TAX BASIS FOR PENSION FUNDS	M E Atkinson John Creedy David Knox
62	JUNE 1998	ACCELERATED SIMULATION FOR PRICING ASIAN OPTIONS	Felisa J Vázquez-Abad Daniel Dufresne
63	JUNE 1998	AN AFFINE PROPERTY OF THE RECIPROCAL ASIAN OPTION PROCESS	Daniel Dufresne
64	AUG 1998	RUIN PROBLEMS FOR PHASE-TYPE(2) RISK PROCESSES	David C M Dickson Christian Hipp
65	AUG 1998	COMPARISON OF METHODS FOR EVALUATION OF THE n-FOLD CONVOLUTION OF AN ARITHMETIC DISTRIBUTION	Bjørn Sundt David C M Dickson
66	NOV 1998	COMPARISON OF METHODS FOR EVALUATION OF THE CONVOLUTION OF TWO COMPOUND R_1 DISTRIBUTIONS	David C M Dickson Bjørn Sundt
67	NOV 1998	PENSION FUNDING WITH MOVING AVERAGE RATES OF RETURN	Diane Bédard Daniel Dufresne
68	DEC 1998	MULTI-PERIOD AGGREGATE LOSS DISTRIBUTIONS FOR A LIFE PORTFOLIO	David C M Dickson Howard R Waters
69	FEB 1999	LAGUERRE SERIES FOR ASIAN AND OTHER OPTIONS	Daniel Dufresne

(The person in the back seat does not have supernatural powers).

In the loss reserving context, the 'straight stretches' are the stable trends in the (incremental) payments. If the trends have been stable in past years, we are confident (but not absolutely sure) in supposing the same trends in the future. This almost perfect analogy will be used throughout the paper.

The mechanisms by which claim sizes, frequencies and delays are generated are invariably complex. When a model is constructed, it is not intended to be an accurate description of every aspect of the claims processes. The aim is to simplify the underlying processes in such a way that the essential features are brought out. According to Milton Friedman [7]: *'A hypothesis is important if it 'explains' much by little...'*. Similar views are expressed by Popper [13]; *'Simple statements... are to be prized more highly than less simple ones because they tell us more; because their empirical content is greater, and because they are better testable.'*

From the statistical point of view, the key feature of a simple model is that it contains a small number of parameters. This is known as the principle of parsimony. Moreover, a simple model is testable. There is no need to model every basic element of the claims process. Instead, we construct a simple model that identifies the trends and deviations (random fluctuations) about the trends in the (aggregate) payments.

It is useful to think of data (measurements) as comprising two components: a **signal** or a **message** which is distorted by a second component, termed **noise**. The **signal** is regarded as deterministic and the **noise** as random. Therefore, a mathematical model of the data combining both signal and noise is stochastic (probabilistic) and is called a statistical model.

Another way of thinking of a statistical model is to consider the signal component as a mathematical description of the main features of the data, and the noise component as all those characteristics not 'explained' by the signal component.

Typically the mathematical description of the signal involves several unknown constants, termed **parameters**.

In the loss reserving context the signal itself has three components of interest, viz., the trends in the three directions, **development year, accident year** and **payment/calendar year** of a "loss development array" described in Section 1.2. For each direction there are trend parameters. The fourth component is the noise, equivalently, the random fluctuations or deviations about the trends. The random fluctuation component is just as important as the three trend components and is necessarily an integral part of the model. The data or transform thereof are decomposed thus:

DATA = TRENDS + RANDOM FLUCTUATIONS

The concept of **trends** and **random fluctuations** about trends is over two hundred years old. These concepts have been widely used in analysing (and forecasting) univariate time series such as sales, stock market prices, interest rates, consumption, energy and so on.

The principal aim of analysing a loss development array is to estimate the trends in the past, especially in the payment/calendar year direction, and determine the random fluctuations about the trends. In this way it can be best judged which assumptions should be used for future trends (and random fluctuations). The probability distributions of the random fluctuations are also computed.

IF THE TRENDS IN THE DATA ARE STABLE THEN THE (OPTIMAL) MODEL WILL VALIDATE WELL AND BE STABLE. If the trends in the data are unstable then the decision about future trends is no longer straight forward. Instability in trends with little random variation about the trends makes data less predictable than stable trends with much random fluctuation. See Sections 4.2 and 4.3 for real life examples.

The 'best' identified model contains assumptions (equivalently, information). All the assumptions must be tested to ensure they are supported by the data (experience).

As we proceed through the model identification strategy we are extracting information (about trends and stability thereof and the amount of random variation) and we 'hope' that the 'best' identified model tells us that the calendar year trend in the data is stable (especially more recently). If trends are not stable then we may not necessarily use the optimal statistical model for forecasting. See Section 5.0.

None of the numerous models contained in the MODELLING FRAMEWORK actually represent explicitly the underlying claims generating processes. The multitude of variables involved in generating the claims are invariably complex. What we attempt to achieve is the identification of a parsimonious model in terms of the simple components of interest for which all the assumptions inherent in the (probabilistic) model are supported by the data. It is subsequently argued that the experience (data) can be regarded as a sample (path) from the identified probabilistic model. The multitude of variables that are the determinants of the claims processes are proxied by the TRENDS and the (residual) variance of the RANDOM FLUCTUATIONS.

The principal advantage of an explicit statistical model is that it makes the assumptions clear. Other advantages include improved accuracy and quantification of variability required for assessment of risk based capital and testing of solvency.

0.4 Loss Reserving Myths and Uncertainty

We debunk a number of loss reserving myths by employing simulation studies and a number of real life examples. In spite of the critical importance of loss reserving in insurance, the statistical foundations are not well understood by many insurance experts. Misstatements and fallacies regarding loss reserving and related concepts are pervasive, ranging from insurance courses in the classroom to insurance cases in the courtroom.

Myth 1

If we know the (exact) probabilistic model including the values of the parameters generating the paid losses, there is no variability or uncertainty.

Reality 1

There is variability or uncertainty even when we know the exact probabilistic model generating the losses. (Recall the die rolling example). This variability (uncertainty) is called **process variability** (uncertainty). See Section 3.1.

Myth 2

Variability or uncertainty is inversely proportional to the size of the insurer's exposure base.

Reality 2

There is no relationship. It is only the process uncertainty (noise) that may reduce with increasing exposure. In Section 4.2 we analyse an experience of a large U.S. insurer with a large exposure base where the paid loss experience has a major shift in trend, and accordingly there is much uncertainty about the future. By contrast, in Section 4.3, we consider a company with a relatively small exposure base where the paid losses fluctuate widely, BUT, the trend is relatively stable and so the future experience is not as uncertain.

Myth 3

Large fluctuations in paid losses implies instability in trends and so the future experience is very uncertain.

Reality 3

Large fluctuations may be due to the "random" component, equivalently, the "noise", not an instability in trends. See Section 4.3 for a real life example where the paid losses fluctuate widely but due to stability in trends, the model estimated three years earlier would have "predicted" the last three years experience and would have yielded the same estimates statistically of the outstanding claim liabilities, as the model estimated at valuation date.

Myth 4

Escalation in payments is due to "claims closing faster", and so less will be paid later.

Reality 4

This is one of the "great lies" in loss reserving. Some insurance practitioners have used this argument to explain the rapid escalation in the claims experience for the new CTP NSW Scheme. See Section 6.0 for a description of the "relationship" between aggregate payments and closure rates in the CTP NSW industry experience, and Section 4.3 for the "relationship" in the (individual) AMP General Insurance CTP (NSW) experience.

Charles McLenahan, a distinguished U.S. GI actuary in referring to Myth 4 remarked:

> *"If I had a nickel for every time I heard this as an explanation for increasing loss development factors, I wouldn't have nearly enough to cover the reserve deficiency of the company which believes it. In twenty-five years, the only situation in which I have ever witnessed a material speedup of claims closure was a company in liquidation. The teller of this untruth is usually armed with various recently-instituted changes in claims handling policies and procedures which account for the change."*

Predictability is intimately related to the concept of uncertainty which abounds in everyday business life. The various components or sources of uncertainty or variability are discussed in Section 5.0.

Uncertainty (variability) is modelled in terms of probability (statistical) distributions. There are four principal sources of uncertainty that are interrelated. These sources of uncertainty determine the predictability of the outstanding claim liabilities.

1. Noise or random variation

   This is called **process uncertainty** or **process risk.** It represents the inherent variability in the process. We have no control over it and cannot reduce it.

2. Estimation error

A statistical model contains parameters that are estimated from data. Due to sampling variation (noise) the parameters are not known exactly.

3. Trend stability or lack thereof

Based on Andrew Harvey's car example in Section 0.3, uncertainty about future trends is related to stability or lack thereof of past trends.

Assumptions about future trends are based on identification of past trends and other sources of information, including business knowledge. See Section 5.0.

This type of uncertainty is commonly referred to as **risk parameter** uncertainty and is intimately related to the next type of uncertainty.

4. The future ain't what it used to be

This source of uncertainty may be difficult to measure statistically, but that does not mean we should ignore it. The future may be very different to the past. A pricing actuary working in 1975 would have had no way of predicting the explosion in pollution liability claims that would have occurred in the 1980's, in respect of claims incurred in 1975. However, had he been aware that there is a strong probability that the environment, legal and economic, may change, judgemental changes to the parameters of the model, could have been made.

The quantification of the first three sources of uncertainty are dependent on the information extracted from the historical experience.

0.5 CTP NSW Industry Experience

The NSW Government established a new Scheme, which commenced in July 1989, for compensating people injured as a result of the fault of others in motor vehicle accidents. Compulsory Third Party Personal Injury

Insurance (or CTP) provides funds to compensate people injured in motor vehicle accidents.

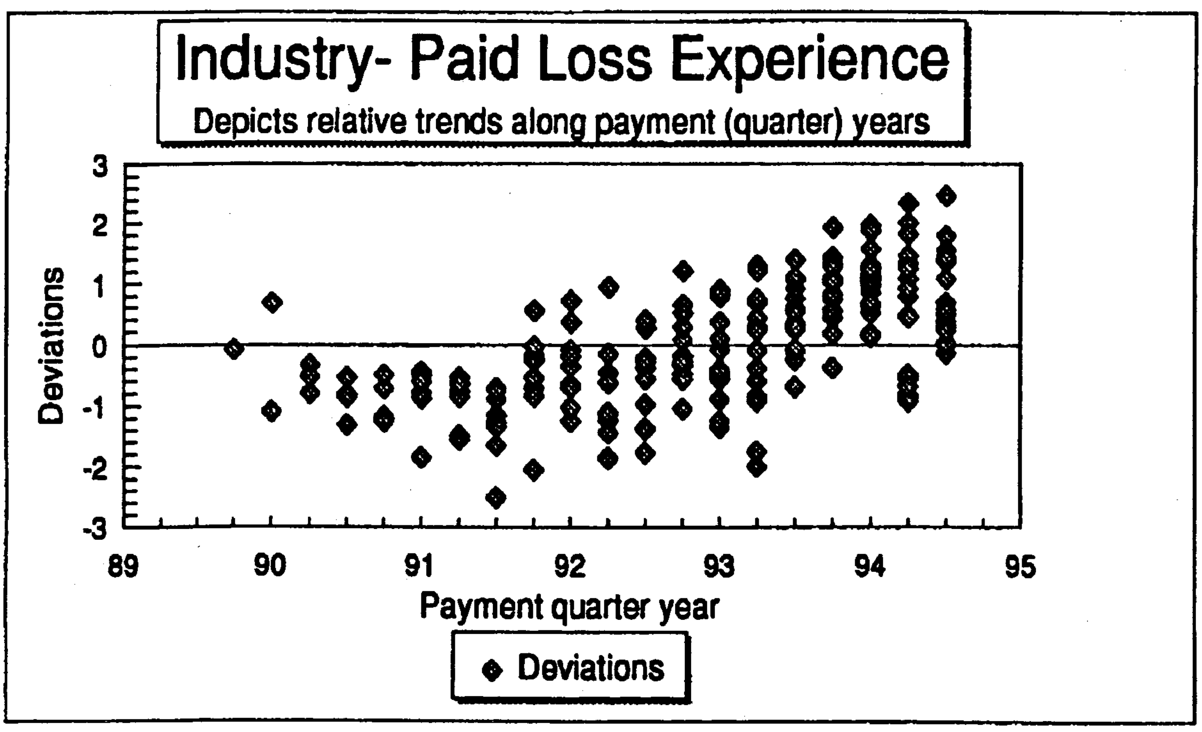
The Scheme includes the following features:

- Each registered owner of a vehicle in NSW is required to insure, with an insurer licensed under The Motor Accidents Act 1988.

- Insurers are licensed under the Act and must file with the Motor Accident Authority, at least once a year, a full set of premiums it proposes to charge for third party policies.

Applying the statistical modelling framework described in the paper, we have determined an alarming claims escalation (deterioration) in the paid losses that has existed for over three years.

The graph below displays the trends in the paid losses across the payment quarter years from the September quarter of 1989 (3-89) to the June quarter of 1994 (2-94). We observe a favourable experience (zero trend) from 3-89 to 2-91. Thereafter, there is a distinct positive trend estimated as 8.42% ± 1.38% per quarter year. That is, 40% p.a.!



The alarming trend has existed for over three years, so that, if the 'optimal' statistical model is estimated at payment quarter year end 3-92 (omitting the 4-92 to 2-94 paid losses), for example, it "predicts" the paid losses for the payment quarter years 4-92 to 2-94 and yields statistically the same outstanding paid losses, as the model estimated at payment quarter year end 2-94! See Section 6.5 for detailed analysis.

The alarming deterioration in claims experience has resulted in substantial underwriting losses for NSW based CTP business, in the current and recent financial years. Some industry practitioners have attributed the deterioration to "claims closing faster". That this contention (Myth 4) is not supported by the experience is demonstrated in Section 6.4.

If we apply the Harvey [9] motor car analogy, even though we have identified an inordinate high inflation rate in the CTP experience, we are not guaranteed it will continue. Indeed, given that the principal reason for the inflation, is a continued trend towards litigation as an avenue by claimants for higher award payments, one would expect and hope to reach 'saturation' in the near future. A downward adjustment to the future trend ought to be made to reflect the fact that the legal environment will stabilise.

In any event, the trends arising from the deterioration in claim costs have only been marginally factored into the current pure premium (before allowing for expenses, profit margins and other contingencies) of $190 (per vehicle). Assuming a discount rate of 8% - 10% p.a., it could only be substantiated by a less than 4% p.a. combined AWE + Superimposed inflation in the future.

In my view, unless there are major upgrades to the industry pure premium and to the provisions carried by companies writing this line, then losses will continue and are indeed likely to increase.

The companies that are inadequately reserved are attracting taxes on profits that are very unlikely to emerge.

Section 6.0 of the paper presents more detailed analysis of CTP NSW.

0.6 Organisation of the paper

The problem of estimation of outstanding claim liabilities including accounting concepts is discussed in Section 1.0. Probabilistic concepts and statistical models in general are presented in Section 2.0.

The probabilistic modelling framework is introduced in Section 3.0 with a few simple examples using simulated data. The difference between process uncertainty and estimation error is illustrated. Model validation, stability and updating are also discussed.

In Section 4.0 we describe a number of loss reserving myths that are related to the measurement of uncertainty. The myths are debunked using real life examples. Formulation of assumptions about the future based on extraction of information from the historical experience, business knowledge and judgment are discussed in Section 5.0.

In Section 6.0 we use the statistical modelling framework to analyse the NSW based CTP experience, in order to illustrate the power and flexibility of the statistical methodology. We also show that the CTP experience is subject to an alarming claims escalation rate.

1.0 THE PROBLEM

One of the major challenges to the General Insurance (GI) actuary is the estimation of the necessary financial provisions for the unpaid outstanding claim liabilities of an insurer to claimants. The determination of the provisions is essential to the long term management of a GI company. Accurate assessment is required for solvency considerations, as well as premium setting.

1.1 Outstanding claims

A GI policy is a short term contract, usually one year. However, the insurer's liability may not necessarily cease at the expiry of the (one year) risk period.

For a 'long tail' line of business there are delays between the time period for which insurance protection is afforded (risk period) under the policy, and the actual claim payments. Accordingly, the insurer may take many years to discharge its obligations assumed under the policy.

An insurer's outstanding claim liabilities at a given date are the amounts which it is liable to pay, after that date, for claims which arose on or before that date.

We define the 'accident year' as the 'year of origin' in which the incident leading to a claim occurred. The year in which a payment is made is referred to as 'payment year' and the difference between 'payment year' and 'accident year' is referred to as the 'development year'.

Each 'accident year' gives rise to a stream of payments in emerging years.

1.2 Triangulation (Loss Development Array)

The claim experience of an insurer in respect of a particular class of business can be summarised in a run-off triangle exemplified below. 'Year of origin' is the year in which the incident leading to a claim occurs.

Incremental Paid Losses ($000)

Year of Origin	Development year (Delay) 0	1	2	3	4
1990	580	1079	131	80	25
1991	494	993	118	91	
1992	551	1060	129		
1993	648	1312			
1994	746				

The diagonals in the array represent the payment years. For example, in respect of claims originating in 1992, payments totalling $129,000 were made in 1994 (development year 2).

Run-off triangles for other (aggregate) data types including number of claims notified, number of claims closed and case estimates can also be created.

The objective is to complete the rectangle in order to compute the total ultimate incurred cost for each year of origin (accident year).

1.3 Accounting Concepts

In order to understand the necessity for the estimation of outstanding claim liabilities, it is helpful to have a conceptual understanding of the basic accounting principles applicable to insurers.

The accounting process produces two important statements, the balance sheet and the income statement, that document the financial position and performance of a firm respectively. The reliability and usefulness of both these statements are dependent on the accuracy and interpretation of the provisions (for outstanding claim liabilities) shown in the insurer's accounts.

1.4 Accrual basis of accounting

The accrual basis of accounting recognises revenue as it is earned. Likewise, costs are reported as expenses in the same period as revenues giving rise to these costs are recognised. This results in an income statement that more appropriately matches costs with appropriate revenues.

