The Effects of Uncertainty on Macroeconomic Performance: The importance of the conditional covariance model[*]

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Abstract

We study the effects of growth volatility and inflation volatility on average rates of output growth and inflation for post-war U.S. data in a multivariate asymmetric GARCH-M model. Our statistical model differs from other work in that we allow the conditional covariance of inflation and growth to be both non-diagonal and asymmetric. We show that the data reject diagonality and symmetry restrictions frequently imposed in the literature. Our results on the macroeconomic effects of uncertainty also differ from those in other recent studies using a more restrictive covariance model. Specifically, we find that increased growth uncertainty is associated with significantly higher average growth, and that higher inflation uncertainty is significantly negatively correlated with lower output growth and lower average inflation.

1. Introduction

The relationship between inflation and real activity is one of the fundamental empirical issues in macroeconomics. Recently, much attention has been focussed on relationships between uncertainty about these variables and their average outcomes, with researchers using a variety of time series models of the conditional variances to measure uncertainty. However, the great majority of empirical work is either univariate, or else uses a restrictive model of the covariance process. Univariate models by definition do not allow study of the joint determination of the two series, and popular covariance-restricted multivariate models can be subject to severe specification error (see Kroner & Ng 1998). Thus existing inferences about how inflation uncertainty and growth uncertainty affect average growth or inflation performance are possibly standing on shaky ground.

In this paper we develop and estimate a fairly general model of the conditional covariance of inflation and output growth. Using post-war US data, we test (and reject) the validity of the restrictions implied by several popular GARCH based covariance models, and further test (and fail to reject) the adequacy of our covariance structure for explaining the data. We use this structure as part of an asymmetric, multivariate GARCH-M model to test four hypotheses about the effects of these conditional variances on the conditional means of output growth and inflation.

Our main findings are (1) the conditional covariance process for inflation and growth is not diagonal. That is to say, innovations in output growth (inflation) significantly affect the conditional variance of inflation (output growth). This finding implies that univariate models of either inflation or output growth volatility are misspecified. (2) The conditional covariance process displays significant asymmetries that cannot be captured by conventional multivariate GARCH models. Positive and negative innovations of a given magnitude result in differing levels of inflation and output volatility. In particular our finding of covariance asymmetry suggests that the constant correlation specification often adopted in the literature is not a valid conditional characterisation of the data. (3) Increases in the conditional variance of inflation significantly lower both average output growth and average inflation. (4) Increases in the conditional variance of output growth significantly raise average output growth.

The paper is organized as follows. The next section describes the economic hypotheses we will test. Section 3 outlines the statistical model. The fourth section describes our data and the testing process we use to choose our covariance structure. Section 5 describes the conditional variance-covariance parameterisation used in this paper. In section 6 we report estimation results and diagnostic tests for model adequacy. Section 7 discusses the results of our hypothesis tests on the effects of uncertainty on inflation and output growth. Section 8 presents information about the quantitative effects of uncertainty in the model along with nature of the asymmetric effects of inflation and output growth shocks on uncertainty. The final section contains our conclusions and suggestions for further work.

2. The effect of uncertainty on average macro performance

In this section we present the four economic hypotheses that are tested below in the empirical section of the paper. First we consider the effect of increased output growth uncertainty on average growth. While standard business cycle models assume these factors to be independent, there are theories that imply a positive relationship and others that imply a negative relationship.[1] The literature on irreversible investment and the option value of waiting predicts a negative relationship between growth uncertainty and average growth. In these models, an increase in uncertainty about future profits raises the value of waiting, thus delaying investment and lowering growth.

However, the work of Fisher Black (1987) implies a positive relationship between growth volatility and average growth. He argues that technology choices are made from a menu of possibilities where the average rate of return and return volatility are positively correlated. In his model, technology that produces faster average growth is inherently more risky. Another argument in favor of a positive relation comes from the theory of precautionary savings, where increased risk raises desired savings and thus investment and growth.[2]

Second, we test the hypothesis that inflation uncertainty lowers output growth. As in the case of output growth volatility and average growth, standard macro models view growth as independent of the conditional variance of inflation. However, if increased inflation uncertainty also increases the risk associated with future profits, the irreversible investment literature referred to above implies that increased inflation uncertainty should delay investment and lower growth.[3]

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[1] Ramey & Ramey (1995) and Grier & Perry (2000) discuss these issues in some detail.

[2] Previous work testing this hypothesis is extremely mixed. Using cross-country data, Ramey & Ramey (1995) find a significant negative relationship between the standard deviation of growth and average growth, while Kormendi & Meguire (1985) and Grier & Tullock (1989) find a significant positive relationship. Using a univariate GARCH model on US data, Caporale & McKiernan (1998) find a positive effect, while Henry & Olekalns (2001) find a negative relation using an asymmetric univariate GARCH model. Grier & Perry (2000) find no effect in a symmetric bivariate GARCH model of inflation and output growth, and Dawson & Stephenson (1997) reach the same conclusion from an examination of state level data.

[3] The idea that inflation uncertainty can lower output growth is often attributed to Friedman (1977) and Okun (1971) who present intuitive arguments to support their views.

The third and fourth hypotheses under consideration concern the effects of inflation uncertainty and output growth uncertainty on average inflation. These effects are predicted by some recent political economy models of monetary policy. Cukierman (1992), and Cukierman & Meltzer (1986) show that if the money supply process has a stochastic element and the public is uncertain about the objective function of the policymaker, then a strategic policy maker will react to an increase in uncertainty about the supply process by raising the average level of inflation. Thus in their models, increased inflation uncertainty should raise average inflation. [4]

Deveraux's (1989) model also assumes that the Fed dislikes inflation but would like to raise output, and that there is a stochastic component to the money supply. He then shows that an exogenous increase in the unpredictability of real shocks will cause workers to lower the degree of indexing in labor contracts. To the Fed, a lower degree of indexing makes surprise inflation a more effective tool to raise output. In equilibrium, the average level of inflation will rise. Deveraux's model thus predicts that increased real uncertainty should raise average inflation.

In what follows below, we test these hypotheses in a multivariate, asymmetric GARCH-M model of inflation and output growth. The two existing papers closest to ours are Grier & Perry (2000) and Henry & Olekalns (2001). Grier & Perry test the same four hypotheses with US data, using a restricted covariance model that we show can be rejected by the data. Henry & Olekalns estimate an asymmetric univariate GARCH-M model for US output growth. This univariate approach does not allow inflation (output growth) residuals to influence the conditional variance of output growth (inflation), an assumption that is also rejected by the data. Beyond estimating

[4] On the other hand, Holland (1994) argues that if high inflation raises uncertainty and increased uncertainty harms growth, then the Fed has a motive to stabilize inflation when uncertainty rises. Grier & Perry (1998) using a univariate GARCH, two step process find that some G-7 countries exhibit a

a more general covariance process, our model gives answers that differ from those previously reported for some of the above hypotheses.

3. Econometric Model and Data Description

In section 4 below, we test these hypotheses in a single simultaneously estimated model. This section describes the data used in the empirical work, and then presents our model, explaining along the way why we think non-diagonality and asymmetry are possible outcomes in the conditional covariance matrix and how we test for their absence.

A. Data

The data used in this study are for the US, and were obtained from the FRED database at the Federal Reserve Bank of Saint Louis. The sample is monthly data over the period April 1947 to October 2000. We measure inflation, π_t, as the annualized, monthly difference of the logarithm of the producer price index. Similarly we measure output growth, y_t, as the annualized, monthly difference of the logarithm of the index of industrial production. These data are shown in Figure 1, and summary statistics for these data are presented in Table 1.

-Figure 1 about here-

- Table 1 about here -

Both output growth and inflation are positively skewed and display significant amounts of excess kurtosis with both series failing to satisfy the null hypothesis of the Bera-Jarque (1980) test for normality. A battery of augmented Dickey-Fuller unit root tests and Kwiatkowski, Phillips, Schmidt and Shin (1992) tests for stationarity suggest that both are I(0) series.

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positive relationship, some a negative relationship and in some cases, there is no significant

However a series of Ljung-Box tests for serial correlation suggests that there is a significant amount of serial dependence in the data. Similarly a Ljung-Box test for serial correlation in the squared data provides strong evidence of conditional heteroscedasticity in the data. Visual inspection of the time series plots of the data in Figure 1 would tend to support the view that the variances of output growth and inflation are not constant.

B. Statistical Model

Equation 1 gives the specification we use for the means of inflation (π_t) and output growth (y_t). It is a VARMA (vector autoregressive moving average), GARCH in Mean model, where the conditional standard deviations of output growth and inflation are included as explanatory variables in each equation

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\begin{aligned}
&Y_t = \mu + \sum_{i=1}^{p} \Gamma_i Y_{t-i} + \Psi \sqrt{h_t} + \sum_{j=1}^{q} \Theta_j \varepsilon_{t-j} + \varepsilon_t \\
&\varepsilon_t \sim \left(0, H_t\right) \\
&H_t = \begin{bmatrix} h_{y,t} & h_{y\pi,t} \\ h_{y\pi,t} & h_{\pi,t} \end{bmatrix}
\end{aligned} \tag{1}
$$

Where $Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}; \varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{bmatrix}; \sqrt{\mathrm{h}_\mathrm{t}} = \begin{bmatrix} \sqrt{h_{y,t}} \\ \sqrt{h_{\pi,t}} \end{bmatrix}; \mu = \begin{bmatrix} \mu_y \\ \mu_\pi \end{bmatrix};\ \Gamma_i = \begin{bmatrix} \Gamma_{11}^{(i)} & \Gamma_{12}^{(i)} \\ \Gamma_{21}^{(i)} & \Gamma_{22}^{(i)} \end{bmatrix};$

$\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}$ and $\Theta_j = \begin{bmatrix} \theta_{11}^{(j)} & \theta_{12}^{(j)} \\ \theta_{21}^{(j)} & \theta_{22}^{(j)} \end{bmatrix}$.[5]

relationship.

