Modelling and Replicating Hedge Fund Returns

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March 1, 2010

Abstract

This paper provides discussion on factor-based modelling for hedge fund returns, and demonstrates replication via both rolling windows and Kalman filters. In particular, we focus on estimating time-varying hedge fund returns exposure through various asset-based style (ABS) factors. It is shown that certain hedge fund strategies are more susceptible to cloning, suggesting a higher likelihood of creating transparent liquid replication products as either an alternative investment vehicle or as a benchmarking/ style analysis tool for institutional investors.

1 Introduction

Modelling and replicating hedge fund returns are encountering growing interest. This motivation is, in part, driven by the high fees charged by existing hedge fund managers and their lack of transparency. Managers charging 2% base and 20% performance fees argued that their returns were unique and uncorrelated to traditional markets, hence warranting the fees charged. As of such, hedge fund returns were widely labelled as 'absolute return' investment vehicles, providing stable returns with low systematic risk. However, events such as the collapse of LTCM in 1998 and their general performance during the global financial crisis in 2008, have drawn to many criticisms from investors. It would seem hedge fund returns had significant correlations to general markets.

An interesting point raised by Takahashi and Yamamoto (2008) is that whilst the hedge fund industry has become increasingly competitive, their fees however, remained at high levels. This is peculiar in comparison to the manufacturing industry, where once a new product has been developed, rival companies releasing similar products would gradually reduce price – a simple supply and demand concept. The reason for this peculiarity, they conclude, is the difficulty in which evaluation can be performed on hedge fund returns. Each hedge fund manager claims their own unique ability at delivering alpha to their investors,

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hence deserving the high fees charged. We find this similar to how firms differentiate themselves in monopolistic competition. Hedge funds, in a sense, could be compared to luxury goods manufacturers.

In terms of modelling, it has been generally accepted that hedge fund returns can be separated into alpha and beta components; the former attributable to manager skill and the latter being generated from market exposure. A multi-factor model is often used to distinguish the two. By increasing granularity, the beta returns itself can be separated into two major families. Traditional beta explains traditional exposures to assets such as stocks and bonds, and alternative beta explains returns generated from dynamic trading strategies. Modelling hedge fund returns primarily involves estimating the magnitude and direction of these alpha and beta components. We note it is the beta components that can be replicated.

In this paper we discuss the evaluation of hedge fund strategies via asset-based style (ABS) factor modelling. Furthermore, using this framework, we illustrate how replication of hedge fund returns through a variety of common assets is possible. Successful replication casts doubts on the justifiability of high fees. Moreover, a successful factor-based framework provides not only provides investors with a cheaper alternative, but also provides the industry with a clear and transparent benchmark or comparable.

2 Literature Review

Asset class factor modelling for mutual funds style analysis was pioneered by Sharpe (1992). He showed that by using a limited number of asset classes, it was possible to explain the sources of performance for US mutual funds. However, Sharpe's model is less effective for hedge funds, which employ dynamic trading strategies such as derivatives and short selling, and are also generally more heavily leveraged. Whilst mutual fund managers are often concerned over investor withdraws after short-term performance downturns or an increase in tracking error, hedge fund managers are less disturbed by this as many have longer lock-up periods and a significantly more flexible mandate. Noting this, Fung and Hsieh (1997) employed a multi-factor approach, using three equity classes (US equities, non-US equities and emerging markets), two bond classes (US government and non-US government), commodities (gold) and currencies (Federal Reserve trade-weighted USD) to model returns. Ennis and Sebastian (2003) used four equity classes (by further splitting US equities into small and large capitalizations), credit and duration as factors for explaining hedge fund returns.

Since Fung and Hsieh's (1997) seminal work, various ABS methods were employed to describe hedge fund returns. To account for dynamic trading strategies, Capocci (2001) used Fama and French factors, momentum (Carhart, 1997) and credit spreads to describe hedge funds returns.

Criticisms that non-linearities were not being captured through simple linear regression models provided motivation for assessing non-linear factors. Earlier works on mutual funds by Treynor and Mazuy (1966) and Merton and Henriksson (1981), used a quadratic model (1) and an option-like payoff factor (2) respectively, quantify a manager's timing ability. Supposing a manager could 'time' market conditions, their returns would be more likely to exhibit all of the upside during bull runs and less of the downside in bear markets.

$$R_{Fund,t} = \alpha + \beta R_{Mkt,t} + \gamma R_{Mkt,t}^2 + \varepsilon_t \tag{1}$$

$$R_{Fund,t} = \alpha + \beta R_{Mkt,t} + \gamma max\left(R_{Mkt,t}, 0\right) + \varepsilon_t \tag{2}$$

Non-linear factors were therefore implemented by Agarwal and Naik (2004) and Fung and Hsieh (2004) in their multi-factor model. Agarwal and Naik (2004) note that a portion of the returns of Event Driven and Relative Value strategies were explained by shorting 'out of the money' put options on the S&P 500 composite index. Similarly, Convertible Arbitrage and Short Bias strategies were significantly explained by shorting 'at the money' puts and 'out of the money' calls respectively.

