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DISTRIBUTIONAL LIMITS AND THE GINI COEFFICIENT

by

Justin van de Ven

Department of Economics
The University of Melbourne
Melbourne Victoria 3010
Australia.

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Abstract

This paper examines the Gini (1912) coefficient of concentration using the framework set out by Dalton (1920) for evaluating a measure of inequality. Particular attention is paid to limited distributions and the associated concept of 'perfect inequality'. It is argued that a rescaled version of the Gini coefficient may be desirable for distributions that are subject to limits which depart from the standard assumption of non-negativity. A scaling parameter is derived and the rescaled Gini coefficient is used to analyse the inequality of wealth in Australia.

1 Introduction

This paper considers the practical implications associated with the use of the Gini (1912) coefficient[1], G, when measuring the inequality exhibited by a distribution that is subject to the general limits, $[x_{\min}, x_{\max}]$. Economic studies of inequality commonly assume limits defined by $[0, \infty)$, which imposes distributional non-negativity on a population. Under these conditions, G satisfies all of the principles that are currently used to characterise a suitable measure of inequality.[2] For any other distributional limits, however, G fails to satisfy the principle of normalisation.[3] In response to this inadequacy, an adjusted Gini coefficient is defined that satisfies all of the principles of inequality measurement, including the principle of normalisation, when subject to general population limits.

The practical significance of this study is directly related to the relevance of distributions that depart from the standard non-negativity constraint, $[0, \infty)$. In the literature that examines inequality, this constraint is typically imposed with little or no comment. When comment is made it usually focuses on justifying the exclusion of negative observations on the grounds that such observations are either unrepresentative, or arise due to data contaminants. In contrast, this paper considers the distributional effects of population limits explicitly, and in doing so provides a framework for associated analysis.[4]

To examine the practical importance of distribution limits other than the standard non-negativity constraint, it is useful to begin by recognising that there are many important, and economically relevant distributions that

[1] The ratio of the mean difference to two times the population mean was suggested as an inequality measure by Gini (1912), although it had been discussed earlier by Helmert (1876), and von Andrae (1872), cf. David (1968).

[2] There are seven principles for evaluating a measure of inequality (see, for example, Dagum, 1983): the Pigou (1912) - Dalton (1920) principle of transfers, the principle of proportionate additions to incomes, the principle of equal additions to incomes, the principle of proportionate additions to persons, the principle of symmetry, the principle of operationallity, and the principle of normalisation.

[3] The principle of normalisation is defined in section 2.

[4] See, for example, Cowell (1995, pp. 155-156) on the exogenous assumption of the non-negative domain and associated distributional issues.

are subject to limits which are *more restricting* than $[0, \infty)$. Consider, for example, the distribution of wage and salary incomes, for which a statutory minimum is imposed. In this case no observation can take a value that is less than the statutory minimum, and hence the lower limit is greater than zero. Maintaining a focus on income[5], it is often useful to impose specific restrictions on the individual values of the populations used. One example is when different distributional analysis is adopted for different quantiles of the population. For instance, poor individuals may be analysed separately from the non-poor, where the division is made with respect to a given 'poverty line'. In this case the populations above and below the poverty line are subject, respectively, to lower and upper bounds. The issue of whether the population below the poverty line should also be subject to a lower bound (equal to zero) is case dependent.

The choice of whether to censor negative observations from an income distribution relates to the definition of income adopted and the associated focus of analysis. Dalton (1920, p. 348) states that "the economist is primarily interested, not in the distribution of income as such, but in the effects of the distribution of income upon the distribution and total amount of economic welfare, which may be derived from income." Taking a broad interpretation, income may be defined as aggregate gains net of any losses, which permits the existence of negative observations. Conversely, taking a narrow interpretation of income as the *inflow* of wealth rules out the possibility of negative observations. If the welfare framework advocated by Dalton is adopted, then it can be argued that the broad interpretation is preferable to the narrow on the basis that the associated distribution is likely to reflect more closely the distribution of economic welfare, a view that is supported by the emphasis on 'net income' in the literature.[6] The issue of whether negative values should be censored from income distributions that are defined using the broad in-

[5] It is evident that there are many distributions of economic interest for which negative observations are likely to be of greater importance compared with income. See, for example, Cowell (1995, p. 155 *et seq.*). The focus on income adopted in this paper reflects its predominant use in the literature.

[6] See, for example, the 1955 Minority Report of the Royal Commission of the Taxation of Profits and Income (p. 355).

terpretation must be determined based upon the specific focus of analysis. Where the analysis examines the distribution of income for the entire population, negative values may be seen as unrepresentative. This view is likely to be less justified, however, for an analysis that focuses expressly on the poor.[7]

Alternatively, Clark and Oswald (1996) find results that offer statistical support for the hypothesis that utility depends on income relative to an individual specific reference level. The reference level of income used by Clark and Oswald (1996) is obtained using a conventional earnings equation. If this hypothesis is accepted, then Dalton's (1920) welfare framework implies that income should be measured relative to the reference level when examining inequality. One method by which this can be achieved is to subtract reference level from observed income prior to the measurement of inequality. In this case, negative observations will form an important part of the total population, and their omission will consequently have a significant effect on the measure of inequality obtained.

