Public Infrastructure Spillovers and Growth: Theory and Time Series Evidence for Australia*

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In this paper, growth of per capita income can be exogenous and/or endogenous due to aggregate public infrastructure spillover. The deterministic Glomm and Ravikumar (1994) *model is augmented in this paper to produce a stochastic growth counterpart, which has useful time series implications. In particular, the model implies certain testable cointegration properties that have a bearing on the role of public capital and endogenous growth. The postulation of strict endogenous growth is tested empirically for Australia using a constructed annual data set for the period 1930/31 to 1990/91. This hypothesis is rejected, as there is evidence of a long-run cointegrating relationship about a deterministic trend implying exogenous growth with public capital spillovers. Short-run impulse response analysis is performed using a reduce- form model incorporating the cointegrating equation as the error correction term. There is further short run evidence of the role for public capital accumulation in contributing to permanent increases in the levels of per capita income and private capital.*

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I Introduction

In this paper, the role played by public infrastructure in the long- and medium-term growth of an economy is considered. This issue is of particular interest in an era of shrinking government spending and privatisation. The conventional textbook argument is that a government is able to provide certain public goods that benefit an economy as a whole and which no private enterprise will be willing to produce without monopoly gain. Furthermore, it may be that certain categories of public infrastructure do enter directly as inputs into private production. (Examples of these are roads, highways and also legal courts.) If so, models from the New Growth literature such as Glomm and Ravikumar (1994) and Barro (1990) may be relevant to the issue. This class of models shows that growth in the long run can be perpetual and

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this is due to the endogenous effect of government spending driving the growth in private productivity.

In fact the role of public capital as distinct from private capital in fostering growth in the context of theoretical growth models has received some attention as early back as Arrow and Kurz (1970). Recent models are found in Glomm and Ravikumar (1994) and Barro (1990). In particular, Glomm and Ravikumar (1994) show in a deterministic model that allowing for the possibility of varying degrees of non-rivalry or congestion in what is otherwise non-exclusive public infrastructure does not affect the optimal tax rate in a system of uniform capital and labour taxation.

The influence of public infrastructure on private production in the empirical literature is known as the public capital debate, which began with the seminal work of Aschauer (1989a). Aschauer's method of estimating a single aggregate production function, which incorporates public capital stock, was first adapted for Australian studies by Otto and Voss (1994a). Both papers found that there was a significantly large elasticity of output with respect to public capital (in the order of 0.40). Their methodology was not without criticism. The critiques range from claims of possible endogeneity of the public capital variable to the ad hoc nature of imposing a production function.[1]

It is maintained in this paper that the production function approach is still valid and not *ad hoc*, albeit subject to a different interpretation. As argued by Flores de Frutos *et al* (1998) the production function can be interpreted as a long-run relationship between output, and the private and public inputs. In order to facilitate this interpretation, the Glomm and Ravikumar (1994) model is modified into a stochastic growth model, which then yields testable time series properties. That is, the stochastic growth model with public infrastructure spillovers implies cointegrating relationship(s) between per capita output, private capital and public infrastructure capital which can be tested using the method of Johansen (1991, 1995). In doing so Lau and Sin (1997) will be followed in an application to Australian data. However, Lau and Sin (1997) consider a similar model for U.S. data. Furthermore, this paper compares results for short- to medium-term effects of public infrastructure with existing VAR studies such as Otto and Voss (1996) from the perspective of a vector error correction model nesting a theory-implied cointegrating relationship. There have not been any published studies utilising the vector error correction to study this issue for Australia.[2]

The paper is thus arranged: A stochastic growth version of the Glomm and Ravikumar model and the time series basis of the production function framework is derived in Section II. Section III contains the estimation and test of the hypothesis of strict endogenous growth within the cointegrating relationship. Impulse response analysis and variance decomposition using the vector error-correction structure is enumerated in Section IV. The paper concludes with Section V.

II A Stochastic Growth Model with Public Capital Spillovers

A stochastic growth version of the Glomm and Ravikumar model is presented in this section. In this framework, a representative household-worker chooses an optimal consumption path to maximise expected lifetime utility, given resource constraints and taking government policy as given. The public policy objective is assumed to be the maximisation of the household's optimal path of consumption, subject to technological constraints and a periodic balanced budget à la Barro (1990).

(i) Technology and household choice

Assume that the aggregate production function takes the Cobb-Douglas form

$$Y_t = AK_t^{\alpha}\left[(1+x)^t L_t\right]^{1-\alpha}\widetilde{G}_t^{\theta}\varepsilon_t^{P} \qquad ;\alpha,\theta \in (0,1) \qquad (1)$$

which yields the production function in per worker terms as

$$y_t = A(1+x)^{(1-\alpha)t}k_t^{\alpha}\widetilde{G}_t^{\theta}\varepsilon_t^{P} \qquad ;\alpha,\theta \in (0,1) \qquad (1')$$

where the lower case variables, y and k, denote per worker (L) output and private capital respectively. The Harrod-neutral rate of technological progress is denoted by x. Thus, the model nests the possibilities of exogenous and/or endogenous growth. The production technology is subject to random shocks, ε_t^{P}, assumed to be multiplicative in this model. Aggregate public capital, $\widetilde{G}_t$, enters as an input into production (implying the spillover or externality effect) and it is taken by the representative agent as given. Further, aggregate public capital is subject to

[1] See Sturm (1998, pp. 57-65) for a survey. See e.g. Lynde and Richmond (1991) and Berndt and Hansson (1991).