Despite the evident increase in R^2 fit by using option-based factors, Bianchi at al. (2008) point out that strike price determination of options-based factors was conceptually ad hoc. Amin and Kat (2003) argues, somewhat earlier, that it is uncertain how many options and what strike prices should be included. Only a small number of ordinary puts and calls can be included in regression, limiting the range and type of non-linearities captured. Furthermore, from a replication point of view, option-factors become unwieldy and unnecessarily complicated for investors. Deiz de los Rios and Garcia's (2008) research provides striking evidence against option-based factors - they find that at the index level they could not reject linearity at all. They then proceed to evaluate funds on an individual basis, and find only 1/5 of the universe under Lipper/TASS can reject the null hypothesis of linearity. Breaking down the hedge fund universe into arbitrage strategies, market neutral and directional strategies, they find only 20%, 10 to 15% and 20% of these groups exhibited non-linearity to the market respectively.

Moreover, Roncalli and Weisang (2008) point out that dynamic trading of assets would in itself result in non-linear return profiles. Hence instead of specifying option-based factors, they highlighted the importance of capturing time-varying beta. As it is well known that hedge fund strategies are dynamic, time-invariant factor loadings are unrealistic and simplistic.

Research in time-varying beta was conducted mostly by academics and practitioners, whom by now, were seeing the possibilities of taking earlier factor models to the next level – i.e. replication. They were interested in constructing a cheap 'clone' based on ABS factor modeling to replicate the median hedge fund. Earlier works by Hasanhodzic and Lo (2007) and Darolles and Mero (2007) estimated time-varying betas through rolling window regression, the former using 24 months and the latter 36 months. However, the selected window is ad hoc, largely a compromise between the ability to capture recent exposures more quickly and statistical accuracy.

Recently, Kalman filtering was suggested to overcome issues with the rolling window approach. Takahashi and Yamamoto (2008) applied both rolling window and Kalman filters for estimating time-varying exposure of the AsiaHedge Asia ex-Japan Index to the Morgan Stanley Capital International (MSCI) Asia Pacific ex-Japan Index. They note on average the Kalman filter captures changes in exposure earlier than the rolling window method. Roncalli and Teïletche (2008) estimated time-varying beta on six underlying factors for the Hedge Fund Research (HFR) composite index, HFRI FOF index and the Credit Suisse Tremont index. They found Kalman filter estimates were notably more stable than rolling regression estimates, as well as its clones having higher mean returns.

Furthermore, Roncalli and Weisang (2008) discuss the use of particle filters, where Gaussian assumptions on the state space model, apparent in the Kalman filter, are relaxed. The idea of employing particle filters is not new; Kobayashi et al. (2005) had proposed a general state space model and Monte Carlo filter for style analysis on Japanese mutual funds. However, Roncalli and Weisang (2008) conclude that the use of particle filters was somewhat disappointing. They find that in order to capture higher moments of hedge fund returns, higher tracking error was experienced. Furthermore, they found little evidence of non-linearities in the majority of hedge fund returns distributions.

Similarly, state space modelling was used by Racicot and Théoret (2009) to determine hedge fund conditional alpha as well as beta. Focusing on modeling rather than replication, their macroeconomic Kalman filter allowed for alpha and market beta to be conditional, whilst factor loadings on Fama French size (SMB) and book to market (HML) are left time-invariant.

$$R_{HF,t} - R_t^f = \alpha_t + \beta_{1,t}\left(R_{Mkt,t} - R_t^f\right) + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_t, \tag{3}$$

$$\alpha_t = \alpha_{t-1} + \varphi_1 r_{t-1} + \varphi_2 mkt_{t-1} + \varphi_5 mkt_{t-1}^2 + \xi_t,$$

$$\beta_{1,t} = \beta_{1,t-1} + \varphi_3 r_{t-1} + \varphi_4 mkt_{t-1} + \varphi_6 mkt_{t-1}^2 + \nu_t.$$

Results suggested whilst conditional alpha is not sensitive to macroeconomic factors, conditional market beta appeared to respond positively to market risk premium (mkt) and negatively to the level of interest rate (r).

In light of previous research, we provide a brief summary of various exposures for the broader range of hedge fund strategies listed in the HFR using an ABS factor model. Following from this, we present and discuss replication techniques via both the traditional rolling windows and the state space approach.

3 Methodology

3.1 Evaluation of Hedge Fund Returns

A linear case ABS factor model is explored to determine the exposures on hedge fund returns and the relevant factors for replication. For our analysis, regression (using monthly data from January 1990 to December 2009) is conducted on the hedge fund indices with the following ten factors (accumulation indices; denominated in USD):

1. SP = Standard & Poor's 500 Index
2. EAFE = Morgan Stanley Capital International EAFE Index
3. EM = Morgan Stanley Capital International Emerging Markets Index
4. SMB = Size differential in the NYSE, AMEX and NASDAQ
5. HML = Book to market differential in the NYSE, AMEX and NASDAQ
6. UMD = Momentum differential in the NYSE, AMEX and NASDAQ
7. BOND = Citigroup World Government Bond Index
8. HY = Barclays Capital Global High Yield Index
9. CMDTY = Goldman Sachs Commodity Index
10. USD = Federal Reserve trade-weighted USD Index

SP, EAFE and EM are employed to estimate exposure on various world equity markets. SMB, HML, UMD are long/short factors used in explaining potential investment styles. SMB is calculated as the average return of three small-cap stock portfolios minus the average return of three large-cap portfolios. (value, neutral and growth describes the book-to-market (BM) ratio of the stocks)

$$SMB = \frac{1}{3}\left[s.value + s.neutral + s.growth\right] - \frac{1}{3}\left[b.value + b.neutral + b.growth\right].$$

