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**Hours of Work: A Demand Perspective**

by

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## **Hours of Work: A Demand Perspective\***

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Abstract : In Australia, and in other countries, we observe at any one time a wide distribution of hours worked per week. We develop a cost-minimising model to explain employer choices over the number of employees and their hours of work. An important finding is that hours of work and the number of employees are not perfect substitutes. We show that this has important implications for the way economists model labour demand and measure productivity. We show that estimates using total hours worked as the measure of labour input implicitly assumes perfect substitution of persons and hours and results, inter alia, in an overestimation of the rate of labour and multifactor productivity growth in Australia and especially in the period prior to the so called 'productivity slow-down'.

Keywords: Employment, Hours, Production Function, Total Factor Productivity

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### *1 Introduction*

In Australia, and in other countries, we observe at any one time a wide distribution of hours worked per week, including part time employment, full time employment and overtime, these distributions vary by gender, industry and occupation, and they vary over time. In May 2007 in Australia, 30 per cent of persons were part time workers, just 45 per cent worked around standard hours of 35 to 40 hours per week, and 17 per cent worked long hours of more than 50 hours a week (ABS, 2007a). Among OECD countries, Australia lies in the upper one-third in terms of the share of employees working part time or long hours, but in neither case is it at the extreme (OECD, 2004). Within Australia, the distribution of hours of work varies markedly from industry to industry (with, for example, 50 per cent of employees in accommodation, cafes and restaurants work part time down to a negligible level for electricity, gas and water) and between occupations (with, for example, 63 per cent of elementary clerical, sales and service workers part time down to 10 per cent for managers) (ABS, 2007a). Over time, the distributions of hours of work have varied both in a trend sense and over the business cycle. For example, the proportion of part time employees increased from 10 per cent in 1970 to nearly 30 per cent by the mid-1990s and since then has been relatively stable (ABS, 2007a and various years). This paper seeks to provide a demand side explanation of differences in employer demands for labour by hours of work.

We develop a labour services cost minimising model to explain employer choices over the number of employees and their hours of work. The model explicitly recognises that labour costs involve a mixture of fixed costs of hiring, training and firing as well as variable costs of wages (and other labour on costs such as superannuation and payroll tax), and that overtime wages involve a premium beyond a legislated normal working week. Employee labour productivity depends on the hours of work, and in particular ultimately decreasing productivity sets in. Differences in the employer desired mix of number of employees and the hours they work vary with the mix of fixed and variable labour costs, the overtime premium, the statutory normal hours, and the productivity relationship with hours worked. It is argued that these factors help to explain differences in the observed patterns of hours worked across industries and occupations, and over time.

An important finding from our model is that hours of work per employee and the number of employees are not perfect substitutes. Yet, in many important economic studies it is assumed that the number of employees and hours of work are perfect substitutes in providing labour services. At its simplest, the perfect substitution argument says labour services  $L$  can be expressed as the product of the number of employees  $N$  times the average hours worked per employee  $H$  to give  $L = NH$ . Then, for example, many labour demand studies seek to explain the determinants of  $L$ , and studies of labour and total factor productivity use  $L$  as a labour input measure. Given the imperfect substitutability of the number of employees and the hours they work, it is argued that demand and productivity studies using a total hours worked series, that is  $L = NH$  with the implicit assumption of perfect substitutability of  $N$  and  $H$ , are likely to incur undesirable biases.

An explicit focus of this paper to explain the pattern of hours worked, and some of its implications, is on the demand side of the labour market. Of course, labour supply factors also may be important, but they are not considered in this paper. Apart from the need to contain the size of the analysis, our focus on the demand side is justified on the tenable assumption that because of the pervasiveness of unemployment in recorded data over the last 30 years, the data we observe on hours worked is on the demand side rather than the supply side of the labour market model. Further support for a demand side focus is provided by the observation by Wooden and Drago (2007) from analysis of the Household Income and Labour Dynamics for Australia survey that at any one time about 40 per cent of employees, including those working part time, normal hours and long hours, would prefer to work different hours to those offered to them.

The rest of the paper is organised as follows. Section II provides some statistical facts on the variation of hours of work per employee for Australia which we seek to explain. Section III provides a microeconomic employer labour services cost minimising model for the long run equilibrium to explain the reasons for different choices of hours of employment. Section IV uses the Section III model to hypothesise reasons for the variation in observed hours of work by occupation and industry in Australia. A summary of earlier literature on differences in the time series properties of hours of work over the business cycle, or short run responses, relative to number of employees and output, and in turn how these patterns vary across skill levels, is provided in Section V. Section VI illustrates the desirability in many economic studies

of labour demand and of productivity growth to recognise the imperfect substitutability of number of employees and hours, rather than to use total hours worked with its implicit assumption of perfect substitutability. A final Section VII provides some conclusions.

## *II Some Statistical Facts*

An overview picture of the wide range of normal hours worked per week in May 2007 by males, females and persons is provided in Table 1. At the aggregate level, 30 per cent would be classified as part time working less than 35 hours a week, 42 per cent work the standard working week of 35 to 40 hours, and 25 per cent work more than 45 hours a week, with 17 per cent working 50 and more hours which often is used as the measure of long hours. Females, relative to males, are more likely to be part time and not to work long hours, and they are slightly more likely to work standard hours.

[TABLE 1 NEAR HERE]

Table 2 describes large differences in the pattern of hours worked across different occupations and industries with the proportion of the workforce who are part time workers and those who work long hours. Industries and occupations are ranked in ascending order by the share of part time employees at May 2007. At the industry level, compared to the economy average of 28 per cent part time employment, four industries (accommodation, cafes and restaurants, retail trade, culture and recreational services, and health and community services) have more than 40 per cent part time employees, and four industries (construction, manufacturing, mining, and electricity, gas and water) have less than 15 per cent part time employees. Industries with a relatively high proportion of long-hours employees (more than 20 per cent) are agriculture, transport, wholesale trade, mining and electricity, while health and government have a relatively low proportion (less than 10 per cent) of long-hours employees. The correlation between the shares of industry employees with part time and long hours is low at -0.45.

[TABLE 2 NEAR HERE]

Turning to occupations, the clerical, sales and services occupations (elementary especially but also intermediate and advanced clerical, sales and services occupations) have more than 40 per cent part time, and a relatively low share of long hours employees. By contrast, less than 20 per cent of associate professionals, tradespersons

and managers & administrators are part time, and the occupations with relatively high shares of long-hours workers are managers & administrators, intermediate production workers and associate professionals. In the case of the occupations, unlike the industries, there is a relatively high correlation of -0.72 between occupations with part time and long-hours employees.

A picture of changes over time in the distribution of hours worked is given in Table 3 for the period 1980 to 2006 for men and women. From 1980 to about the turn of the century, the share of part time employment increased from 50 to 58 per cent for women and from 25 to 31 per cent for men, and since then the part time share has stabilised. The share of employees working long hours increased from 18 to 26 per cent for men and from 5 to 9 per cent for women up to about 2002, and in recent years the share working long hours has fallen. There has been a trend decline in the share of employees working the normal or statutory working week, and primarily because of the expansion of part time employment.

[TABLE 3 NEAR HERE]

A long term picture of changes in the make-up of the workforce in terms of persons employed and average hours worked per employee is provided in Figure 1. The two series run from 1966-67 to 2004-05 and are shown in index form with 2003–04 = 100. Over the period, the number of employees has almost doubled, with cyclical downfalls, particularly in 1983 and 1991. By contrast, the series for average hours worked has trended downwards by about 10 per cent, with the largest falls in the late 1970s, and again with some cyclical variation.