[2] c.f. Otto and Voss (1994b). However, their results were not so robust to impulse response analysis and the VEC model was later rejected in Otto and Voss (1996).

congestion from its use by private production (where G_t is aggregate public infrastructure investment):

$$\widetilde{G}_t = \frac{G_t}{K_t^{\phi}\left[(1+x)^t L_t\right]^{1-\phi}} \qquad ;\phi \geq 0\,. \tag{2}$$

This is contrary the usual notion that public goods are non-exclusive and non-rival.

The equilibrium of the economy in this model is solved by an artificial social planner's problem.[3] To be able to obtain solutions that contain a stationary equilibrium equation (1) can be written in per efficiency unit worker terms as:

$$\hat{y}_t = A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P} \qquad ;\alpha,\theta,\phi \in (0,1) \tag{3}$$

where per efficiency unit variables are denoted by a hat, "^". Assume that there is 100 per cent depreciation at the end of each period for private capital. This assumption merely facilitates analytical tractability in the model. Then private per capita investment will give the following period's capital stock

$$k_{t+1} = i_t \varepsilon_{t+1}^{K} \tag{4}$$

where i is per capita investment and ε_{t+1}^{K} is a random shock to investment. Similarly, aggregate public infrastructure investment is assumed to depreciate fully at the end of the period:

$$G_{t+1} = I_t^{G}\varepsilon_{t+1}^{G} \qquad ;G_0 \ given \tag{5}$$

where I^G is aggregate public expenditure on infrastructure.

The household maximises expected lifetime utility subject to resource constraints conditional on available information and supplies one unit of labour inelastically to production. Thus the household solves

$$V(k_0) = \max_{\hat{c}_t,\hat{k}_{t+1}} \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t \ln(\hat{c}_t)\right\} \qquad ;\beta \in (0,1) \tag{6}$$

subject to

a) $\hat{k}_{t+1} = (1-\tau_t)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P} - \hat{c}_t$

b) $\hat{k}_0$, $\hat{g}_0$ given

c) $\hat{c}_t, \hat{k}_{t+1} \geq 0 \qquad \forall$ t

[3] At equilibrium, each agent's desired level of the public good, G_{it}, is consistent with all other agent's votes for the same such that

$$G_{it} = G_t \qquad \forall i\,.$$

where τ is the uniform income tax rate. It is shown in Appendix A, by restating the problem in equation (6) as a dynamic program, that the solution to the household problem taking government policy as given, yields the optimal paths of consumption and private capital:[4]

$$\hat{c}_t = (1-\tau_t)\left[1-(\alpha-\theta\phi)\beta\right]A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P} \tag{7}$$

and

$$\hat{k}_{t+1} = (1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P} \tag{8}$$

(ii) Public Sector

The government budget is given by expenditure on public investment which is financed by what is equivalently a uniform tax on capital and labour incomes in a decentralized equilibrium:

$$I_t^G = \tau_t Y_t . \tag{9}$$

The optimal policy is determined by choosing $\{\tau_t\}_0^\infty$, the optimal path of the tax rate, to

$$\max \sum_{t=0}^{\infty} \beta^t \ln\left[(1-\tau_t)(1-(\alpha-\theta\phi)\beta)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}\right] \tag{10}$$

subject to

a) $\tau_t \in [0,1]$

b) $\hat{g}_{t+1} = \tau_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}$

c) $\hat{k}_{t+1} = (1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}$;

d) $\hat{k}_0, \hat{g}_0$ given.

That is, the benevolent government maximises household welfare when it maximises household consumption growth. A further assumption is that the sequence $\{\widetilde{G}_t\}_{t=0}^{\infty}$ is bounded above by $\{\eta^t \widetilde{G}_t\}_{t=0}^{\infty}$, for some value of $\eta \geq 1$, to ensure that the infinite horizon household objective is bounded above for all feasible consumption paths. In other words, the optimal paths in equations (7) and (8) will be unique: see Glomm and Ravikumar (1994).

[4] See generally Stokey and Lucas (1989).

(iii) Optimal Public Policy

Solving the government's problem by dynamic programming (Appendix B), it is found that the optimal tax rate is a function of constants.[5] Specifically, the optimal tax rate is defined by the function

$$\tau_t = \theta\beta; \qquad \forall t \tag{11}$$

Thus, the optimal tax rate is equal to the one-period discounted share of public capital in output, where the government faces the same subjective discount rate, β, as the household.

Second, given the optimal choice of public policy, the evolution of private capital per efficiency unit worker in equation (8) can be described by the first-order stochastic difference equation

$$\hat{k}_{t+1} = (1-\theta\beta)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^P \tag{12}$$

The evolution of public capital per efficiency unit worker is

$$\hat{g}_{t+1} = \theta\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^P . \tag{13}$$

Consequently, the ratio of the optimal paths for private and public capital stays constant over time. This can be observed by taking the ratio of equation (13) to equation (12), which yields

$$\frac{\hat{g}_{t+1}}{\hat{k}_{t+1}} = \frac{\theta}{(1-\theta\beta)(\alpha-\theta\phi)} \qquad \forall t . \tag{14}$$

(iv) Long-run growth

Substitution of equation (14) into (12) gives the essential difference equation for the evolution of private capital

$$\hat{k}_{t+1} = \left[(1-\theta\beta)(\alpha-\theta\phi)\right]^{1-\theta}\theta^{\theta}\beta A\hat{k}_t^{\alpha+(1-\phi)\theta}\varepsilon_t^P . \tag{15}$$

Under constant returns to scale to reproducible factors, where $\alpha+(1-\phi)\theta = 1$, the steady-state ($\varepsilon_t^P = 1; \forall t$) growth rate of private capital will be given by $\left[(1-\theta\beta)(\alpha-\theta\phi)\right]^{1-\theta}\theta^{\theta}\beta A$, which is perpetual and non-explosive.[6] Also, output and public capital will grow at the same rate as private capital, with constant returns to scale Cobb-Douglas technology.

