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Dynamic Location Games

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Abstract

We study a location game where consumers are distributed according to some density f and where market entry is costly and occurs sequentially. This permits an endogenous determination of the number of active firms, their locations and the sequence in which these locations are occupied. While in general the analysis of such games is complicated by the fact that equilibrium locations and the sequence of settlement must be determined simultaneously, we show that they can be independently derived for certain classes of densities including monotone and, under some additional restrictions, hump-shaped and U-shaped ones. For these classes we characterize the subgame perfect equilibrium outcome. Moreover, when f is monotone and concave the equilibrium locations in areas where the density is larger tend to be more profitable. When f is uniform the number of firms entering in equilibrium is minimal.

JEL-Code: D43, L13, D21

Keywords: Spatial competition, product differentiation, dynamic games, entry deterrence.

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1 Introduction

Explaining the determinants of product differentiation has a long tradition in economics. Theoretical research typically focuses on firms' location patterns in product space and on how these depend on the market environment. Recent empirical research aims at assessing the impact of the size of different groups of consumers on firms' choices of product characteristics. For example, should a firm cater larger (or economically more important) groups of consumers where it also expects fierce competition, or is it better off by targeting remote areas of the product spectrum where competition tends to be less intense?

The canonical framework for analyzing these issues is due to Hotelling (1929). The theoretical literature on Hotelling models can broadly be separated into three categories. First, there are the location-cum-price models where two exogenously given firms first choose locations and then prices (e.g. Hotelling, 1929; d'Aspremont, Gabszewicz, and Thisse, 1979; Anderson, Goeree, and Ramer, 1997). Second, there are the static models in which an exogenously given number of players simultaneously choose locations only (e.g. Lerner and Singer, 1937; Downs, 1957; Eaton and Lipsey, 1975; Osborne, 1995). Common to both types of models is that they are not easily amendable to the analysis of entry decisions.[1] The desire to study market entry has led Prescott and Visscher (1977, PV hereafter) to introduce a third variant of location models, to which we refer as dynamic location games, where firms enter sequentially, bear a fixed setup cost and cannot relocate once they have chosen their locations.

Almost exclusively, the theoretical literature studies models with uniformly distributed consumers. The notable exception among location-cum-price models with an exogenously given number of firms is Anderson, Goeree, and Ramer (1997).[2] As for sequential location-only models, Palfrey (1984) and Callander (2005) are the only exceptions we are aware of where

[1]This is trivially true for the first approach. As for the second, it is well-known that difficulties such as non-existence of pure strategy equilibria may arise (see e.g. Eaton and Lipsey, 1975).

[2]They note (p.101) that "one assumption, which is clearly unrealistic, has been left virtually untouched by the tools of theorists. This is the condition that consumers are uniformly distributed...".

two incumbent players are concerned with deterring entry by a third one, where the underlying distributions are non-uniform, but symmetric.[3]

From a theoretical perspective, this focus on uniform distributions is not satisfactory as some of the most interesting issues cannot be tackled. For example, whether firms should first enter where there are many consumers and whether areas with more consumers are more intensively catered by firms necessarily requires a departure from the uniform distribution. Empirically, the focus on the uniform case is even more problematic since there is by now ample evidence for non-uniform consumer preferences.[4]

Relaxing the uniform assumption in dynamic location games is the point of departure of this paper. Specifically, we extend the framework of PV by considering larger classes of densities. In a first step we develop the necessary concepts to analyze equilibrium behavior in this generalized framework. Using these concepts, we then derive some general equilibrium properties which do not qualitatively depend on the underlying distribution. A full equilibrium characterization (i.e. how many firms enter at which locations and in which order) of this seemingly simple game is non-trivial because in general, equilibrium locations cannot be determined without knowing the sequence of settlement, i.e. the sequence in which locations are occupied, which in turn depends on the equilibrium locations themselves. However, a first contribution of our paper is that we show that for certain classes of densities, equilibrium locations and the equilibrium sequence of settlement can be independently determined. This makes the equilibrium characterization tractable. In particular, we show that the subgame perfect equilibrium locations are *independent* of the sequence of settlement when the density is (i) monotonically

[3]In Loertscher and Muehlheusser (2008b), a fixed number of players enter in distinct markets, where the distribution across markets may be non-uniform.

