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FRANCIS YSIDRO EDGEWORTH 1845-1926

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1 Biographia

Francis Ysidro Edgeworth (1845-1926) was born in Edgeworthstown in County Longford, Ireland. The background into which he was born was dominated by the 'larger than life' figure of his grandfather Richard Lovell Edgeworth (1744-1817), whose life was documented in a two-volume memoir (1820) by his oldest daughter, the famous novelist Maria Edgeworth (1767-1849).[1] Richard Lovell's many scientific and mechanical experiments were helped by his strong association with the Lunar Society of Birmingham, whose members included Watt, Bolton, Wedgwood, Priestley, Darwin, and Galton. In addition, Maria's scientific acquaintances also included Davy, Humboldt, Herschel, Babbage, Hooker and Faraday. The marriage of F. Y. Edgeworth's cousin Harriet Jessie Edgeworth (daughter of Richard Lovell's seventh and youngest son Michael Pakenham, 1812-81) to Arthur Gray Butler provided links with another large and eminent academic family. These connections extend even further since A. G. Butler's sister, Louisa Butler, married Francis Galton, a cousin of Charles Darwin.

Richard Lovell's sixth son, and seventeenth surviving child, was Francis Beaufort Edgeworth (1809-46), who met his wife, Rosa Florentina Eroles, the daughter of a Spanish refugee from Catalonia and then aged sixteen, while on the way to Germany to study philosophy; they married within three weeks

*This is a forthcoming entry for the *New Palgrave Dictionary of Economics*. I am grateful to Denis O'Brien and Steven Durlauf for comments on an earlier draft.

[1] On the family background, see also Butler and Butler (1927). For a full-length treatment of Edgeworth's work, see Creedy (1986).

in 1831. F. Y. Edgeworth was their fifth son. With his family background and his knowledge of French, German, Spanish and Italian, Edgeworth had wide international sympathies.

Edgeworth was educated by tutors in Edgeworthstown until the age of 17, when in 1862 he entered Trinity College Dublin to study languages. In 1867 Edgeworth entered Exeter College, Oxford, but after one term transferred to Magdalen Hall. He transferred to Balliol in 1868, where in Michaelmas 1869 he obtained a first in Literae Humaniares. He was called to the bar in 1877, the same year in which his first book, *New and Old Methods of Ethics*, was published. Edgeworth applied unsuccessfully for a Professorship of Greek at Bedford College, London, in 1875, but later lectured there on English language and literature for a brief period from late 1877 to mid 1878. He had earlier lectured on logic, mental and moral sciences and metaphysics to prospective Indian civil servants, at a private institution run by a Mr Walter Wren. In 1880 he applied for a chair of philosophy, also unsuccessfully, but began lecturing on logic to evening classes at King's College London. Soon after the publication of his second book, *Mathematical Psychics*, in 1881, he applied for a professorship of logic, mental and moral philosophy and political economy at Liverpool. Testimonials for two of Edgeworth's applications were given by Jevons (see Black, 1977c, pp. 98, 145) and Marshall.

Edgeworth had to wait until 1890 until he obtained a professorial appointment: this was at King's College London, where he succeeded Thorold Rogers in the Tooke Chair of Economic Science and Statistics. In the next year, 1891, he again succeeded Rogers, this time to become Drummond Professor and Fellow of All Souls' College, Oxford, a position he held until his retirement in 1922. Edgeworth therefore finally settled in Oxford at the age of 46 in what was to become one of the most illustrious British chairs in economics. At the same time he became the first editor of the *Economic Journal*.[2] After a tremendously creative period of the late 1870s and 1880s, Edgeworth had become firmly established as the leading economist, after

[2] He was editor or co-editor from its first issue until his death. He was supported by Henry Higgs from 1892 to 1905 (when the latter became the Prime Minister's Private Secretary), with further assistance provided at a later stage by Alfred Hoare. Keynes was a co-editor for 15 years.

Marshall, in Britain.

In addition to his work in economics, Edgeworth began a series of statistical papers in 1883, and was secretary to the British Association *Report on Index Numbers* (1887, 1888, 1889). He was President of section F of the British Association in 1889, a position he held again in 1922. Edgeworth's work on mathematical statistics took an increasingly important role. Indeed, of about 170 papers which he published, approximately three-quarters were concerned with statistical theory. He became a Guy Medalist (Gold) of the Royal Statistical Society in 1907 and was President of the Society during 1912-14. His main contributions to statistics concern work on inference and the law of error', the correlation coefficient, transformations (what he called 'methods of translation'), and the 'Edgeworth expansion'. The latter, a series expansion which provides an alternative to the Pearson family of distributions, has been widely used (particularly since the work of Sargan, 1976) to improve on the central limit theorem in approximating sampling distributions.[3] His third and final book was *Metretike: or the Method of Measuring Probability and Utility* (1887). These contributions are not examined here; see Bowley (1928) and Stigler (1978).

2 Approach to Economics

A dominant characteristic of Edgeworth's approach to economics is that it is mathematical, characterised by an original use of techniques, although he does not appear to have received a formal training in mathematics. However, he came to economics from moral philosophy. The central question of distributive justice, rather than simply the application of mathematics, dominated his attitude towards economics. His main argument was that mathematics provided powerful assistance to 'unaided' reason, and could check the conclusions reached by other methods. Thus:

> He that will not verify his conclusions as far as possible by

[3] It has also been used to provide support for the bootstrap in providing an Edgeworth correction. Edgeworth's work in probability and statistics has been collected by McCann (ed.) (1996).

> mathematics, as it were bringing the ingots of common sense to be assayed and coined at the mint of the sovereign science, will hardly realise the full value of what he holds, will want a measure of what it will be worth in however slightly altered circumstances, a means of conveying and making it current. (1881, p.3)

Edgeworth's approach contrasting sharply with Marshall, who commented to Bowley that 'Edgeworth might have done something great at it [economics]: but he has crushed his instincts between the cog wheels of his mathematical machinery'. The contrast between Edgeworth and Marshall was neatly summarized by Pigou as follows:

> During some thirty years until their recent deaths in honoured age, the two outstanding names in English economics were Marshall ... and Edgeworth ... Edgeworth, the tool-maker, gloried in his tools ... Marshall, on the other hand, had what almost amounted to an obsession for hiding his tools away. (Pigou and Robertson, 1931, p.3)

Although both men turned to economics from mathematics and moral philosophy, Marshall generally used biological analogies, and was concerned with developing maxims. In contrast Edgeworth generally used mechanical analogies, and was more concerned with developing theorems.

In the 1880s and 1890s the deductive method encountered a great deal of criticism, especially from the 'historical school' of economists. Edgeworth's defence of the deductive method often involved showing how other economists had advocated its use. His interest in the natural sciences often led him to make comparisons with scientific laws, and especially to show that the physical sciences also relied on abstraction and approximation.

Edgeworth argued carefully that the assumptions used in economics are often untestable, and he therefore took precautions against the accusation of 'plucking assumptions from the air'. He was conscious of the fact that the difficulty is in making the crucial abstractions which make the particular problem under consideration tractable, but which are not question begging.

His attitude to many a priori assumptions was directly related to his approach to statistical inference. In *Mathematical Psychics*, for example, he referred to 'the first principle of probabilities, according to which cases about which we are equally undecided ... count as equal' (1881, p.99). This was then transferred to economics. The appropriate assumption was that all feasible values, say of elasticities, were equally likely, until evidence is obtained. Hence, 'There is required, I think ... in order to override the a priori probability, either very definite specific evidence, or the concensus of high authorities' (1925, p.391). This also illustrates Edgeworth's attitude to authority and his many allusions to the views of other leading economists. Price (1946, p.38) refered to his frequent 'reference to authority for ... support of tentative opinion waveringly advanced'.