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[5] See Theorem 4.5 in Stokey and Lucas (1989, 76-77).

[6] See Lau and Sin (1997, p.129)

(v) Time series properties of the model

Equations (12) and (13) can be written in natural logarithm and substitution of these into the stochastic investment equations in (3) and (4) yields:

$$\ln \hat{k}_{t+1} = \ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A] + (\alpha-\theta\phi)\ln\hat{k}_t + \theta\ln\hat{g}_t + \ln\varepsilon_t^P + \ln\varepsilon_{t+1}^K \qquad (16)$$

and

$$\ln\hat{g}_{t+1} = \ln(\theta\beta A) + (\alpha-\theta\phi)\ln\hat{k}_t + \theta\ln\hat{g}_t + \ln\varepsilon_t^P + \ln\varepsilon_{t+1}^G \qquad (17)$$

Multiplying equation (16) by $(1-\theta L)$ on both sides, where L is the lag operator, and substituting for $(1-\theta L)\ln\hat{g}_t$ from equation (17) yields an equilibrium dynamic equation of the log of per capita private capital expressed in terms of its own lags and the external shocks,

$$\begin{aligned}\{1-[\alpha+(1-\phi)\theta]L\}(\ln k_t - xt) = {} & \{(1-\theta)\ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A] + \theta\ln(\theta\beta A)\} \\ & + \theta(L)\ln\varepsilon_t^G + (L)\ln\varepsilon_t^P + (1-\theta L)\ln\varepsilon_t^K\end{aligned} . \qquad (18)$$

Similarly, multiplying equation (17) by $[1-(\alpha-\theta\phi)L]$ on both sides, and substituting for $[1-(\alpha-\theta\phi)L]\ln\hat{k}_t$ from equation (16) yields the equilibrium path for aggregate public capital,

$$\begin{aligned}\{1-[\alpha+(1-\phi)\theta]L\}(\ln g_t - xt) = {} & \{\ln(\theta\beta A) \\ & + (\alpha-\theta\phi)\ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A]\} \\ & + [1-(\alpha-\theta\phi)L]\ln\varepsilon_t^G + (L)\ln\varepsilon_t^P \\ & + (\alpha-\theta\phi)(L)\ln\varepsilon_t^K\end{aligned} . \qquad (19)$$

Also, taking logs of the equation for the private production function in equation (3), multiplying this by $\{1-[\alpha+(1-\phi)\theta]L\}$ and expressing this in per worker terms, yields

$$\begin{aligned}\{1-[\alpha+(1-\phi)\theta]L\}(\ln y_t - xt) = {} & \{1-[\alpha+(1-\phi)\theta]L\}\ln A \\ & +(\alpha-\theta\phi)\ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A] \\ & +\theta[(1+\alpha-\theta\beta)\ln(\theta\beta A)] \\ & +(\alpha-\theta\phi)\ln\varepsilon_t^K + \theta\ln\varepsilon_t^G + \ln\varepsilon_t^P\end{aligned} . \qquad (20)$$

This equation describes the equilibrium path of the log of output per worker, $\ln y_t$.

(a) Perpetual and stable growth at steady state

In this growth model, growth in per capita output or income depends on the coefficient of the lagged output variable, $\alpha + (1-\phi)\theta$. This is also the sum of all the exponents (or what is loosely known as the factor shares in neoclassical terms) of the private and public inputs into production. There will be no perpetual growth in the per capita variables once the economy reaches the steady-state path, if $\alpha + (1-\phi)\theta < 1$, since the effects of past disturbances decay successively in equation (20). Conversely, the steady-state growth path will be explosive if $\alpha + (1-\phi)\theta > 1$. In this case there is increasing returns to all inputs.

To obtain perpetual growth with stability in the model, it is a requirement that $\alpha + (1-\phi)\theta = 1$ and $x = 0$. This is the strict endogenous growth case. Thus, even if private production displays diminishing returns to private inputs, overall it experiences constant returns to scale due to the spillover effect from public capital. Hence there are two empirical properties to be expected of the variables in the endogenous growth case. First, the sequences $\{k_t\}_{t=1}^{\infty}$, $\{g_t\}_{t=1}^{\infty}$ and $\{y_t\}_{t=1}^{\infty}$ will be exact unit root processes. Second, and consequently, the first difference of the logs of the per capita variables will be white noise processes, if the linear combinations of the shocks in (18) to (20) are *I*(0).