[FIGURE 1 NEAR HERE]

### *III Theoretical Model: Long Run Equilibrium*

We build a simple labour cost minimising model for a representative competitive firm. The firm has to choose the number of employees  $N$  and the hours worked per employee  $H$  to minimise the cost of providing a quantum of labour services  $L$ .

Consider a labour services production function of the form

$$L = f(N, H) \tag{1}$$

$$= g(H)N \quad (1')$$

The general form is given by (1). We will use a more specific function (1') which was proposed by Ehrenberg (1971), and has been used by Bell (1980), Calmfors and Hoel (1988 & 1989), Booth and Ravallion (1993), Kapteyn et al. (2004) and others. It imposes constant returns to scale and marginal productivity for the number of employees  $N$ , and in essence assumes an infinite supply of clone workers available for hire, but it attaches a more general production function to hours of work per employee  $H$ . The function  $g(H)$  may initially have an increasing marginal productivity phase to reflect, say, learning by doing and skill acquisition, but ultimately a phase of declining productivity sets in to reflect, for example, tiredness or boredom. Declining productivity per hour worked also may reflect the inability of the employer to provide suitable work given the importance of timeliness in the supply of some products, and in particular services which cannot be stored. More formally, on  $g(H)$  in (1'), we impose the conditions  $g(0) = 0$ , and beyond a particular level of  $H > H^*$ ,  $g'(H) > 0$ , and  $g''(H) < 0$ .<sup>1</sup>

The labour services production function (1') can be shown as an isoquant map in number of employees, hours of work, or N-H, space. The isoquant has a marginal rate of technical substitution, *MRTS*

$$MRTS = \partial N / \partial H = -(Ng'(H) / g(H)) \quad (2)$$

Given the assumed properties of  $g(H)$  and  $g'(H)$  in (1'), the isoquant will have nice convex to the origin properties. More informative than the *MRTS* is the elasticity of substitution of hours for employees in the provision of labour services,  $\sigma_L$ :

$$\begin{aligned} \sigma_L &= (\partial N / \partial H)(H / N) = -g'(H)H / g(H) \\ &= -MP_h / AP_h \end{aligned} \quad (3)$$

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<sup>1</sup> A particular example of (1') with these properties often used in the literature is  $L = NH^\alpha$  with  $\alpha < 1$ , or this could be generalised to allow a set of hours  $H^\#$  before decreasing returns set in as  $L = N(H - H^\#)^\alpha$ . In general, economic efficiency means we would choose a set of hours where the  $MP_h$  is declining, ie the condition  $g''(H) < 0$ , and the  $MP_h < AP_h$ .

where,  $MP_h = g'(H)$  is the marginal product per hour worked, and  $AP_h = g(H)/H$  is the average product per hour worked. Then, as we substitute hours for employees in providing labour services, the elasticity of substitution is  $< -1$  when  $MP_h > AP_h$  if skill acquisition requires at least a minimal set of hours, but ultimately when decreasing returns set in and  $MP_h < AP_h$  the elasticity of substitution is  $> -1$  and approaches zero.

A special case of (1) often used in studies of labour demand and productivity assumes perfect substitution of the number of employees and hours of work labour services giving the specific special case production function

$$L = NH \quad (1'')$$

This function implicitly assumes a constant marginal product per hour of work regardless of the number of hours worked, and in this case the marginal and average products are equal. In this special case, the elasticity of substitution,  $\sigma_L = -1$ , is constant and always equal to minus unity.

The total cost of labour services,  $C$ , is given by

$$C = (WH + W^o(H - HS) + F)N \quad (4)$$

where,  $H$  and  $N$  as before refer to hours per employee and number of employees,  $W$  is the variable labour cost per hour up to standard hours  $HS$ ,  $W^o$  is the overtime wage premium, and  $F$  is the fixed cost per worker. From (4) we can form an isocost function in  $N$ - $H$  space with a marginal rate of transformation,  $MRT$ ,

$$MRT = \partial N / \partial H = -(N/H)((WH)/(WH + F)) \text{ for } H < HS \text{ and} \quad (5)$$

$$= -(N/H)((WH + W^o(H - HS))/(WH + W^o(H - HS) + F)), \text{ for}$$

$$H > HS. \quad (5')$$

Note from (5) and (5') that the isocost function for labour services has two convex segments with a discontinuity at the standard hours  $HS$ , with the  $MRT$  increasing in absolute value at the  $HS$  kink. The  $MRT$  depends on the ratio of variable costs to total labour costs, for example in (5)  $(WH/(WH + F)) < 1$  by definition.



As with the isoquant for labour services, also for the isocost function, it will be convenient to consider the elasticity of substitution rather than the slope or  $MRT$ . The elasticity of substitution of hours for employees along the cost function ( $\sigma_c$ ) is:

$$\sigma_c = (\partial N / \partial H) / (H / N) = -(WH + W^o(H - HS)) / (WH + W^o(H - HS) + F) \quad (6)$$

We note that  $\sigma_c$  is just minus the share of variable wage costs in total labour costs. Then,  $\sigma_c > -1$ , it declines to  $-1$  as the number of hours increase and spread the fixed costs, and it initially becomes steeper for hours in excess of the standard hours when the overtime premium is payable.

A labour cost minimising firm will choose the combination of number of employees ( $N$ ) and hours per employee ( $H$ ) where the  $MRTS = MRT$ , or where  $\sigma_L = \sigma_c$ . Using the latter, and substituting from (3) and (6), the least-cost combination of  $N$  and  $H$  is given where

$$(WH + W^o(H - HS)) / (WH + W^o(H - HS) + F) = (MP_h / AP_h) \quad (7)$$

That is, employers will choose the  $N$  and  $H$  combination where the ratio of variable to total labour costs, the LHS of (7), equals the ratio of the marginal and average products of the production function for hours worked, the RHS of (7). As we explore in more detail in Figures 2, 3 and 4 below, from (7) we can picture the circumstances where employers demand part time, regular hours, and long hours (including overtime pay), respectively.

Figures 2, 3 and 4 show the employer decision choice problem over hours per worker ( $H$ ) and number of employees ( $N$ ) to minimise the cost of providing a given set of labour services ( $L$ ). A convex isoquant represents the production function (1') with the marginal rate of technical substitution and elasticity of substitution defined in (2) and (3), respectively. The cost function information of (4) is represented by the isocost function, with two convex segments joined at the point of standard hours  $HS$ , and the marginal rate of transformation and elasticity of substitution are defined in (5) and (6), respectively. The cost minimising combination of  $H$  and  $N$  is given where the isoquant and isocost curves are tangent to each other at the  $H$  and  $N$  combination defined by (7).

Figure 2 describes the choice of part time employment. Here, employers minimise labour costs by choosing  $H < HS$  at  $E^{PT}$  where

$$WH/(WH + F) = (MP_h / AP_h) \leq 1 \quad (8)$$

A necessary condition for part time employment is either that the marginal productivity per hour worked be declining for  $H < HS$ , or that the fixed costs of employment be zero. That  $F = 0$  seems unlikely, although  $F$  may be small, so (8) is strictly less than unity and therefore  $MP_h < AP_h$  and marginal productivity per hour is falling. If marginal productivity per hour is not falling, full time employment, or  $H \geq HS$ , to spread the fixed costs would be a dominant cost minimising choice. From (8), and Figure 2, part time employment is an employer cost minimising decision for employees with the combinations of low fixed employment costs and with declining marginal productivity per hour setting in before the standard working week hours.