Edgeworth was also prone to stress negative results. For example, in discussing taxation, where the criterion of minimum sacrifice does not alone provide a simple tax formula. He stated:

> Yet the premises, however inadequate to the deduction of a definite formula, may suffice for a certain negative conclusion. The ground which will not serve as the foundation of the elaborate edifice designed may yet be solid enough to support a battering-ram capable of being directed against simpler edifices in the neighbourhood. (1925, p.261)

Edgeworth's position as editor of the enabled him to combine both his critical attitude and his appetite for a wide range of reading. He contributed 32 book reviews, and in sending books to other reviewers he would include 'apposite remarks on particular points in the text' (Bowley, 1934, p.123). These reviews should also be placed beside his 17 reviews in the Academy, and 131 articles in the original *Palgrave's Dictionary of Economics*. Furthermore, Edgeworth's later articles in the *Economic Journal*, such as those on international trade and on taxation, took the form of extended commentaries on contemporary work.

3 Early Work in Moral Philosophy

Before turning to economics, Edgeworth published a brief note in *Mind* in 1876, and his first (privately printed) book on *New and Old Methods of Ethics* in 1877. The description by Keynes of Edgeworth's first book could just as well be applied to his other two books:

> Edgeworth's peculiarities of style, his brilliance of phrasing, his obscurity of connection, his inconclusiveness of aim, his restlessness of direction, his courtesy, his caution, his shrewdness, his wit, his subtlety, his learning, his reserve - all are there full-grown. Quotations from the Greek tread on the heels of the differential calculus. (Keynes, 1972, p.257)

The main focus of this early work, strongly influenced by the great Cambridge philosopher Henry Sidgwick (1838-1900), was to examine in detail the implications of utilitarianism for the optimal distribution of resources. Edgeworth's special and original contribution was to apply advanced mathematics to this problem. Edgeworth's approach was dominated by his utilitarianism, but the influence of contemporary psychological research and the impact of evolutionary ideas can also be traced. Both aspects led to explicit consideration of differences between individuals and changes which take place over time.

Edgeworth was also influenced by the major fierce debates in the last half of the nineteenth century between egoism, evolutionism, idealism, intuitionism, and of course utilitarianism. His brand of utilitarianism became extremely eclectic, and embraced the majority of the above principles (except for those of the Hegelian idealists) while regarding utilitarianism as the 'sovereign principle'. His note in *Mind* discussed Matthew Arnold's views of Joseph Butler, who had examined egoist at great length. Arnold had argued that Butler's term 'self love' should be interpreted to mean 'the pursuit of our temporal good'. However, Edgeworth argued that egoism and utilitarianism could be subsumed under the same principle. He believed Butler to be saying, 'duty and interest are perfectly coincident; for the most part in this world, but entirely and in every instance, if we take in the future and the whole' (1876, p.571).

Edgeworth generally distinguised between 'impure' and 'pure' utilitarianism. In the latter case individuals are assumed to be concerned with the welfare of society as a whole. The former case in fact corresponds more closely with a 'short term' version of egoism. Economic exchange can usefully be analysed in terms of 'jostling egoists', but he believed that ultimately individuals would evolve to become pure utilitarians. A reason for believing that individuals would make such a transition was later to be developed by Edgeworth in the form of his contractarian justification of utilitarianism as the appropriate principle of distributive justice.

Edgeworth's early utilitarianism was influenced by his wide knowledge of work in experimental psychology. In his books of 1877 and 1881 there are many references to the work of Delboeuf, Fechner, Helmholtz, Weber and Wundt. These references occur in the context of discussing the nature of utility functions and, although Edgeworth at this time was not aware of the earlier work of Jevons, the same range of psychological work was also important to Jevons. Edgeworth in 1877 explicitly suggested, in connection with Fechner, that an additive form would not be appropriate.

A further aspect of Edgeworth's utilitarianism is his attitude towards authority. An important issue for early utilitarians involved the nature of inductive evidence about the consequences of acts. Most people cannot know the full consequences of their acts, so that rules of moral conduct must be followed (in contrast with intuitionism where individuals are assumed to have immediate consciousness of moral rules). In arriving at such rules, the opinions of highly regarded individuals are taken to be credible though it may not be possible to show conclusively that they are 'correct'. Edgeworth argued, for example, that 'we ought to defer even to the undemonstrated dicta and opinions of the wise, who have a power of mental vision acquired by experience' (1925, ii, p.149).

Edgeworth defined the problem of determining the optimal utilitarian distribution as follows: 'given a certain quantity of stimulus to be distributed among a given set of sentients ... to find the law of distribution productive of the greatest quantity of pleasure' (1877, p.43). In treating this problem mathematically, Edgeworth used Lagrange multipliers, without any explanation, and concluded that, 'unto him that hath greater capacity for pleasure shall be added more of the means of pleasure' (1877, p.43). In using Lagrange multipliers, Edgeworth was also careful to discuss possible complications, referring to the possibility of multiple solutions and explicitly discussing corner solutions and inequality constraints.

Further complexities were then examined, where Edgeworth emphasize that utilitarianism implies equality of the 'means of pleasure' only under a special set of assumptions, and in the general case the prescribed solution will be some form of inequality. In dealing with the distribution of effort, he argued not surprisingly that most work should be provided by those most capable of providing it. In a yet more general treatment of the problem, Edgeworth used the calculus of variations, but again provided the reader with virtually no help in following his mathematical argument. Edgeworth's analysis of the utilitarian optimal distribution was continued in his paper on 'The hedonical calculus' (1879), which was later reprinted as the third part of *Mathematical Psychics*.

4 Early Work in Economics

The turning point in Edgeworth's work was his introduction to Jevons in 1879 by a mutual friend James Sully, who in 1878 moved to Hampstead, where Edgeworth had lodgings in Mount Vernon and where Jevons also lived; see Sully (1918, pp. 180, 223). His first knowledge of Marshall came from Jevons who, 'highly praised the then recently published Economics of Industry' (in Pigou, ed, 1925b, p.66). Edgeworth became interested in the problem of the indeterminacy of the rate of exchange, arising from the existence of only a small number of transactors. This led rapidly to Edgeworth's second and most important book *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (1881), which was clearly written in a state of considerable enthusiasm for his new subject. This slim volume of 150 pages was known only to a small group of experts. Marshall's review began, ‘this book shows clear signs of genius, and is a promise of great things to come’ (Whitaker, ed, 1975, p.265). Jevons began by stating that ‘whatever else readers of this book may think about it, they would probably all agree that it is a very remarkable one’ (1881, p.581). It was not until the middle of the 20th century that many of its central ideas began to be more fully appreciated.

Part 1 of *Mathematical Psychics* (1881, pp. 1-15) was devoted mainly to a justification of the use of mathematics in economics where precise data are not available. There is probably no other ‘apology’ in the whole of economic literature which compares with Edgeworth’s plea for the application of mathematics. For example, when considering individual utility maximization:

> Atoms of pleasure are not easy to distinguish and discern; more continuous than sand, more discrete than liquid; as it were nuclei of the just-perceivable, embedded in circumambient semi-consciousness. We cannot count the golden sands of life; we cannot number the ‘innumerable smile’ of seas of love; but we seem to be capable of observing that there is here a greater, there a less, multitude of pleasure-units; mass of happiness; and that is enough. (1881, pp. 8-9)

Great stress was placed on comparison with Lagrange’s ‘Principle of least action’ in examining the overall effects produced by the interactions among many particles. The connection with Edgeworth’s analysis of competition, involving interaction among a large number of competitors to produce a determinate rate of exchange, is central here. The fact that in the natural sciences so much could be derived from a single principle was important for both Jevons and Edgeworth. But Edgeworth took this to its ultimate limit in arguing that the comparable single principle in social sciences, that of maximum utility, would produce results of comparable value. Referring to Laplace’s massive work, *Mecanique Celeste*, he suggested that:

> ‘Mécanique Sociale’ may one day take her place along with ‘Mécanique Celeste’, throned each upon the double-sided height

> of one maximum principle, the supreme pinnacle of moral as of physical science . . . the movements of each soul, whether selfishly isolated or linked sympathetically, may continually be realising the maximum energy of pleasure, the Divine love of the universe. (1881, p.12)

Jevons's work in the *Theory of Political Economy* involved the application of very basic mathematics, and of psychological research, to the analysis of exchange in competitive markets. In addition to this direct stimulus, Edgeworth was also influenced by an anonymous review of Jevons's book in the *Saturday Review* (1871).