(b) Derivation of cointegrating relationships

If there are three *I*(1) variables in the system, there can be a maximum of two linearly independent cointegrating vectors. For non-explosive, perpetual endogenous growth, it was concluded that $\alpha + (1-\phi)\theta = 1$. Using this fact in equations (19) and (20), and then subtracting the former from the latter, and performing the same again on equation (18) and (20) gives the cointegrating space as

$$\ln y_t - \ln k_t - \Lambda_1 = \ln \varepsilon_t^P - \theta \ln \varepsilon_t^K + \theta \ln \varepsilon_t^G \qquad (21)$$

$$\ln y_t - \ln g_t - \Lambda_2 = \ln \varepsilon_t^P + (1-\theta) \ln \varepsilon_t^K - (1-\theta) \ln \varepsilon_t^G \qquad (22)$$

where

$$\Lambda_1 = \left\{ (\alpha + (1-\phi)\theta - 1) \ln\left[(1-\theta\beta)(\alpha - \theta\phi)\beta A\right] + \theta\left[(\alpha - \theta\beta)\ln(\theta\beta A)\right] \right\} (1-L)^{-1}$$

$$\Lambda_2 = \left\{ \theta\left[(1 + \alpha - \theta\beta) - 1\right] \ln(\theta\beta A) \right\} (1-L)^{-1}.$$

It is assumed that the linear combination of the external impulses is *I*(0). If the cointegrating space in equation (21) and (22) is rejected, then there may be at most one cointegrating vector. This cointegrating equation is a linear combination of all the variables. This is shown by multiplying equation (21) on both sides by $(1-\theta)$, and equation (22) on both sides by θ, and then summing the two equations, to obtain

$$\ln y_t - (1-\theta)\ln k_t - \theta \ln g_t - \left[(1-\theta)\Lambda_1 + \theta\Lambda_2\right] = \ln \varepsilon_t^P \quad (23)$$

The cointegrating equation in (23) also represents the production function at steady state with non-explosive, perpetual growth. In general, without assuming $\alpha + (1-\phi)\theta = 1$, the single unrestricted cointegrating equation can be derived from equation (3) yielding:

$$\ln y_t - (\alpha - \theta\phi)\ln k_t - \theta \ln g_t - \left(\left[1-(\alpha+(1-\phi)\theta)\right]x\right)t - \ln A = \ln \varepsilon_t^P \quad (24)$$

Note that (23) is a nested case of (24) where (23) was derived under the hypothesis of $\alpha + (1-\phi)\theta = 1$. These possible cointegrating relationships will be tested in Section III of this paper.

III. Evidence for Australia

(i) Data

The empirical analysis in this Part uses annual time series from 1930/31 to 1990/91 for Australia. It is important, for the purposes of testing for cointegrating relationships, to have a longer series as opposed to a more frequently sampled series.[7] Real GDP ($mil) at 1966/67 constant prices is constructed from two series obtained from the *Reserve Bank of Australia Preliminary Annual Database*, Table R7701-3 and the *Australian Bureau of Statistics Time-Series*, Table 5204.01. Data for net public capital stock ($mil at 1966/67 constant prices) is obtained from the *RBA Preliminary Annual Database*, Table R7701-10. From this Table, public capital stock is taken to be the stock of plant and equipment, and railway. Netting these of depreciation, and summing up, yields the net public capital stock estimates. This series is appended to another series from the *ABS* Catalogue 5221.7 and 5221.8, for public enterprises and general government net stocks of non-dwelling construction and equipment capital, respectively. The sources of private net capital stock ($mil at 1966/67 constant prices) are the *RBA Preliminary Annual Database*, Table R7701-8

[7] See Hakkio and Rush (1991).

and the *ABS* Catalogue 5221.5. The former provides for private stocks of gross plant and equipment, non-dwelling construction capital and depreciation. The latter contains net stocks of non-dwelling construction and equipment capital. Finally, data on civilian employment (`000 persons) is obtained from the *RBA Australian Economic Statistics*, Table 4.10c.

(a) Weak stationarity of the series

Using the Augmented Dickey-Fuller (1979, 1981) unit root test, all the variables appear to be first-difference stationary. This is reported in Table 1.

(ii) Cointegration and output elasticity estimates

In this section, the long-run relationship between the series, ln *y*, ln *k* and ln *g*, will be determined. The Johansen (1991, 1995) multivariate cointegration method is

Table 1: Unit root tests

	$\Delta \ln y_t$	$\Delta \ln k_t$	$\Delta \ln g_t$
Lags for ADF test	2	0	0
	with constant	with constant	with constant and trend
ADF statistic	-5.680	-3.014	-3.925
5% critical value	-2.913	-2.911	-3.486
AIC	-3.741	-5.034	-3.083

used. It should be noted that the Johansen test is sensitive to the specification of the lag-length for the Vector Error Correction (VEC) framework. A VEC(2) framework is chosen after preliminary tests of up to four lags in the VEC yields the smallest Schwarz Information Criteria (SIC = -11.68) with the VEC(2) specification. The second step involves testing for the existence of the cointegrating equations. The single cointegrating equation (24) is estimated as

$$\ln y_t = \beta_2 \ln k_t + \beta_3 \ln g_t + \beta_4 t + \beta_5 . \qquad (25)$$

The test for this cointegrating relationship is reported in Table 2.

Table 2: Johansen cointegration test

Series: $\ln y \ln k \ln G$

VAR(2)

Eigenvalue	λ_{trace}	5 per cent Critical Value	1 per cent Critical Value[ℵ]	Hypothesized No. of CE(s)
0.4511	62.41	42.44	48.45	r = 0 **
0.2659	27.61	25.32	30.45	r ≤ 1 *
0.1538	9.685	12.25	16.26	r ≤ 2

Note: [ℵ] Critical values as tabulated in Osterwald-Lenum (1992). *(**) represents rejection of the null hypothesis at 5%(1%).