Value stocks are known to have high BM while growth stocks have lower BM. HML is the average return of two value portfolios minus the average return of two growth portfolios.

$$HML = \frac{1}{2}\left[s.value + b.value\right] - \frac{1}{2}\left[s.growth + b.growth\right].$$

Furthermore, a proxy for momentum is used, similar to Carhart (1997) and Capocci (2001). Momentum is the prior month's high return portfolio minus low return portfolio. Fama and French calculate this by splitting the portfolios into small and large capitalisations as well as high and low returns.

$$UMD = \frac{1}{2}\left[s.high + b.high\right] - \frac{1}{2}\left[s.low + b.low\right].$$

The return series for our three spreads were obtained from French's website. BOND and HY are two investable factors that describe investment into investor grade and high yield bonds. Similar to Fung and Hsieh (1997) we believe commodities and currency may be significant factors. However, unlike Fung and Hsieh (1997) we use the Goldman Sachs Commodity Index as a proxy rather than the price of gold. Furthermore, monthly returns on the trade-weighted US dollar index have been used to provide exposure to foreign exhange movements relative to the dollar.

$$\begin{aligned} R_{HF,t} - R_t^f &= \alpha + \beta_1\left(SP_t - R_t^f\right) + \beta_2\left(EAFE_t - R_t^f\right) \\ &+\beta_3\left(EM_t - R_t^f\right) + \beta_4 SMB_t + \beta_5 HML_t \\ &+\beta_6 UMD_t + \beta_7\left(BOND_t - R_t^f\right) + \beta_8\left(HY_t - R_t^f\right) \\ &+\beta_9\left(CMDTY_t - R_t^f\right) + \beta_{10}\left(USD_t - R_t^f\right) + \varepsilon_t. \end{aligned} \tag{4}$$

We note the risk-free rate does not apply for long/short factors. This is because the risk-free rate cancels out for the two returns that determine the spread. This is consistent with Fama and French (1993), Carhart (1997) and Capocci (2001).

3.2 Rolling Windows for Capturing Hedge Fund Beta

A simple and applicable technique to account for dynamic changes in exposure is a rolling window regression. Beta exposures at time t is determined by regressing return series from $t-\tau$ to $t-1$. The determination of the window,τ, is somewhat ad hoc. As mentioned earlier, Hasanhodzic and Lo (2007) uses 24-months lag window (τ =24), whilst Darolles and Mero (2007) prefers a 36-months lag window (τ =36). By using smaller windows, we are able to capture recent exposures more quickly, but at a cost of statistical accuracy.

The rolling regression used to determine time-varying beta exposures on K significant ABS factors is expressed as,

$$R_{HF,t-j} = \textstyle\sum_{k=1}^{K}\beta_{k,t-j}R_{k,t-j} + \varepsilon_{t-j}, \qquad \forall j = 1 ... \tau. \tag{5}$$

For our analysis, varying windows (18-month, 24-months, 30-months and 36-months) are conducted and compared. Similar to Darolles and Mero (2007), the sum of the coefficients of the regression is not restricted to 1 for better interpretation of the weights.

3.3 Kalman Filtering for Capturing Hedge Fund Beta

Rolling window regressions are unable to capture immediate changes in exposures as the estimated parameters are lagged, depending upon the length of the estimation window. Kalman filtering overcomes the issues apparent in rolling window regression. Here we present a random walk Kalman filter state space model.

Let the state equation be,

$$\beta_t = I\beta_{t-1} + \nu_t, \tag{6}$$

where is β_t a $m \times 1$ state vector, whereas m is the number of ABS factors employed. In matrix form (6) with 5 factors can be expressed as,

$$\begin{bmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \\ \beta_{4,t} \\ \beta_{5,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \\ \beta_{4,t-1} \\ \beta_{5,t-1} \end{bmatrix} + \begin{bmatrix} \nu_{1,t} \\ \nu_{2,t} \\ \nu_{3,t} \\ \nu_{4,t} \\ \nu_{5,t} \end{bmatrix}.$$

The process noise ν_t is assumed to be additive, white and Gaussian. It's covariance matrix is defined by,

$$E\left[\nu_t \nu_s^T\right] = \begin{cases} Q & t = s \\ 0 & t \neq s \end{cases},$$

where Q is a diagonalised matrix,

$$Q = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix}.$$

We note that the covariance matrix describes the dynamics of the deviations of the states from their mean.
Let the measurement equation be,

$$HF_t = F_t\beta_t + \varepsilon_t, \tag{7}$$

whereas HF_t is the hedge fund return at time t, F_t is a vector of $1 \times m$ factor returns, β_t is the state vector of $m \times 1$ factors and ε_t is the one dimensional error where,

$$\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right).$$

The Kalman filter is a recursive method, in which we predict and then update our estimated betas. The following briefly defines and describes the estimation procedure in a Kalman filter state space model.