[5] We choose the values of p and q that minimize the Akai and Swartz information criteria. In the results below p=q=2.

Under the assumption $\varepsilon_t \mid \Omega_t \sim (0, H_t)$, where Ω_t represents the information set available at time *t*, the model may be estimated using Maximum Likelihood methods, subject to the requirement that H_t, the conditional covariance matrix, be positive definite for all values of ε_t in the sample.

The difficulty of checking, let alone imposing, such a restriction led Engle and Kroner (1995) to propose the following parameterisation, usually referred to as the *BEKK* model:

$$H_t = C_0^{*'} C_0^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* \tag{2}$$

The *BEKK* parameterisation requires estimation of only 11 free parameters in the conditional covariance structure and guarantees H_t positive definite.

Our covariance model allows for the innovations of inflation and output growth to have both non-diagonal and asymmetric effects on the conditional variances of each series and the conditional covariance. The model nests simpler diagonal and symmetric models and we can provide a statistical test of their appropriateness. While the answer to the question of what covariance model is correct is ultimately empirical, there are good reasons to expect that the inflation – growth conditional covariance may be characterized by non-diagonality and asymmetry.

A non-diagonal covariance matrix simply means that shocks to inflation (growth) affect the future predictability of growth (inflation). Since the means of these series are frequently modeled as co-determined, both theoretically and empirically, it stands to reason that their conditional covariances might be codetermined as well.

The BEKK model does allow for non-diagonality, however it is commonly imposed on the model by assuming that, $\alpha_{ij}^* = \beta_{ij}^*$=0 for *i,j=1,2* and *i*≠j in equation 2 above. Some popular multivariate covariance models also impose further restrictions on the diagonal model (e.g. the constant correlation model of Bollerlsev (1990) or the Factor GARCH model). If the diagonal covariance restriction is invalid, imposing it on the data creates a potentially serious specification error.[6]

The idea that the covariance matrix may be asymmetric is not new. The EGARCH (Nelson 1991) GJR (Glosten, Jagannathan, & Runkle 1993) models both allow the sign of the lagged innovation as well as its size to affect uncertainty. It is a common finding in finance that "bad news" about stock returns (which is to say, a negative residual from the forecasting equation) raises the conditional variance by more than do equal sized positive residuals. Further, Henry & Olekalns (2001) find the conditional variance of output growth to be significantly asymmetric in a univariate context. Our empirical work allows for the possibility that macroeconomic "bad news" has asymmetric effects on average performance.

If inflation is higher than expected, we take that to be bad news. In this case, the inflation residual will be positive. By contrast if output growth is lower than expected, we consider that to be bad news. Thus bad news about output growth is captured by a negative residual. We therefore define $\xi_{y,t}$ as $\min\{\varepsilon_{y,t},0\}$ which is to say the negative innovations, or bad news about growth. Similarly let $\xi_{\pi,t}$ be the $\max\{\varepsilon_{2,t},0\}$ (i.e. the positive inflation residuals), thus capturing bad news about inflation.[7] We allow for asymmetric responses in the BEKK model in (2) using

$$H_t = C_0^{*'}C_0^{*} + A_{11}^{*'}\varepsilon_{t-1}\varepsilon_{t-1}^{'}A_{11}^{*} + B_{11}^{*'}H_{t-1}B_{11}^{*} + D_{11}^{*'}\xi_{t-1}\xi_{t-1}^{'}D_{11}^{*} \qquad (3)$$

6 Kroner & Ng (1998) review the properties of many widely used multivariate GARCH models.

7 As a preliminary test, we subject each of the two series to an Engle & Ng (1993) test for asymmetry in volatility, finding that output growth does exhibit negative sign and size bias while inflation exhibits positive size bias. Thus there is initial indicative evidence that allowing for asymmetry may be important and that macroeconomic "bad news" matters more than good news.

where $C_0^* = \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix}; A_{11}^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{21}^* & \alpha_{22}^* \end{bmatrix}; B_{11}^* = \begin{bmatrix} \beta_{11}^* & \beta_{12}^* \\ \beta_{21}^* & \beta_{22}^* \end{bmatrix}; D_{11}^* = \begin{bmatrix} \delta_{11}^* & \delta_{12}^* \\ \delta_{21}^* & \delta_{22}^* \end{bmatrix}$

and $\xi_t^2 = \begin{bmatrix} \xi_{y,t}^2 \\ \xi_{\pi,t}^2 \end{bmatrix}$.[8]

The symmetric *BEKK* model (5) is a special case of (6) for $\delta_{ij} = 0$, for all values of i and j. Just as in the case of diagonality, the symmetry restriction should be tested rather than imposed, because the invalid imposition of the restriction creates a potentially serious specification error.

Non-diagonality and asymmetry can occur separately, together and also synergistically, which is to say that the innovations of inflation (output growth) could enter the conditional variance of output growth (inflation) in an asymmetric fashion.[9]

4. Results

Table 2 reports parameter estimates for the full model given by equations (1) and (3) above. Preliminary results suggest that the assumption of normally distributed

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[8] Brooks and Henry (2000) and Brooks Henry and Persand (2001) have used this model.

[9] Kroner and Ng (1998) identify three possible forms of asymmetric behaviour. Firstly, the covariance matrix displays *own variance asymmetry* if $h_{y,t}\left(h_{\pi,t}\right)$, the conditional variance of $y_t\left(\pi_t\right)$, is affected by the sign of the innovation in $y_t\left(\pi_t\right)$. Secondly, the covariance matrix displays *cross variance asymmetry* if the conditional variance of $y_t\left(\pi_t\right)$ is affected by the sign of the innovation in $\pi_t(y_t)$. Finally if the conditional covariance, $h_{y\pi,t}$ is sensitive to the sign of the innovation for either variable then the model is said to display *covariance asymmetry*. It is only through a multivariate approach that the full range of potential asymmetries can be examined.

standardised innovations, $z_{j,t} = \varepsilon_{j,t} / \sqrt{h_{j,t}}$, for $j = y, \pi$, may be tenuous. We thus follow Weiss (1986) and Bollerslev and Wooldridge (1992) who argue that asymptotically valid inference regarding normal quasi-maximum likelihood estimates may be based upon robustified versions of the standard test statistics.[10]

- Table 2 about here –

A. Specification tests

Before discussing the results of our macroeconomic hypothesis tests, we consider tests on the form of the conditional covariance and the adequacy of the specification. First, there is significant conditional heteroskedasticity in these data. Homoskedasticity requires the A_{11}^{*}, B_{11}^{*} and D_{11}^{*} coefficient matrices to be jointly insigificant, and they are actually jointly and individually significant at the 0.01 level.

Second, the hypothesis of a diagonal covariance process requires the off-diagonal elements of the same three coefficient matrices to be jointly insignificant and these estimated coefficients are jointly significant at the 0.05 level or better. To be more specific, the insignificance of the non-diagonal coefficients in the A_{11}^{*} matrix indicates that allowing for non-diagonality does not increase the persistence of the conditional variances. However, the significance of the analogous coefficients in the B_{11}^{*} and D_{11}^{*} matrices, shows that the lagged squared innovations in each series do impact the conditional variance of the other series in some manner.

Third, the hypothesis of a symmetric covariance process requires the coefficient matrix D_{11}^{*} to be insignificant, and in our model all elements save δ_{12}^{*} are

[10] Maximum likelihood estimation assuming a conditional Students-t distribution was also performed. The results were qualitatively unchanged and are not reported to conserve space. Details are available from the second author upon request.

individually significant, and the overall coefficient matrix is significant, at the 0.01 level. In particular, the significance of α_{22}^{*} coupled with the significance of δ_{22}^{*} indicates that inflation displays own variance asymmetry, implying that, *ceteris paribus*, a positive inflation innovation leads to more inflation volatility than a negative innovation of equal magnitude. In a similar manner, the fact that both α_{11}^{*} and δ_{11}^{*} are significant suggests that, *ceteris paribus*, the response of output growth displays own variance asymmetry, negative growth shocks raise uncertainty more than positive shocks.

The non-diagonal nature of our covariance structure allows the possibility of cross-variance asymmetry, where innovations to inflation (growth) affect the conditional variance of growth (inflation) asymmetrically. In our model the significance of α_{12}^{*} coupled with the insignificance of δ_{12}^{*} implies an absence of cross variance asymmetry between inflation uncertainty to growth uncertainty. However the converse is true for the spillovers of growth uncertainty on inflation uncertainty; the insignificance of α_{12}^{*} coupled with the significance of δ_{21}^{*} indicates cross variance asymmetry. Bad news about growth will tend to raise inflation uncertainty.

In sum, for these US postwar data, the inflation – output growth process thus is strongly conditionally heterskedastic, innovations to inflation (output growth) significantly influcence the conditional variance of output growth (inflation) and the sign as well of the size of both inflation and growth innovations are important.