[FIGURE 2 NEAR HERE]

Figure 3 shows the case of an employer choice of overtime hours of work to minimise labour costs. Here the equilibrium condition at  $E^O$  is given by

$$((W + W^O)H - W^OHS)/((W + W^O)H - W^OHS + F) = (MP_h / AP_h) < 1 \quad (9)$$

Then, a cost minimising choice of overtime hours will be positively related to the importance of fixed costs in total labour costs, to shorter and binding standard hours, and to the hours production function having an extended period of hours worked before declining marginal productivity per hour sets in, and then only at a very gradual rate; and the choice will be negatively related to the wage premium paid for overtime relative to the standard wage rate.

[FIGURE 3 NEAR HERE]

Employer choice of a standard working week with  $H = HS$  is illustrated in Figure 4. Here, at  $E^F$  the equilibrium condition is

$$WHS/(WHS + F) < (MP_h / AP_h) < ((W + W^O)HS)/((W + W^O)HS + F) < 1 \quad (10)$$

The standard working week before the overtime wage premium is triggered clearly is a dominant determining variable. The relative importance of fixed costs in total labour costs, the relative mark up of the overtime wage premium, and the point and rate

at which the marginal productivity per hour worked affect employer choices for a part time or an overtime appointment are also important explanatory variables.

[FIGURE 4 NEAR HERE]

An important result from our model of employer cost-minimising decisions over the choice of hours and number of employees is the unsuitability of the implicit assumption in (1'') that hours and employees are perfect substitutes, or that the elasticity of the isoquant is minus one. Brechling (1965), Feldstein (1967) and others have raised these concerns, but with little recognition in subsequent empirical work on, for example, studies of productivity and the demand for labour, both of which we explore further in section 6 below. Under the perfect substitutability assumption, so long as there is a fixed cost element in the labour cost, it clearly is inefficient for an employer to hire any part time workers. Unless the fixed cost component is a very large share of labour costs, it is inefficient to hire overtime workers, and then if you chose overtime the cost effective choice becomes an infinite level of overtime hours. Put another way, the observed importance of part time employment and of a few hours per week of above standard hours of work requires the more general assumption that at some point the marginal product per hour falls (and is falling further). A number of writers, including Hart (1984) and Hamermesh (1993) provide empirical evidence to support the hypothesis that the number of employees and hours worked per employee are imperfect substitutes.

#### *IV Differences in Hours Worked Across Occupations and Industries*

The different patterns of hours worked by occupation and industry observed in Table 2 and over time observed in Table 3 and Figure 1 can be explained in part using the model of Section III above by focusing on differences in the relative importance of fixed costs in total labour costs, and hypotheses about differences in the marginal productivity functions per hour of work. Differences in standard hours of work, especially over time, and institutional and social differences affecting the rigor with which standard hours are imposed in initiating payment of an overtime premium, also are likely to be important in some cases. Limited evidence of systematic variation in the overtime premium, both over time and across industries and occupations, from the

general “time and a half” make it difficult to attribute a significant explanatory role for this term for Australian data.

While there are few, if any, comprehensive empirical studies for Australia on the relative importance of the fixed and variable costs for labour, our guess is that the ratio is similar as for other OECD countries. Hart (1984) suggests for both the US and the UK it is reasonable to put “fixed labour costs at roughly 20 per cent of total variable (labour) costs” (p 19), and Martins (2004) suggests that the ratio of variable costs in total costs is around 75 per cent for Portugal. A number of empirical studies provide support for the argument that an increase in the share of fixed costs in the total labour cost does induce longer hours. Cutler and Madrian (1998) found that increases in health insurance costs in the US resulted in longer hours of work by workers covered by health insurance, and Dolfin (2006) found that higher costs of hiring, training and firing were associated with longer employee hours.

It seems reasonable to hypothesise important differences in the relative importance of fixed costs across occupations and industries. Rosen (1968) for example argues that the fixed cost share rises with the occupational skill level, and his empirical work with data for the US railroad industry supports this contention. Some of the connections between more skilled occupations and a higher share of fixed costs in total labour costs include more time and effort required in the job search and matching process for hiring managers and professionals, more staff investment in training and provision of firm specific human capital, and higher costs of monitoring, and if required firing, skilled workers. More capital intensive industries, relative to labour intensive industries, are likely to invest more in the training and monitoring of their workforce to protect the on-going value of their more expensive capital assets per employee.

In the case of the production function for hours of work, we have no hard evidence, but it seems reasonable to hypothesise variations across occupations and industries. Declining marginal productivity per hour worked associated with employee boredom, lack of interest and motivation, and from the monotony of repetition is likely to set in at a smaller number of hours for the less skilled occupations. It also seems likely that declining marginal productivity will set in at a smaller number of hours for the labour intensive service industries due to the employer peak/off-peak product and derived labour demand fluctuations, whereas the goods industries can smooth production and sales to a greater extent through the holding of inventories.

A number of institutional factors are likely to influence the employer cost-minimising choice of hours per employee via the effect of these factors on the labour cost and productivity variables in (8), (9) and (10). The way in which the standard hour binds becomes important. For example, a rather loose restriction applies in the case of most managers and professional occupations on salaries, at least relative to more tightly enforced restrictions for the more unionised blue collar trades and for others on wages. The relative importance of fixed costs in labour costs is influenced by the industrial relations system, including the conditions for and costs of firing workers, both absolutely but also relatively between part time and full time employees.

Different characteristics of different occupations and industries shown in Table 2 can be related to the conditions in which employers are likely to choose part time, normal or long hours. Occupations and industries with a high proportion of low skill and repetitive jobs are more likely to see an early productivity decline per hour worked than those with a high proportion of high skilled and challenging jobs, and hence involve part time hours. In many service industries, when compared with goods production industries, the variability of labour demand by time of day and by day effectively means relatively low productivity of hours outside the peak periods, and favouring a part time choice for some employees. Industries and occupations which are skill intensive, or capital intensive, or both, are likely to incur high investment costs in training and familiarisation, and then employers want to hold employees in secure full time employment, and sometimes on long hours, to capitalise on the investments made. A relatively low share of fixed costs in total labour costs favouring part time employment seems to be the case in the low skill and labour intensive occupations and industries, relative to high skilled and capital intensive industries and occupations where full time and overtime hours dominate.

#### *V Short Term Adjustments to Hours of Work*

While the model of Section III and its application in Section IV was addressed primarily at long run equilibrium decisions on the number of employees and the hours of work to minimise the cost of providing labour services, an early strand of the labour economics literature, including Becker (1962), Oi (1962) and Rosen (1968), focussed on the short run response of employment numbers and hours worked to changes in

output, including the responses of employment and hours worked over the business cycle. In these models, quasi-fixed labour costs of employment resulted in hours following a cyclical pattern, and in employment being a lagging indicator and less volatile than hours and output. Further disaggregation of labour by skill and the relative importance of fixed costs in total labour costs, and hypothesising that the more skilled labour was less substitutable with the (short term) fixed capital stock, was used to explain differences in the time series properties of output, and hours and employment by skill level.