The crucial development following Edgeworth's contact with Jevons was not simply the realization that mathematics could be used to examine equilibrium in exchange. Rather it was that in his analysis Jevons explicitly assumed, through his 'law of indifference', that all individuals take the equilibrium prices as given, that is outside their control. In using this law as 'one of the central pivots of the theory', Jevons stated that, 'there can only be one ratio of exchange of one uniform commodity at any moment' (1957, p.87). His theory was explicitly limited to the static equilibrium conditions. He deliberately excluded the role of the number of competitors from his analysis via the awkward notion of the 'trading body', following correspondence with Fleeming Jenkin (1833-85), who raised the question of indeterminacy with just two traders; see Black (1977a, pp. 166-78). Jenkin could not see why two isolated individuals should accept the price-taking equilibrium, whereas Jevons wished to consider the behaviour of two typical individuals in a large market.

In a section on 'Failure of the Laws of Exchange', Jevons discussed cases in which some indeterminacy would result. His most notable example was of house sales, where it was suggested that indeterminacy would result from the discrete nature of the good being exchanged. The *Saturday Review* article took exception to this, suggesting that indeterminacy 'is really owing in our opinion to the assumed absence of competition' (see Black, 1981, p.157). The stress on indeterminacy was also influence by Marshall's discussion of wage bargaining: Edgeworth (1881, p.48, n.1) referred to Thornton's comparison of the determination of prices in Dutch and English auctions, and cited Alfred and Mary Paley Marshall's joint book on the *Economics of Industry* (1879).

It was this gap in Jevons's analysis that Edgeworth set out to fill. His achievement was to show the conditions under which competition between buyers and sellers, through a barter process, leads to a 'final settlement' which is equivalent to one in which all individuals act independently as price takers. As he later stated (1925, p.453), 'the existence of a uniform rate of exchange between any two commodities is perhaps not so much axiomatic as deducible from the process of competition in a perfect market'.

5 Exchange and Contract

Having argued that, 'the conception of Man as a pleasure machine may justify and facilitate the employment of mechanical terms and Mathematical reasoning in social science' (1881, p.15), Edgeworth moved on to the analysis of the 'economical calculus', the starting point of which was the assumption that, 'every agent is actuated only by self-interest' (1881, p.16).

In modern economic analysis the analytical tools invented by Edgeworth in 1881, such as the indifference map and the contract curve, are now used in a vast range of contexts. They were introduced by Edgeworth to examine the nature of barter among individuals. He wanted to see if a determinate rate of exchange would be likely to result in barter situations where it is assumed only that individuals wish to maximize their own utility, considered solely as a function of their own consumption. With full knowledge of individuals' utility functions, and their initial endowments of goods, would it be possible to work out a 'determinate' rate of exchange at which trade would take place? Edgeworth's direct statement of the problem is as follows:

> The PROBLEM to which attention is specially directed in this introductory summary is: How far contract is indeterminate – an inquiry of more than theoretical importance, if it show not only that indeterminateness tends to [be present] widely, but also

> in what direction an escape from its evils is to be sought. (1881, p.20)

Edgeworth began his analysis of this problem by taking the simplest case of two individuals exchanging fixed quantities of two goods. The basic framework is that described by Jevons, where the first individual holds all of the initial stocks of the first good, the second individual holds all the stocks of the second good. He wrote the utility functions of each individual in terms of the amounts exchange, rather than consumed. He then immediately defined the general utility function ('utility is regarded as a function of the two variables, not the sum of two functions of each', 1881, p.104), the contract curve and indifference curves, in that order.

In the sentence which follows Edgeworth's introduction of the general utility function, he raised the question of the equilibrium which may be reached with, 'one or both refusing to move further'. In barter the conditions of exchange must be reached by voluntary agreement, or contract, between the two parties, and of course it is fundamental that no egoist would agree to a contract which would make him worse off than before the exchange. The question thus concerns the nature of the settlement reached by two contracting parties. He immediately answered that contract supplies only part of the answer so that, 'supplementary conditions ... supplied by competition or ethical motives' are required, and then wrote the equation of his famous contract curve (1881, pp. 20-1).

The problem of obtaining the equilibrium values of x and y which, 'cannot be varied without the consent of the parties to it' was stated as follows: 'It is required to find a point (x, y) such that, in whatever direction we take an infinitely small step, $[U_A]$ and $[U_B]$ do not increase together, but that, while one increases, the other decreases' (1881, p.21). The locus of such points, 'it is here proposed to call the contract-curve'. Edgeworth's alternative derivations of the contract curve involved the movement, from an arbitrary position, along one person's indifference curve; 'motion is possible so long as, one party not losing, the other gains' (1881, p.23). He thus used the Lagrange multiplier method of maximizing one person's utility subject to the condition that the other person's utility remains constant.

In the diagram drawn by Edgeworth (1881, p.28) he did not use a box construction. Furthermore the only indifference curves shown fully were those which each individual is able to reach in isolation, and which therefore specify the limits beyond which each is not prepared to move. Also part of the offer or reciprocal demand curves of each individual were drawn on the same diagram, although they were not defined until ten pages later.

After presenting the results for the two-person two-good case, Edgeworth (1881, p.26) examined the contract curve in the case where three individuals exchange three goods, stated, that it is given by the 'eliminant', and then gave three lines of three sets of partial derivatives. In fact the contract curve in this context is defined by $\left|\frac{\partial U_i}{\partial x_j}\right| = 0$, where $\frac{\partial U_i}{\partial x_j}$, is the marginal utility of person i with respect to good j, but Edgeworth did not use the modern notation for determinants and did not set the Jacobian equal to zero. This early use of determinants in economics would probably have confused many of his readers.

5.1 The Problem of Indeterminacy

The concepts of indifference curves and the contract curve therefore help to specify a range of 'efficient exchanges' of goods between individuals. The essential feature of the analysis from Edgeworth's point of view is precisely that there is a range, rather than a unique point: 'the settlements are represented by an indefinite number of points' (1881, p.29). At any particular settlement, the rate of exchange is expressed simply in terms of the amount of one good which is given up in order to obtain a specified amount of the other good. Hence the existence of a range of efficient contracts means that the rate of exchange is 'indeterminate'. The rate of exchange achieved in practice will thus depend to a large extent on bargaining strength. It was this result which led Edgeworth to make his often quoted remark that, 'an accessory evil of indeterminate contract is the tendency, greater than in a full market, towards dissimulation and objectionable arts of higgling.' (1881, p.30).

Edgeworth argued that his analysis of indeterminacy in contract between two traders could be applied to a very wide variety of contexts. In particular, the tendency of large groups to form 'combinations', as in the case of trade unions and employers' associations, would serve to increase the extent of indeterminacy. The general applicability of his analysis of contract and indeterminacy was summarized by Edgeworth as follows:

> What it has been sought to bring clearly into view is the essential identity (in the midst of diversity of fields and articles) of contract; a sort of unification likely to be distasteful to those excellent persons who are always dividing the One into the Many, but do not appear very ready to subsume the Many under the One. (1881, p.146)[4]

Having shown the possibilities of indeterminacy, Edgeworth then went on to show how 'the escape from its evils' requires either competition or arbitration.

6 Competition and the Number of Traders

The central question which Edgeworth was trying to resolve in the second part of Mathematical Psychics was that of the conditions necessary to remove the indeterminacy which exists in the case of barter between two traders. The question naturally arises as to the extent to which this indeterminacy is the result of the absence of competition in the simple two-person market. Edgeworth thus quickly moved on to the introduction of further traders.