Refer to Figure 1

Let the information set be,

$$\Omega_t = \{HF_1, \ldots, HF_{t,} F_1, \ldots, F_t\}.$$

Let the predicted state exposure at time t using the information available at time t be defined as,

$$\beta^-_{t+1|t} = E\left[\beta_{t+1}|\Omega_t\right].$$

Let the estimated state exposure at time $t+1$be defined as,

$$\beta^-_{t+1} = E\left[\beta_{t+1}|\Omega_{t+1}\right].$$

Then the priori and posteriori state exposure errors are as follows,

$$e_{t+1|t} = \beta^-_{t+1|t} - \beta_{t+1},$$

$$e_{t+1} = \beta^-_{t+1} - \beta_{t+1}.$$

The priori and posteriori error covariance are defined as,

$$V_{t+1|t} = E\left[e_{t+1|t}e^T_{t+1|t}\right],$$

$$V_{t+1} = E\left[e_{t+1}e^T_{t+1}\right].$$

The prediction for the next period is defined by the state estimate propagation and the error covariance estimate propagation,

$$\beta^-_{t+1|t} = \beta^-_t,$$

$$V_{t+1|t} = V_t + Q.$$

The estimated state exposureβ^-_{t+1}, can be expressed as a linear combination of our predicted state exposure $\beta^-_{t+1|t}$ and an adjustment of K_{t+1}, the Kalman gain, on the error of the measurement equation. The state exposure estimate update is as shown,

$$\beta_{t+1}^{-} = \beta_{t+1|t}^{-} + K_{t+1}\left(HF_{t+1} - F_{t+1}\beta_{t+1|t}^{-}\right) \tag{8}$$

whereas the Kalman gain is defined to be,

$$K_{t+1} = \frac{V_{t+1|t}F_{t+1}^{T}}{F_{t+1}V_{t+1|t}F_{t+1}^{T} + \sigma_{\epsilon}^{2}}.$$

The Kalman gain can be derived by trying to minimize posteriori error covariance V_{t+1}.

The measurement update on the error covariance is as shown,

$$V_{t+1} = \left(1 - K_{t+1}F_{t+1}\right)V_{t+1|t}. \tag{9}$$

This represents the improvement of state estimation accuracy. We estimate the parameters Q and σ_{ε}^{2} by maximum likelihood, following which we can obtain the exposures to our various factors.

As with any recursive process, one must specify the inital estimates which is defined to be,

$$\beta_{0}^{-} = E\left(\beta_{0}\right),$$

$$V_{0} = E\left[\left(\beta_{0} - E\left(\beta_{0}\right)\right)\left(\beta_{0} - E\left(\beta_{0}\right)\right)^{T}\right].$$

However, in practise it may be difficult to specify initialisation parameters, as initial exposures are not known and needs to be approximated. We can use rolling window estimates as a guide in such circumstances.

3.4 Clone construction

Construction of the clone from beta exposures estimated on m asset factors and n spreads is as follows,

$$R_{Clone,t} = \sum_{k=1}^{m}\beta_{k,t}R_{k,t} + \sum_{l=m+1}^{m+n}\beta_{l,t}R_{l,t} + \left(1 - \sum_{k=1}^{m}\beta_{l,t}\right)R_{t}^{f}. \tag{10}$$

It is assumed borrowing and lending is at the risk-free rate (USD 1 month T-bills is used as a proxy). Furthermore, it is decided that no initial capital is required to invest in long/short factors. This assumption assumes the ability to perform naked shorts which may be restricted in actual trading. By construction for long/short factors, the weight for the long component is exactly identical to the weight for the short component. The net weight is therefore zero. Supposing capital is required to invest in the spreads, clone returns would be reduced.

In order to have a fair comparison, the cloned returns need to be renormalised so that it has the same volatility as the actual hedge fund. Ideally with equal variances, we are then able to compare the returns to see how much of it was replicated. For this paper, we propose the following method to gear up or down our weights in order to achieve the same variance as the hedge fund index. By introducing a renormalisation factor K wheres,

$$K \geq 0.$$

The renormalised clone can be defined such that,

$$R_{Clone,t}^{renorm} = K\left(\sum_{k=1}^{m}\beta_{k,t}R_{k,t} + \sum_{l=m+1}^{m+n}\beta_{l,t}R_{l,t}\right) + \left(1 - K\sum_{k=1}^{m}\beta_{l,t}\right)R_t^f. \quad (11)$$

K can be solved to satisfy,

$$\sigma^2\left(R_{Clone,t}^{renorm}\right) = \sigma^2\left(R_{HF,t}\right),$$

in which the clone variance equates to the hedge fund variance.

For our comparison, we have used the period January 1993 to December 2009 to determine the respective variances. We note the in-sample nature of this estimation procedure. For example, at 1994, we would not have information set from 1995 to 2009 to estimate the value of the renormalisation factor K. In practice, out-of-sample estimates of K_t need to be conducted using a rolling window approach. However, with this approach, the variance of the clone for the sample period studied would not exactly equate to that of the clone. Therefore, for comparison purposes and not for practical implementation, we have chosen an unconditional estimate of K. From a replication perspective, the renormalisation is not a critical element, and used merely as a way of comparing returns from a similar level of variance. Whilst Hasanhodzic and Lo (2007) suggested a rolling window approach for the estimation of the renormalisation factor, Takahashi and Yamamoto (2008) and Roncalli and Teiletche (2008) did not discuss about renormalising clones. For this paper, we use it simply for ease of comparison, and as a method to illustrate the ability for us to gear the exposures of our clone, in order to change the variance accordingly.