Consider in more detail the model and supporting empirical results provided by Rosen, especially for comparing the time series properties for labour with different skill levels<sup>2</sup>. His simplest model of short term labour services hire decisions has a fixed capital stock and two categories of labour inputs, namely high skilled and low skilled. The high skilled have a relatively higher share of fixed costs in total labour costs, and their elasticity of substitution with capital is relatively lower. A short run fall in output is shown to induce labour services cost minimising decisions involving a smaller percentage reduction in labour services, and then initially most of the reduction in hours per worker rather than in fewer employees, with employee numbers falling only with a lag. These effects are driven by the quasi-fixed labour costs which, once incurred (especially for recruitment and training), are sunk costs. In terms of the composition of labour services by skill level, the greater reduction in labour input falls on the lower skilled, in part because the lower share of fixed costs means a relatively smaller fall in the (short run relevant) relative labour cost for the low skilled, and in part because of the greater elasticity of substitution of low skilled labour with the fixed capital input. That is, compared with the time series for output, the time series for labour services is less variable, and then the time series for number of employees is less variable again and a lagging indicator, and this reduction in volatility and magnitude of the lag is greater for the more skilled relative to the low skilled labour types. Rosen finds these implications to be consistent with actual data for the US railways.

Aggregate time series data on number of employees and average hours of work, such as in Figure 1, include these industry effects discussed by Rosen, and in addition there are likely to be industry composition effects. For example, over the business

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<sup>2</sup> Compared with the model of Section III where declining marginal productivity per hour was important, the Rosen model assumes constant marginal productivity at around current hours of work.

cycle, different product demands are affected differently, including necessities versus discretion purchase products, and some sectors are better able to smooth production with fluctuating demands, for example storable goods versus many consumer services.

### *VI Some Implications*

A key finding of the preceding analysis is that the hours of work per employee and the number of employees are imperfect substitutes. Rejection of the simplified production function for labour services (1''), or  $L = NH$ , in favour of the more general function (1'), or  $L = Ng(H)$ , where beyond a basic number of hours worked the marginal productivity per hour ( $g'(H)$ ) declines, is required to explain the mix of observed part time, standard hours and overtime hours in employer cost minimising decisions. In this section we raise some model specification issues and the interpretation of estimates of productivity and labour demand functions using time series data prepared under the simplifying assumption that  $N$  and  $H$  are perfect substitutes, and in particular that  $g'(H)$  is a constant for all hours worked, in providing labour services.

#### *(i) Productivity Growth*

Many estimates of indexes of labour productivity and of multifactor productivity, and the derived rates of growth of productivity, use a measure of total hours worked, that is  $L = HN$ , as the labour input measure. The Australian Bureau of Statistics (1999 and 2000, and the regular publication in Catalogue 5404.0) is just one of many examples. Here we consider only the measure of the labour input, and we ignore other important issues often raised with the measures of output and the capital input.

To illustrate the potential biases in estimates of productivity growth using this simplified measure rather than the more general function of (1'), consider the labour productivity index  $LP$

$$LP = Y/L \tag{11}$$

where,  $Y$  is output and  $L$  is labour services measured as hours worked, namely  $L = NH$ .

The labour productivity growth rate,  $dLP/LP$  can be expressed as the difference in the growth rates of output and the labour services input

$$dLP/LP = dY/Y - dL/L \quad (12)$$

Under the simplifying assumption that the number of employees and hours of work are perfect substitutes, that is (1''), we can expand (12) as

$$dLP/LP = dY/Y - dN/N - dH/H \quad (12')$$

But, if we use the more general labour services function  $L = g(H)N$  of (1'), then (12) becomes

$$dLP/LP = dY/Y - dN/N - \alpha(dH/H) \quad (12'')$$

where

$$\alpha = MP_h/AP_h < 1 \quad (13)$$

and in equilibrium, as shown in (7),  $\alpha$  equals the share of variable labour costs in total labour costs and, as before,  $MP_h = g'(H)$  is the marginal product per hour worked, and  $AP_h = g(H)/H$  is the average product per hour worked.

Comparing (12') and (12''), when hours and employees are not perfect substitutes, and when there has been a decline in hours worked over time, as observed in Australia and shown in Figure 1, and in many other countries, the use of (12') rather than (12'') over-estimates the growth of labour productivity. The over-estimate given by  $(\alpha - 1)(dH/H)$ , will be greater the larger the reduction over the time period of average hours worked and the smaller is the term  $\alpha$  in (13) which in turn depends primarily on the relative importance of variable labour costs in total costs (see the employer cost minimising condition (7)).

A similar story can be told for the multifactor productivity (MFP) measure. For multifactor productivity, the bias in estimation will be given by  $s(\alpha - 1)(dH/H)$ , where  $s$  is the share of labour costs in total factor costs, and, as before,  $\alpha$  is the share of variable labour costs in total labour costs and  $dH/H$  is the rate of change in average hours worked. Again, with declining (increasing) average hours worked per employee, the conventional MFP measure based on the assumption of perfect substitutability of



persons overestimates (underestimates) the rate of MFP growth when compared with a measure based on the more realistic assumption of imperfect substitution, or of declining productivity per hour worked. The simplified measure attributes some of the productivity change to the employer choice of different working hours.

Pictures of the pattern and magnitude of differences in time series estimates of labour productivity (LP) and multifactor productivity (MFP) assuming hours and persons are perfect substitutes (the conventional ABS estimates) and allowing that they are imperfect substitutes with  $\alpha = 0.83$  are shown in Figures 5 and 6. The Figures show the difference between the two series for LP and MFP growth per annum for the market sector of Australia over the period 1966-67 through 2004-05. Given that hours per worker have fallen over this time period, the perfect substitution assumption estimate has on average exceeded the estimates based on the assumption that hours and persons are imperfect substitutes. The overestimate was relatively large in the 1970s when the fall in average hours was most marked (Figure 1), and as much as 0.3 percentage points per year in the case of LP and 0.2 percentage points for MFP, with another blip in 2003-04.

[FIGURES 5 AND 6 NEAR HERE]

A disaggregated by industry comparison of productivity estimates prepared under different assumptions on the substitutability of hours and employees is shown in Table 4.<sup>3</sup> Here the compound average annual percentage changes in LP and MFP over the period 1985-86 through 2005-06 are reported. Given that this is a period of relative stability of hours worked, it is not surprising that the differences, or the bias, are small. What however is of particular interest is that a more disaggregated level, in this case different industries, the bias varies in sign. Over the last twenty years average hours

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<sup>3</sup> Source of data is Australian Bureau of Statistics (2007b and c). The data provided includes indices of total hours worked in each industry which we have decomposed into its average hours worked and number of persons employed by comparing the total hours worked series with figures for the numbers employed in each industry obtained from the Labour Force Survey. Data on fixed costs of employment across industries (especially industries other than manufacturing) are extremely difficult to come by. The only estimates we have found are in Hart and Kawasaki (1999, p 67, Table 4.4). Based on the figures given there we estimate the ratio of variable costs to total costs (and thus our estimate of  $\alpha$  for each industry) to be:<sup>3</sup> Mining, 0.91; Manufacturing, 0.94; Electricity, gas & water, 0.90; Construction, 0.96; Wholesale trade, 0.95; Retail trade, 0.95; Accommodation, cafes & restaurants, 0.95; Transport & storage, 0.95; Communication services, 0.95; Finance & insurance, 0.93, and; Cultural & recreation services, 0.96. It will be seen that these figures are all quite high and show very little variation across industries.

worked per employee rose in the mining, electricity, gas and water, wholesale trade and finance and insurance industries, while they were constant in the construction, transport and storage industries, and fell in the other four industries. Then, depending on the industry experience with average hours worked, productivity estimates based on the assumption of perfect substitutability of persons and hours can underestimate or overestimate productivity growth based on the more plausible assumption of imperfect substitution.