In Edgeworth's earlier problem of two traders exchanging two goods, the definition of a range of efficient exchanges (along the contract curve) is of course analytically separate from the question of whether or not two isolated traders would actually reach a settlement on the contract curve. However, these two aspects were not clearly separated by Edgeworth because at the beginning of his analysis he introduced his stylized description of the process

[4] Plato's expression 'the one in the many' was later used by Marshall as the motto for his 1919 book on *Industry and Trade.*

of barter: this is the famous 'recontracting' process. Edgeworth did not wish to assume that individuals initially have perfect knowledge. Instead, he supposed that, 'There is free communication throughout a normal competitive field. You might suppose the constituent individuals collected at a point, or connected by telephones – an ideal supposition, but sufficiently approximate to existence or tendency for the purposes of abstract science' (1881, p.18). The knowledge of the other traders' dispositions and resources could be obtained by the formation of tentative contracts which are not assumed to involve actual transfers, and can be broken when further information is obtained. Edgeworth introduced this in typical style:[5]

> 'Is it peace or war?' asks the lover of 'Maud', of economic competition, and answers hastily: it is both, pax or pact between contractors during contract, war, when some of the contractors without the consent of others recontract. (1881, p.17)

An important role of the recontracting process is thus to disseminate information among traders. It allows individuals who initially agree to a contract, which is not on the contract curve, to discover that an opportunity exists for making an improved contract according to which at least one person gains without another suffering.

However, the real importance of the recontracting process lies in the fact that it allows for Edgeworth's analysis of the role of the number of individuals in a market. With numerous individuals, the recontracting process makes it possible to analyse the use of collusion among some of the traders. Individuals are allowed to form coalitions in order to improve bargaining strength. Recontracting enables the coalitions to be broken up by outsiders who may attract members of a group away with more favourable terms of exchange.

Edgeworth's analysis was extremely terse and the following discussion does not therefore follow his own presentation. The analysis begins by introducing a second person A and a second person B. The new traders are assumed to be exact replicas of the initial pair, with the same tastes and

[5] The allusion here is to Alfred Tennyson's poem 'Maud; A Monodrama', part 1, verse VII.

endowments. This simplification is useful because the dimensions of the Edgeworth box and the utility curves are identical for each pair of traders. Hence, it enables the same diagram to be used as in the case when only two traders are considered in isolation. Two basic points can be stated immediately. First, in the final settlement all individuals will be at a common point in the Edgeworth box. Second, the settlement must be on the contract curve. The first point arises because if two individuals have identical tastes then their total utility is maximised by sharing their resources equally. It is useful to consider other types of contract which will eventually be broken, in order to illustrate the way in which the introduction of additional traders provides a role for some kind of competitive process.

The major question at issue is whether the range of indeterminacy along the contract curve is reduced by the addition of these traders. Consider Figure 1 and suppose that when A_1 and B_1 are trading independently of A_2 and B_2, trader B_1 has all the bargaining power and is able to appropriate all the gains from trade by pushing A_1 to the limit of the contract curve at point C. Suppose also that the same applies to A_2 and B_2. If the two pairs of traders are then able to communicate with each other, A_2 can now simply refuse to trade with B_2 at C. With no transaction costs, A_2 was previously indifferent between trading at C and consuming at the endowment point, E. This endowment position is effectively the 'threat point' of the As: it is the position in which they would find themselves if the bargaining process were to break down. But A_2 no longer needs to remain in isolation after refusing to trade with B_2, and instead can trade with A_1, after A_1 has traded with B_1 at C and has therefore obtained some of good Y. The two As can share their stocks of X and Y equally, ariving at point P; such an equal division maximises their total utility.

By reaching point P, half way between C and E, the convexity of the indifference curves implies that they are both better off than anywhere on the no-trade indifference curve. The two As would be on a higher common indifference curve, and thus better-off, if they could consume at a point along the CPE which is to the north-west of point P. However, they do not have enough resources to move beyond the half-way point P.

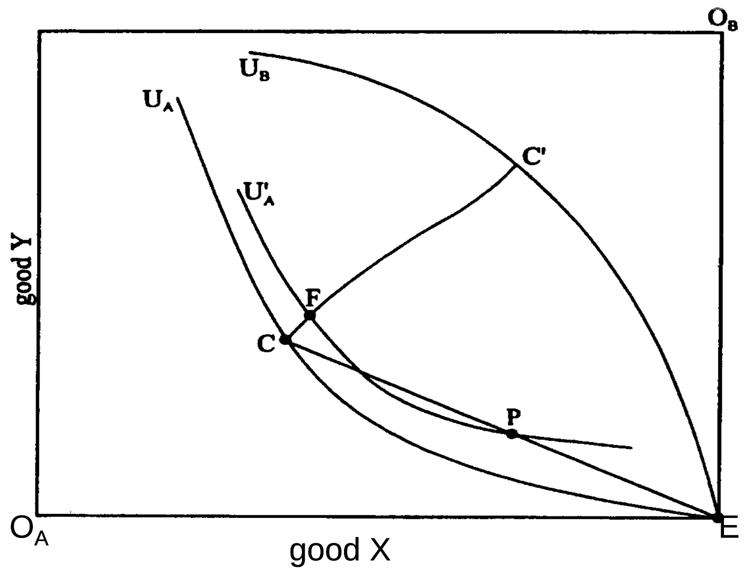

Figure 1: Two Pairs of Traders

Trader B_2, who has been isolated, cannot prevent such a bargain. Thus B_1 is at C, both As are at P and B_2 is at the initial endowment point E. In this situation B_1 has no incentive to change, but B_2 has a strong incentive to offer a better deal to one of the As than the one offered by trader B_1. So long as B_2 offers one of the As, say A_2, a trade on the contract curve which allows A_2 to reach a higher indifference curve than U'_A, the initial agreement with B_1 will be broken and recontracting will take place.

The implication is that the ability of the As to turn to someone else, rather than deal with a single trader, means that the Bs now compete against each other. However trader B_1, who cannot prevent the recontracting, has an incentive to make yet a better offer. Hence, the recontracting process continues. The stylised process of recontracting with the two Bs competing against each other will produce a final settlement at the point C^* in Figure 2. This has the property that the indifference curve U'''_A passes through C^* and P^*, where P^* is half way between C^* and E. This means that the two As are indifferent between C^* and P^* , and since they cannot both reach any point between C^* and P^* along the line C^*E, they are unable to improve on C^*. Hence there is no need to leave one of the Bs in isolation and the two

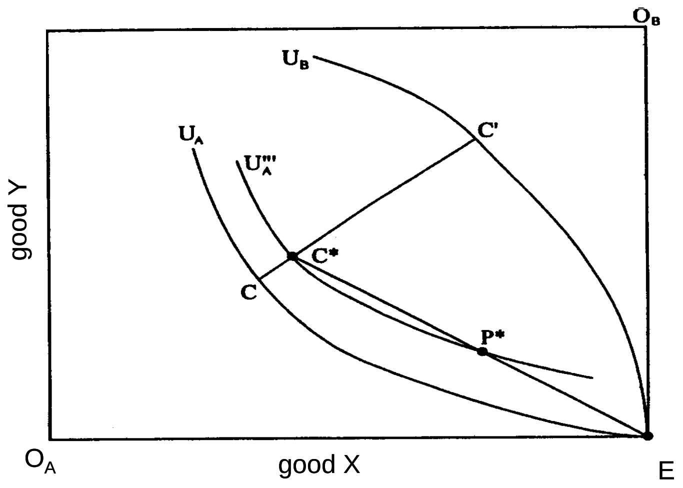

Figure 2: The New Limit to the Contract Curve

Bs will trade with the two As at point C^*.

This argument has shown that at the final settlement all traders are at a common point on the contract curve and the limit has moved inwards along the old contract curve. The analysis can be repeated by starting with an alternative situation whereby the As are initially assumed to be able to appropriate all the gains from trade. The point C' would then no longer qualify as a point on the new contract curve. The introduction of the additional pair of traders means that the contract curve shrinks, and the range of indeterminacy involved in barter is correspondingly reduced.

