EXPLORING UNKNOWN QUANTITIES

DEVELOPMENT AND APPLICATION OF A STOCHASTIC CATASTROPHE MODEL WITH OUTPUT AND SENSITIVITIES

by

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Abstract

Stochastic catastrophe models are now widely used to assess and manage catastrophe exposure. In Australia, natural perils are the focus of modelling, having historically caused substantial insured losses. The proprietary nature of commonly used models has resulted in limited public scrutiny of their workings – despite the existence of significant inconsistencies in cross-model output. This paper adds to the currently limited body of publicly available literature regarding the detail of catastrophe model development. This is done through a thorough presentation of a theoretical model and the application of this model to the peril of hailstorm in the Sydney region for commercial property insurance. It is found that data is difficult to obtain, placing constraints on the model design. Additionally, the sensitivity of output to changes in assumptions and parameters is highly significant – supporting the argument for greater cross-model comparison. Finally, it is suggested that increased co-operation and openness would help to address the causes of model inconsistency and improve the overall standard of catastrophe modelling.

1. Introduction

The most severe hailstorm on record strikes the heart of metropolitan Sydney. Claims flood into insurers, amounts inflated by a dramatic surge in demand for materials and labour; the total cost is in the billions. Such an event is unpredictable, but the impact for the insurer can be mitigated by a pre-event assessment of risk. In today's insurance industry, this comes in the form of catastrophe modelling.

According to the American Academy of Actuaries Catastrophe Management Workgroup (2001):

> *'…[catastrophes are] infrequent events that cause severe loss, injury or property damage to a large population of exposures.'*

In practice, an insurer defines catastrophe relative to the insurer's position. This paper is concerned with events that dramatically concentrate the frequency and severity of claims, resulting in aggregate claims that are extreme relative to the usual experience of an insurer. Figure 1 gives a visual representation.

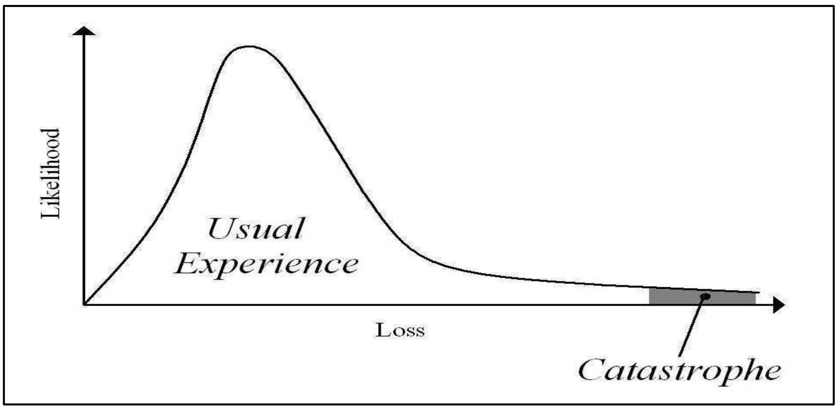

Figure 1 An example density of an insurer's aggregate claims over a given time period

Catastrophes may be caused by natural or man-made events. Natural catastrophe risk is of particular interest to Australian general insurers, whose portfolios are exposed to a number of perils that have historically caused substantial insured losses (IDRO Disaster List, 2004). This paper concentrates on modelling natural catastrophe risk for a general insurer, although some of the ideas may be more widely applicable.

1.1 Catastrophe Modelling Objectives

For the general insurer there are multiple objectives that can be met by catastrophe modelling. Pricing of insurance policies requires a proper assessment of all risks, including those that are remote. The costs of catastrophes need to be incorporated into premiums, or a competitive market will ensure that those costs cannot be recovered in the future (Minty, 1997). An insurer that bases its premiums on periods when it has had favourable experience with regard to catastrophes will have diminished profitability in the long run. However, incorporating a loading for expected catastrophe losses does not necessarily improve the insurer's capacity to sustain these losses. In the years when catastrophe losses are low, the premium loading will be released as profit; in the few years when catastrophe losses are very large, they will likely dwarf the premium catastrophe loading.

While pricing is a matter for the insurer alone, in Australia, reserving and reinsurance are areas in which the Australian Prudential Regulation Authority (APRA) is interested. A general insurer must satisfy APRA that it is prepared for a catastrophe by holding adequate capital to cover a remote loss, net of reinsurance recoveries, and including the cost of one reinstatement of the reinsurance arrangement (APRA, 2002). This is called an insurer's 'maximum event retention' (MER), and is often calculated by using catastrophe models. *Explicit* catastrophe reserves could also be determined using catastrophe models. However, current accounting practices do not recognise catastrophe reserves as liabilities, resulting in taxation disincentives. This is a matter of on-going controversy (HIH Royal Commission, 2002).

In addition to reserves, a general insurer enters reinsurance treaties to deal with catastrophe risk. Reinsurance is purchased to a level that is commensurate with the risk preferences of the insurer, and catastrophe modelling is commonly used to help determine appropriate levels and types of reinsurance (Musulin & Rollins, 2001).

In relation to an insurer's product development and promotion, modelling may be used to help make choices about promoting (or not promoting) certain lines of insurance cover or locations. Catastrophe modelling can highlight ways to reduce the risk posed by policy concentration (Kozlowski & Mathewson, 1995).

1.2 Catastrophe Models

According to Musulin (1997), there are two types of catastrophe model, deterministic and stochastic. Historically, deterministic models have been used. These models usually produce single figure measures of catastrophe risk and often rely heavily on historical experience.

In Australia, the first catastrophe model to be widely used was the Insurance Council of Australia (ICA) Risk Zones Accumulation Guide outlined by Walker (1997). This deterministic model divides Australia into 49 "ICA zones", each with an attached percentage that is multiplied by the insurer's exposure to measure likely maximum losses. Walker points out that the zones are inadequate as a modern method of risk management – essentially because of a lack of sophistication. For further examples of deterministic models see Pielke et al (2000) and Woo (2002).

Over recent years, stochastic models have become widely used. Typically they simulate hypothetical events using an understanding of real-world phenomena to generate distributions of multiple aspects of an insurer's catastrophe exposure (Sanders et al, 2002). Stochastic models can present output in a probabilistic format and often rely on historical experience to a lesser extent than deterministic models. These attributes are generally considered to make them superior to their deterministic counterparts (Walker, 1997; Rüttener & Geissbühler, 2004). Stochastic models are therefore the focus of this paper.

Commercial modelling providers typically develop stochastic models. For Australian interests, firms such as EQECAT, RMS and Risk Frontiers have produced models of perils such as hailstorm, flood, cyclone, and earthquake for various locations. Millions of dollars may be spent on model development (Boyle, 2002), and the basic fee for modelling is in the order of $20,000. To prevent their value from diminishing, commercial catastrophe models are shrouded in confidentiality agreements.

In addition to the available commercial models, insurers and reinsurers develop internal models. These are also not open to public scrutiny. In fact, greater mystery may surround internal models because there is no publicly available documentation that relates to them whatsoever.

1.3 Cross-Model Inconsistencies

Parties modelling the same phenomena have access to different data, producing models independently of one another. There may even be active effort on the part of some to avoid model comparisons. The proprietary nature of models is a well-recognised problem for regulators in US insurance markets (Florida Insurance Council, 1998; Musulin, 1997). However, the "black box" label of catastrophe models (Keykhah, 2000; Malmquist, 1997) is a global phenomenon, and signifies the difficulty in making a proper assessment of model workings (Dorland, 1999). It is unsurprising that industry sources claim that there are significant inconsistencies in model output. For example, the chief underwriting officer of a major US insurer made this comment in regard to terrorism catastrophe models (Hyle, 2004):

'*We looked at the three modelling firms, and they all had good things to say, but the output is significantly different… They have different assumptions, so they predict different results.*' – Jim McDonald of ACE INA

In Florida, USA, the formation of the Florida Commission on Hurricane Loss Projection Methodology aimed to overcome the black box nature of hurricane models through scrutiny and approval of models (Musulin, 1997). This falls short of cross-company co-operation because model details are not divulged to competitors or the public (Musulin, 1997). However, it has led to a convergence of results in Florida (Sanders et al, 2002). It is an approach that has been resisted by APRA, which is model non-specific in its guidance regarding calculation of the MER (APRA, 2002).

1.4 The Need for Open Exploration

Catastrophe models provide valuable insight to the user, which has resulted in great amounts of effort and resources being invested in their development. Far less work appears to have been done in scrutinising and comparing different models. Model inconsistencies exist and result in disparities in the approaches of insurers facing similar risks. Further, they are indicative of the improvement in modelling that might be possible if cross-model comparisons took place.

An exploration of the causes of model inconsistency is impossible without access to a model, and so one must be built. The information barrier described above inhibits this process, but once a model is constructed, the extent to which assumption changes and differences in data impact on output can be examined. This is a matter of public interest because the output of models is used in managing catastrophic risks that are even more devastating when mismanaged.