[TABLE 4 NEAR HERE]

In effect, when the number of employees and the hours worked per employee are not perfect substitutes, as we argue is reality, and hours worked per employee vary, the conventional measures of labour productivity and of multifactor productivity confound two sources of gains in productivity in the conventional estimates based on the assumption of perfect substitutability. These two sources are on the one hand technical changes, better work and management practices, increases in worker effort and intensity and so forth generally regarded as efficiency improvements, and on the other hand changes in productivity due to changes in hours worked. Of course, for some questions the distinction between the two sets of productivity gain may be of little merit or policy interest, and in other cases the magnitudes of differences may be small and of no consequence. But for other questions, and particular sample periods, including the 1970s, the distinction will be important. Further, using (12'') for LP, and an analogous formulae for MFP, it is easy to disentangle the two sets of sources of productivity gains.

*(ii) Aggregate Labour Demand*

When the number of employees and the hours they work can be regarded as perfect substitutes, it is appropriate to derive and estimate labour demand functions with labour hours,  $L = NH$ , as the demand dependent variable. But, if the dependent variable instead is specified to be the number of employees  $N$ , then  $H$  should be included as an explanatory variable, and under the perfect substitution assumption, with a restricted elasticity of minus unity. This approach is adopted in the labour demand equation in the TRYM model for example (Taplin and Parameswaran (1993), Stacey and Downes (1995) and Downes and Bernie (1999)). Once we allow for imperfect substitutability of  $N$  and  $H$  for the reasons argued in this paper, the elasticity parameter on the  $H$  explanatory variable is given by minus the share of variable labour costs in total labour

costs (or the ratio of the marginal and average products per hour worked), which is less than unity. Alternatively, in other studies  $H$  is omitted as an explanatory variable (or with an implied elasticity of zero), for example in the labour demand studies of Russell and Tease (1991), Debelle and Vickery (1998) and Lewis and MacDonald (2002). For such specified models, if  $H$  varies over the sample period and is correlated with included explanatory variables, estimates of the parameters on the included explanatory variables will be biased. Using a more general model specification of the demand for the number of employees with  $H$  as an explanatory variable where the parameter on the variable  $H$  is estimated, for example in Dixon et al. (2005), the estimated elasticity of the number of employees  $N$  with respect to average hours worked  $H$  is found to be about -0.75, a number which closely corresponds to the estimated share of variable costs in total labour costs reported for the US and UK by Hart (1984) and Martins (2004) for Portugal.

A further specification and estimation issue for a labour demand function when the number of employees and hours worked per employee are imperfect substitutes is the issue of simultaneity. Since both  $N$  and  $H$  are decision variables, both  $N$  and  $H$  are jointly endogenous variables. This requires a simultaneous equation model with both  $N$  and  $H$  as endogenous variables, and the use of an appropriate estimator.

## *VII Conclusions*

A model of firm labour services cost minimisation is used to explain the observed diverse range of hours of work from part time, normal hours and long hours. The model explicitly recognises an important fixed cost component in total labour costs and a premium for overtime hours, and it allows labour productivity per hour worked to decline after a number of hours. At the cost minimising mix of number of employees and hours per employee, the employer equates the share of variable labour costs in total labour costs, around 0.8 but varying with occupations and industries, with the ratio of the marginal and average products per hour worked, which also varies with circumstances.

Part time employment is more common in those occupations and industries where fixed costs are a relatively small share of total labour costs, and where marginal

productivity per hour worked declines before normal working hours. These include the lower skilled occupations and the service industries.

Long hours of work employment is more prevalent where fixed costs are a relatively large share of total labour costs, where productivity per hour starts to fall only after many hours, and where institutional restraints triggering the overtime premium are weak. These conditions are found with the managerial and professional occupations and in the capital intensive industries.

Quasi-fixed labour costs, and difference across labour categories in the relative importance of fixed costs in total labour costs and in the substitutability of the different labour categories with the fixed capital stock, result in different time series properties for the number of employees and hours relative to short term product output changes, including over the business cycle. In general, as a consequence of output changes, hours respond earlier and with greater volatility than do the number of employees. The different time series properties are more marked for the relatively more skilled workers and for those with higher fixed cost shares.

A key implication of the paper is that the number of employees and hours worked per employee cannot be treated as perfect substitutes in the supply of labour services. Recognition of imperfect substitution, and in particular the declining marginal productivity per hour of work around the cost minimising labour services choice by employers, has important implications for the way economists model, estimate and interpret labour demand studies and measures of productivity. Measures of labour and multifactor productivity using total hours worked as the labour input, and implicitly assuming perfect substitution of persons and hours, confound the effects of efficiency gains with, for example technology and better practices, and changes in hours worked. In a trend historical sense, the efficiency gains have been over estimated. Labour demand studies using total hours worked as the dependent variable are found wanting. A more desirable model involves a simultaneous equation model to explain both hours and number of employees, and the elasticity parameter on the hours explanatory variable in the number employed equation will have a value less than unity and equal to the share of variable costs in total labour costs.

## REFERENCES

- Australian Bureau of Statistics (1990), *Occasional Paper: Estimates of Multifactor Productivity*, Cat. No. 5233.0, Canberra.
- Australian Bureau of Statistics (2000), *Australian National Accounts: Concepts, Sources and Methods*, Cat. No. 5216.0, Canberra.
- Australian Bureau of Statistics (2007a), *Australian Labour Market Statistics*, May 2007, Cat No 6105.0, Canberra.
- Australian Bureau of Statistics (2007b), *Information paper: Experimental Estimates of Industry Multifactor Productivity*, Cat No 5260.0.55.001, Canberra.
- Australian Bureau of Statistics (2007c), *Experimental Estimates of Industry Multifactor Productivity- Data Cube*, No 5260.0.55.002.
- Becker, G. (1962), 'Investment in Human Capital: A Theoretical Analysis', *Journal of Political Economy*, **52**, 9-49.
- Bell, D. (1982), 'Labour Utilization and Statutory Non-Wage Costs', *Economica*, **49**, 335-43.
- Brechling, F. (1965) 'The Relationship Between Output and Employment in British Manufacturing Industries', *Review of Economic Studies*, **32**, 187-216.
- Booth, A. and Ravallion, M. (1993), 'Employment and length of the working week in a unionized economy in which hours of work influence productivity', *Economic Record*, **69**, 428-36.
- Calmfors, L. and Hoel, M. (1988), 'Work Sharing and Overtime', *Scandinavian Journal of Economics*, **90**, 45-62.
- Calmfors, L. and Hoel, M. (1989), 'Work Sharing, Employment and Shiftwork', *Oxford Economic Papers*, **41**, 758-73.
- Commonwealth Treasury (1996) *Documentation of the Treasury Macroeconomic (TRYM) Model of the Australian Economy*, Canberra.
- Craine, R. (1973), 'On the Service Flow from Labour', *Review of Economic Studies*, **40**, 39-46.