From Table 2, the null hypothesis that there is no cointegrating equation can be rejected at even the 1 per cent level, since the trace statistic from the likelihood ratio test exceeds the 1 per cent critical value. The null that there is at most one cointegrating relationship, as against the alternative of two cointegrating equations is rejected at the 5 per cent level but cannot be rejected at the 1 per cent level. Using the more stringent 1 per cent test, it can be concluded that there is only one cointegrating equation of the form in equation (24). The cointegrating relationship, normalised with respect to $\ln y$, is

$$\ln y_t = 0.0019 \ln k_t + 0.1021 \ln g_t + 0.0173t + 0.7877 \qquad (25')$$

The model estimated in (25') can be compared with the restricted (endogenous growth) model under the case where $\beta_2 + \beta_3 = 1$ and $\beta_4 = 0$. These parameter restrictions correspond to the restriction $\alpha + (1-\phi)\theta = 1$ on equation (24). The calculated likelihood ratio statistic is 10.92, which follows a χ^2 distribution with two degrees of freedom ($\chi^2_{0.05,2} = 5.99$). Therefore, the hypothesis of the endogenous growth model in (23) can be rejected in favour of the exogenous growth model with public infrastructure spillovers in (24).

The parameter $\theta = \hat{\beta}_3 = 0.10$ represents the size of the spillover effect of public capital on output. This value is about a quarter of the estimates by Aschauer (1989) and Otto and Voss (1994a). It is close to the estimate of the elasticity of output with respect to public infrastructure for the US in Lau and Sin (1997).

In this section, it was found that there is evidence of public capital spillover effects on aggregate production. However, this effect is small compared to claims in earlier studies for the US and Australia such as Aschauer (1989) and Otto and Voss

(1994a), respectively. In the following section, the VEC(2) model incorporating (25') is used to study short-run dynamics and interactions between the variables.

IV. Short-run dynamics and impulse response

Aschauer (1989a) pointed out the possibility of reverse causation between the level of public capital expenditure and production. That is, ln g responds to rises in ln y. This example of Wagner's Law arises if expenditure on public infrastructure or public goods is a superior good.[8]

Furthermore, there may also be interactions between ln g and ln k. On the one hand, public capital expenditure may be seen as the springboard for private investment. This runs counter to standard elementary macroeconomic argument that government expenditure tends to crowd out private investment. However, it may be that public capital increases the marginal product of private capital. An obvious example is the provision of better highways, which results in less wear and tear of private vehicles while goods are transported more efficiently. On the other hand, public capital expenditure may be seen as responding to private investment demands.

Impulse response analysis of the VEC(2) model is used to provide some insights into the interactions between these variables in the short run. In particular, the interest is in the effect of ln g on ln k and ln y.

(ii) Impulse response analysis and variance decomposition

In performing an impulse response of the VEC(2) model, a simulation period of up to fifty years and an ordering of (ln g, ln k, ln y) is used. Since the variables are in logarithms, each 0.01 unit change in the response functions denotes a 1 per cent change. From Figure 1, ln y responds negatively to a one-period (positive) shock to g and, after the initial six periods, positively to g. Time taken for agents to readjust to capture the positive externality of public infrastructure investment may possibly explain the initial negative response of ln y to ln g.[9] The response of ln y to ln g reaches a 2 per cent (permanently) higher level after 20 years. The ln g shock has permanent effects on ln y due to the existence of a unit root in the series.

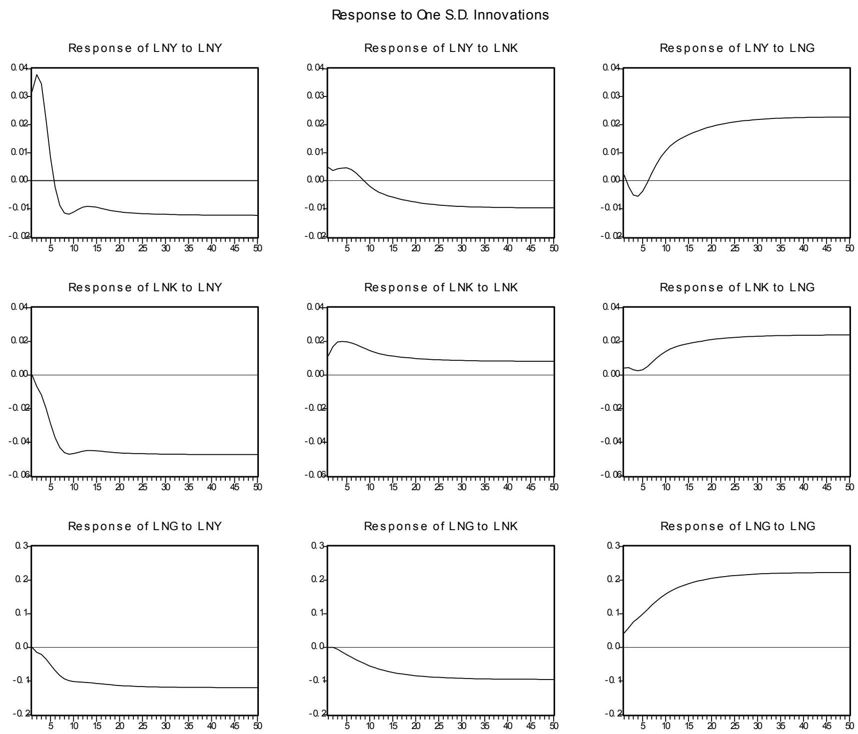
Also, ln k responds positively to ln g from the beginning. Thus, there is evidence of public infrastructure crowding in private investment, which affirms

[8] See Olekalns (1999) for interest.