[4]Waldfogel (2003), George and Waldfogel (2003), George and Waldfogel (2006) and Waldfogel (2007) provide evidence that preferences for media products differ vastly across ethnic groups so that aggregate distributions will be generically non-uniform. Similarly, consider the market for pharmaceutical drugs, where consumers' "preferences" are given by the prevalence of various diseases. Since some diseases are rare and others very prevalent in any given population, the distribution of consumer preferences will typically not be uniform.

increasing or decreasing, (ii) hump-shaped and satisfies an additional joint condition on the entry cost and the density and (iii) U-shaped and satisfies an additional symmetry condition around its minimum.

Our model exhibits the following, intuitive comparative statics properties. First, larger markets, or equivalently, markets with lower fixed costs, attract more entry and generate more product variety. This is clearly consistent with the available empirical evidence.[5] Second, firms locate closer to each other in more densely populated segments of the product spectrum, which reflects the findings in Anderson, Goeree, and Ramer (1997, p.111) that "tight density functions are a force of agglomeration". Thus, consumers with similar preferences exert positive externalities on each other. The existence and significance of such preference externalities is empirically well documented (see e.g. George and Waldfogel, 2003; Waldfogel, 2003, 2007). Third, despite the fiercer competition, equilibrium profits in these segments tend to be larger than those in less densely populated areas. Therefore, somewhat loosely speaking, firms should first enter where there are many consumers, despite the fiercer competition this involves. Fourth, we discuss an extension of the model where firms are allowed to operate multiple outlets. We show that this extension is without consequences in that the equilibrium locations in a model with only single-outlet firms are also equilibrium locations in a model with multi-product firms. In other words, there is no product proliferation in the sense that collusive and competitive outcomes coincide under the threat of entry. Therefore, the paper also contributes to the classical debates on product proliferation (see e.g. Schmalensee, 1978; Bonanno, 1987) and on the effect of horizontal mergers on product variety (see e.g. Berry and Waldfogel, 2001; Federal Communications Commission, 2001; Sweeting, 2008).

Moreover, our analysis sheds new light on the uniform case by showing that it is rather special in many important respects. First, it is the distribution that induces the smallest number

[5]See e.g. Berry and Waldfogel (1999) and Waldfogel (2003) for media markets or Hsieh and Moretti (2003) for real estate brokers.

of active firms in equilibrium.[6] Second, as observed by PV the uniform distribution exhibits a multiplicity of subgame perfect equilibria, both with respect to locations and the sequence of settlement. This indeterminacy is unique to the uniform case insofar as the equilibrium locations and the sequence of settlement are generically pinned down for the families of densities we consider. Third, the multiplicity of equilibria under the uniform led PV to focus on a particular, symmetric equilibrium, where the sequence of settlement occurs from outside in. We show that the resulting equilibrium locations are the ones that arise when the uniform distribution is considered as the limit case of a symmetric hump-shaped density. While this result gives some justification for PV's equilibrium selection, the equilibrium sequence of settlement tends to be from inside out rather than outside in.[7]

Throughout, we abstract from consumer price competition after locations have been chosen. The main reason for doing so is analytical tractability. The difficulties arising in models with location-cum-price competition are well known even for the uniform case with two exogenously given firms (see e.g. d'Aspremont, Gabszewicz, and Thisse, 1979). Here they are exacerbated not only by the fact that the distribution is non-uniform, but mainly by the inevitable problem that one needs to know the pricing equilibrium with three or more active firms. To the best of our knowledge this problem has not yet been solved and therefore, we confine attention to pure location games in this paper.[8]

Of course, the extent to which our focusing on location choices is a good approximation to real world markets is an empirical question. One natural application of our model are media

[6]This mirrors another result in the locations-cum-price model with two exogenously given firms of Anderson, Goeree, and Ramer (1997, p.105,125), namely that the uniform puts an upper bound on the degree of equilibrium product differentiation.

[7]More generally, PV's claim (see their Footnote 5) that their outside-in principle would also be appropriate for non-uniform distributions finds no support for the classes of densities we consider.

