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The Composition of Government Expenditure in an Overlapping Generations Model

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Abstract

This paper examines the choice of government expenditure on public goods and transfer payments (in the form of pension) in an overlapping generations model, in which individuals live for two 'periods' and expenditure is financed on a pay-as-you-go (PAYG) basis. The condition required for majority support of the social contract involved in the PAYG scheme is established and shown to be independent of tax rates and expenditure levels. The choice of expenditure composition can thus be made conditional on acceptance of the social contract. Two decision mechanisms regarding the choice of government expenditure are considered. The first is positive and involves majority voting and the second is normative and involves maximizing a social welfare function. In each case the ratio of the transfer payment to public goods expenditure depends, among other things, on the ratio of median to mean income. A reduction in the skewness of the income distribution is associated with a reduction in this ratio, at a decreasing rate.

JEL code: D72, H41, H53, H11

Keywords: Overlapping Generations, Equilibrium Growth, Median Voter, Optimal Expenditure, Public Goods, Pensions

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1 Introduction

There are substantial variations among countries in their tax structures and composition of government expenditure.[1] This paper examines the choice of the composition of government expenditure in the context of an overlapping generations model in which a pure public good and a transfer payment, in the form of a pension, are tax-financed on a pay-as-you-go (PAYG) basis. The unconditional pension therefore involves a decision regarding effective income shifting within the life cycle as well as intra- and inter-generational redistribution. Two decision mechanisms are considered. The first is positive and involves majority voting by members of each cohort regarding desired pension and public good expenditure during the retirement period, on the understanding that during the working period each cohort finances the expenditure previously agreed by the preceding cohort and voters are aware of the nature of the government budget constraint.[2] There is therefore a social contract in which each generation, in the retirement period, is able to benefit from the income and population growth of the following generation. The majority voting equilibrium exhibits balanced growth with pensions and public goods expenditure per capita both growing with population and income growth, while their ratio remains constant.

The majority voting outcome reflects the selfish preferences of the median voter, who holds the balance of power, and involves a 'dictatorship' of the minority by the majority. In the second approach, this allocation is compared with a normative mechanism in which a decision-maker maximises a social welfare function expressed in terms of the lifetime utilities of members of each cohort; in relative terms all cohorts are treated equally along the balanced growth path. The welfare function makes explicit the value judgements,

[1] For empirical evidence, see Creedy and Moslehi (2007a).

[2] Hassler *et al.* (2003, 2007), and Hassler *et al.* (2005) consider two-period overlapping generations models in which individuals are born identical but become successful or unsuccessful. Young individuals are able to affect the probability of becoming successful by making private investments. In their private behaviour, all individuals take the actions of others as given and choose their best actions or investments. In their public behaviour, individuals take into account how current political choice affects the distribution of voters and future policies.

involving for example inter-personal comparisons of utility, of the judge. The situation in which a judge would agree with the median voter – though for entirely different reasons – can thus be established. This approach, though involving steady growth, contrasts with the large literature concentrating on optimal expenditure in a growth framework. In such studies, a social planner maximises the multi-period welfare function involving a representative agent to obtain the Pareto optimal inter-temporal allocation: for example, see Chen (2006) and Agénor (2008).

Relatively little attention seems to have been given to the analysis of the composition of government expenditure. Nevertheless, there is a substantial politico-economic literature on decisions involving government expenditure. A majority of this literature focuses on one type of government expenditure, either public goods expenditure or redistributive transfer payments. [3] Since the single type of government expenditure is financed by income taxation, the choice of government expenditure in these models is determined by the tax rate, which is often chosen by majority voting, stemming from the early work of Meltzer and Richard (1981). Several recent studies consider more than one type of government expenditure. Bearse *et al.* (2001) study majority voting over a transfer payment and public education in a static framework. Hassler and *et al.* (2007) consider redistribution policy as well as provision of public goods financed by imposing a tax on the rich, which indicates the extent of redistribution. Creedy and Moslehi (2007a) examine majority voting over government expenditure on transfer payments as well as public goods, within a static framework. This paper follows this line of research, with a focus on the composition of government expenditure within an overlapping generations framework.

Since the emphasis of the present study is on the composition of expenditure rather than its total, as in Bearse *et al.* (2001), the income tax rate is assumed to be exogenously fixed; it is not a decision variable. This ensures

[3] For example, Tridimas (2001), Grossmann (2003) and Tridimas and Winer (2005) consider expenditure on public goods, while Meltzer and Richard (1981), Grossman and Helpman(1998), Hassler *et al.* (2003), Hassler *et al.* (2005), Krusell and Rios-Rull (1999), Azzimonti *et al.* (2006), and the survey by Borck (2007) concentrate on transfer payments.

that voting is only over one dimension.[4] After pointing out that this is a common assumption, Tridimas (2001, p. 308) suggested that, 'This is less restrictive than it first appears, since in practice governments are often constrained in the policy instruments that they may vary at anyone [*sic*] time'. In practice, tax and expenditure policies are usually debated separately and stronger constraints are usually imposed on changes in income tax rates.

In the present dynamic context, explicit solutions for the components of government expenditure can be found in which more inequality gives rise to the transfer payment (here the pension) forming a larger proportion of total government expenditure. This result is consistent with models examining majority voting over the tax rate, with an unconditional transfer payment, in which a consistent result is that more basic inequality leads to the choice of a more redistributive tax and transfer structure. For examples, see Meltzer and Richard (1981) and Krusell and Rios-Rull (1999). However, empirical evidence concerning this relationship, based on cross-sectional data for a range of countries, has been found to be mixed. By considering the composition, rather than absolute levels, of expenditure in a dynamic setting, the present study shows that in view of the wide range of factors affecting the composition of expenditure, it may indeed in practice be difficult to observe a simple relationship between redistribution and basic inequality in cross-country comparisons.

The paper is arranged as follows. Section 2 describes the framework of analysis. In view of its central role, this section also examines the condition under which a majority of the members of any cohort support the required social contract between generations, whereby each generation agrees to finance the pensions of those currently retired (and who, in view of population growth, form a minority of the population at any time) knowing that the next generation will do the same. In the present model there is thus no conflict between generations. This involves extending the condition obtained by Aaron (1966) and Samuelson (1958).

[4]In some voting models involving more than one dimension, a two-stage procedure is envisaged. Other studies use intermediate preferences and consider a multidimensional policy with unidimensional conflict. However, the condition of intermediate preferences is restricted; see Borge and Rattsø, J. (2004) for examples of both approaches.

Support for the social contract does not actually depend on the precise expenditure composition, so agreement can be obtained prior to voting over the composition. Section 3 then characterizes the voting equilibrium regarding government expenditure where voting takes place in each period when individuals vote only on the pension to be paid during the next period, for a given tax rate. Hence, there is no incentive for members of the old cohort to vote (as their preferences are entirely selfish). Those currently retired do not have a second vote over the public good expenditure, from which they benefit (but do not help to finance), during the period, since this has already been determined by their vote in the previous period.[5] The preferences of members of the young cohort are shown to be single-peaked, satisfying the sufficient condition for the existence of a majority-voting equilibrium. Closed-form solutions for expenditure on pensions and public goods are obtained. It is also shown that the voting equilibrium is a balanced-growth equilibrium such that all endogenous variables grow at the same rate.

Comparative static properties of the model and some numerical examples are examined in Section 4. Section 5 examines the optimal composition of government expenditure, defined as the composition that maximises a social welfare function expressed in terms of the lifetime utility of each generation. Along a balanced growth path all cohorts are treated equally, so only one cohort needs to be considered. Using a general form of welfare function, it turns out that the optimal composition takes a similar form to the majority voting outcome, except that median income is replaced by a welfare-weighted average income measure. Brief conclusions are in section 6.

2 The Economic Environment

This section begins by describing the overlapping generations model with a public sector, in subsection 2.1. In views of its central role, the condition under which a majority of each cohort supports the intergenerational social

[5] Hassler *et al.* (2007) use a similar assumption (each generation only votes once) to find the political equilibrium in an overlapping generations context. In their study, agents vote on tax at each period and also decide how the revenue should be spent.

contract is established in subsection 2.2.