Section 2 examines the effects of distributional limits on perfect inequality and the associated Gini coefficient. Section 3 describes an adjusted Gini coefficient that satisfies all of the principles of inequality measurement for general distributional limits, and discusses associated issues of interpretation. A practical application is provided in section 4. Section 5 concludes.

2 The Gini Coefficient, Limited Distributions, and the Principle of Normalisation

The principle of normalisation specifies that the range of an inequality measure should be in the interval [0,1], with zero (one) for perfect equality (inequality). Perfect equality arises when all of the individual incomes in a distribution take the same value. For non-negative income, perfect inequality is defined as the distribution in which an infinitely small proportion of

[7]This issue is returned to in section 4.

the population earn the entire population's income.[8] To determine the value of the Gini coefficient under perfect equality and inequality when subject to distributional limits $[0, \infty)$, consider a population that is divided into two groups, one of which is comprised of individuals earning zero income, while the individuals of the other group earn $x > 0$. The Gini coefficient for this population is characterised by[9]:

$$\begin{aligned} G &= \frac{1}{2\mu}\{2(x-0) f(x)[1-f(x)]\} \\ &= \frac{1}{\mu} x f(x)[1-f(x)] \\ &= 1-f(x) \end{aligned} \tag{1}$$

Equation (1) indicates that when perfect equality is observed, such that $f(x) = 1$, $G = 0$; and when perfect inequality is observed, such that $f(x) \to 0$, $G \to 1$. Hence the Gini coefficient satisfies the principle of normalisation for a non-negative distribution.

The singular advantage of the principle of normalisation is to describe the dispersion of a distribution relative to the extremes of perfect equality and inequality. Consider a large population with a continuous distribution for which individual incomes must lie between a specified minimum, $x_{\min}$, and maximum, $x_{\max}$, such that $\mu > 0$.[10] Furthermore, assume that $0 < f(0) < F(0) = \alpha$. The fact that $x_{\min} < 0$ implies that the Gini coefficient associated with this distribution satisfies all but the principle of normalisation.

To see why the Gini coefficient fails to satisfy the principle of normalisation, it is useful to start by considering exactly what is meant by 'perfect

[8] This definition of perfect inequality may be interpreted in terms of the welfare loss due to inequality, following Atkinson (1970). For any welfare function that exhibits diminishing marginal returns to income, the welfare loss due to inequality is maximised, subject to distribution non-negativity, where the entire population's income is allocated to an infinitely small subgroup.

[9] obtained using the absolute difference relation; $G = \frac{1}{2\mu}\int_0^\infty \int_0^\infty |y - x| f(y)\, dy f(x)\, dx$

[10] Setting $\mu > 0$, is an innocuous assumption made for ease of exposition. If $\mu < 0$, then the order of the income parade is reversed, and the value of the associated concentration index consequently obtains $G(-\mu) = -G(\mu)$. It is, however, necessary that $\mu \neq 0$, since the Gini coefficient is otherwise undefined. When measuring the inequality of a distribution for which $\mu < 0$, it is recommended that the mean should be divided through the income parade and the Gini coefficient calculated for the subsequent distribution.

equality' and 'perfect inequality'. The least contentious of the two concepts is that of perfect equality, defined above as the distribution in which all individual values are equal to the population mean; that is, $f(\mu) = 1$. The definition of perfect inequality for a distribution that is subject to general limits is, however, more opaque. Specifically, it will be evident that the definition of perfect inequality provided above is particular to a distribution specified on the interval $[0, \infty)$. In fact, the published literature provides no general definition of perfect inequality due to the predominant role that the non-negative distribution has played with respect to inequality analysis.

The principal characteristic underlying the definition of perfect inequality is that it should describe a distribution that maximises inequality subject to the distribution constraints. This condition is satisfied by the distribution in which the maximum proportion of the population, q, earn x_{min}, and the remainder earn x_{max}, subject to μ. Hence, for $x_{\text{min}}, x_{\text{max}} \in \mathbb{R}$:

$$q x_{\text{min}} + (1 - q)\, x_{\text{max}} = \mu \tag{2}$$

$$\therefore q = \frac{x_{\text{max}} - \mu}{x_{\text{max}} - x_{\text{min}}} \tag{3}$$

To see that inequality is maximised by this distribution, note that the only inequality affecting redistribution that is possible, subject to the distribution limits, is a transfer from an individual earning x_{max} to an individual earning x_{min}. By the Pigou-Dalton principal of transfers this redistribution will decrease inequality. In the general case, the distribution of perfect inequality is therefore defined by:

$$x_p = \begin{cases} \frac{\mu - (1-q)x_{\text{max}}}{q} = x_{\text{min}} & 0 \leq p < q \\ \frac{\mu - q x_{\text{min}}}{(1-q)} = x_{\text{max}} & q \leq p \leq 1 \end{cases} \tag{4}$$

where x_p defines the income of the pth proportional rank in the income distribution.