[8]In addition, it is well-known that models with price competition are very sensitive to the functional form of consumers' preference costs (see e.g. d'Aspremont, Gabszewicz, and Thisse, 1979). In contrast, our results do not depend on whether these costs are, for example, linear or quadratic; all we need is that they are symmetric and monotone in distance.

markets, with firms being newspapers, radio stations or TV broadcasters.[9] Our reading of the relevant empirical literature here is that location choice in product space is indeed of first order importance while prices are not.[10] Similarly, in on-air radio and TV broadcasting consumers are not charged any direct prices.[11]

Apart from media markets, our model is also applicable to markets where prices are administered or where consumption decisions are made largely irrespective of prices. Pharmaceutical products are natural examples of such markets since prices are typically regulated by the government. Moreover, in many countries consumers only pay a small fraction of the price of the drugs they consume due to mandatory or private health insurance plans. Another industry where price competition is essentially absent is real estate brokerage. Brokers do not set prices, but rather commission fees that are levied on the transaction price. Empirically, these fees exhibit almost no variance across markets and over time (see e.g. Hsieh and Moretti, 2003). Thus, competition between brokers will mainly be in product location choices. Similarly, retailers who face binding retail price maintenance contracts will by and large behave as price taking firms who compete in location choice.

Media firms have most recently received a lot of attention in the two-sided markets literature. If one assumes that media firms do not compete for consumers by setting prices, then

[9] Price competition is also naturally absent when considering product differentiation in the context of political economy (see e.g. Downs, 1957; Palfrey, 1984; Osborne, 1995; Callander, 2005).

[10] For example, in their empirical analysis of the effect of the New York Times on local newspapers' content, George and Waldfogel (2006) abstract completely from price competition. Indeed, when comparing, say, the New York Times to the New York Post, the crucial distinguishing feature tends to be their (different) locations in product space, and not price differences. Moreover, there is remarkably little variation in prices over time or across outlets and markets. According to George and Waldfogel (2000), 75% of general interest newspapers in the U.S. were sold at 50 cent per copy in 2000. This is in stark contrast with the variance in newspaper circulation.

[11] Another question in the media context concerns advertisements where firms do compete in prices and which is often by far the most important source of revenue for media firms (see e.g. George and Waldfogel, 2000). Borrowing ideas from Anderson and Coate (2005), we show in Appendix B that the model with media firms who compete first for consumers and then for advertisement revenue is easily amendable to price competition for advertisements.

our paper also relates to this literature; see e.g. Gabszewicz, Laussel, and Sonnac (2001), Rysman (2004), Anderson and Gabszewicz (2005), Anderson and Coate (2005), Rochet and Tirole (2006) or Ambrus and Reisinger (2006).

The remainder of the paper is organized as follows. Section 2 introduces the model. In Section 3 we develop the crucial concepts for the subsequent equilibrium analysis. Section 4 contains some general equilibrium properties. In Sections 5 and 6 we analyze monotone and non-monotone densities, respectively. In Section 7 we first discuss the uniform case in some detail, and then allow firms to locate at multiple locations. Section 8 concludes. All proofs are in Appendix A. In Appendix B, we provide an extension of the model where media firms compete in prices for advertisement revenue after choosing their locations.

2 The model

Consider a product market with a unit mass of consumers distributed along the $[0, 1]$-interval according to the cumulative distribution function $F(x)$ with density $f(x) > 0$ for all $x \in [0, 1]$. There is a large number of firms who can potentially enter the market. Firms move sequentially in an exogenously given order.[12] If firm i is given the move, it decides whether or not to enter the market. If it enters, it incurs a fixed cost $K > 0$ and chooses a location in $[0, 1]$. In either case, its decision is observed by all firms moving subsequently. Once a location is chosen, it is prohibitively costly to change it ex post.[13] Each consumer patronizes the closest firm. The profit of each active firm, gross of the entry cost K, is equal to the mass of consumers it attracts. To rule out trivial cases, we assume $K < 1/2$ such that the market can at least support two firms. Apart from the possibility that later entrants may face a less attractive choice set, no costs are associated with entering later. Finally, for convenience we assume that firms stay out when indifferent.

[12]For the uniform case, Anderson and Engers (2001) analyze a model where the order of entry is endogenous.

[13]Such costs may include physical relocation costs, or advertisement costs to change the brand image of a firm (see e.g. PV).

3 Concepts

In this section, we develop several concepts which are crucial for the following equilibrium analysis. We begin with the optimal location of a firm in a given interval under the assumption of no subsequent entry, and then turn to the issue of entry deterrence. Throughout the paper, we refer to an interval (L, R) as one where the points L and R are already **occupied** by competitors, and which is **empty** in the sense that no firm is located in its interior.