1.5 Provision

An outstanding claim provision is an amount set aside in the insurer's accounts, to provide for outstanding claim liabilities.

1.6 The Balance Sheet and Underwriting Statement

The balance sheet reports on the financial position of the firm at a specific point in time. It shows the levels of assets and liabilities, and the status of the shareholders' equity, or surplus, for the insurer.

Assets = Liabilities + Shareholders' Equity

The liabilities include the outstanding claims liabilities defined in Section 1.1. Through common usage the term "loss reserve" or "claims reserve" has come to denote the GI company's provision in the balance sheet for its outstanding claims liability.

In the Underwriting statement

Incurred Claims = Paid Losses + End of year Loss Reserve
- Beginning of year Loss Reserve

The Incurred Claims are the total claim costs incurred in the underwriting year. The Paid Losses represent the total payments made in the underwriting year, in respects of claims incurred in the current and all prior underwriting years.

1.7 Solvency and Income

Any change in the (financial) provisions in the accounts have a direct impact on Shareholders' Equity and accordingly solvency, and Incurred Losses and accordingly income.

1.8 Loss (claims) reserving

Loss or claims reserving is the process of estimating the amount of the company's outstanding claim liabilities.

2.0 LOSS (CLAIMS) RESERVING METHODS

The basic goal of this paper is to introduce and describe a unified statistical approach to loss (claims) reserving with its principal advantages and benefits. At the core, is the paradigm shift, from the non-statistical standard actuarial techniques to the statistical actuarial techniques.

In spite of the critical importance of loss reserving in insurance, the statistical foundations are not well understood by many insurance experts. Misstatements and fallacies regarding loss reserving and related concepts are pervasive, ranging from insurance courses in the classroom to insurance cases in the courtroom.

Paradigm Shift on the Port Bow

The following true story of a naval cammander's brush with a new reality occurred some years ago during US navy practice manoeuvres. The ship in question was steaming just after dark in heavy fog when a light was reported by a lookout.

The captain ordered his signalman to flash the message 'We are on a collision course. Advise you change course 20 degrees'. The reply came back through the fog, 'Advise you change course'.

The next signal said 'I am a captain. Change course 20 degrees'. The reply was 'I am a seaman, 2nd class. You had better change course'.

The captain, now infuriated, sent back, 'I'm a battleship. Change course 20 degrees'. The reply? 'I'm a lighthouse'.

Stephen R. Covey [5]

The statistical approach to loss reserving requires a totally different perception and/or framework. It requires a paradigm shift.

Celebrated actuaries have suggested "statistical thinking" as the principal approach to solving GI problems.

In his Presidential address (Institute of Actuaries, London) in 1888, William Sutton expressed the wish that insurance offices other than life offices should benefit from the practical application of the doctrine of probabilities.

More recently, the celebrated American actuary Arthur Bailey, in spite of his mathematical cum statistical brilliance, had a way of presenting ideas so lucidly that even lay people could get his message. For example, in his 1942 paper, *"Sampling Theory in Casualty Insurance"*, he said:

"Thus the losses paid by an insurer never actually reflect the hazard covered, but are always an isolated sample of all possible amounts of losses which might have been incurred. It is this condition, of never being able to determine, even from hindsight, what the exact value of the inherent coverage was, that has brought the actuary into being."

2.1 Claims Processes

The mechanisms by which claim severities, frequencies and delays are generated are invariably complex. When a model is constructed, it is not intended to be an accurate description of every aspect of the claims processes. The aim is to simplify the underlying processes in such a way that the essential features are brought out. According to Milton Friedman [8]: *'A hypothesis is important if it 'explains' much by little....'.* Similar views are expressed by Popper [14]: *'Simple statements... are to be prized more highly than less simple ones because they tell us more; because their empirical content is greater, and because they are better testable'.*

From the statistical point of view, the key feature of a simple model is that it contains a small number of parameters. This is known as the principle of parsimony. Moreover, a simple model is testable.

The purpose of constructing a statistical model is to systematically account for as much of the variation in the observations with as few parameters as possible.

The "essential features" of the data in the loss reserving context are the trends and the random fluctuations about the trends. We decompose the data thus:

Log 'payments' = Trends + Random Fluctuations

Another way of thinking of this statistical model is to regard the Trends as a mathematical description of the main features of the data and the Random Fluctuations (or error or noise component) as all of those characteristics not 'explained' by the Trends. All the complex mechanisms involved in generating the data are implicitly included in the model as creating the Trends plus the residual variance in the Random Fluctuations.

The final identified model that 'explains' the data does not represent explicitly the underlying claims generating process. The model has probabilistic properties for which the data may be regarded as a sample (path) from it. Another classical modelling example in insurance where the same kind of modelling concepts are used is when we fit a Pareto distribution, say, to loss sizes. We do not assume that the Pareto distribution represents the underlying generating process. Whatever is driving the claims is very complex and depends on many factors. All we are saying is that our experience (sample) can be regarded as a random sample from the estimated Pareto distribution. The estimated Pareto distribution describes the variability in the loss sizes.

2.2 Statistics, Statistical Models and Forecasting

The best way to suppose what may come, is to remember what is past.

George Savile, Marquis of Halifax.

In this section we discuss a number of fundamental statistical forecasting concepts including which salient features of the data should be used to *"remember what is past"*.

Statistics can be defined as the art and science of gathering, analysing and making inferences from data. Statistics is the study and modelling of variability and uncertainty.

The basic principles of statistical inference necessarily involve probabilities. Indeed, statistics is primarily concerned with the application of probability theory to data. The statistical approach to modelling is based on the construction or estimation of a probabilistic model. The model does not necessarily represent the underlying generating process of the losses. Whatever generates the losses is complex and depends on a myriad of factors. Instead, the statistical model is simple, and defines the probabilistic mechanisms (or laws) which are regarded as being capable of having produced the data (observations). If the model were to generate several sets of data (or observations), each data set would be different but they would all obey the same probabilistic laws.

Forecasting

Indeed it (forecasting) has been likened to driving a car blindfolded while following directions given by a person looking out the back window. Nevertheless, if this is the best we could do, it is important that it should be done properly, with the appreciation of the potential errors involved. In this way it should at least be possible to negotiate straight stretches of road without a major disaster.

Andrew C. Harvey [10]

In the loss reserving context the 'straight stretches' are the stable trends in the (incremental) payments. If the trends have been stable in past years, we are confident (but not absolutely sure) in supposing the same trends in the future. This almost perfect analogy will be used throughout the paper.

Predictability, as will become apparent, is intimately related to stability of trends.

3.0 PROBABILISTIC MODELS

We use probabilistic models to extract information from data. Based on:

- information extracted from the incremental paid losses development array (triangle);
- information extracted from other data types;

and

- business knowledge,

the actuary determines the most appropriate assumptions about the future. Information extracted from the loss development arrays will necessarily involve (i) validation analysis, (ii) stability analysis, (iii) sensitivity analysis and (iv) 'what if?' analysis.

3.1 Example of A Statistical Model Generating A Loss Development Array (Triangle)

We describe a (simple) probabilistic model representing the generation of incremental paid losses in a loss development array (triangle).

Consider first, only one accident year, and denote by p(d) the incremental paid loss in respect of development year d.

d	:	0	1	2	......	s
p(d)	:	p(0)	p(1)	p(2)	......	p(s)

Assume p(d) is generated by the trend curve $\exp(\alpha + \gamma d)$, an exponential curve. So,

$$\begin{aligned} y(d) &= \ln p(d) \\ &= \alpha + \gamma d. \end{aligned}$$

That is, y(d), the logarithm of p(d) is generated by the constant trend line $\alpha + \gamma d$.

The parameter α (alpha) represents the intercept whereas the parameter γ (gamma) represents the slope or trend.

Note that logarithms are like percentages and are used to measure trends.

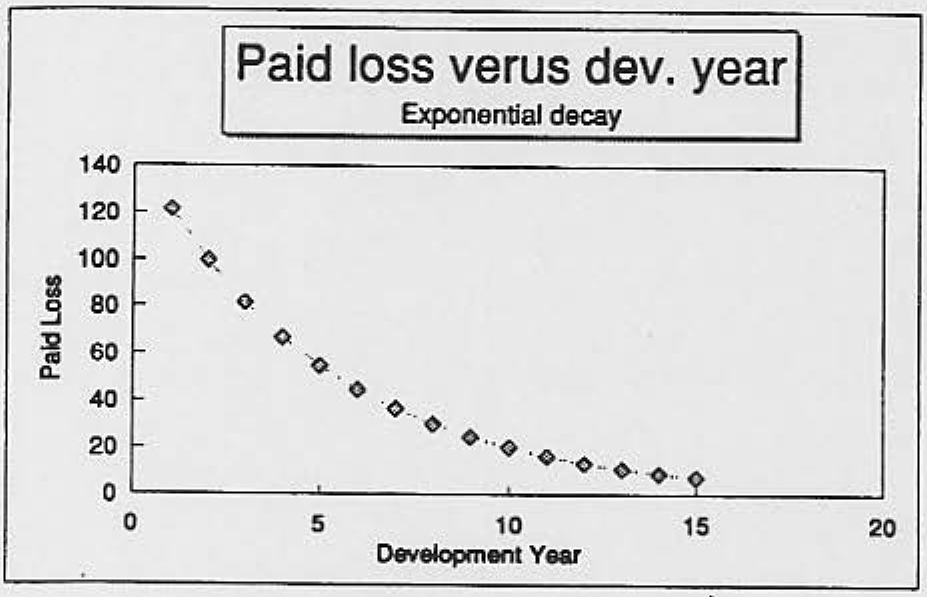


Figure 3.1.1

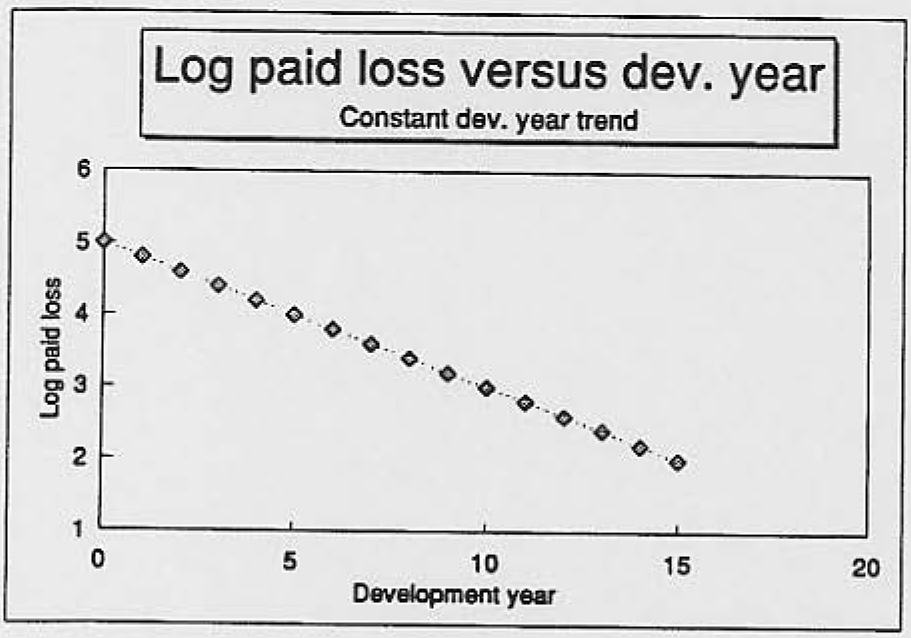


Figure 3.1.2

Figure 3.1.1 depicts the exponential curve $\exp(\alpha+\gamma d)$ and Figure 3.1.2 the corresponding logarithm.

Hitherto, we have assumed that each y(d) value sits on the straight line $\alpha + \gamma d$.

Suppose that in actual fact the observations y(d) fluctuate about the line $\alpha + \gamma d$, such that positive fluctuations (deviations) are as likely as negative fluctuations (deviations). Indeed, the deviations of y(d) about $\alpha + \gamma d$ can be described by the symmetric bell-shaped normal distribution. That is, the deviations or fluctuations follow a particular type of probabilistic law, depicted in Figure 3.1.3.

The symmetric bell shaped curve about the trend line represents the (relative) frequency of the deviations.

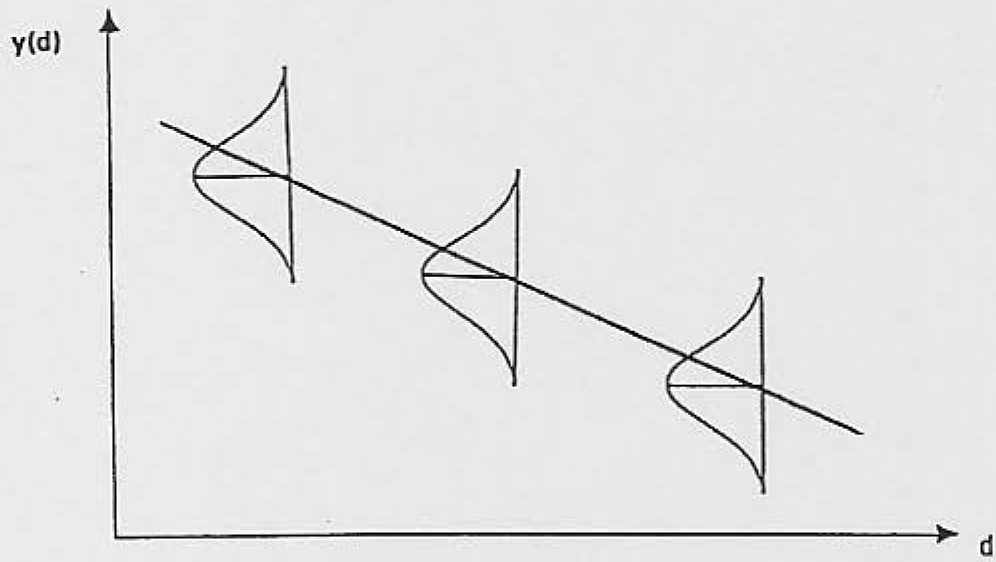


Figure 3.1.3

The model can now be written

$$y(d) = \alpha + \gamma d + \epsilon(d), \tag{3.1.1}$$

where the "error" or "deviation" $\epsilon(d)$ of y(d) from the straight line $\alpha + \gamma d$ is a random selection from a normal distribution with mean zero (average deviation = 0) and variance σ^2, say. (The Greek letter, σ, denotes the standard deviation of the "deviations").

If we estimate (or fit) the above model to one accident year's observed experience we are not only estimating the straight line $\alpha + \gamma d$ but also estimating the normal distribution of the deviations of the observed from the fitted line. The estimated normal distribution with means lying on the estimated line define the probabilistic mechanisms which are regarded as being capable of having produced the observations.

Note that the model assumes that the mean effective yearly trend (on the $ scale) between any two development years is constant and equal to $\exp(\gamma) - 1$. The mean continuous trend (like force of mortality, or force of inflation) is represented by the parameter γ. We call γ a development factor on a log scale.

Suppose now that this constant development year trend model (on a log scale) applies to every accident year in the triangle with the same parameters α, γ and σ^2 for each accident year.

So what does this model containing only two mean parameters α and γ, and one variance parameter σ^2, assume about the observations in a triangle?

Succinctly, it assumes complete homogeneity of accident years with a constant trend along development years. More specifically,

(A1): The mean trend between any two development years is constant and is the same for each accident year. Accident years are homogeneous in respect of development year trends.

(A2): Accident years are homogeneous in respect of mean level. The same parameter α applies to each accident year.

(A3): The deviations of the (log) observations from the trend line follow a normal distribution with mean zero and constant variance σ^2.

N.B. The distribution of the deviations, equivalently, the random fluctuations, about the trend is an integral part of the model.

The model decomposes the (log) observations into trend plus deviations or random fluctuations.

DATA = TREND(S) + DEVIATIONS (RANDOM FLUCTUATIONS)

The above model contains very few assumptions (parameters). It is only useful for projections if all the assumptions contained in the model are supported by the data.

Violations of Assumptions

1. A constant trend along development years

   The violation of this assumption can be detected diagnostically by fitting (estimating) a constant trend to development years and examining the graph of observed deviations (residuals) versus development years for any residual trends.

   For example, the display of observed deviations versus development years in Figure 3.1.4 below exhibits non-randomness. Therefore, the trend along development years is not constant. Indeed, there appear to be four distinct trends.

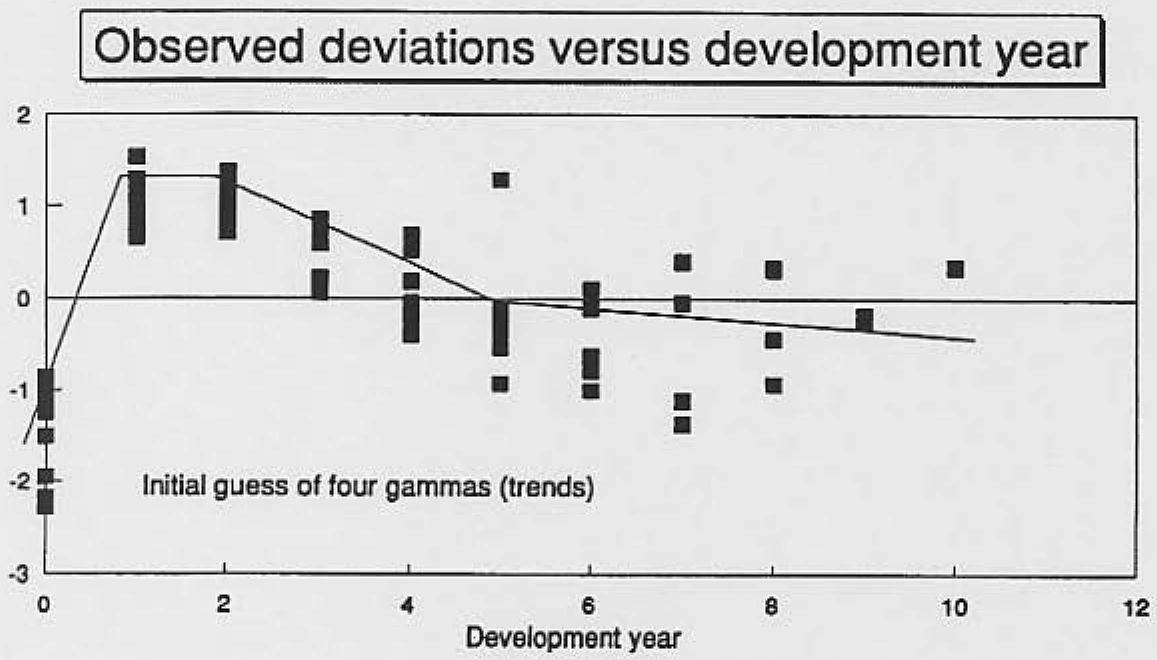


Figure 3.1.4

2. Accident years are homogeneous in respect of levels

The violation of this assumption can be detected diagnostically by examining the graph of observed deviations (residuals) versus accident years for systematic patterns.