Under the standard restriction of a distribution to the non-negative domain, $x_{\text{min}} = 0$ and x_{max}, tends toward (positive) infinity. Applying these bounds, it can be seen from equation (3), that q tends toward 1, and so, from

(4);

$$x_p = \begin{cases} 0 & 0 \leq p < 1 \\ \infty & p = 1 \end{cases} \tag{5}$$

Equation (5) implies that the entire population's income is earned by an infinitely small subgroup, consistent with the definition of perfect inequality associated with the standard restricted distribution, as discussed previously.[11]

It remains to determine the values of G for the general continuous distribution defined at the beginning of this section, and for the associated distribution of perfect inequality. With regard to the general continuous distribution, it is evident that the Lorenz curve will have a negative slope for all $F(.) < \alpha$, after which it's slope will become positive. Furthermore, by definition, the Lorenz curve must pass through the (0,0), and (1,1) coordinates, and exhibit non-negative curvature for its entire domain. Hence, for the general continuous distribution considered, the Lorenz curve will take a form characterised by L in figure 1. Similarly the Lorenz curve of perfect inequality and perfect equality are depicted respectively by L_{eq}, and L_{ueq}.

The slope of L_{ueq} is defined by the conditions described in (4). Specifically below q, the associated population earn the minimum income $x_{\min}$, and hence the slope of L_{ueq} equals $\frac{x_{\min}}{\mu}$. Similarly, above q, the slope of L_{ueq} equals $\frac{x_{\max}}{\mu}$. These slopes fully characterise L_{ueq}, given the requirement that it pass through the $(0,0)$ and $(1,1)$ coordinates, where q is identified at the point where the two straight lines (denoted $l_{\min}$ and $l_{\max}$ in figure 1) intersect. Examining figure 1 reveals that increasing $x_{\min}$ and $x_{\max}$ respectively rotate $l_{\min}$ up through the origin and $l_{\max}$ up through the (1,1) coordinate, both of which increase q. It can also be seen from the figure that, as $l_{\min}$ is rotated up, *ceteris paribus*, the area $(A + B)$ is decreased, while the opposite is true for the upward rotation of $l_{\max}$.

The standard methods used for calculating the Gini coefficient obtain a measure equal to two times the area captured between L_{eq} and L, whether

[11]Note that, for a finite population of size N, the maximum value that q can take is $\frac{(N-1)}{N}$. In this case, equation (4) implies that the entire population's income is earned by a single individual and hence the maximum observable inequality is less than perfect inequality as it is defined here.

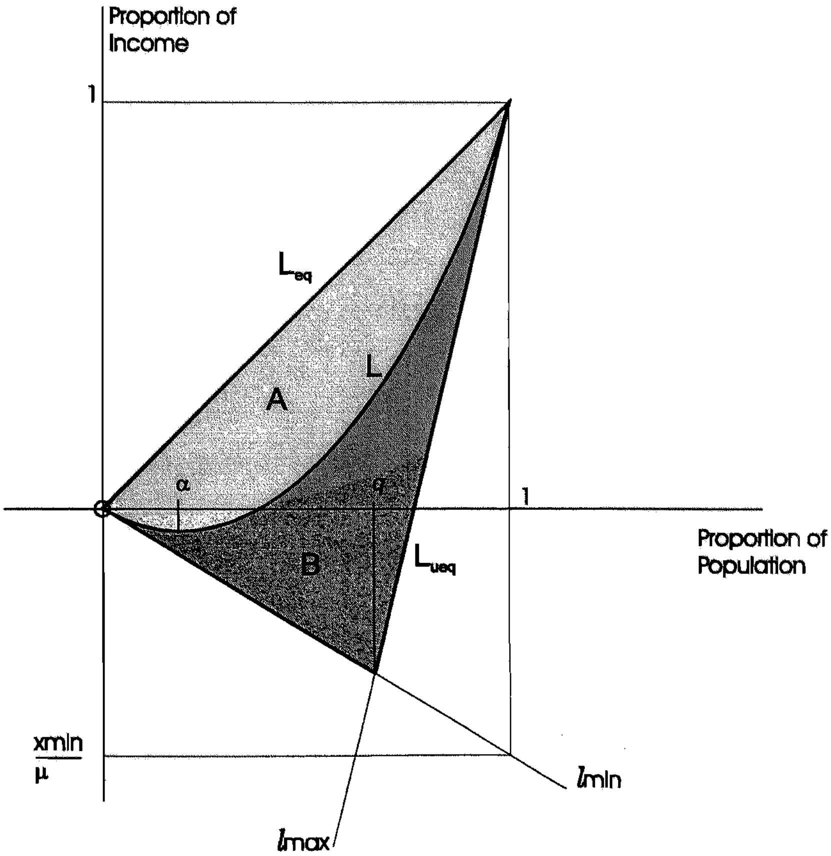

Figure 1: Lorenz Curves of the General Continuous Distribution, Distribution of Perfect Equality, and Distribution of Perfect Inequality

or not the income distribution includes negative incomes[12]. Specifically, the Gini coefficient associated with L in figure 1 can be calculated using any one of the following standard measures (specified in discrete form);

$$G = \begin{cases} \left(1 + \frac{1}{N} - \frac{2}{N^2} \sum_{i=1}^{N} (N + 1 - i) \frac{x_i}{\mu}\right) \\ \left(\frac{2}{\mu}\right) Cov\left(x, F(x)\right) \\ \frac{1}{2\mu N^2} \sum_i \sum_j |x_i - x_j| \end{cases} \tag{6}$$