Overall, the model appears well specified. The standardised residuals, $z_{jt} = \varepsilon_{jt} / \sqrt{h_{jt}}$ for $j = y, \pi$, and their corresponding squares, satisfy the null of no fourth order linear dependence of the *Q(4)* and *Q^2(4)* tests. Similarly there is no evidence, at the 5% level, of twelfth order serial dependence in $z_{y,t}$ and $z_{y,t}^2$.[11] We also subject the standardized residuals to a series of tests based on moment conditions. In a well-specified model $E(z_{it}) = 0$ and $E(z_{it}^2) = 1$. These conditions are supported at any level of significance. The model also significantly reduces the degree of skewness and kurtosis in the standardised residuals when compared with the raw data. Similarly the model predicts that $E\left(\varepsilon_{i,t}^2\right) = h_{i,t}$ for $i = y, \pi$ and $E\left(\varepsilon_{y,t}\varepsilon_{\pi,t}\right) = h_{y\pi,t}$. These conditions are not rejected by the data at the 0.05 level

Appendix A reports the results of applying a battery of specification tests to variance covariance structure of the estimated model which we summarize here by noting that on balance, the estimated multivariate asymmetric GARCH-M model appears to provide a very reasonable characterisation of the data.

- Figure 2 about here -

In Figure 2, we plot the respective conditional variances for the rates of inflation and output growth, as well as the conditional covariance, implied by our estimates. For output growth, volatility appears highest, on average, during the 1950s. The well-documented decline in output growth volatility over the 1990s is also apparent in these data. For inflation, the period of greatest volatility occurs in the mid-1970s, with the most benign volatility outcomes coming during the 1960s and mid 1990s.

[11] On the basis of *Q^2(12)* though, there is some evidence of twelfth order dependence in the squared

B. Hypotheses tests

The hypotheses we presented in section 2 imply that the conditional variances of inflation or output growth significantly affect the evolution of average inflation or output growth. The relevant coefficients for testing these hypotheses are found in the estimated Ψ matrix of GARCH-M effects in Table 2. The first hypothesis, whether increased conditional volatility of output growth lowers or raises average growth, concerns the sign and significance of ψ_{11}, the upper left element of the coefficient matrix. This coefficient is positive and significant at all usual confidence levels with an asymptotic t-statistic of around 13.0. We thus find strong evidence in favor of the correlation implied by Fisher Black's ideas about technological adoption or the effects of uncertainty on optimal saving. The hypothesis that increased output volatility lowers growth is clearly rejected in these data.

The second macroeconomic hypothesis, whether inflation uncertainty lowers output growth or not, is tested by the sign and significance of ψ_{12} the second element in the first row of the coefficient matrix. This coefficient is negative and again significant at all usual levels with a t-statistic of over 20.0. We thus find confirmation of Friedman's and Okun's informal arguments about the pernicious real effects of inflation uncertainty.[12]

The relevant coefficient for testing the hypothesis that the Fed reacts to increased inflation uncertainty by raising the average inflation rate as in Cukierman & Meltzer is ψ_{22}, the lower right hand element of the of Ψ matrix. This coefficient is

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standardised residuals of inflation

[12] Given these first two results, it is difficult to evaluate the hypothesis that uncertainty delays investment and thus growth. Either inflation uncertainty is relevant for investment decisions and real uncertainty is not, or both inflation uncertainty and real uncertainty depress investment, but in the case of real uncertainty, the Black effect or the precautionary savings effect outweighs the delayed investment effect. Another possibility is that uncertainty of the type measured here does not delay investment in either case.

negative and significant at the 0.01 level, indicating that higher inflation uncertainty is associated with lower, rather than higher, average inflation.[13] The fourth hypothesis, that increased growth uncertainty raises average inflation receives no support from the data as can be seen from the negative, but small and only marginally significant coefficient of ψ_{21}, the first element in the second row of the of the Ψ coefficient matrix.[14]

C. On the quantitative importance of uncertainty

The tests presented above establish the statistical significance of inflation and growth uncertainty for explaining the behavior of average inflation and growth, and explain which theoretical models receive statistical support. Now we proceed to give an idea of the quantitative importance of inflation and growth uncertainty. Given the generality of our covariance matrix and the non-linearity and feedback that exists in the model, the task is not a simple one. Appendix B derives the individual equations for the conditional variances of each series and the conditional covariance.

Figure 3 reports the effect of a 5 unit negative (bad news) growth residual in our model under the following simplifying assumptions. (1) All other shocks are set to zero, (2) insignificant coefficients in the model are set to zero, and (3) the MA terms in the mean equations are set to zero. The top half of the figure illustrates the strong effect of negative growth shocks on inflation uncertainty we find in the data, in that

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[13] In a series of univariate models for each of the G7 countries, Grier & Perry (1998) find the same result. They argue that if higher inflation raises uncertainty, a stabilizing Fed would react to increased uncertainty by lowering inflation. They found a similar result for the UK and Germany, and found results consistent with the models of Cukierman and Meltzer for Japan and France. Holland (1995) also finds that increased inflation uncertainty lowers average inflation in US data, using a survey based uncertainty measure.

[14] To see the importance of allowing for non-diagonal and asymmetric responses of uncertainty to innovations, it is instructive to compare the above results with those in Grier & Perry (2000) who investigate similar hypotheses using a bivariate GARCH-M model with diagonality and symmetry restrictions. They too find that higher inflation uncertainty lowers growth, but the rest of their GARCH-M coefficients are insignificant. By relaxing their restrictions we find strong support for the hypothesis that real uncertainty and average growth are positively correlated and that inflation uncertainty and average inflation are negatively correlated.

the rise of the conditional variance of inflation from the growth shock is almost as large proportionately as the rise in the conditional variance of growth. Given that a higher inflation variance lowers growth, but a higher growth variance raises it, and the growth shock substantially raises both variances, the net effect of the bad growth news on growth is small. As shown in the bottom half of the figure, the growth rate fall initially from 3.6 to 3.5, after nine months the growth rate is slightly higher than average, and from that point the effect disappears slowly. The inflation rate is hardly affected by the shock, falling only a very small amount and slowly returning to average afterward.

Figure 4 shows the effects of a 5 unit positive inflation residual in our model under the same three assumptions listed above. As can be seen in the top half of the figure, the bad inflation news causes a large (tiny) jump in the conditional variance of inflation (output growth) which decays slowly as these estimated conditional variances are somewhat persistent. These movements in the variances cause the movements in output growth and inflation depicted in the bottom half of the figure. Growth falls from its sample average of around 3.6 down to 3. This .5 percentage point drop is a 14% decline in the growth rate. The effect disappears within 15 months. The inflation rate falls slightly from its mean of 3 down to 2.9 (only a 3.5% decline). The effect disappears very slowly because the inflation process is very persistent.

It is important to note that while the size of these individual shocks may seem large; they do not increase the conditional standard deviations by a large amount relative to its sample movements. That is to say, even though it may appear that a big *inflation shock* has a relatively small effect on output growth, it is not correct to infer that *inflation uncertainty* has a relatively small effect. In the second experiment above the conditional standard deviation of inflation increased by roughly 1 unit (from 2.34 to 3.3), which is a small move given its sample variability. A swing of 7 or 8 units is not at all uncommon (see Figure 2 above). Swings of this type would have large effects on the means of inflation and growth, but they cannot be generated from a single isolated shock. In our model, the inflation residual, the growth residual, and the cross product of the residuals each raise the conditional variances of inflation and output growth. Thus while the effect of a single isolated shock is relatively weak, uncertainty is derived from multiple shocks, and the overall effect of uncertainty is large.

Consider an unspecified combination of shocks at a single point in time that raises the conditional standard deviation of inflation by 8 units. Using the same assumptions employed in the examples above, growth would fall by over 3 percentage points in 6 months, and after 20 months, the growth rate would still be depressed by around 1 percentage point.

D. News Impact Surfaces

The effect of innovations on the conditional variances is increasingly complex when we allow shocks to inflation and growth of different magnitudes, and the joint arrival of shocks. Therefore, we simulate the model over a range of different shocks to assess the relative importance of these asymmetries. Following Kroner and Ng (1998), we treat innovations as a collective measure of news arriving between the end of period *t-1* and the end of period *t*, and define the relationship between such innovations and the conditional variance-covariance structure as the news impact surface (a multivariate form of the news impact curve of Engle and Ng (1993)).

In the figures that follow, we provide news impact surfaces, evaluated in the region $\varepsilon_{j,t} = [-5,5] \quad j = y, \pi$ (following Engle and Ng 1993 and Kroner and Ng

1998). While the absence of extreme outliers in the data suggests that some caution should be exercised in interpreting the news impact surfaces for extreme values of $\varepsilon_{j,t}$, the asymmetry in variance and covariance is clear from each figure.

Figures 5, 6 & 7 about here

5. Conclusions

This results in paper imply that virtually all existing ARCH or GARCH models of inflation or output growth are misspecified and therefore are suspect with regard to their inferences. We have shown that for the United States, the conditional volatilities of inflation and output growth exhibit significant non-diagonality and asymmetry with respect to the impact of lagged innovations. Volatility in one series spills over into volatility in the other, and the size and sign of the innovation (our distinction between good and bad news) has a differential impact upon the estimated conditional variance-covariance matrix.

Having found an adequate conditional characterization of the data, we then test 4 recent macroeconomic hypotheses about the effects of inflation and output growth uncertainty on the conditional means of the two series. We find strong evidence in favor of the propositions that increased growth uncertainty is associated with a higher average rate of growth and that increased inflation uncertainty is associated with lower average growth rates. Contrary to the prediction that higher inflation uncertainty induces policymakers to raise the average inflation rate, we find that higher inflation uncertainty is associated with lower average inflation rates. We find no evidence that increased growth uncertainty increases the average rate of inflation.