For example, the display of observed deviations versus accident years in Figure 3.1.5 below indicates a systematic pattern (trend).

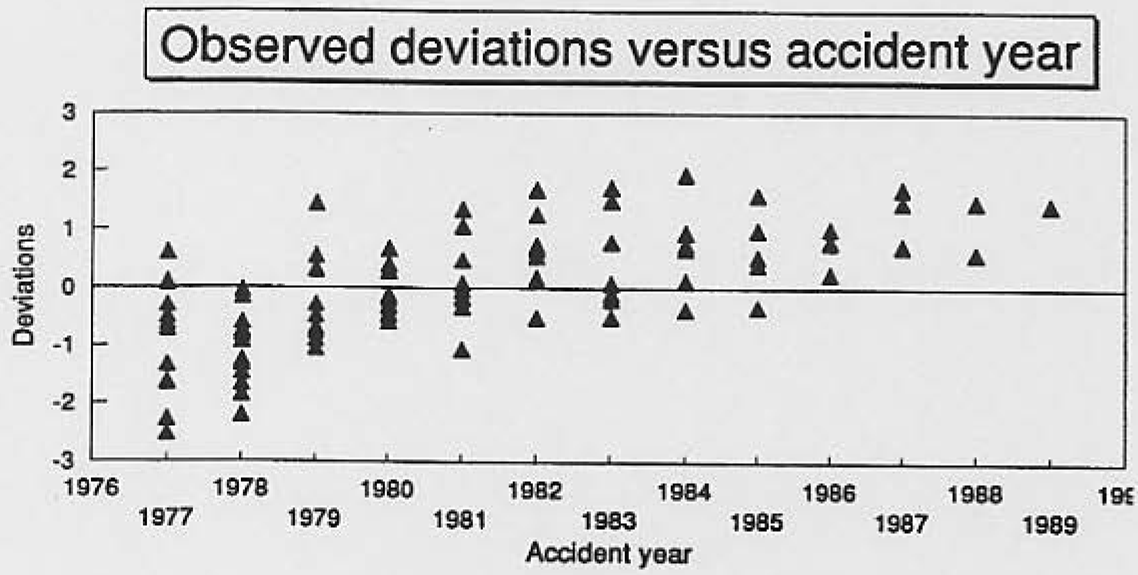


Figure 3.1.5

3. Payment/Calendar year trend is zero

The most important dimension or direction in the triangle is the payment/ calendar year direction, equivalently, the diagonals in the triangle. Model (3.1.1) assumes that the trend between any two contiguous payment/calendar years is zero. The violation of this assumption can be detected diagnostically by examining the graph of observed deviations versus the payment years.

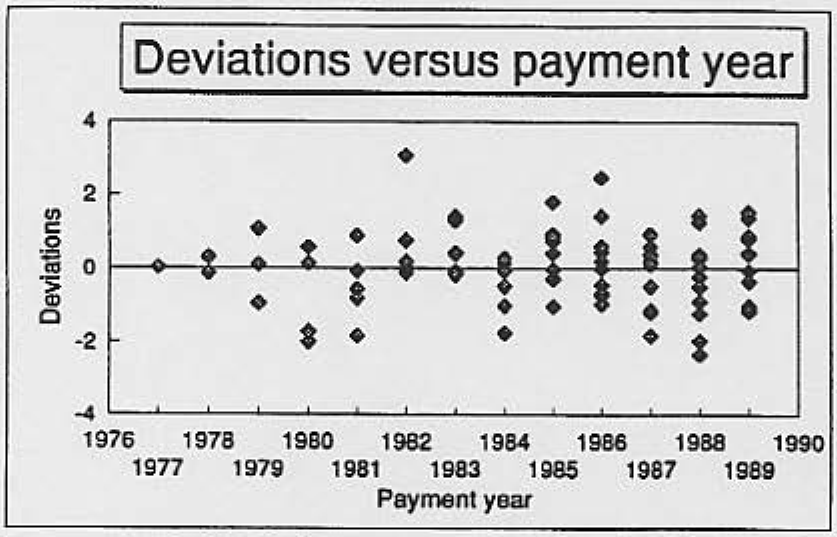


Figure 3.1.6

For example, the observed deviations in Figure 3.1.6 indicate diagnostically a zero trend whereas the observed deviations in Figure 3.1.7 indicate diagnostically a positive constant trend.

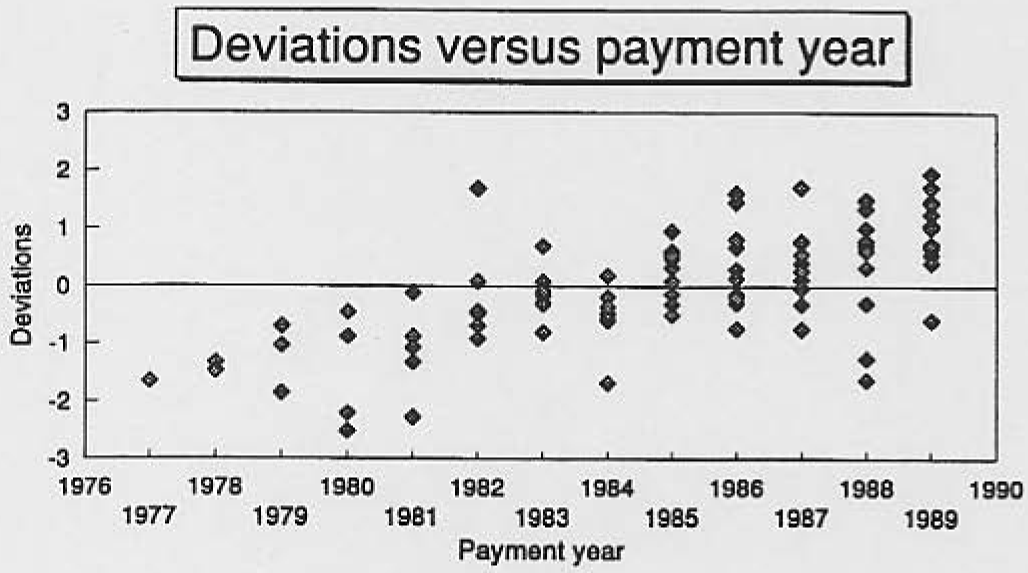


Figure 3.1.7

The most important assumption is concerned with the distribution of the deviations. The distribution of the deviations is assumed to be normal with mean zero and constant variance. This assumption must also be tested.

3.2 A Model with Three Inflation Parameters

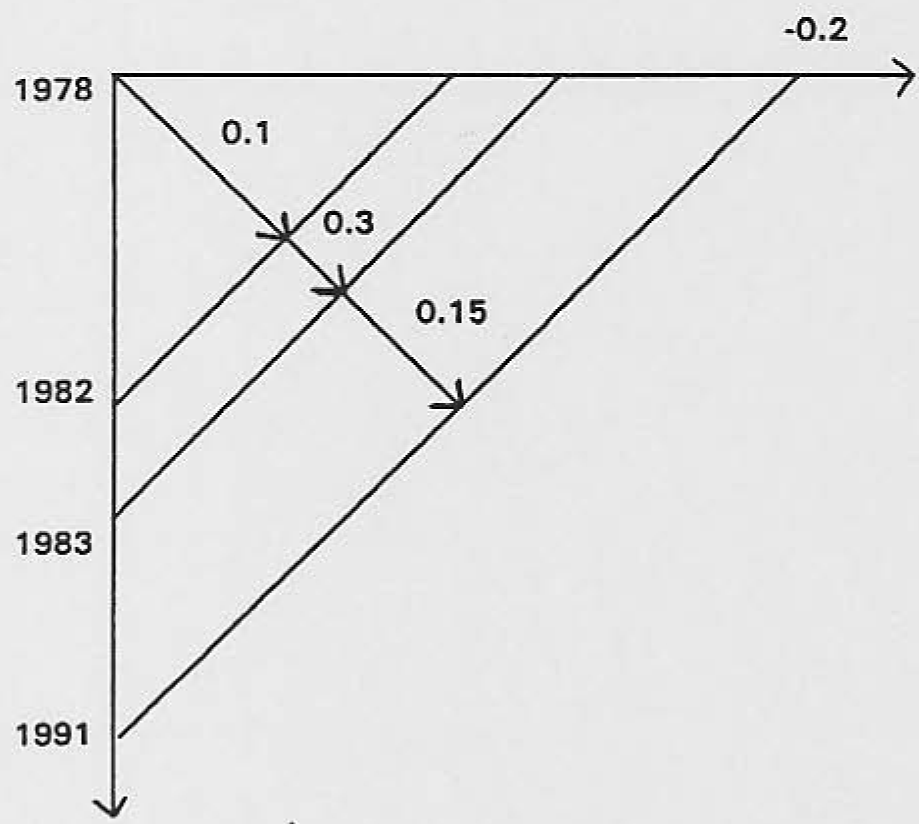
The data in Appendix A1 to Appendix A9 are generated as follows.

First, we create payments based on the formula:

$$p(w,d) = \exp(\alpha - 0.2*d).$$

That is, each accident year w is generated by the same exponential curve with γ (gamma) or decay factor equal to -0.2. The Greek letter α (alpha) represents the intercept, level or (log) "exposure". See Appendix A1 for a display of the data.

<u>TRENDS</u>



On a log scale we introduce payment/calendar year trends thus: 10% trend from 1978-82, 30% trend from 1982-83 and 15% trend from 1983-91. The logarithms of the payments with these trends are given in Appendix A2.

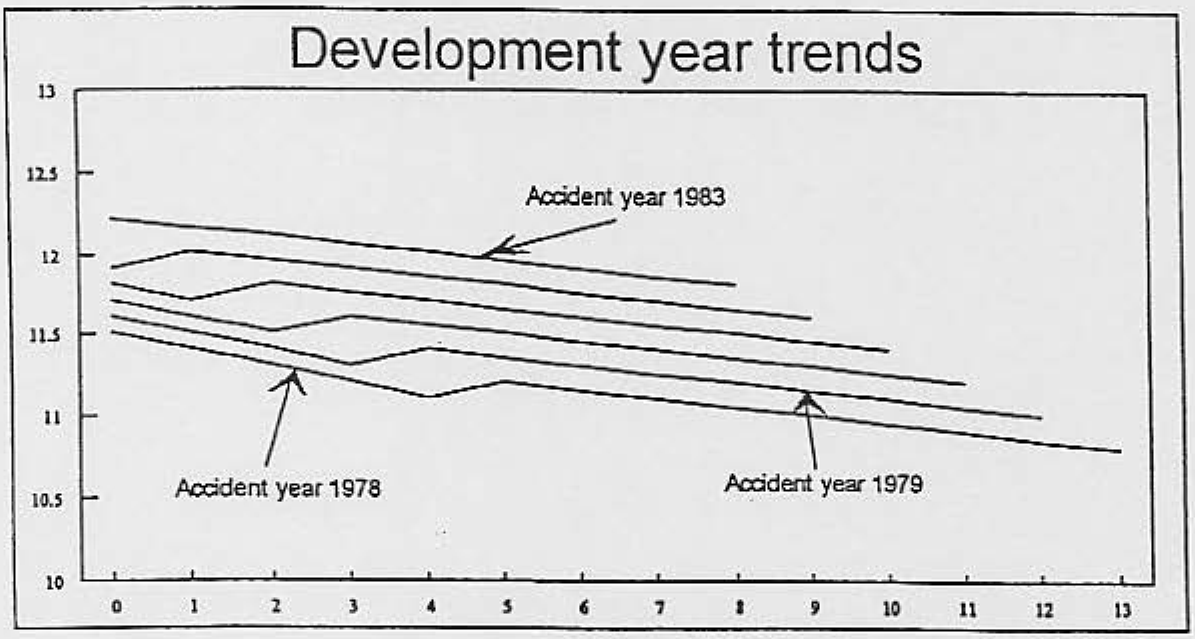


Figure 3.2.1

Figure 3.2.1 displays the graph of the log paid losses versus development year for the first six accident years. (The log paid losses are presented in Appendix A2).

Observe how payment/calendar year trends project onto development years and accident years. Each of the first six accident years has a different run-off development.

Consider the first accident year 1978. The 10% calendar year trend projects onto the development year, so that the resultant trend from development year 0 to development year 4 is -0.2 (the gamma) + .1 (the payment year trend) = -.1. The 30% trend between calendar years 1982 and 1983 also projects onto the development year so that the trend between development year 4 and 5 is -0.2+0.3=+0.1. Thereafter the trend is -.2+.15=-.05. Since .15 is larger than .1, the resultant decay in the tail is less rapid (-.05>-.1).

Consider the next accident year 1979. First up to development year 3, this accident year is 10% higher than the previous one since the 10% calendar year trend also projects onto the accident years. The 10% upward trend is one development year earlier than in previous accident year since the 30% trend is a calendar year change.

So, changing payment/calendar year trends can cause some interesting development year patterns. The run-off pattern is different for each accident year. The calendar year trends cannot be determined by the link ratios (age-to-age development factors) displayed in Appendix A4.

The patterns became much more complicated in the presence of random fluctuations superimposed on the trends.

The model describing the data we have constructed can be represented pictorially thus:

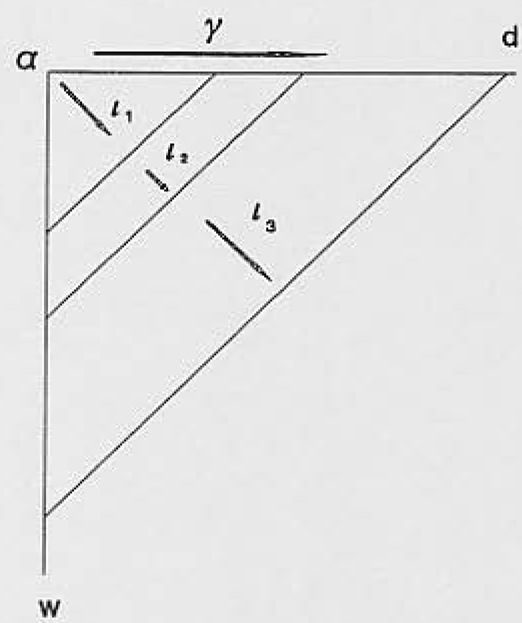


Figure 3.2.2

where γ=-0.2, ι_1=0.1, ι_2=0.3 and ι_3=0.15. The Greek letter ι (iota) represents a trend or inflation along payment/calendar years.

Writing the equations explicitly is not necessary. Indeed, it is too complicated.

We note that the resultant trend (age-to-age development factor) between development years j-1 and j is the (base) development factor γ between the two development years plus the payment year trend ι (iota) between the two corresponding payment years.

We now introduce random fluctuations or deviations from trends.

To all the log "payments" in the triangle we add random numbers from a normal distribution with mean zero. Equivalently, to the trends depicted in Figure 3.2.1, we add random numbers from a normal distribution displayed in Appendix A5. The sum of trends (Appendix A2) plus random fluctuations (Appendix A5) is displayed in Appendix A6.

The graph of the first six accident years of the data in Appendix A6 is given in the Figure 3.2.3 below.

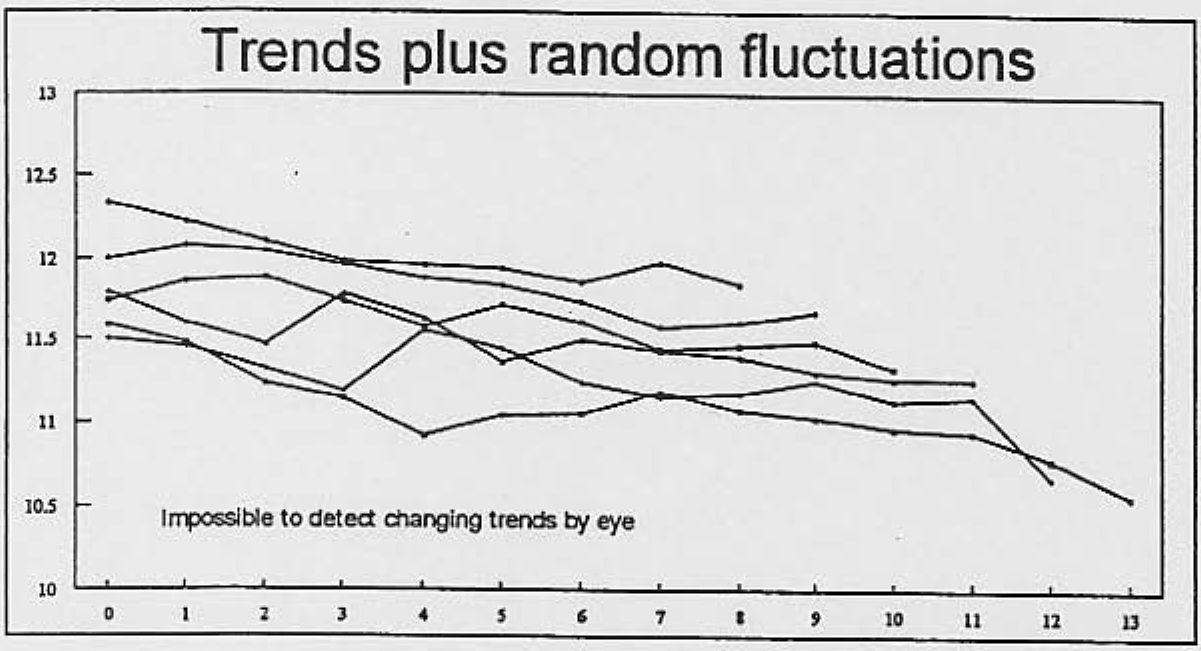


Figure 3.2.3

NOTE that it is impossible to determine the trends and/or changes in trends by eye or from the age-to-age link ratios of the cumulative payments (Appendix A9). See Appendices A7 - A9. THE TRENDS CAN ONLY BE DETERMINED BY USING REGRESSION.