Aschauer (1989b). As an aside, there does not appear to be evidence of Wagner's Law, if not to the contrary (as ln *g* responds negatively to ln *y*).

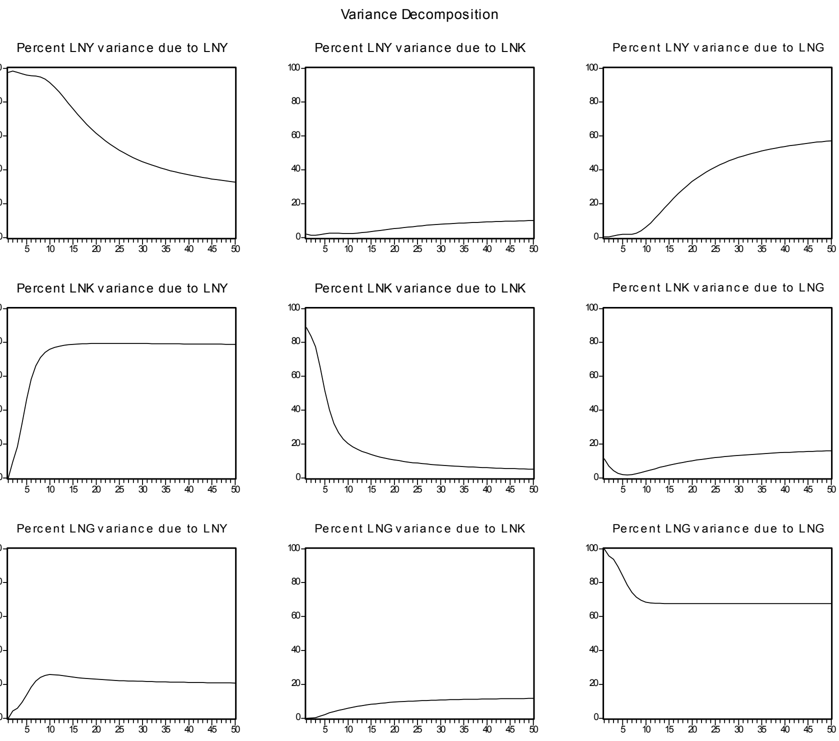
The influence of ln *g* can further been seen in the variance decomposition of the 50-period forecast error of the variables in the system in Figure 2. It can be observed that up to about 50 per cent of the forecast error in ln *y* is due to the innovation to ln *g* and about 70 per cent of the forecast error of ln *g* is due to its own innovation. There is a relatively lower contribution of ln *g* to the forecast error of ln *k*. Therefore, it can be concluded from the impulse response analysis and variance decompositions that public infrastructure investment does impact positively on per worker income in the short to medium term.

Figure 1: Impulse response functions with ordering (ln *g*, ln *k*, ln *y*).



9 See e.g. David (1990).

Figure 2: Variance decompositions with ordering (ln g, ln k, ln y).



V. Conclusions

It was the aim in this paper to study the effect of public infrastructure on the aggregate economy in terms of long-run growth and short-run effects. In particular, the issue was whether growth was determined in the long run, in part, by the accumulation of the stock of public infrastructure. A simple stochastic growth model nesting exogenous and endogenous growth with public capital spillovers was considered in Section II of the paper.

The long-run implication of this model was tested empirically for Australia in Section III. It was found that there was evidence of cointegration between per worker output, per worker private capital and per worker public capital. A nested test of the strictly endogenous growth model was rejected in favour of the exogenous growth model with public infrastructure spillovers as evidenced by the existence of a time trend.

Lastly, the cointegrating relationship was incorporated into a vector error-correction model to study the short-run behaviour of the variables in Section IV. It was found that innovations to public infrastructure induce permanently higher levels of output and private investment in both the short and the long run. The forecast error variance of per capita output is also largely due to that of public infrastructure, under a variance decomposition of the statistical model. Therefore, there is evidence that the accumulation of public capital can have positive short to long term effects on economic growth, a conclusion which is in line with the existing empirical literature.

APPENDIX A: ***Dynamic programming for the household problem***

The method of solving the household's intertemporal utility maximization problem subject to given constraints and public policy in equation (6) is as follows. Bellman's (1957) principle of optimality dictates that if the sequence of $\{\hat{c}_t, \hat{k}_{t+1}\}_{t=0}^{\infty}$ is maximising, then it must also be the case that it maximises the functional over $\{\hat{c}_0, \hat{k}_1\}$ and $\{\hat{c}_t, \hat{k}_{t+1}\}_{t=1}^{\infty}$. Hence the problem in equation (6) can be written as

$$V(\hat{k}_t, \varepsilon_t) \equiv \max_{\hat{c}_t, \hat{k}_{t+1}} \left[\ln(\hat{c}_t) + \beta \mathrm{E}_t V(\hat{k}_{t+1}) \right] \tag{A.1}$$

subject to the constraints (6)(a)-(c).

A guess of the solution to (A.1) is

$$V(\hat{k}_0, \varepsilon_0) = B_0 + B_1 \ln(\hat{k}_0) + B_2 \ln(\varepsilon_0) \tag{A.2}$$

Substituting the form of equation (A.2) into (A.1) gives

$$V(\hat{k}_t, \varepsilon_t) \equiv \max_{\hat{c}_t, \hat{k}_{t+1}} \left\{ \ln(\hat{c}_t) + \beta \mathrm{E}_t V \left[B_0 + B_1 \ln(\hat{k}_{t+1}) + B_2 \ln(\varepsilon_{t+1}) \right] \right\} \tag{A.3}$$

subject to constraints (6)(a)-(c).