2. Literature Review and the Aims of this Paper

There does not appear to be any *detailed* and *public* record of model construction with output and sensitivities. Broad issues in model fitting are addressed by a number of papers, for example, Dorland et al. (1999), Keykhah (2000) and Malmquist (1997). These authors explain the difficulties associated with obtaining quality data, but are unable to offer resolutions to this problem.

Cross-model inconsistency is an issue frequently raised, but not often analysed in detail (for overseas examples see Musulin (1997), Keykhah (2000), Watson & Johnson (2003), Major (1999), and Sanders et al. (2002)). There appears to be no reference that compares the output of Australian-based models, but the phenomenon of inconsistency can be expected to be universal.

Inspiration for model construction is limited. Some basic information is outlined in academic papers and publications of reinsurance and modelling companies (see, for example, Sanders et al (2002), Benfield Greig (2001), Smith (2000), Kilmartin (2003), GenRe (2004), Boyle (2002), Leigh & Kuhnel (2001), Blong et al (2001)). A more detailed reference is Kozlowski & Mathewson (1995), which outlines a catastrophe model structure, but does not provide specific functional forms.

There is a general model structure that is common in the literature (see, for example, Kozlowski & Mathewson (1995), Benfield Greig (2001), and Watson & Johnson (2003)). The structure consists of three modules:

(i) *The Science Module*: uses historical data and statistical modelling to generate realisations of a peril and determine the intensity of the event at each location.

(ii) *The Engineering Module*: translates intensity into property damage at each location.

(iii) *The Insurance Coverage Module*: applies the conditions of the insurer's issued policies and reinsurance arrangements to determine claims for each event.

The available literature paints a picture of the catastrophe modelling process with a broad brush. A detailed and publicly available account of stochastic catastrophe model construction that includes output and sensitivities appears to be absent from the literature.

The Aims of this Paper

The aims of this paper are twofold. First, present a publicly available account of the catastrophe modelling process, including the issues involved and problems encountered. Second, use the developed model to explore potential causes of cross-model inconsistencies by presenting results as output sensitivities to parameters and assumptions. To achieve these aims, a theoretical model is built that is consistent with the known structure of catastrophe models, and is applied to the peril of hailstorm in the Sydney region.

Before presenting the model, the reader may wish to be acquainted with some important catastrophe modelling terminology in Section 3. Sections 4 to 6 present the model and findings. The final two sections discuss limitations, further work, and conclusions. The middle sections of the paper are deliberately structured for easy reference to the work completed.

3. The Terminology of Catastrophe Modelling Output

This section describes basic terminology for the reader unfamiliar with catastrophe modelling output. In particular, the following are examined:

- Probable Maximum Loss: a single figure measure of an insurer's greatest exposure (analogous to the concept of value-at-risk);
- Return Period: a measure of the time that is expected to elapse between extreme events; and
- Loss/Return Period and Exceedence Probability curves: these map exposure against different likelihood tolerances.

A definition of an *insured risk* is required for what follows. An insured risk is denoted X if the distribution of annual losses from that risk follows the distribution of the random variable X.

3.1 Probable Maximum Loss

A Probable Maximum Loss (PML) is a value of *annual claims* that will be exceeded with a very low probability (low exceedence probability) (American Academy of Actuaries, 2001). PML and variations on it are commonly measured quantities. For example, in calculating the MER, APRA Guidance Note GGN 110.5 states that an insurer must calculate the greatest loss to which it is exposed due to policy concentration using a return period (defined below) of at least 250 years (APRA, 2002). Another application of PMLs is in determining layers of reinsurance cover.

The PML, denoted Λ, is defined as the result of evaluating the inverse of the distribution function of an insured risk *X*, denoted F_X^{-1}, at some high probability *p* so that $\Lambda = F_X^{-1}(p)$. The exceedence probability of Λ is then 1 – *p*.

This definition is concerned with annual losses and corresponds with premium collection and some reinsurance arrangements. Examination of reinsurance arrangements that cover excess losses from single events requires a measure of single event loss risk. This paper uses PML in the context of annual losses.

3.2 Return Period

PMLs are ordinarily associated with a return period, rather than probability. There is a one-to-one relationship between return period and exceedence probability.

For an insured risk *X*, the return period ρ of losses *l* is defined by

$$\rho(l) = \frac{1}{\bar{F}_X(l)} \tag{1}$$

where $\bar{F}_X(x) = 1 - F_X(x)$, is the exceedence probability of a loss of *x*.

This definition is a commonly used practical definition for return period that differs from the mathematical definition (that is equivalent to expected waiting time) (Embrechts et al, 1997). Since PMLs are estimated by the upper sample quantiles of annual loss distributions (Musulin & Rollins, 2001), return period is more easily measured by inverting an event's annual exceedence probability (Natural Disaster Coalition, 1995; Musulin & Rollins, 2001). The practical definition given by (1) is therefore assumed for this paper. In any case, it can be shown that under standard assumptions the definition alternatives give very similar answers.

3.3 Loss/Return Period Curve

For a given insured loss, the Loss/Return Period Curve (LRPC) relates return periods to their associated PMLs (Walker, 1997). The curve is useful for demonstrating an insurer's exposure at different return period tolerances. The Loss/Return Period Curve of an insured risk *X*, denoted *LRPC*, is defined by

$$LRPC(\rho) = F_X^{-1}\left(1 - \frac{1}{\rho}\right). \tag{2}$$

A related curve is the exceedence probability curve of an insured loss *X*, denoted *EP*, and defined by

$$EP(x) = \bar{F}_X(x). \tag{3}$$

3.4 Single and Multiple Peril PMLs

One may specify a portfolio PML or LRPC in terms of one peril, multiple perils or all perils, depending on the purpose of the modelling (Walker, 1997). When measuring catastrophe risk for a portfolio exposed to multiple perils and spread across different locations, it is important to incorporate the contribution of all risks. Using a single peril PML for a portfolio exposed to multiple perils can substantially underestimate the true portfolio PML at a given return period tolerance.

The model in this paper is concerned with a single peril and location. Techniques such as convolution or addition of exceedence probabilities may be employed to determine a multiple peril or location PML.

4. A Catastrophe Model - The Zone Percentage Loss Model

Without access to proprietary information, construction of a catastrophe model is a largely unguided exercise. The general principles outlined by Kozlowski & Mathewson (1995) form a broad foundation for the model design that follows. In this section, the theoretical design of what will be called the *Zone Percentage Loss Model* (ZPLM) is developed.

4.1 Model Objectives

The ZPLM models total claims made under the policies of an insurer's portfolio when a perilous event occurs. The model is general, in that it may be applied to a range of perils, for example, earthquake, hailstorm, or hurricane. The theoretical framework forms the basis of a simulation technique developed in Section 5.

4.2 Definitions and Assumptions

Preliminary Definitions

D4.1 A *perilous event* occurs when a predefined set of circumstances is present that directly causes claims under an insurer's issued policies.

D4.2 The *area of interest* is the geographical area in which the insurer's relevant exposures are located and is divided into *n zones* of equal geographical area.

D4.3 The *percentage loss* (PL) of a collection of insurance claims is the total claim amount expressed as a percentage of the total of the sums insured of the underlying policies.

D4.4 An *affected zone* is a zone that has a non-zero PL as a result of a perilous event. Together, the affected zones comprise the *affected area*.

D4.5 The *severity* of a perilous event is a function of the *severity factors* that characterise a peril.

Preliminary Assumptions

A4.1 When a perilous event occurs the exposures in an affected zone each sustain the same PL. Unaffected zones have a PL equal to zero.

A4.2 The affected area is comprised of a continuous block of affected zones.

A4.3 The PL for two policies issued in the same zone is the same in all circumstances.

A4.4 Perilous events of fixed severity and with a fixed number of affected zones result in exactly the same pattern of PLs, regardless of the location of the affected area.

A4.5 Severity is an increasing function of each of the severity factors.

A4.6 PL is an increasing function of both proximity to the centre of a perilous event and severity of an event.

Comments

The use of PL is inspired by Kozlowski & Mathewson (1995), whose generic model determined dollar losses as percentage damage multiplied by exposure. In their hurricane model, the percentage damage at each location depended on an estimated wind-speed at that location and the nature of the affected property. In the ZPLM, a zone's PL is determined by the severity of the overall event and the proximity of the zone to the event's centre, regardless of the insured property within the zone.

Assumption A4.1 is driven by the practical limitation of insurers' claims data that is often most easily extracted at a zone level. Assumption A4.3 implies that the location of insured property within a zone is irrelevant with regard to PL. A study that examined the impact of this assumption on the output of a catastrophe model found that it can be significant, particularly where the affected area is small (Risk Frontiers, 2003b). The ZPLM is therefore better suited to perils that affect large areas relative to zone size.