3.1 Optimal locations absent further entry

Consider a firm entering in an interval (L, R).[14] Under the assumption of *no further subsequent entry* in this interval, when entering at location $x \in (L, R)$, the firm's profit is

$$\pi(x, L, R) := F\left(\frac{x+R}{2}\right) - F\left(\frac{x+L}{2}\right).$$

That is, it attracts all customers between the midpoints between its own location and the locations of its competitors to the right and left, respectively. Note that the "reach" of the firm's customer base, or its market coverage, denoted by $\Delta(L, R)$, is simply half the interval length, and thus independent of x:

$$\Delta(L, R) := \frac{R-L}{2}.$$

So choosing an optimal location within a given interval (L, R) is equivalent to finding a location x that maximizes the integral over $\Delta(L, R)$.

Lemma 1 *For any location $x \in (L, R)$, $\pi(x, L, R)$ strictly decreases in L and strictly increases R. Moreover,*

$$\frac{\partial \pi(x, L, R)}{\partial L} = -\frac{1}{2} f\left(\frac{x+L}{2}\right) < 0 \quad \textit{and} \quad \frac{\partial \pi(x, L, R)}{\partial R} = \frac{1}{2} f\left(\frac{x+R}{2}\right) > 0.$$

[14] Note that while the interval (L, R) is open by definition, firms are not a priori prohibited to choose identical locations. As shown below, however, such behavior is inconsistent with equilibrium.

The lemma is obvious and requires no proof. It says that the profit of a firm, whose location is fixed at some $x \in (L, R)$, is strictly increasing in the distance to its closest competitors.

Some additional notation is useful:

Definition 1 *Denote by*

(i) $X^*(L, R)$ *the set of optimal locations in the interval* (L, R)*:*

$$X^*(L, R) := \arg \max_{x \in (L,R)} \pi(x, L, R).$$

An element of this set is denoted by $x^*(L, R)$.[15]

(ii) $\pi^*(L, R)$ *the firm's profit when locating optimally:*

$$\pi^*(L, R) := \pi(x^*, L, R) \quad \textit{for} \quad x^* \in X^*(L, R).$$

(iii) $\hat{X}(\underline{z}, \overline{z}, L, R)$ *the set of optimal locations in the interval* (L, R) *when the choice set is restricted to some interval* $[\underline{z}, \overline{z}] \subset (L, R)$*:*

$$\hat{X}(\underline{z}, \overline{z}, L, R) := \arg \max_{x \in [\underline{z}, \overline{z}]} \pi(x, L, R)$$

An element of this set is denoted by $\hat{x}(\underline{z}, \overline{z}, L, R)$.

(iv) $\hat{\pi}(\underline{z}, \overline{z}, L, R)$ *the firm's profit when choosing one of these optimal (restricted) locations:*

$$\hat{\pi}(\underline{z}, \overline{z}, L, R) = \pi(\hat{x}, L, R) \quad \textit{for} \quad \hat{x} \in \hat{X}(\underline{z}, \overline{z}, L, R).$$

(v) $L^+ := L + \epsilon$ *and* $R^- := R - \epsilon$ *for arbitrarily small* $\epsilon > 0$*, the smallest and the largest possible locations in* (L, R)*, respectively.*[16]

[15] For example for the uniform distribution, $X^*(L, R) = (L, R)$.

[16] Of course, since the interval (L, R) is open, L^+ and R^- are not well-defined in a continuous framework. We follow the standard notion in the literature where the continuous case emerges as the limit of a discrete choice set with "grid size" ϵ, where $\epsilon \to 0$.

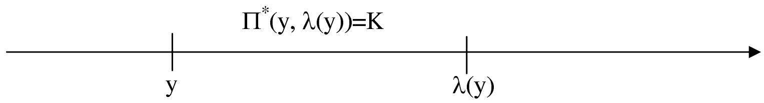

Figure 1: y and $\lambda(y)$.

Lemma 2 *Assume $x^*(L,R)$ is interior, satisfying $f\left(\frac{x^*+L}{2}\right) = f\left(\frac{x^*+R}{2}\right)$ and $f'\left(\frac{x^*+L}{2}\right) > 0 > f'\left(\frac{x^*+R}{2}\right)$. Then:*

$$-1 < \frac{\partial x^*}{\partial L} < 0 \quad \textit{and} \quad -1 < \frac{\partial x^*}{\partial R} < 0.$$

Intuitively, when $x^*(L,R)$ is determined by a first-order condition, and the left-hand neighbors comes closer, then it is optimal for the firm at $x^*(L,R)$ to also move in the direction of that neighbor, but less so. As a result, by moving right the firm at L would still gain costumers to its right if its right-hand neighbor at $x^*(L,R)$ is not already there.