2.1 The Two-Period Framework

Each individual is assumed to live for two periods, a working and a retirement period, so that the economy is populated by two overlapping cohorts in any given period. Individuals have identical preferences but are heterogenous with respect to income endowments. A young individual i, born at time t, works in the first period and receives an exogenously fixed income, $y_{i,t}$. If the objective were to examine the choice of income tax rate, the assumption of exogenous incomes would make no sense, but the emphasis here is solely on the composition of expenditure. Income is taxed at the rate τ, which is the same for all individuals and is assumed to be exogenously determined.

In period t, a young individual, i, allocates disposable income between current consumption, $c_{1i,t}$, and savings, $s_{i,t}$. In the second period of life, the individual finances consumption of private goods, $c_{2i,t+1}$, using the unconditional and untaxed pension from the government, b_{t+1}, and the return on savings, $(1+r)s_{i,t}$, where r is the constant interest rate at which individuals can borrow or lend.[6] Government expenditures on pure public goods in t and $t+1$ are denoted as G_t and G_{t+1}. Prices are normalised to unity, so that c denotes private consumption expenditure.

The transfer payment is referred to here as a pension, since it is received in the second period of the life cycle.[7] However, it may be thought of more broadly as a standard type of income transfer since it augments the exogenously fixed income. Assuming perfect capital markets, some low-income people may wish to vote for a high value of b_{t+1}, part of which is used to repay a loan in the first period while the remainder finances consumption in the second period. Conversely high-income individuals may prefer a low, or even zero, transfer while making positive savings during the working period.

In view of the inter-generational transfers in addition to the income shifting and intra-generational redistribution, it is necessary to allow for popula-

[6] For simplicity it is assumed that there is no tax on interest income.

[7] The introduction of an explicit transfer received in the working period would involve considerable problems arising from multidimentional voting.

tion and income growth. Suppose the average income of young individuals grows at a constant rate of ω over time, so that:

$$\frac{\bar{y}_{t+1}}{\bar{y}_t} = 1 + \omega. \tag{1}$$

Also, there is positive and constant growth, at the rate n, in the population, so that:

$$\frac{N_{t+1}}{N_t} = 1 + n, \tag{2}$$

where N_t denotes the number of individuals born in period t. Here it is assumed that income growth involves an equal proportional change in all incomes and that population growth involves an equal increase in frequencies at each income level, so that ω and n are independent.

2.2 Majority Support for The Social Contract

This section examines the condition required for the majority of each generation to be better off with the PAYG arrangement, thus ensuring the co-operation between generations. Consider a situation in which there is no transfer payment. For any given value of expenditure on the public good, G, and its associated tax rate, the question concerns the condition under which an individual (with given $y_{i,t}$) is better off with a transfer payment (and its associate higher tax rate) compared with a framework in which retirement income is provided by private savings alone, involving income shifting without the intra- and inter-generational redistribution of the transfers. Clearly, those who are most likely to prefer a private system are those with relatively high incomes. It is shown here that the condition required for each person's support of a PAYG system is independent of the given value of G and of the transfers (and thus the tax rate). Thus, having established support for the social contract, it is then possible to consider voting over the precise composition of expenditure, for a given tax rate.

The context here is thus somewhat different from the basic treatment of social insurance by Samuelson (1958) and Aaron (1966), who essentially considered the conditions under which the average consumption of each generation is higher with a PAYG system. Such a system allows each generation to share in the benefits arising from population and productivity growth, since later generations will be both larger and richer, so long as those gains are sufficient to outweigh the returns from private investment plans. They found that social insurance can increase average welfare 'if the sum of the rates of growth of population and real wages exceeds the rate of interest' (Aaron, 1966, p. 372).[8] The context, like the present treatment, is nevertheless confined to a partial equilibrium analysis, so no consideration is given to the possibility that different regimes imply different private returns to saving, arising because total savings in the two systems are substantially different.

By considering whether individual i supports a PAYG social contract as an addition to the finance of a given value of public good expenditure, comparisons between public and private systems are made such that they provide the same expenditure on public goods in each period. Let a superscript, P, be used to denote values in the PAYG system with a basic pension (which may of course be augmented by private savings), while F indicates values in the scheme with private pension funds only. Thus, $G_t^P = G_t^F = G_t$, while tax rates in the two system are obviously different, at τ^P and τ^F. The transfer payment to be received in period $t+1$ is b_{t+1}^P.

From the lifetime budget constraint, the present value of the ith individual's life time income under the PAYG system is given by $(1 - \tau^P)y_{i,t} + b_{t+1}^P / (1 + r)$. Consider next the privately funded system where there is no transfer payment and income tax finances only the provision of public goods. Individual i's lifetime income is simply $(1 - \tau^F)y_{i,t}$.

Since both systems provide the same amount of public good under the two systems, a sufficient condition for utility to be higher in the PAYG system compared with private funding is that lifetime income is higher. This requires

[8]Creedy and van de Ven (2000) extend the Aaron-Samuelson analysis by allowing for labour supply effects of taxation and for increasing longevity. Walque (2005) uses non-cooperative game theory to examine an overlapping generations model with productivity and population growth. He assumes that some public transfer payment is proposed and voters decide to accept or reject it through their vote. The young form the majority at each time, since population growth is positive. He finds similar conclusions to Aaron (1966).

$(1-\tau^P)y_{i,t} + \frac{b^P_{t+1}}{1+r} > (1-\tau^F)y_{i,t}$, or equivalently:

$$(1+r) < \frac{b^P_{t+1}}{y_{i,t}(\tau^P - \tau^F)}. \tag{3}$$

The government budget constraint at time $t+1$ in the privately funded system is:

$$G^F_{t+1} = \tau^F N_{t+1}\bar{y}_{t+1}, \tag{4}$$

and in the PAYG system it is given by:

$$G^P_{t+1} + N_{t-1}b^P_{t+1} = \tau^P N_{t+1}\bar{y}_{t+1}. \tag{5}$$

Combining these two constraints, and using $G^F_{t+1} = G^P_{t+1} = G_{t+1}$, gives the pension as:

$$b^P_{t+1} = (1+n)\bar{y}_{t+1}\left(\tau^P - \tau^F\right). \tag{6}$$

Substituting b^P_{t+1} from (6) into (3), and using $\bar{y}_{t+1} = \bar{y}_t(1+\omega)$ gives the following condition under which the ith individual is better-off in the PAYG system:

$$(1+n)(1+\omega)\frac{\bar{y}_t}{y_{i,t}} > (1+r). \tag{7}$$

An important feature of this condition is that it is independent of G, b and τ. Hence agreement by a majority of each cohort for the use of a PAYG scheme and its associated social contract is established prior to considerations regarding the choice of actual expenditure levels. For a positively skewed income distribution, median income, $y_{m,t}$, is less than the arithmetic mean, $\bar{y}_t$: hence the greater the skewness, the more likely is the condition above to be satisfied for given values of the relevant rates.

This can be compared with the basic Aaron and Samuelson condition in a model without heterogeneity and public goods. The basic Aaron and Samuelson condition is $(1+n)(1+\omega) > (1+r)$.[9] From (7) the condition for the 'average' individual, for whom $y_{i,t} = \bar{y}_t$, is precisely the same as the Aaron-Samuelson condition. The higher-income individuals are more likely to prefer the privately funded scheme, which involves only income shifting

[9] It is by ignoring cross-product terms that the condition is often stated as $r < n + \omega$.

rather than redistribution within and between generations. A majority of the cohort of workers at time t are more likely to prefer the PAYG scheme even if the Aaron-Samuelson condition is not satisfied.

All individuals must of course comply with the majority voting outcome. Hence if $(1+n)(1+\omega)\frac{\bar{y}_t}{y_{m,t}} > (1+r)$, a PAYG system is maintained. Those high-income individuals who would prefer only private funding are not allowed to 'contract out' of, or withdraw from, the social contract and the pension system. Without this compulsory element there would be an adverse selection problem arising from the gradual reduction in average income, of those remaining in the system, as the richest individuals gradually contract out.