Assumptions A4.2, A4.4 and A4.6 limit the use of the model to perils that have a 'centre' and for which the spread of damages is not altered by the nature of the affected area. Bushfire, for example, may be inappropriate because it tends to pierce urban communities from the edges and certain locations are subject to significantly greater risks.

Assumptions A4.5 and A4.6 are intuitive and similar assumptions appear in other models (for an example related to proximity, see Risk Frontiers (2003b)). See Figure 2 for visualisation of A4.6 (for a given severity). The use of zones results in a cruder version of Figure 2, with a constant PL across each zone.

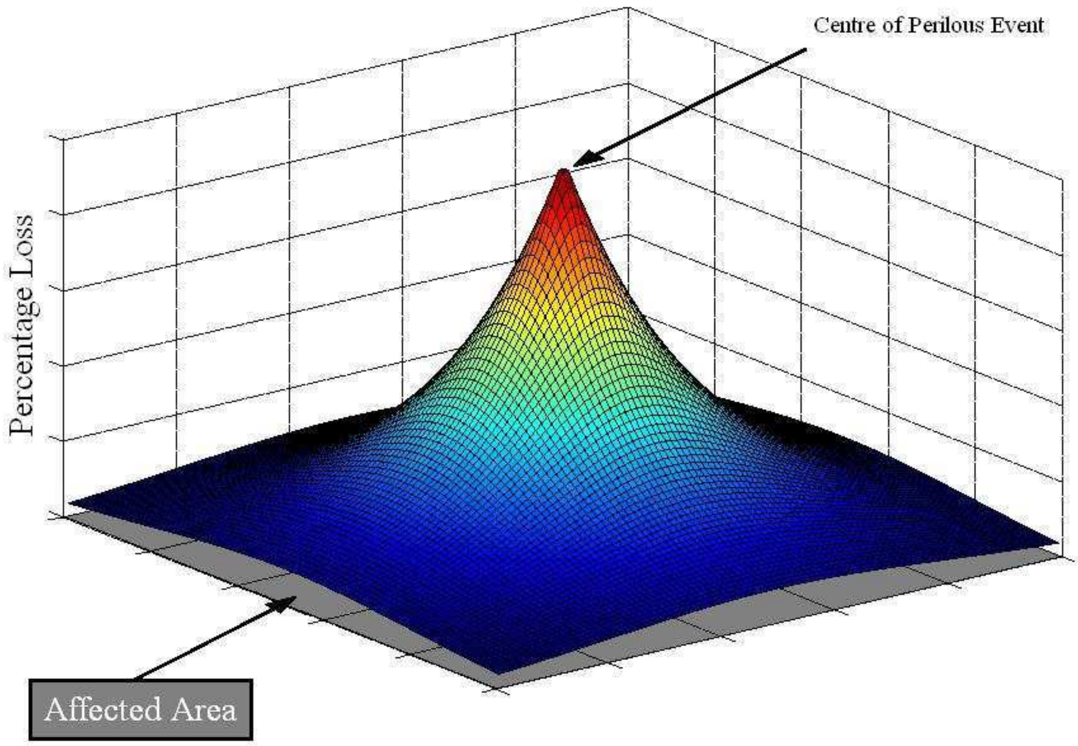

Figure 2 Percentage loss increases with proximity to the centre of the perilous event

4.3 The Theoretical Framework

Some Definitions and Notation

Let $z_1, z_2, \ldots, z_n$ denote the n zones.

Let $SI(z)$ denote the total sum insured of all policies issued by an insurer in zone z.

An insurer's portfolio (within the area of interest) is denoted $\{SI(z_1), SI(z_2), \ldots, SI(z_n)\}$.

Let severity, denoted S, be equal to a function f of the severity factors denoted $\beta_1, \beta_2, \ldots, \beta_b$. Then

$$S = f(\beta_1, \beta_2, \ldots, \beta_b)$$
$$\frac{\partial f}{\partial \beta_i} > 0, \quad i = 1, 2, \ldots, b \tag{4}$$

Let $A \leq n$ be the (integer) number of zones in the affected area.

Let Z be a subset of $\{z_1, z_2, \ldots, z_n\}$, with elements ordered by increasing proximity to the centre of the perilous event, so that $Z = \{z^{(1)}, z^{(2)}, \ldots, z^{(A)}\}$. The subset Z is divided into two further subsets of *core* and *non-core* (*nc*) zones denoted Z_{core} and Z_{nc} respectively.

Let $\gamma \in [0,1]$ denote the proportion of zones that are core (subject to rounding). Then

$$Z_{nc} = \{z^{(1)}, z^{(2)}, \ldots, z^{(d)}\}$$
$$Z_{core} = \{z^{(d+1)}, z^{(d+2)}, \ldots, z^{(A)}\} \tag{5}$$

where, for ease of notation, define

$$d = \lfloor (1-\gamma) A \rfloor \tag{6}$$

As an example, the non-core zone closest to the centre of the event is $z^{(d)}$.

For an event of severity *S*, the PL for the k^{th} element of *Z*, denoted $L\left(S, z^{(k)}\right)$ is assumed to take the following form:

$$L\left(S, z^{(k)}\right) = \begin{cases} g_{nc}(S,k) & , \quad k \le d \\ g_{core}(S,k) & , \quad k > d \end{cases} \tag{7}$$

for some functions g_i $(i = core, nc)$ such that

$$\frac{\partial g_i}{\partial S} > 0 , \quad \frac{\partial g_i}{\partial k} > 0 \text{ and } g_{nc}(S,d) = g_{core}(S,d) .$$

Comments

The model may be generalised further by including an arbitrary number of subsets of *Z*. The use of two subsets is a result of examining the available claims data described in section 5.

The conditions placed on derivatives in (4) and (7) are a result of A4.5 and A4.6. In (7), the function *L* is continuous at *d*. Continuity of derivatives is not required, but may be imposed if desired.

Percentage Loss Calculation

Using the above framework, the total claims (denoted *T*) arising from a perilous event with severity *S* and number of affected zones *A*, is given by (8).

$$\begin{aligned} T &= \sum_{j=1}^{A} SI\left(z^{(j)}\right) L\left(S, z^{(j)}\right) \\ &= \sum_{j=1}^{d} SI\left(z^{(j)}\right) g_{nc}(S,j) + \sum_{j=d+1}^{A} SI\left(z^{(j)}\right) g_{core}(S,j) \end{aligned} \tag{8}$$

4.4 Model Fitting – General Concepts

The theoretical framework outlined above is not highly prescriptive. To apply the ZPLM to a particular peril, appropriate functional forms must be determined and relevant parameters estimated. Data that relates to historical percentage losses by zone and the frequency and severity of perilous events is required. Data for highly rare events, for example earthquakes in Australia, may not exist. In these cases scientific opinion, or datasets that relate to comparable locations, must be utilised.

5. Application of the Zone Percentage Loss Model to Hailstorms in Sydney, New South Wales

5.1 Introduction

Hailstorm is considered to be the single most important insured natural peril to which Australia is exposed, accounting for 34% of insured losses from major events (Schuster & Blong, 2004) and being the cause of the single most costly event to date: Sydney, 14 April 1999. The relatively high frequency with which extreme hailstorm events have occurred in the Sydney region makes the peril and location an attractive choice for application of the ZPLM. The model will be called the *Sydney Hail Model* (SHM) and is used to meet the objectives outlined in Section 1.1, and estimate the quantities defined in section 3.

Outline of the Sydney Hail Model

The SHM simulates annual experience, with each event during a year being modelled by the ZPLM. A simulation of one year is comprised of the following:

- Number of perilous events during the year is determined, denoted N
- For *each* event, the following are determined:
    - Severity factor values: $\beta_1, \beta_2, ..., \beta_b$
    - Severity: S
    - Affected area: A, Z_{core}, Z_{nc}
    - Percentage losses: $L\left(S, z^{(k)}\right),\ k = 1, 2, ..., A$
- Incorporating the insurer's exposure data, aggregate claims for each event are calculated: $T_i,\ i = 1, 2, ..., N$
- Annual aggregate claims can then be calculated as $\sum_{i=1}^{N} T_i$

Large numbers of observations of the random variable $\sum_{i=1}^{N} T_i$ are simulated to estimate the insurer's annual loss distribution for extreme hail events.

SHM Definition of a Perilous Event

The following will be the SHM definition of a perilous event:

a weather event where the maximum observable hailstone size exceeds 2cm in diameter.

This choice of definition corresponds to the Bureau of Meteorology (BoM) severe storm hail definition; events are compulsorily reported only when the definition is met. The SHM is therefore not concerned with the more minor levels of damage that events with hailstones smaller than 2 cm can inflict (Blong, 1997). An insurer might incorporate these less severe events into a general storm model.

Model Definition of Area of Interest

For the hail model the area of interest is defined as:

all postcodes located in ICA zones 41, 42 and 43

This is an area of 3,050 km^2 (Natural Hazards Research Centre) and contains 243 postcodes. ICA zones are commonly used in other models, allowing for cross-model comparison (c.f. Risk Frontiers, 2003a).