The extent to which the contract curve shrinks when the additional pair of traders is introduced is influenced by the fact that the As cannot get further than half way along a ray from a point on the contract curve to the endowment position. However, if there are three pairs of As and three pairs of Bs, the repetition of the above analysis involves two of the As dealing with two of the Bs at a point on the contract curve. The two As then share their resources equally with the remaining A while the third B is isolated. The As are able to consume together at a point which is two-thirds of the way along the ray from the initial endowment position to the point on the contract curve where the trade involving the two As and two Bs takes place.

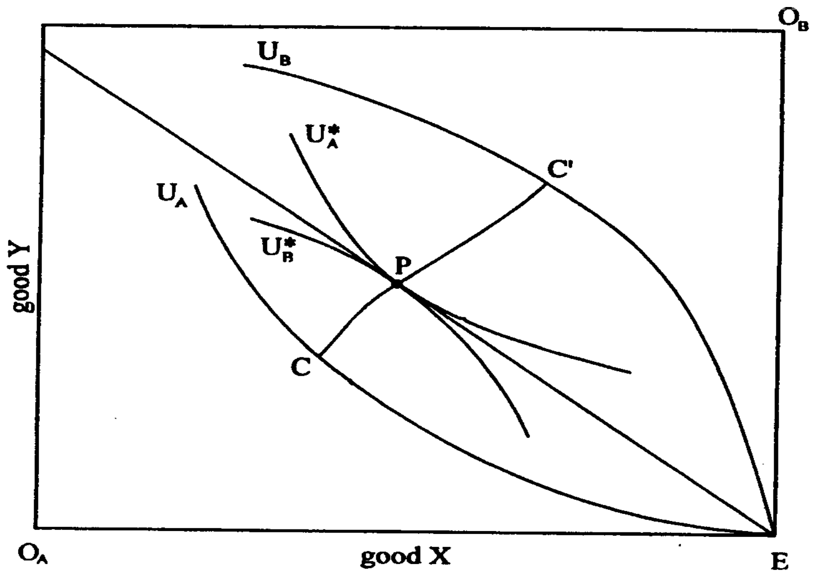

Figure 3: Final Settlement with Many Traders

With N pairs, the As can reach a proportion $\frac{(N-1)}{N}$ of the way from the endowment point to the contract curve. Thus as N increases, the values of k approaches unity. This means that the As can reach all the way from E to the contract curve, so that the final settlement must be such that the indifference curve is tangential to the ray from the origin. A final settlement with many traders is therefore shown in Figure 3 as point P on the contract curve. The effect of working in from the point C' would lead to an equivalent result for an indifference curve of the Bs, shown as U_B^*.

The result is that the final settlement looks just like a price-taking equilibrium. The figure illustrates the case where there is a single price-taking equilibrium. If there are multiple equilibria, the recontracting process causes the number of final settlements, with sufficiently large N, to shrink to the number of price-taking equlibria.[6] This argument relating to the shrinking contract curve, first established by Edgeworth, is often referred to as the *limit theorem*.

After Edgeworth's terse discussion, he stated:

[6] For discussion of utility functions involving multiple equilibria, and comparison of bargaining, competitive and utilitarian solutions, see Creedy (1994a).

> If this reasoning does not seem satisfactory, it would be possible to give a more formal proof; bringing out the important result that the common tangent to both indifference curves ... is the vector from the origin. (1881, p.38)

The price-taking solution is necessarily on the contract curve. This gives rise to what is now referred to as the 'first fundamental theorem' of welfare economics – that a price-taking equilibrium is Pareto efficient. Furthermore, the use of price-taking provides a considerable reduction in the amount of information required by traders when compared with the recontracting process. Given an equilibrium set, individuals only need to know the prices of goods, whereas in the recontracting process they have to learn a considerable amount of information about other individuals' preferences and endowments. But Edgeworth placed more stress on the equivalence of the competitive price-taking solution with a recontracting barter process involving large numbers.

Given that coalitions among traders are allowed in the recontracting process, a price-taking equilibrium cannot be blocked by a coalition of traders. In this sense the competitive equilibrium is robust. The argument that a complex process of bargaining among a large number of individuals produces a result which replicates a price-taking equilibrium, allowing for the free flow of information using recontracting and enabling coalitions of traders to form and break up, is an important result that is far from intuitively obvious. The recontracting process can be said to represent a competitive process, and the contract curve shrinks essentially because of the competition between suppliers of the same good, although it is carried out in a barter framework in which explicit prices are not used (although rates of exchange are equivalent to price ratios).

The price-taking equilibrium, in contrast, does not actually involve a competitive process. Individuals simply believe that they must take market prices as given and outside their control. They respond to those prices without any reference to other individuals. But the result is that the price-taking equilibrium looks just like a situation in which all activity is perfectly co-ordinated.

Edgeworth suggested that similar results apply when some of the assumptions are relaxed. Thus, 'when we suppose plurality of natures as well as persons, we have to suppose a plurality of contract-curves ... Then, by considerations analogous to those already employed, it may appear that the quantity of final settlements is diminished as the number of competitors is increased' (1881, p.40). He then briefly considered different numbers of As and Bs, concluding that, 'the theorem admits of being extended to the general case of unequal numbers and natures' (1881, p.43). However, some of the results do not hold in the general case; for example equality within the group of As no longer holds when there are unequal numbers of As and Bs. A considerable number of articles have been written, since the late 1950s, examining various aspects of the Edgeworth recontract model under different assumptions.

6.1 Reciprocal Demand Curves

It has been mentioned that Edgeworth included in his diagram (1881, p.28) the reciprocal demand curve, or offer curve, of each individual, although such curves were then called 'demand-and-supply curves'. Edgeworth mentioned them only briefly in the text (1881, p.39), but the lack of emphasis understandable, since in imperfect competition they are not relevant. Edgeworth's contribution was to provide the basic 'analytics' of the offer curve in terms of indifference curves, whereby it is 'the locus of the point where lines from the origin touch curves of indifference' (1881, p.113).

When there is a lack of competition, giving rise to indeterminacy, there is nothing to ensure that individuals will trade on their offer curves and, as Edgeworth argued, 'the conceptions of demand and supply at a price are no longer appropriate' (1881, p.31). It is this general preference, in favour of the analysis of barter in non-competitive situations, to which Marshall objected and which led to the controversy discussed below.

7 The Utilitarian Calculus

Having shown how indeterminacy can be removed by increasing the number of traders, Edgeworth turned to consider the role of arbitration in resolving the conflict between traders, in a 'world weary of strife' (1881, p.51). The principle of arbitration examined was, not surprisingly, the utilitarian principle, which Edgeworth had earlier used to examine the optimal distribution. However, the new context of indeterminacy led him to a deeper justification of utilitarianism as a principle of distributive justice. Having arrived at this new link between 'impure' (egoistic) and 'pure' utilitarianism, Edgeworth had only to reorientate his earlier analysis of optimal distribution, contained in his paper in *Mind* of 1879.

The need for arbitration with indeterminacy had been stated by Jevons as follows:

> The dispositions and force of character of the parties ... will influence the decision. These are motives more or less extraneous to a theory of economics, and yet they appear necessary considerations in this problem. It may be that indeterminate bargains of this kind are best arranged by an arbitrator or third party. (1957, pp. 124-5)

Edgeworth's statement of the same point was as usual rather less prosaic: 'The whole creation groans and yearns, desiderating a principle of arbitration, and end of strifes' (1881, p.51). Edgeworth argument involved two steps. First, he showed that the principle of utility maximisation places individuals on the contract curve, because the first-order conditions are equivalent to the tangency of indifference curves.