The incremental paid losses we have generated in Appendix A7 were generated by five trend parameters (α , γ , ι_1 , ι_2 , ι_3) and one variance (noise, randomness) parameter σ^2 .

Since the incremental paid losses possess a stable trend (15%) along the payment years from 1983 to 1991 we would expect that the estimated model will validate well and be stable. See Section 3.3. Basically, when we look out the back window we determine that the road has been straight for quite a long distance.

3.3 Model Validation and Stability

Suppose we generate a sample triangle using model (3.3.1) with selected values of the three parameters $\alpha=10$, $\gamma=-0.3$ and $\sigma^2=0.4$.

When we use the generated observations in the triangle to estimate the parameters α, γ and σ^2, our estimates, due to sampling variation, will not be identical to the actual selected values of the parameters.

What should we expect if we re-estimate the model from a sub-sample of the triangle, say the triangle excluding the last three diagonals?

First, we would expect the estimates of the three parameters to be stable. That is, the estimates based on the sub-sample should not be statistically different to those based on the whole triangle. This is because when we look out the back window of our car (along the diagonals) we determine that the road has been straight (stable (zero) trend) for many years.

Secondly, we would expect the estimated model based on the sub-sample to accurately forecast the observations in the last three diagonals. When we use the estimated model for forecasting we are projecting not only the (average) trends for the future but most importantly the distribution of the deviations of the observations from the trends. We would expect that the observed deviations of the actual observations from the forecast trends to be governed by the probabilistic mechanisms of the forecast distributions. Moreover, the completion of the rectangle should be statistically non-different to basing our projections on the estimated model from the whole triangle.

The following table displays results of estimating the model $\alpha + \gamma d + \epsilon$ from a sample triangle.

TABLE 3.3.1

Payment yrs in Estimation	Estimate of gamma %	Forecast
1978-1994	-28.67±1.26	299,660±35,487
1978-1993	-28.58±1.46	303,980±37,885
1978-1992	-28.65±1.66	302,601±38,843
1978-1991	-29.26±1.95	304,711±42,149
1978-1990	-29.40±2.28	296,650±43,625

The true model for which $\alpha=10$, $\gamma=-0.3$ and $\sigma^2=0.4$ yields a (true) mean reserve forecast of 284,125 and a (true) standard deviation of 30,970. The standard deviation of 30,970 is referred to as process uncertainty. So,

EVEN IF WE KNOW ALL THE PARAMETERS OF THE TRUE MODEL, THERE IS STILL UNCERTAINTY OR VARIABILITY THIS IS CALLED PROCESS UNCERTAINTY

We now give a summary of the analysis of the incremental paid losses array generated by the probabilistic model of Section 3.2. The model contains a stable payment year trend of 15% since 1983.

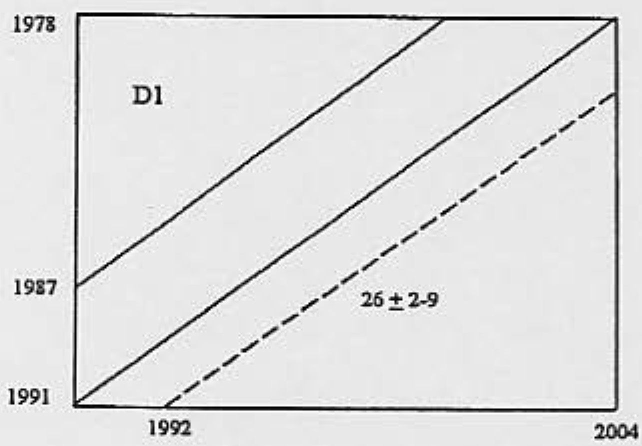
The model has four parameters, one α = 11.51293, one γ = -0.2, and three inflation parameters ι_1 = 0.1 for payment years 1978-82, ι_2 = 0.3 for payment years 1982-83 and ι_3 = 0.15 for payment years 1983-91. The incremental paid losses array appears in Appendix A7.

The following table gives estimates of parameters and outstanding total payments (i) using the whole triangle, (ii) using the triangle up to year end 1990, (iii) using the triangle up to year end 1989, (iv) using the triangle up to year end 1988 and finally (v) using the triangle up to year end 1987. In case (v) we do not use over 50% of the data points.

TABLE 3.3.2

Payment yrs in Estimation	Estimate of gamma (in tail) %	Estimate of iota (since 1983) %	Forecast $M
(i) 1978-91	-20.62±0.33	14.46±0.46	23±0.9
(ii) 1978-90	-20.75±0.36	15.27±0.51	25±1.2
(iii) 1978-89	-20.86±0.42	15.12±0.64	25±1.5
(iv) 1978-88	-21.19±0.45	15.75±0.75	26±2.0
(v) 1978-87	-21.31±0.55	15.63±1.03	26±2.9

Case (v)



D1 represents data points in the 1978-1987 payment years.

When we use the data set D1 to estimate the model parameters and then project payments for payment/calendar years 1992-2004, we obtain the same answer (26±2.9) statistically as when we use the whole triangle (23±0.9), that is data points from 1978-1991. Moreover, at year end 1987 the estimated model accurately forecasts the distribution of the deviations of the observed values from projected trends for payment/calendar years 1988-1991.

The identified model informs us that there is a stable trend in the data since 1983.

3.4 Model Maintenance and Updating

Once a model has been identified up to year end 1993, say, one year later at year end 1994, there is no need to analyse the history again. A number of post-sample predictive tests are conducted and the model is subsequently updated.

Consider the analysis in Section 3.3 of the data generated in Section 3.2.

At year end 1991 the model is stored. Next year, 1992, on receipt of the 1992 experience the same model is restored and zero weight assigned to the 1992 experience, in order to determine whether the estimated model at year end 1991 forecast the 1992 experience, and if not why not? Which assumption is the culprit if the answer is in the negative? Is it, for instance, that inflation between 1991 and 1992 is not 14.46% ± 0.46%. The post-sample predictive testing is a type of forward validation analysis.

3.5 Family of Models

Hitherto, we have assumed a constant trend γ (gamma) across development years. This is often not the case. It is usually the case that development year trends change in the early development years and become constant somewhere in the tail. Hence, we need the flexibility to determine the trends between every two contiguous development years and whether they change significantly. Accordingly, the modelling framework allows for a γ_j parameter between development years j-1 and j.

Similarly, the modelling framework allows for an inflation parameter ι_t (iota) between payment year t-1 and payment year t and a level parameter α_w for accident year w. Each parameter is readily interpretable.

<u>Development factors</u>

γ_j - trend between development years j-1 and j represents the development factor (on a log scale) between development years j-1 and j.

<u>Inflations</u>

ι_t - trend between payment/calendar years t-1 and t represents the inflation (superimposed inflation) between payment/calendar years t-1 and t. If the data are adjusted by some kind of CPI index then the trend represents **superimposed inflation**, otherwise the sum of the two effects: economic and social.

Exposures

α_w - level for accident year w represents the log "exposure".

The model of Section 3.2 has one γ parameter (constant base trend along development years), one α parameter (one constant 'exposure') and three iota parameters (inflation parameters). It also has a parameter σ^2 that represents the variance of deviations about the trends. So, the paid losses of Section 3.2 were only created by six parameters.

4.0 LOSS RESERVING MYTHS AND UNCERTAINTY

In the present paper we debunk some persistent loss reserving myths including a number of misconceptions concerning uncertainty. We use real life data to demonstrate our assertions.

4.1 Uncertainty

It is part of the actuary's task to respond to uncertainty, both as a technical matter and in the presentation of results.

There are a number of components of this uncertainty.

(i) **Process uncertainty (Noise or random variation)**

Even if we know the (exact) probabilistic model including the values of the parameters generating the paid losses, there is variability or uncertainty. See model (3.3.1) and the discussion in Section 3.3 for an example of process uncertainty. (If a coin, that is unbiased, is to be tossed 100 times, we know the probabilistic model generating the number of heads but we do not know how many heads we will observe).

(ii) **Estimation error or uncertainty**

The parameters of a probabilistic model are estimated from the historical experience. Accordingly, there is uncertainty associated with the true values of the parameters. See Section 3.3.

(iii) **Trend stability or lack thereof**

Based on Andrew Harvey's car example in Section 0.3, uncertainty about future trends is related to stability or lack thereof of past trends.

Assumptions about future trends are based on identification of past trends and other sources of information, including business knowledge. See Section 5.0.

This type of uncertainty is commonly referred to as **risk parameter** uncertainty and is intimately related to the next type of uncertainty.

(iv) **The future ain't what it used to be**

This source of uncertainty may be difficult to measure statistically, but that does not mean we should ignore it. The future may be very different to the past. A pricing actuary working in 1975 would have had no way of predicting the explosion in pollution liability claims that would have occurred in the 1980's, in respect of claims incurred in 1975. However, had he been aware that there is a strong probability that the environment, legal and economic, may change, judgemental changes to the parameters of the model, could have been made.

An actuary working in 1975 would have had no way of predicting the explosion in pollution liability that would have occurred in the 1980's (in respect of claims incurred in 1975).

4.2 Myth 1

Smooth data and/or smooth age-to-age link ratios means stability of trend in (incremental) payments.

The principal objectives of the analysis of the real life data in this section are to demonstrate that:

1. Age-to-age link ratios (or development factors) based on the cumulative paid losses give no indication about the trends and random fluctuations in the (incremental) payments.

2. Smooth data may have major shifts in payment year trends.

3. A large company's run-off payments are not necessarily stable in respect of payment year trends, even though the payments may be extremely smooth (with very little random fluctuations about the trends). So, even a large company with a large exposure base can have significant problems.

The data (save a multiplicate factor in order to preserve confidentiality) come from a large insurer and are given in Appendix C1. Accident year exposures, (from memory), represent earned premium (relativities). As we shall see in the next section, the exposures are not that important.

The age-to-age link ratios presented in Appendix C2 are relatively smooth. For the early development years they tend to decrease slightly in the middle accident years and then increase in the latter payment years.

The graph below is that of the residuals of the (statistical) chain ladder model. The model adjusts the data for the average trends between every two contiguous development years and every two contiguous accident years. We use the model as a powerful diagnostic tool to determine the relative payment/calendar year trends. Note a major shift in trend around 1984-1985. The trend changes quite alarmingly from 8% to 16%.

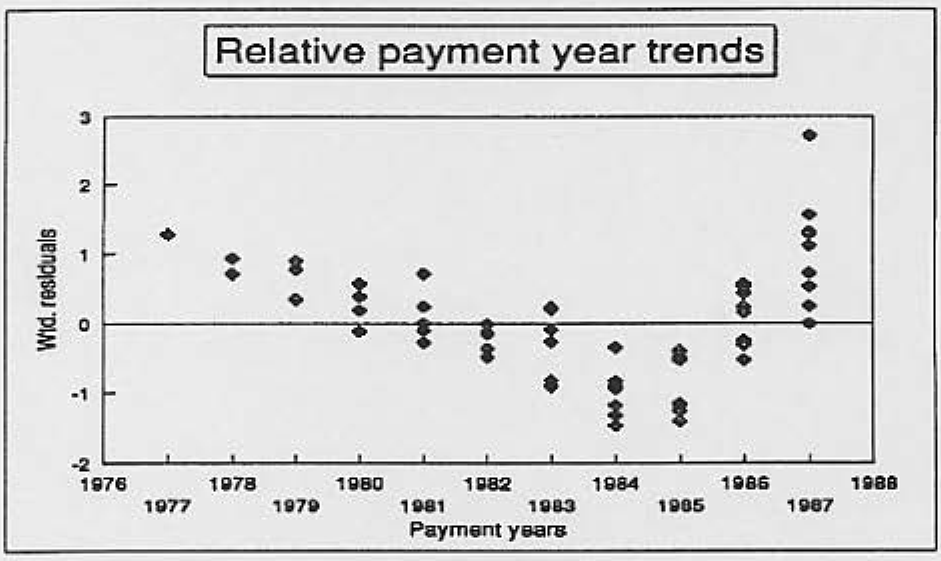


Figure 4.2.1

4.3 Myth 2

Rough data and/or age-to-age link ratios means instability in trend in incremental payments.

In the present sub-section we present some of the results of the analysis of a real (incremental) paid loss development array which we name GLD1. The exposure base is very small relative to example in Section 4.2.

The paid losses, presented in Appendix E1, are rough (i.e., have much variation), have rough link ratios or age-to-age development factors (Appendix E3), YET the payment/calendar year trend is essentially stable. Had we estimated the optimal model at year end 1989, we would have forecast accurately the trends and the distribution of deviations about the trends for the years 1990 to 1992, and moreover, the estimate of the outstanding liabilities beyond 1992 would be statistically the same as estimating the model to all the years at year end 1992.

So, loss (claims) reserve myth number two of the next Section is debunked, namely, **rough paid losses and/or rough link ratios (age-to-age development factors) imply payment year trend instability in the paid losses.**

It turns out that the identified optimal model has one (constant) trend parameter along the payment/calendar years. The estimate of this (average) inflation parameter is 12.77%±3.93%.

A graph of the deviations (residuals) of the observed from the fitted trends versus payment years is presented in Figure 4.3.1 below.

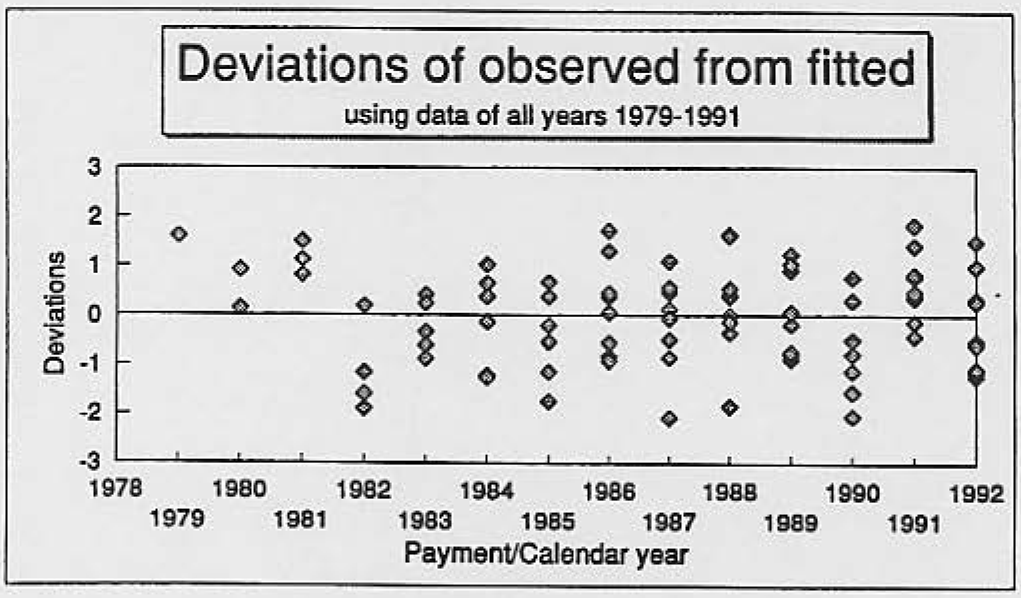


Figure 4.3.1

We observe some oscillations in the deviations about the zero line in the years 1989 to 1992. The deviations for the year 1992 appear almost symmetric about the zero line, whereas the deviations for 1991 are mostly positive, the deviations for 1990 mostly negative and those for 1989 almost symmetric.

So, when we remove the payments in 1992 from the estimation of the model we may expect the estimate of average trend (inflation) to increase slightly. Removal of 1991 and 1992 is expected to decrease the average trend and removal of 1990, 1991 and 1992, is not easy to call but we do not expect the estimate to change significantly.

We now present the validation and stability analysis results.

TABLE 4.3.1

Years in Estimation		Inflation estimate (%)	Trend estimate along develop yrs. 4-8 (%)
(i)	1979-1992	12.77±3.93	-59.53±10.42
(ii)	1979-1991	13.83±4.28	-63.52±11.26
(iii)	1979-1990	11.68±4.54	-65.78±12.19
(iv)	1979-1989	14.18±4.63	-65.14±13.09

TABLE 4.3.2

Years in Estimation		Estimate of resultant dev. yr. trend 4-8 (%)	Forecast (mean±s.e.) $M
(i)	1979-1992	-46.76± 9.59	202±53
(ii)	1979-1991	-49.69±10.57	212±62
(iii)	1979-1990	-54.10±11.76	164±52
(iv)	1979-1989	-50.97±12.87	222±82

Note that both the inflation and development year parameters are statistically stable. In case (iii), since we assume a much lower mean trend for the future, the mean forecast drops considerably. This essentially results from excluding one high year 1991 from the estimation and including one low year 1990 in the estimation.

So, what assumption do we invoke for the trends along the future payment/calendar years? We determined the past trend to be essentially stable. Three years earlier, at year end 1989, had we assumed a similar trend, 14.18%±4.63%, we would have forecast accurately the experience of 1990, 1991 and 1992 and our estimate of the outstanding liability beyond 1992 would not have been statistically different.

It is therefore reasonable to assume for the future 12.77%±3.93%, as in case (i). That is, actual trend in the future is a random value from a normal distribution with mean (average) 12.77% and standard deviation 3.93%. This implies that the mean payment in the future is not derived by just using a trend of 12.77%. Inflating the payments by 12.77% will yield the median payment not the mean payment. The mean payment is obtained by using inflation of 12.77% + ½ x $(3.93\%)^2$ = 12.85%.

