Using a Multiplicative Intensity Process to Forecast Firm Failure

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Abstract

A continuous-time multiplicative intensity model is applied to US firm failure. Using this semi-parametric framework allows macroeconomic influences without explicit specification. It is proposed that the relative risk of failure experienced by individual firms can be estimated by both market and company account driven covariates. Also, industry is considered as a further possible distinction between firms.

A strong relative risk measure is found by partial likelihood estimation, with support from out-of-sample forecasting. The continuous-time multiplicative intensity model accepts coefficient estimates that were rejected in discrete-time parametric models proposed in Shumway (2001). Industry heterogeneity is present in a market-wide sample, and alternatives for modelling this are analysed, but the appropriate technique for considering industry distinction remains unclear.

Keywords: bankruptcy; firm failure; industry heterogeneity; partial likelihood estimation; relative risk.

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The author would like to recognise David Pitt for his continued support and guidance, and Richard Heaney for a thorough and helpful peer review. The author remains responsible, however, for any errors,

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1. Introduction

A firm's demise affects a financial asset derived from that firm, whether equity, debt or some other instrument. A model for predicting firm failure is useful for providing insight into the risks associated with holding these assets, and could possibly be extended to contribute to asset valuation.

There have been numerous and diverse contributions to predicting firm failure, beginning with a simple univariate analysis by Beaver (1966). Naturally, this developed into analysing predictors collectively, such as in multiple discriminant models (Altman, 1968) and conditional probability models (Ohlson, 1980). Other techniques, such as recursive partitioning (Frydman et al., 1985) and rough sets theory (McKee, 2000) have also been tested with some success. The first explicit use of survival analysis, the approach that is adopted here, was by Lane et al. (1986), who applied a time-invariant proportional hazards model to forecast bank failures. Survival analyses continue to gain favour, with both Shumway (2001) and Chava and Jarrow (2004) using a hazard model to accurately predict bankruptcy.

This paper develops a hazard-style model, focusing on the relative risk of failure experienced by a firm. This is in contrast to previous hazard models by Shumway (2001) and Chava and Jarrow (2004) who focused on absolute risk, which is a measure of the risk of failure in comparison to observed firms irrespective of time. By measuring relative risk, we isolate external influences shared by all observed firms and thus confine our analyses to a comparison at the time of interest. For example, while a recession could be reasonably thought to increase the absolute risk of failure of most firms, it would have no effect on the relative risk of failure, and so the relative risk model is subject to fewer disruptions.

Unlike these previous studies, firm failure will be considered in continuous time. The time of firm failure can be measured to the nearest day, which we considered a small enough interval to warrant a continuous-time model. The justifications for the discrete-time analysis used in Shumway (2001) involve the annually or quarterly reporting of company accounts, but it is this study's conjecture that possessing all available public information is sufficient to use a continuous-time analysis. In particular, the covariates changing over time are seen as continuous step processes, with jumps when new information becomes available.

For performing our survival analyses, we propose that the life of a firm follows a multiplicative intensity process, which adheres to our requirements above. Under this process, every firm shares an equal underlying probability of failure at any instant, before multiplying each firm's probability by a selection of individual risk factors—our covariates. Thus the chosen covariates scale, either up or down depending on the favourability of the covariate, the risk of failure for each individual firm. This enables us to estimate the impact that publicly-available information has on a firm's survival, and subsequently use this information to predict firm failure. The multiplicative intensity process is adopted for the following reasons: First, it permits the measurement of the failure *intensity* relative to other firms (with the intensity being the rate of transition). This means that we can compare firms without needing to recognise pressures on survival beyond firm-specific effects—such as macroeconomic impacts. Second, it is a generalisation of that used in firm survival analyses by Shumway (2001) and Chava and Jarrow (2004), who have previously demonstrated strong results, and so the model is favoured at both extending on and comparing with previous research. We produce accurate models for the failure intensity relative to other firms by using improved specifications on these previous studies, demonstrable by forecasting outside the initial data sample. Using partial likelihood estimation (Cox, 1975), the *relative risk* is examined in isolation from other effects (with the relative risk being the failure intensity relative to all other firms at a time). We are able to estimate this risk for each firm by using the relative risk process, which is a component of the multiplicative intensity process.

Care must be taken when defining failure. A great majority of firms will either survive until the study period is complete, or leave the study by a decrement other than firm failure (e. g. mergers and acquisitions). All of these firms, however, must be included in the sample to avoid selection bias. For this reason, upon exit or cessation of observation, we treat all non-failed firms as being right-censored, which enables us to incorporate into the analyses the important information that they did *not* fail. Liquidation, bankruptcy and delisting under extreme financial distress are all categorised as failure, which is a broader definition than used by Shumway (2001) and Chava and Jarrow (2004). We consider this approach necessary because the alternative treatment of right-censoring assumes that the decrement is non-informative of failure, which is clearly false, since we can reasonably expect that the probability of delisting under extreme financial distress is highly correlated to the probability of bankruptcy.

We test market and company account data as time-dependent covariates in the relative risk process for determining whether they are informative of a firm's failure intensity. Thus, it is these covariates which enable us to discriminate between firms with regard to their risk of failure. These time-dependent covariates are more suitable than time-invariant covariates because they are able to acknowledge a firm's financial position, and thus probability of failure, changing over time (LeClere, 2005). The covariates are similar to those chosen in previous research (Altman, 1968; Zmijewski, 1984; Shumway, 2001).

Another consideration is industry heterogeneity, which we model using two distinct methods: For the first method, we apply industry indicator covariates (Chava and Jarrow, 2004), which allow a direct measurement of the impact of a firm's industry on the failure intensity, under an assumption of proportionality. For the second method, we categorise firms into industry strata, which leaves the industry effects unspecified and unmeasured, and therefore less assumptive and informative than using indicator covariates. Both of these methods allow an investigation into how a firm's industry may affect the failure intensity indirectly, through the firm-specific covariates.

Section 2 discusses the source, motivation and calculation of the covariates, as well as the assumptions adopted to achieve a usable data set. The multiplicative intensity process is presented in Section 3, with further mathematics left for the interested reader in Appendix C . The results are displayed in Section 4 to Section 6, with simple models of varying forecast length proposed before testing the improved assumptions and introducing industry heterogeneity. Section 7 discusses the limitations associated with the multiplicative intensity process used in predicting firm failure before the findings are summarised in Section 8.

2. The covariates

The covariates tested, shown in Table 2.1, resemble those used in Altman (1968), Zmijewski (1984) and Shumway (2001), with each representing an attribute of firm health. Market and failure data are sourced from CRSP and company account data are sourced from Compustat (see Appendix A). In a set of univariate models (not shown), the covariates, with the exception of

Table 2.1: Covariates

Covariate	Abbrev.	Source
excess return	return	Shumway (2001)
relative size	size	Shumway (2001)
standard deviation of excess return	volatility	Shumway (2001)
$\frac{\text{working capital}}{\text{total assets}}$	WC/TA	Altman (1968)
$\frac{\text{retained earnings}}{\text{total assets}}$	RE/TA	Altman (1968)
$\frac{\text{earnings before interest and tax}}{\text{total assets}}$	EBIT/TA	Altman (1968)
$\frac{\text{market equity}}{\text{total liabilities}}$	ME/TL	Altman (1968)
$\frac{\text{sales}}{\text{total assets}}$	SALES/TA	Altman (1968)
$\frac{\text{net income}}{\text{total assets}}$	NI/TA	Zmijewski (1984)
$\frac{\text{total liabilities}}{\text{total assets}}$	TL/TA	Zmijewski (1984)
$\frac{\text{current assets}}{\text{current liabilities}}$	CA/CL	Zmijewski (1984)

SALES/TA, have a statistically significant impact on failure prediction in the direction we would expect.

We truncate all covariates at the first and ninety-ninth percentiles of their distribution over all time, which reduces unacceptable influence from an individual datum and is consistent with Shumway (2001). Although such truncation violates the filtration argument (see Appendix B), it is a minor corruption of the assumption and is acceptable for simplicity in estimation. Regardless, non-truncated covariate models produce similar results.

When new data are not available, the most recent information is used. Where no new data are released in excess of two years, the field is left vacant and the observation is excluded from the analyses, with the firm regarded as unobservable and thus not at risk. We exclude approximately 0.5 percent of firm exposure for this reason. This is another departure from that proposed by Shumway (2001) and Chava and Jarrow (2004), where a missing datum has been replaced with the mean of the covariate over all time, which assumes that the event of missing data is random across all firms. This is contestable given that delays in financial results and suspension of trading are events that occur mostly to troubled firms. Also, such an assumption corrupts the filtration argument.

2.1. market driven covariates

The market driven covariates we use in this study were first proposed in Shumway (2001), albeit with slightly different calculation, and relate to the market value of the firm's common equity. These covariates are relative to an aggregation of the American Stock and Options Exchange

(AMEX), New York Stock Exchange (NYSE) and later, the National Association of Securities Dealers Automated Quotations (NASDAQ). The market driven covariates are measured relative to an index so they are exclusive to the firm-specific effects, recalling that common effects are acknowledged separately from firm-specific effects.

- *return* is the firm's continuously compounding rate of return in excess of the index, inclusive of dividends. This return is usually calculated over three months, but varies from one month to one year for some calibrations. We would expect that return has a negative impact on the firm failure intensity.
- *size* is the natural logarithm of the ratio of the firm's equity value relative to the total index value, which is an identical definition to that proposed by Shumway (2001). Larger firms, we expect, are less likely to fail.
- *volatility* is the standard deviation of the firm's daily continuously compounded rate of return in excess of the index, with the calculation made over the same period as the return. Failures are more frequent from firms with higher volatility.

Where data are missing, we adopt the *most recent information* convention for the size covariate, meaning relative size is assumed constant. Consequently, the firm's growth must be equal to the market, and thus instead of adopting the same convention, missing data for the return covariate (excess return over the index) must equal zero. The same rationale is not followed for the volatility, however, as this would lead to the perverse outcome of decreasing volatility when data are missing, so the *most recent information* convention is used.

2.2. company account driven covariates

Account driven data are intermittent, with most reported quarterly. Using our *most recent information* argument, the covariate processes can be seen as piecewise constant, with value changes when new information is observed. To avoid heterogeneity between covariate definitions, each of the following account ratios are subject to reasonably consistent reporting standards.

- WC/TA measures the proportion of assets which are liquid. That is, the proportion of assets which are neither tied-up nor attributable to current liabilities. WC/TA will diminish for firms with consistently poor profitability, and thus is a stable but informative gauge for distress.
- RE/TA measures cumulative profits. This is a complex but popular ratio for analysis of corporate distress. Intuitively, RE/TA is an unreliable risk gauge because of strong interaction with other account ratios and easy corporate manipulation, where both a firm with low profitability and a firm with a strong dividend pay-out policy have low ratios. Statistically, however, a low RE/TA precedes financial distress in a univariate setting, possibly because it is typically low for young firms, and thus remains in our analyses for consideration.
- EBIT/TA approximates the productivity of a firm's assets. This is one of the most popular ratios for predicting financial distress because it is not distorted by firm leverage. If an asset's ability to generate income cannot satisfy liabilities, the firm will become insolvent.