- Cutler, D. and Madrian, B. (1998), 'Labor Market Responses to Rising Health Insurance Costs: Evidence on Hours Worked', *RAND Journal of Economics*, **29**, 509-30.
- Debelle, G. and Vickery, J. (1998), 'The Macroeconomics of Australian Unemployment', in G. Debelle and J. Borland (eds), *Unemployment and the Australian Labour Market*, Reserve Bank of Australia, Sydney, 235-65.
- Dixon, R., Freebairn, J. and Lim G. C. (2005), 'An Employment Equation for Australia', *Economic Record*, **81**, 204-14.
- Dolfin, S. (2006), 'An Examination of Firms' Employment Costs', *Applied Economics*, **38**, 861-78.
- Downes, P. and Bernie, K. (1999), The Macroeconomics of Unemployment in the Treasury Macroeconomic (TRYM) Model, *TYRM Related Paper No. 20*, Commonwealth Treasury, Canberra.
- Ehrenberg, R. (1971), *Fringe Benefits and Overtime Behavior*, Lexington Books, Lexington, Mass.
- Feldstein, M. (1967), 'Specification of the Labour Input in the Aggregate Production Function', *Review of Economic Studies*, **34**, 375-86.
- Hamermesh, D. (1993), *Labor Demand*, Princeton University Press, Princeton.
- Hart, R. (1984), *The Economics of Non-Wage Labour Costs*, Allen & Unwin, London.
- Hart, R. and Kawasaki, S. (1999), *Work and Pay in Japan*, Cambridge, Cambridge University Press .
- Kapteyn, A., Kalwij, A. and Zaidi, A. (2004), 'The Myth of Worksharing', *Labour Economics*, **11**, 293-313.
- Lewis, P. and MacDonald, G. (2002), 'The Elasticity of Demand for Labour in Australia', *Economic Record*, **78**, 18-30.
- Martins, A. (2004), 'The Employment-Hours Trade-off: Theory and an Application to the Portuguese Case', *Labour*, **18**, 465-502.
- OECD, (2004), *OECD Employment Outlook 2004*, Organisation for Economic Cooperation and Development, Paris.

- Oi, W. (1962), 'Labor as a Quasi-Fixed Factor', *Journal of Political Economy*, **70**, 538-55.
- Parham, D. (2004), 'Sources of Australia's Productivity Revival', *Economic Record*, **80**, 239 -57.
- Rosen, S. (1968), 'Short-Run Employment Variation on Class-I Railroads in the U.S., 1947-1963', *Econometrica*, **36**, 511-29.
- Russell, B. and Tease, W. (1991) 'Employment, Output and Real Wages', *Economic Record*, **67**, 34-45.
- Stacey, G. and Downes, P. (1995) 'Wage Determination and the Labour Market in the Treasury Macroeconomic (TRYM) Model', *TYRM Related Paper No. 13*, Commonwealth Treasury, Canberra.
- Taplin, B. and Parameswaran, P. (1993), 'Employment, Investment, Inflation and Productivity: Decisions by the Firm', *TYRM Related Paper No. 3*, Commonwealth Treasury, Canberra.
- Wooden, M. and Drago, R. (2007), 'The Changing Distribution of Working Hours in Australia', Melbourne Institute Working Paper Series, Working Paper No. 19/07, The University of Melbourne.

TABLE 1

*Percentage Distribution of Usual Hours Worked per Week by Gender, May 2007*

Usual Hours Worked per week	Males	Females	Persons
0-15	6.2	18.1	11.5
16-29	6.4	20.6	12.7
30-34	3.2	8.2	5.4
35-39	20.2	21.7	20.9
40	24.7	16.6	21.0
41-44	3.7	2.3	3.1
45-49	10.9	4.7	8.1
50 and over	24.7	7.9	17.1

*Source:* Compiled from ABS, Australian Labour Market Statistics, May 2007, 6105.0, Table 2.8.



TABLE 2

*Relative Importance of Part Time Employees and Long Hours Employees as a Percentage of the Workforce by Industry and Occupation, May 2007*

Industry or Occupation	Percentage of Employees Part Time	Percentage of Employees Long Hours
All	28.4	17.2
<u>Industry:</u>		
Accommodation, cafes and restaurants	49.9	15.6
Retail trade	46.5	12.8
Culture and recreational services	42.4	14.3
Health and community services	41.4	8.2
Education	36.4	15.6
Personal and other services	29.8	14.8
Property and business services	25.4	19.1
Agriculture, forestry and fishing	25.4	40.6
Finance and insurance	20.2	16.2
Transport and storage	18.2	24.9
Wholesale trade	16.6	21.1
Government administration and defence	16.7	7.6
Communication services	16.2	16.5
Construction	13.9	24.2
Manufacturing	12.8	17.6
Mining	2.3	36.2
Electricity, gas and water	0.0	20.7
<u>Occupation:</u>		
Elementary clerical, sales and service	62.6	4.8
Advanced clerical and service	44.2	6.3

Intermediate clerical, sales and service	40.8	6.7
Intermediate production and transport	39.6	21.5
Professionals	30.0	18.4
Labour and related	20.3	11.0
Associate professionals	18.2	24.8
Tradespersons and related	12.0	19.0
Managers and administrators	10.3	41.9

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*Source:* Compiled from ABS, 6291.0.55.003, Labour Force, Australia, Detailed, Quarterly, Table 11 Employed persons by Actual hours worked, Industry and Sex and Table 12 Employed Persons and Actual Hours Worked, Occupation and Sex.

TABLE 3  
*Percentage Distribution of Employed Persons by Actual Hours Worked per Week by  
 Gender, 1980 to 2006*

	Actual Weekly Hours Worked					
	Zero	1-15	16-34	35-40	41-49	50+
<u>Males</u>						
1980	7.4	3.1	14.4	42.1	14.7	18.4
1985	7.3	3.6	16.8	38.6	14.3	19.4
1990	7.2	4.9	15.2	35.1	14.9	22.7
1995	6.5	6.1	14.6	31.7	15.1	26.0
2000	6.3	6.6	14.5	31.3	15.0	26.3
2006	7.2	6.8	17.5	31.0	14.5	23.0
<u>Females</u>						
1980	7.7	16.6	25.9	38.2	6.6	5.0
1985	7.9	17.4	28.1	34.2	6.9	5.4
1990	7.8	18.7	28.2	31.1	7.7	6.6
1995	7.6	19.2	28.4	28.2	8.3	8.3
2000	7.5	18.7	28.7	27.7	8.9	8.5
2006	8.7	17.2	31.9	26.3	8.2	7.7

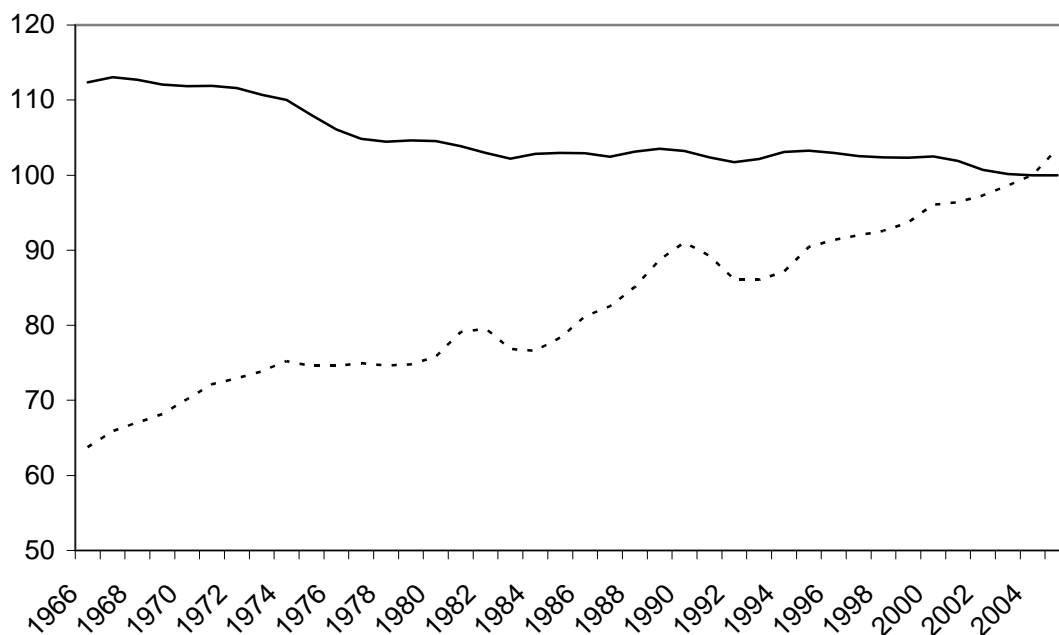
*Source:* From Wooden and Drago (2007), Table 1, in turn from ABS, Labour Force, Australia, Detailed – Electronic Delivery, ABS Catalogue No. 6291.0.55.-001 (Table 09: Employed persons and actual hours worked by sex).