Lemma 3 *$\pi^*(L,R)$ and $\hat{\pi}(\underline{z},\overline{z},L,R)$ strictly decrease in L and strictly increase in R.*

3.2 Entry-deterring locations

The following concepts are useful for addressing the issue of entry deterrence and for determining equilibrium configurations. A distinction has to be made between entry deterrence (i) with respect to an already occupied location inside (0,1), and (ii) with respect to one of the (in any equilibrium unoccupied) boundary points $\{0,1\}$ of the product spectrum.

Definition 2 *(i) Define $\lambda(y)$ and $\rho(y)$ such that*

$$\pi^*(y,\lambda(y)) = K \quad \textit{and} \quad \pi^*(\rho(y),y) = K,$$

for any occupied locations $y \in [0,\rho(1)]$ and $y \in [\lambda(0),1]$, respectively.

(ii) Let

$$\lambda_B := F^{-1}(K) \qquad \textit{and} \qquad \rho_B := F^{-1}(1-K).$$

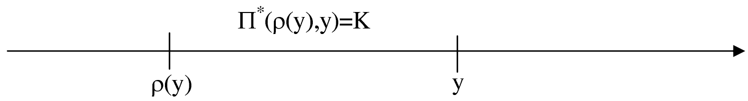

Figure 2: $\rho(y)$ and y.

Note first that $\lambda(\cdot)$ and $\rho(\cdot)$ also depend on K. As for part (i), consider Figures 1 and 2 for illustrations and note that $\lambda(y) > y$ and $\rho(y) < y$. Observe also that $\lambda(\cdot)$ is the inverse of $\rho(\cdot)$, i.e. $\rho(\lambda(y)) = y = \lambda(\rho(y))$.[17] Intuitively, with competitors located at y and $\lambda(y)$, an entrant would get exactly K when locating optimally in the interval $(y, \lambda(y))$ and, consequently, prefers not to enter. The intuition for $\rho(y)$ is analogous.

Because $\pi(x, L, R)$ strictly decreases in L and strictly increases in R for any $x \in (L, R)$, $\lambda(\cdot)$ and $\rho(\cdot)$ are unique. As will be shown below, for any occupied location y, $\lambda(y)$ is therefore the largest entry-deterring location to the right of y. Analogously, $\rho(y)$ is the smallest entry-deterring location to the left of y.

Part (ii) is an appropriate adaption of these definitions to unoccupied boundary points: If the left boundary point 0 is not occupied while a firm is located at λ_B, an entrant would just be deterred from entering in the interval $[0, \lambda_B)$.[18] An analogous argument applies to the right boundary point 1. Note that our assumption $K < \frac{1}{2}$ implies $\lambda_B < \rho_B$.[19]

Lemma 4 *(i) $\lambda(y)$ and $\rho(y)$ strictly increase in y.*

(ii) For any two occupied locations L, R with $L < R$,

$$\lambda(L) > R \Leftrightarrow \rho(R) < L \Leftrightarrow \pi^*(L, R) < K.$$

[17] Thus, $\lambda(y) \leq 1$ holds for all $y \leq \rho(1)$ and $\rho(y) \geq 0$ holds for all $y \geq \lambda(0)$.

[18] Note that an entrant's optimal location in this case would be λ_B^- for any distribution, since he gets the whole hinterland. Moreover, we have $\lambda_B < \lambda(0)$, because the only difference refers to whether or not the end point 0 is occupied. By definition of λ_B, this is not the case, and so a firm locating at some $x \leq \lambda_B$ gets the whole hinterland to the left of x. However, when the end point 0 is occupied as is the case by definition of $\lambda(0)$, this hinterland is shared with the firm at 0. Analogous reasoning establishes that $\rho(1) < \rho_B$.

[19] It is now clear why the case $K > \frac{1}{2}$ is trivial: either $\frac{1}{2} < K < 1$ (i.e. $0 < \rho_B < \lambda_B < 1$), so that the first firm would optimally enter somewhere in the interval $[\rho_B, \lambda_B]$ thereby forestalling further entry. Or $K \geq 1$ (i.e. $\rho_B \leq 0 < 1 \leq \lambda_B$), in which case the market could not even support one firm.