- ME/TL measures the distance to insolvency. Altman (1968) included this ratio for its ability to measure how much the value of a firm's assets can fall before insolvency. It was found to predict bankruptcy well, and so is included in our more complex analyses.
- SALES/TA is the capital-turnover ratio—another representation of productivity. We would expect the risk of failure to decrease with an increasing SALES/TA. Interestingly, however, SALES/TA fails to predict failure in a univariate model, although it is often found useful in multivariate applications, and, as we will see in Section 6, is particularly relevant for particular industries. This is not surprising since SALES/TA is traditionally used to compare the competitiveness of two firms of different size in the same industry.
- NI/TA is a common representation for return on assets. This Zmijewski (1984) account ratio is very similar to EBIT/TA from Altman (1968). Its application in this paper is used often as a comparison to EBIT/TA in predicting failure, although it is useful in its own right for industries where earnings before interest and tax are not released.
- TL/TA measures financial leverage. This is the most forward-looking of our account ratios, with it representing the firm's ability to satisfy its obligations in the long run. If the firm is unable to satisfy these obligations, it will become insolvent.
- CA/CL is a representation of liquidity. This signifies whether the firm is capable of servicing its immediate obligations, with fail to do so likely to result in liquidation.

It is clear that these account ratios are strongly related, with some representative of similar facets of a firm's operations. We will distinguish which covariates add value to a prediction of failure.

3. The multiplicative intensity model

This section introduces the multiplicative intensity. For the unfamiliar and interested, the counting process framework for survival analysis is presented in Appendix B and the multiplicative intensity model estimator is derived in Appendix C. Although we encourage an inspection of the multiplicative intensity model, Appendix B and Appendix C are unnecessary to appreciate the results.

We allow our covariate processes for firm i to be denoted by the column vector $\mathbf{X}_i$, with $\mathbf{X}_i(t)$ containing all the applicable firm-specific information known just prior to time t that will be used to scale the underlying firm failure intensity at time t.

The impact that a selection of covariates has on the underlying failure intensity is estimated linearly. In other words, each covariate is multiplied by a coefficient to determine its individual impact, and all of these impacts are summed. These coefficients are represented in column vector $\boldsymbol{\beta}$, and so this linear sum can be expressed as $\boldsymbol{\beta}'\mathbf{X}_i(t)$. This linear sum is transformed exponentially, $\exp\{\boldsymbol{\beta}'\mathbf{X}_i(t)\}$, so each covariate can be thought of as scaling the failure intensity individually (i. e. $\exp\{\beta_1 X_1(t) + \beta_2 X_2(t)\} = \exp\{\beta_1 X_1(t)\}\exp\{\beta_2 X_2(t)\}$) and for ease of computation. Regardless, a monotonically increasing transform such as the exponential does not affect the ranking of the riskiness of firms, which is a primary objective of this study.

This gives us what we refer to as our relative risk for firm i,

$$rr_i(t) = Y_i(t) \exp\left\{\boldsymbol{\beta}' \mathbf{X}_i(t)\right\} ,$$

where $Y_i(t)$ equals one if the firm is being observed just prior to time t and zero otherwise. Y_i is referred to as the *at risk* process and is introduced to acknowledge that only firms which are being observed can be observed to fail. The relative risk for firm i is multiplied by the underlying firm failure intensity to gain the firm-specific failure intensity.

This underlying firm failure intensity process is referred to as the baseline intensity process, and is represented by $d\Lambda_0$. Intuitively, the value of this process must be non-negative. For the purposes of our model, however, the baseline intensity needn't be specified, since we apply an estimation technique that ignores this process. This convenience is why the multiplicative intensity model is so attractive; we can concentrate our analyses entirely on the estimation of how firm-specific information influences the underlying failure intensity.

Thus, the multiplicative intensity model can be expressed as

$$d\Lambda_i(t) = Y_i(t) \exp\left\{\boldsymbol{\beta}' \mathbf{X}_i(t)\right\} d\Lambda_0(t) ,$$

with $d\Lambda_i(t)$ the failure intensity specific to firm i at time t.

This article is concerned with the continuous relative risk of firm failure and therefore we focus on the estimation of the coefficient vector, $\boldsymbol{\beta}$ (see Appendix C).

In addition to the usual tests for coefficient estimate and overall model statistical significance, we complete numerous other statistical test to ensure the model assumptions are appropriate. Particularly, the multiplicative intensity model assumes that the covariates impact the failure intensity proportionally, and we later draw attention to where this assumption fails.

4. Simple models

The results in this section are modelled from firms listed on the AMEX and NYSE that are not finance, insurance or real estate (*financials*) or public administration firms. The samples consist of US firms from 1 January 1962 to 31 December 1999, which permits comparison with Chava and Jarrow (2004).

We refer to the models where all covariates are tested (and either accepted or rejected) as Optimal models, whereas the Shumway models consist of only the covariates suggested by Shumway (2001): return, size, volatility, NI/TA and TL/TA. Lastly, the Market and Account models test only the market and company account driven covariates respectively. The distance the covariates are lagged is equivalent to the length of the forecast, with a one year covariate lag meaning the covariates are used to predict failure one year hence.

Models are fitted by excluding the covariates with the most statistically insignificant coefficient estimates in succession (*backwards exclusion*), until all coefficient estimates in the model have p-values less than five percent. Still, the p-value is displayed for an excluded covariate, and is based on the χ^2-statistic immediately before rejection. We complete many diagnostics tests, but discuss them only when required. For example, individual firm influence is minimal for all calibrations, so its presentation is unnecessary, with the exclusion of any firm always changing $\hat{\beta}$ by less than twenty percent of one standard deviation. However, nonproportionality is of occasional concern, and it is discussed for this reason. All calibrations are displayed with the quantity of failures, firms and exposure in the sample period, where exposure is the aggregate years of observation of all firms in the sample.

All models are accompanied with an out-of-sample forecast, which quantifies its success, with calibration performed from 1 January 1962 to 31 December 1990, and tested for accuracy from 1 January 1991 to 31 December 1999. Again, these periods correspond with Chava and Jarrow (2004). For the out-of-sample forecasts, we rank firms according to their relative risk value at a time of firm failure, and so a good estimate for relative risk would have majority residency in the higher percentiles—most firm failures occur from the upper end of the relative risk scale. We are primarily concerned with the residency above the ninetieth percentile (highest decile) and the eightieth percentile (highest quintile), as these can be directly compared to results given by Shumway (2001). All out-of-sample forecast graphics contain an *uninformative* model that assumes all firms suffer only the baseline failure intensity.

Also of concern in the out-of-sample forecasts are failures of firms with low estimated failure intensities. Thus, for an overall measure of forecasting power that considers both where model succeeds *and* fails, we measure the area under the out-of-sample forecast plot. This measure can be seen as analogous to measuring ROC (receiver operating characteristic) curves, but in a relative risk environment. The area under an out-of-sample forecast curve, labelled RROC (relative ROC) area, allows another comparison between models. A RROC area close to one-hundred percent suggests very strong overall predictive power, with failure always occurring to the firm ranked riskiest, while a RROC area of fifty percent indicates the estimated relative risk process is *uninformative*.

4.1. one year covariate lag

The coefficient estimates of four models with a one year covariate lag are displayed in Table 4.1, calibrated on a maximum sample of 564 failures. This sample diminishes, however, when account driven covariates are introduced, which is a reflection of the availability of data. In all of these models, the period of calculation for return and volatility is three months.

All market driven coefficient estimates are highly significant in the Optimal model with, as expected, greater volatility increasing the risk of failure, but higher return and size having the opposite effect. Of the company account ratios, WC/TA and EBIT/TA are meaningful, with coefficient estimates statistically significant with a p-value below 0.01 percent. Such findings are contrary to those of Shumway (2001), who found that the coefficient estimates for NI/TA and TL/TA were the only statistically significant account driven covariates. Inspection of the Shumway model reveals that the NI/TA and TL/TA coefficient estimates are highly significant in a model when WC/TA and EBIT/TA are excluded. Also, because both models have similar exposure, comparison of the model fit is acceptable, where the log likelihood ratios show similarity. This change in account driven covariates cannot be explained by the use of a continuous-time multiplicative intensity model (which is confirmed in Section 5.2), as it is simply a generalisation of that previously used. It is more likely that the sample period and data assumptions caused this change. Although Chava and Jarrow (2004) used the same sample period, they did not re-test covariates that Shumway

Table 4.1: One Year Covariate Lag Calibrations. Optimal, Market, Account and Shumway model calibrations with one year covariate lag. The sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1999. The left column of the major panel is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

Covariates	Optimal		Market		Account		Shumway	
#Failures	367		564		343		370	
#Firms	3503		3757		3404		3518	
Exposure	51604		52482		48382		51679	
return	-1.2648	(< 0.0001)	-1.2407	(< 0.0001)			-1.2789	(< 0.0001)
size	-0.5068	(< 0.0001)	-0.4843	(< 0.0001)			-0.5307	(< 0.0001)
volatility	31.8053	(< 0.0001)	34.8917	(< 0.0001)			31.8283	(< 0.0001)
WC/TA	-0.6395	(< 0.0001)				(0.7409)		
RE/TA		(0.2412)				(0.4177)		
EBIT/TA	-0.4667	(0.0008)			-1.3572	(< 0.0001)		
ME/TL		(0.3418)				(0.1432)		
SALES/TA		(0.7068)			0.1673	(0.0022)		
NI/TA		(0.5218)				(0.2640)	-0.2602	(0.0510)
TL/TA		(0.1915)			0.8269	(< 0.0001)	0.4303	(< 0.0001)
CA/CL		(0.7135)				(0.6723)		
Model Fit	4311	(< 0.0001)	6876	(< 0.0001)	4729	(< 0.0001)	4328	(< 0.0001)

(2001) rejected; WC/TA and EBIT/TA.

The change of account driven covariates is hardly critical, because similarities exist between NI/TA and EBIT/TA, and TL/TA and WC/TA. Both NI/TA and EBIT/TA measure the productivity of a firm's assets, and have a Pearson correlation coefficient of 0.7764. While TL/TA and WC/TA share a weaker relationship (with a -0.5455 Pearson correlation coefficient), they both compare assets to liabilities; the former with a long-term perspective, and the latter with a short-term perspective. This difference in perspective is worth remembering, because we will see evidence of it when we adjust the forecast length. Figure 4.1 supports the Optimal model, with it exceeding the Shumway model in accuracy by almost three percent over the highest quintile, with a residency of ninety-seven percent. The RROC area is very high for both the Optimal and Shumway models, indicating that the covariate selections describe the failure intensity well. Importantly, the coefficient estimate pertaining to TL/TA does display statistically significant time dependence, suggesting nonproportionality, which may account for the marginally inferior results, although the Shumway model still performs well overall.