3 The Voting Equilibrium

This section examines the majority voting equilibrium composition of expenditure, assuming that the condition required for support of the PAYG system, established above, is satisfied. Voting concerns only on the pension to be received in the next period, for the given tax rate. Individuals are assumed to have full information about the government budget constraint. The resulting public good expenditure is determined from the government budget constraint, and before voting the working cohort has been committed to pay for public good expenditure and pensions decided by the previous cohort. Since there is no incentive for the old to vote they are absent from the election. Therefore, the median member of the young cohort is the decisive voter. As part of the social contract, young individuals understand that they must finance the pension of those currently retired (and of course the majority of them recognise that they are better off by doing this). Individual preferences, where the lifetime indirect utility function is expressed in terms of the transfer payment, public good expenditure and the tax rate, are examined in subsection 3.1. The voting equilibrium is derived in subsection 3.2, and subsection 3.3 shows that it is a balanced-growth equilibrium.

3.1 Individual Preferences

Each individual is assumed to have the following Cobb-Douglas lifetime direct utility function, expressed in logarithmic form:

$$U_{i,t} = \log c_{1i,t} + \gamma \log G_t + \beta \left(\log c_{2i,t+1} + \gamma \log G_{t+1} \right), \tag{8}$$

where $0 < \beta = \frac{1}{1+\rho} < 1$ is the discount factor and ρ is the time preference rate, and γ is the weight attached to consumption of public goods. An alternative approach would be to express utility in terms of the quantity, Q_G, of the public good consumed by each person, rather than the total government expenditure on it. If the good is produced at a constant cost per unit of p, then $G = pQ_G$ must continue to appear in the government's budget constraint below. However, it can be shown that the results are identical to those obtained by using the present specification.[10]

The lifetime budget constraint of an individual is given by:

$$c_{1i,t} + \frac{c_{2i,t+1}}{(1+r)} = (1-\tau) y_{i,t} + \frac{b_{t+1}}{(1+r)} \equiv M_{i,t}. \tag{9}$$

This form allows for the fact that tax-financed public goods are non-excludable so that individuals are not charged at the point of consumption. To examine the voting equilibrium it is necessary to obtain each individual's indirect utility function, $V_{i,t}$, as follows.

The consumption plans, conditional on the values of public expenditure and the pension, are given, using the standard properties of Cobb-Douglas utility functions, as:

$$c_{1i,t} = \frac{M_{i,t}}{(1+\beta)}, \tag{10}$$

$$c_{2i,t+1} = \frac{\beta (1+r) M_{i,t}}{(1+\beta)}. \tag{11}$$

[10] This arises from the homothetic nature of the Cobb-Doublas utility function, which also ensures that consumption does not become concentrated on a single good as population size increases. For a detailed analysis of this point in a single-period framework, see Creedy and Moslehi (2007a). In a general equilibrium model, it is of course expected that the unit cost, p, would vary as the output share varies.

Hence planned private savings of the young individual, $s_{i,t}$, are:

$$s_{i,t} = \frac{\beta(1-\tau)y_{i,t}}{(1+\beta)} - \frac{b_{t+1}}{(1+r)(1+\beta)}. \tag{12}$$

From (12) an increase in the tax rate and the pension reduces savings, while an increase in the interest rate increases savings. Without the pension, the income and substitution effects of changes in the interest rates would offset each other such that savings would be independent of the interest rate.[11] Nevertheless, with a pension, the substitution effect outweighs the income effect such that an increase in the interest rate increases private savings. According to the saving function (12) the individual borrows if:

$$y_{i,t}(1-\tau) < b_{t+1}\left(\frac{1+\rho}{1+r}\right). \tag{13}$$

That is, borrowing takes place if disposable income is low in relation to the pension. These low income individuals would borrow to finance their first period consumption and repay their debt with the pension received in the retirement period.

The indirect utility function, $V_{i,t}$, is obtained by substituting the optimal $c_{1i,t}$ and $c_{2i,t+1}$ into the direct utility function (8), whereby:

$$V_{i,t} = \log\left(\frac{M_{i,t}}{1+\beta}\right) + \beta\log\left(\frac{\beta(1+r)M_{i,t}}{(1+\beta)}\right) + \gamma\left(\log G_t + \beta\log G_{t+1}\right). \tag{14}$$

However, the pension and public goods expenditure in each period are financed on a pay-as-you-go (PAYG) basis. Hence the values of G_t and G_{t+1} can be expressed in terms of b_t and b_{t+1} using the government budget constraint; there is only one degree of freedom in policy choices. The government budget constraint is given by:

$$G_t + N_{t-1}b_t = \tau N_t \bar{y}_t. \tag{15}$$

Substituting into (14) gives indirect utility:

$$\begin{aligned} V_{i,t} = {} & \log\left(\frac{(1+r)(1-\tau)y_{i,t} + b_{t+1}}{(1+r)(1+\beta)}\right) \\ & +\beta\log\left(\frac{\beta\left((1+r)(1-\tau)y_{i,t} + b_{t+1}\right)}{1+\beta}\right) \\ & +\gamma\log\left(\tau N_t\bar{y}_t - N_{t-1}b_t\right) + \beta\gamma\log\left(\tau N_{t+1}\bar{y}_{t+1} - N_t b_{t+1}\right). \end{aligned} \tag{16}$$

[11] This is a particular property of Cobb-Douglas utility functions.

Indirect utility is therefore a function of b_{t+1} along with other variables and preference parameters.

3.2 Majority Voting

From (16), voting involves only one dimension, the value of b_{t+1}, because all other variables determining an individual's indirect utility are either predetermined or exogenously given. If the indirect utilities for all young individuals are single-peaked, it is known that the majority voting outcome is dominated by the median voter, who in the present context is the individual with median income, $y_{m,t}$.

Single-peakedness is guaranteed if the relationship between $V_{i,t}$ and b_{t+1} is strictly concave for all individuals, that is, if $\partial^2 V_{i,t}/\partial b_{t+1}^2 < 0$ for all i. This condition is confirmed by differentiation of (16). Consequently, maximizing the indirect utility function with respect to b_{t+1} gives the majority choice of pension at time $t+1$, $b_{m,t+1}$. The first-order condition is

$$\frac{\partial V_{m,t}}{\partial b_{m,t+1}} = \frac{1+\beta}{(1+r)(1-\tau)y_{m,t} + b_{m,t+1}} - \frac{\beta\gamma N_t}{\tau N_{t+1}\bar{y}_{t+1} - N_t b_{m,t+1}} = 0, \quad (17)$$

and $b_{m,t+1}$ is solved as:

$$\begin{aligned} b_{m,t+1} &= \bar{y}_t \left(\frac{1+\beta}{1+\beta+\beta\gamma}\right) \\ &\quad \times \left\{(1+n)(1+\omega)\tau - \left(\frac{\beta\gamma}{1+\beta}\right)(1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}\right\}. \quad (18) \end{aligned}$$

By substituting $b_{m,t+1}$ into the government budget constraint at time $t+1$, the majority choice of public goods expenditure, $G_{m,t+1}$, can be solved as:

$$\frac{G_{m,t+1}}{N_t} = \bar{y}_t \left(\frac{\beta\gamma}{1+\beta+\beta\gamma}\right) \left\{(1+n)(1+\omega)\tau + (1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}\right\}. \quad (19)$$

The focus here is on the ratio of the total expenditure on pension to that on public goods, $R_{m,t+1}$, which is given by:

$$R_{m,t+1} = \frac{\frac{(1+\beta)}{\beta\gamma}(1+n)(1+\omega)\tau - (1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}}{(1+n)(1+\omega)\tau + (1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}}. \quad (20)$$

This result shows that $R_{m,t+1}$ depends, *inter alia*, on the ratio of median income to mean income at time t and parameters regarding population growth, income growth, tax rate and preference. Changing absolute income levels by a shift in the distribution of income does not affects the majority choice of the composition of expenditure; only the *ratio* of median to mean income matters. The growth rates of population and incomes, n and ω, also appear in a symmetric fashion in (20); they both have the same effect on $R_{m,t+1}$.

Substituting (18) into (13) gives the criterion for borrowing by individual i, with the majority choice of pension, as:

$$\frac{y_{i,t}}{\bar{y}_t} < \frac{(1+\beta)(1+n)(1+\omega)\tau - \beta\gamma(1+r)(1-\tau)\frac{y_{m,t}}{\bar{y}_t}}{\beta(1+r)(1+\beta+\beta\gamma)(1-\tau)}. \tag{21}$$

This result shows that individual i borrows to finance first-period consumption if the ratio of i's income to average income is less than some critical value, which is determined by the ratio of median to mean income, among other parameters.