Model Zone Definition

Australia Post postcodes will be adopted as the zone definition. Insurance claims and sum insured data are easily extracted by postcode. Postcode areas are relatively small so that assumptions such as A4.1 are reasonable, and are similar in size, so that D4.2 is approximately satisfied (Sydney Postcode Map, 2002). Further, without detailed knowledge of exposure locations, postcode is the only practical zone definition.

Insured Property Definition

It is reasonable to expect that functional forms and parameter values change depending on the underlying property that is insured. Analysis is restricted to:

> *commercial property insurance for both building and contents (excluding business interruption).*

Commercial lines of insurance are of interest because the available literature and modelling tends to focus on domestic lines (for example, Blong (1997)). Additionally, the latest release of a Risk Frontiers model, HailAUS, only includes domestic household and motor vehicle insurance (Risk Frontiers, 2003a).

Despite the application to commercial insurance in this paper, the model may be re-fitted for other lines of insurance.

5.2 SHM Additional Assumptions

In addition to A4.1-4.6, some additional assumptions are made.

<u>A5.1</u> Relative proximity of postcodes can be determined by the numerical order of postcode numbers.

<u>A5.2</u> The number of perilous events in successive years is a sequence of independent and identically distributed random variables.

<u>A5.3</u> Past experience in relation to model variables is a reasonable basis for estimation of probability distributions of future realisations of these variables.

Comments

Assumption A5.1 is reasonable on inspection of a postcode map (Sydney Postcode Map, 2002). Assumption A5.2 ignores the possibility of long cyclical weather phenomena that may change event frequency, for example, the Southern Oscillation Index mentioned in Leigh & Kuhnel (2001).

Assumption A5.3 means that the model does not take account of climate change despite strong evidence to suggest that climate changes are occurring and that the frequency and severity of extreme weather events is increasing (Swiss Re, 2004). This is a situation that is expected to deteriorate at an increasing rate (Coleman et al, 2004).

Climate modelling may be used to measure expected changes in hail event variables and make adjustments to historic data.

Assumption A5.3 also allows the use of historic insurance data to model claims for current exposures. It is not clear exactly how insurers' overall exposure is changing, though there is evidence to suggest that property is becoming more susceptible to damage (Dlugolecki, 1999).

5.3 Data

Sophisticated catastrophe modelling is a data intensive exercise. Data relating to extreme events and insurance claims are guarded possessions of modelling and insurance companies. The model has been built with this in mind. For example, data availability is one of the main reasons for the use of postcode as the zone definition.

Data was obtained from a variety of public and private sources. The following are the datasets used to build and fit the SHM:

- Event Generation
  - Bureau of Meteorology severe storm and hail datasets (Sydney Region, 1990 to 2004)
  - Bureau of Meteorology wind-speed records (Sydney Region, 1946 to 2004)
  - Hailstorm occurrence by postcode from an anonymous source (Sydney Region, 1969 to 1995)
  - Emergency Management Australia disaster database (various records)
- Percentage Losses
  - Commercial insurance claims data from an anonymous source (Sydney, 14 April 1999)

5.4 Functional Forms and Distributions

Event Frequency

Assumption A.2 implies that an appropriate distributional assumption for annual event frequency must be determined. One possibility is to assume a Poisson distribution, which is in line with the approaches of a Benfield Greig Wind model (Sanders et al, 2002) and a model outlined by Dorland (1999). The data suggests that this assumption is questionable because across the years 1990 to 2004, the sample mean and variance of the number of annual events were quite different, equal 4.7 and 2.6 respectively; the Poisson distribution has a mean and variance that are equal. However a goodness-of-fit test yields a p-value of 0.21, indicating that the Poisson distribution assumption is acceptable.

Let N be the number of perilous events over one year. Then

$$N \sim \text{Poisson}(\lambda). \tag{9}$$

The higher variance of the Poisson distribution has the advantage of simulating years where there are clusters of events due to factors such as the Southern Oscillation Index. An alternative to be examined in section 6 is the Binomial distribution, which may have a mean greater than its variance.

Severity Factors

There are a number of candidates for the severity factors, for example:

- maximum or average hailstone size;
- duration of hailfall;
- density of hailfall;
- maximum or average wind-speeds;
- rate or duration of rainfall.

Each of these storm attributes can lead to damage. Hailstone size is important because larger hail reaches higher speeds and causes greater damage (Blong, 1997). Longer duration of hailfall results in greater likelihood of roof collapse or gutter-blocking that floods roofs. Wind can drive hail through glass windows as well as cause tree-fall and structural collapse. Penetration of roofing may lead to subsequent flooding and water damage (Blong, 1997).

The need to model all of the listed severity factors can be reduced by taking advantage of the results of meteorological and damage studies. Paul (1968) showed that larger hailstone size is associated with larger storms that take longer to pass over the ground and therefore result in longer hailfall duration. Additionally, damage to structures is more strongly related to maximum hailstone size than other measures of hailfall (Summers & Wojtiw, 1971; Changnon, 1977). Blong (1997), in reviewing the work of Changnon, noted that 'man-made structures are rather insensitive to the density of hailfall'. Additionally, maximum hailstone size was the measure most prominent in defining hailfall in the hail model outlined by Leigh & Kuhnel (2001).

In the case of wind-speeds, choice is limited to some extent by the available data. It is wind shear, or the way that wind varies with height, that is most important in determining hailstorm severity. However, many of the BoM records only record wind-speed at fixed times and places during the day. Unless storms occur at these times then the wind-speed during storms cannot be determined. Instead, daily maximum wind gust can be used as a proxy for prevailing wind-speeds.

Rainfall can give an indication of likely water damage. While flood is not ordinarily covered by insurance policies, some water damage is covered. Inclusion of a rainfall factor, however, would require more comprehensive datasets than those available.

Given the relationships and limitations demonstrated above, the severity factors to be used in the hail model are:

- maximum hailstone size (diameter in cm), denoted *m*; and
- maximum wind-speed on storm day (km/h), denoted *w*,

so that $b = 2$, with $\beta_1 = m$ and $\beta_2 = w$.

Storm Severity

In examinations of hailstorm damage, roof damage makes up a great proportion of total damage, for example, 75% in a St. Louis study (Changnon, 1972). In the 18 March 1990 Sydney storm, however, only 23% of total damage was to tiling. A greater proportion (33%) was to windows and screens (Blong, 1997). Hail that is driven by strong winds causes this latter damage and is an important component of severity. The functional form of *f* must take account of the way in which the severity factors result in damage.

A Physics-Based Severity Calculation

The severity calculation seeks to measure the likelihood of damage caused by a storm, and there are many ways that this could be done. In this paper, some basic physics will be used to justify the form of the severity calculation and develop a functional form for severity that varies reasonably with its arguments. However, physics is not the focus of this paper, and the form used is primarily for demonstration purposes.

The basis of the severity calculation is the quantity of momentum, defined as mass multiplied by velocity. Momentum is commonly used in physics for the study of collisions and has the property of being conserved in all collisions (Serway & Jewett, 2004). The extent to which momentum is transferred in hailstorms varies according to the hailstones and the material they collide with. In the absence of this detailed information, it is assumed that a collision with greater momentum results in greater momentum transfer and subsequent damage.

There are two components to the severity calculation, *downward momentum* resulting from falling hailstones, and *horizontal momentum* due to wind-driven hailstones. For each storm the severity is calculated as the sum of the theoretical maximum observed values of these two components. This means that the severity represents the highest potential sum of horizontal and vertical momenta that could be present in the hailstorm.

In calculating mass, a quantity proportional to mass is adequate, since any constant multiple will cancel in later calculations (see *The Severity Ratio* below). An assumption that hailstone density is constant, and that hailstones are spherical, allows the use of the sphere volume formula to determine mass. Munich Re (1984) noted that hailstone density decreases with size, so that assuming constant hailstone density tends to overstate the momentum of large hail. The mass calculation is given by (10):

$$M = \frac{4}{3}\delta\pi\left(\frac{1}{2}m\right)^3 \tag{10}$$

where hailstone mass is denoted M and hailstone density is denoted δ.

The velocity of a hailstone on impact is assumed to be its terminal velocity (denoted V_{term}). That is, the hailstone has travelled far enough so that its acceleration has been overcome by the drag of the air and it has reached a maximum speed. Research has found that a hailstone's terminal velocity is roughly proportional to the square root of its diameter, with a diameter of 1 cm corresponding to a terminal velocity of 50 km/h (Munich Re, 1984). There is some suggestion that this relationship underestimates terminal velocities for larger hailstones because they have lower drag coefficients (Doswell, 1985).