Note: Rolling window K_t is described below.

If the length of our window was z months, our renormalised clone would be,

$$R_{Clone,t}^{renorm} = K_t\left(\sum_{k=1}^{m}\beta_{k,t}R_{k,t} + \sum_{l=m+1}^{m+n}\beta_{l,t}R_{l,t}\right) + \left(1 - K_t\sum_{k=1}^{m}\beta_{l,t}\right)R_t^f, \quad (12)$$

whereas K_t is determined through the following constraint,

$$\frac{1}{z-1}\sum_{j=1}^{z}\left(R_{Clone,t-j}^{renorm} - \bar{R}_{Clone,t}^{renorm}\right)^2 = \frac{1}{z-1}\sum_{j=1}^{z}\left(R_{HF,t-j} - \bar{R}_{HF,t}\right)^2 \quad (13)$$

whereas,

$$\bar{R}^{renorm}_{Clone,t} = \frac{1}{z}\sum_{j=1}^{z} R^{renorm}_{Clone,t-j},$$

$$\bar{R}_{HF,t} = \frac{1}{z}\sum_{j=1}^{z} R_{HF,t-j}.$$

4 Data

Monthly returns from January 1990 to December 2009 on the HFR database (Hedge Fund Research, Inc) are used for hedge fund returns. HFR indices contain over 2,000 funds and are equal-weighted and net of fees. We look into 8 broad strategies as listed in Table 1. It is noted that these indices are non-investable. Details on the definition of the strategies are discussed in the appendix A.

Refer to Table 1

Prior to conducting analysis, we shall mention briefly the biases apparent with hedge fund indices. As this paper focuses more on the replication procedures, we shall provide only a simple discussion; an interested reader may follow up on the references provided.

Firstly, survivorship bias is present when funds that have died are not included in the index because returns do not exist for the period studied. As discussed by Capocci (2001), there is a risk of overestimating mean returns, as funds that ceased to exist because of poor performance would not be taken into account. Fung and Hsieh (2000) claim survivorship bias to be at 3% pa using the Lipper TASS database. Furthermore, hedge fund managers are not legally obliged to report to databases. If returns are provided, it is solely of a voluntary basis. Many hedge funds include previously unreported performances to the database when they first start reporting, causing backfill bias. Generally only successful managers would find the incentive to report on databases to promote their achievement, this is especially so, given the legal restrictions on hedge fund advertising. According to Posthuma and van der Sluis (2003), more than 50% of all returns in the Lipper TASS database are backfilled returns. Fung and Hsieh (2000) deleted the first 12 month (since the median lag between fund inception and the date of registration into the database is 343 days – also known as the incubation period) of all fund reported returns to account for possible backfill bias, and found returns had fallen 1.4% pa. Generally various biases in hedge fund databases tend to overstate the returns of these strategies represented.

5 Results

This section presents some of the results obtained by employing the methods described in section 3 on the HFR indices.

5.1 Evaluation of Hedge Funds

We find that a large percentage of the hedge fund index could be explained in terms of a few asset based style (ABS) factors. Strategies with high exposure to equity factors are directional as they are more affected by market movements. We find these strategies (Quantitative Directional, Short Bias and Emerging Markets) tend to have higher adjusted R^2 in comparison to non-directional funds (Equity Market Neutral, Relative Value and Macro) and are easier to replicate using a factor approach.

Refer to Table 2

Regression coefficients from Fama French factors suggest hedge fund managers have a tendency to select small capitalisations and value stocks. With the case of Short Bias, its negative exposure to SMB and UMD suggests the likelihood to short small capitalisations and short momentum to be part of the strategy. On the other hand, momentum seems to be a significant positive factor for a broad range of other strategies with exception to relative value. We note the lack of significance of bond and commodity factors in most strategies, with exception to macro and emerging market funds where the coefficient suggests exceptionally high exposure. Similar to existing academic research, we find an ABS factor model is capable to explain up to 80.5% of directional hedge fund returns, however subsequently it lacks the ability to explain non-directional funds, with the adjusted R^2 for Equity Market Neutral at 32.2%.

Leveraging on these results, one is able to replicate hedge fund indices using either of the two methodologies supplied in section 4. It is noted that Relative Value, Equity Market Neutral and Macro strategy funds showed poor fit to our factor model and therefore inappropriate to replicate. Below is our list of hedge fund strategies and the factors employable.

Hedge fund strategy	ABS factors employed
Quantitative Directional	SP, EM, SMB, HML, UMD, HY
Short Bias	SP, EAFE, SMB, HML, UMD
Event Driven	SP, SMB, HML, UMD, BOND, HY
Emerging Markets	EM, UMD, HY, USD
Fund of Funds	EAFE, EM, UMD, HY, CMDTY

5.2 Factor-based Replication

We ran both rolling window regression and Kalman filtering to determine time-varying exposures of the hedge fund strategies listed above. For illustrative purposes, beta exposures using these methods for Quantitative Directional are shown.