To examine how the population limits impact upon the extreme values that G can take, consider a population where a proportion k take the value $x_{\min}$, and a proportion $(1 - k)$ take the value $x_{\max}$. From the absolute difference

[12]see, for example, Chen *et al.* (1982).

equation described in (6):

$$G = \frac{2\,(Nk)\,(N\,(1-k))\,(x_{\max}-x_{\min})}{2\mu N^2} \tag{7}$$

Perfect equality is obtained when $x_{\max} = x_{\min}$, such that every individual earns the same income. From equation (7), the Gini coefficient associated with this distribution is equal to 0, which is consistent with the principle of normalisation. Substituting $k = q = \frac{x_{\max}-\mu}{x_{\max}-x_{\min}}$ from equation (3) into equation (7), and simplifying obtains:

$$G = \frac{(x_{\max}-\mu)\,(\mu - x_{\min})}{\mu\,(x_{\max}-x_{\min})} \tag{8}$$

In this case the value of G is dependent entirely upon the relative values of $x_{\max}$, $x_{\min}$, and μ, and is potentially unconstrained, which is in direct violation of the principle of normalisation.

3 Normalising the Gini Coefficient

The discussion of the preceding section suggests that the Gini coefficient can be rescaled to comply with the principle of normalisation by using the limit described by equation (8). Specifically, define the 'adjusted Gini', $\tilde{G}$, as the area, A, captured between L_{eq} and L, divided by the area, $(A+B)$, captured between L_{eq} and L_{ueq} in figure 1. This is equivalent to dividing the Gini coefficient associated with L, by the Gini coefficient associated with L_{ueq}, such that:

$$\tilde{G} = \frac{\mu\,(x_{\max}-x_{\min})}{(x_{\max}-\mu)\,(\mu - x_{\min})}G \tag{9}$$

where G is the standard Gini coefficient associated with L.

From the preceding discussion, it is evident that $0 \leq G \leq \frac{(x_{\max}-\mu)(\mu-x_{\min})}{\mu(x_{\max}-x_{\min})}$, where the equality with zero applies for a distribution that exhibits perfect equality, and the equality with $\frac{(x_{\max}-\mu)(\mu-x_{\min})}{\mu(x_{\max}-x_{\min})}$ applies for a distribution that exhibits perfect inequality. Hence, $\tilde{G}$ satisfies the principle of normalisation for the general limits $[x_{\min}, x_{\max}]$. In addition, the fact that $\tilde{G}$ is a reparameterisation of the Gini coefficient implies that, like G, it satisfies the other six principles that characterise a suitable measure of inequality.

3.1 Interpreting the Adjusted Gini Coefficient

A common criticism of the Gini coefficient is that it suffers from "the disadvantage of being affected very much by the mean measured from some arbitrary origin." (Kendall *et al.*, 1987, p. 60). The standard requirement of a relative measure of inequality is that it should satisfy the contemporary interpretation of the principle of proportionate additions to incomes.[13] For a statistic to comply with this requirement it must rescale individual incomes to cancel any associated units of measure. The precise rescaling required is, however, not specified, and may consequently be drawn from an infinite set. Kendall *et al.*'s criticism regarding the 'arbitrary origin' used to calculate the Gini coefficient is an issue that relates to this indeterminacy.

It is useful to specify intuitively desirable criteria for limiting the indeterminacy to which the rescaling factor of a relative measure of inequality is subject. One such requirement, which complements the analysis undertaken in section 3, is that a measure of inequality should assess dispersion with respect to the associated distribution constraints; that is, with regard to *both the limits and the mean* of the distribution. When a distribution is subject to the limits $[0, \infty)$, it is natural that the factor used to adjust incomes should measure the mean relative to zero. The same is not true, however, for the general limits $[x_{\text{min}}, x_{\text{max}}]$, and hence the rescaling factor used by the Gini coefficient may be considered arbitrary within this framework, consistent with the criticism of Kendall *et al.*.

The adjustment suggested in section 3 for the Gini coefficient may be interpreted as a method of tailoring the inequality measure to the limits of the distribution of interest. Consider the following characterisation of $\tilde{G}$, derived by dividing the discrete absolute difference form for A from equation (6) by $(A + B)$ as described by equation (8):

$$\tilde{G} = \frac{1}{2N^2} \left[\frac{(x_{\text{max}} - x_{\text{min}})}{(x_{\text{max}} - \mu)(\mu - x_{\text{min}})} \sum_i \sum_j |x_i - x_j| \right] \tag{10}$$

Equation (10) indicates that individual incomes are rescaled by the factor,

[13] This principal is the subject of some contention. See, for example, Kolm (1976, p. 419).