(iii) For any occupied locations $y \in [0, \rho(1)]$ and $y \in [\lambda(0), 1]$, respectively,

$$K < F(\lambda(y)) - F(y) \leq 2K \quad \text{and} \quad K < F(y) - F(\rho(y)) \leq 2K.$$

(iv) For y given, $\lambda(y)$ is increasing, and $\rho(y)$ is decreasing in K.

Entry-deterring locations when F is uniform In the uniform case, any location in a given interval (L, R) yields the same payoff of $\frac{F(R)-F(L)}{2} = \frac{R-L}{2}$. Thus, by definition of $\lambda(y)$, $\frac{\lambda(y)-y}{2} = K$ must hold, implying $\lambda(y) - y = F(\lambda(y)) - F(y) = 2K$. Analogously, $y - \rho(y) = 2K$ holds. It then follows from part (iii) of Lemma 4 that the mass of consumers between two entry deterring locations is maximum in the uniform case. As Theorem 9 below shows, this also implies that the equilibrium number of active firms will be minimum for the uniform distribution.

4 General equilibrium properties

The remainder of the paper characterizes subgame perfect equilibria, to which we simply refer as "equilibria". We first derive some general equilibrium properties, starting with an implication of Lemma 4:

Corollary 1 *Three occupied locations L, x, R, where $L < x < R$ are not consistent with equilibrium if*

$$\rho(R) \leq L < x < R \leq \lambda(L). \tag{1}$$

When (1) holds, then $\pi(x, L, R) \leq K$ for all $x \in (L, R)$ follows from Lemma 4. So the firm at location x could profitably deviate by staying out of the market.

4.1 Number of entrants in a given interval

Denote by # the number of firms entering in equilibrium in a given interval (L, R).

Theorem 1 *In any equilibrium,*

(i) $\# = 0$ *if* $\rho(R) \leq L < R \leq \lambda(L)$

(ii) $\# \in \{1, 2\}$ *if* $L < \rho(R) < \lambda(L) < R$

(iii) $\# \geq 2$ *if* $L < \lambda(L) < \rho(R) < R$.

Intuitively, in part (i) the market size in a given interval is too small to support profitable entry. As the market size increases, so that we are in the case described by part (ii), at least one entrant can profitably enter in the interval. Whether this first entrant optimally forestalls further entry or invites entry by one more firm depends on the distribution of consumers. As the market size increases even further, so that we are in the case described by part (iii), the first entrant can no longer deter further entry, so that in this case at least two firms enter.

4.2 Distance between neighboring firms

Apart from the number of firms entering in equilibrium in a given interval, we can also say something about distances between firms in any equilibrium. We refer to two firms at locations L and R as **neighbors** when the interior of the interval (L, R) is empty. We start with the following corollary of Theorem 1:

Corollary 2 *In any equilibrium, two firms at locations L and R are neighbors if and only if $\pi^*(L, R) \leq K$.*

Part (i) of Theorem 1 implies that there will be no further entry if $\pi^*(L, R) \leq K$, which, as will be recalled, is equivalent to $\rho(R) \leq L \Leftrightarrow R \leq \lambda(L)$. To see that there is entry if $\pi^*(L, R) > K$, observe first that $\pi^*(L, R) > K \Leftrightarrow \rho(R) > L \Leftrightarrow R > \lambda(L)$, for which case(s) parts (ii) and (iii) of Theorem 1 say there will be entry. Hence, L and R cannot be neighbors if $\pi^*(L, R) > K$. The import of Corollary 2 (and Theorem 1) is that there is entry in (L, R) whenever $\pi^*(L, R) > K$, so that there are no "black holes". That is, it cannot happen in equilibrium that a firm does not enter in an interval (L, R) because it correctly fears that it would subsequently not break even because of further entry whereas it would pay to enter in (L, R) were it the only entrant.

Theorem 2 *For any three neighboring equilibrium locations L, x, R satisfying $L < x < R$, the following condition must hold:*

$$\rho(x) \leq L < \rho(R) \leq x \leq \lambda(L) < R \leq \lambda(x). \tag{2}$$

Theorem 2 is a statement about distances between neighboring firms in any equilibrium: First, the *minimum distance* between the firms located at x and L must be strictly larger than $x - \rho(R)$. Otherwise, we would have $L > \rho(R)$ (i.e. when L is "too close"), then by Corollary 1, the firm at x does not break even. The same argument applies to the right-hand side when $R < \lambda(L)$.