The Market model accepts all covariates proposed, and the out-of-sample forecast shows a slightly weaker predictive capacity when compared to the Optimal model. This similarity in results between a Market model and an Optimal model is similar to the findings of Shumway (2001) and Chava and Jarrow (2004).

The Account model delivers unconvincing results, where statistically insignificant coefficient estimates from the Optimal model are accepted here, and WC/TA is rejected. Thus it is not surprising that Figure 4.1 shows the Account model has poor predictive power, which we can attribute to

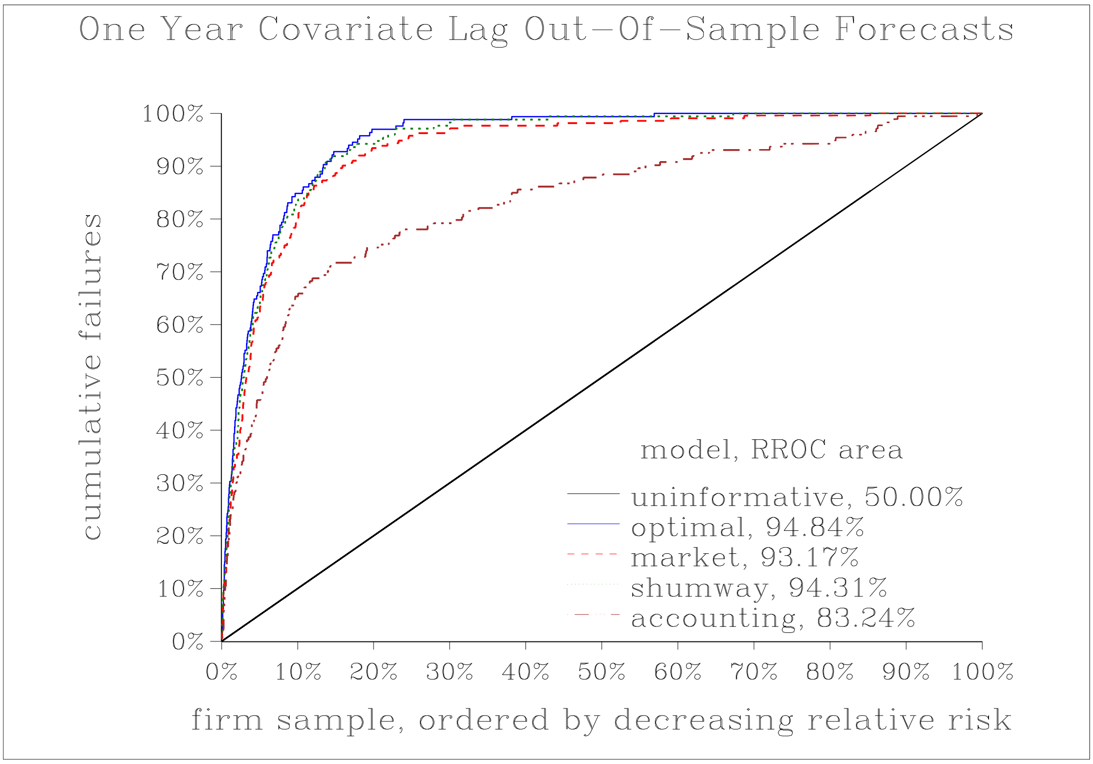

Figure 4.1: Optimal, Market, Account and Shumway model out-of sample forecasts with one year covariate lag. The calibration sample consists of all observable firms from the AMEX and NYSE with the exclusion of financial and public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

nonproportionality. Although it is not shown, the effect of TL/TA on predicting failure diminishes over the sample period, with no recognisable offset from the other covariates to retain proportionality. The forecast shows that many firms failed despite being modelled to be less risky than other firms, this is reflected in the relatively low RROC area of 83.24 percent.

4.2. short term covariate lags

In Table 4.2, the Optimal model from Section 4.1 is extended to consider more imminent failure, using the same estimation procedure, but with covariate lags of one month, three months, six months and one year. The one month covariate lag model, however, differs in specification from the other models in two ways. Firstly, the return and volatility are calculated over a one month period, which is more appropriate with only a short time until potential firm failure, whereas other models use a three month calculation period. Secondly, the covariate lag for the account driven covariates remains at three months to compensate for reporting delays. The sample size decreases as the covariate lag increases because the minimum period of observation is increasing, so very young firms are excluded.

By inspecting the coefficient estimates, we can see that the impact of a change in the WC/TA and size covariates on the failure intensity diminishes as the forecast lengthens, and the same

Table 4.2: Short Term Covariate Lags Calibrations. Optimal model calibrations with one month, three month, six month and one year covariate lag. The sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1999. The left column is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

	Optimal Covariates			
Covariate Lag	One Month[1]	Three Month	Six Month	One Year
#Failures	378	377	374	367
#Firms	3695	3598	3577	3503
Exposure	55264	54278	53748	51604
return	-1.1435 (0.0006)	-1.1490 ($<$ 0.0001)	-1.2065 ($<$ 0.0001)	-1.2648 ($<$ 0.0001)
size	-0.7619 ($<$ 0.0001)	-0.6241 ($<$ 0.0001)	-0.5753 ($<$ 0.0001)	-0.5068 ($<$ 0.0001)
volatility	28.9624 ($<$ 0.0001)	34.9585 ($<$ 0.0001)	34.5753 ($<$ 0.0001)	31.8053 ($<$ 0.0001)
WC/TA	-0.8524 ($<$ 0.0001)	-0.8424 ($<$ 0.0001)	-0.7659 ($<$ 0.0001)	-0.6385 ($<$ 0.0001)
EBIT/TA	-0.3501 (0.0019)	-0.3706 (0.0012)		-0.4667 (0.0008)
NI/TA			-0.2615 (0.0254)	
Model Fit	4045 ($<$ 0.0001)	4077 ($<$ 0.0001)	4197 ($<$ 0.0001)	4311 ($<$ 0.0001)

1. company account covariates have minimum of three months covariate lag

appears to be true of volatility after three months. EBIT/TA and return, however, strengthen their effect on the failure intensity. The equivalence of EBIT/TA and NI/TA is again highlighted, which is especially evident between the three month and six month covariate lag models, where the statistically significant coefficient estimates of the account driven covariates change.

As the covariate lag grows, the predictive power lessens and approaches *uninformative*, although the predictive power for the one and three month covariate lags are very similar, as shown in Figure 4.2. The forecast accuracy three months prior to failure is one percent inferior in the highest decile to one month prior to failure, but superior at lesser thresholds; with over ninety-nine compared to ninety-six percent residency in the highest quintile. This similarity in forecast accuracy is highlighted further by the RROC areas, 97.72 percent for one month and 97.32 percent for three months. Also, the multiplicative intensity model remains a very powerful predictor for firm failure at six and twelve months.

We find that the size coefficient estimate displays statistically significant linear time dependence when the covariate lag is less than one year. The other relevant coefficient estimates, however, demonstrate opposing (albeit statistically insignificant) time dependence, suggesting the package of covariates and coefficients retains the proportional effect on the failure intensity, despite the strengthening impact of the size covariate over the sample period. This argument is corroborated by the favourable out-of-sample forecasting results.

For all measured short term covariate lags, the Optimal model performs better in the out-of-sample forecasts than the Shumway model with respect to both higher decile residency and RROC area. When applying a one month covariate lag model with only market driven covariates, we find the out-of-sample forecast performance is comparable to the corresponding Optimal model. Therefore, this indicates that the market will fully reflect the financial position of a firm one month

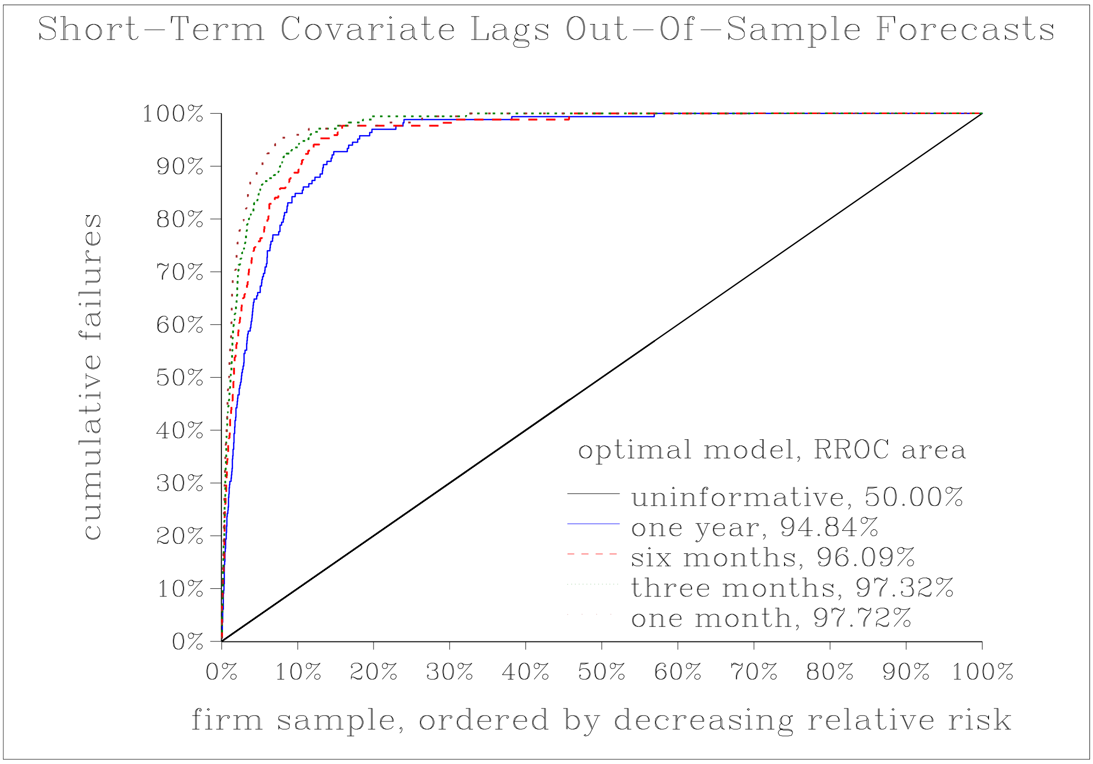

Figure 4.2: Optimal model out-of-sample forecasts with one month, three month, six month and one year covariate lag. The calibration sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

prior to failure—account driven covariates contribute little information. For brevity, neither the Shumway nor Market models are shown.

4.3. long term covariate lags

We now measure the predictive capacity of the multiplicative intensity model over long periods by increasing the covariate lag to between two and five years. As with Section 4.2, the analysis is completed for the Optimal model. A one year calculation period for return and volatility is adopted, as this length is more appropriate for the models with longer covariate lags because long term financial health is the primary interest. The calibrations and out-of-sample forecasts for the Optimal model are displayed in Table 4.3 and Figure 4.3. For comparison, the one year covariate lag Optimal model is also included in Figure 4.3, where the RROC area is marginally weaker than in previous sections because of a different calculation period for return and volatility.