Refer to Figure 2

With exception to Short Bias, most hedge fund strategies tend to have positive exposure to equities, which includes factors SP, EAFE and EM. Similar to Roncalli and Teiletche (2008), we note most funds tend to have decreased exposure to SP towards the end of 1997. In addition, we note net long funds tend to have a positive exposure on small capitalisations, whilst Short Bias funds tend to bet against them. Furthermore, we find notable differences with the Kalman estimates in comparison to the rolling window estimates. In many cases, the Kalman filter reacted to changes in beta exposures quicker than rolling windows. Furthermore, Kalman time-varying estimates were less volatile for traditional asset classes. For long/short factors (SMB, HML, UMD) they were dynamic, suggesting managers are more active in taking bets on the direction of size, book to market and momentum spreads. Furthermore, we note Kalman filters reacted faster than regression techniques. Whilst rolling 18 month windows reacted to changes in exposure quicker than rolling 36 month windows, estimated parameters are also more volatile. It can be argued that it may not be optimal to quickly change exposures if the underlying exposures themselves are highly unstable. On the other hand, a long window may lead to a slower reaction on what may be persistent trends.

Cloned results show that Kalman filter clones of Quantitative Directional, Short Bias, Emerging Markets and Fund of Funds, have higher returns than the rolling window approach, with the ability to clone up to 75.3%, 295.3%, 109.4% and 79.9% of actual returns respectively. Furthermore, it generated higher correlations is most cases, reaching as high as 90% for Emerging Markets. However, cloned returns for Event Driven were lower using the Kalman filter method. Whilst it produced a higher unconditional correlation of 79.8% to the hedge fund index, a rolling 18 month regression produced the highest return, replicating 69.3% of monthly returns. Results are shown in table 3.

Refer to Table 3

It is also observed that cloned return distributions are generally more negatively skewed with higher levels of kurtosis in comparison to their actual hedge fund counterparts. This suggests cloned return distributions are generally inferior to actual returns as they exhibit heavier negative tails.

Refer to Figure 3

In figure 3, we plot the rolling 12 months of actual and cloned funds using the Kalman filter method. It can be realised that the gap between actual returns and clones have narrowed across time. Cloned returns generated in the last 5 years for Quantitative Directional, Short Bias, Fund of Hedge Funds and Emerging Market tracked the actual index quite closely. Large excess returns over the clone that existed during the 90s have seemingly disappeared. One explanation on the reduction of alternative alpha is due to the 'population explosion' of hedge funds, and hence diluting returns and reducing the opportunities for arbitrage. The lack of opportunity means managers have to rely more on beta returns. As discussed in Smedts and Smedts (2006), when hedge fund markets become more competitive, only beta returns will survive. This seems to be the case as our clones have caught up to the returns of actual funds.

6 Conclusions

We have shown how selected hedge fund strategy returns can be modelled and replicated. By using either rolling windows or a Kalman filter, investors are able to gain a greater understanding into the exposures exhibited by these strategies, which would be a step forward in terms of transparency. Moreover, the factors employed by our clones are liquid, which offers potential investors an opportunity to invest in a more flexible and cheaper option to real hedge funds. One might find them useful from an asset allocation point of view: to provide temporary exposure to hedge fund returns as a bridge to finding the opportunity to invest in a particular 'top quartile' hedge fund or as a method to obtain generic return characteristics of the average hedge fund. The fact that clones have been able to replicate a significant portion of the hedge fund index is suggestive that hedge fund managers as an entity, on average, cannot justify their high fees.

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Table 1: Descriptive statistics on monthly returns of hedge fund strategies

	Reletive Value	Equity Market Neutral	Quantitative Directional	Short Bias	Emerging Markets	Event Driven	Macro	Fund of Funds
Mean	0.009	0.006	0.011	0.003	0.012	0.010	0.011	0.007
Median	0.009	0.006	0.017	-0.002	0.0165	0.013	0.008	0.008
Maximum	0.057	0.036	0.107	0.228	0.148	0.051	0.079	0.069
Minimum	-0.08	-0.029	-0.133	-0.212	-0.21	-0.089	-0.064	-0.075
Std. Dev.	0.013	0.009	0.039	0.056	0.042	0.020	0.022	0.017
Skewness	-2.216	-0.173	-0.484	0.159	-0.894	-1.374	0.450	-0.725
Kurtosis	16.847	4.218	3.607	4.929	6.622	7.288	3.820	7.014
Jarque-Bera	2113.742	16.04431	13.0653	38.23649	163.1973	259.4613	14.84115	182.1338
Probability	0	0.000328	0.001455	0	0	0	0.000599	0

Table 2: Regression on ABS factor model

Linear regression as described in section 3.1 is performed on various HFR hedge fund strategies between January 1990 to December 2009. The hedge fund strategies are namely, Relative Value (RV), Equity Market Neutral (EMN), Quantitative Directional (QUANT), Short Bias (SHORT), Emerging Markets (EMKT), Event Driven (EVENT), Global Macro (MACRO) and Fund of Funds (FOF). The factors are the S&P 500 Index (SP), MSCI EAFE Index (EAFE), MSCI Emerging Markets Index (EM), Fama and French size differential, book to market differential and momentum factors (SMB, HML & UMD), Citigroup World Government Bond Index (BOND), Barclays High Yield Index (HY), Goldman Sachs Commodities Index (CMDTY) and the Federal Reserve Trade-weighted Dollar Index (USD). Note indices are accumulation, as opposed to price.