The proportional relationship is assumed with a constant drag coefficient – to some extent offsetting the assumption of constant hailstone density. Thus,

$$V_{term} = 50\sqrt{m} \tag{11}$$

Summing downward and horizontal momentum results in a formula for the severity of a storm (denoted *S*):

$$\begin{aligned} S &= MV_{term} + Mw \\ &= M\left[50\sqrt{m} + w\right] \end{aligned} \tag{12}$$

An alternative severity calculation that is based on the idea that momentum is a vector, with both direction and magnitude, is considered in section 6.

A Modification

Equation (12) assumes that horizontal and downward momentum contribute comparably to storm severity. Blong (1997) suggested that it requires smaller hail to break windows (2 cm), than to penetrate roofing (4 cm). A factor θ may be included to take account of differences in horizontally and vertically driven damage, giving

$$S' = M\left[50\sqrt{m} + \theta w\right] \tag{13}$$

where S' is an alternative severity calculation.

Estimation of θ is not straightforward. For example, damage to windows may occur more easily than to roofing, but this does not translate directly to higher repair cost. The parameter θ must therefore be calibrated using historical experience. In the absence of the required datasets, θ is assumed to be equal to 1.

Maximum Hailstone Size

Examination of hail data for hailstone size greater than 2 cm shows that the sample distribution of maximum hailstone size is positively skewed (coefficient of skewness 0.4). It is useful to think of distribution fitting for *m* as fitting a tail to the distribution of all hail sizes, since the vast majority of hail events have a maximum hailstone size less than 2 cm.

The Use of Extreme Value Theory

An approach to modelling the tail of the distribution of maximum hailstone size is to use Extreme Value Theory, and in particular, Peaks over Thresholds. The basis of this technique is a theorem that states for a wide class of distributions, the distribution of the tail over a threshold tends to the Generalised Pareto Distribution (GPD) as the threshold tends to infinity. Details are outlined in Embrechts et al (1997) and Finkenstadt & Rootzen (2004). The concept is represented in Figure 3.

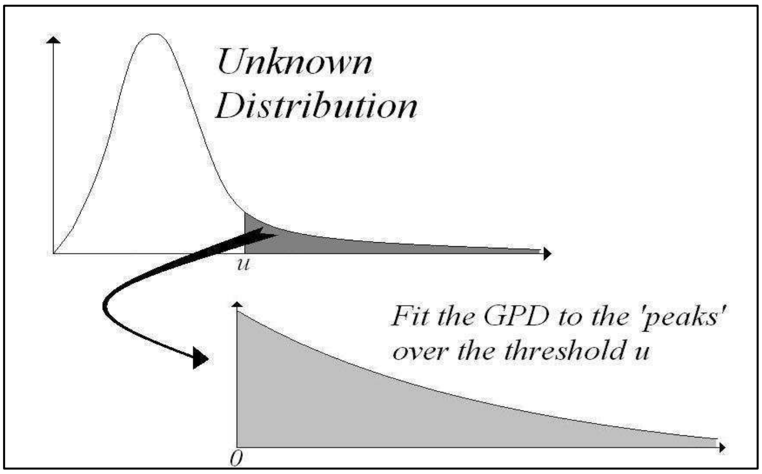

Figure 3 The GPD models the tail of a wide class of distributions

The distributional assumption for hailstone size is then the GPD, with the following functional form (for some random variable *Y*):

$$\Pr\left[Y-u_Y \le x \middle| Y > u_Y\right] = 1-\left(1+\frac{\xi_Y x}{\sigma_Y}\right)^{-1/\xi_Y}, \quad x \ge 0 \tag{14}$$

where ξ_Y, σ_Y, u_Y are parameters, with $\sigma_Y > 0$.

For maximum hailstone size, replace *Y* with *m* in (14), and 2 cm is a natural choice for the threshold, so that $u_m = 2$. Figure 4 reveals that the fit is reasonable, though the model gives a greater weight to smaller hail. However, for more extreme hail sizes the fit is very close. It may be possible to find a distribution that provides a better fit at the smaller hail sizes, but this will likely come at the cost of the fit at the more important, higher values.

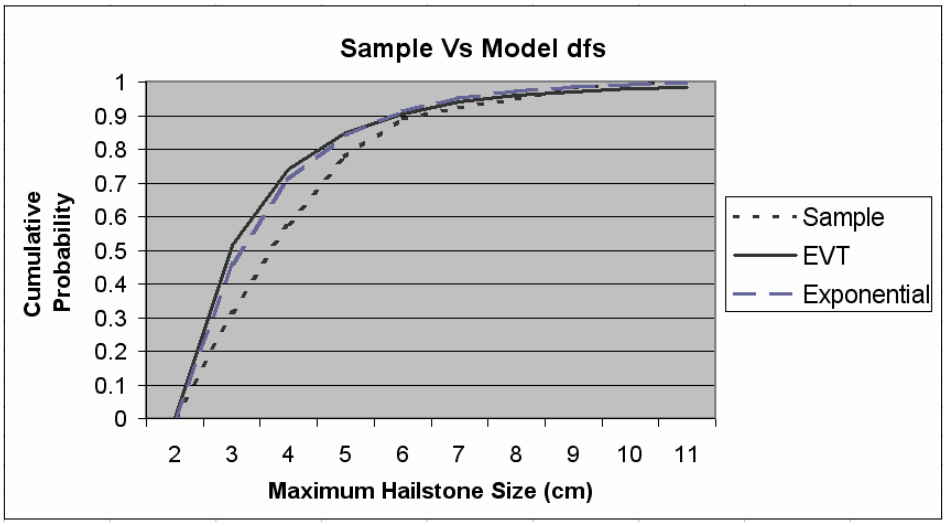

Figure 4 The sample and model distribution functions of maximum hailstone size.

Alternative assumptions to the GPD include an exponential distribution or distribution based on sample frequencies, which are each depicted in Figure 4. The exponential tail decays quickly and may understate the risk of large hailstones, while the sample frequency data will only incorporate events that occurred over the period of observation.

In using the GPD it is necessary to truncate the distribution to avoid absurd results (the largest hail on record in the world is 15.2 cm (Blong, 1997)). The density to the right of some extreme value can be uniformly assigned to the range of values below it. In the SHM, drawings that exceed 15 cm are re-sampled according to a uniform distribution between 0 and 15.

Maximum Wind-speed

Maximum wind readings for hailstorm days have a skewed distribution and the log-data passes standard tests for normality – indicating a lognormal distribution is appropriate. As with hailstone size, the density is truncated at some high value and the remaining density uniformly assigned. While winds in the order of 150 km/h have been observed (EMA website), the available data suggests that wind-speeds during Sydney hailstorms do not exceed 200 km/h and drawings beyond this are uniformly re-sampled.

Storm Affected Area

The available data relating to number of affected postcodes includes a wider range of hail events than the definition of a perilous event given above. The dataset is scaled back to include events that satisfy the definition by excluding the smallest events in accordance with the correlation between affected area and hailstone size (to be discussed below).

The modified data suggests a distribution of affected postcodes that is highly concentrated on the low integers, with some moderate numbers of observations from 5 to 10, and then three outliers at 83, 137 and 166. These outliers are a strong argument for the use of EVT. An appropriate choice of threshold is found by using the methods outlined in Embrechts et al (1997). A distribution based on observed frequency is appropriate for the few values below the threshold.

Correlations

The positive correlation between storm size and hailstone size is a well-documented phenomenon (Paul, 1968). Leigh & Kuhnel (2001) note that some of the event attributes in the HailAUS model are independent, while others are correlated. Sample data of the number of affected postcodes and maximum hailstone size suggests a significant relationship, with a sample correlation coefficient of 0.57.

There is less evidence to suggest that hailstone size is related to thunderstorm winds given that a storm occurs. Data provided by the Bureau of Meteorology does not show a significant relationship between these variables. Wind-speed is therefore assumed to be independent of both hailstone size and affected area.

Standard Severity

If the only available and reliable catastrophe claims data relates to a few events then it is useful to define a standard severity.

<u>D5.1</u> A *standard severity* (denoted S_{std}) is calculated as the severity of a single event (a *standard storm*) for which the claims experience is known.

Where data relating to a number of significant events is available, multiple standard severities may be defined, each over a different range of severity. For example, a severity of S in the range (S_{j-1}, S_j) might be associated with a standard severity $S_{std,j}$. Despite this generalisation, data availability restricts the analysis of this paper to a single standard storm.

The Severity Ratio

The severity of a storm is compared to a standard severity via the severity ratio. The severity ratio then forms the basis of the PL calculation.

D5.2 The definition of the *severity ratio* of a storm with severity S, denoted ψ, is

$$\psi = \frac{S}{S_{std}} \tag{15}$$

A severity ratio greater/less than 1 indicates a more/less severe storm than the standard, and for a fixed storm size and proximity, PLs are higher/lower.