The coefficient estimates for WC/TA and EBIT/TA are not statistically significant (and thus not included) in the Optimal model beyond a one year covariate lag, and are not displayed for this reason. As the covariate lag lengthens, TL/TA becomes more meaningful in comparison to WC/TA, despite still failing to qualify at five percent significance beyond a three year covariate lag. As

Table 4.3: Long Term Covariate Lags Calibrations. Optimal model calibrations with two year, three year, four year and five year covariate lag. The sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1999. The left column is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

	Optimal Covariates			
Covariate Lag	Two Year	Three Year	Four Year	Five Year
#Failures	326	309	391	363
#Firms	3105	2908	2887	2700
Exposure	45896	42855	40561	37974
return	-0.7975 (< 0.0001)	-0.4830 (0.0010)	-0.3006 (0.0262)	
size	-0.5301 (< 0.0001)	-0.5114 (< 0.0001)	-0.4250 (< 0.0001)	-0.4454 (< 0.0001)
volatility	23.8736 (< 0.0001)	20.8397 (< 0.0001)	25.5455 (< 0.0001)	22.3348 (< 0.0001)
TL/TA	0.4979 (< 0.0001)	0.45729 (0.0021)		
Model Fit	3980 (< 0.0001)	3884 (< 0.0001)	5052 (< 0.0001)	4699 (< 0.0001)

previously discussed, TL/TA is a long term comparison of assets to liabilities, so this result is not surprising. Also, as the covariate lag increases, the linear time dependence (nonproportionality) of the TL/TA coefficient estimate decreases, and is not statistically significant for a covariate lag greater than one year. Beyond the Optimal model with a three year covariate lag, we find only market driven coefficient estimates are accepted, with company account ratios deemed uninformative of long term failure. Furthermore, a firm's return in excess of the index is of insignificant consequence when analysing failure over four years hence, with only a firm's size and the volatility of the returns in excess of the index remaining relevant.

Again, we are shown by Figure 4.3 that as the covariate lag increases, the out-of-sample forecasting accuracy of the Optimal models decrease toward *uninformative*. Although lessened at four and five years, the predictive power of the multiplicative intensity model is commendable considering it is regressed on only three and two covariates respectively, with over seventy percent of failed firms ranked in the highest quintile. Also, there are very few failures from firms with low relative failure intensities even with a forecast as far as five years. Recall, the relative risk process is not explicitly affected by influences beyond the firm level, so it is more accurate over longer periods than an absolute risk measure, because we do not need to explicitly considered events common to all firms.

The multiplicative intensity model has proven to be both flexible and accurate when predicting failure for firms listed on AMEX and NYSE.

5. Analyses of the model assumptions

We have introduced numerous improvements over previous models, particularly the proposition of a relative risk process in a continuous time environment, and the subsequent choice of covariates.

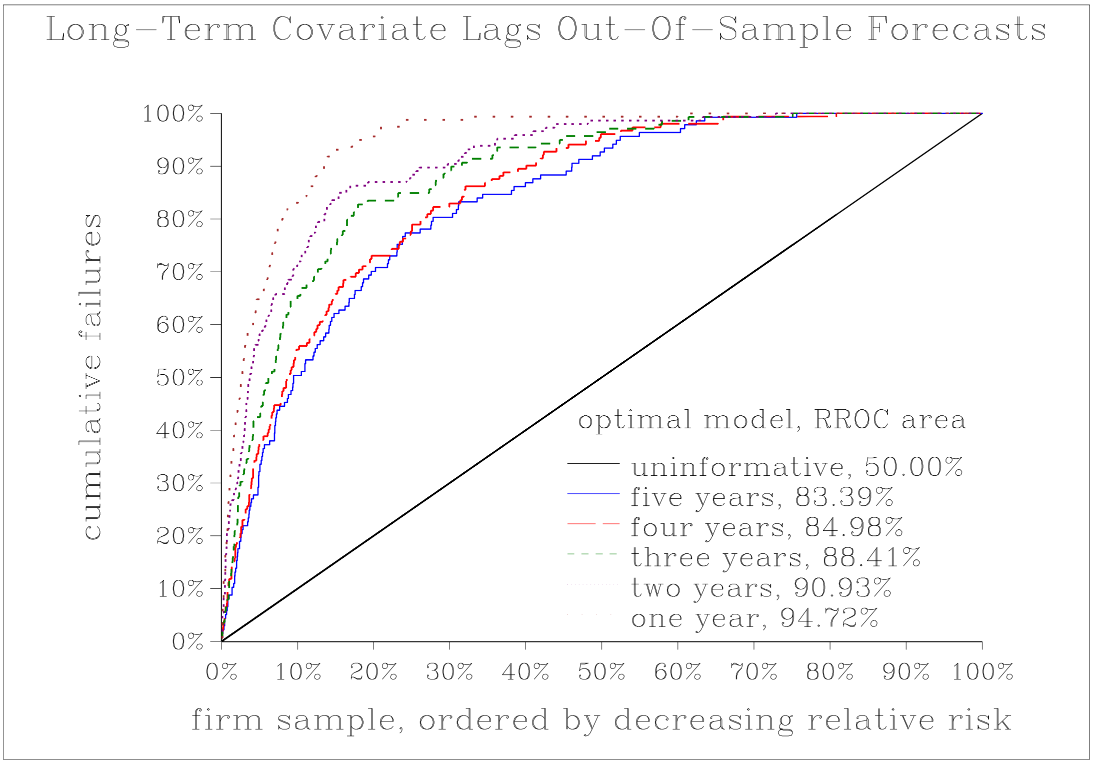

Figure 4.3: Optimal model out-of-sample forecasts with one year, two year, three year, four year and five year covariate lag. The calibration sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

In this section, we replicate the analyses performed by Shumway (2001), thus allowing a direct comparison with the assumptions made. Minor changes in results appear, however, because of different failure definitions and data handling techniques, but such exceptions are necessary to ensure comparability between the analyses.

5.1. discrete *versus* continuous time

The accuracy of a continuous-time multiplicative intensity model with a one year covariate lag is compared to the equivalent discrete-time model.

For the discrete-time multiplicative intensity model

$$\frac{d\Lambda_i^*(t)}{1-d\Lambda_i^*(t)} = \frac{Y_i(t)\exp\{\alpha'\mathbf{X}_i(t)\}\,d\Lambda_0^*(t)}{1-d\Lambda_0^*(t)} \ ,$$

which we construct using one-year intervals (Shumway, 2001). $d\Lambda_i^*(t)$ is the non-zero failure probability process for the period t, and $d\Lambda_0^*(t)$ the baseline probability process, as adapted from Cox (1972). The coefficient vector, $\boldsymbol{\alpha}$, is determined by the same procedure as $\boldsymbol{\beta}$ in the continuous-time multiplicative intensity model. The covariate processes, however, are lagged six months prior

Table 5.1: Discrete *versus* Continuous Time Calibrations. Optimal model relative risk calibrations in a discrete time setting with one year time periods and a six month covariate lag, and in a continuous time setting with a one year covariate lag. The sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1999. The left column of the major panel is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$ and $\hat{\alpha}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

	Optimal Model	
Time Measure	Discrete - Annual	Continuous
#Failures	367	367
#Firms	3445	3503
Exposure	52490	51604
return	-0.9899 (0.0001)	-1.2648 ($<$ 0.0001)
size	-0.5824 ($<$ 0.0001)	-0.5068 ($<$ 0.0001)
volatility	24.0890 ($<$ 0.0001)	31.8053 ($<$ 0.0001)
WC/TA	-1.1085 ($<$ 0.0001)	-0.6395 ($<$ 0.0001)
EBIT/TA	-0.9691 ($<$ 0.0001)	-0.4667 (0.0008)
Model Fit	3100 ($<$ 0.0001)	4311 ($<$ 0.0001)

to the year of interest in acknowledgement a period of one year to the average failure time, and thus ensuring comparability.

The calibration of the discrete-time Optimal model, shown in Table 5.1, finds the same coefficient estimates being accepted as statistically significant as with a continuous-time measure. The impact on the firm failure intensity of movement in the company account driven covariates, EBIT/TA and WC/TA, is greater in a discrete time setting, as is movement in size. On the other hand, changes in return and volatility have a lesser effect in a discrete time setting. The out-of-sample forecasting in Figure 5.1 lends evidence to the superiority of a continuous-time measure, with residency in the highest quintile surpassing the annual discrete-time measure by four percent. Thus, we consider the continuous time setting as preferable to the discrete time setting under a relative risk model.

5.2. constant baseline intensity

Filtration arguments aside, Shumway (2001) and Chava and Jarrow (2004) used a discrete-time multiplicative intensity model assuming a constant baseline probability process—a generalised linear model with binomially distributed error and logit link function. We fit this logit model as used by Shumway (2001) and Chava and Jarrow (2004), and compare it to the continuous-time multiplicative intensity model with a constant baseline intensity process—a generalised linear model with binomially distributed error and complementary log-log (*cloglog*) link function. Thus, we are able to seek justification for the optimal covariate selection and continuous time setting under specification similar to that used by Shumway (2001). Also, to minimise divergence in methodology from Shumway (2001), unconditional maximum likelihood estimation is used to calibrate the absolute risk process in the continuous-time and discrete-time multiplicative intensity models.

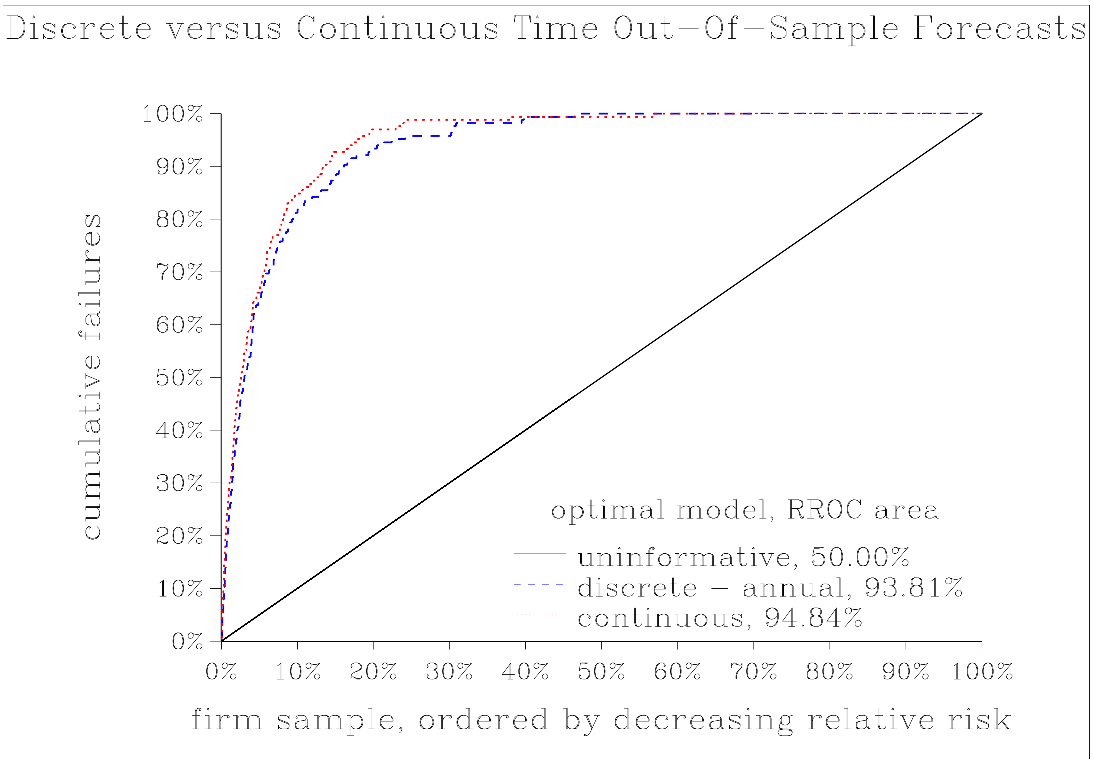

Figure 5.1: Optimal model out-of-sample forecasts in both a discrete time setting with one year time periods and a six month covariate lag, and a continuous time setting with a one year covariate lag. The calibration sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

A constant baseline intensity process is not an ideal assumption, which is demonstrated by the cloglog model containing different coefficient estimate values in Table 5.2 than when the models were calibrated with the baseline intensity process left undefined. We do not, however, investigate the true nature of the baseline intensity process. The logit model with Shumway covariates demonstrates statistically significant nonproportionality, which suggests it is an inappropriate choice, with diminishing effects of NI/TA and TL/TA on the probability of failure over the sample period.