Second, the *maximum distance* between the firm located at x and its neighbors at L and R, is $x - \rho(x)$ and $\lambda(x) - x$, respectively. Otherwise, by Corollary 2, there would be entry in between, contradicting that the firms at locations x and L (respectively x and R) are neighbors.

Recall that we do not a priori rule out the possibility that firms choose identical locations. However, the following implication of Theorem 2 establishes that this will not happen in equilibrium:

Corollary 3 *In any equilibrium, any location $x \in [0, 1]$ is occupied by at most one firm.*

4.3 Range of product variety

Let a be the leftmost and b be the rightmost location that is occupied in equilibrium. Then:

Theorem 3

$$a \leq \lambda_B \quad \textit{and} \quad b \geq \rho_B.$$

A natural question is whether it is possible to prove the substantially stronger statement that in any equilibrium, $a = \lambda_B$ and $b = \rho_B$. Generally however, one cannot exclude the

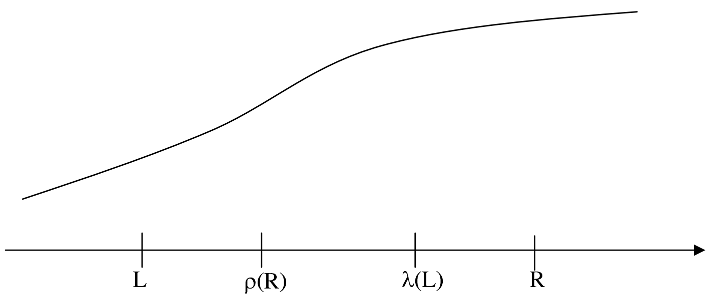

Figure 3: Optimal location and entry-deterrence for increasing f.

possibility that a firm might want to locate to the left (right) of λ_B (ρ_B) so as to induce its closest neighbor to locate substantially further away (which it might if the neighbor's location is given by a first order condition, see Lemma 2).

4.4 Equilibrium properties for quasiconcave densities

We end this section with a few properties that hold for the class of quasiconcave densities.

Lemma 5 *Let f be quasiconcave over an interval (L, R) and consider any $x^*(L, R)$ and any locations y, z.*

(i) If $L < x^*(L, R) < y < z \leq R$, then $\pi(y, L, R) \geq \pi(z, L, R)$ holds.

(ii) If $L \leq y < z < x^*(L, R) < R$, then $\pi(y, L, R) \leq \pi(z, L, R)$ holds.

Consider now an interval (L, R) satisfying $L < \rho(R) < \lambda(L) < R$. From Theorem 1 we know that at least one and at most two firm(s) will enter in this interval. The next result establishes that when f is quasiconcave, exactly one firm will enter:

Lemma 6 *Let f be quasiconcave over an interval (L, R) satisfying $L < \rho(R) < \lambda(L) < R$, exactly one additional firm will enter. This firm locates at $\hat{x} = \lambda(L)$ if f is increasing, at $\hat{x} = \rho(R)$ if f is decreasing and at $\hat{x} \in [\rho(R), \lambda(L)]$ if f is hump-shaped, in which case the exact location depends on the specifics of f, K, L and R.*

The result is illustrated in Figure 3 for the case where f is increasing. Note first again the difference between optimal locations in a given interval *absent* further entry, and optimal entry-deterring ones: Clearly, $x^*(L,R) = R^-$ which, however, would invite further entry. As a result, the first entrant optimally chooses the best entry-deterring location, $\hat{x}(\rho(R), \lambda(L), L, R) = \lambda(L)$, thereby earning profit $\hat{\pi}(\cdot) < \pi^*(\cdot)$. Note also that the entrant's optimal location $\lambda(L)$ only depends on the location of its left-hand neighbor L, but not on the location of its right-hand neighbor at R.