> It is a circumstance of momentous interest that one of the in general indefinitely numerous settlements between contractors is the utilitarian arrangement ... the contract tending to the greatest possible total utility of the contractors. (1881, p.53)

Edgeworth recognized that this result was not sufficient to justify the use of utilitarianism as a principle of arbitration. It is only a necessary condition of a principle of arbitration that it should place the parties somewhere on the contract curve. Edgeworth's justification for utilitarianism as a principle of justice, comparing points along the contract curve, was as follows:

> Now these positions lie in a reverse order of desirability for each party; and it may seem to each that as he cannot have his own way, in the absence of any definite principle of selection, he has about as good a chance of one of the arrangements as another ... both parties may agree to commute their chance of any of the arrangements for ... the utilitarian arrangement. (1881, p.55)

The important point to stress about this statement is that Edgeworth clearly viewed distributive justice in terms of choice under uncertainty. He argued that the contractors, faced with uncertainty about their prospects, would choose to accept an arrangement along utilitarian lines. A crucial component of this argument, also clearly stated by Edgeworth in this quotation, is the use of equal a priori probabilities.

The importance to him of this new justification of utilitarianism cannot be exaggerated. Indeed the whole of *Mathematical Psychics* seems to be imbued with a feeling of excitement generated by his discovery of a justification based on a 'social contract'. This provided the crucial link between 'impure' and 'pure' utilitarianism in a more satisfactory way than his earlier appeal to evolutionary forces.

Edgeworth believed that he had provided an answer to an age old question, stating, 'by what mechanism the force of self-love can be applied so as to support the structure of utilitarian politics, neither Helvetius, nor Bentham, nor any deductive egoist has made clear' (1881, p.128). Nevertheless this argument was neglected until re-statements along similar lines were made by Harsanyi (1953, 1955) and Vickrey (1960). The maximization of expected utility, with each individual taking the a priori view that any outcome is equally likely, was shown to lead to the use of a social welfare function which maximizes the sum of individual utilities. This approach is now usually described as 'contractarian neo-utilitarianism'.

In discussing the utilitarian solution as a principle of arbitration in indeterminate contract, Edgeworth did not clearly indicate in 1881 that the utilitarian solution of maximum total utility could specify a position which makes one of the parties worse off than in the no-trade situation. This was nevertheless later made explicit when, after proposing arbitration along utilitarian lines, he added, 'subject to the condition that neither should lose by the contract' (1925, ii, p.102). This possibility of course depends largely on the initial endowments of the individuals.

8 Later Work in Economics

After the publication of *Mathematical Psychics*, Edgeworth concentrated increasingly on mathematical statistics, in particular on the problem of statistical inference, but following his appointment to the Drummond Chair at Oxford, Edgeworth again made important contributions to economics although this work mainly involved reactions to, and discussions arising from, the later work of other authors.

8.1 Demand and Exchange

In the *Principles of Economics* (1890, Appendix F) Marshall included a brief discussion of Edgeworth's analysis of barter, and produced a figure showing the contract curve. During the following year, in the course of a review written in Italian, Edgeworth criticized Marshall for not having dealt sufficiently with the problem of indeterminacy. The basic problem was that Marshall, using a model in which a series of trades are allowed to take place at disequilibrium prices, believed he had shown that prices will eventually settle at the price-taking equilibrium. However, the argument was not transparent. The adjustment process involves moving from the initial endowment point in a series of trades, where trading at 'false' prices is allowed at each step. The process must conclude with both individuals at a point on the contract curve. A feature of the process is the assumption that each stage or iteration

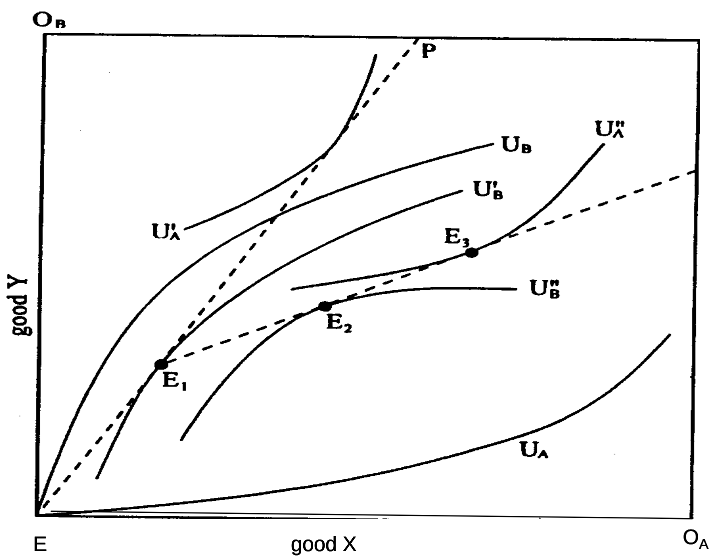

Figure 4: Disequilibrium Trades

of the sequence involves Pareto improvements: individuals trade only if it makes them better off. Furthermore, it involves trading at the 'short end' of the market, that is, the minimum of supply and demand. This arises from the impossibility of forcing any individual either to buy or sell more than desired at any price.

An example of two disequilibrium trades is shown in Figure 4, where the endowment moves from E to E_1, and then to E_2. With a price line represented by EP, there is an excess supply of good X as person A tries to reach the indifference curve U'_A and person B wishes to reach U'_B. Trade takes place at E_1, the short-end of the market. Point E_1 then becomes the new endowment point. At the second trading stage, the price of X must be lowered to induce person B to purchase more. At a price represented by the line E_1P_1 through the new endowment point, the excess supply is lower than formerly and trade takes place at E_2. Comparing U'_A and U'_B with U''_A and U''_B respectively, it can be seen that E_2 is a Pareto-improvement relative to E_1. It is also clear that person A is better off, the slower is the fall in the price of X relative to Y at each stage.

The combination of Pareto-efficient moves at each stage, combined with an adjustment process such that an excess supply leads to a price reduction, and vice versa, produces a stable process that converges to an equilibrium somewhere on the contract curve.[7]

The basic problem was that Marshall believed that his assumption of an additive utility function, combined with the assumption that the marginal utility of one good is constant for both individuals, guaranteed a determinate price, if the good having constant marginal utility was money. Indeed, this case was mentioned by Edgeworth (see 1925, ii, p.317, n.1). The contract curve is a straight line parallel to the y axis (where this good is the one with constant marginal utility), along which the rate of exchange is constant. So the equilibrium price does not depend on the sequence of trades. However, Edgeworth's point was that the total amount spent on good x remains indeterminate.

There was a later, though much milder, disagreement between Marshall and Edgeworth over the so-called Giffen good. In a book review, Edgeworth argued that, 'even the milder statement that the elasticity of demand for wheat may be positive, though I know it is countenanced by high authority, appears to me so contrary to a priori probability as to require very strong evidence (1909, p.104). The 'authority' was of course Marshall (1890, p.132), who replied directly to Edgeworth that, 'I don't want to argue ... But ... the matter has not been taken quite at random' (Pigou, 1925, p.438). Marshall gave a numerical example involving a journey travelled by two methods, where the distance travelled by the cheaper and slower method must increase when its price increases.[8]

It has been mentioned that Edgeworth introduced the generalized utility function. An implication is that it allows for complementarity, although Edgeworth did not explicitly consider this in 1881. The first formal definition of complementarity is attributed to Auspitz and Lieben, and it was used by Edgeworth in his paper on the pure theory of monopoly, and also by Pareto: this amounts to what is now called 'gross' complementarity, defined in terms

[7] This type of sequence of disequilibrium trades was later used by Launhardt; see Creedy (1994b).

[8] For further details, see Creedy (1990).

of cross-price elasticities. It is also sometimes referred to, using the initials of the four people mentioned above, as ALEP complementarity.

The first major criticism came from Johnson (1913), who pointed out that the criterion was not invariant with respect to monotonic transformations of the utility function. His treatment was extended by Hicks and Allen (1934), so that the modern definition involves 'net' complements in terms of compensated price changes. There is no symmetry between gross substitutes and complements as only the matrix of (compensated) substitution elasticities is assumed to be symmetric.