The out-of-sample forecasts in Figure 5.2 are based on the absolute failure intensity as approximated using unconditional maximum likelihood estimation. Hence, instead of ranking observations based on the relative risk at a time, we rank observations over the entire forecast period, which produces the traditional ROC area. The cloglog model is superior to the logit model for the Optimal covariate selection and for the covariates proposed by Shumway (2001): it exceeds the logit model in residency in the highest decile by five percent with the Optimal covariates; and by over three percent with the Shumway covariates. Adding to this, the ROC area is higher for the cloglog model for both the Optimal and Shumway covariates. Since the assumptions are now directly comparable, the results presented here should remove any doubt as to the appropriateness of a continuous time setting and the superiority of the Optimal covariates. Both out-of-sample forecasting performance and satisfying the assumption of proportionality is preferable in the continuous-time multiplicative intensity model.

Table 5.2: Unconditional Maximum Likelihood Calibrations. Optimal and Shumway Covariates models with absolute risk calibrations in both a discrete time setting (logit) with one year time periods and a six month covariate lag, and a continuous time setting (cloglog) with a one year covariate lag. The sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1999. The left column of the major panel is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$ and $\hat{\alpha}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

	Optimal Covariates[1]				Shumway Covariates[2]			
Model Link	logit		cloglog		logit		cloglog	
#Failures	367		367		370		370	
#Firms	3445		3503		3457		3518	
Exposure	52490		51604		52799		51679	
log(baseline)	-12.1644	(< 0.0001)	-14.0524	(< 0.0001)	-13.2932	(< 0.0001)	-14.8588	(< 0.0001)
return	-0.9447	(0.0001)	-1.0903	(< 0.0001)	-1.0010	(< 0.0001)	-1.1352	(< 0.0001)
size	-0.6267	(< 0.0001)	-0.5173	(< 0.0001)	-0.6549	(< 0.0001)	-0.5458	(< 0.0001)
volatility	21.0696	(< 0.0001)	27.7790	(< 0.0001)	29.0085	(< 0.0001)	28.0695	(< 0.0001)
WC/TA	-1.0879	(< 0.0001)	-0.6743	(< 0.0001)				
EBIT/TA	-1.0353	(< 0.0001)	-0.5146	(< 0.0001)				
NI/TA					-0.5740	(0.0003)	-0.3085	(0.0135)
TL/TA					0.6866	(< 0.0001)	0.4477	(< 0.0001)
Model Fit	3343	(< 0.0001)	5316	(< 0.0001)	3354	(< 0.0001)	5329	(< 0.0001)

1. return, size, volatility, WC/TA & EBIT/TA

2. return, size, volatility, NI/TA & TL/TA

6. Expanded sample models with industry heterogeneity

We now expand our model to include firms listed on the NASDAQ; with the sample now including AMEX, NYSE and NASDAQ firms from 1 January 1962 to 31 December 1999. Unlike in Sections 4 and 5, the sample includes financials, but public administration firms remain excluded. Models with covariate lags at three months and one year are featured in this section, with other covariate lags excluded for brevity.

With similar criteria for firm inclusion, the expanded sample increases the number of failures on Chava and Jarrow (2004) three-fold. Therefore, we can conclude that this paper has a broader definition of failure. Despite burdening the forecasting results, widening the definition of failure is an imperative compromise to minimise informative right-censoring. That is, by excluding failures similar to bankruptcy within the definition, the assumption of non-informative censoring is corrupted, because correlation exists between the probability of bankruptcy and delisting due to financial distress.

We split the data into four general industry classes for modelling industry heterogeneity (Table 6.1). These classes are fewer and thus wider than labelled by the market, but such simplification is to ensure sufficient residency in each class and is consistent with Chava and Jarrow (2004). Agriculture, construction, wholesale and retail trade, and service industries ($IND4$) form the largest industry group, with thirty-nine percent of the sample exposure. Mineral and manu-

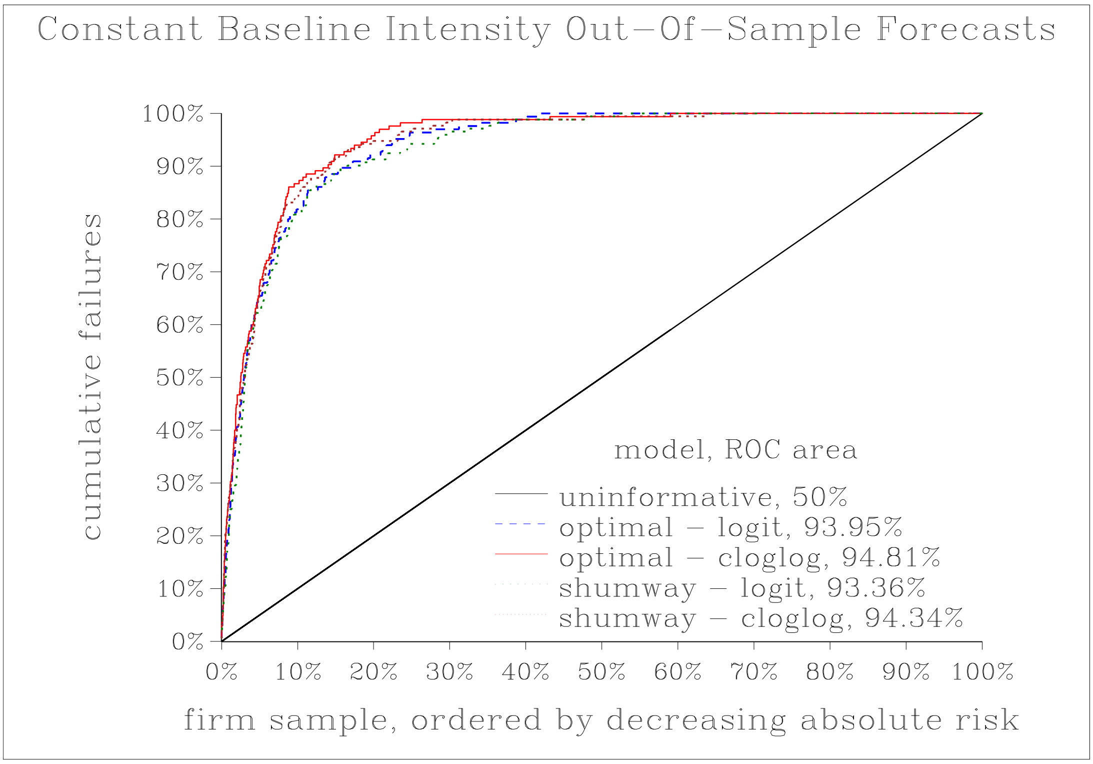

Figure 5.2: Optimal and Shumway Covariates models with absolute risk out-of-sample forecasts in both a discrete time setting (logit) with one year time periods and a six month covariate lag, and a continuous time setting (cloglog) with a one year covariate lag. The calibration sample consists of all observable firms from the AMEX and NYSE with the exclusion of financials and public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

facturing industries (*IND*3) follow with thirty-eight percent. Finance, insurance and real estate (*IND*1) firms contribute fifteen percent, and the remaining exposure (8%) can be attributed to transportation, communication and utilities (*IND*2).

Industry heterogeneity is modelled by two methods. In the first of these methods, the relative risk for a firm i in industry group I is scaled by a constant value, κ_I, thus assuming that industry classification has a proportional effect on the intensity process. The value of κ_I is calibrated by introducing the indicator covariates $INDI = \mathbb{1}_{i \in I}$ for $I = 1,2,3$ ($I = 4$ is redundant, since industry group four is implied by $IND1 = IND2 = IND3 = 0$). Coefficients for these covariates are estimated in the usual way, with $\kappa_I = exp\{\beta_{INDI}\}$. This style of model is referred to as a *proportional effects* model. For the second method, the baseline intensity process is allowed to vary between industry groups, with $\hat{\beta}$ left identical between industries. This generalises the previous model to allow a non-proportional effect of industry group on the multiplicative intensity, and is referred to as a *baseline effects* model. Lastly, the coefficient estimates are allowed to differ between industry groups, thus implying that the impact of firm-specific covariates differ between industries. This style of model is completed for both methods, and is referred to as possessing *covariate effects*. We fit the Optimal models with no covariate effects, all covariate effects, and Chava and Jarrow (2004) covariate effects (where NI/TA and TL/TA are the only account driven covariates).

Table 6.1: Industry Classification

Group	SIC	Covariate	Exposure Contribution
finance, insurance and real estate	6000-6799	IND1	15%
transportation, communication and utilities	4000-4999	IND2	8%
mineral and manufacturing	1000-1499, 2000-4000	IND3	38%
agriculture, construction, wholesale and retail trade, and service industries	1-999, 1500-1799, 5000-5999, 7000-8899	IND4	39%

EBIT/TA and WC/TA are excluded for the analysis on the expanded sample, because many of the firms previously excluded, particularly financials, do not supply readily available information to calculate these covariates on a consistent basis. This is an unfortunate detraction for the models presented in this section, because EBIT/TA and WC/TA are the account driven covariates which were found most appropriate in Section 4 and are the most cohesive with the assumption of proportionality. CA/CL is withdrawn for the same reason.

6.1. three month covariate lag model

The expanded sample models with a three month covariate lag, displayed in Table 6.3, are fitted by the backwards exclusion procedure described in Section 4, with all coefficients (excluding those relating to EBIT/TA, WC/TA and CA/CL) tested. The Model Fit statistics (likelihood ratios) are comparable in the cases with identical exposure, although not between the proportional and baseline effects models.