Appendix A: further specification testing

Define the generalised residual $v_{y\pi,t} = \varepsilon_{y,t}\varepsilon_{\pi,t} - h_{y\pi,t}$. The distance between the covariance news impact surface and the realised data is measured by $v_{y\pi,t}$. Likewise define $v_{y,t} = \varepsilon^2_{y,t} - h_{y,t}$ and $v_{\pi,t} = \varepsilon^2_{\pi,t} - h_{\pi,t}$ to measure the distances between the variance news impact surfaces and the realised data. Kroner and Ng (1998) suggest the use of indicator variables to detect misspecification of the conditional variance covariance matrix. Three types of misspecification may be detected using this method. Firstly, bias due to the sign of innovations is examined using the indicators m_1, which identify negative innovations. Secondly, four quadrants $\left(\varepsilon_{y,t-1} < 0,\ \varepsilon_{\pi,t-1} < 0\right)$, $\left(\varepsilon_{y,t-1} > 0,\ \varepsilon_{\pi,t-1} < 0\right)$, $\left(\varepsilon_{y,t-1} < 0,\ \varepsilon_{\pi,t-1} > 0\right)$ and $\left(\varepsilon_{y,t-1} > 0,\ \varepsilon_{\pi,t-1} > 0\right)$ may be defined for the innovations. Indicator variables, labeled m_2, may be used to detect quadrant bias. Finally a set of indicators, labeled m_3, may be used to detect sensitivity to the sign and size of the innovations to the variables in the state vector, Y_t .

Table A1 presents the results of the robust conditional moment tests proposed by Kroner and Ng (1998). In the main, the model is well specified. Only two of the thirty generalised residual test statistics are significant at the 5% level. The indicator $m_3^{\pi,y}$, used to detect bias to the magnitude of $\varepsilon_{y,t-1}$ when $\varepsilon_{\pi,t} < 0$ is significant for $v_{y,t}$. Similarly for the conditional variance of inflation only the indicator m_1^{y} is significant indicating some bias to forecasts of inflation volatility when growth innovations are negative. The conditional covariance equations display no evidence of quadrant and size/sign misspecification.

-Table A1 about here-

Appendix B: The impact of news on the variance-covariance matrix

The conditional variance-covariance structure may be written as:

$$H_t = \begin{bmatrix} h_{y,t} & h_{y\pi,t} \\ h_{y\pi,t} & h_{\pi,t} \end{bmatrix}$$

And

$$H_t = C_0^{*'} C_0^{*} + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^{*} + B_{11}^{*'} H_{t-1} B_{11}^{*} + D_{11}^{*'} \xi_{t-1} \xi_{t-1}^{'} D_{11}^{*} \qquad \text{(A.1)}$$

Where $C_0^{*} = \begin{bmatrix} c_{11}^{*} & c_{12}^{*} \\ 0 & c_{22}^{*} \end{bmatrix}; A_{11}^{*} = \begin{bmatrix} \alpha_{11}^{*} & \alpha_{12}^{*} \\ \alpha_{21}^{*} & \alpha_{22}^{*} \end{bmatrix}; B_{11}^{*} = \begin{bmatrix} \beta_{11}^{*} & \beta_{12}^{*} \\ \beta_{21}^{*} & \beta_{22}^{*} \end{bmatrix}; D_{11}^{*} = \begin{bmatrix} \delta_{11}^{*} & \delta_{12}^{*} \\ \delta_{21}^{*} & \delta_{22}^{*} \end{bmatrix}$

Following Engle and Ng (1993) and Ng and Kroner (1998) we hold information at time t-1 and before constant, and evaluate the lagged elements of the conditional variance – covariance matrix at their corresponding unconditional levels, for example $h_{y,t} = \sigma_y^2$. The focus is therefore on the impact at time t of an innovation at time t-1.

1. Impact of News on Output Growth Volatility

Expanding A(1) for the 1,1 element of H_t yields

$$\begin{aligned} h_{y,t} &= c_{11}^{*\,2} + \alpha_{11}^{*\,2} \varepsilon_{y,t-1}^2 + 2\alpha_{11}^{*} \alpha_{12}^{*} \varepsilon_{y,t-1} \varepsilon_{\pi,t-1} + \alpha_{21}^{*\,2} \varepsilon_{\pi,t-1}^2 \\ &+ \beta_{11}^{*\,2} h_{y,t-1} + 2\beta_{11}^{*} \beta_{12}^{*} h_{y\pi,t-1} + \beta_{21}^{*\,2} h_{\pi,t-1} \\ &+ \delta_{11}^{*\,2} \xi_{y,t-1}^2 + 2\delta_{11}^{*} \delta_{12}^{*} \xi_{y,t-1} \xi_{\pi,t-1} + \delta_{21}^{*\,2} \xi_{\pi,t-1}^2 \end{aligned} \qquad \text{(A.2)}$$

The impact of bad news about growth in period t-1 on growth volatility in period t, $h_{y,t}$, is given by

$$\frac{\partial h_{y,t}}{\partial \varepsilon_{y,t-1}} = 2\alpha_{11}^{*\,2}\varepsilon_{y,t-1} + 2\alpha_{11}^{*}\alpha_{12}^{*}\varepsilon_{\pi,t-1} + 2\delta_{11}^{*\,2}\xi_{y,t-1} + 2\alpha_{11}^{*}\alpha_{12}^{*}\xi_{\pi,t-1}$$

If there is no news about inflation this reduces to:

$$\frac{\partial h_{y,t}}{\partial \varepsilon_{y,t-1}}\Big|_{\varepsilon_{\pi,t-1}=0} = 2\alpha_{11}^{*\,2}\varepsilon_{y,t-1} + 2\delta_{11}^{*\,2}\xi_{y,t-1} \qquad \text{(A.4)}$$

Where there is "good news" about growth, $\varepsilon_{y,t} > 0$, and $\xi_{y,t} = 0$, the impact on output volatility is:

$$\frac{\partial h_{y,t}}{\partial \varepsilon_{y,t-1}}\Big|_{\varepsilon_{\pi,t-1}=0} = 2\alpha_{11}^{*\,2}\varepsilon_{y,t-1} \qquad \text{(A.5)}$$

Thus for a unit shock to growth the impact of a negative innovation exceeds the impact of good news by the quantity $2\delta_{11}^{*\,2}$.

Similarly the period t impact of bad news about inflation in period t-1 on output volatility is given by

$$\frac{\partial h_{y,t}}{\partial \varepsilon_{\pi,t-1}} = 2\alpha_{11}^{*}\alpha_{12}^{*}\varepsilon_{y,t-1} + 2\alpha_{21}^{*}\varepsilon_{\pi,t-1} + 2\delta_{21}^{*\,2}\xi_{\pi,t-1} + 2\alpha_{11}^{*}\alpha_{12}^{*}\xi_{y,t-1}$$
(A.6)

Again, for simplicity, make the usual ceteris paribus assumption, that is $\varepsilon_{y,t} = 0$ and the impact of bad news in period t-1 about inflation in period t is

$$\frac{\partial h_{y,t}}{\partial \varepsilon_{\pi,t-1}}\Big|_{\varepsilon_{y,t-1}=0} = 2\alpha_{21}^{*\,2}\varepsilon_{\pi,t-1} + 2\delta_{21}^{*\,2}\xi_{\pi,t-1} \qquad \text{(A.7)}$$

For shocks to inflation and output growth of equal magnitude arriving in period t-1, bad news about inflation will therefore generate higher output growth volatility in period t if ${\alpha_{21}^{*}}^{2} + {\delta_{21}^{*}}^{2} > {\alpha_{11}^{*}}^{2} + {\delta_{11}^{*}}^{2}$.

2. Impact of news on inflation volatility

Expanding A(1) for the 2,2 element of H_t yields

$$\begin{aligned} h_{\pi,t} &= {c_{12}^{*}}^{2} + {c_{22}^{*}}^{2} + {\alpha_{12}^{*}}^{2}\varepsilon_{y,t-1}^{2} + 2\alpha_{12}^{*}\alpha_{22}^{*}\varepsilon_{y,t-1}\varepsilon_{\pi,t-1} + {\alpha_{22}^{*}}^{2}\varepsilon_{\pi,t-1}^{2} \\ &+ {\beta_{12}^{*}}^{2} h_{y,t-1} + 2\beta_{12}^{*}\beta_{22}^{*} h_{y\pi,t-1} + {\beta_{22}^{*}}^{2} h_{\pi,t-1} \\ &+ {\delta_{12}^{*}}^{2}\xi_{y,t-1}^{2} + 2\delta_{12}^{*}\delta_{12}^{*}\xi_{y,t-1}\xi_{\pi,t-1} + {\delta_{22}^{*}}^{2}\xi_{\pi,t-1}^{2} \end{aligned} \tag{A.8}$$

The impact of bad news about inflation in period t-1 on inflation volatility in period t, $h_{\pi,t}$, is given by

$$\frac{\partial h_{\pi,t}}{\partial \varepsilon_{\pi,t-1}} = 2\alpha_{12}^{*}\alpha_{22}^{*}\varepsilon_{y,t-1} + 2{\alpha_{22}^{*}}^{2}\varepsilon_{\pi,t-1} + 2\delta_{12}^{*}\delta_{22}^{*}\xi_{y,t-1} + 2{\delta_{22}^{*}}^{2}\xi_{\pi,t-1} \tag{A.9}$$

Again, imposing the ceteris paribus assumption, then the differential impact of bad news about inflation, $\varepsilon_{\pi,t-1} > 0$ is $2{\delta_{22}^{*}}^{2}$. The impact of bad news about growth in period t-1 on period t inflation volatility is given by

$$\frac{\partial h_{\pi,t}}{\partial \varepsilon_{y,t-1}} = 2{\alpha_{12}^{*}}^{2}\varepsilon_{y,t-1} + 2\alpha_{12}^{*}\alpha_{22}^{*}\varepsilon_{\pi,t-1} + 2\delta_{12}^{*}\delta_{22}^{*}\xi_{\pi,t-1} + 2{\delta_{12}^{*}}^{2}\xi_{y,t-1} \tag{A.10}$$

For a unit shock to growth the impact of a negative innovation exceeds the impact of good news by the quantity $2{\delta_{12}^{*}}^{2}$. For shocks to inflation and output growth of equal magnitude, bad news about inflation will therefore generate higher levels of inflation volatility if ${\alpha_{22}^{*}}^{2}+{\delta_{22}^{*}}^{2}>{\alpha_{12}^{*}}^{2}+{\delta_{12}^{*}}^{2}$.