Percentage Loss Function for the Standard Storm

Examination of PL data from the 14 April 1999 storm (the 1999 storm) reveals that the ordered PLs closely follow a piecewise exponential curve. The data shows a distinct change in curve behaviour; the growth in PLs is more rapid over the final 30% of observations. Separate exponential curves provide an excellent fit to these data (R^2 of 0.99 in each case).

The function L_{std} for standard storm PLs is then:

$$L_{std}\left(z^{(k)}\right) = \begin{cases} a_{nc} \exp\left\{\mu \dfrac{k}{d}\right\} & , \quad 0 < k \le d \\ a_{core} \exp\left\{\nu \dfrac{k-d}{A-d}\right\} + b_{core} & , \quad d < k \le A \end{cases} \tag{16}$$

where a_{nc}, a_{core}, b_{core}, μ, and ν are parameters to be estimated.

The following condition ensures that L_{std} is continuous at d:

$$b_{core} = a_{nc} e^{\mu} - a_{core} \tag{17}$$

The parameter γ is found by observing the point at which the PL versus proximity curve changes dramatically and may be optimised by maximising the fit of each piece of L_{std}.

The Severity Ratio Function

A further assumption is now required in order to utilise the standard storm.

A5.4 The PLs of the standard storm can be used to determine PLs for non-standard storms.

D5.3 The *severity ratio function*, denoted $h(\psi)$ relates the PLs of the standard storm to those of non-standards storms via the relationship

$$L\left(S, z^{(k)}\right) = h(\psi) L_{std}\left(z^{(k)}\right). \tag{18}$$

By definition, it must be that $L\left(S_{std}, z^{(k)}\right) = L_{std}\left(z^{(k)}\right)$, so that $h(1) = 1$.

Finding the functional form of *h* requires data relating to multiple catastrophic events. In the absence of this data a particular functional form must be assumed. The following is a possible form for *h*:

$$h(\psi) = \begin{cases} \psi^{p} , & 0 \leq \psi \leq 1 \\ \psi^{q} , & \psi > 1 \end{cases} \tag{19}$$

The hybrid form of (19) allows for a different relationship between PL and severity ratio depending on the range in which the severity ratio lies. For example, percentage loss might increase steadily, and then more rapidly, as 1 is reached. Beyond 1 there may be 'diminishing marginal percentage loss' from increasing the severity ratio so that $p > 1$ and $q < 1$.

The surge in demand for building materials and labour (and hence inflation of claims costs) following a catastrophe is not modelled directly, but indirectly, through the use of the 1999 storm data when such a demand surge occurred (EMA website). A potential extension to the SHM is to incorporate the effects of demand surge into *h*, so that it is also a function of *A*.

The reader may have noticed that the determination of an appropriate functional form for the severity calculation is made obsolete by a perfectly estimated severity ratio function. In theory, if *h* were accurately determinable regardless of the functional form of *S*, then the form of *S* would not matter. However, there is likely to be inadequate data to fully estimate *h*, particularly at the upper extreme. Instead, what can be done is to use a well-justified physical basis for *S* and then detect a trend in *h* based on the data. For example, realising that there is diminishing marginal PL helps to develop a functional form for *h*.

5.5 Parameter Estimation

Parameter estimation was performed using maximum likelihood estimation. Table 5.1 contains parameter estimates.

Table 5.1

	Parameter	Estimate
Event Frequency (*N*)	λ	4.66
GPD – Hailstone Size (*m*)	ξ_m	0.23132
	σ_m	1.26770
Lognormal Distribution for Wind-speed (*w*)	μ_w	4.07
	σ_w	0.387
PL Function for the Standard Storm (*L*)	γ	0.3
	a_{nc}	0.0000713
	μ	4.8327
	a_{core}	0.000174
	b_{core}	0.008779
	v	6.0472

Comments on Table 5.1

Notice that v is greater than μ. This is expected, because PLs increase faster in core zones than non-core zones as proximity increases.

The lognormal distribution of maximum wind-speed has a mean of 63 km/h with a standard deviation of 25.

Storm Affected Area

For $A \leq 4$, the sample distribution in Table 5.2 was derived.

Table 5.2

No. of affected postcodes (*A*)	Empirical probability
1	0.0579
2	0.3802
3	0.1488
4	0.1653
> 4	0.2479

Beyond 4, a GPD was fitted with parameters ξ_A = 0.88344 and σ_A = 3.69811, and threshold u_A = 4. The coefficient of determination associated with a linear fit of the QQ plot is 0.84, showing that this fit is not ideal. However, given high irregularity in observations of the variable, a better fit may be an unreasonable expectation. Note that beyond 4, rounding is required to generate an integer number of affected zones.

Correlations

Generating the desired correlation between m and A is not straightforward. Rather than trying to simulate from a bi-variate distribution with the appropriate correlation, it is simpler to adjust the simulation process. For a given number of generated storms, the observations of *m* and *A* are sorted in ascending order and paired up so that $m^{(k)}$ corresponds with $A^{(k)}$, where (*k*) denotes the k^{th} order statistic.

This formulation results in high positive correlation between *m* and *A* values. To reach the desired level of correlation, a proportion of the hailstone sizes is extracted and randomly rearranged. The higher the proportion that is randomly rearranged, the lower the correlation. It was found by trial and error that rearranging one in four of the ordered $m^{(k)}$'s results in a correlation coefficient of 0.56 on a consistent basis for large numbers of storm simulations. This is close to the desired correlation of 0.57 and this method is therefore incorporated into simulation. Rearrangement of one in three and one in five of the ordered observations results in correlations of 0.48 and 0.61 respectively.

This method should be used with some care because correlation is a measure of linear dependence and the relationship between the variables need not be linear.

Standard Severity

In practice it is difficult to determine whether claims are due to a particular event unless a code is reliably attached to such claims. Only for the largest events are codes reliably recorded and there may be only one such event. A method for obtaining claims data from an un-coded catastrophic event is to take the total claims for each zone for a date of loss and subtract an average daily loss in typical circumstances.

Data limitations mean that the 1999 storm is the only standard storm that can be used in the SHM. It had the following attributes (EMA website): *m* = 9 cm, *w* = 85 km/h, so that $S_{std} = 89,700\delta$.

Severity Ratio Function

In the absence of more detailed data, it is assumed that *p* = *q* = 1. This means that a storm with a severity ratio of ψ_1 has PLs that are ψ_1 times those of the standard storm for a given affected area. This assumption may overestimate claims for storms significantly more severe than the standard because marginal percentage loss is likely to diminish as the severity ratio increases. For example, once a roof tile is broken, replacement cost does not change with a further crack in the tile.

Standard storm PLs relative to other storms under this assumption can be visualised by plotting severity (see Figure 5).

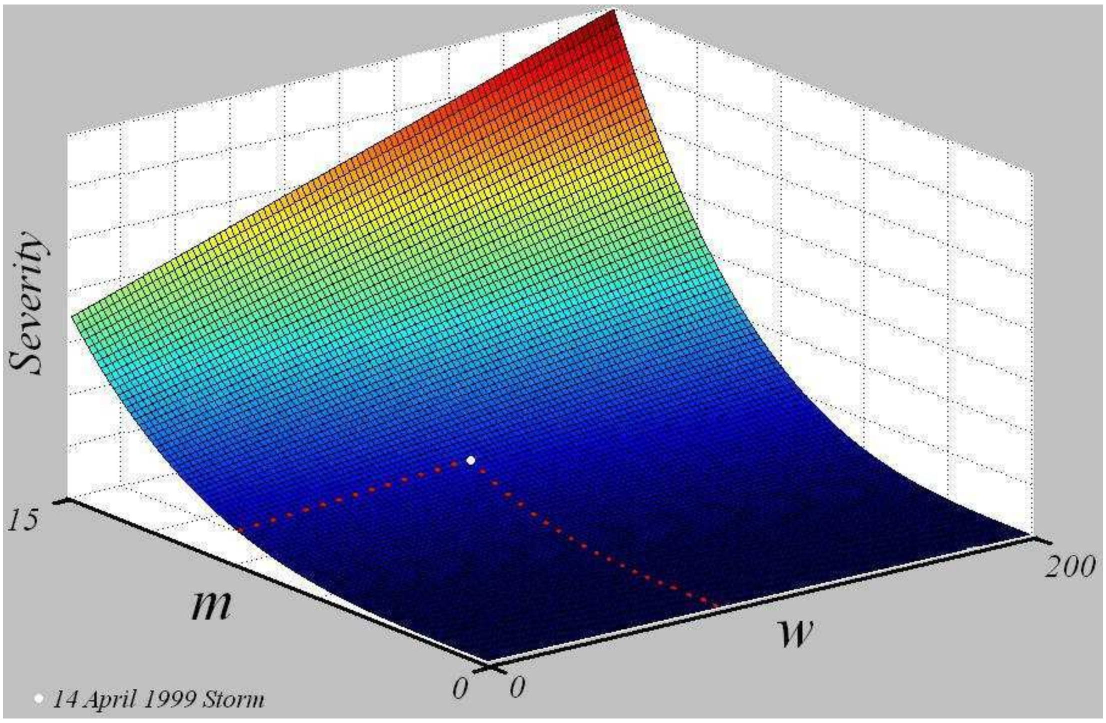

Figure 5 Storms may be generated with PLs that are many times those of the standard storm.