Consider case (iv) where the model is estimated at year end 1989. Figure 4.3.2 below displays the deviations of observed from the fitted trends for the years 1979-1989 and the deviations (prediction errors) of the observations from the predicted trends for years 1990 to 1992. Note that the deviations from fitted for the years 1990 to 1992 in Figure 0.7.1 are very similar to deviations (prediction errors) from predicted for the years 1990 to 1992 in Figure 4.3.2.

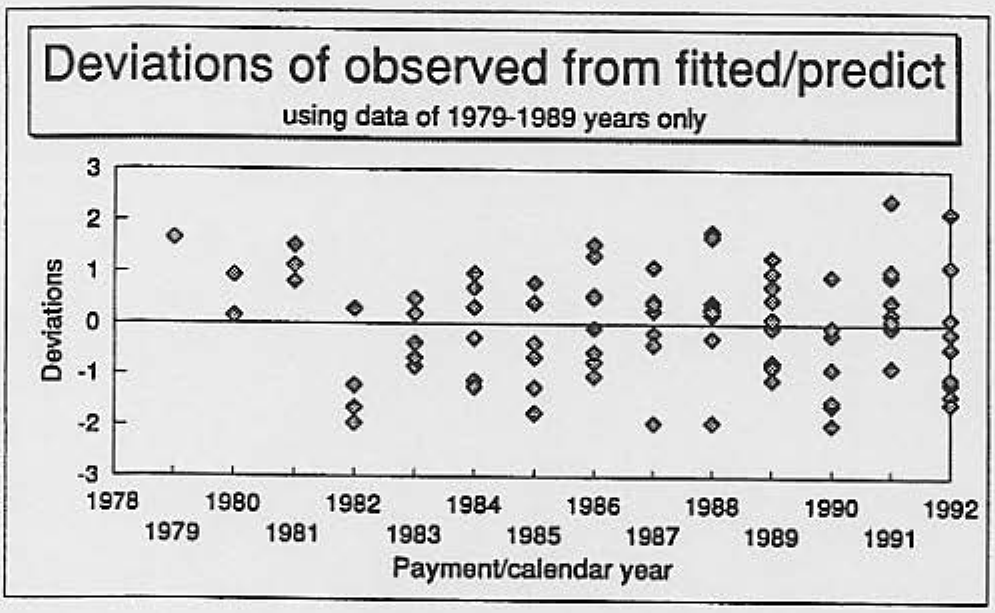


Figure 4.3.2

The model as at year end 1989 also predicts normal distributions for the deviations of the (log) observations from predicted trends for the years 1990-1992. Figure 0.7.3 below displays a normal probability plot of the deviations (prediction errors) for the years 1990-1992, as at year end 1989.

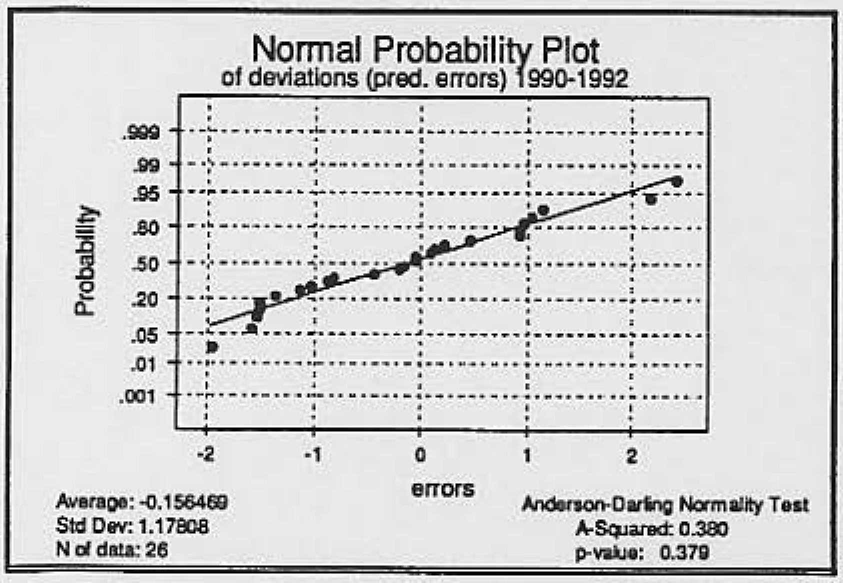


Figure 4.3.3

So there is considerable evidence that the predicted deviations for the years 1990-1992 are values selected at random from the predicted normal distribution.

Most importantly, all our calculations about the future are conditional on our assumptions for the future remaining true. For example, the assumption regarding future payment/calendar year inflation is 12.77%±3.93%. If, in some years in the future, inflation turns out to be 28%, say, then our assumption is violated and our subsequent calculations do not apply. This is analogous to seeing an almost straight road out the back window, but the road may not remain as straight in the future.

4.4 **Myth 3**

Increase in incremental payments is associated with increase in speed of closures (finalisations) of claims.

Often the contrary of the above statement is true. Figure 4.4.1 below displays the residuals of the statistical chain ladder applied to the incremental paid losses of CTP NSW for AMP General Insurance. Note there is a favourable trend in the early years and thereafter a relatively stable trend. In 1987 the payments are relatively lower.

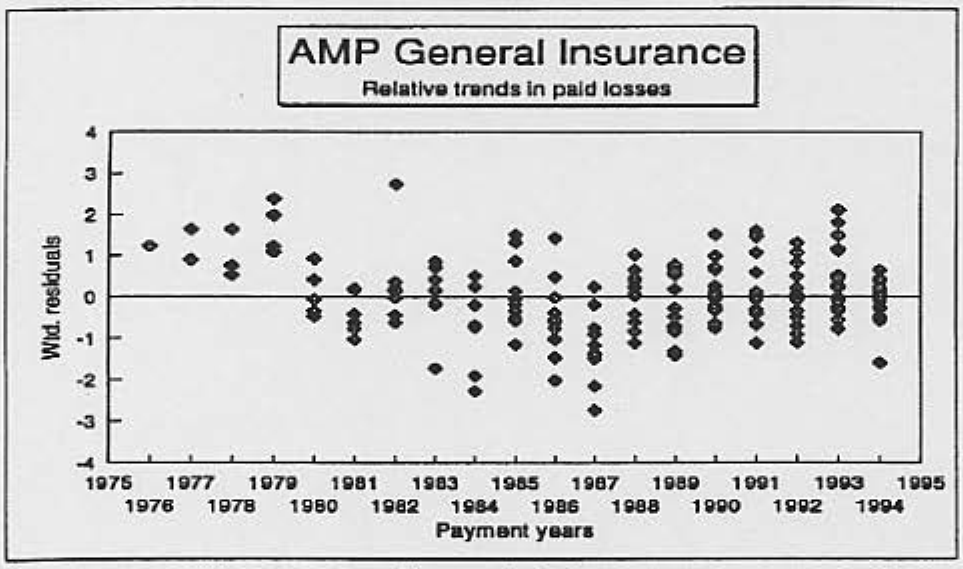


Figure 4.4.1

Figure 4.4.2 displays the relative payment/calendar year trends for the closed claim counts.

Note:

(i) Higher closures do not imply higher payments.

(ii) Closed claim counts are less stable than the paid losses in terms of trends.

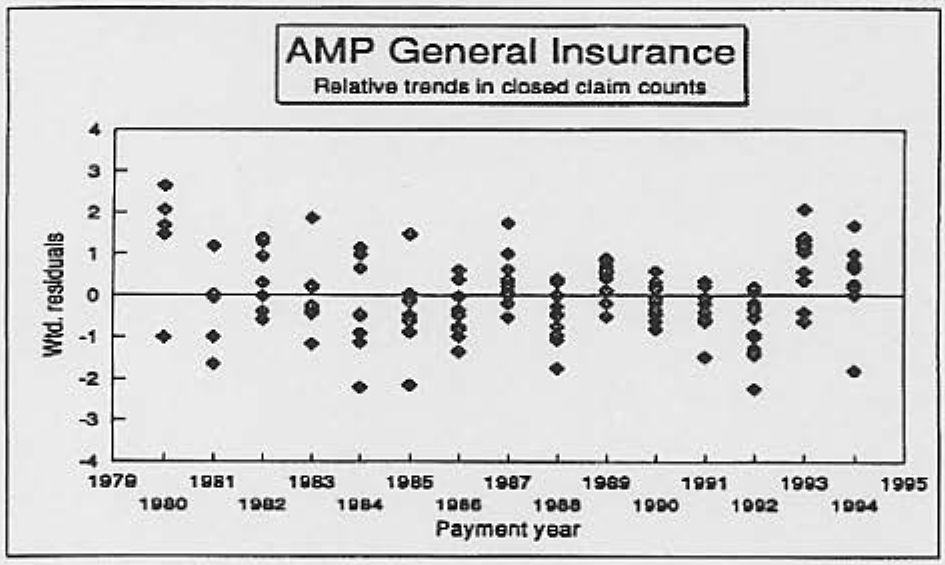


Figure 4.4.2

4.5 Myth 4

Closed (finalised) claim counts have more stable trends than incremental payments and are so better predictable.

Displays 4.4.1 and 4.4.2 demonstrate that the incremental paid losses have more stable trends than the closed claim counts and so are better predictable. Indeed, according to the author's expeience, it is rare that closed claim counts have stable payment/calendar year trends.

5.0 STABILITY, ASSUMPTIONS ABOUT FUTURE JENSEN'S INEQUALITY AND PREDICTABILITY

In this section we discuss related issues of trend stability, assumptions about the future and Jensens' inequality.

5.1 Stability

Returning to our example of Section 4.3, we ask the question whether at year end 1989 our completion of the rectangle should be materially different from completion at year end 1992. The answer, as was demonstrated, is in the negative since trends, especially in the payment year direction are stable. (Applying the Harvey motor car analogy, the road was essentially straight).

We illustrate with another four examples. (There are numerous others that occur in practice.)

Example 1: Suppose payment year trends (after adjusting for trends in the other two directions) are as depicted in Figure 5.0.1 below. The trend is stable and suppose its estimate is 10% $\pm$ 2%. How do we know that the trend is stable? Well, as we remove the more recent payment years from the estimation, the estimates of trends do not change (significantly). For example, after removing 1990 and 1991, the estimate of trend is 9.5% $\pm$ 2.1%, say. Alternatively, we could estimate a new trend parameter from 1989-1991 and examine whether the trend has changed significantly.

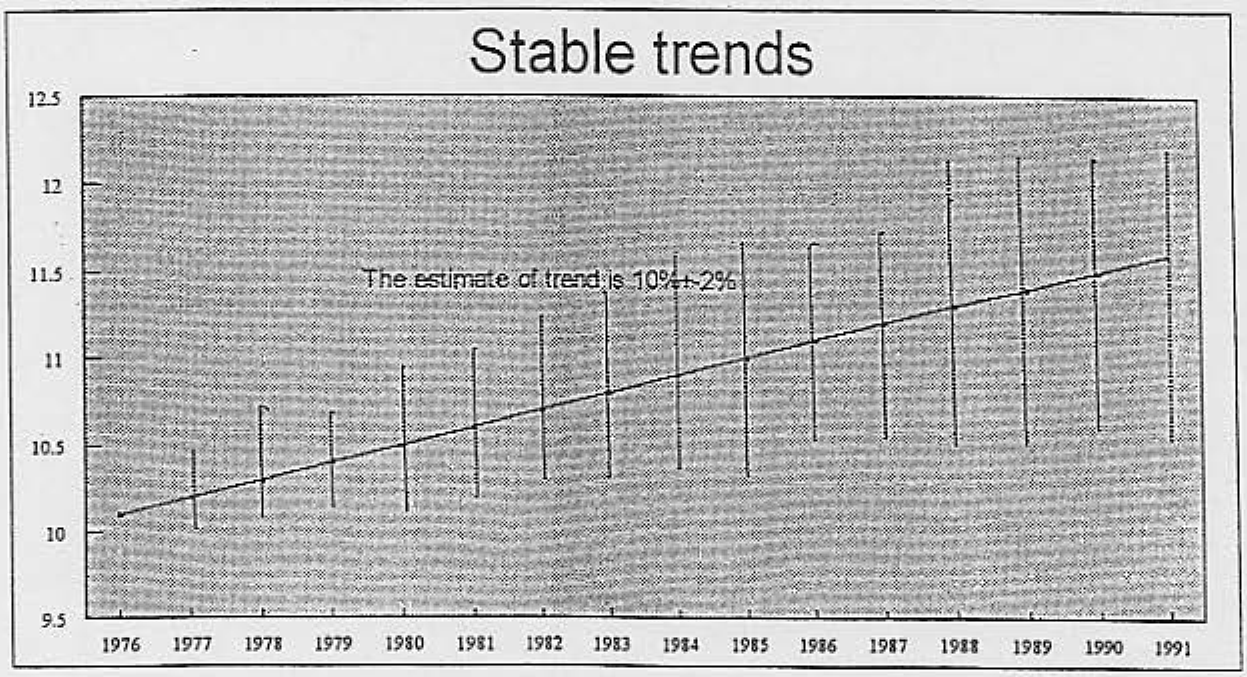


Figure 5.0.1

Typically, if the payment/calendar year trend is stable, the model will also validate well. Here our estimates of outstanding payments do not change significantly as we omit recent years.

Example 2: Consider the payment year trends depicted in Figure 5.0.2 below.

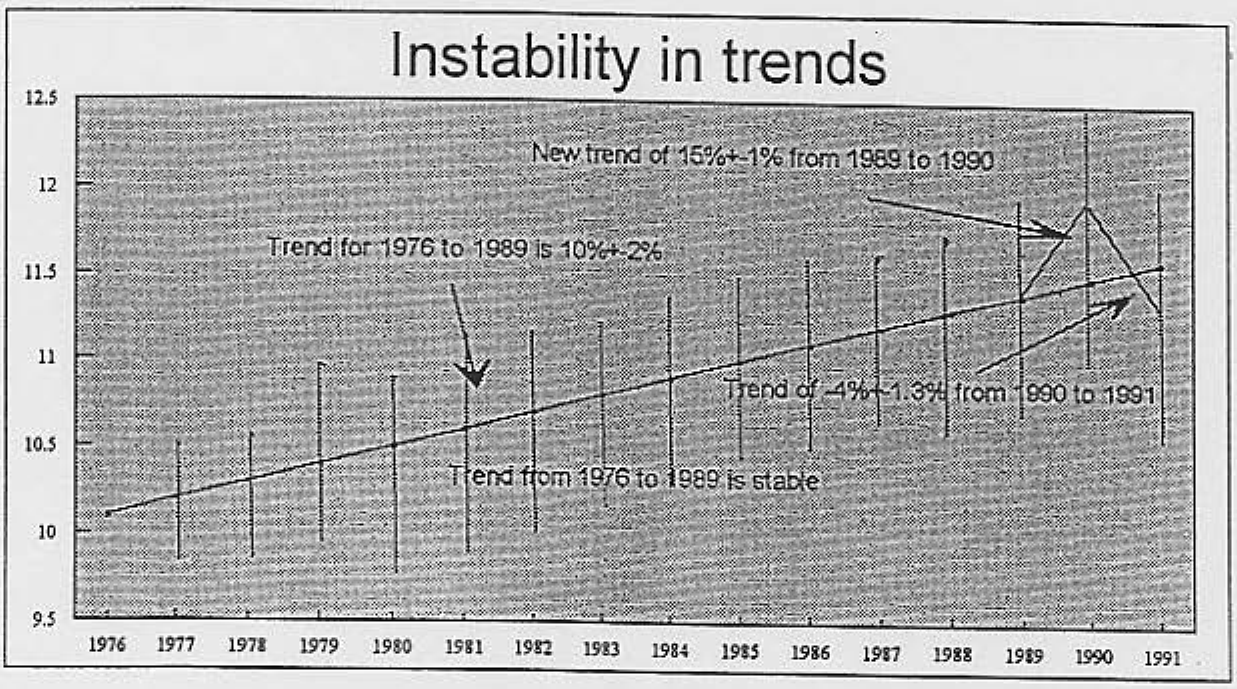


Figure 5.0.2

The trend in the years 1976 to 1989 is relatively stable. Its estimate is 10% ± 2%, say. However, the trend from 1989 to 1990 is higher at 15% (± 1%) and from 1990 to 1991 it is -4% (± 1.3%), say. This information is extracted from the "optimal" statistical model. The shifts in trends is a property of the data (determined through the model). A question now emerges as to which trend assumption do we make for the future, first in the absence of any other information. It would be foolhardy to assume the estimate between the last two years of -4% ± 1.3%. The most reasonable assumption (for the future) is a mean trend of 10% with a standard deviation of 2%, that which was estimated for the years 1976-1989.

Suppose we also have access to another data type, the number of closed claims development array. We find utilising the modelling framework that the additional 5% above the 10% between 1989 and 1990 can be explained by a corresponding increase in speed of closures of claims and the -15% from 1990 to 1991 below the 10% from 1976-1989 can be explained by a corresponding decrease in the speed of closures of claims. What assumption about future trends in payments should we adopt then? I would still recommend 10% ± 2% for the future. That's a decision based on my judgement and experience. The instability in trends in the last few years means that the model will not validate well. At year end 1990, we would not have forecast the distributions for 1991, for example.

Example 3: It is possible to have a transient change in trend. Consider Figure 5.0.3. The business has been moving along 10% ± 2% but between the last two calendar years 1990 and 1991 the trend increases to 20% ± 3%. What do we assume for the future? Well, that depends on the explanation for the increase in trend. Suppose its a "transient" change that can be explained by a new level of benefits that apply retrospectively. Then it is reasonable to assume 10% ± 2% for the future. Suppose instead that subsequent to analysis of claims closed triangle, the trend change is explained by increase in severities. That's a problem, because this means that it is now more likely that the new trend will continue.