TABLE 4  
*Compound Annual Percentage Changes in Labour Productivity (LP) and Multifactor Productivity (MFP) by Industry, 1985-86 – 2005-06.*

Industry	LP	Bias	LP*	MFP	Bias	MFP*
Mining	2.12	-0.04	2.16	0.36	-0.01	0.37
Manufacturing	2.13	0.01	2.12	0.88	0.01	0.87
Electricity, gas & water	3.67	-0.08	3.75	1.37	-0.03	1.40
Construction	0.77	0.00	0.77	0.64	0.00	0.64
Wholesale trade	2.74	-0.04	2.78	1.52	-0.02	1.54
Retail trade	1.49	0.04	1.45	0.83	0.03	0.80
Accommodation, cafes & restaurants	0.42	0.02	0.40	-0.10	0.02	-0.12
Transport & storage	2.47	0.00	2.47	1.80	0.00	1.80
Communication services	6.70	0.00	6.70	3.56	0.00	3.56
Finance & insurance	3.38	-0.01	3.39	1.98	-0.01	1.99
Cultural & recreation services	-0.02	0.03	-0.05	-0.66	0.02	-0.68

*Notes:* LP is labour productivity while MFP is multifactor productivity estimated by ABS on the assumption that hours per employee and number of employees are perfect substitutes. LP\* is labour productivity while MFP\* is multifactor productivity estimated assuming hours and employees are imperfect substitutes with  $\alpha = 0.83$  as described in the text. Bias = LP – LP\* and MFP – MFP\*, with more details in the text.

FIGURE 1.  
*Indices for the Number Employed (broken line) and Average Hours Worked per Employee (solid line) for the Aggregate Market Sector (2003/04 = 100).*



*Notes:* Dates are for the financial year ending in the year shown. We are grateful to Paul Roberts from the ABS for providing us with unpublished data on aggregate market sector total hours worked and number employed for the period 1978/79 – 2004/05 which allowed us to compute the implied average hours for the market sector over the period. For the period prior to 1978/79 we have used the analogous TRYM market sector indices (which are highly correlated with the ABS series over the period 1978/79 – 2004/05). We splice the TRYM data for 1966/67 – 1978/79 onto the ABS series for N and H over the period 1978/79 – 2004/05 to generate a series for N and H for the whole of the period 1966/67 – 2004/05.