5.6 Putting the Model Together

Putting together all the discussed model components, the evolution of simulation can be described as follows. Suppose a number of years of experience is to be simulated. For each year there is a random drawing from the Poisson distribution, which determines the number of storms satisfying the perilous event definition that will occur in that year. For each of these perilous events, a triplet is drawn from three separate distributions: maximum wind-gust, maximum hailstone size, and number of affected postcodes. The latter two are correlated variables. The former two variables are used to determine the overall severity of the storm.

The severity is then compared with the appropriate standard to give the severity ratio, which is the argument of the severity ratio function, the result of which is an argument of the percentage loss function. Given the storm size, a vector of percentage losses according to proximity is developed. Depending on the storm centre, the affected postcodes are determined and a vector of corresponding sums insured, ordered by proximity, is generated. The percentage losses vector multiplies this vector and the sum of the result is the gross loss to the insurer from the event. Note that single event catastrophe reinsurance contracts may be applied to the data at this point. Annual losses are then found by summing event losses and this allows the application of annual stop-loss reinsurance arrangements.

Once net annual losses have been determined for each year of simulation, the data may be used in a similar fashion to any sample data; one may produce summary statistics, sample pdfs and dfs, fit distributions, develop Loss/Return Period Curves and so on.

6. Output, Sensitivities and Discussion

The hail model was programmed in MATLAB™ using the structure outlined in 5.1. In this section the output of the SHM is presented, with a particular emphasis on model sensitivity. The analysis is intended to highlight the potential for highly variable output when only moderate (and justifiable) differences in model design or inputs exist.

6.1 Preliminaries

Portfolios

Three different portfolios are analysed to explore the effect of geographical concentration of sum insured on model output. Each portfolio has a total sum insured of $10 billion spread across ICA zones 41, 42 and 43.

Portfolio A: equal amount of sum insured in each of 243 postcodes.

Portfolio B: sums insured in each postcode proportional to those of the commercial portfolio underlying the loss data specified in section 5.3 (across 210 postcodes).

Portfolio C: $200 million sum insured in each of 50 postcodes.

Standard Output

Sample quantiles and averages are used to derive the following quantities, denoted Q, relating to annual aggregate claims.

Q1	Average annual loss
Q2	Median annual loss
Q3	250-year PML estimate
Q4	500-year PML estimate
Q5	Maximum simulated loss

A Base Model

To demonstrate model sensitivities, the following base model is defined so that the effects of changes in assumptions and parameters can be examined:

- a maximum PL (Max PL) of 1;
- $\theta = 1$ in (13);
- $p = q = 1$ in (19);
- Corr(m, A) = 0.56;
- all other parameters as estimated.

6.2 Model Validation against Data

To validate the model using the available data, the 14 April 1999 storm is simulated and the insured loss is determined for Portfolio B – an event for which the actual answer is known.

Table 6.1

	Loss ($m)	Error
Actual	114.3	-
Model Prediction (i)	110.1	-3.8%
Model Prediction (ii)	150.5	+31.7%

Model prediction (i) in Table 6.1 uses the post-event knowledge of the ordered PLs in each postcode and is therefore essentially a test of the PL curve fit, which is good, with only a small percentage error. Model Prediction (ii) uses cruder data of the storm centre and number of affected postcodes, and gives an error of 31.7%. This error is due to the model assumptions regarding PL behaviour and postcode proximity from the storm centre. It is reasonable to expect that the assumptions will sometimes result in understatement and sometimes in overstatement of loss, so that a large number of simulations mitigate the resulting level of error.

6.3 Variability in Simulation Output

The use of simulation means that there is error in estimating the 'true' distribution of annual losses under model assumptions. To measure this variation, 10 x 1,000 and 10 x 10,000 simulations are run, and confidence intervals determined. Figure 6 depicts the output in terms of loss return period curves under two different severity ratio function assumptions.

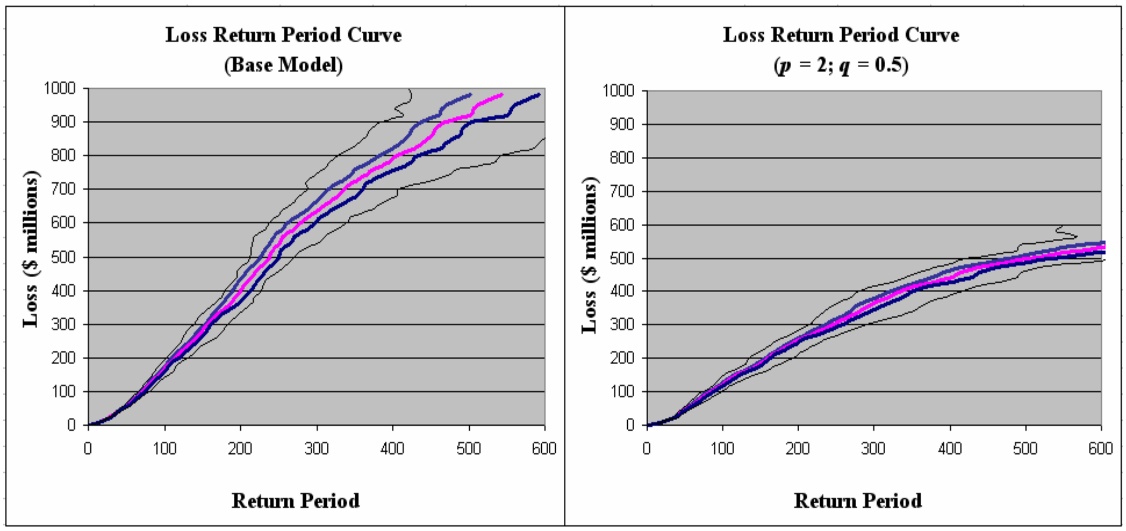

Figure 6 Best estimates of LRPCs with associated confidence intervals.

In Figure 6, the middle line is the best estimate of the LRPC, while the next two sets of lines represent confidence intervals based on 100,000 and 10,000 simulations respectively. Increasing the number of simulations narrows the confidence interval of PMLs at each return period considerably. This demonstrates that the model is stable, with the use of 100,000 runs being reasonable on inspection of Figure 6. More broadly, it demonstrates a need to run a catastrophe model for an adequate number of simulations. Figure 6 also shows that the variability of the simulations changes depending on parameter values, with the base model simulations noticeably more variable than the simulations of the modified model (depicted on the right).

In figure 6 the base model produces only a slightly concave LRPC. However, with $p = 2$ and $q = 0.5$, the dampening of large event losses results in a distinctly concave curve. This is reminiscent of available residential portfolio hail model output (Blong et al, 2001) and so it appears that consistency with other models will require $q < 1$.

6.4 Portfolio Sensitivity

The output from 100,000 simulations of the base model for each portfolio are in Table 6.2.

Table 6.2 (all quantities are in millions of dollars)

Quantity	Portfolio A	Portfolio B	Portfolio C
Q1	9.05	8.95	9.00
Q2	0.5727	0.4504	0.2510
Q3	556	544	514
Q4	967	932	979
Q5	2,537	2,571	3,348

Comments on Table 6.2

Geographical diversification within the area of interest appears to have little influence on PMLs in this model (c.f. Blong et al, 2001). Some effect is demonstrated with Portfolio C, where **Q5** is significantly larger than the other two portfolios. The only other notable difference is in **Q2**. This occurs because increased sum insured concentration leads to an increased number of storms with zero insured loss.

For brevity of presentation, the remaining analysis uses Portfolios A and B.

6.5 Severity Ratio Function and Maximum PL Sensitivities

In Table 6.3 the maximum PL and values of p and q are varied to give an indication of the reliance of model output on these parameters for Portfolio A and using 100,000 simulations.

Table 6.3 (all quantities are in millions of dollars)

Quantity	Base Model	Max PL = 0.1	Max PL = 1 $p = 2; q = 0.5$	Max PL = 0.1 $p = 2; q = 0.5$
Q1	9.05	7.70	4.90	4.79
Q2	0.5727	0.5711	0.0251	0.0249
Q3	556	442	301	295
Q4	967	744	483	458
Q5	2,537	1,942	1,200	1,133

Comments on Table 6.3

Catastrophe models may include a maximum loss (10% of sum insured in this case) to avoid extreme simulations overstating catastrophe risk. The use of a maximum PL has differing effects depending on the shape of the severity ratio function. Capping loss must be appropriately justified because, as shown with the base model, output can change significantly. Simply because historical PLs have not exceeded some fixed number does not imply that this will continue in the future.