3. Impact of News on Inflation-Output Growth Covariance

Expanding A(1) for the 1,2 or 2,1 element of H_t yields

$$
\begin{aligned}
h_{y\pi,t} &= c_{11}^{*}c_{12}^{*}+\alpha_{11}^{*}\alpha_{12}^{*}\varepsilon_{y,t-1}^{2}+\left(\alpha_{21}^{*}\alpha_{12}^{*}+\alpha_{11}^{*}\alpha_{22}^{*}\right)\varepsilon_{y,t-1}\varepsilon_{\pi,t-1}+\alpha_{21}^{*}\alpha_{22}^{*}\varepsilon_{\pi,t-1}^{2}\\
&+\beta_{11}^{*}\beta_{12}^{*}h_{y,t-1}+\left(\beta_{21}^{*}\beta_{12}^{*}+\beta_{11}^{*}\beta_{22}^{*}\right)h_{y\pi,t-1}+\beta_{21}^{*}\beta_{22}^{*}h_{\pi,t-1}\\
&+\delta_{11}^{*}\delta_{12}^{*}\xi_{y,t-1}^{2}+\left(\delta_{21}^{*}\delta_{12}^{*}+\delta_{11}^{*}\delta_{22}^{*}\right)\xi_{y,t-1}\xi_{\pi,t-1}+\delta_{21}^{*}\delta_{22}^{*}\xi_{\pi,t-1}^{2}
\end{aligned}
\tag{A.11}
$$

The impact of bad news about growth in period t-1 on the growth-inflation covariance in period t, $h_{y\pi,t}$, is given by

$$
\begin{aligned}
\frac{\partial h_{y\pi,t}}{\partial \varepsilon_{y,t-1}} &= 2\alpha_{11}^{*}\alpha_{12}^{*}\varepsilon_{y,t-1}+\left(\alpha_{21}^{*}\alpha_{12}^{*}+\alpha_{11}^{*}\alpha_{22}^{*}\right)\varepsilon_{\pi,t-1}\\
&+2\delta_{11}^{*}\delta_{12}^{*}\xi_{y,t-1}+\left(\delta_{21}^{*}\delta_{12}^{*}+\delta_{11}^{*}\delta_{22}^{*}\right)\xi_{\pi,t-1}
\end{aligned}
\tag{A.12}
$$

Enforcing the ceteris paribus assumption shows that the differential impact of a negative shock to output growth in period t-1 on $h_{y\pi,t}$, over a positive shock of equal magnitude is $2\delta_{11}^{*}\delta_{12}^{*}$. Similarly the impact of news in period t-1 about inflation on the inflation growth covariance is

$$
\begin{aligned}
\frac{\partial h_{y\pi,t}}{\partial \varepsilon_{\pi,t-1}} &= \left(\alpha_{21}^{*}\alpha_{12}^{*}+\alpha_{11}^{*}\alpha_{22}^{*}\right)\varepsilon_{y,t-1}+2\alpha_{21}^{*}\alpha_{22}^{*}\varepsilon_{\pi,t-1}\\
&+\left(\delta_{21}^{*}\delta_{12}^{*}+\delta_{11}^{*}\delta_{22}^{*}\right)\xi_{y,t-1}+2\delta_{21}^{*}\delta_{22}^{*}\xi_{\pi,t-1}
\end{aligned}
\tag{A.13}
$$

The differential impact of a positive shock to inflation in period t-1 on $h_{y\pi,t}$ over a negative shock of equal magnitude is $2\delta^*_{21}\delta^*_{22}$. For shocks to inflation and output growth of equal magnitude, bad news about inflation will therefore matter more if $\alpha^*_{21}\alpha^*_{22} + \delta^*_{21}\delta^*_{22} > \alpha^*_{11}\alpha^*_{12} + \delta^*_{11}\delta^*_{12}$ in the sense that a positive period t-1 inflation innovation leads to a higher level of conditional covariance than a shock to growth of equal magnitude.

It follows from the above that bad news can raise uncertainty more than good news if any or all of the elements of D^*_{11} matrix are significant. In this sense no news, $\varepsilon_{y,t} = \varepsilon_{\pi,t} = \xi_{y,t} = \xi_{\pi,t} = 0$, is good news as it leads to the minimum level of uncertainty in the following periods.

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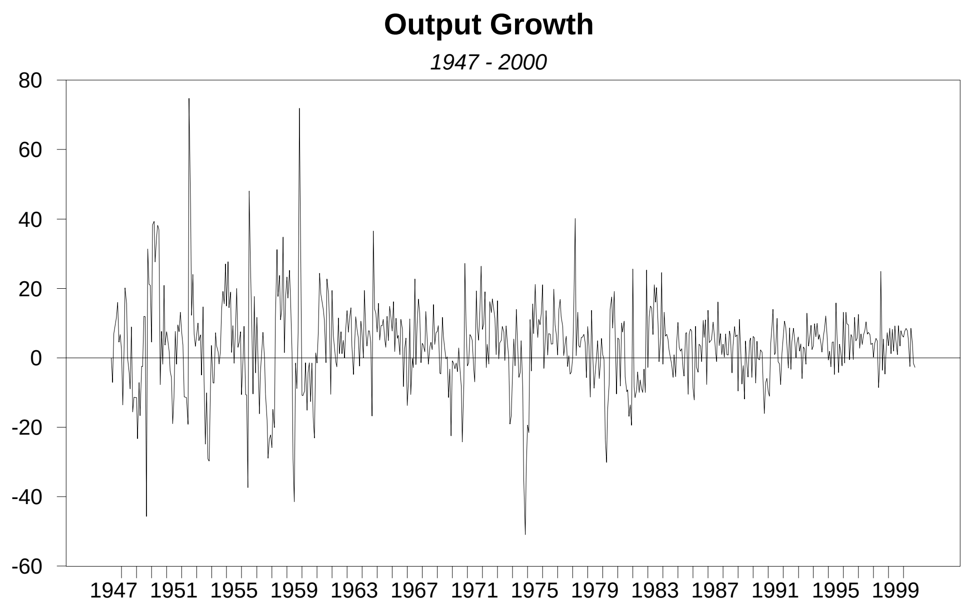