At time *t*, the control variables, $\hat{c}_t$ and $\hat{k}_{t+1}$, and the state variables, ε_t and $\hat{k}_t$, are all known. Further, with the assumption that $\ln(\varepsilon_t)$ is independently and identically distributed such that $\mathrm{E}_t \ln(\varepsilon_{t+1}) = 0$, the terms in the curly brackets of equation (A.3) can be reduced to

$$\ln(\hat{c}_t) + \beta \mathrm{E}_\mathrm{t} \left[B_0 + B_1 \ln(\hat{k}_{t+1}) \right] \tag{A.4}$$

Define the Lagrangian as

$$L = \ln(\hat{c}_t) + \beta \left[B_0 + B_1 \ln(\hat{k}_{t+1}) \right] + \lambda \left[(1-\tau_t) A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} - \hat{c}_t - \hat{k}_{t+1} \right]$$

and the first-order conditions for maximisation are

$$\frac{1}{\hat{c}_t} = \lambda \tag{A.5}$$

$$\frac{\beta B_1}{\hat{k}_{t+1}} = \lambda \tag{A.6}$$

$$(1-\tau_t) A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} = \hat{c}_t + \hat{k}_{t+1} \tag{A.7}$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t \hat{k}_{t+1} = 0 .$$

Substitute equations (A.5) and (A.6) into (A.7) to get:

$$\hat{c}_t = \frac{1}{(1+\beta B_1)}(1-\tau_t) A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^P \qquad \text{(A.8)}$$

Use the natural constraint (A.7) and (A.8) to derive the stochastic difference equation for private capital per efficiency unit worker:

$$\hat{k}_{t+1} = \frac{\beta B_1}{(1+\beta B_1)}(1-\tau_t) A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^P \qquad \text{(A.9)}$$

Substitute equations (A.8) and (A.9) into the RHS of the Bellman equation (A.3) to verify that the LHS of equation (A.3)

$$V\left(\hat{k}_t, \varepsilon_t\right) \equiv B_0 + B_1 \ln(\hat{k}_{t+1}) + B_2 \ln(\varepsilon_{t+1})$$

is equal to the RHS of equation (A.3), which is

$$\ln\left[\frac{1}{(1+\beta B_1)}(1-\tau_t) A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^P\right]$$
$$+\beta\left\{B_0 + B_1 \ln\left[\frac{\beta B_1}{(1+\beta B_1)}(1-\tau_t) A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^P\right]\right\}$$

Expanding terms on the RHS:

$$\beta B_0 - (1+\beta B_1)\ln(1+\beta B_1) + \beta B_1 \ln(\beta B_1) + (1+\beta B_1)\ln\left[(1-\tau_t) A \hat{g}_t^{\theta}\right]$$
$$+ (\alpha-\theta\phi)(1+\beta B_1)\ln(\hat{k}_t) + (1+\beta B_1)\ln(\varepsilon_t^P)$$

For the functional to be valid, the LHS must, *inter alia*, satisfy the condition that

$$B_1 = (\alpha-\theta\phi)(1+\beta B_1)$$

and thus, the guess in (A.4) will be correct if

$$B_1 = \frac{\alpha-\theta\phi}{1-(\alpha-\theta\phi)\beta} \qquad \text{(A.10)}$$

Substitute (A.10) into equations (A.8) and (A.9) to obtain the optimal household consumption and investment paths with given public policy:

$$\hat{c}_t = (1-\tau_t)\left[1-(\alpha-\theta\phi)\beta\right] A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^P \qquad (7)$$

$$\hat{k}_{t+1} = (1-\tau_t)(\alpha-\theta\phi)\beta A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^P \qquad (8)$$

APPENDIX B: ***The government's problem and optimal outcomes***

The government's infinite horizon problem is defined here as a dynamic program. That is, the maximized value of the household's "welfare" is

$$v(\hat{k}_t, \hat{g}_t, \varepsilon_t) \equiv \max_{\hat{k}_{t+1}, \hat{g}_{t+1}, \tau_t} \left\{ \ln\left[(1-\tau_t)(1-(\alpha-\theta\phi)\beta) A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} \right] \right. \\ \left. + \beta \mathrm{E}_t v(\hat{k}_{t+1}, \hat{g}_{t+1}) \right\} \tag{B.1}$$

subject to (10)(a)-(d).

Second, guess that the solution is of the form below:

$$v(\hat{k}_0, \hat{g}_{0,} \varepsilon_0) \equiv B_0 + B_1 \ln(\hat{k}_0) + B_2 \ln(\hat{g}_0) + B_3 \ln(\varepsilon_0)\,. \tag{B.2}$$

Utilising the guess in (B.2), re-write equation (B.1) as

$$v(\hat{k}_t, \hat{g}_t, \varepsilon_t) \equiv \max \left\{ \ln\left[(1-\tau_t)(1-(\alpha-\theta\phi)\beta) A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} \right] \right. \\ \left. + \beta \mathrm{E}_t \left[B_0 + B_1 \ln(\hat{k}_{t+1}) + B_2 \ln(\hat{g}_{t+1}) \right] \right\} \tag{B.3}$$

subject to (B.1)(a) to (d). It is also assumed here that $\ln(\varepsilon_{t+1}^{P})$ is white noise and therefore, $\mathrm{E}_t \ln(\varepsilon_{t+1}^{P}) = 0$.