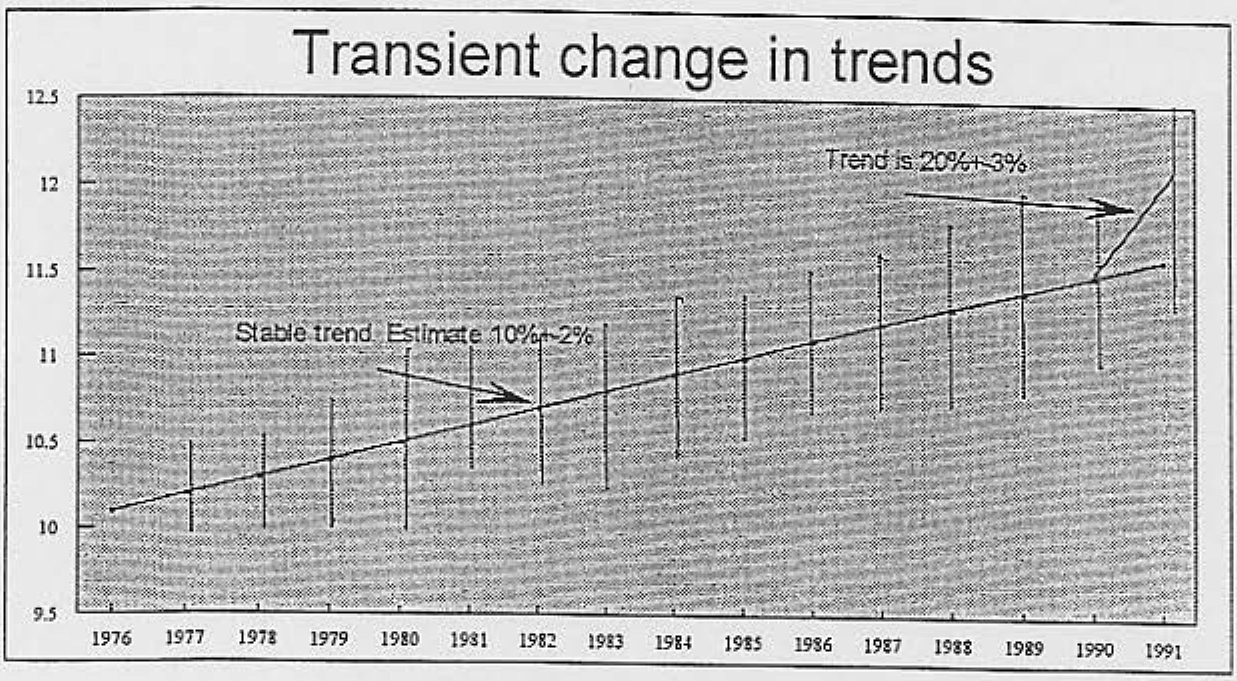


Figure 5.0.3

So the decision making process about the future becomes more complicated when trends are unstable. We are talking about trends in the (incremental) payments not age-to-age link ratios.

The last example illustrates an 'unpredictable' loss development array.

Example 4: The payment year trends are depicted in Figure 5.0.4 below.

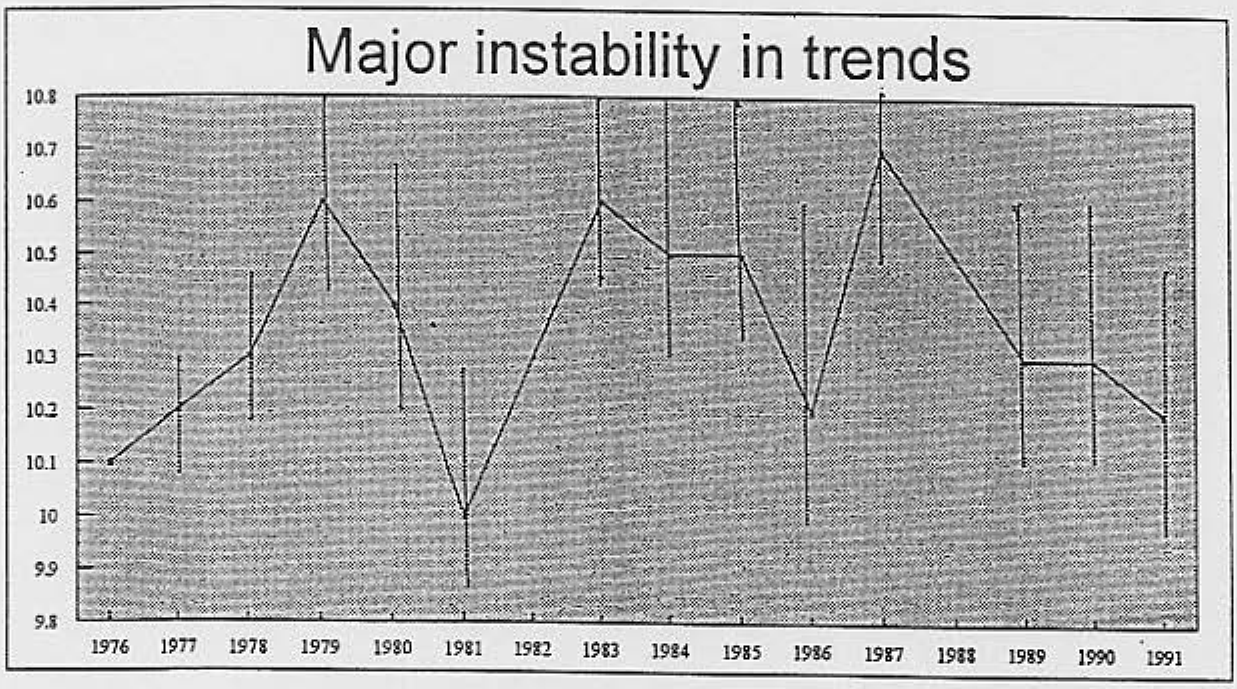


Figure 5.0.4

Note the instability in the trends. At year end 1989, would anyone be able to predict a flat trend for the next year and a downward trend for the following year?

Here, maybe, one could calculate $\hat{\imath}$, a weighted average of trends estimated in the past with a weighted variance $\hat{\sigma}^2$ and assume for the future a mean trend of $\hat{\imath}$ with standard deviation of trend $\hat{\sigma}$. Since $\hat{\sigma}$ will be relatively large, mean forecasts will be well above the median forecasts and the standard deviation of the distributions will be relatively large.

It is instructive to relate the foregoing discussion with the quote from Harvey [9] given at the end of Section 2.2.

5.2 Assumptions about the Future and Jensen's Inequality

We demonstrated in Section 5.1 that if payment/calendar year trend has been stable, then the assumption about the future trend is relatively straightforward, although we may not be absolutely sure how long the trend may continue. It may be helpful to determine what factors are driving the trend, in order to make appropriate assumptions about the future. This determination may be based on analysing data types other than the paid losses and knowledge of the business.

If on the other hand, payment/calendar year trend has been unstable, as in examples 2 and 3 of Section 5.1, the nature of the instability, analysis of other data types and business knowledge would be critical in formulating assumptions about the future. Of course, business knowledge should be combined with the objective facts in formulating sound judgment.

Application of Jensen's inequality tells us that the mean payment using a variable inflation rate is higher than if we just inflate by the mean inflation. Indeed, ignoring this result is dangerous, especially, if the standard error of inflation is large.

6.0 CTP NSW AGGREGATE EXPERIENCE

6.1 Background

The NSW Government established a new Scheme, which commenced in July 1989, for compensating people injured as a result of the fault of others in motor vehicle accidents. Compulsory Third Party Personal Injury Insurance (or CTP) provides funds to compensate people injured in motor vehicle accidents.

The Scheme includes the following features:

- Each registered owner of a vehicle in NSW is required to insure, with an insurer licensed under The Motor Accidents Act 1988.

- Insurers are licensed under the Act and must file with the Motor Accident Authority, at least once a year, a full set of premiums it proposes to charge for third party policies.

The Scheme has only five years experience in its current form, and accordingly the principal loss development arrays analysed have a quarterly sampling period.

6.2 Relevant Findings Based on Probabilistic Models

The following findings are relevant:

1. The incremental paid (loss) experience has a high inflation (combined AWE + superimposed) rate of 8.42% $\pm$ 1.38% (continuous per quarter) since payment quarter year 2-91. The inflation has been relatively stable, save for some seasonality.

2. The high inflation rate is <u>not</u> explained by the speed of closures of claims, although there has been a relatively small increase in closure rates especially in the early development quarter years.

3. There is a small increase in the number of claims notified, but only for the early development quarter years.

4. The high trends (inflation) in the case estimates and incurred losses parallel those in the paid losses.

5. There is some evidence that the tail in the incremental paid loss experience is beginning to decay.

6. Projections of payments outstanding based on quarterly incremental paid losses data are statistically the same as projections based on yearly incremental paid losses data.

6.3 Inflation in the Paid Loss Experience

The graph presented below (Figure 6.3.1) depicts the relative payment quarter year (diagonal) trends after adjusting the paid loss data only in the development quarter year direction.

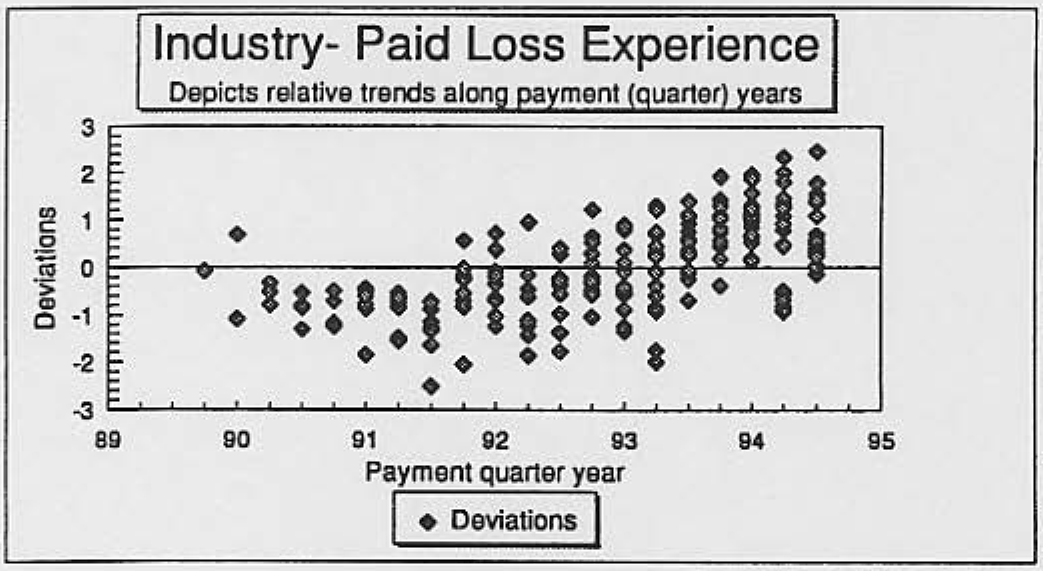


Figure 6.3.1

From payment quarter 3-89 to 2-91 (first two years) there is a relatively favourable experience.

Thereafter, the trend is relatively stable but very steep, indicating a <u>high</u> level of inflation.

6.4 Inflation in the Paid Losses and Claim Settlement Rates

Much of the high trend in the paid loss experience is not explained by the trends (speed of closure) in the closed claim count experience. Figure 6.3.2 below depicts the trends along the payment quarter years (diagonals) in the closed claim count experience.

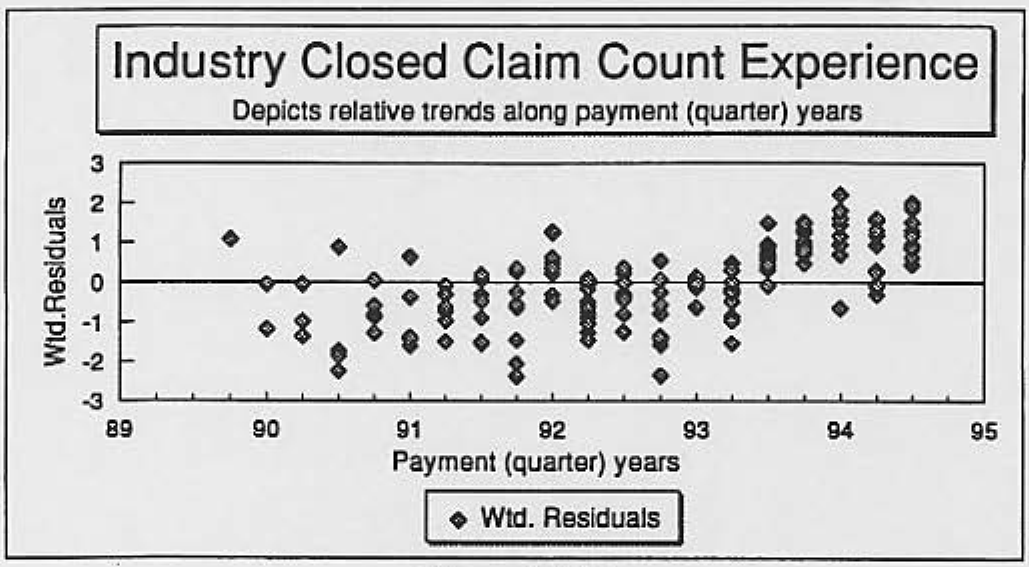


Figure 6.4.1

Note positive trend in closed claims only from payment quarter 1-93 to 4-93, which also partly explains the higher payments in those quarter years. However, the higher payments in other years including 2-94 are not explained by the speed of closures of claims. The trends in the closed claim counts are not as high as those in the paid losses and are mainly in the early development quarter years where payments are low.

Accordingly we now present two displays showing the relative payment quarter year trends in the paid losses and the closed claim counts beyond development quarter 4, respectively.

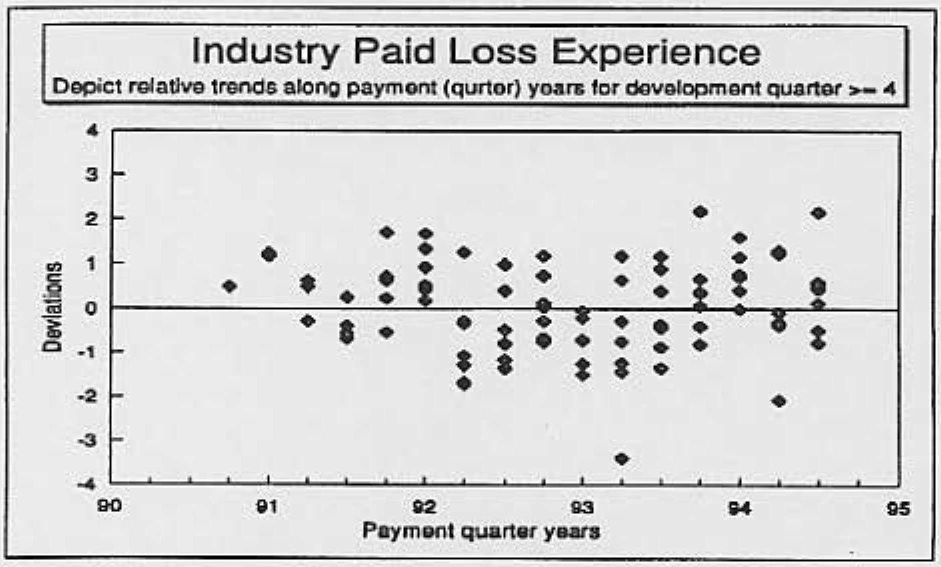


Figure 6.4.2

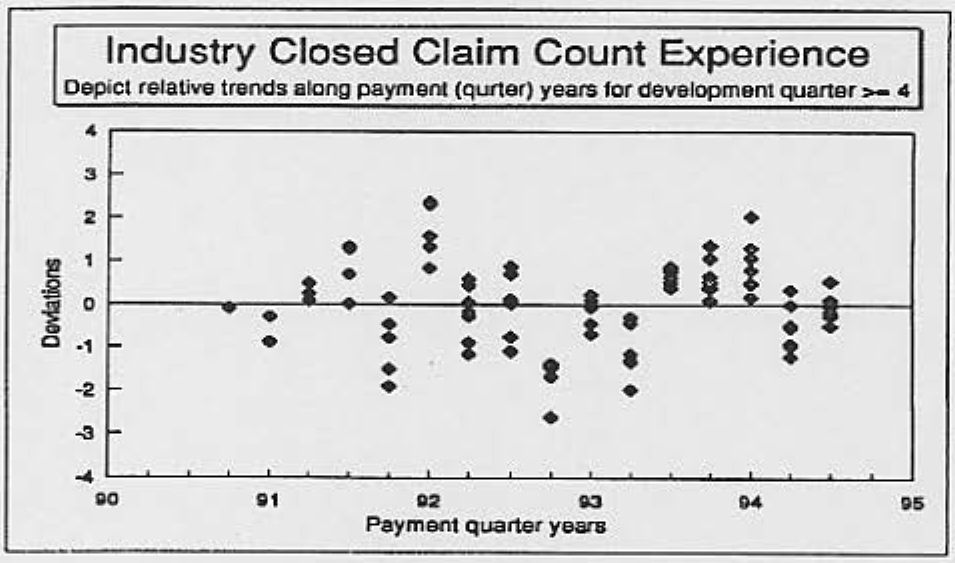


Figure 6.4.3

We observe:

- Paid losses experience has more stable trend and accordingly is better predictable.

- The trends in the paid losses are <u>not</u> positively correlated with the trends in the closed claim counts.

The latter observation means that Taylor's See Saw hypothesis is valid: "The faster claims are closed the less is paid per claim". As a result of the validity of this hypothesis we expect the first observation to be true.

6.5 Validation Analysis and Stability of Inflation

We examine the behaviour of the estimated model as we remove payment quarter years (diagonals) from the estimation process. The important question is whether the model estimated one to seven payment quarter years ago would have predicted the most recent experience.

If we estimate the best model a number of times, each time removing more of the recent experience, the following results are obtained.

Table 6.5.1

Projections to quarter 20			
Quarter Years used in estimation	**Trend Estimate Since 1-92 (Inflation) %**	**Mean Forecast $M**	**Standard Error $M**
3-89 to 2-94	8.42 ± 1.38	2,963	225
3-89 to 1-94	9.01 ± 1.59	2,931	282
3-89 to 4-93	10.25 ± 1.56	3,459	393
3-89 to 3-93	9.94 ± 1.82	3,236	461
3-89 to 2-93	8.24 ± 2.09	2,654	457
3-89 to 1-93	6.60 ± 2.33	2,356	490
3-89 to 4-92	7.70 ± 2.49	2,675	686
3-89 to 3-92	6.68 ± 2.77	2,804	945

Changes in mean projections as years are removed from the estimation process are closely reflected by the change in assumption as to future "inflation". The assumed future payment quarter year inflation is based on that estimated since payment quarter year 1-92.