3.3 The Balanced Growth Path

In this subsection the voting equilibrium is shown to be a balanced-growth equilibrium in which all endogenous variables, including total expenditure on public goods and pensions, total consumption for young individuals and for old individuals, and total savings by young individuals, grow at the same rate.

First, the crucial ratio, $y_{m,t}/\bar{y}_t$, is constant, since income growth is assumed to involve an equal proportional increase at all income levels. This implies by equation (18) that the majority choice of pension per old individual, $b_{m,t+1}$, grows at the same rate as the average income of individuals, $\bar{y}_t$, and consequently total expenditure on the pension, $N_t b_{m,t+1} \equiv B_{m,t+1}$, grows at the same rate as the total income of individuals. That is, pension per capita grows at rate ω and total pension expenditure grows at the rate, $n + \omega$.[12] Similarly, by equation (19) total expenditure on public goods, $G_{m,t}$, grows at the rate, $n + \omega$, and public goods expenditure per capita,

[12]This ignores the term $n\omega$ in the expansion of $(1+\omega)(1+n)$.

$G_{m,t}/(N_{t-1}+N_t)$, grows at the rate, ω. As a result, the ratio of pension expenditure to public goods expenditure is a constant, as also confirmed by equation (20).

The period-t total consumption of young individuals, $N_t \sum_{i=1}^{N_t} c_{1i,t}$, total savings of young individuals, $N_t \sum_{i=1}^{N_t} s_{i,t}$, and total consumption of old individuals, $N_{t-1} \sum_{i=1}^{N_{t-1}} c_{2i,t}$, can be expressed by substituting $b_{m,t+1}$ into (10), (12) and (11). All these variables grow at the rate, $n+\omega$, and their per capita terms grow at the rate, ω.

Hence, the voting equilibrium is characterised by a balanced growth path, along which all aggregate endogenous variables grow at the same rate as aggregate income, and per capita variables grow at the same rate as average income. For simplicity incomes are assumed to be exogenously determined in the model. But it can be easily and naturally incorporated into the model that income growth results from exogenous technological progress. Hence these implications regarding the balanced growth path are consistent with those of neoclassical growth models with exogenous PAYG social security, despite the fact that the social security is determined by majority choice in the present model.

4 Some Comparative Statics

This section presents some comparative static properties of the model. The aim is to examine how total expenditures on pension, $B_{m,t+1}$, public goods, $G_{m,t+1}$, and their ratio, $R_{m,t+1}$, change in response to changes in parameters of the model. As shown in equations (18),(19) and (20), these relations are nonlinear. The signs of first and second derivatives of these variables with respect to each parameter are reported in Table 1.

A key determinant is $y_{m,t}/\bar{y}_t$, the ratio of median to average income. A rise in $y_{m,t}/\bar{y}_t$ represents a fall in inequality. First, $B_{m,t+1}$ and $G_{m,t+1}$ are linearly decreasing and increasing in $y_{m,t}/\bar{y}_t$, respectively, suggesting that higher inequality causes voters to vote on higher pension expenditure and lower public goods expenditure. This implies that a rise in inequality would lead to a higher ratio of pension expenditure to public goods expenditure.

Table 1: Comparative Statics of Median Voter's Choice of Expenditure on Pension and Public Goods and Their Ratio

	$y_{m,t}/\bar{y}_t$	τ	γ	β	r	ω	n
First Derivative							
$B_{m,t+1}$	$-$	$+$	$-$	$-$	$-$	$+$	$+$
$G_{m,t+1}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$R_{m,t+1}$	$-$	$+$	$-$	$-$	$-$	$+$	$+$
Second Derivative							
$B_{m,t+1}$	0	0	$+$	$+$	0	0	$+$
$G_{m,t+1}$	0	0	$-$	$-$	0	0	$+$
$R_{m,t+1}$	$+$	$-$	$+$	$+$	$+$	$-$	$-$

This is confirmed by a negative first derivative of $R_{m,t+1}$ with respect to $y_{m,t}/\bar{y}_t$. The second derivative of $R_{m,t+1}$ with respect to $y_{m,t}/\bar{y}_t$ is positive, implying that the ratio of pension expenditure to public goods increases at an increasing rate as inequality rises.[13]

It is also of interest to consider how the majority choice of total expenditure on pensions and public goods change with the tax rate. Clearly $B_{m,t+1}$ is linearly increasing in τ.[14] However, $G_{m,t+1}$ is also linear in τ, but increases with τ only if $(1+n)(1+\omega)-(1+r)\frac{y_{m,t}}{\bar{y}_t}>0$. This condition is the same as that derived above for majority support for the PAYG social security system. The majority choice of the composition of government expenditure, $R_{m,t+1}$ is increasing in τ unambiguously, but at a decreasing rate, if again the condition for majority support for the social contract is satisfied. These results suggest that an increase in the tax rate gives the government more income to spend on both types of expenditure, but the increase in pension expenditure is relatively higher than the increase in public goods expenditure.[15]

The comparative static results with respect to γ suggest that an increase in the weight on public goods in the utility function unambiguously increases the total expenditure on public goods, but decreases total expenditure on

[13] A negative relationship between basic equality and the transfer payment relative to pubic good expenditure was also found by Creedy and Moslehi (2007a,b) in a static model.

[14] If labour supply were affected by the tax rate, this property would clearly be modified.

[15] However, Creedy and Moslehi (2007a, b) find a concave relationship between the expenditure ratio and the tax rate. This arises because they allow for adverse incentive effects arising from the tax and transfer system.

pensions and the ratio of pension to public goods expenditure.

The results with respect to β suggest that an increase in the discount factor has a negative effect on the total expenditure on pensions and on the ratio of pensions to public goods, but increases the total expenditure on public goods. This result can be understood from a positive relationship between a young individual's savings and the discount factor; see equation (12). A higher discount factor, that is a higher weight on second-period utilities, leads to more private savings by individuals at a young age, such that individuals tend to vote on a lower public-saving-pension, which results in a higher public goods expenditure and a lower ratio of pension to public goods expenditure.

An increase in the interest rate has similar effects to those of an increase in the discount factor. It increases public goods expenditure, but decreases expenditure on pensions and its ratio to public goods expenditure. This is clear from the condition for majority support for the social security system, in equation (7). An increase in the interest rate raises the return on private savings such that individual are more likely to prefer the privately funded scheme; that is, individuals are more likely to vote on a lower pension expenditure.

Regarding parameters governing the growth of income and population, the results show that increases in ω and n increase total expenditure on pension and on public goods, and also their ratio. With income growth or population growth, tax revenues of the government are increased such that the government is able to spend more on both types of expenditure. However, the increase in the expenditure on pensions is higher than the expenditure on public goods because a higher n and ω make individuals more likely to prefer the PAYG system, as shown in (7).

Consider the special case where $n = \omega = 0$ and $r = \rho = 0$. This no-growth and no-discounting situation implies a steady state in which endogenous variables remain constant over time, and substitution in (20) gives:

$$R_{m,t+1} = \frac{\frac{\tau}{\gamma/2} - (1-\tau)\frac{y_{m,t}}{\bar{y}_t}}{\tau + (1-\tau)\frac{y_{m,t}}{\bar{y}_t}}. \tag{22}$$

Then:

$$\frac{dR_{m,t+1}}{d\left(\frac{y_{m,t}}{\bar{y}_t}\right)} = -\frac{(1-\tau)}{\tau + (1-\tau)\frac{y_{m,t}}{\bar{y}_t}}\left(1 + R_{m,t+1}\right). \tag{23}$$

This may be compared with an economy with $r = \rho = 0$ but with positive growth, such that $(1+n)(1+\omega) = \xi > 1$, and:

$$\frac{dR_{m,t+1}}{d\left(\frac{y_{m,t}}{\bar{y}_t}\right)} = -\frac{(1-\tau)}{\xi\tau + (1-\tau)\frac{y_{m,t}}{\bar{y}_t}}\left(1 + R_{m,t+1}\right). \tag{24}$$

Hence, the growing economy – with the same tax rate and initial values of $\frac{y_{m,t}}{\bar{y}_t}$ and $R_{m,t+1}$ – has a smaller absolute response of the expenditure ratio to a change in $\frac{y_{m,t}}{\bar{y}_t}$.[16] Furthermore, it can be shown that a non-zero value of ρ produces a result similar to (24) except that $\xi\tau$ in the denominator is replaced by a term in which τ is multiplied by a constant depending on ρ and ξ; this is lower than ξ and therefore implies a slightly higher response, compared with the growing economy with zero ρ, than otherwise (again given that the two economies initially have the same τ, $\frac{y_{m,t}}{\bar{y}_t}$ and $R_{m,t+1}$). This contrasts with non-zero values of r, which do not affect this response. A comparison with a single-period version of the model is made in Appendix A.