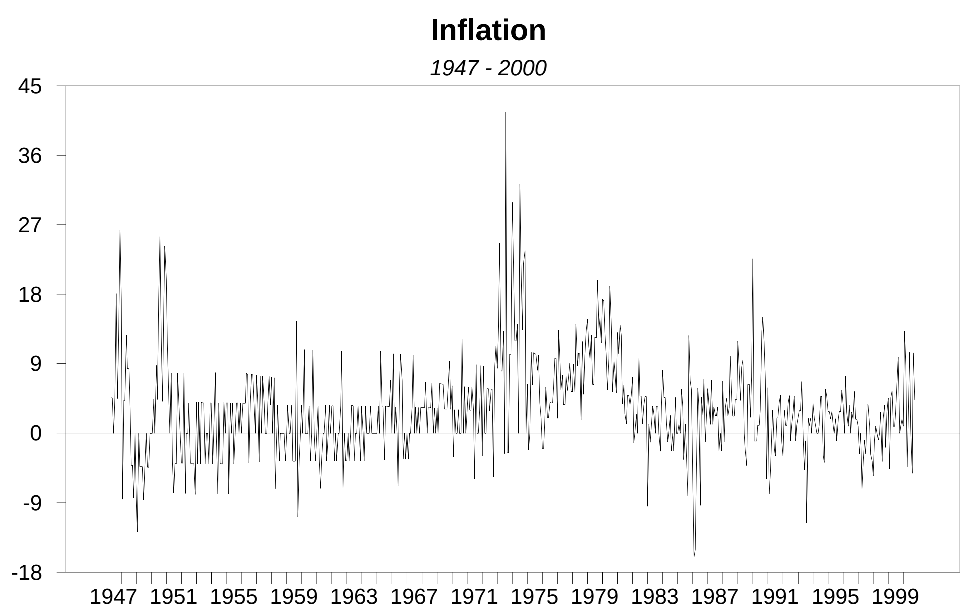

Figure1: The Data

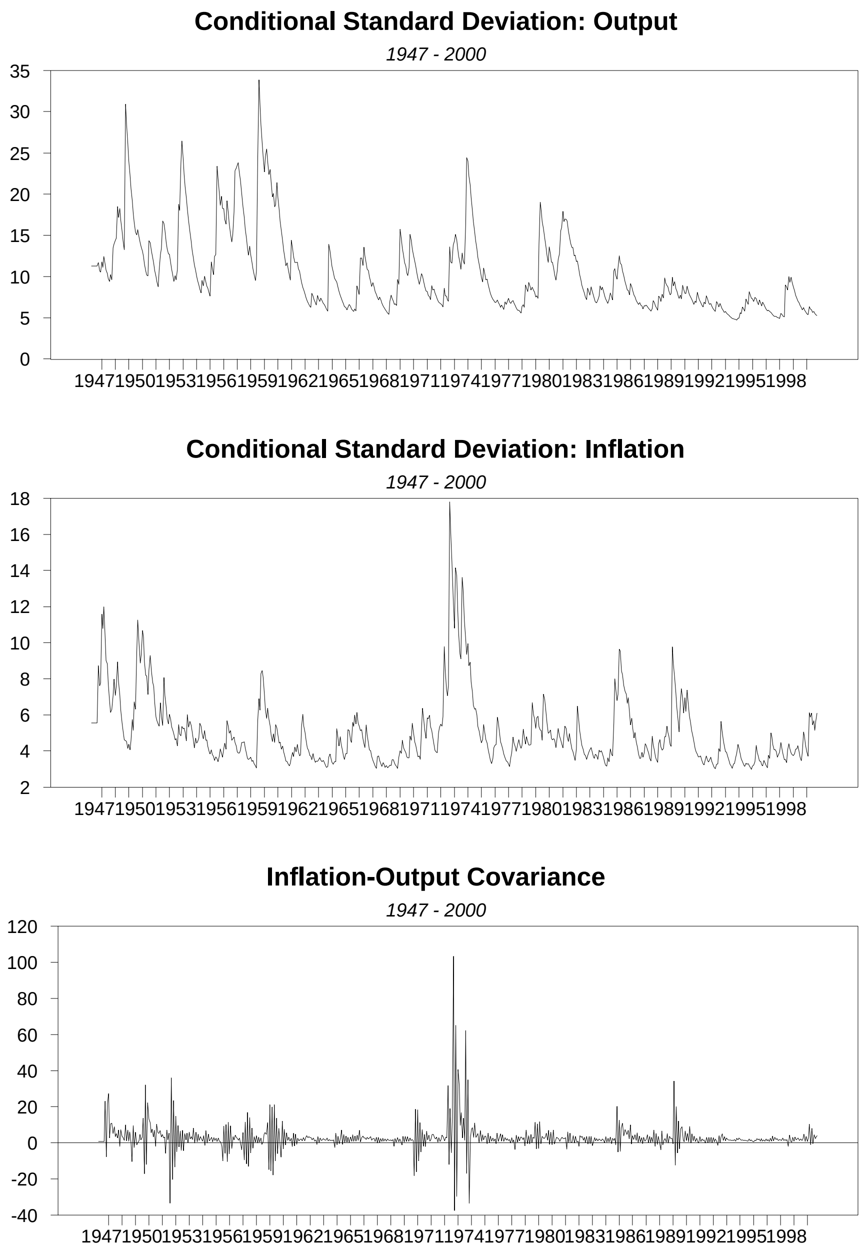

Figure 2: Estimated Conditional Standard Deviations and Conditional Covariance

Figure 3.

The effect of bad news about growth on the CV's and Means of growth and inflation

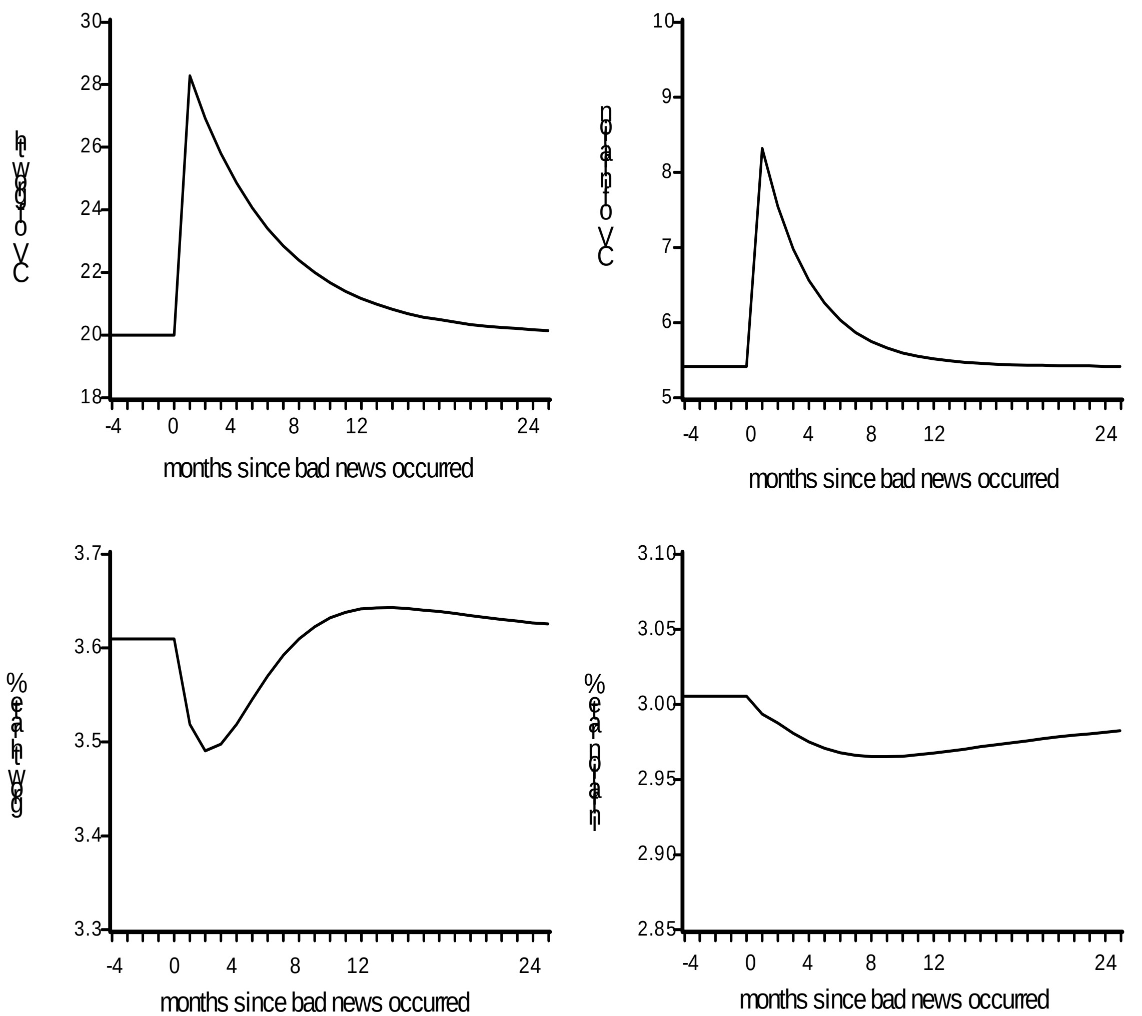

Figure 4.

The effect of bad news about inflation on the CV's and Means of growth and inflation

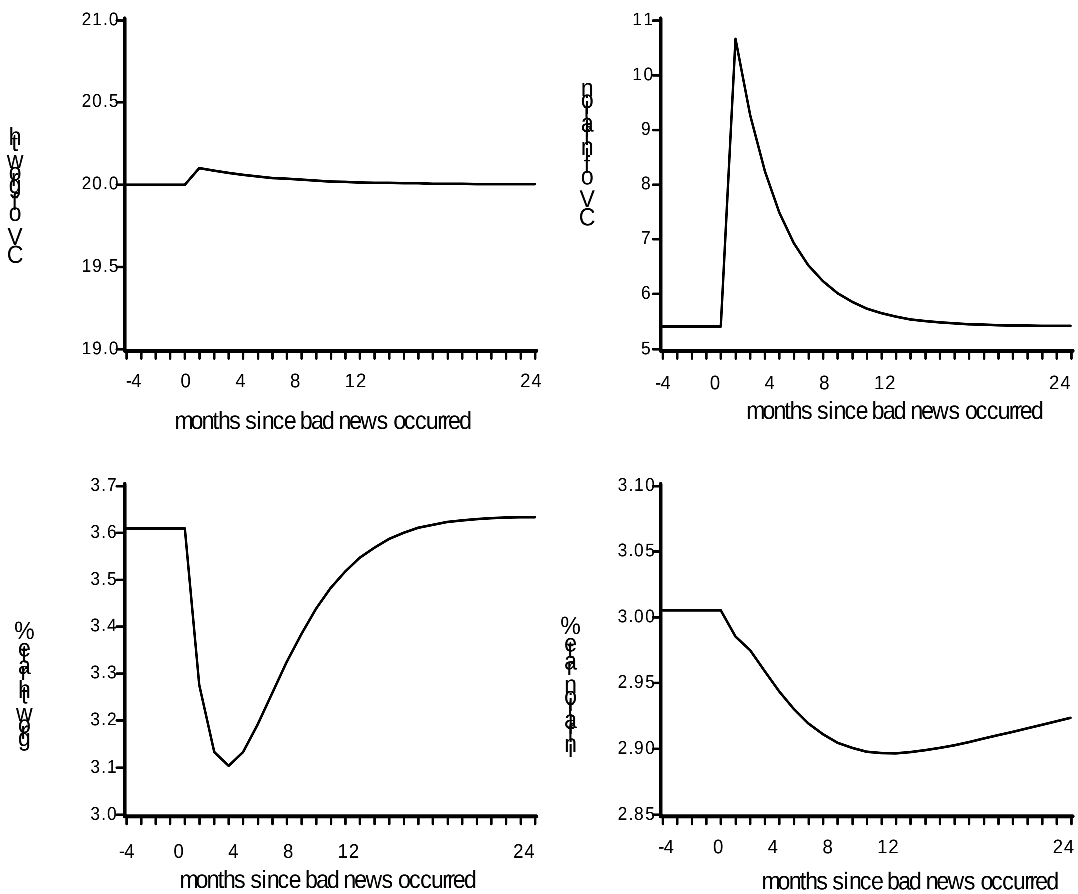

Table 1: Summary Statistics

	Mean	Variance	Skewness	Excess Kurtosis	Bera-Jarque Normality
Y	3.6054	155.7047	0.2428	4.5962	562.4889 [0.0000]
π	3.0559	37.5103	1.1579	4.4310	658.2563 [0.0000]
			Unit Root and Stationarity Tests		
	ADF(τ)	ADF(μ)	ADF	KPSS(μ)	KPSS(τ)
Y	-12.4483	-12.4438	-11.6179	0.07595	0.03498
π	-5.4309	-5.3842	-4.3728	0.4664	0.3975
5 % C.V.	-3.4191	-2.8664	-1.9399	0.463	0.146
			Tests for Serial Correlation and ARCH		
	Q(4)	*Q(12)*	$Q^2(4)$	$Q^2(12)$	ARCH(4)
Y	165.3173 [0.0000]	192.0829 [0.0000]	88.1327 [0.0000]	97.4497 [0.0000]	52.1685 [0.0000]
π	321.3849 [0.0000]	682.6248 [0.0000]	136.8077 [0.0000]	463.0983 [0.0000]	62.7177 [0.0000]

Notes to Table1: Marginal significance levels displayed as [.]