Define the Lagrangian as:

$$\begin{aligned} L = {} & \ln\left[(1-\tau_t)(1-(\alpha-\theta\phi)\beta) A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} \right] \\ & + \beta \left[B_0 + B_1 \ln(\hat{k}_{t+1}) + B_2 \ln(\hat{g}_{t+1}) \right] \\ & + \mu \left[\tau_t A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} - \hat{g}_{t+1} \right] \\ & + \psi \left[(1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} - \hat{k}_{t+1} \right] \end{aligned}$$

The necessary first-order conditions for maximization of the Lagrangian are:

$$L_{\hat{k}_{t+1}} = \frac{\beta B_1}{\hat{k}_{t+1}} - \psi = 0 \tag{B.4}$$

$$L_{\hat{g}_{t+1}} = \frac{\beta B_2}{\hat{g}_{t+1}} - \mu = 0 \tag{B.5}$$

$$L_{\tau_t} = -\frac{1}{(1-\tau_t)} + \mu A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} - \psi(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} = 0 \tag{B.6}$$

$$L_{\mu} = \tau_t A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} - \hat{g}_{t+1} = 0 \tag{B.7}$$

$$L_{\psi} = (1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \varepsilon_t^{P} - \hat{k}_{t+1} = 0 \tag{B.8}$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t \hat{k}_{t+1} = 0 .$$

Express the constraints (B.7) and (B.8) in terms of $\hat{g}_{t+1}$ and $\hat{k}_{t+1}$ respectively, and substitute these into equations (B.4) and (B.5) to obtain

$$\frac{\beta B_1}{(1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}} = \psi \tag{B.4'}$$

$$\frac{\beta B_2}{\tau_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}} = \mu \tag{B.5'}$$

Substitute equations (B.4') and (B.5') into equation (B.6) gives

$$\tau_t = \frac{\beta B_2}{1+\beta B_1 + \beta B_2} . \tag{B.9}$$

Further substitution of equation (B.9) back into constraints (B.7) and (B.8) results in

$$\hat{g}_{t+1} = \left(\frac{\beta B_2}{1+\beta B_1+\beta B_2}\right) A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P} \tag{B.10}$$

and

$$\hat{k}_{t+1} = (\alpha-\theta\phi)\beta\left(\frac{1+\beta B_1}{1+\beta B_1+\beta B_2}\right) A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P} . \tag{B.11}$$

Next, substitute (B.10) and (B.11) into (B.3) and compare with the form of (B.2):

$$\begin{aligned}
&B_0 + B_1\ln(\hat{k}_{t+1}) + B_2\ln(\hat{g}_{t+1}) + B_3\ln(\varepsilon_{t+1}) \equiv \\
&\ln\left[(1-(\alpha-\theta\phi)\beta)\left(\frac{1+\beta B_1}{1+\beta B_1+\beta B_2}\right)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}\right] \\
&+\beta\left\{B_0 + B_1\ln\left[(\alpha-\theta\phi)\beta\left(\frac{1+\beta B_1}{1+\beta B_1+\beta B_2}\right)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}\right]\right. \\
&\left.+B_2\ln\left[(\alpha-\theta\phi)\beta\left(\frac{\beta B_2}{1+\beta B_1+\beta B_2}\right)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^{P}\right]\right\}
\end{aligned} \tag{B.12}$$

Expand the RHS of equation (B.12) and collect terms

$$\beta B_0 + \ln\left[\frac{(1-(\alpha-\theta\phi)\beta)(1+\beta B_1)}{1+\beta B_1+\beta B_2}\right] + \beta B_1 \ln\left[\frac{(\alpha-\theta\phi)\beta(1+\beta B_1)}{1+\beta B_1+\beta B_2}\right]$$

$$+\beta B_2 \ln\left[\frac{(\alpha-\theta\phi)\beta^2 B_2}{1+\beta B_1+\beta B_2}\right] + (1+\beta B_1+\beta B_2)\ln\left[A\right]$$

$$+(\alpha-\theta\phi)(1+\beta B_1+\beta B_2)\ln(\hat{k}_t) + \theta(1+\beta B_1+\beta B_2)\ln(\hat{g}_t) + (1+\beta B_1+\beta B_2)\ln(\varepsilon_t^P)$$

For the functional to be valid, that is the LHS=RHS in (B.12), it must be that the coefficients on the LHS of (B.12) satisfy, *inter alia*

$$B_1 = (\alpha-\theta\phi)(1+\beta B_1+\beta B_2)$$

$$B_2 = \theta(1+\beta B_1+\beta B_2).$$

Solving for B_1 and B_2 yields

$$B_1 = \frac{\alpha-\theta\phi}{1-\theta\beta-(\alpha-\theta\phi)\beta} \tag{B.13}$$

$$B_2 = \frac{\theta}{1-\theta\beta-(\alpha-\theta\phi)\beta}. \tag{B.14}$$

Substitution of equation (B.13) and (B.14) into (B.9), (B.10) and (B.11) gives the equations of the evolution of private and public capital:

$$\tau_t = \theta\beta \qquad \forall t \tag{11}$$

$$\hat{k}_{t+1} = (1-\theta\beta)(\alpha-\theta\phi)\beta A \hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^P \tag{12}$$

$$\hat{g}_{t+1} = \theta\beta A \hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\varepsilon_t^P. \tag{13}$$

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