4.1 Numerical Examples

The comparative static analysis provides a general idea of whether the composition of government expenditure would increase or decrease following a change in a given parameter of the model. However, it does not show the precise sensitivity of the composition with respect to changes in parameters. This subsection reports some numerical examples to illustrate these properties, which may help identify some important factors underlying the observed variations in the composition of government expenditure across countries.

To set a baseline value for each parameter, assume the length of a time period in the model is 20 years. The average annual growth rate of world GDP per capita and world population during the period 1986-2006 are 0.016

[16] The assumption that they have the same initial values of R here requires the preference parameter to differ between economies.

and 0.014, respectively.[17] Therefore, ω is set to 0.37 (given by $(1+0.016)^{20}-1$). Similarly, n is 0.33 (given by $(1+0.014)^{20}-1$). The annual real interest rate per year is roughly 4 per cent, so r is 1.19 (given by $(1+0.04)^{20}-1$). Assuming that the time preference rate is equal to the interest rate , $\rho = r$, then β is set to 0.46 (given by $1/(1+0.04)^{20}$). The baseline value for the tax rate, τ, is set to 0.35. The preference parameter, γ, is chosen such that, with other parameter values determined as above, the expenditure ratio matches the average ratio for a sample of 24 democratic countries.[18] This yields a value of γ equal to 0.75.

Figures 1 and 2 illustrate variations in the relationship between $R_{m,t+1}$ and $y_{m,t}/\bar{y}_t$, for given values of the other parameters. As expected, increasing $y_{m,t}/\bar{y}_t$ reduces $R_{m,t+1}$ at a decreasing rate in all figures, demonstrating that higher equality is consistently associated with a lower ratio of pension expenditure to public goods expenditure. The three diagrams of Figure 1 show in turn the effects on the relationship of varying ω, n and r around their baseline values. The variations considered are 15 per cent and 30 per cent changes around the baseline value. By taking percentage variations it is possible to make statements about the relative sensitivity to different parameter changes. The three diagrams of Figure 2 show the effects of varying τ, β and γ respectively. As each parameter is changed, the relationship between $R_{m,t+1}$ and $y_{m,t}/\bar{y}_t$ is found to shift, though not in a parallel fashion.

As shown in the top two diagrams of Figure 1, increasing income and population growth rates produces upward shifts in the profile of the ratio of expenditure on pensions to public goods. However, these impacts are quantitatively small, implying that the composition of government expenditure is not sensitive to variations in the growth rates of income and population. The bottom diagram of Figure 1 shows that increasing the interest rate shifts downwards the expenditure ratio profile, and this effect is quantitatively more pronounced than the shifts arising from growth rate variations.

Figure 2 shows that increasing the discount factor and the preference pa-

[17] The source of both data is World Development Indicators 2007.

[18] See Creedy and Moslehi (2007a) for the data on public goods and transfer payments. They find that the average value of ratio of expenditure on transfer payment to public goods for 24 democratic countries is 0.63, with an average wage ratio of 0.85.

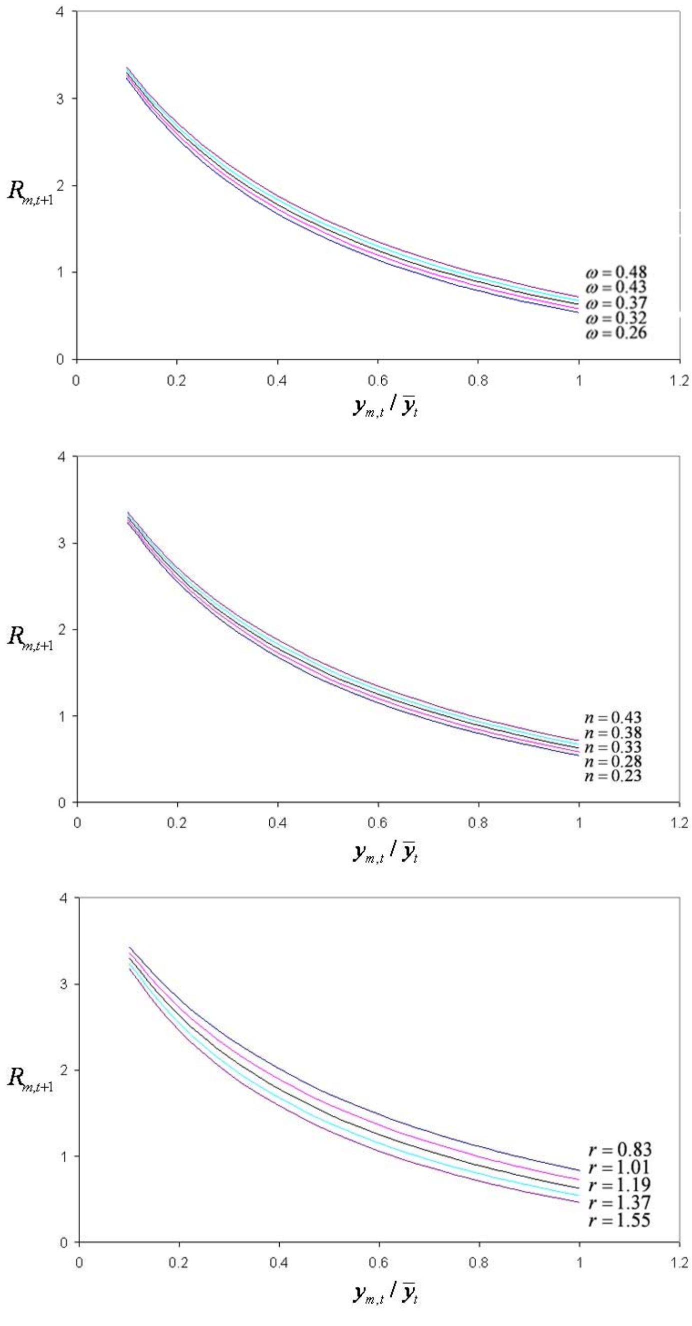

Figure 1: Variation in Expenditure Ratio for Alternative Growth Rates and Interest Rates