The directions of the coefficient estimates in the models with no covariate effects support intuition, with the exception of RE/TA. We cannot be too hasty, however, in concluding that higher net income retention is indicative of failure. Recall, the covariate processes must be considered as a package, particularly, RE/TA should be analysed alongside NI/TA, with which it shares a strong positive relationship. NI/TA is usually larger in magnitude than RE/TA, and its coefficient estimate also dominates. So considered in tandem, they display a strong negative effect of earnings on the failure intensity; although retaining earnings (rather than returning earnings to investors) lessens this effect within a three month covariate lag model. The model quantifies the intuitive result, with failure in the short term predicted as more likely for firms that retain rather than distribute profits.

There are two other estimates worth noting. First, the SALES/TA coefficient estimate is statistically significant for the proportional effects model, but not the baseline effects model. Recall also that SALES/TA is not informative in a univariate model. We contend that this ambiguity occurs because, as we will see when covariate effects are allowed, SALES/TA has a strong (but different) effect in all industry groups except the largest—agriculture, construction, wholesale and retail trade, and service industries. Second, the proportional effects model suggests a higher failure intensity for the transportation, communication and utilities industry group. It does not, however, find a statistically significant difference between all other industry groups. As we will discuss

Table 6.3: Expanded Sample Three Month Covariate Lag Calibrations. Optimal Covariates models allowing industry heterogeneity at a covariate lag of three months. The sample consists of all observable firms from the AMEX, NYSE and NASDAQ with the exclusion of public administration firms, 1 January 1962 to 31 December 1999. The left column of the major panel is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

	Optimal Covariates					
Covariate Effects	None		All		Chava and Jarrow (2004)	
Industry Effects	Proportional	Baseline	Proportional	Baseline	Proportional	Baseline
#Failures	3457	3457	3457	3457	3499	3499
#Firms	17302	17302	17302	17302	17484	17484
Exposure	147356	147356	147356	147356	152193	152193
return	-0.9022 ($<$ 0.0001)	-0.9031 ($<$ 0.0001)	-0.9014 ($<$ 0.0001)	-0.8847 ($<$ 0.0001)	-0.9542 ($<$ 0.0001)	-0.9534 ($<$ 0.0001)
size	-0.8007 ($<$ 0.0001)	-0.8008 ($<$ 0.0001)	-0.8055 ($<$ 0.0001)	-0.8068 ($<$ 0.0001)	-0.7926 ($<$ 0.0001)	-0.7948 ($<$ 0.0001)
size $\times$ IND1			0.0415 (0.0002)			
volatility	14.3000 ($<$ 0.0001)	14.2456 ($<$ 0.0001)	13.5857 ($<$ 0.0001)	13.4302 ($<$ 0.0001)	14.4700 ($<$ 0.0001)	14.4585 ($<$ 0.0001)
volatility $\times$ IND1			5.9148 (0.0003)	6.8892 ($<$ 0.0001)		
NI/TA	-0.7119 ($<$ 0.0001)	-0.7283 ($<$ 0.0001)	-0.6381 ($<$ 0.0001)	-0.6576 ($<$ 0.0001)	-0.45924 ($<$ 0.0001)	-0.4665 ($<$ 0.0001)
NI/TA $\times$ IND3			-0.2166 ($<$ 0.0001)	-0.2049 ($<$ 0.0001)	-0.2416 ($<$ 0.0001)	-0.2437 ($<$ 0.0001)
TL/TA	0.5304 ($<$ 0.0001)	0.0334 ($<$ 0.0001)	0.5107 ($<$ 0.0001)	0.5254 ($<$ 0.0001)	0.5104 ($<$ 0.0001)	0.5185 ($<$ 0.0001)
TL/TA $\times$ IND2			0.6119 ($<$ 0.0001)	0.8310 ($<$ 0.0001)		
RE/TA	0.0457 ($<$ 0.0001)	0.0467 ($<$ 0.0001)	0.0457 ($<$ 0.0001)	0.0488 ($<$ 0.0001)		
RE/TA $\times$ IND2			0.1195 (0.0005)	0.1222 (0.0002)		
ME/TL	-0.0273 ($<$ 0.0001)	-0.0255 ($<$ 0.0001)	-0.0225 ($<$ 0.0001)	-0.0215 ($<$ 0.0001)		
ME/TL $\times$ IND1			0.0306 (0.0009)	0.0321 (0.0005)		
ME/TL $\times$ IND3			-0.0800 ($<$ 0.0001)	-0.0826 ($<$ 0.0001)		
SALES/TA	-0.0344 (0.0494)					
SALES/TA $\times$ IND1			0.2478 (0.0003)	0.2014 (0.0063)		
SALES/TA $\times$ IND2			-0.1579 (0.0142)	-0.2640 (0.0007)		
SALES/TA $\times$ IND3			-0.0658 (0.0077)	-0.0943 (0.0083)		
IND2	0.2310 (0.0019)				0.2434 (0.0012)	
IND3					-0.1690 ($<$ 0.0001)	
Model Fit	48251 ($<$ 0.0001)	40822 ($<$ 0.0001)	48123 ($<$ 0.0001)	40679 ($<$ 0.0001)	49037 ($<$ 0.0001)	41477 ($<$ 0.0001)

below, these differences are better explained by allowing covariate effects.

By including covariate effects, the industry indicators alone are no longer relevant, hence different failure intensities between industries are fully reflected through firms' covariate processes. Although the improvement in adequacy of fit by allowing covariate effects is minimal, with the introduction of such complexities changing the Model Fit by a statistically insignificant amount in both the proportional and baseline effects models.

The all covariate effects models provide an indication of how covariates impact on the failure intensity between industries. For example, it is seen that high volatility is a prelude to firm failure in the finance, insurance and real estate industries more so than other industries. Indeed, many coefficient estimates specific to the finance, insurance and real estate sector justify the industry's exclusion in previous analyses, with counterintuitive results for some coefficient estimates, such as an improved SALES/TA increasing the failure intensity. Although we have argued that coefficient estimates should not be analysed in isolation, such patently unnatural results prompt suspicion as to the validity of the inclusion of this industry class or the SALES/TA covariate in a model assuming proportionality.

As mentioned, we fail to accept coefficient estimates for solitary industry indicator covariates, meaning that a firm's industry hasn't a direct statistically significant impact of the failure intensity. Instead, industry heterogeneity is better explained indirectly by the different impacts of the covariate processes on a firm's failure intensity.

When Chava and Jarrow (2004) covariate effects are considered, we can conclude that NI/TA has a stronger effect on reducing the mineral and manufacturing industries' failure intensity than other industries, which is true for the proportional and baseline industry effects. Also, by not allowing the wide selection of covariates, the proportional effects model now accepts the coefficient estimates for $IND2$ and $IND3$.

Some improvement is expected with the baseline effects model lessening constraints, but the degree of improvement supports the baseline effects assumption, although specific tests on the Model Fit statistics are forbidding on a data sample of this size. The inappropriateness of industry indicators in the intensity function is confirmed, however, upon testing such covariates for proportionality, with the coefficient estimates for $IND1$ and $IND3$ demonstrating statistically significant time dependence. Moreover, the $IND1$ coefficient estimate actually changes direction over the course of the sample. That periods of high and low absolute failure intensity differ across industry classes is not a surprise.

Despite the additional covariates calibrated, Figure 6.1 displays slightly less accurate results compared to Chava and Jarrow (2004), but for a three month instead of a one month covariate lag and a broader definition of failure. The RROC areas show little discernible difference between the models. The Chava and Jarrow (2004) covariate effects models perform the best over the highest quintile and have the strongest RROC areas, which are likely to be due to the exclusion of other account driven covariates, with ME/TL demonstrating statistically significant nonproportionality. Although residency between ninety and ninety-two percent in the highest quintile is a reduction from ninety-nine percent in the three month *simple* model (Figure 4.2), the forecast period of 1991 to 1999 now constitutes approximately two thirds of the sample of failures, and so calibration is performed on a relatively small sample. The worst forecasting performance is the baseline industry effects model with covariate effects (92.21 percent RROC area), which is surprising given it

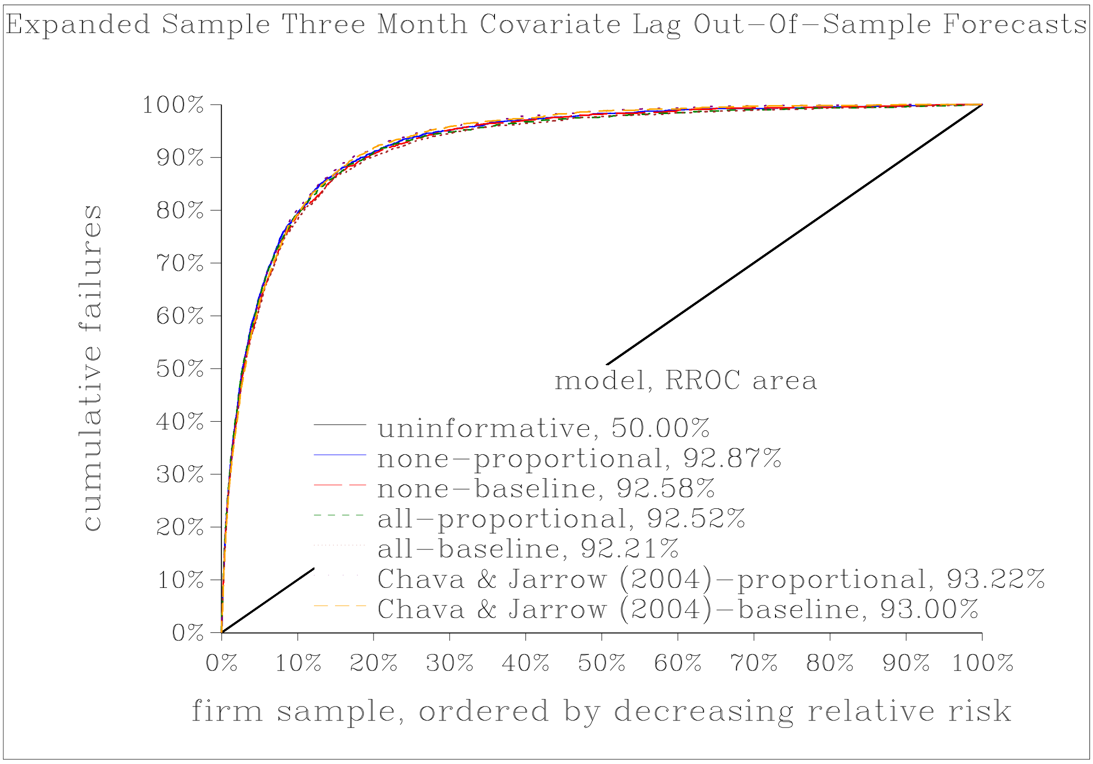

Figure 6.1: Optimal Covariates models allowing industry heterogeneity at a covariate lag of three months. The calibration sample consists of all observable firms from the AMEX, NYSE and NASDAQ with the exclusion of public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

displayed the best Model Fit statistic of those comparable. Such a seemingly contradictory finding is testament to the nonproportionality in these expanded sample models.