FIGURE 2  
*Part-time Employment*

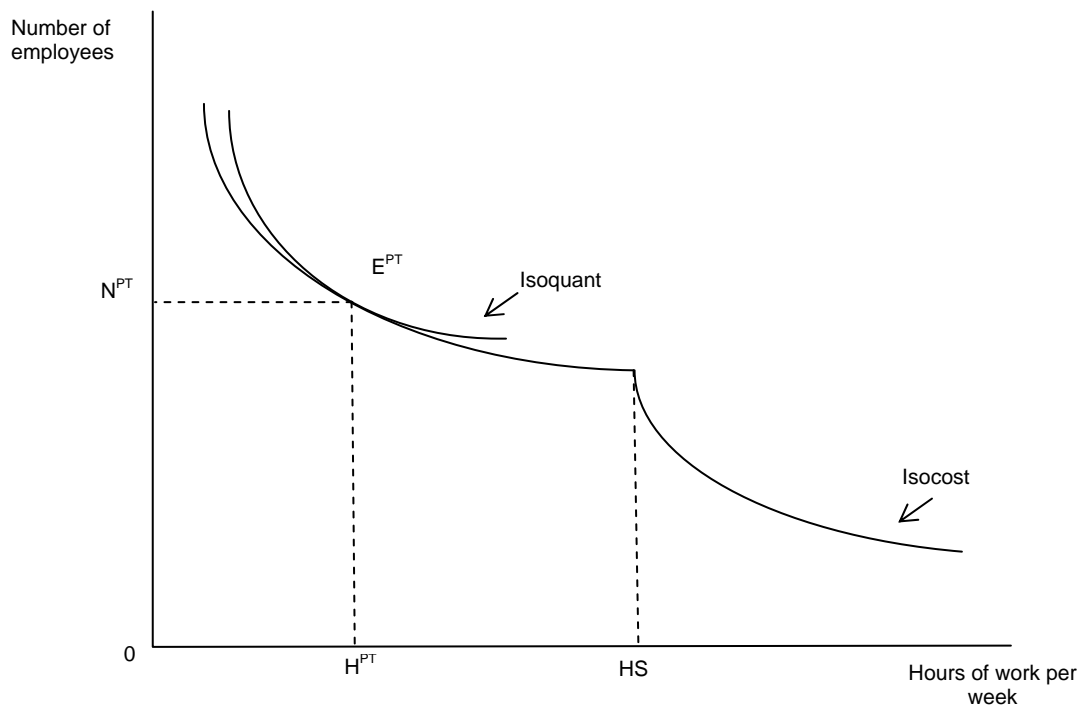


FIGURE 3  
*Overtime Employment*

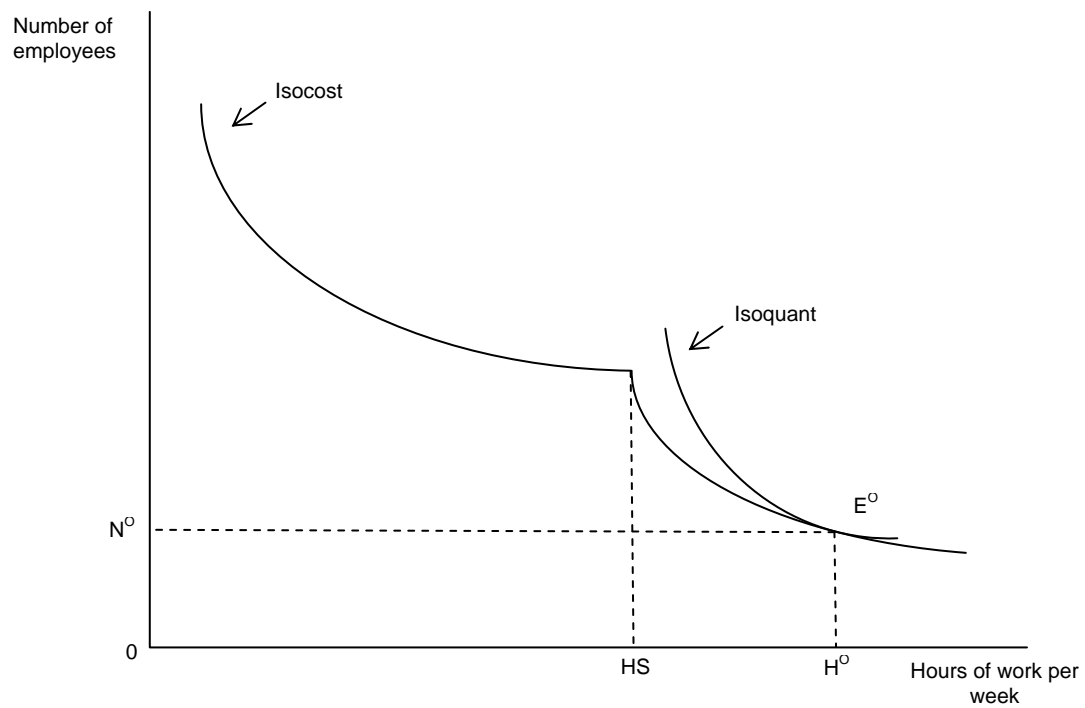


FIGURE 4  
*Normal Hours Full-time Employment*

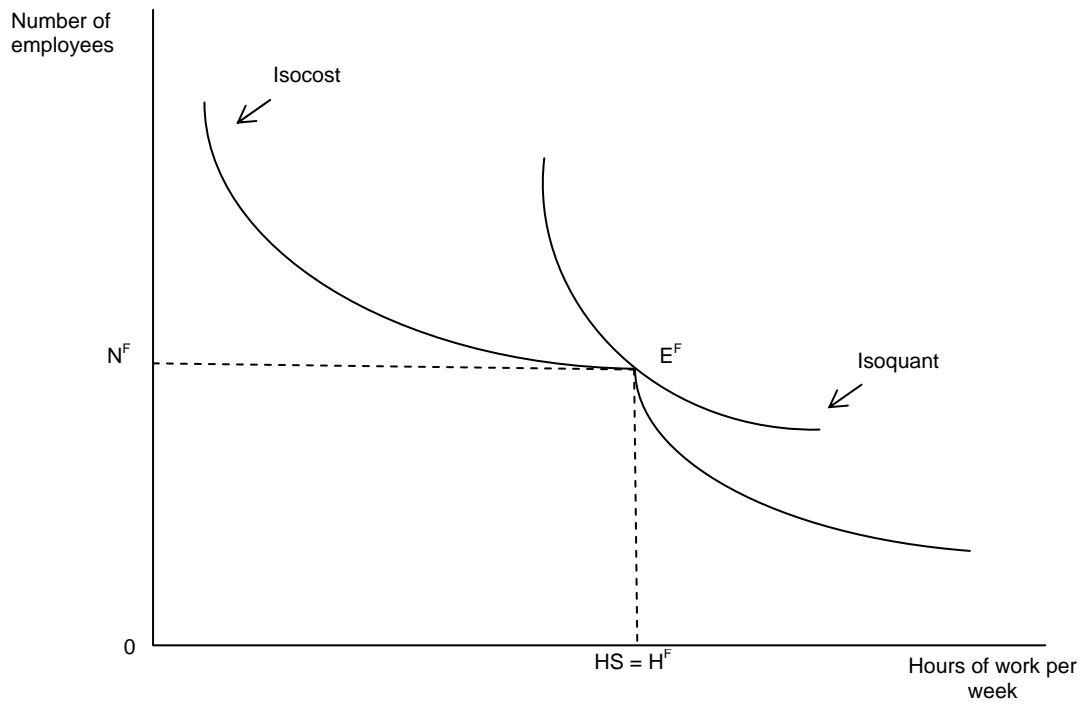
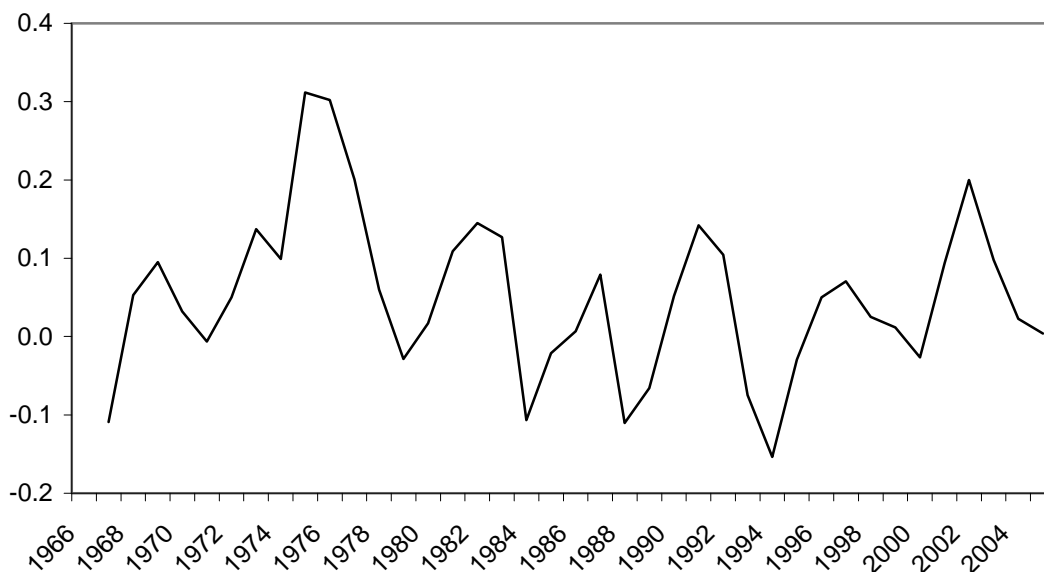




FIGURE 5

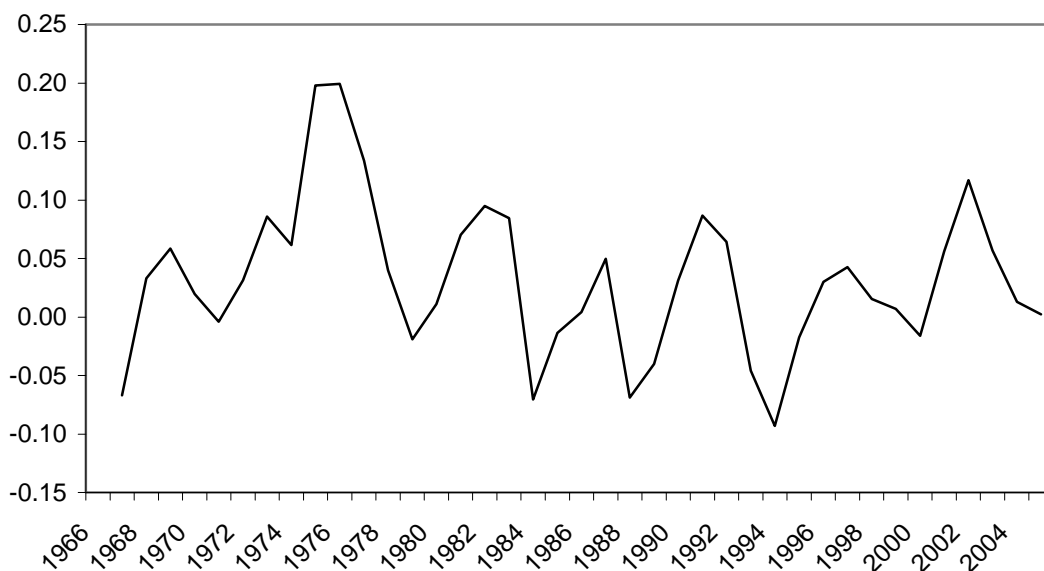
*Bias in calculating the year on year before rate of growth in labour productivity, calculated as described in the text using  $\alpha = 0.83$ .*



*Note:* See n3, p 16.

FIGURE 6

*Bias in calculating the year on year rate of growth in MFP, calculated as a Divisia index using  $\alpha = 0.83$  and labour cost shares data provided by the ABS.*



*Note:* See n3, p 16.