8.2 Monopoly and Oligopoly

In a paper first published in Italian in 1897, and not translated until the collected *Papers* (1925), Edgeworth examined several problems relating to monopoly. He began his discussion with Cournot's (1838) example of the 'source minerale' in which there are 'two monopolists' (that is, duopolists), each owning a spring of mineral water. It would be natural for Edgeworth to expect an indeterminate price in this 'small numbers' context. Cournot had arrived at a determinate solution for price and output, but Edgeworth showed that, 'when two or more monopolists are dealing with competitive groups, economic equilibrium is indeterminate' (1925, p.116). The daily output from each spring was assumed to be limited to identical fixed amounts, delivery costs were zero and all consumers had the same demand curve (purchasing one unit only of output). Hence demand is $n(1-p)$ where n is the number of customers and p is the price. Cournot's solution was that the price would be $p = 1/4$, but Edgeworth argued that one of the 'monopolists' had an incentive to raise the price back to $p = 1/2$, which is the revenue maximising price, so that there is not a determinate price. He argued that:

> at every stage . . . it is competent to each monopolist to deliberate whether it will pay him better to lower his price against his rival as already described, or rather to raise it to a higher . . . for that remainder of customers of which he cannot be deprived by his rival. . . . Long before the lowest point has been reached,

that alternative will have become more advantageous than the course first described' (1925, p.120)

Edgeworth went on to say, 'the matter may be put in a clearer light', and he then defined what are now called the reaction curve and isoprofit lines (in that order) for variations in prices. However, it was not until Bowley's (1924) discussion that these matters began to be presented in a more transparent manner.

Edgeworth then considered the case of complementary demand within the context of 'bilateral monopoly', where the two goods are demanded in fixed proportions for use in the production of a further article. An interesting feature is that he wrote the equations of the reaction curves and explicitly dealt with what are now called conjectural variations, reflecting the extent to which one duopolist is expected to change price in response to changes made by the second duopolist. In discussing this problem Edgeworth also introduced the further important concept of the 'saddle point', which he called the 'Hog's Back', clearly indicating its importance for stability.

8.3 The No-profit Entrepreneur

Walras (1874, p.225) had introduced the concept of the entrepreneur who neither gains nor loses. This result applied only to the competitive equilibrium, where there are no incentives for entrepreneurs to enter any industry. This does not of course mean that there are no profits, in the accounting sense, since the returns to homogeneous units of inputs of organization and management services are subsumed in the costs of the firm.

Edgeworth's criticisms of this concept of the no-profit entrepreneur, reproduced in his *Papers* (1925), recognized that with Walras's assumptions there was nothing illogical about the argument. The theory simply means that nothing remains, 'after the entrepreneur has paid a normal salary to himself' (1925, pp.26, 30). Furthermore, 'if [the general expenses] are taken into account, the argument becomes a fortiori. For why should not a substantial remuneration for the entrepreneur be included in the general expenses of the business' (1925, ii, p.469). Edgeworth's difference with Walras was to some extent 'only verbal', but he was also unhappy with the idea that entrepreneurship is homogeneous and divisible.

8.4 The Theory of Taxation

In the 1890s Edgeworth produced two surveys of considerable importance. These surveys, of the pure theory of taxation and of the pure theory of international values, were both published in the *Economic Journal* and subsequently reproduced (with alterations) in his *Papers* (1925, vol. ii). Each survey consisted of three separate parts, and displayed a staggering breadth of knowledge and command of the subject. They represent his most serious attempts to produce any kind of synthesis of a branch of economic literature. Edgeworth began his survey with the rather strong statement that, 'the science of taxation comprises two subjects to which the character of pure theory may be ascribed; the laws of incidence, and the principle of equal sacrifice' (1925, p.64). He then considered a variety of special cases and contexts of tax incidenc. The basic framework for incidence analysis was the simple partial equilibrium approach, still used in many basic textbooks, in which the incidence depends on the relative values of supply and demand elasticities.

The basic approach to incidence analysis actually stemmed from the important paper by Jenkin (1871). It suggests that in general the price of the taxed good will either remain constant (in the extreme case of inelastic supply) or will increase. However, this result ignores interrelationships among commodities. Edgeworth showed that when such interrelationships are explicitly allowed, there are some circumstances in which the price of the taxed good will actually fall. When discussing this 'paradox', Edgeworth reproduced his argument which had in fact been explored in more detail in his paper on monopoly, published in Italian in the same year, 1897. Edgeworth first stated his 'tax paradox' in the following terms:

> when the supply of two or more correlated commodities – such as the carriage of passengers by rail first class or third class – is in the hands of a single monopolist, a tax on one of the articles – e.g. a percentage of first class fares – may prove advantageous to

> the consumers as a whole. ... The fares for all the classes might be reduced. (1925, p.139)

Edgeworth regarded this result as an example of a situation where, 'the abstract reasoning serves as a corrective to what has been called the "metaphysical incumbus" of dogmatic laisser faire' (1925, i, p.139; see also 1925, ii, pp.93-4). Essentially the two commodities must be substitutes in consumption and production, and the result is partly brought about by the fact that the monopolist has an incentive to increase the supply of the untaxed commodity. Edgeworth also recognized that the result could occur in competitive markets (see 1925, p.63). As with many of Edgeworth's original results, this tax paradox was not a subject of continuous development. Its main practical importance perhaps arises from the fact that in the early 1930s it attracted the attention of Hotelling (1932).[9]

The section of the taxation survey which attracted most immediate attention was Edgeworth's discussion of the various 'sacrifice' theories of the distribution of the tax burden, and his qualified support for progressive taxation. Edgeworth's attitude to taxation was similar to that of the major classical economists in that he rejected a benefit approach, on the argument that taxation is not an economic bargain governed by competition. Thus in his view the problem was to determine, 'the distribution of those taxes which are applied to common purposes, the benefits whereof cannot be allocated to particular classes of citizens' (1925, p.103). A principle of justice is thus required. His approach can be seen as marking a crucial stage in the transition towards a 'welfare economics' view of public finance, rather than using a special set of 'tax maxims' such as the famous criteria laid down by Adam Smith.

Not surprisingly, Edgeworth (1925, p.102) argued along neo-contractarian lines set down in *Mathematical Psychics* that the utilitarian arrangement would be accepted by individuals uncertain of their own prospects and taking an equal a priori view of the probabilities. He suggested that, 'each party may reflect that, in the long run of various cases ... of all the principles of

[9] For further discussion of the paradox, see Creedy (1988).

distribution which would afford him now a greater, now a smaller proportion of the sum-total utility obtainable ... the principle that the collective utility should be on each occasion a maximum is most likely to afford the greatest utility in the long run to him individually'. Having established the use of utilitarianism as a principle of distribution justice, Edgeworth then succinctly stated the main argument:

> The condition that the total net utility procured by taxation should be a maximum then reduces to the condition that the total disutility should be a minimum ... it follows in general that the marginal disutility incurred by each taxpayer should be the same. (1925, p.103)

The implication is that if all individuals have the same cardinal utility function, after-tax incomes would be equalized. Edgeworth also clearly recognized that if there is considerable dispersion of pre-tax incomes relative to the total amount of tax to be raised, where there is, 'not enough tax to go around' (1925, ii, p.103), the equi-marginal condition cannot be fully satisfied unless there is a 'negative income tax' which raises the incomes of the poorest individuals to a common level. Thus, 'the acme of socialism is for a moment sighted' (1925, p.104). But Edgeworth immediately considered the practical limitations to such high progressive taxation. The following quotation illustrates one of Edgeworth's favourite metaphors, his respect for Sidgwick, his attitude to authority, his views on utilitarianism and the applicability of pure theory, and of course his unmistakable style:

> In this misty and precipitous region let us take Professor Sidgwick as our chief guide. He best has contemplated the crowning height of the utilitarian first principle, from which the steps of a sublime deduction lead to the high tableland of equality; but he also discerns the enormous interposing chasms which deter practical wisdom from moving directly towards that ideal. (1925, p.104)

Among the various limitations, Edgeworth noted differences in individual utility functions, population effects, the disincentives to work, growth of culture and knowledge, savings, and of course the problem of evasion.