V:

$$V = \frac{(x_{\max} - \mu)(\mu - x_{\min})}{(x_{\max} - x_{\min})} \tag{11}$$

when generating $\tilde{G}$, as apposed to the calculation of G, which rescales by μ. From equation (11), V is a quadratic function of μ, with a minimum when $\mu = x_{\min}$ or $x_{\max}$, and a maximum when $\mu = (\frac{x_{\max}+x_{\min}}{2})$[14]. When μ is close to either of the distribution limits the values that individual observations can take are highly restricted. Similarly, when the mean is close to the mid-point of the distribution limits, then individual observations are subject to the least restraint, given the distribution limits. The rescaling factor, V, can consequently be interpreted as a measure of the 'variability' to which individual values of a distribution are subject, given the distribution limits and associated mean. Equation (10) therefore makes the relationship between *ex ante* population variability, and inequality explicit, where $\tilde{G}$ varies inversely with V.

The adjusted Gini coefficient complies with all of the principles that are used to evaluate the appropriateness of an inequality measure, including the principle of normalisation. In addition, it can be calculated for any mean depending on the limits to which the distribution is subject, *including the case where* $\mu = 0$. This property may come as some surprise, given that $\tilde{G}$ was derived using the Lorenz curve, which is clearly not defined when $\mu = 0$. It is evident from equation (10), however, that $\tilde{G}$ may be calculated so long as $\mu \neq x_{\min}$ or $x_{\max}$.[15] The result is attributable to the shift of focus away from the origin of zero, and arises mathematically due to the fact that, as $\mu \to 0$, $A \to \infty$ at the same rate as $(A + B)$.

There is, however, a cost associated with the suggested adjustment to the Gini coefficient. Specifically in return for obtaining a measure of inequality that is specified relative to perfect inequality for given distribution limits,

[14] given the constraint that $x_{\min} \leq \mu \leq x_{\max}$.

[15] When $\mu = x_{\min}$ or $x_{\max}$, the individual values of the associated distribution are subject to no 'variability', as indicated by the fact that $V = 0$. In this case, it is evident that inequality relative to the distribution constraints is undefined. This is the same effect that is responsible for the fact that the standard Gini coefficient can not be calculated for $\mu = 0$.

comparisons between two distributions that are subject to different limits are complicated by the use of different scaling factors. Hence using adjusted Gini coefficients means that no association is maintained with regard to Lorenz dominance, unless the two distributions compared happen to be subject to the same restrictions regarding their respective domains.[16] It is pertinent to bear this issue in mind when determining whether, and how, a Gini coefficient should be rescaled.

4 A Practical Application

The distribution of wealth in Australia is analysed to examine the practical significance of the suggested adjustment to the Gini coefficient. Studies of the distribution of wealth in Australia are quite rare, owing mainly to the lack of suitable data. The raw data used in this paper are derived from Woon (2000), where measures of 'net worth' are imputed for individuals based on the 1994 to 1997 Income and Housing Costs Surveys (IHCS).[17] These surveys, which are the four most recent editions to be published by the Australian Bureau of Statistics (ABS), provide demographic and income microdata for a representative sample of the Australian population. Woon (2000) imputes the wealth of individuals by aggregating annual earnings (comprising own business and wage and salary post tax income), own housing wealth (net of mortgage liabilities), imputed interest bearing (savings deposits), dividend bearing (company stock holdings), and rent bearing (real estate property holdings) assets, and superannuation.

For the analysis undertaken here, the wealth data derived from Woon (2000) were aggregated by income unit, which is defined by the ABS as

[16]The same criticism is true of the adjusted Gini coefficient suggested by Chen *et al.* (1982), which is also bound within the range [0,1] for distributions that include some negative incomes. Unlike the present paper, however, Chen *et al.* (1982) explicitly avoid the conceptual issues associated with the definition of perfect inequality when deriving their suggested adjustment, which complicates *any* interpretation of the coefficient that they advocate.

[17]Woon (2000) refers to the imputed measures as 'non-human wealth' or 'net worth'. For clarity, the distribution is referred to throughout the subsequent discussion as wealth. See Piggott (1984) with regard to the distinction between human and non-human wealth.

either, married couples (with or without children), sole parents, or single individuals. This distribution was adjusted for family size and composition using a variant of the equivalence scale examined by Buhmann *et al.* (1988), and the measures of 'adult equivalent wealth' were allocated to all individual family members following Danziger and Taussig (1979) on the related issue of income, and Sen (1997, p.30) on the related issue of poverty measurement.