Note from Table 6.5.1 above, that had the best model been estimated at payment (quarter) year end 3-92, the estimated total outstanding of $2,804M ± $945M is statistically not different to $2,963M ± $225M, that obtained by the model using the experience to payment quarter year end 2-94. Indeed, the two answers are remarkably close especially that $2,804M ± $945M is obtained after removing 56% of the most recent experience.

We now explore whether the model estimated at quarter year end 3-92 "predicts" the subsequent paid losses in payment quarter years of 4-92 to 2-94.

Figure 6.5.1 displays the deviations based on the model estimated at payment quarter year end 3-92. For payment quarter years 3-89 to 3-92, the deviations are the observed values minus the fitted values, whereas for payment quarter years 4-92 to 2-94 (last seven payment quarter years!) the deviations represent the prediction errors. It is remarkable how the prediction errors for the last seven payment quarter years are centred around zero.

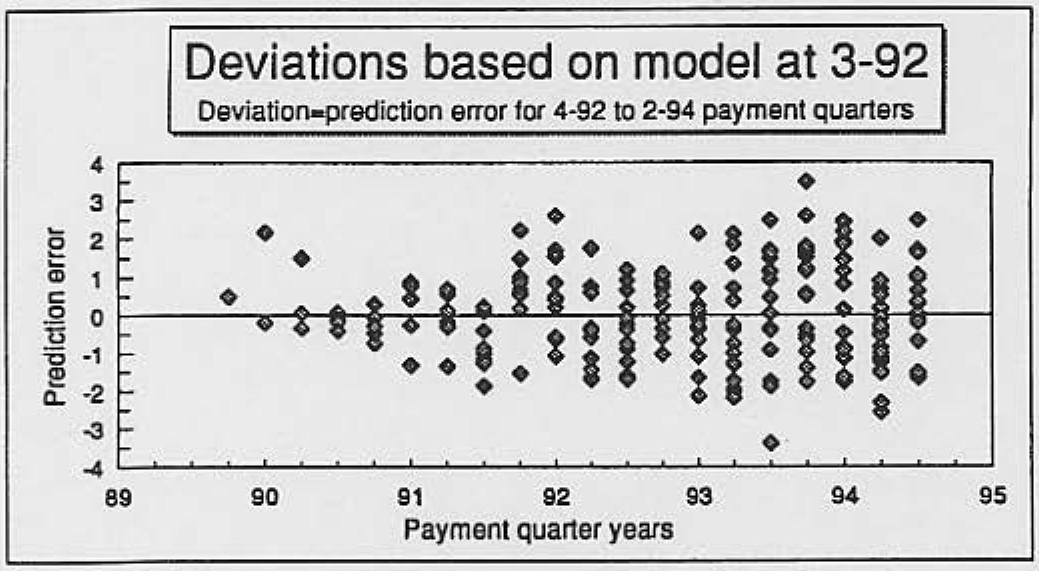


Figure 6.5.1

Figure 6.5.2 presents the prediction errors for payment quarter years 4-92 to 2-94 based on the model estimated at year end 3-92. Note the slight curvature due to higher payments in 3-93 and 4-93 and lower payments in 1-94.

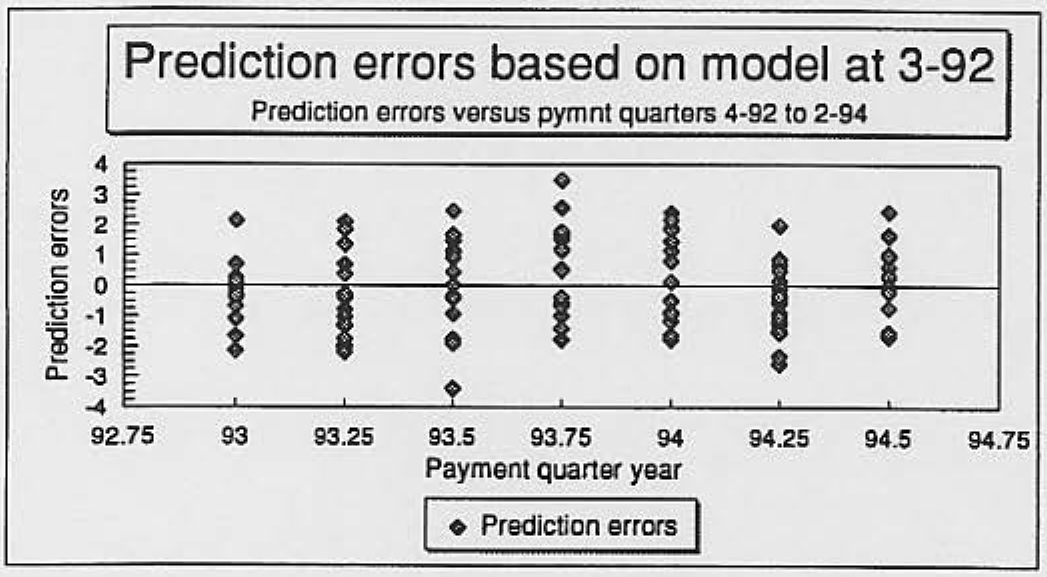


Figure 6.5.2

Finally, the normality plot for the prediction errors of observations in payment quarters 4-92 to 2-94, based on model estimated at payment quarter year 3-92 is displayed below and one can see it is in good shape.

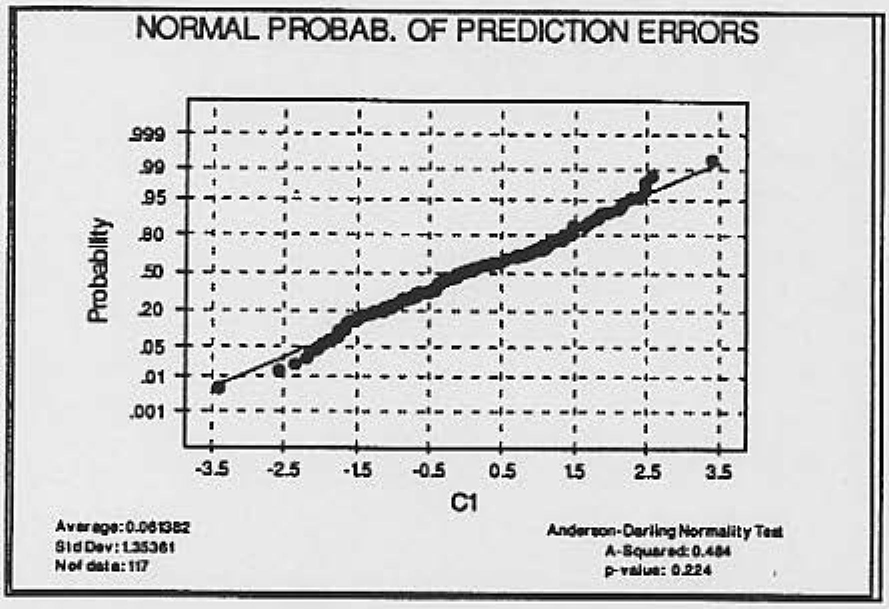


Figure 6.5.3

By way of summary:

- The paid loss experience contains a high inflation rate not explained by the speed of closures of claims.

- The high inflation rate has existed since 2-92 and has been essentially stable, save for some seasonality.

- The model estimated at payment quarter year end 3-92, predicts accurately the next seven quarter payment years experience and yields the same outstanding claims estimate statistically as the model estimated at quarter year end 2-94.

6.6 Premium and Future Inflation

If we apply the Harvey [9] motor car analogy, even though we have identified an inordinate high inflation rate in the five year CTP experience, we are not guaranteed it will continue. Indeed, given that the principal reason for the inflation is a continued trend towards litigation as an avenue by claimants for higher award payments one would expect and hope to reach 'saturation' in the near future.

In any event, the current pure premium of approximately $190 (per vehicle), as advised by a number of actuarial consultants is substantially too low. Assuming a discount rate of 10% - 11% p.a., it could only be substantiated by a less than 4% combined AWE + Superimposed inflation in the future, and moreover we would have to ignore Jensen's inequality.

The principal reason for the high inflation seems to be a continued trend towards litigation as an avenue by claimants for higher award payments, although some practitioners are arguing that the inflation is principally driven by increase in speed of closures of claims.

7.0 CONCLUSIONS

We have argued that the four components of interest regarding a loss development array are the trends in the three directions and the distributions of the deviations (random fluctuations) about the trends.

A MODELLING FRAMEWORK was introduced where each model contained therein possesses the four components of interest. The modelling approach offers the actuary a way of fitting (estimating) distributions to the cells in a loss development array and predicting (forecasting) distributions for future years that affords numerous advantages including:

- simplicity;
- clarity of assumptions;
- testing of assumptions;
- assessment of loss reserve variability;
- asset/liability matching;
- model maintenance and updating.

We showed how the identified optimal statistical model for the (incremental) payments conveys information about the loss experience to date. In applying the model to predicting distributions of future payments the actuary may (need to) adjust some of the parameters to reflect knowledge about the business and to incorporate his view of the future. View of the future may be based on analysis of other data types, especially if there are instabilities in the trends in the paid losses in the recent payment years.

A prediction interval computed from the forecast distributions is <u>conditional</u> on the assumptions made about the future remaining true.

In passing we have debunked a number of pervasive loss reserving perceptions concerning data types, age-to-age link ratios, stability, forecasting and regression.

Methods based on age-to-age link ratios do not (and cannot) separate trends from random fluctuations and moreover do not satisfy the <u>basic fundamental</u> property of additivity of trends.

REFERENCES

[1] Abraham, B. and Ledholter, J. (1983). Statistical Methods for Forecasting, John Wiley, New York.

[2] Bailey R.A., "Sampling theory in casualty insurance", PCAS, 1942, pp.29-50.

[3] Catterjee, S. and Price, B. (1977). Regression Analysis by Example. John Wiley, New York.

[4] Christofidies, S. (1980). "Regression models based on log-Incremental Payments". Institute of Actuaries Loss Reserving Manual, Volume 2., Chapter 2.

[5] Covey, S.R. (1989). The 7 Habits of Highly Effective People. Fireside, Simon & Schuster.

[6] Foundations of Casualty Actuarial Science, 1989, Casualty Actuarial Science, New York.

[7] Fisher, W.H. and Lange, J.T. (1973). "Loss reserve testing: a report year approach", PCAS, 60, pp.189-207.

[8] Friedman, M. (1953). "The methodology of positive economics", in Essays in Positive Economics, University of Chicago Press, pp.3-43.

[9] Gauss, K.F. (1809). Theoria Motus corpurum coelstium, Werke 7, Hamburg.

[10] Harvey, A.C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press, New York.

[11] Laurenzano, V. (1992). "Balancing capital with risks", Best's Review, June, 1992.

[12] Loss Reserving Manual, 1990, Institute of Actuaries, London.

[13] Muth, J.F. (1960). "Optimal properties of exponentially weighted forecasts". J. Amer, Statist. Assoc., 55, 299-306.

[14] Popper, K.F. (1959). The Logic of Scientific Discovery. Hutchinson.

[15] Zehnwirth, B. (1985). "Linear Filtering and Recursive Credibility Estimation", ASTIN Bulletin, Vol. 15, No. 1, 19-35.

Appendix A1 (Lotus Worksheet M3INF)

Model is p = exp(alpha-.2d) with no error or randomness
alpha = 11.51293

Year/delay

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080	9072	7427
1979	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080	9072	
1980	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080		
1981	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534			
1982	100000	81873	67032	54881	44933	36788	30119	24660	20190	16530				
1983	100000	81873	67032	54881	44933	36788	30119	24660	20190					
1984	100000	81873	67032	54881	44933	36788	30119	24660						
1985	100000	81873	67032	54881	44933	36788	30119							
1986	100000	81873	67032	54881	44933	36788								
1987	100000	81873	67032	54881	44933									
1988	100000	81873	67032	54881										
1989	100000	81873	67032											
1990	100000	81873												
1991	100000													

Appendix A2 (Lotus Worksheet M3INF)

y=log(p) plus .1 inf. from 1978-82, .3 inf. from 1982-83 and .15 inf. from 1983-91

Year\delay	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	11.5129	11.4129	11.3129	11.2129	11.1129	11.2129	11.1629	11.1129	11.0629	11.0129	10.9629	10.9129	10.8629	10.8129
1979	11.6129	11.5129	11.4129	11.3129	11.4129	11.3629	11.3129	11.2629	11.2129	11.1629	11.1129	11.0629	11.0129	
1980	11.7129	11.6129	11.5129	11.6129	11.5629	11.5129	11.4629	11.4129	11.3629	11.3129	11.2629	11.2129		
1981	11.8129	11.7129	11.8129	11.7629	11.7129	11.6629	11.6129	11.5629	11.5129	11.4629	11.4129			
1982	11.9129	12.0129	11.9629	11.9129	11.8629	11.8129	11.7629	11.7129	11.6629	11.6129				
1983	12.2129	12.1629	12.1129	12.0629	12.0129	11.9629	11.9129	11.8629	11.8129					
1984	12.3629	12.3129	12.2629	12.2129	12.1629	12.1129	12.0629	12.0129						
1985	12.5129	12.4629	12.4129	12.3629	12.3129	12.2629	12.2129							
1986	12.6629	12.6129	12.5629	12.5129	12.4629	12.4129								
1987	12.8129	12.7629	12.7129	12.6629	12.6129									
1988	12.9629	12.9129	12.8629	12.8129										
1989	13.1129	13.0629	13.0129											
1990	13.2629	13.2129												
1991	13.4129													

Appendix A3 (Lotus Worksheet M3INF)

Cumulative data (on a $ scale) derived from Appendix A2

100000	190484	272357	346439	413471	487552	558021	625053	688816	749469	807164	862045	914250	963908
110517	210517	301001	382874	473358	559428	641302	719182	793263	863732	930764	994527	055180	
122140	232657	332657	443174	548302	648302	743425	833908	919979	001852	1079732	1153814		
134986	257126	392112	520515	642655	758838	869355	974482	1074482	1169605	1260089			
149182	314055	470886	620068	761975	896961	1025363	1147504	1263687	1374204				
201375	392929	575141	748467	913339	1070170	1219352	1361259	1496245					
233965	456519	668219	869594	1061148	1243360	1416685	1581557						
271828	530399	776359	1010324	1232878	1444578	1645954							
315819	616236	902001	1173829	1432400	1678360								
366930	715964	1047976	1363795	1664212									
426311	831831	1217574	1584504										
495303	966450	1414619											
575460	1122855												
668589													

Appendix A4 (Lotus Worksheet M3INF)

Age-to- age link ratios of the cumulative losses of Appendix A3

1.904837	1.429816	1.272002	1.193488	1.179170	1.144535	1.120124	1.102011	1.088054	1.076981	1.067992	1.060558	1.054316
1.904837	1.429816	1.272002	1.236327	1.181830	1.146351	1.121440	1.103008	1.088834	1.077607	1.068505	1.060986	
1.904837	1.429816	1.332224	1.237213	1.182381	1.146726	1.121712	1.103213	1.088994	1.077736	1.068611		
1.904837	1.524979	1.327463	1.234652	1.180786	1.145639	1.120925	1.102618	1.088529	1.077362			
2.105170	1.499375	1.316812	1.228856	1.177152	1.143152	1.119119	1.101248	1.087456				
1.951229	1.463726	1.301361	1.220279	1.171712	1.139400	1.116378	1.099162					
1.951229	1.463726	1.301361	1.220279	1.171712	1.139400	1.116378						
1.951229	1.463726	1.301361	1.220279	1.171712	1.139400							
1.951229	1.463726	1.301361	1.220279	1.171712								
1.951229	1.463726	1.301361	1.220279									
1.951229	1.463726	1.301361										
1.951229	1.463726											
1.951229												

Appendix A5 (Lotus Worksheet M3IR5)

Random error random from Normal with mean 0

Year\delay	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	0.083	0.075	-0.076	-0.065	-0.188	-0.164	-0.101	0.078	0.021	0.029	0.005	0.03	-0.073	-0.241
1979	-0.113	-0.049	-0.086	-0.123	0.148	0.09	-0.06	-0.099	-0.032	0.096	0.028	0.1	-0.331	
1980	0.086	-0.007	-0.037	0.17	0.071	-0.138	0.047	0.022	0.036	0.003	0.004	0.058		
1981	-0.071	0.147	0.067	-0.028	-0.132	0.049	0	-0.117	-0.042	0.026	-0.078			
1982	0.081	0.059	0.073	0.048	0.025	0.029	-0.023	-0.133	-0.044	0.066				
1983	0.117	0.059	-0.017	-0.081	-0.051	-0.024	-0.048	0.124	0.033					
1984	-0.024	-0.026	0.134	0.214	0.071	0.193	-0.022	0.012						
1985	0.022	0.015	0.076	-0.028	-0.004	0.155	0.032							
1986	-0.043	0.181	0.184	-0.192	-0.16	-0.048								
1987	0.07	0.106	0.144	0.032	-0.102									
1988	0.056	-0.195	0.032	0.041										
1989	0.145	0.187	-0.159											
1990	0.001	-0.153												
1991	-0.142													

Appendix A6 (Lotus Worksheet M3IR5)