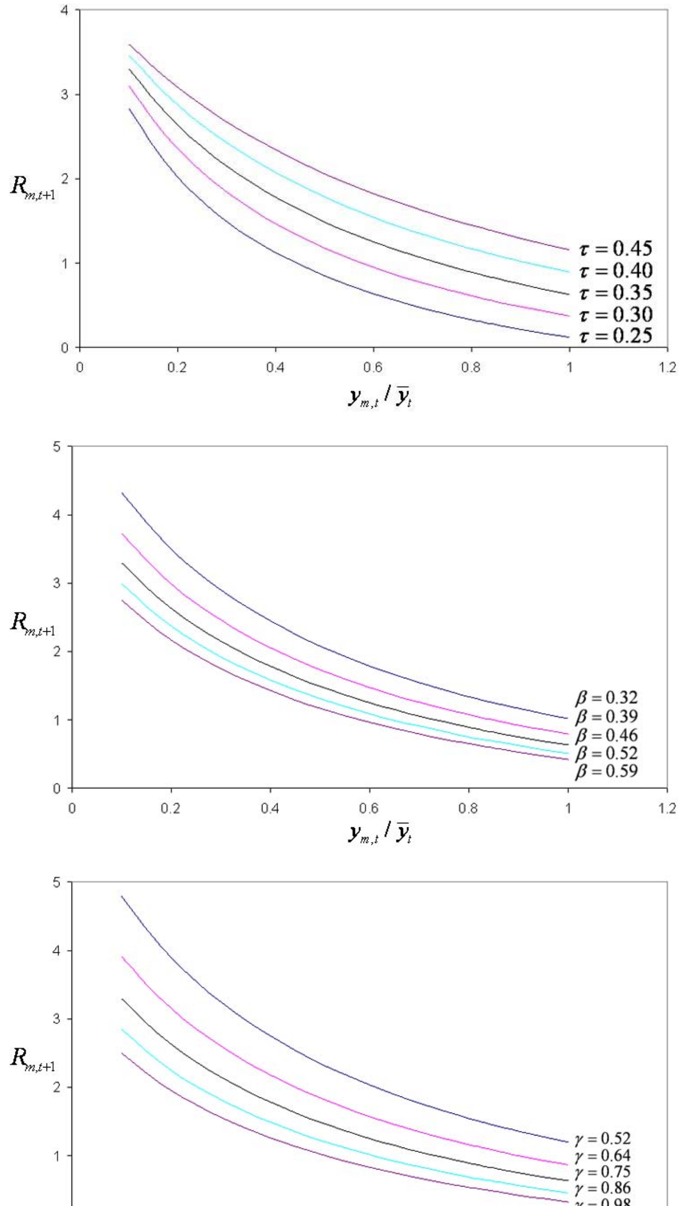

Figure 2: Variations in Expenditure Ratio for Alternative Tax Rates, Discount Factors and Preference Parameters

rameter, γ, considerably shifts downwards the relationship between $R_{m,t+1}$ and $y_{m,t}/\bar{y}_t$. Increasing the tax rate shifts the expenditure ratio profile upwards, and the effect is quantitatively relatively large. Comparing these figures highlights the point that the majority choice of the composition of government expenditure is quite sensitive to variations in the tax rate, discount factor and the weight of public goods in the utility function, while less sensitive to changes in the interest rate and growth rates of population and income. This indicates the importance of preference parameters in shaping the composition of expenditure.

These results perhaps throw some light on the mixed nature of empirical studies, using cross-sectional data for a range of countries, which have attempted to test the conclusion of earlier tax models regarding the relationship between redistribution and basic inequality. The results suggest first that, over the most relevant range of $y_{m,t}/\bar{y}_t$, the response of the expenditure ratio to a change in inequality is expected to be very low. Secondly, the expenditure ratio is sensitive to a range of variables (other than the tax rate) which are typically not included in the empirical investigations. For example, although inequality in Scandinavian countries is lower than in other countries, such as the USA, these countries have more redistributive policies.[19] This may be related to the different preferences of people in Scandinavia compared with the USA, rather than any inadequacy of the underlying model.[20]

5 Maximising a Social Welfare Function

The majority choice of expenditure proportions, examined in previous sections, is determined by the selfish preferences of the person with median income. Given the unchanged ratio of y_m to $\bar{y}$ in steady state growth, the actual situation of the rich and poor is irrelevant. It is therefore of interest

[19] Creedy and Moslehi (2007a) found that the average of $y_{m,t}/\bar{y}_t$ $(2000-2006)$ for Denmark, Finland, Norway, and Sweden were 0.91, 0.89, 0.90 and 0.90 respectively. However, the average $y_{m,t}/\bar{y}_t$ $(2000-2006)$ for USA was 0.75.

[20] Nevertheless, Lind (2005) discussed various complexities not treated by the models, such as the existence of multiple social contracts, prospect of upward income mobility, multi-dimensional policies, race and redistribution versus social insurance.

to evaluate the outcome in terms of a social welfare function, expressed in terms of the utilities of all individuals. This section examines the optimal composition of expenditure, defined as that which maximises a social welfare function defined in terms of the lifetime utilities of individuals in each cohort. Only a single cohort needs to be considered because in equilibrium growth, the same composition applies to each cohort and the social contract is also stable.[21] The social welfare function reflects the value judgements of an independent judge or decision maker, and is maximised subject to the government PAYG budget constraint. The optimal allocation of expenditure is obtained as a point of tangency of the highest social indifference curve which can be reached subject to the government's budget constraint. Indifference curves are first examined, followed by the tangency solution

The social welfare function at time t is considered to be a general function of lifetime indirect utility of the young cohort at time t, so that:

$$W_t = W\left(V_{1,t}, ..., V_{i,t}, ...V_{N_t,t}\right), \tag{25}$$

where $V_{i,t}$ is lifetime indirect utility of individual i as defined in (14). This function is individualistic, Paretean and concave with respect to b_{t+1} and G_{t+1}. The total differential of W_t with respect to b_{t+1} and G_{t+1} gives:

$$dW_t = \left(\sum_{i=1}^{Nt} \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial b_{t+1}}\right) db_{t+1} + \left(\sum_{i=1}^{Nt} \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial G_{t+1}}\right) dG_{t+1}. \tag{26}$$

Define $v_{i,t} = \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial b_{t+1}}$ as the welfare weight attached to an increase in i's income. The derivative of W_t with respect to G_{t+1} can thus be written as:

$$\sum_{i=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial G_{t+1}} = \sum_{i=1}^{N_t} v_{i,t} \left(\frac{\partial V_{i,t}/\partial G_{t+1}}{\partial V_{i,t}/\partial b_{t+1}}\right). \tag{27}$$

Social indifference curves describe combinations of G_{t+1} and b_{t+1} for which W_t is constant. Hence, setting $dW_t = 0$, and writing $\tilde{v}_{i,t} = v_{i,t}/\sum_{i=1}^{n} v_{i,t}$ gives:

$$\left.\frac{db_{t+1}}{dG_{t+1}}\right|_{W_t} = -\sum_{i=1}^{N_t} \tilde{v}_{i,t} \left(\frac{\partial V_{i,t}/\partial G_{t+1}}{\partial V_{i,t}/\partial b_{t+1}}\right). \tag{28}$$

[21] Hence it is not necessary to consider a transition from, say, a private pension to a PAYG transfer payment.

The slope of social indifference curves is therefore a weighted sum of the ratio of $\partial V_{i,t}/\partial G_{t+1}$ to $\partial V_{i,t}/\partial b_{t+1}$. From equation (14), these partial derivatives are:

$$\frac{\partial V_{i,t}}{\partial G_{t+1}} = \frac{\beta\gamma}{G_{t+1}}, \tag{29}$$

$$\frac{\partial V_{i,t}}{\partial b_{t+1}} = \frac{1+\beta}{(1+r)(1-\tau)y_{i,t} + b_{t+1}}. \tag{30}$$

Substituting into (28) and writing $\tilde{y}_t = \sum_i^{N_t} \tilde{v}_{i,t} y_{i,t}$ gives:

$$\left.\frac{db_{t+1}}{dG_{t+1}}\right|_{W_t} = -\frac{\beta\gamma\left\{(1+r)(1-\tau)\tilde{y}_t + b_{t+1}\right\}}{G_{t+1}\left(1+\beta\right)}. \tag{31}$$

The term $\tilde{y}_t$ is a weighted average of incomes, with weights, $\tilde{v}_{i,t}$.

From (15), the slope of the government's budget constraint for a given tax rate is:

$$\left.\frac{db_{t+1}}{dG_{t+1}}\right|_{\tau} = -\frac{1}{N_t}. \tag{32}$$

Equating the slope in (31) with (32) gives the tangency solution for the optimal pension, $b_{W,t+1}$, as:

$$G_{t+1}\left(1+\beta\right) = \beta\gamma N_t\left((1+r)(1-\tau)\tilde{y}_t + b_{t+1}\right). \tag{33}$$

Substituting for G_{t+1} from the government budget constraint gives:

$$\begin{aligned} b_{W,t+1} &= \frac{\bar{y}_t\left(1+\beta\right)}{\left(1+\beta+\beta\gamma\right)} \\ &\quad \times \left\{\left(1+n\right)\left(1+\omega\right)\tau - \frac{\beta\gamma}{\left(1+\beta\right)}(1+r)(1-\tau)\frac{\tilde{y}_t}{\bar{y}_t}\right\}. \end{aligned} \tag{34}$$

Comparing this result with the median voter's choice of $b_{m,t+1}$ in (18) shows that the two expressions are identical except for the fact that the majority choice depends on $y_{m,t}/\bar{y}_t$ whereas maximization of a social welfare function depends on $\tilde{y}_t/\bar{y}_t$. The only difference is in the relevant income ratios. The same feature must of course apply to expenditure on the public good and the expenditure ratio.