The equivalence scale used is characterised by:

$$z = (n_a + \Phi n_c)^{\theta} \tag{12}$$

where n_a and n_c refer respectively to the number of adults and children in the family, and $0 \leq \theta, \Phi \leq 1$. The value of Φ affects the weight given to children relative to adults and θ, called by Buhmann *et al.* (1988) the 'equivalence elasticity', is a measure of economies of scale, which are increased as θ is reduced. Buhmann *et al.* (1988) considered the scale where $\Phi = 1$, such that both the age and gender of family members are ignored, and showed that with suitable choice of the parameter, $0 \leq \theta \leq 1$, an approximation can be made to a wide range of equivalence scales currently in use.[18] Parameter values of $\theta = 0.5$ and $\Phi = 0.6$ are exogenously assumed for the analysis undertaken.

While it is acknowledged that this distribution of wealth is subject to criticism, the focus here is upon the practical implications of the suggested adjustment of the Gini coefficient, not an analysis of the distribution of wealth in Australia *per sè*. For a detailed description of the data used and associated distributional analysis see Woon (2000).

The wealth of 16,608 individuals is analysed, where the distribution is ranged between -\$528,382.67 and \$6,808,348.73, with a mean of \$110,477.46.[19] There are 408 individuals, 2.5 per cent of the entire survey population, that possess negative wealth, and 1,888 individuals, 11.4 per cent of the population, that possess non-positive wealth.

[18]The formulation used in this study has also been applied by, for example, Cutler and Katz (1992), Banks and Johnson (1994) and Jenkins and Cowell (1994).

[19]All wealth figures are measured in terms of 1996 Australian adult equivalent dollars.

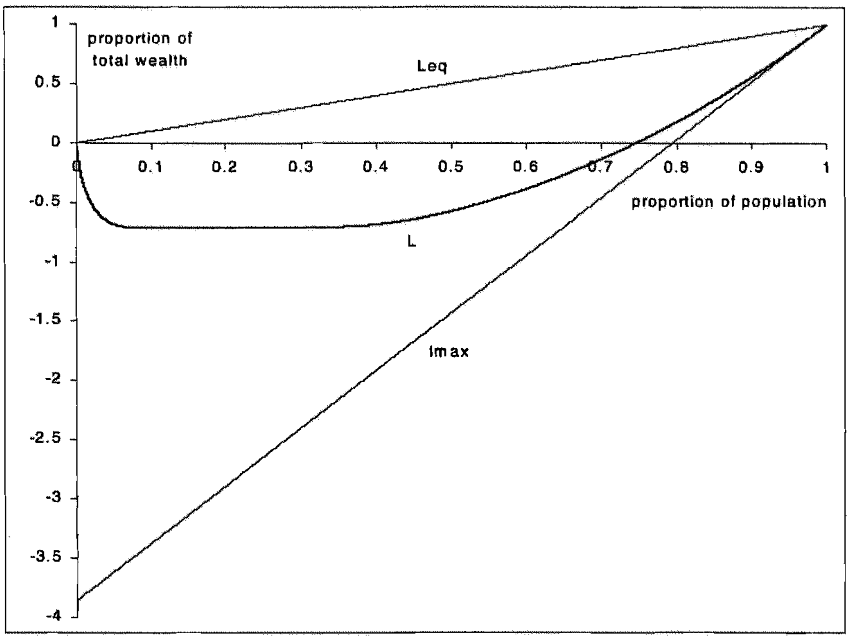

Figure 2: Lorenz curves for Lower Third of the Population based upon Measured Wealth

Figure 2 displays the Lorenz curve associated with the distribution of wealth for the least-wealthy third of the total survey population. The fact that this Lorenz curve does not have a positive slope for more than one third of the population is expected, given the distribution statistics provided above. The steep negative slope of the Lorenz curve at the origin and rapid levelling out displayed in figure 2 indicates that the -$528,382.67 wealth possessed by the least wealthy individual is 'extreme'. The proportion of the population that own negative wealth do, however, have an important impact on the inequality of the lower-third wealth distribution. This observation is highlighted by the fact that the Lorenz curve of figure 2 does not indicate positive aggregate wealth until the upper three quarters of the proportion of population axis. In addition, the standard Gini coefficient associated with the Lorenz curve of figure 2 is equal to 1.5850, which lies outside of the bounds required by the principle of normalisation.

Given that the population underlying figure 2 is subject to a maximum and no minimum limit, $L_{ueq} = l_{\max}$. Compared with the standard population limits, figure 2 indicates that the limits imposed on the population allow greater distribution variability, as discussed in section 3.1[20]. Specifically, for the imposed limits of $x_{\min} \rightarrow -\infty$, and $x_{\max} = \$37,200.50$, V from equation (11) is equal to $(x_{\max} - \mu) = \$29,538.61$, which is greater than the population mean ($\mu = \$7,661.89$) that is applicable for the standard non-negativity constraint. The adjusted Gini coefficient is consequently substantially less than the standard Gini. From equation (9), $\tilde{G} = 0.2594G = 0.4111$.