Table 2: The Multivariate Asymmetric GARCH-in-Mean model

Conditional Mean Equations

$$Y_t = \mu + \sum_{i=1}^{p} \Gamma_i Y_{t-i} + \Psi\sqrt{h_t} + \sum_{j=1}^{q} \Theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix};\ \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix};\ \Gamma_i = \begin{bmatrix} \Gamma^i_{11} & \Gamma^i_{12} \\ \Gamma^i_{21} & \Gamma^i_{22} \end{bmatrix};\ \Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix};$$

$$\sqrt{h_t} = \begin{bmatrix} \sqrt{h_{y,t}} \\ \sqrt{h_{\pi,t}} \end{bmatrix};\ \varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{bmatrix};\ \Theta_j = \begin{bmatrix} \theta^j_{11} & \theta^j_{12} \\ \theta^j_{21} & \theta^j_{22} \end{bmatrix}$$

$$\mu = \begin{bmatrix} 1.2584 \\ (0.0545) \\ \\ 0.0913 \\ (0.0172) \end{bmatrix} \quad \Gamma_1 = \begin{bmatrix} 0.4385 & 0.04768 \\ (0.0121) & (0.0102) \\ & \\ 0.0072 & 0.7794 \\ (0.0053) & (0.0047) \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} 0.3339 & -0.1126 \\ (0.0117) & (0.0109) \\ & \\ 0.0233 & 0.1939 \\ (0.0045) & (0.0046) \end{bmatrix}$$

$$\Theta_1 = \begin{bmatrix} -0.2525 & -0.1897 \\ (0.0243) & (0.0467) \\ & \\ 0.0012 & -0.6225 \\ (0.0085) & (0.0246) \end{bmatrix} \quad \Theta_2 = \begin{bmatrix} -0.3131 & 0.0170 \\ (0.0274) & (0.0559) \\ & \\ -0.0171 & -0.2006 \\ (0.0075) & (0.0241) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.0846 & -0.2385 \\ (0.0065) & (0.0113) \\ & \\ -0.0036 & -0.0209 \\ (0.0017) & (0.0037) \end{bmatrix}$$

Residual Diagnostics

	Mean	Variance	*Q(4)*	$Q^2(4)$	*Q(12)*	$Q^2(12)$
$\varepsilon_{1,t}$	0.0140	0.9932	2.8898	6.1466	21.4150	11.7959
	[0.7225]	[0.9969]	[0.5764]	[0.1885]	[0.0446]	[0.4622]
$\varepsilon_{2,t}$	0.0265	1.0088	1.9639	5.6143	11.4304	26.9583
	[0.5035]	[0.9991]	[0.7474]	[0.2298]	[0.4924]	[0.0078]

Moment Based Tests

$E(\varepsilon^2_{y,t}) = h_{y,t}$	$E(\varepsilon^2_{\pi,t}) = h_{\pi,t}$	$E(\varepsilon_{y,t}\varepsilon_{\pi,t}) = h_{y\pi,t}$
0.6317	3.6123	2.0114
[0.4267]	[0.0574]	[0.1561]

Notes: Standard errors displayed as (.). Marginal significance levels displayed as [.]. *Q(p)* and $Q^2(p)$ are are Ljung_Box tests for p[th] order serial correlation in $z_{j,t}$ and $z^2_{j,t}$ respectively for $j = y_t, \pi_t$.

.

Table 2 Continued: Estimates of the Multivariate Asymmetric GARCH Model

Conditional Variance-Covariance Structure		
$H_t = C_0^{*'} C_0^{*} + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^{*} + B_{11}^{*'} H_{t-1} B_{11}^{*} + D_{11}^{*'} \xi_{t-1} \xi_{t-1}^{'} D_{11}^{*}$		
$\varepsilon_{t-1} = \begin{bmatrix} \varepsilon_{y,t-1} \\ \varepsilon_{\pi,t-1} \end{bmatrix}$; $\xi_{t-1} = \begin{bmatrix} \min(\varepsilon_{y,t-1}, 0) \\ \max(\varepsilon_{\pi,t-1}, 0) \end{bmatrix}$		
$C_0^* = \begin{bmatrix} 1.8064 & 0.6612 \\ (0.0817) & (0.1595) \\ 0 & 1.2033 \\ & (0.0977) \end{bmatrix}$ $B_{11}^* = \begin{bmatrix} 0.9155 & 0.0024 \\ (0.0026) & (0.0213) \\ -0.1414 & -0.8567 \\ (0.1088) & (0.0064) \end{bmatrix}$		
$A_{11}^* = \begin{bmatrix} -0.0741 & 0.0627 \\ (0.0255) & (0.0139) \\ 0.0202 & 0.3844 \\ (0.0818) & (0.0179) \end{bmatrix}$ $D_{11}^* = \begin{bmatrix} -0.5711 & 0.0123 \\ (0.0147) & (0.0176) \\ 0.3409 & 0.2479 \\ (0.0745) & (0.0518) \end{bmatrix}$		
Diagonal VARMA	$H_0 : \Gamma_{12}^i = \Gamma_{21}^i = \theta_{12}^i = \theta_{21}^i = 0$	[0.0000]
No GARCH-M	$H_0 : \psi_{ij} = 0$ for all i, j	[0.0000]
No asymmetry:	H_0: $\delta_{ij}=0$ for $i,j=1,2$	[0.0000]
Diagonal GARCH	$H_0 : \alpha_{12}^* = \alpha_{21}^* = \beta_{12}^* = \beta_{21}^* = \delta_{12}^* = \delta_{21}^* = 0$	[0.0000]

.

Table A1: Robust conditional moment tests

Indicator	$v_{y,t}=\varepsilon_{y,t}^2-h_{y,t}$	$v_{y\pi,t}=\varepsilon_{y,t}\varepsilon_{\pi,t}-h_{y\pi,t}$	$v_{\pi,t}=\varepsilon_{\pi,t}^2-h_{\pi,t}$
m_1^y	0.2002 [0.6546]	1.1889 [0.2756]	6.0239 [0.0014]
m_1^π	0.0007 [0.9789]	0.5253 [0.4686]	0.1048 [0.7461]
$m_2^{-,-}$	4.4018 [0.0359]	0.4363 [0.5089]	0.2990 [0.5845]
$m_2^{-,+}$	0.8892 [0.3457]	2.4581 [0.1169]	1.2379 [0.2659]
$m_2^{+,-}$	1.2342 [0.2666]	1.4946 [0.2215]	1.4946 [0.2215]
$m_2^{+,+}$	0.0004 [0.9844]	0.1814 [0.6701]	1.7098 [0.1910]
$m_3^{y,y}$	0.1471 [0.7014]	1.5014 [0.2204]	4.3499 [0.0370]
$m_3^{y,\pi}$	0.1358 [0.7125]	0.1792 [0.6721]	3.2139 [0.0730]
$m_3^{\pi,y}$	0.8974 [0.3435]	0.0001 [0.9941]	0.5373 [0.4636]
$m_3^{\pi,\pi}$	0.7223 [0.3954]	0.6679 [0.4138]	1.0869 [0.2972]

Notes: All tests are distributed as $\sim\chi^2(1)$. Marginal significance levels displayed as [.]. The misspecification indicator is defined where *I(*)* takes the value 1 if the expression in the parentheses below is satisfied and zero otherwise.

Sign Misspecification	Quadrant Misspecification	Size/ Sign Misspecification
$m_1^y=I\left(\varepsilon_{y,t-1}<0\right)$	$m_2^{-,-}=I\left(\varepsilon_{y,t-1}<0,\varepsilon_{\pi,t-1}<0\right)$	$m_3^{y,y}=\varepsilon_{y,t-1}^2 I\left(\varepsilon_{y,t-1}<0\right)$
$m_1^\pi=I\left(\varepsilon_{\pi,t-1}<0\right)$	$m_2^{+,-}=I\left(\varepsilon_{y,t-1}>0,\varepsilon_{\pi,t-1}<0\right)$	$m_3^{y,\pi}=\varepsilon_{y,t-1}^2 I\left(\varepsilon_{\pi,t-1}<0\right)$
	$m_2^{-,+}=I\left(\varepsilon_{y,t-1}<0,\varepsilon_{\pi,t-1}>0\right)$	$m_3^{\pi,y}=\varepsilon_{\pi,t-1}^2 I\left(\varepsilon_{y,t-1}<0\right)$
	$m_2^{+,+}=I\left(\varepsilon_{y,t-1}>0,\varepsilon_{\pi,t-1}>0\right)$	$m_3^{\pi,\pi}=\varepsilon_{\pi,t-1}^2 I\left(\varepsilon_{\pi 2,t-1}<0\right)$