6.2. one year covariate lag model

Similar coefficient estimates are found statistically significant when lengthening the expanded sample to a one year covariate lag. Table 6.4 contains these calibrations, and Figure 6.2 displays the corresponding out-of-sample forecasts. Many of the same conclusions can be reached, with baseline industry effects demonstrating a better fit than the proportional effects. Again, forecast results favour the Chava and Jarrow (2004) covariate effects over others by a small margin of approximately two percent in the highest quintile. Allowing covariate effects does not significantly improve the Model Fit; in particular, allowing all covariate effects diminishes the forecasting accuracy under proportional industry effects. This can be explained by the nonproportionality in the effects of both ME/TL and the industry indicators.

An increased heterogeneity is found amongst industries in the effect of NI/TA and TL/TA on the failure intensity one year hence in the Chava and Jarrow (2004) models. A strong NI/TA is particularly favourable to finance, insurance and real estate firms, while the transportation, communication and utilities industries have a relatively poor response to a high TL/TA. As with the

Table 6.4: Expanded Sample One Year Covariate Lag Calibrations. Optimal Covariates models allowing industry heterogeneity at a covariate lag of one year. The sample consists of all observable firms from the AMEX, NYSE and NASDAQ with the exclusion of public administration firms, 1 January 1962 to 31 December 1999. The left column of the major panel is the covariate, with all other columns displaying the corresponding coefficient estimates, $\hat{\beta}$, and p-values. The Model Fit panel refers to the likelihood ratio test with p-value.

	Optimal Covariates - One Year Lag					
Covariate Effects	None		All		Chava and Jarrow (2004)	
Industry Effects	Proportional	Baseline	Proportional	Baseline	Proportional	Baseline
#Failures	3253	3253	3253	3253	3297	3297
#Firms	16407	16407	16407	16407	16606	16606
Exposure	134891	134891	134891	134891	139590	139590
return	-0.6312 (< 0.0001)	-0.6293 (< 0.0001)	-0.7093 (< 0.0001)	-0.6186 (< 0.0001)	-0.6795 (< 0.0001)	-0.6788 (< 0.0001)
return × IND1			0.2486 (0.0328)			
size	-0.5967 (< 0.0001)	-0.6007 (< 0.0001)	-0.5641 (< 0.0001)	-0.5685 (< 0.0001)	-0.5841 (< 0.0001)	-0.5849 (< 0.0001)
size × IND3			-0.09272 (0.0005)	-0.0892 (0.0010)		
volatility	10.6719 (< 0.0001)	10.4476 (< 0.0001)	9.9457 (< 0.0001)	9.6872 (< 0.0001)	10.8919 (< 0.0001)	10.7236 (< 0.0001)
volatility × IND1			6.7324 (0.0002)	8.5236 (< 0.0001)		
NI/TA	-0.6627 (< 0.0001)	-0.6603 (< 0.0001)	-0.6619 (< 0.0001)	-0.6590 (< 0.0001)	-0.4273 (< 0.0001)	-0.4263 (< 0.0001)
NI/TA × IND1					-0.2722 (0.0293)	-0.3494 (0.0109)
NI/TA × IND3					-0.1758 (0.0010)	-0.1920 (0.0004)
TL/TA	0.3671 (< 0.0001)	0.3835 (< 0.0001)	0.3910 (< 0.0001)	0.3883 (< 0.0001)	0.2972 (< 0.0001)	0.30641 (< 0.0001)
TL/TA × IND2				0.3367 (0.0434)	0.2616 (0.0024)	0.33199 (< 0.0285)
RE/TA	0.0533 (< 0.0001)	0.0539 (< 0.0001)	0.0589 (< 0.0001)	0.0587 (< 0.0001)		
RE/TA × IND1			-0.0638 (0.0309)	-0.0702 (0.0246)		
ME/TL	-0.0196 (< 0.0001)	-0.0195 (< 0.0001)	-0.0138 (0.0009)	-0.0136 (0.0012)		
ME/TL × IND3			-0.0301 (0.0024)	-0.0298 (0.0026)		
SALES/TA	-0.1988 (< 0.0001)	-0.2053 (< 0.0001)	-0.1807 (< 0.0001)	-0.1828 (< 0.0001)		
SALES/TA × IND1			0.3678 (< 0.0001)	0.3109 (0.0005)		
SALES/TA × IND3			-0.1714 (0.0007)	-0.1785 (0.0005)		
IND1	-0.3344 (< 0.0001)		-0.8669 (< 0.0001)		-0.1887 (0.0068)	
IND3	-0.1406 (0.0002)		-1.0644 (0.0023)		-0.1977 (< 0.0001)	
Model Fit	49683 (< 0.0001)	42606 (< 0.0001)	49613 (< 0.0001)	42625 (< 0.0001)	50526 (< 0.0001)	43413 (< 0.0001)

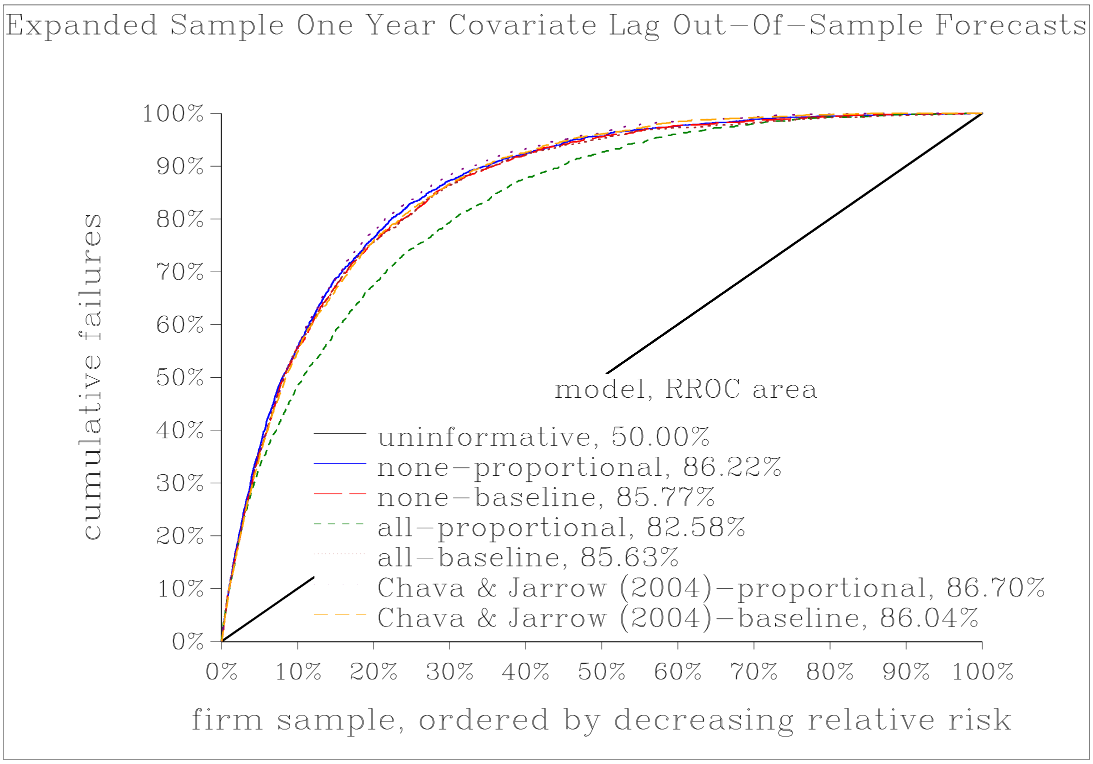

Figure 6.2: Optimal Covariates models allowing industry heterogeneity at a covariate lag of one year. The calibration sample consists of all observable firms from the AMEX, NYSE and NASDAQ with the exclusion of public administration firms, 1 January 1962 to 31 December 1990. The out-of-sample forecast is performed from 1 January 1991 to 31 December 1999.

three month covariate lag model, NI/TA also has a strong negative effect on the failure intensity of mineral and manufacturing firms.

Chava and Jarrow (2004) found the coefficient estimate of NI/TA as unique for all four industry classes, and a high TL/TA more detrimental to mineral and manufacturing firms. The Chava and Jarrow (2004) models estimated here, however, are dissimilar to the discrete-time absolute risk model presented in Chava and Jarrow (2004). Since the definition for firm failure also differs, with this study increasing the sample of failures three-fold, different results are expected.

7. Limitations

The multiplicative intensity process analyses we present in this paper are not without limitations. While improvements over previous research have been investigated, scope remains to refine the model further, particularly in versatility of application. We outline possible extensions to the analyses below.

All experiments in this paper aim toward calibrating and testing a firm failure intensity predictor at some future time, as determined by the covariate lag. Coefficient estimate values change over different covariate lags, and covariates gain and lose relevance, which is difficult when seeking to apply the multiplicative intensity process at a covariate lag not represented in the model, or when attempting to observe the change in the failure intensity over a continuous time scale. Whilst calibrating with any covariate lag is possible, it would be burdensome, and a more practical solution would be to perform a further regression on the estimated failure intensities.

Alternatively, one could recognise the stochastic nature of the covariate processes and seek to project these values forward, generating a failure intensity distribution at future times. Such an approach, however, is problematic because, as this paper has shown, the statistically significant coefficient estimates change with the time to failure. Additionally, caution is advised because the covariate processes are *internal*, with the failure intensity potentially exciting the covariate processes (Kalbfleisch and Prentice, 2002).

Lastly, to formulate an absolute failure intensity, the baseline intensity process requires specification, otherwise the results are limited in use to comparisons of failure intensities between firms. Whilst it is beyond the scope of this paper to estimate the nature of such specifications, future research may extend the analyses presented here to design a baseline intensity process. As suggested earlier, the baseline intensity enables the recognition of regressors common to all firms, and the design of this process should be based on intuition regarding these influences. In particular, we propose that economic forecasts and business cycle prediction would be the best basis.

8. Conclusion

A multiplicative intensity process is a powerful tool for assessing an AMEX or NYSE listed firm's propensity to fail. Conditioned on the careful selection of covariates, the models presented in this paper have demonstrated their suitability in ranking firms most likely to fail at a specified future time.

With out-of-sample forecasting, we have shown that the multiplicative intensity model performs best over shorter time periods, although the predictive capacity remains strong for failure over one year hence. As the time horizon extends, market driven covariates such as size and volatility become the optimal predictors of failure over company account driven covariates. Moreover, the forecasting power is an improvement of that presented by Shumway (2001) and Chava and Jarrow (2004) for the AMEX-NYSE sample.

The transfer from the discrete time setting proposed by Shumway (2001) to the continuous time setting adopted in this article is intuitively more acceptable, and has also delivered more accurate forecasting results. This is true for the relative failure intensity from a maximised partial likelihood and for the corresponding absolute failure intensity from a maximised unconditional likelihood.