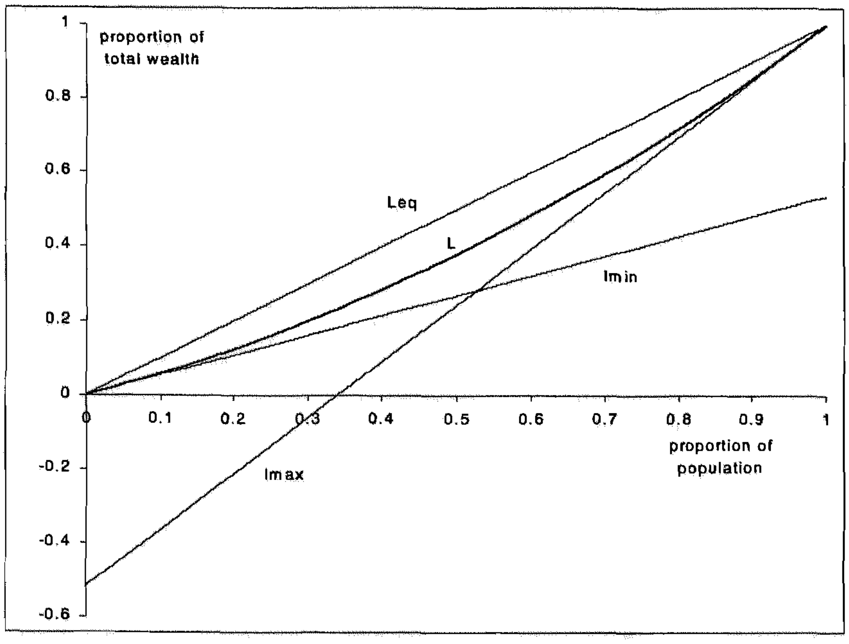

Figure 3: Lorenz curves for Middle Third of the Population based upon Measured Wealth

Figure 3 displays the Lorenz curve associated with the distribution of wealth for the middle third of the population. Given that this distribution is subject to both upper and lower limits, $l_{\min}$ and $l_{\max}$ are both depicted in this figure. The lines $l_{\min}$ and $l_{\max}$ indicate that the limits imposed on the distribution of wealth of the middle third of the population significantly

[20]since the area within which the associated Lorenz curve must be defined (between L_{eq} and $l_{\max}$) is larger than for the standard non-negativity constraint.

restrict the distribution of wealth compared to the limits of the standard non-negativity constraint. Comparing the measures of V characterised by equation (11), for example, the standard limits imply a value of $(\mu) = \$69,565.71$, whereas the imposed limits imply a value of $\left(\frac{(105429.52-69565.71)(69565.71-37232)}{(105429.52-37232)}\right)$ $= \$17,003.70$. Consequently, the standard Gini coefficient, which equals 0.1611 for the distribution underlying the Lorenz curve in figure 3, is less than the associated adjusted Gini of 0.6591.

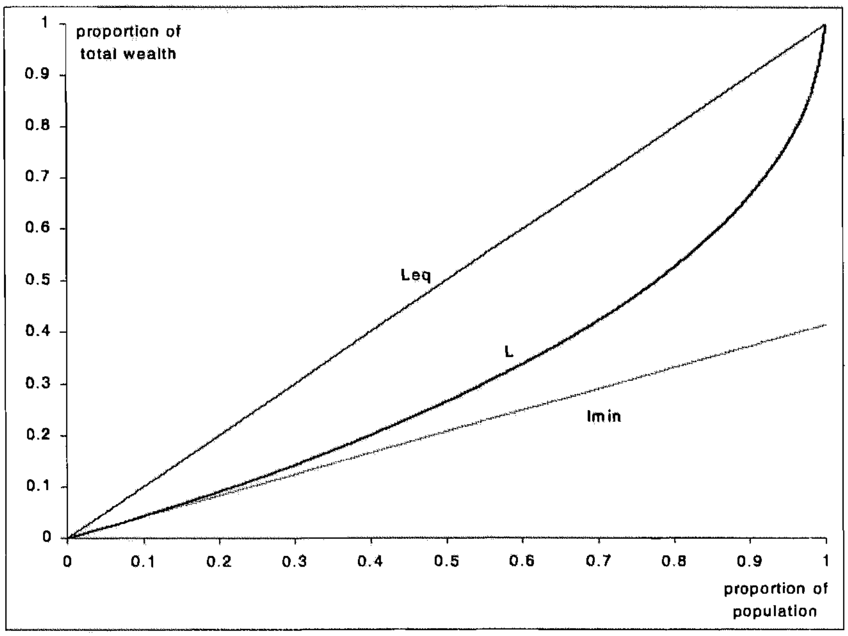

Figure 4: Lorenz curves for Upper Third of the Population based upon Measured Wealth

Figure 4 completes the span of the survey population by focusing on the wealth of the upper third of the distribution. This population is subject only to a minimum limit, and consequently, $l_{\min} = L_{ueq}$. Given that the minimum limit is greater than zero and there is no upper limit imposed on the population, the distribution underlying the Lorenz curve displayed in figure 4 is subject to less variability than the distribution underlying the standard non-negativity constraint. The adjusted Gini coefficient is consequently greater than the standard Gini by a factor of 1.7089, taking the value 0.6369, as opposed to 0.3727.