6.6 Parameter, Correlation and Assumption Sensitivities

Tables 6.4 and 6.5 display output for various alterations to the base model using Portfolio B and 100,000 simulations. In each case the change indicated is the only change made to the base model.

Table 6.4 (all quantities are in millions of dollars)

Quantity	Base Model	Adjustment					
		(a)	(b)	(c)	(d)	(e)	(f)
Q1	8.95	4.47	18.11	8.50	9.03	10.44	7.35
Q2	0.4504	0.1553	1.1268	0.4477	0.4634	0.4735	0.4223
Q3	544	259	1,148	569	534	624	429
Q4	932	499	1,959	974	994	1,062	806
Q5	2,571	1,524	4,170	1,551	2,220	2,707	2,354

Adjustments to the base model:

(a) $\lambda = 2.33$
(b) $\lambda = 9.32$
(c) Weighted average of output from a fixed number of 4 and 5 events per annum to artificially construct $E(N) = 4.66$ and $Var(N) = 0$
(d) $N \sim$ Binomial(11, 0.4236); $E(N) = 4.66$; $Var(N) = 2.69$
(e) Estimate of ξ_m increased by one standard error (= 0.30432)
(f) Estimate of ξ_m decreased by one standard error (= 0.15832)

Comments on Table 6.4

Adjustments **(a)** and **(b)** show that changes in λ flow through approximately proportionally to changes in all quantities. This implies that under/overestimation of average event frequency results in similar under/overestimation of catastrophe risk quantities. It also suggests that years with a higher expected event frequency would have similarly higher PMLs.

Adjustment **(c)** simulates a fixed number of events per annum with the same mean as the Poisson distribution of the base model. All quantities except **Q5** are similar to the base model, indicating that the inclusion of variability in the number of events per year only affects the far right hand tail of the annual loss distribution.

For adjustment **(d)** the variance of the annual number of events has a negligible effect on all quantities except **Q5**. This highlights the influence of a small number of extreme events, rather than a multiplicity of events, in determining the highest annual losses. For this model, the inclusion of climactic cycles through higher variance does not translate into higher PMLs.

For adjustments **(e)** and **(f)**, the effect on the 250-year PML of varying ξ_m by its standard error is in the order of a 20% change. This shows that differences in estimation of parameters like ξ_m (for example due to data differences) can have large effects on output.

Table 6.5 (all quantities are in millions of dollars)

Quantity	Base Model	(g)	(h)	(i)	(j)	(k)	(l)	(m)
Q1	8.95	6.81	2.63	8.43	9.43	7.80	11.42	9.55
Q2	0.4504	0.5111	0.6975	0.4867	0.4223	0.3530	0.6315	0.4492
Q3	544	364	57	508	567	477	680	567
Q4	932	535	84	879	1,056	835	1,165	1,056
Q5	2,571	2,065	335	2,392	2,845	2,258	3,236	2,789

Adjustments to the base model:

(g) No Extreme Value Theory used in the distributions of hailstone size and number of affected postcodes. An exponential distribution and distribution based on sample frequency was used for these variables instead.
(h) Corr(m, A) = 0
(i) Corr(m, A) = 0.48 (see Section 5.5 for an explanation of the choice of correlation values)
(j) Corr(m, A) = 0.61
(k) $\theta = 0.5$
(l) $\theta = 2$
(m) An alternative severity calculation described below.

Comments on Table 6.5

The profound effect that the use of EVT has on output is seen for adjustment **(g)**. Models that use EVT will generate substantially different answers to those that use standard distributional assumptions. The use of EVT must be justified since it generally attaches a heavy tail to model variables, which may not be warranted.

Adjustments **(h)** to **(j)** portray the vital role that variable correlation plays. Quantities under **(h)** are alarmingly smaller than for **(i)** and **(j)**. Models that (incorrectly) do not incorporate variable correlation may seriously mis-estimate potential loss. For **(i)** and **(j)**, a moderate change in assumed correlation is shown to have a significant, but undramatic impact.

Adjustments **(k)** and **(l)** show that the effect of halving or doubling θ has a moderate effect on results. The factor only applies to wind, and as was seen in Figure 5, changes in wind-speed have a more restrained influence on severity than do changes in maximum hailstone size. Nevertheless, the noticeable sensitivity in output indicates that reliable estimates of model parameters like θ are important.

Adjustment **(m)** involves the use of an alternative severity calculation. If the quantities of horizontal and vertical momentum are considered as vectors with magnitude and direction, then we can calculate the magnitude of the sum of the vectors, giving the following alternative severity function (S^*):

$$\begin{aligned} S^* &= \sqrt{\left(\frac{4}{3}\pi\delta\left(\frac{1}{2}m\right)^3 * 50\sqrt{m}\right)^2 + \left(\frac{4}{3}\pi\delta\left(\frac{1}{2}m\right)^3 w\right)^2} \\ &= \frac{4}{3}\pi\delta\left(\frac{1}{2}m\right)^3 \sqrt{2500m + w^2} \end{aligned} \tag{20}$$

The use of this alternative calculation does not have a very large impact on results. This demonstrates that the exact way in which momentum is calculated is probably not as important as the decision to use momentum as a measure of severity in the first place.

6.7 Summary of Findings

For a given set of functional forms and parameters, the SHM was shown to be relatively stable. However, significant sensitivities to assumptions, parameters, and therefore data, have been shown to exist. While some model components are more influential than others, overall it is clear that small changes can have profound effects on output. This sensitivity is observed within the framework of a single model. Alternative models may also have substantially different structures, adding another potential cause of model output disparity.

7. Limitations and Further Work

7.1 Limitations

The primary limitation of the paper is an inability to access information. Without detailed knowledge of the models and data that are used in practice, findings include an element of speculation. Nevertheless, this limitation is itself a finding. The difficulties encountered during catastrophe model construction are indicative of information barriers that plague the insurance and modelling industries.

Limited access to data results in the model presented relying on one major event. In general it would be expected that commercial and internal modellers have greater access to historical information. However, reliance on a single event is not unusual. Sanders et al. (2002) discuss a Benfield Greig UK flood model that simulates the effects of a single 1953 flood on modern day portfolios. Two different, and more sophisticated models, produce PMLs that vary by a factor of eight – showing that for these flood models, other model differences are of more concern than a reliance on a single event!

The simplifying assumptions of the SHM ensure that it is relatively uncomplicated compared to what is known about other models. This is both a strength and a weakness. Given the uncertainty that surrounds model variables, a more complex model may simply contain more statistical ‘noise’ rather than produce more accurate results. Additionally, the available data is not reliable enough to justify the use of extremely precise modelling techniques. On the other hand, more sophisticated modelling that correctly incorporates subtle portfolio or peril attributes may lead to greater accuracy. However, even a complex model is likely to contain parameters like ξ_m, and use assumptions on which results hinge – it is an inevitability of modelling the extreme.

In addition to the specific limitations of using the SHM to draw conclusions about catastrophe modelling, there are weaknesses that apply to any analysis of this kind. The uncertainty inherent in modelling catastrophes cannot be fully remedied through wider agreement on methodologies and access to datasets. For instance, all models are slaves to the past, each containing at least some element of parameter estimation and calibration that relies on historical data. Issues such as climate change and the changing nature of exposures raise the question: do the catastrophe models of today realistically model the catastrophes of tomorrow?

Catastrophe modelling is an uncertain exercise; there are no ‘correct’ answers. To aim for a convergence of output, as in Florida, is not to aim for improved modelling. The latter requires further work and co-operation.

7.2 Further Work

In regard to this paper, further work includes complete estimation and calibration of the SHM followed by a thorough comparison with other models. In addition, the ZPLM could be applied to different perils and locations. More broadly and importantly, further work includes the preparation of similar papers to this one, examining alternative models, perils and locations.

Completing this work will require improved co-operation on information. There are some simple ideas to encourage this to occur. In regard to data availability, one idea is to establish a central data exchange whereby insurers anonymously submit documented datasets to a central body and in return are able to access submissions by others. To address the proprietary nature of models, regulator-sponsored conferences would allow model developers to share ideas and discuss controversial model components, improving modelling standards that should result in greater consistency.

8. Conclusions

Catastrophes eventually impact on all insurers and indeed members of society. When models of the same phenomenon produce different answers, a productive response is to seek an explanation. This process may uncover errors or weaknesses in a model that can be remedied or improved upon. Even disagreement on model components is productive, informing modellers of the stance taken by their peers. However, in the current world of catastrophe modelling the co-operation and openness needed to achieve this outcome does not occur.

This paper has provided a publicly available exhibition of catastrophe model construction, output and sensitivities that will hopefully be added to in the future. The difficulties encountered in model construction and fitting largely result from an inability to access information. The considerable sensitivities in model output that have been demonstrated show why cross model comparison is so important. Improved co-operation would address data and structural deficiencies in models, enhancing our capacity to assess and manage catastrophic risks.

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