On expansion of the data sample to include both NASDAQ listed and finance, insurance and real estate firms, the forecasting accuracy of the multiplicative intensity process diminishes, with methods for handling industry heterogeneity showing varying success. Leaving the direct industry influence unspecified suggests a superior fit when compared to assuming proportionality, although this is not reflected in the out-of-sample forecasts. When coefficient estimates are permitted to vary between industry classes, no significant benefit is achieved in either the Model Fit or the out-of-sample forecasting. Unfortunately, the success of the out-of-sample forecasting in this expanded sample is limited because of the unavailability of the account driven covariates that were relevant in the AMEX-NYSE sample. The expanded sample still delivers comparable forecasting results to Chava and Jarrow (2004), which is encouraging given the broader definition of failure presented in this study.

With these more inclusive failure definitions, some loss in accuracy is expected, so this amplifies the superiority of the multiplicative intensity process in the AMEX-NYSE sample. We have presented a system for comparing firms in regard to their failure intensity a future time. Highly accurate forecasting has testified to the applicability of the multiplicative intensity process in this system.

Appendices

A. Data Schedule

CRSP		
Field	Use in Analysis	Notes
Static Data		
npermno	identification	
cusip	merging sets	the primary identify for merging databases
linkdt	merging sets	Used to find where merging links correspond
linkenddt	merging sets	Used to find where merging links correspond
linktype	merging sets	Used to find where merging links correspond
dlstdt	failure/censor time	
dlstcd	failure/censor reason	Failure Codes: 400-499, 552-561, 572, 574-587
siccd	industry identification	see table 6.1 on page 19
primexch	exchange identification	Includes AMEX, NYSE and NASDAQ
Dynamic Data		
sasdate	date	
prc	size	
ret	return and volatility	
shrout	size	
shrsdt	size	
shrenddt	size	
Indices Data		
caldt	date	
vwretd	return and volatility	For both simple and expanded samples
totval	size	For both simple and expanded samples

Compustat		
Field	Use in Analysis	Notes
Static Data		
GVKEY	identification	
DLDTE	merging sets	Used to corroborate merging link
DLRSN	merging sets	Used to corroborate merging link
SIC	merging sets	Used to corroborate merging link
Dynamic Data		
DATADATE	date	
ACT	CA/CL	ACTQ (quarterly equivalent) also used
AT	WC/TA, RE/TA, EBIT/TA, SALES/TA, NI/TA and TL/TA	ATQ (quarterly equivalent) also used
CEQ	ME/TL	CEQQ (quarterly equivalent) also used
LCT	CA/CL	LCTQ (quarterly equivalent) also used
LT	ME/TL and TL/TA	LTQ (quarterly equivalent) also used
NI	NI/TA	NIQ (quarterly equivalent) also used
RE	RE/TA	REQ (quarterly equivalent) also used
SALE	SALES/TA	SALEQ (quarterly equivalent) also used
WCAP	WC/TA	WCAPQ (quarterly equivalent) also used

B. survival analysis and the counting process format

This appendix deals with the background material required to understand the multiplicative intensity process estimator derivation in Appendix C.

The observed pair $(\min(T,C), \mathbb{1}_{T\le C})$ is often taken as our primary observations to be applied to some estimation, which can be seen as the time of observable exit for each firm and the mode of this exit. Here, for the ith firm, T_i is the time of firm failure, C_i is the time of right-censoring, and $\mathbb{1}_{T_i\le C_i}$ indicates that firm failure has been observed, for $i = 1,\dots,q$. We transform this observed pair into a more flexible counting process formulation, (N,Y) (Andersen and Gill, 1982).

For the multivariate counting process $N = (N_1,\dots,N_q)$, each process N_i begins at zero and counts the unit jumps of the ith firm, with a jump representing observed firm failure. Importantly, $N_i(t)$ cannot be observed for $t > T_i$. Intuitively, this is not restrictive since the observed process N_i is strictly limited to 1 by the definition of firm failure—a firm can fail only once. When dealing with events other than firm failure, this specification can easily be relaxed. Assume $dN_i(t)dN_j(t) = 0$ for $j \ne i$ where $dN_i(t) = \lim_{dt\searrow 0}(N_i(t^- + dt) - N_i(t^-))$; no two jumps can occur simultaneously. Also, $N_i(t)$ must be finite for some finite t, which is clearly satisfied here.

For the multivariate process $Y = (Y_1,\dots,Y_q)$, Y_i is an indicator process of a firm being observed. That is, $Y_i(t) = 1$ if the firm is under observation at time t^-, and zero otherwise. Y_i is referred to as the *at risk* process.

We define the ith firm's covariate processes as the $(p\times 1)$ column vector $\mathbf{X}_i$ with $\mathbf{X} = (\mathbf{X}_1,\dots,\mathbf{X}_q)$. Any vector $\mathbf{X}_i(t)$ contains some function of observable (possibly lagged) information on firm i at times up to but not including t.

The observed triple $(N,Y,\mathbf{X})$ generates an increasing right-continuous filtration $\{\mathcal{F}_t\}_{t\ge 0}$ of σ-algebras on the probability space $(\Omega,\mathcal{F},\mathbb{P})$. N_i is cadlag; $N_i(t)$ is known at exactly time t. Y_i and $\mathbf{X}_i$ are left-continuous and thus $\{\mathcal{F}_t\}_{t\ge 0}$-predictable; $Y_i(t)$ and $\mathbf{X}_i(t)$ are known at time t^-. Y_i and $\mathbf{X}_i$ have right-side limits and are adapted, and so are locally bounded processes. We have

$$\mathcal{F}_t = \sigma\{N_i(s), Y_i(s^+), \mathbf{X}_i(s^+); i = 1,\dots,q; 0 < s \le t\} \ .$$

N has the cumulative intensity process $\Lambda = (\Lambda_1,\dots,\Lambda_q)$. $\mathbb{E}[dN_i(t)|\mathcal{F}_{t^-}] = d\Lambda_i(t)$ where, in firm failure application, $dN_i(t) = \mathbb{1}_{\{t\le\min(T_i,C_i)<t+dt, T_i\le C_i\}}$. Thus $d\Lambda_i(t)$ can be viewed as the probability of firm i failing in the very small time interval $[t,t+dt)$. We model $d\Lambda_i$ based on the most recent information available at that time, and also require a firm to be at risk to be observed to fail. Thus it can be seen that

$$\begin{aligned} d\Lambda_i(t) &= \mathbb{E}[dN_i(t)|\mathcal{F}_{t^-}] \\ &= \mathbb{P}[dN_i(t) = 1|N_i(s^-), Y_i(s), X_i(s), 0 < s \le t] \\ &= \mathbb{P}[dN_i(t) = 1|N_i(t^-), Y_i(t) = 1, X_i(t)] \\ &= \mathbb{P}[dN_i(t) = 1|Y_i(t) = 1, X_i(t)] \end{aligned}$$

since the information $N_i(t^-)$ is contained in $Y_i(t) = 1$.

C. partial likelihood estimation

We propose the multiplicative intensity model

$$d\Lambda_i(t) = Y_i(t)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_i(t)\right\}d\Lambda_0(t) \ , \tag{1}$$

where $\boldsymbol{\beta}$ is a column vector of coefficients corresponding to the covariate process $\mathbf{X}_i$ (Andersen, 1982). Put simply (or simpler), the failure intensity, $d\Lambda_i$, is modelled as the some underlying failure intensity common to all firms, $d\Lambda_0$, multiplied by the exponential of a linear function of firm-specific covariates (the relative risk process).

We are able to discard Λ_0 by considering the partial likelihood (Cox, 1975) process of equation (1) giving

$$\mathcal{L}(\boldsymbol{\beta},t) = \prod_i \prod_{0<s\leq t}\left(\frac{\exp\{\boldsymbol{\beta}'\mathbf{X}_i(s)\}}{\sum_j Y_j(s)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_j(s)\right\}}\right)^{dN_i(s)} \quad \text{, and} \tag{2}$$

$$\begin{aligned}\log\mathcal{L}(\boldsymbol{\beta},t) &= \sum_i \int_0^t \left[\boldsymbol{\beta}'\mathbf{X}_i(s) - \log\left(\sum_j Y_j(s)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_j(s)\right\}\right)\right]dN_i(s) \\ &= \sum_i \int_0^t \boldsymbol{\beta}'\mathbf{X}_i(s)dN_i(s) - \int_0^t \log\left(\sum_j Y_j(s)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_j(s)\right\}\right)d\bar{N}(s) \end{aligned} \tag{3}$$

where $\bar{N}(s) = \sum_i N_i(s)$.

To simplify formulation, consider the scalar, q-vector and $q\times q$-matrix:

$$\begin{aligned} S^{\{0\}}(\boldsymbol{\beta},t) &= q^{-1}\sum_i Y_i(s)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_i(t)\right\} \ , \\ S^{\{1\}}(\boldsymbol{\beta},t) &= q^{-1}\sum_i \mathbf{X}_i(t)Y_i(s)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_i(t)\right\} \text{ , and} \\ S^{\{2\}}(\boldsymbol{\beta},t) &= q^{-1}\sum_i \mathbf{X}_i(t)^{\otimes 2}Y_i(s)\exp\left\{\boldsymbol{\beta}'\mathbf{X}_i(t)\right\} \ , \end{aligned}$$

and allow $\mathcal{E}(\boldsymbol{\beta},t) = \frac{S^{\{1\}}(\boldsymbol{\beta},t)}{S^{\{0\}}(\boldsymbol{\beta},t)}$. Then $\mathcal{E}(\boldsymbol{\beta},t)$ can be interpreted as the expectation of $\mathbf{X}_i(t)$, weighted by the relative risk of each firm. Similarly, $\mathcal{V}(\boldsymbol{\beta},t) = \frac{S^{\{2\}}(\boldsymbol{\beta},t)}{S^{\{0\}}(\boldsymbol{\beta},t)} - \mathcal{E}(\boldsymbol{\beta},t)^{\otimes 2}$ can be interpreted as the covariance matrix of $\mathbf{X}_i(t)$, weighted by the relative risk of each firm.

We further our analysis by differentiating equation (3) to find the score vector

$$\mathcal{U}(\boldsymbol{\beta},t) = \sum_i \int_0^t \mathbf{X}_i(s)dN_i(s) - \int_0^t \mathcal{E}(\boldsymbol{\beta},s)\,d\bar{N}(s) \ ,$$

and differentiating again with respect to $\boldsymbol{\beta}$ and multiplying by -1 to find the information matrix

$$I(\boldsymbol{\beta},t) = -\int_0^t \mathcal{V}(\boldsymbol{\beta},s)\,d\bar{N}(s) \ .$$

Lastly, we find the $\boldsymbol{\beta}$-estimator, $\hat{\boldsymbol{\beta}}$, on the sample period $0 < t \leq \tau$ by solving the equation $\mathcal{U}(\boldsymbol{\beta},\tau) = 0$. $\hat{\boldsymbol{\beta}}$ is asymptotically normally distributed with a mean $\boldsymbol{\beta}$ and an approximate variance of $\mathbb{E}\left[I(\boldsymbol{\beta},\tau)\right]^{-1}$ (Therneau and Grambsch, 2000, chap. 3.1).

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