These results highlight the importance of taking into consideration the actual limits to which a distribution is subject when interpreting inequality relative to the distributional extremes. In particular, comparing the adjusted with the standard Gini coefficients discussed above reveals that very little can be said about how close inequality is to either one of the distributional extremes when the limits to which the distribution is subject are not explicitly taken into consideration. In the case of the lower third of the population based on wealth, a high standard Gini coefficient coincides with a distribution that is closer to absolute equality than inequality, as it is defined by the adjusted Gini coefficient. Conversely, for the middle and upper third populations, a low standard Gini coefficient coincides with distributions that are closer to absolute inequality.

Figure 5, displays the Lorenz curve for the full, unrestricted distribution of wealth. From this figure, it can be seen that Lorenz curve of the entire population falls below the 'proportion of total population' axis only slightly due to the 2.5 per cent of the population that possess negative wealth. The effect of this sub-group on the standard Gini coefficient is consequently slight, increasing it by 6.2 per cent from 0.5788 to 0.6147.

The limiting lines l_{min} and l_{max} are based respectively upon the minimum and maximum measures of wealth existing in the unrestricted distribution and, as such, define the most severe distributional constraint that can be imposed without requiring the omission of sample observations. The associated adjusted Gini coefficient based upon these limits is, however, 81.1 per cent less than the standard Gini coefficient, taking a value of 0.1164. This large change is driven by the fact that the proportion of the total survey population that possess negative wealth is small, and their inclusion in the distributional analysis requires the lower population limit to be relaxed considerably. The negative observations may consequently be described as 'extreme' or 'unrepresentative' in an analysis of the complete distribution of wealth, in which case it may be preferable to impose the standard non-negativity population limits. For the distributions underlying the Lorenz curves of figures 2 to 4, however, assuming the standard non-negativity limits would seem less justified.

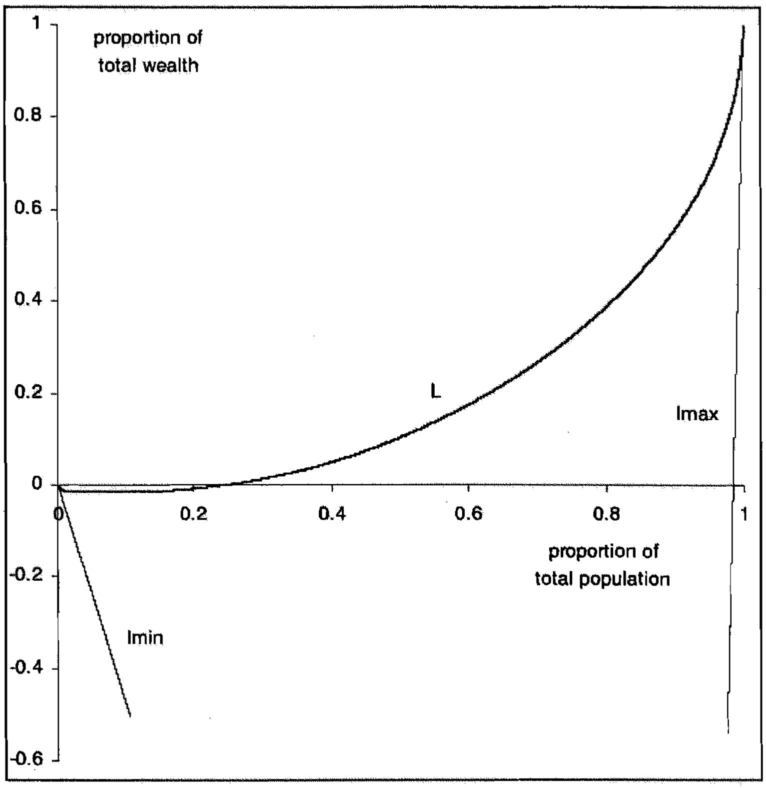

Figure 5: Lorenz curve of Wealth Distribution for Full Population

5 Conclusion

This paper has examined the implications of measuring inequality for distributions that are subject to general limits using the evaluative framework advocated by Dalton (1920). It is noted that the Gini coefficient satisfies all of the standard principles that characterise a useful measure of inequality under the assumption of distribution non-negativity. The Gini coefficient fails to satisfy the principle of normalisation, however, for any population that is subject to other distributional limits. An adjustment to the Gini coefficient based upon a stated definition of 'perfect inequality' is suggested, which produces a statistic that satisfies all of the principles to which an inequality measure is subject for any general distributional limits.

The adjusted Gini coefficient requires the distribution limits to be stated explicitly. Although this property enhances transparency, the added flexibility that is implied comes at a cost. Specifically, the connection between the measures of inequality of two distributions and Lorenz dominance is severed if the distributions are subject to different population limits. In addition, where there exists no 'natural' distribution limits, there exists a trade-off between inclusion of observations, and disparity between standard and adjusted Gini coefficients. Choice of the limits imposed must be based on the distribution of interest and with reference to the associated focus of analysis. The degree to which this choice is arbitrary, however, may be considered to impinge on the ability of the adjusted Gini coefficient to satisfy the principle of operationallity.

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