8.5 International Trade

Edgeworth's survey of the pure theory of international values was in some ways responsible for a change of emphasis in the approach to trade theory, despite the fact that it contained few original analytical contributions. Indeed, he said that, 'Mill's exposition of the general theory is still unsurpassed' (1925, p.20), and acknowledged further that, 'what is written ... after a perusal of [Marshall's] privately circulated chapters ... can make no claim to originality' (1925, p.46). Edgeworth saw trade theory as an application of the general theory of exchange:

> The fundamental principle of international trade is that general theory ... the Theory of Exchange ... which ... constitutes the 'kernel' of most of the chief problems in economics. It is a corollary of the general theory that all the parties to a bargain look to gain by it ... This is the generalised statement of the theory of comparative cost. (1925, p.6)

Thus the gains from trade are analogous to the gains from exchange in simple barter and, 'It is useful ... to contemplate the theory of distribution as analogous to that of international trade proper' (1925, p.19). Hence trade theory is to Edgeworth simply one more application of the general method of *Mathematical Psychics*. In directly applying the theory of exchange to that of trade, Edgeworth was quite content to use community indifference curves without clearly specifying how aggregation might be carried out. He said only that, 'by combining properly the utility curves for all the individuals, we obtain what may be called a collective utility curve' (1925, p.293).

One of Edgeworth's criticisms of Mill (1848) was that the latter took as his measure of the gain from trade the change in the ratio of exchange of exports against imports. Thus Mill in this case 'confounds "final" with integral utility' (1925, p.22). The same point had in fact been made by Jevons (1957, pp. 154-6). However, Edgeworth, while preferring total utility, admitted that Mill was not otherwise led to serious error in using his own measure.

Edgeworth's survey was, as always, extremely wide ranging, though for later developments the most interesting parts are concerned with his elucidation of Mill's 'recognition of the case in which an impediment may be beneficial – or an improvement prejudicial – to one of the countries' (1925, p.9). These cases would now be discussed under the headings of the 'optimal tariff' and 'immiserising growth'. In the case of an optimal tariff, a country acts as monopolist and imposes a price which enables that country to attain its highest indifference curve, subject to the other country's offer curve. However, this position is not on the contract curve. The detailed specification of the optimum tariff in terms of elasticities had to wait until Bickerdike (1906), Pigou (1908) and the later revivals of interest in the 1940s. Edgeworth's judgement of Bickerdike was that he had, 'accomplished a wonderful feat. He has said something new about protection' (1925, ii, p.344).

Edgeworth could not of course be expected to support the use of such tariffs in practice. He acknowledged the possibility of retaliation, but also:

> For one nation to benefit itself at the expense of ... others is contrary to the highest morality ... But in an abstract study upon the motion of projectiles in vacuo, I do not think it necessary to enlarge upon the horrors of war. (1925, p.17, n. 5)

The 'highest morality' was, of course, the principal of utilitarianism.

9 Conclusions

It has been seen that Edgeworth did not begin working and writing in economics until his mid thirties, but in common with the majority of neoclassical economists he soon pursued an academic career as a professor of economics. Indeed, in a period which saw the rapid and widespread professionalisation of the subject, Edgeworth held an academic position in England that was regarded as second only to that of Alfred Marshall. In spite of his wide range of reading and sympathies, Edgeworth's work was characterized by the fact that it was virtually all addressed to his fellow professional economists. So uncompromising was he in his view that economics is a very difficult subject offering only remote and nearly always negative policy advice, that it may fairly be said that his work was addressed to just a small number of 'fellow travellers' in the rarefied atmosphere of the 'higher regions' of pure theory. However, Edgeworth imposed no geographical limitations, and with his considerable linguistic skills and international sympathies was in contact with the majority of leading economists around the world.

The distinguishing feature of the neoclassical 'revolution' was its emphasis on exchange as the central economic problem. The success of this shift of focus from production and distribution to exchange was closely associated with the fact that it had as its foundation a model based on utility maximisation. This allowed for a deeper treatment of the gains from exchange and the wider considerations of economic welfare. Schumpeter summarised the point by stating that utility analysis must be understood in terms of exchange as the central 'pivot' and 'the whole of the organism of pure economics thus finds itself unified in the light of a single principle' (1954, p.913). This is indeed the context in which Edgeworth's work in economics must be seen. Schumpeter's remark is merely a more prosaic expression of Edgeworth's view quoted above that '"Méchanique Sociale" may one day take her place along with "Méchanique Celeste", throned each upon the double-sided hieght of one maximum principle'. The central theme of Edgeworth's work is also clear in his revealing statement, taken from his Presidential address to Section F of the Royal Society, that:

> It may be said that in pure economics there is only one fundamental theorem, but that is a very difficult one: the theory of bargain in a wide sense. (1925, ii, p.288)

This perspective helps the major thread which runs through all Edgeworth's work in economics to be seen. His earlier mathematical analysis of the implications of utilitarianism for the optimal distribution, before turning to economics, was not only highly original (and esoteric) but laid the foundation for his work in economics. Thus, the transition from *New and Old Methods of Ethics* to *Mathematical Psychics* was not a shift in major preoccupations but rather a change of emphasis. Distribution was then seen as an important concomitant of exchange, so that the analysis of contract became for Edgeworth central. Edgeworth's emphasis on the indeterminacy (the inability of utility maximisation alone to determine the rate of exchange, only a range of efficient exchanges) which results from the existence of a small number of traders led him to his pathbreaking analysis of the role of numbers in competition, along with the efficiency properties of competitive equilibria.

The analysis of the utilitarian objective as an arbitration rule led Edgeworth directly to his new 'social contract' argument in explaining the acceptance of utilitarianism as a principle of social justice. It was the realisation of this new justification of utilitarianism, using his newly developed analytical tools, which generated the excitement that is clearly evident in his first work in economics. While *Mathematical Psychics* developed the techniques of indifference curves and the contract curve within the 'Edgeworth box' – tools which are now ubiquitous in economic analysis – Edgeworth himself was clearly driven mainly by his ability to link the analysis of private contracts in markets to that of a social contract in which utilitarianism is the 'sovereign principle'. The integration of his analysis of barter, and the effects of the introduction of additional traders into the market, with the demonstration that the utilitarian arrangement prescribes a point on the contract curve of efficient exchanges and is acceptable to risk-averse traders, was to Edgeworth nothing short of 'momentous'.

The results are of course highly abstract. In discussing their ultimate value suggested that:

> Considerations so abstract it would of course be ridiculous to fling upon the flood-tide of practical politics ... it is at a height of abstraction in the rarefied atmosphere of speculation that the secret springs of action take their rise, and a direction is imparted

> to the pure foundation of youthful enthusiasm whose influence will ultimately affect the broad current of events. (1881, p.128)

The intellectual pleasure derived from being able to draw together so many different subjects of analysis, and strands of his enormous range of learning, is clearly evident. However, it is precisely this wide field of vision, combined with the technical level and idiosyncratic style of writing, which made *Mathematical Psychics* so difficult for his contemporaries, and which continue to make the book seem so strange and yet so rewarding to the modern reader.

Selected Publications by Edgeworth

[1] Edgeworth, F. Y. (1876) Mr. Matthew Arnold on Bishop Butler's doctrine of self love. *Mind*, 1, 570-1.

[2] Edgeworth, F. Y. (1877) *New and Old Methods of Ethics: or 'Physical Ethics' and 'Methods of Ethics'*. Oxford: Parker.

[3] Edgeworth, F. Y. (1879) The hedonical calculus. *Mind*, 4, 394-408.

[4] Edgeworth, F. Y. (1881) *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences*. London: Kegan Paul.

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