The expression for $\tilde{y}_t$ actually conceals considerable complexity. Strictly, (34) is not a closed-form solution because $\tilde{y}_t$ actually depends on the optimal values themselves. However, Appendix B examines the relationship between

$y_{m,t}/\bar{y}_t$ and $\tilde{y}_t/\bar{y}_t$ using an approximation in which the weighted income $\tilde{y}_t$ is replaced by the equally-distributed equivalent income. This is the income which, if equally distributed, gives the same social welfare as the actual distribution, for a welfare function defined in terms of incomes. If the distribution of income is lognormal and the welfare function takes the additive iso-elastic form it is shown that the relationship between the two relevant ratios is:

$$\frac{\tilde{y}_t}{\bar{y}_t} = \left(\frac{y_{m,t}}{\bar{y}_t}\right)^{\varepsilon}, \tag{35}$$

where ε denotes the relative inequality aversion of the judge. The optimal expenditure levels and their ratio can thus be expressed in terms of $y_{m,t}/\bar{y}_t$, just as in the majority voting framework, except that there is an additional degree of nonlinearity in the expressions, involving the term ε. By substituting (35) into (34) it can be shown that, as expected, an increase in ε is associated with an increase in the optimal expenditure ratio.[22] Only an independent judge having $\varepsilon = 1$ would evaluate the majority voting outcome as optimal.

6 Conclusions

This paper has examined the composition of government expenditure, considered as the outcome both of majority voting and optimizing by a benevolent government. The main focus was on public goods and a transfer payment, in the form of a pension (which can nevertheless be used to augment first-period consumption). Tax-financed expenditure is financed on a pay-as-you-go basis. A two-period overlapping generation model was constructed in which individuals have similar preferences but different incomes. Individuals work during the first period of life, pay a proportional income tax, and either save or borrow. During the second period of life, the retirement period,

[22] The relationships between optimal pension, public goods and their ratio and $y_{m,t}/\bar{y}_t$ are different from those with majority voting. First, although total expenditure on pension, $B_{W,t+1}$, and public goods, $G_{W,t+1}$, are decreasing and increasing in $y_{m,t}/\bar{y}_t$ respectively, this relationship is not linear. If $\varepsilon < 1$ the second derivative of $B_{W,t+1}$ and $G_{W,t+1}$ with respect to $y_{m,t}/\bar{y}_t$ is positive and negative respectively. The relationship between the optimal ratio and $y_{m,t}/\bar{y}_t$ is negative, but the second derivative is undetermined.

they receive the tax-financed transfer payment. The condition under which there is majority support for the social contract involved in a pay-as-you-go financing structure was established, providing an extension to the familiar Aaron-Samuelson condition. This condition was found to be independent of tax rates and the expenditure levels, so that it is possible to consider the choice of actual amounts, conditional on the social contract being supported.

Using the positive, majority voting, approach, selfish individuals vote in the first period on their pension to be received in the next period, given the tax rate and full information about the nature of the government's budget constraint (including growth rates of incomes and population). Therefore, voting is over one dimension and expenditure on public goods obtained from the government's budget constraint. Preferences are single-peaked and the decisive voter is the young individual with the median income, since the old cohort does not have incentive to vote on next period's pension. The comparative static results of this majority choice show that the ratio of expenditure on pension to public goods falls at a decreasing rate when the ratio of median to mean income increases, and the profile is relatively flat for realistic values of the income ratio. Numerical results show that the ratio of expenditure on pensions to public goods is more sensitive to changes in inequality, preferences parameters and tax rate compared with changes in rates of population and income growth.

In the second and normative approach, a social welfare function in term of individuals' lifetime utilities is maximized subject to the government's budget constraint, again for a given tax rate. The expressions for the choice of expenditure levels and their ratio were found to take the same form as in the majority voting context, except that a welfare-weighted average of income replaces median income. Although this does not provide a closed-form solution, because the weighted average itself depends on optimal values, an approximation using the concept of an equally-distributed equivalent income was obtained in the special case of an iso-elastic welfare function and a log-normal income distribution, allowing the same expression to be used. It was found that an independent judge with constant relative inequality aversion of unity would evaluate the majority voting outcome as also optimal.

Although the analysis has concentrated on the composition of government expenditure, rather than choice of the tax rate, it has in common with many other models the property that a higher degree of basic inequality is associated with a more redistributive structure. However, the comparative static results suggest that, over the relevant range, differences in the ratio of transfer payments to public good expenditure arising from differences in inequality are likely to be small. Furthermore, the ratio is relatively sensitive to preference parameters (the time preference rate and weight attached to public goods in utility functions). This suggests that in a cross-sectional comparison of democratic countries, a simple relationship between redistribution and basic inequality may not necessarily be observed.

The modelling framework, involving two overlapping generations and fixed incomes, is in some ways extremely simple. Nevertheless, consideration of the choice of tax-financed public good expenditure and the level of a pension is substantially complicated by the fact that the pension involves a combination of income-shifting between phases of the life cycle (in addition to that provided by private savings) with both inter-generational and intra-generational transfers. The latter arises because the basic pension is unrelated to income whereas the tax is proportional to income, and the former arises from the pay-as-you-go feature of financing whereby each generation can benefit from productivity and thus income growth accruing to the following generation. It is suggested that the approach therefore offers useful insights into the relevant relationships involved.

Appendix A: A Static Model

This appendix considers a single-period version of the model, in which the transfer payment is received by all individuals, rather than being a pension. The utility function and budget constraint facing individual i are:

$$U_i = \log c_i + \gamma \log G, \tag{A.1}$$

$$c_i = y_i (1-\tau) + b. \tag{A.2}$$

For a population of N, the government budget constraint is:

$$G/N = \tau \bar{y} - b, \tag{A.3}$$

where, as before, $\bar{y}$ is arithmetic mean income. By substituting (A.2) and (A.3) into (A.1) the indirect utility function, V_i, is obtained as a function of, b, so that:

$$V_i = \log \{y_i (1-\tau) + b\} + \gamma \log \{N (\tau \bar{y} - b)\}. \tag{A.4}$$

Since $\partial^2 V_i / \partial b^2 < 0$, preferences are single-peaked. Consequently, the solution for the majority voting outcome is found by maximising the median voter's indirect utility, V_m, with respect to the transfer, b_m.

$$\frac{\partial V_m}{\partial b_m} = \frac{1}{y_m (1-\tau) + b_m} - \frac{\gamma}{(\tau \bar{y} - b_m)} = 0, \tag{A.5}$$

which, after some manipulation, can be solved to give b_m as:

$$b_m = \frac{\bar{y}}{1+\gamma} \left\{ \tau - \gamma \frac{y_m}{\bar{y}} (1-\tau) \right\}. \tag{A.6}$$

The corresponding value of G_m is obtained by substituting b_m into the government budget constraint, and the ratio of per capita expenditure on transfer payment to public goods, $R_m = b_m / (G_m/N)$ is given by:

$$R_m = \frac{\frac{\tau}{\gamma} - (1-\tau) \frac{y_m}{\bar{y}}}{\tau + (1-\tau) \frac{y_m}{\bar{y}}}. \tag{A.7}$$

This may be compared with (22) above, which takes the same form except that the latter has $\gamma/2$ instead of γ.

This allows comparisons to be made between the majority choice of expenditure on public goods, transfer payment and their ratio in this static model with the two-period dynamic model where $n = \omega = 0$ and $r = \rho = 0$. Suppose the preferences parameter, γ, is same in both models. In the static model the majority choice of transfer payments and public goods are respectively lower and higher than two-period overlapping generations model with $n = \omega = 0$ and $r = \rho = 0$.[23] However, care is needed in making such comparisons because, in the static case, all individuals benefit – at the time of voting – from the transfer payment as well as public good expenditure. In the dynamic model it is only the young working cohort that, in the first period, vote on next period's transfer payment conditional on knowing that the next young generation will finance that pension.