		Relative Value Index	Equity Market Neutral Index	Quantitative Directional Index	Short Bias Index	Emerging Markets Index	Event-Driven Index	Macro Index	Fund of Funds Composite Index
Alpha		0.0035	0.0018	0.0025	0.0025	-0.0022	0.0039	0.0039	0.0001
		0.0000	0.0000	0.0400	0.2860	0.1010	0.0000	0.0020	0.9250
SP	Coefficient		0.077	0.550	-0.738		0.218		
	P-value		0.000	0.000	0.000		0.000		
EAFE	Coefficient				-0.199			0.062	0.055
	P-value				0.008			0.082	0.011
EM	Coefficient	0.036		0.119		0.469		0.119	0.108
	P-value	0.002		0.000		0.000		0.000	0.000
SMB	Coefficient	0.031	0.032	0.288	-0.328		0.153		
	P-value	0.083	0.027	0.000	0.000		0.000		
HML	Coefficient	0.039	0.060	-0.099	0.296		0.078		
	P-value	0.036	0.000	0.010	0.000		0.001		
UMD	Coefficient			0.132	-0.142	0.107	0.062	0.134	0.099
	P-value			0.000	0.002	0.000	0.000	0.000	0.000
BOND	Coefficient	-0.061					-0.095	0.237	
	P-value	0.036					0.009	0.000	
HY	Coefficient	0.222		0.137		0.341	0.289	0.149	0.155
	P-value	0.000		0.010		0.000	0.000	0.005	0.000
CMDTY	Coefficient	0.039	0.018						0.049
	P-value	0.000	0.024						0.000
USD	Coefficient					-0.191		0.239	
	P-value					0.064		0.022	
Adjusted R-Squared		**54.2%**	**32.2%**	**80.5%**	**62.3%**	**78.0%**	**70.4%**	**37.0%**	**59.5%**

Notes:
Fama and French factors are based on stocks in the NYSE, AMEX and NASDAQ

Table 3: Comparison between clones and actual hedge funds
Monthly returns of the cloned hedge funds from January 1993 to December 2009 are constructed.

	Actual hedge fund	Kalman filter clone	Rolling 18 month clone	Rolling 24 month clone	Rolling 30 month clone	Rolling 36 month clone
Quantitative Directional						
Return	0.0102	0.0077	0.0067	0.0074	0.0075	0.0073
Standard Deviation	0.0380	0.0380	0.0380	0.0380	0.0380	0.0380
Skewness	-0.5208	-0.9856	-1.0445	-1.0132	-1.1123	-1.2230
Kurtosis	0.7901	2.7563	2.4725	2.4040	2.7468	3.4687
% replicated		75.7%	65.1%	72.0%	73.6%	71.1%
Correlations		84.4%	84.7%	85.5%	82.2%	80.8%
Renormalisation factor		0.9179	0.9442	0.9618	0.9635	0.9552
Short Bias						
Return	0.0017	0.0051	0.0019	0.0017	0.0013	0.0016
Standard Deviation	0.0557	0.0557	0.0557	0.0557	0.0557	0.0557
Skewness	0.2844	0.7176	0.6079	0.7974	0.9129	0.8661
Kurtosis	2.4805	3.9844	1.9494	3.1542	3.3508	3.2306
% replicated		295.3%	112.7%	101.1%	76.3%	95.6%
Correlations		71.3%	67.6%	66.1%	66.4%	66.7%
Renormalisation factor		1.0005	0.9899	1.0331	1.0370	1.0459
Emerging Markets						
Return	0.0112	0.0122	0.0113	0.0111	0.0114	0.0115
Standard Deviation	0.0423	0.0423	0.0423	0.0423	0.0423	0.0423
Skewness	-0.9721	-1.1113	-0.9785	-1.1155	-1.3183	-1.3053
Kurtosis	4.0228	2.9293	2.4555	3.5141	4.7393	4.5357
% replicated		109.4%	101.0%	99.3%	102.0%	103.0%
Correlations		90.0%	87.3%	88.2%	88.6%	89.1%
Renormalisation factor		1.1320	1.0769	1.0729	1.0831	1.0978
Event Driven						
Return	0.0097	0.0058	0.0068	0.0066	0.0061	0.0059
Standard Deviation	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199
Skewness	-1.4014	-1.8224	-1.1648	-1.4718	-1.6872	-2.1117
Kurtosis	4.8279	7.0767	3.0283	5.3409	6.4000	9.0025
% replicated		59.7%	69.3%	68.2%	62.9%	60.3%
Correlations		79.8%	68.8%	76.3%	78.3%	78.0%
Renormalisation factor		0.9190	1.0153	1.0325	1.0366	1.0074
Fund of Funds						
Return	0.0059	0.0047	0.0040	0.0043	0.0044	0.0045
Standard Deviation	0.0179	0.0179	0.0179	0.0179	0.0179	0.0179
Skewness	-0.7192	-2.0632	-1.7755	-1.8555	-2.0728	-2.0620
Kurtosis	3.7973	10.8176	7.0701	7.9815	9.6648	10.0891
% replicated		79.9%	67.9%	73.9%	75.4%	75.9%
Correlations		78.1%	78.6%	79.4%	78.6%	78.0%
Renormalisation factor		0.8810	0.9517	0.9749	1.0194	1.0609