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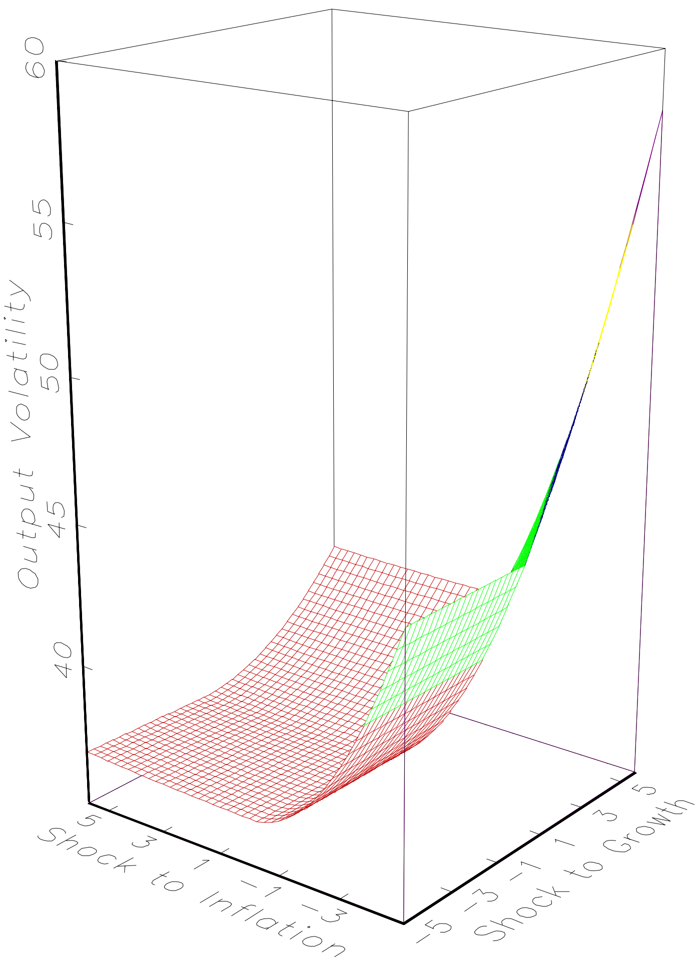

Figure 5: News Impact Surface for output volatility, $h_{y,t}$.

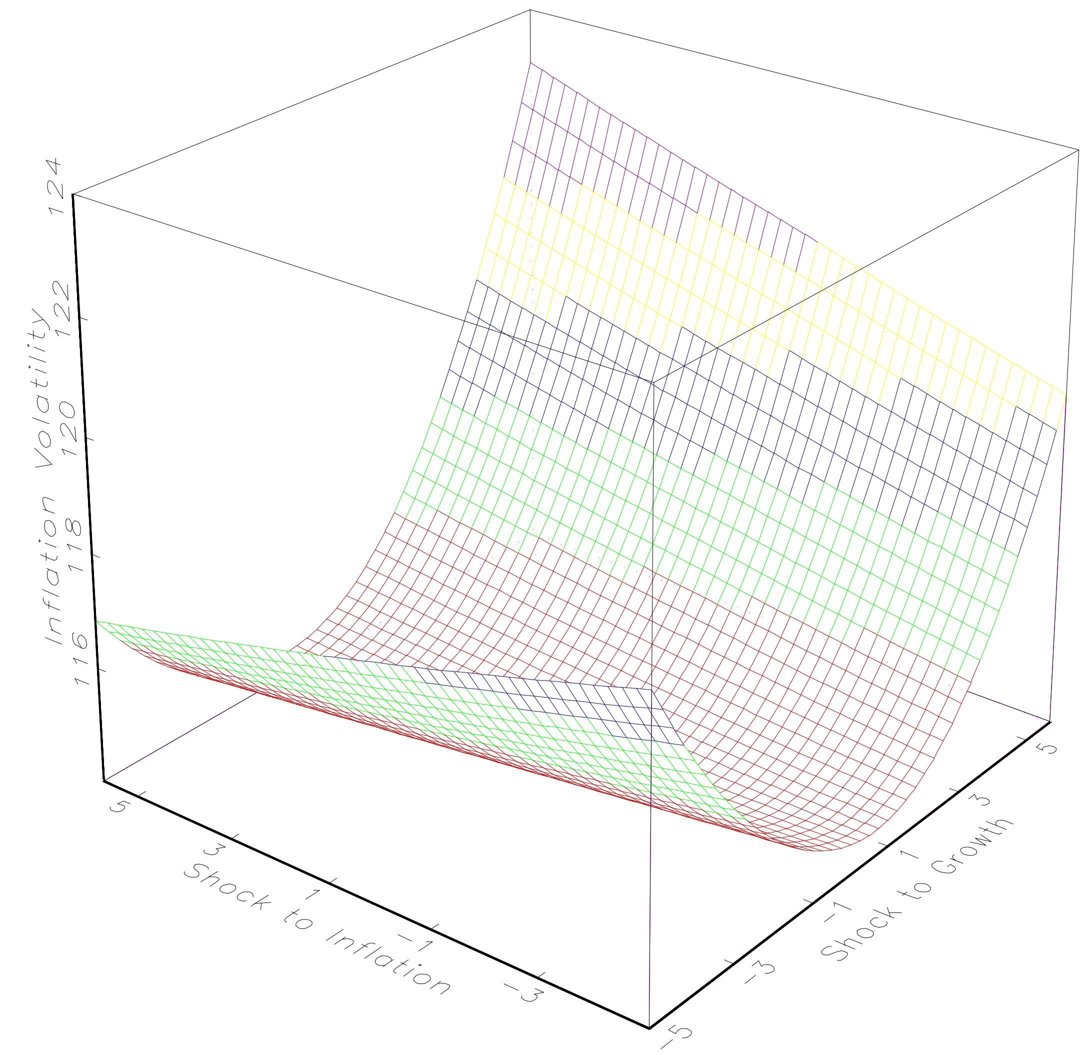

Figure 6: News Impact Surface for Inflation Volatility, $h_{\pi,t}$

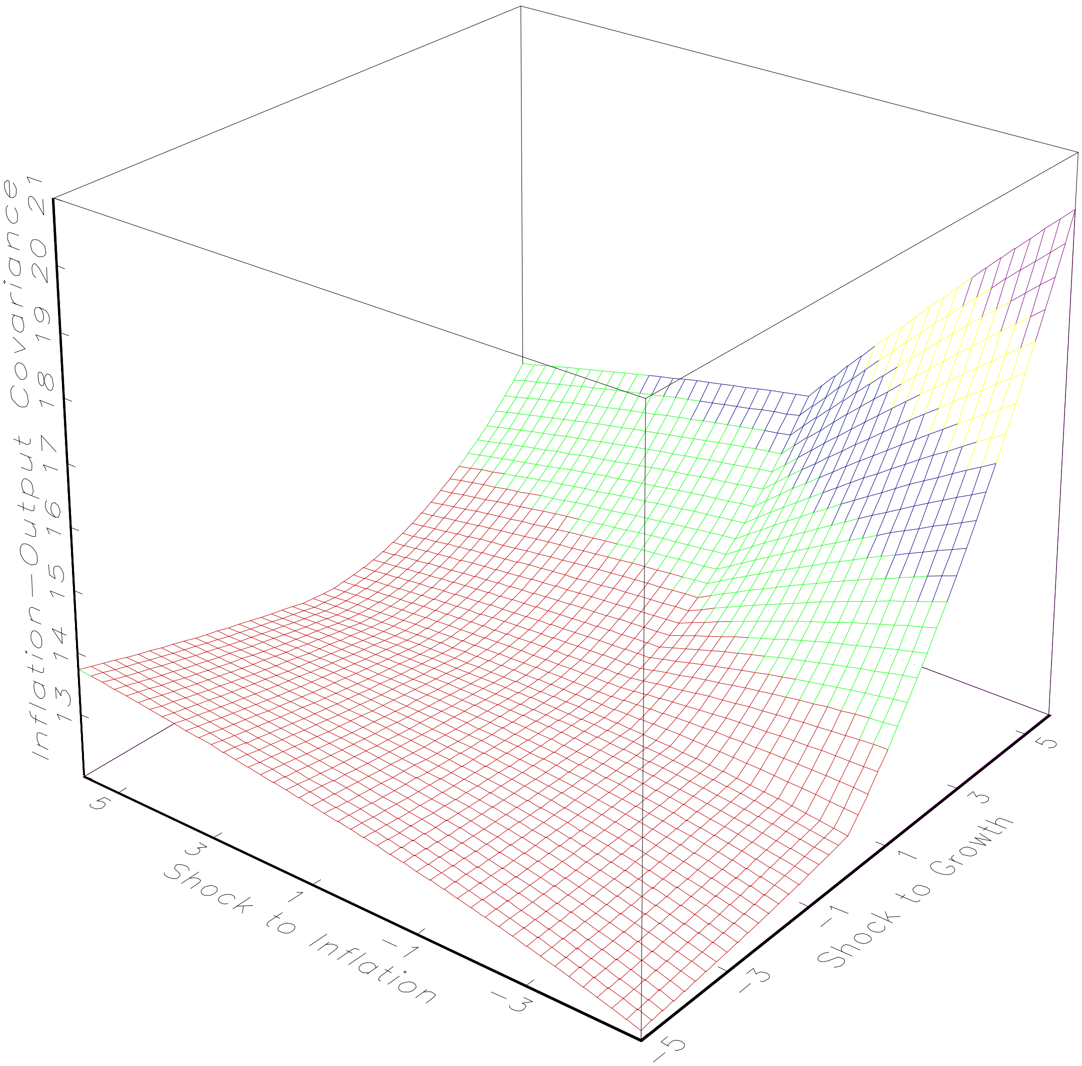

Figure 7: News Impact Surface for Inflation-Output Covariance, $h_{\pi y,t}$.