Sum of data in Appendices A2 and A5 to produce trends + randomness

Year\delay

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1978	11.5959	11.4879	11.2369	11.1479	10.9249	11.0489	11.0619	11.1909	11.0839	11.0419	10.9679	10.9429	10.7899	10.5719
1979	11.4999	11.4639	11.3269	11.1899	11.5609	11.4529	11.2529	11.1639	11.1809	11.2589	11.1409	11.1629	10.6819	
1980	11.7989	11.6059	11.4759	11.7829	11.6339	11.3749	11.5099	11.4349	11.3989	11.3159	11.2669	11.2709		
1981	11.7419	11.8599	11.8799	11.7349	11.5809	11.7119	11.6129	11.4459	11.4709	11.4889	11.3349			
1982	11.9939	12.0719	12.0359	11.9609	11.8879	11.8419	11.7399	11.5799	11.6189	11.6789				
1983	12.3299	12.2219	12.0959	11.9819	11.9619	11.9389	11.8649	11.9869	11.8459					
1984	12.3389	12.2869	12.3969	12.4269	12.2339	12.3059	12.0409	12.0249						
1985	12.5349	12.4779	12.4889	12.3349	12.3089	12.4179	12.2449							
1986	12.6199	12.7939	12.7469	12.3209	12.3029	12.3649								
1987	12.8829	12.8689	12.8569	12.6949	12.5109									
1988	13.0189	12.7179	12.8949	12.8539										
1989	13.2579	13.2499	12.8539											
1990	13.2639	13.0599												
1991	13.2709													

Appendix A7 (Lotus Worksheet M3IR5)

Incremental paids derived from Appendix A6

1978	108651	97529	75879	69418	55542	62875	63697	72468	65114	62436	57983	56551	48528	39023
1979	98706	95216	83025	72396	104914	94174	77103	70538	71747	77567	68934	70467	43560	
1980	133106	109743	96365	130993	112860	87108	99698	92494	89224	82117	78190	78504		
1981	125731	141478	144336	124854	107034	122015	110514	93517	95885	97626	83692			
1982	161765	174888	168704	156514	145495	138954	125480	106927	111179	118054				
1983	226364	203191	179136	159835	156670	153108	142187	160637	139511					
1984	228411	216837	242050	249422	205644	220996	169549	166858						
1985	277868	262472	265375	227499	221660	247187	207918							
1986	302519	360015	343485	224336	220334	234427								
1987	393525	386054	383425	326081	271278									
1988	450855	333667	398276	382277										
1989	572576	568013	382277											
1990	576021	469724												
1991	580068													

Appendix A8 (Lotus Worksheet M3IR5)

Cumulative paids from Appendix A7

1978	108651	206180	282059	351477	407019	469894	533591	606059	671173	733609	791592	848143	896671	935694
1979	98706	193922	2769473	49343	454257	548431	625534	696072	767819	845386	914320	984787	1028347	
1980	133106	242849	339214	470207	583067	670175	769873	862367	951591	1033708	1111898	1190402		
1981	125731	267209	411545	536399	643433	765448	875962	969479	1065364	1162990	1246682			
1982	161765	336653	505357	661871	807366	946320	1071800	1178727	1289906	1407960				
1983	226364	429555	608691	768526	925196	1078304	1220491	1381128	1520639					
1984	228411	445248	687298	936720	1142364	1363360	1532909	1699767						
1985	277868	540340	805715	1033214	1254874	1502061	1709979							
1986	302519	662534	1006019	1230355	1450689	1685116								
1987	393525	781579	1165004	1491085	1762363									
1988	450855	784522	1182798	1565075										
1989	572576	1140589	1522866											
1990	576021	1045745												
1991	580068													

Appendix A9 (Lotus Worksheet M3IR5)

Age-to-age factors (link ratios) of the cumulative payments

1978	1.897635	1.368023	1.246111	1.158024	1.154476	1.135556	1.135811	1.107438	1.093025	1.079038	1.071439	1.057216	1.043519
1979	1.964642	1.428136	1.261407	1.300318	1.207314	1.140588	1.112764	1.103074	1.101022	1.081541	1.077070	1.044232	
1980	1.824478	1.396810	1.386166	1.240021	1.149396	1.148764	1.120141	1.103464	1.086294	1.075640	1.070603		
1981	2.125243	1.540161	1.303378	1.199541	1.189631	1.144378	1.106759	1.098903	1.091636	1.071962			
1982	2.081123	1.501121	1.309709	1.219823	1.172107	1.132597	1.099763	1.094321	1.091521				
1983	1.897629	1.417026	1.262588	1.203857	1.165487	1.131861	1.131616	1.101012					
1984	1.949328	1.543629	1.362902	1.219536	1.193454	1.124361	1.108850						
1985	1.944592	1.491125	1.282356	1.214534	1.196981	1.138421							
1986	2.190057	1.518441	1.222993	1.179081	1.161597								
1987	1.986097	1.490577	1.279896	1.181933									
1988	1.740076	1.507667	1.323197										
1989	1.992030	1.335157											
1990	1.815463												
1991													

One cannot determine changing calendar year trends from the age-to-age link ratios.

APPENDIX C1

	DELAY										
ACC. YEAR	0	1	2	3	4	5	6	7	8	9	10
1977	153638	188412	134534	87456	60348	42404	31238	21252	16622	14440	12200
1978	178536	226412	158894	104686	71448	47990	35576	24818	22662	18000	
1979	210172	259168	188388	123074	83380	56086	38496	33768	27400		
1980	11448	253482	183370	131040	78994	60232	45568	38000			
1981	219810	266304	194650	120098	87582	62750	51000				
1982	205654	252746	177506	12952	96786	82400					
1983	197716	255408	194648	142328	105600						
1984	239784	329242	264802	190400							
1985	326304	471744	375400								
1986	420778	590400									
1987	496200										

ACCI EXPOSURES

YEAR	
1977	2.20
1978	2.40
1979	2.20
1980	2.00
1981	1.90
1982	1.60
1983	1.60
1984	1.80
1985	2.20
1986	2.50
1987	2.60

APPENDIX C2

AGE LINK RATIOS OF CUMULATIVE PAYMENTS

DELAYS

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/10
1977	2.226337	1.393316	1.183505	1.106992	1.067912	1.046848	1.030445	1.023109	1.019622	1.016259
1978	2.268158	1.392381	1.185665	1.106873	1.064853	1.045149	1.030135	1.026712	1.020665	
1979	2.233123	1.401389	1.187119	1.106787	1.064900	1.041831	1.035220	1.027606		
1980	2.198791	1.394403	1.202128	1.101360	1.070173	1.049607	1.039413			
1981	2.211519	1.400420	1.176416	1.109359	1.070629	1.053616				
1982	2.228986	1.387229	1.203681	1.126446	1.095567					
1983	1.291792	1.429568	1.219719	1.133653						
1984	2.373077	1.465360	1.228344							
1985	2.445719	1.470397								
1986	2.403115									

APPENDIX E1 (Project GLD1, Incremental paid losses)

DEVELOPMENT YEAR

ACCI. YEAR	0	1	2	3	4	5	6	7	8
1979	2737400	1818600	1902200	838900	1134000	200600	748000	319600	9300
1980	989500	3285100	464700	413900	348400	12900	127900	268200	145100
1981	2442600	263300	1007100	1530500	1418700	943700	225600	2056500	53800
1982	364500	1710700	2168400	296300	853300	13200	331700	258200	532000
1983	884800	3607600	757900	3630900	3519900	3317200	1856200	28700	615100
1984	1791600	1982000	4784100	2089900	968400	1266400	111700	759300	1068600
1985	1631600	2348500	3247100	2160700	1555700	272800	711300	112200	
1986	2024600	3231300	4406200	4349300	1662500	2647200	622400		
1987	3704500	5610500	6892300	3529400	11132200	6099200			
1988	5272000	7300600	4549400	4573100	2873500				
1989	3643600	6201300	10902500	5080700					
1990	4902900	7321000	7763000						
1991	10358000	10870200							
1992	6435800								

ACCI YR	RELATIVE EXPOSURES
1979	2.75
1980	2.75
1981	2.75
1982	3.00
1983	3.00
1984	3.50
1985	3.50
1986	3.50
1987	4.00
1988	4.00
1989	4.00
1990	5.50
1991	5.50
1992	5.50

APPENDIX E2 (Project GLD1, Payment per claim "incurred")

DEVELOPMENT YEAR

ACCI YEAR	0	1	2	3	4	5	6	7	8
1979	995418	661309	691709	305054	412363	72945	272000	116218	3381
1980	359818	1194581	168981	150509	126690	4690	46509	97527	52763
1981	888218	95745	366218	556545	515890	343163	82036	747818	19563
1982	121500	570233	722800	98766	284433	4400	110566	86066	177333
1983	294933	1202533	252633	1210300	1173300	1105733	618733	9566	205033
1984	511885	566285	1366885	597114	276685	361828	31914	216942	305314
1985	466171	671000	927742	617342	444485	77942	203228	32057	
1986	578457	923228	1258914	1242657	475000	756342	177828		
1987	926125	1402625	1723075	882350	2783050	1524800			
1988	1318000	1825150	1137350	1143275	718375				
1989	910900	1550325	2725625	1270175					
1990	891436	1331090	1411454						
1991	1883272	1976400							
1992	1170145								

APPENDIX E3 (Project GLD1, Age-to-Age link ratios for the cumulative paid losses)

DEVELOPMENT YEAR

ACCI YEAR	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8
1979	1.664353	1.417515	1.129897	1.155404	1.023793	1.086657	1.034074	1.000959
1980	4.319960	1.108712	1.087334	1.067608	1.002345	1.023193	1.047533	1.024549
1981	1.107795	1.372187	1.412200	1.270564	1.141650	1.029661	1.262593	1.005441
1982	5.693278	2.044911	1.069823	1.187956	1.002448	1.061353	1.044997	1.088721
1983	5.077306	1.168707	1.691560	1.396332	1.267492	1.118092	1.001633	1.034943
1984	2.106274	2.267781	1.244213	1.090950	1.109022	1.008671	1.058434	1.077697
1985	2.439385	1.815834	1.298968	1.165713	1.024928	1.063416	1.009407	
1986	2.596019	1.838334	1.450140	1.118653	1.168892	1.033972		
1987	2.514509	1.739914	1.217766	1.564036	1.197584			
1988	2.384788	1.361850	1.267089	1.132449				
1989	2.701971	2.107426	1.244884					
1990	2.493198	1.635067						
1991	2.049450							

RESEARCH PAPER SERIES

No.	Date	Subject	Author
1	MAR 93	AUSTRALIAN SUPERANNUATION : THE FACTS, THE FICTION, THE FUTURE	David M Knox
2	APR 93	AN EXPONENTIAL BOUND FOR RUIN PROBABILITIES	David C M Dickson
3	APR 93	SOME COMMENTS ON THE COMPOUND BINOMIAL MODEL	David C M Dickson
4	AUG 93	RUIN PROBLEMS AND DUAL EVENTS	David CM Dickson Alfredo D Egidio dos Reis
5	SEP 93	CONTEMPORARY ISSUES IN AUSTRALIAN SUPERANNUATION - A CONFERENCE SUMMARY	David M Knox John Piggott
6	SEP 93	AN ANALYSIS OF THE EQUITY INVESTMENTS OF AUSTRALIAN SUPERANNUATION FUNDS	David M Knox
7	OCT 93	A CRITIQUE OF DEFINED CONTRIBUTION USING A SIMULATION APPROACH	David M Knox
8	JAN 94	REINSURANCE AND RUIN	David C M Dickson Howard R Waters
9	MAR 94	LIFETIME INCOME, TAXATION, EXPENDITURE AND SUPERANNUATION (LITES): A LIFE-CYCLE SIMULATION MODEL	Margaret E Atkinson John Creedy David M Knox
10	FEB 94	SUPERANNUATION FUNDS AND THE PROVISION OF DEVELOPMENT/VENTURE CAPITAL: THE PERFECT MATCH? YES OR NO	David M Knox
11	JUNE 94	RUIN PROBLEMS: SIMULATION OR CALCULATION?	David C M Dickson Howard R Waters
12	JUNE 94	THE RELATIONSHIP BETWEEN THE AGE PENSION AND SUPERANNUATION BENEFITS, PARTICULARLY FOR WOMEN	David M Knox
13	JUNE 94	THE COST AND EQUITY IMPLICATIONS OF THE INSTITUTE OF ACTUARIES OF AUSTRALIA PROPOSED RETIREMENT INCOMES STRATEGY	Margaret E Atkinson John Creedy David M Knox Chris Haberecht
14	SEPT 94	PROBLEMS AND PROSPECTS FOR THE LIFE INSURANCE AND PENSIONS SECTOR IN INDONESIA	Catherine Prime David M Knox

31	MAR 1996	RESERVING CONSECUTIVE LAYERS OF INWARDS EXCESS-OFF-LOSS REINSURANCE	Greg Taylor
32	AUG 1996	EFFECTIVE AND ETHICAL INSTITUTIONAL INVESTMENT	Anthony Asher
33	AUG 1996	STOCHASTIC INVESTMENT MODELS: UNIT ROOTS, COINTEGRATION, STATE SPACE AND GARCH MODELS FOR AUSTRALIA	Michael Sherris Leanna Tedesco Ben Zehnwirth
34	AUG 1996	THREE POWERFUL DIAGNOSTIC MODELS FOR LOSS RESERVING	Ben Zehnwirth
35	SEPT 1996	KALMAN FILTERS WITH APPLICATIONS TO LOSS RESERVING	Ben Zehnwirth
36	OCT 1996	RELATIVE REINSURANCE RETENTION LEVELS	David C M Dickson Howard R Waters
37	OCT 1996	SMOOTHNESS CRITERIA FOR MULTI-DIMENSIONAL WHITTAKER GRADUATION	Greg Taylor
38	OCT 1996	GEOGRAPHIC PREMIUM RATING BY WHITTAKER SPATIAL SMOOTHING	Greg Taylor
39	OCT 1996	RISK, CAPITAL AND PROFIT IN INSURANCE	Greg Taylor
40	OCT 1996	SETTING A BONUS-MALUS SCALE IN THE PRESENCE OF OTHER RATING FACTORS	Greg Taylor
41	NOV 1996	CALCULATIONS AND DIAGNOSTICS FOR LINK RATION TECHNIQUES	Ben Zehnwirth Glen Barnett
42	DEC 1996	VIDEO-CONFERENCING IN ACTUARIAL STUDIES – A THREE YEAR CASE STUDY	David M Knox
43	DEC 1996	ALTERNATIVE RETIREMENT INCOME ARRANGEMENTS AND LIFETIME INCOME INEQUALITY: LESSONS FROM AUSTRALIA	Margaret E Atkinson John Creedy David M Knox
44	JAN 1997	AN ANALYSIS OF PENSIONER MORTALITY BY PRE-RETIREMENT INCOME	David M Knox Andrew Tomlin
45	JUL 1997	TECHNICAL ASPECTS OF DOMESTIC LINES PRICING	Greg Taylor
46	AUG 1997	RUIN PROBABILITIES WITH COMPOUNDING ASSETS	David C M Dickson Howard R Waters
47	NOV 1997	ON NUMERICAL EVALUATION OF FINITE TIME RUIN PROBABILITIES	David C M Dickson
48	NOV 1997	ON THE MOMENTS OF RUIN AND RECOVERY TIMES	Alfredo G Egídio dos Reis
49	JAN 1998	A DECOMPOSITION OF ACTUARIAL SURPLUS AND APPLICATIONS	Daniel Dufresne
50	JAN 1998	PARTICIPATION PROFILES OF AUSTRALIAN WOMEN	M. E. Atkinson Roslyn Cornish

51	MAR 1998	PRICING THE STOCHASTIC VOLATILITY PUT OPTION OF BANKS' CREDIT LINE COMMITMENTS	J.P. Chateau Daniel Dufresne
52	MAR 1998	ON ROBUST ESTIMATION IN BÜHLMANN STRAUB'S CREDIBILITY MODEL	José Garrido Georgios Pitselis
53	MAR 1998	AN ANALYSIS OF THE EQUITY IMPLICATIONS OF RECENT TAXATION CHANGES TO AUSTRALIAN SUPERANNUATION	David M Knox M. E. Atkinson Susan Donath
54	APR 1998	TAX REFORM AND SUPERANNUATION – AN OPPORTUNITY TO BE GRASPED.	David M Knox
55	APR 1998	SUPER BENEFITS? ESTIMATES OF THE RETIREMENT INCOMES THAT AUSTRALIAN WOMEN WILL RECEIVE FROM SUPERANNUATION	Susan Donath
56	APR 1998	A UNIFIED APPROACH TO THE STUDY OF TAIL PROBABILITIES OF COMPOUND DISTRIBUTIONS	Jun Cai José Garrido
57	MAY 1998	THE DE PRIL TRANSFORM OF A COMPOUND R_k DISTRIBUTION	Bjørn Sundt Okechukwu Ekuma
58	MAY 1998	ON MULTIVARIATE PANJER RECURSIONS	Bjørn Sundt
59	MAY 1998	THE MULTIVARIATE DE PRIL TRANSFORM	Bjørn Sundt
60	JUNE 1998	ON ERROR BOUNDS FOR MULTIVARIATE DISTRIBUTIONS	Bjørn Sundt
61	JUNE 1998	THE EQUITY IMPLICATIONS OF CHANGING THE TAX BASIS FOR PENSION FUNDS	M E Atkinson John Creedy David Knox
62	JUNE 1998	ACCELERATED SIMULATION FOR PRICING ASIAN OPTIONS	Felisa J Vázquez-Abad Daniel Dufresne
63	JUNE 1998	AN AFFINE PROPERTY OF THE RECIPROCAL ASIAN OPTION PROCESS	Daniel Dufresne
64	AUG 1998	RUIN PROBLEMS FOR PHASE-TYPE(2) RISK PROCESSES	David C M Dickson Christian Hipp
65	AUG 1998	COMPARISON OF METHODS FOR EVALUATION OF THE n-FOLD CONVOLUTION OF AN ARITHMETIC DISTRIBUTION	Bjørn Sundt David C M Dickson
66	NOV 1998	COMPARISON OF METHODS FOR EVALUATION OF THE CONVOLUTION OF TWO COMPOUND R_1 DISTRIBUTIONS	David C M Dickson Bjørn Sundt
67	NOV 1998	PENSION FUNDING WITH MOVING AVERAGE RATES OF RETURN	Diane Bédard Daniel Dufresne
68	DEC 1998	MULTI-PERIOD AGGREGATE LOSS DISTRIBUTIONS FOR A LIFE PORTFOLIO	David C M Dickson Howard R Waters
69	FEB 1999	LAGUERRE SERIES FOR ASIAN AND OTHER OPTIONS	Daniel Dufresne