Appendix B: Comparing Income Ratios

In section 5, it was found that a welfare-weighted average income, $\tilde{y}$, plays a crucial role in determining the optimal composition of expenditure. This measure is highly complex, even for simple social welfare functions. However, it is useful to consider an approximation, for the ubiquitous case where the social welfare function takes the iso-elastic form:

$$W_t = \frac{1}{1-\varepsilon} \sum_{i=1}^{N} V_{i,t}^{1-\varepsilon} \qquad \varepsilon \neq 1, \varepsilon > 0. \tag{B.1}$$

Here it is useful to work with the multiplicative form of the Cobb-Douglas utility function, $U_{i,t} = c_{1i,t} c_{2i,t+1}^{\beta} G_t^{\gamma} G_{t+1}^{\beta\gamma}$.[24] Consequently, indirect utility, from (16), can be rewritten as:

$$V_{i,t} = \frac{\left((1-\tau)y_{i,t} + \frac{b_{t+1}}{(1+r)}\right)^{(1+\beta)}}{(1+\beta)^{(1+\beta)}} \left(\beta(1+r)\right)^{\beta} G_t^{\gamma} G_{t+1}^{\beta\gamma}. \tag{B.2}$$

[23] The absolute difference between public goods and transfer payment in the static and dynamic model with no-growth and no-discountingis equal to $\frac{\gamma\{\tau+(1-\tau)(y_{m,t}/\bar{y}_t)\}}{(1+\gamma)(2+\gamma)}$

[24] Such a monotonic transformation does not of course affect the majority voting outcome, but optimal values are affected.

From (B.1), $\partial W_t / \partial V_{i,t} = V_{i,t}^{-\varepsilon}$. And using $\partial V_{i,t} / \partial b_{t+1}$ from (B.2), the welfare weights, $v_{i,t} = \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial b_{t+1}}$, are:

$$v_{i,t} = \frac{(1+\beta)\left(\frac{\beta^\beta (1+r)^\beta G_t^\gamma G_{t+1}^{\beta\gamma}}{(1+\beta)^{(\beta+1)}}\right)^{1-\varepsilon}}{(1+r)} \left((1-\tau) y_{i,t} + \frac{b_{t+1}}{(1+r)}\right)^{\beta - \varepsilon(1+\beta)} . \quad \text{(B.3)}$$

Suppose b_{t+1} is small relative to $y_{i,t}$. In this case an approximation, denoted $\tilde{y}_{A,t}$, for $\tilde{y}_t = \sum y_{i,t} \left(v_{i,t} / \sum v_{i,t} \right)$, is obtained as:

$$\tilde{y}_{A,t} = \frac{\frac{1}{N_t} \sum y_{i,t}^{(1+\theta)}}{\frac{1}{N_t} \sum y_{i,t}^{\theta}}, \quad \text{(B.4)}$$

with $\theta = \beta - \varepsilon (1+\beta)$. Thus $\tilde{y}_A$ is the ratio of two fractional moments. Suppose further that $y_{i,t}$ is lognormally distributed as $\Lambda(y_{i,t} | \mu_t, \sigma_t^2)$, with mean and variance of logarithms of μ_t and σ_t^2 respectively. Using the properties of the lognormal moment generating function, it can be found that:[25]

$$\tilde{y}_{A,t} = \exp\left(\mu_t + (1-\varepsilon) \frac{\sigma_t^2}{2}\right) \exp\left((2\beta(1-\varepsilon) - \varepsilon) \frac{\sigma_t^2}{2}\right) . \quad \text{(B.5)}$$

The final term in this expression is close to, but less than, unity. This is because β is less than one. Also, reasonable values of ε are small and < 1.[26] However, the use of the assumption that b_{t+1} can be neglected in (B.3) actually attaches too much weight to the lower incomes, and thus imparts a downward bias to the approximation. One approach is thus to 'correct' for this downward bias by excluding the final term in (B.5). This gives:

$$\tilde{y}_{A,t} = \exp\left(\mu_t + (1-\varepsilon) \frac{\sigma_t^2}{2}\right) . \quad \text{(B.6)}$$

A feature of this result in (B.6) is that $\tilde{y}_{A,t}$ is in fact closely related to Atkinson's measure of the inequality of income. Following Atkinson (1970), let $y_{ede,t}$ denote the 'equally distributed equivalent' income, representing the equal income which gives the same welfare as the actual distribution, using a

[25] On the lognormal distribution, see Aitchison and Brown (1957).

[26] In the cross-sectional inequality context, questionnaire studies involving consideration of the 'leaky bucket' experiment found values for respondents which averaged around 0.2; see Amiel, Creedy and Hurn (2001).

welfare function of the form $W_t = \frac{1}{1-\varepsilon} \sum_{i=1}^{N} y_{i,t}^{1-\varepsilon}$. This is the same as the above but with $V_{i,t}$ replaced by $y_{i,t}$. Thus:

$$y_{ede,t} = \left(\frac{1}{N_t} \sum_{i=1}^{N_t} y_{i,t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{B.7}$$

Again using lognormal properties, the term $y_{ede,t}^{1-\varepsilon}$ has mean and variance of logarithms respectively of $(1-\varepsilon)\,\mu_t$ and $(1-\varepsilon)^2\,\sigma_t^2$, so that:

$$y_{ede,t} = \exp\left((1-\varepsilon)\,\mu_t + (1-\varepsilon)^2 \frac{\sigma_t^2}{2} \right)^{\frac{1}{1-\varepsilon}} = \exp\left(\mu_t + (1-\varepsilon)\,\frac{\sigma_t^2}{2} \right). \tag{B.8}$$

Consequently, the ratio of the equally distributed equivalent to the arithmetic mean income, $\bar{y} = e^{\mu_t + \sigma_t^2/2}$, is:

$$\frac{y_{ede,t}}{\bar{y}_t} = \frac{\exp\left(\mu_t + (1-\varepsilon)\,\frac{\sigma_t^2}{2} \right)}{\exp\left(\mu_t + \frac{\sigma_t^2}{2} \right)} = \left(\exp\left(-\frac{\sigma_t^2}{2} \right) \right)^{\varepsilon}. \tag{B.9}$$

Furthermore, as $y_{m,t} = e^{\mu_t}$:

$$\frac{y_m}{\bar{y}_t} = \exp(-\frac{\sigma_t^2}{2}), \tag{B.10}$$

giving:

$$\frac{y_{ede,t}}{\bar{y}_t} = \left(\frac{y_{m,t}}{\bar{y}_t} \right)^{\varepsilon}. \tag{B.11}$$

If $\tilde{y}_t$ is approximated by $y_{ede,t}$, (B.11) gives the required relationship between the two income ratios reported in Section 5 above.

It is important to test the value of the above approximation. Hence values of the expenditure components using the approximation $\tilde{y}_A = y_{ede}$ were compared with those obtained using a simulated population of size 15000 drawn at random from a lognormal distribution with $\mu = 9.0$ and $\sigma^2 = 0.5$.[27] Using the simulated distribution, a range of values of b_{t+1} were investigated. For each b_{t+1} the government budget constraint was used to obtain G_{t+1} and the resulting values were used to calculate each individual's level of utility. These

[27]This implies that $\bar{y} = 10405$ and $y_m/\bar{y} = 0.78$.

were then used to obtain social welfare, using the iso-elastic function with a specified inequality aversion parameter, ε. Finally, given a large number of W_t measures, the maximum was determined, giving the optimal composition. In order to compare welfare values, the expenditure values obtained from the approximation was used with the simulated population. Table 2 gives the results for a range of inequality aversion, ε, with other baseline parameters.

Table 2: Optimal Composition of Expenditure: Alternative Solutions

	Approximation			Simulation			
ε	b_{t+1}	$\frac{G_{t+1}}{(N_t+N_{t+1})}$	R_{t+1}	b_{t+1}	$\frac{G_{t+1}}{(N_t+N_{t+1})}$	R_{t+1}	$\%\Delta W_t$
0.8	3087	1532	0.863	3460	1372	0.925	-0.011
0.5	2907	1609	0.774	3452	1376	0.930	-0.028
0.2	2713	1692	0.687	3442	1380	0.936	-0.043

These results show that the approximation does indeed give values of expenditure levels and ratios which are reasonably close to those obtained using a large simulated population. The percentage difference of the social welfare function using the approximation from that obtained by simulation, $\%\Delta W$, is in each case found to be small, at less than half a percentage point. This reflects the relative flatness of the profile relating W_t to b_{t+1} (for given parameter values) as well as the closeness of the approximation.

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