Figure 1: Kalman filtering

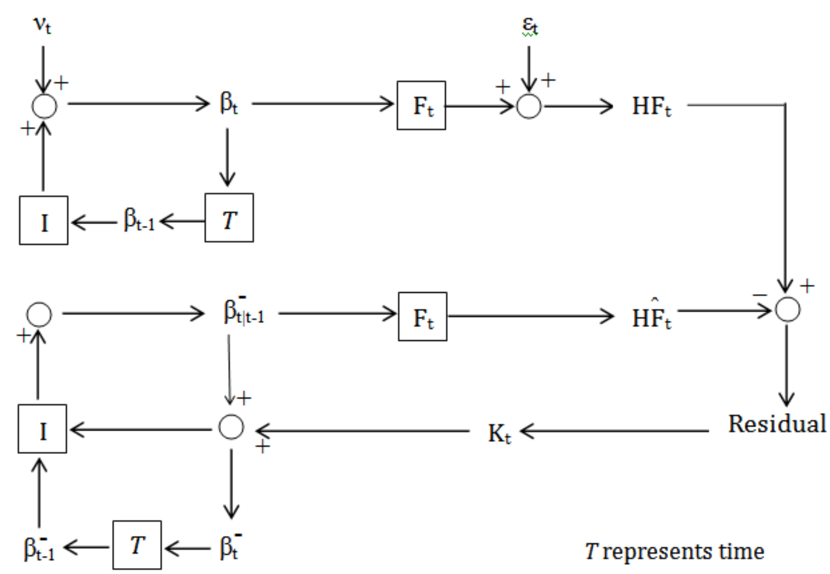

Figure 2: Quantitative Directional time-varying factor exposures
Kalman filtering and rolling regression with 18, 24, 30 and 36 month windows are used to estimate hedge fund factor exposures.

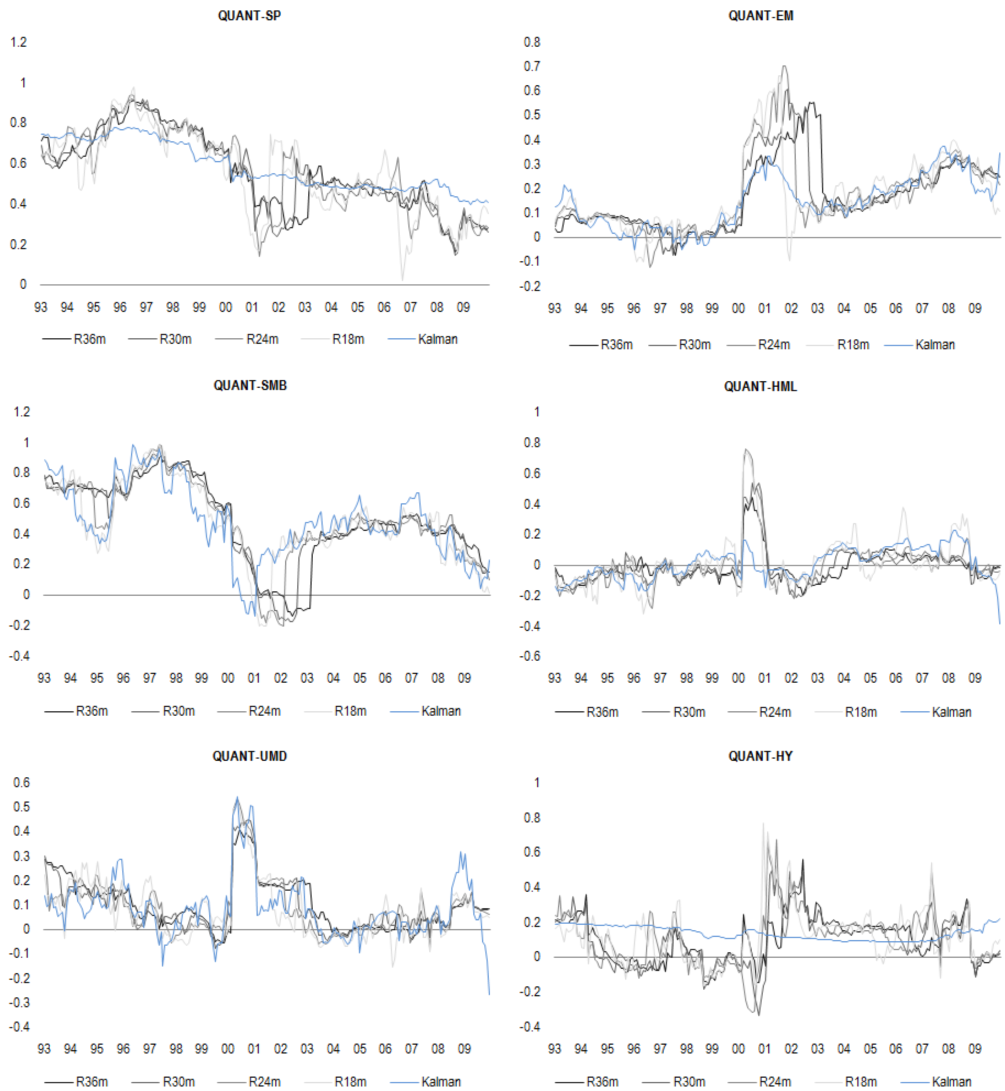

Figure 3: Rolling 12 month returns comparison between Kalman filter clones and actual hedge funds