5 Monotone densities

5.1 Equilibrium locations

Let us now look at the cases where for all $x \in [0,1]$, f is strictly increasing and decreasing, respectively. Let $\lambda^0 \equiv \lambda_B$,

$$\lambda^1 \equiv \lambda(\lambda_B), \quad \lambda^2 \equiv \lambda(\lambda^1), \quad ... \quad , \lambda^{k+1} \equiv \lambda(\lambda^k),$$

$\rho^0 \equiv \rho_B$ and

$$\rho^1 \equiv \rho(\rho_B), \quad \rho^2 \equiv \rho(\rho^1), \quad ... \quad , \rho^{j+1} \equiv \rho(\rho^j).$$

Let $n \geq 0$ and $m \geq 0$ be the largest integers such that

$$\lambda^n < \rho_B \quad \text{and} \quad \lambda_B < \rho^m.$$

That such n and m exist and are unique follows from the monotonicity of $\lambda(\cdot)$ and $\rho(\cdot)$.[20] Throughout we focus on the generic cases where $\lambda^k \neq \rho_B$ and $\lambda_B \neq \rho^k$ for any integer k.

Theorem 4 *When f is monotone over $[0,1]$, the set of equilibrium locations is unique.*

[20]The restriction to strictly increasing (decreasing) functions can be easily relaxed since the analysis goes through if f is flat on some parts of the interval [0,1] as long as it increases (decreases) sufficiently often. To be precise, a sufficient condition is that in any interval $[\lambda^k, \lambda^{k+1}]$ there is an x such that f increases (decreases) at x. This will make sure that the best responses are unique.

(i) When f is increasing, $n+2$ firms enter at locations

$$\{\lambda_B, \lambda^1, .., \lambda^n, \rho_B\}.$$

(ii) When f is decreasing, $m+2$ firms enter at locations

$$\{\lambda_B, \rho^m, .., \rho^1, \rho_B\}.$$

As for part (i), the optimal location of firm i is driven solely by the location of its left-hand neighbor, at location y say. As for the right-hand neighbor, either it is already there in which case, because f is increasing, firm i optimally moves to the right as far as possible without inviting further entry to its left, which is at $\lambda(y)$. When i anticipates its right-hand neighbor to be a subsequent entrant, then a fortiori $\lambda(y)$ is optimal for firm i, because its future right-hand neighbor will also optimally locate at $\lambda(\lambda(y))$, so that i pushes this firm as far to the right as possible.

It follows that it is never optimal for a firm *not* to locate at a λ-location (except for location ρ_B, of course), independent of whether its neighbors to the left and right (who will also optimally locate at λ-locations) are already there or will be future entrants. As a result, for monotone densities *the set of equilibrium locations is independent of when these locations are occupied.* Non-monotone densities will typically not have this property: In general, the set of equilibrium locations will depend on the sequence in which these locations are taken and vice versa, which renders the equilibrium analysis much more complicated.

Theorem 4 also implies that the distance between equilibrium locations becomes smaller as the density increases. Hence, the more densely populated a segment of the product spectrum, the more product variety will emerge in equilibrium in this segment.

Preference Externalities The present framework also exhibits what has become known as “preference externalities” (see e.g. Waldfogel, 2003; George and Waldfogel, 2003) in a very concise and natural way. To see this, assume in line with the empirical evidence presented in these studies that preferences differ according to ethnic background, so that, say, a decreasing (increasing) density describes the distribution of preferences in a population consisting of Hispanics (Whites). Because firm locations are closer together where the density is high, Hispanics have on average lower preference costs in the predominantly Hispanic area than in the predominantly White area and vice versa.

5.2 Sequence of Settlement

In general, for a given distribution of consumers and order of entry, the sequence of settlement is pinned down, where the earlier entrants grasp the larger profits, except when two or more equilibrium profits are the same. Without additional information about f, however, it is generally not possible to determine the full ordering of equilibrium profits. Nonetheless, some results can be derived under fairly general conditions. In the following, we denote by $\pi(y)$ the equilibrium profit of a firm at equilibrium location y.

Recall from Theorem 4 that when f is increasing on $[0, 1]$, the two rightmost equilibrium locations are λ^n and ρ_B, respectively. Analogously, when it is decreasing, the two leftmost equilibrium locations are λ_B and ρ^m, respectively:

Theorem 5 *(i) If f is increasing, then*

$$\pi(\rho_B) > \pi(\lambda^n),$$

such that location ρ_B will be occupied prior to location λ^n in any equilibrium.

(ii) If f is decreasing, then

$$\pi(\lambda_B) > \pi(\rho^m),$$

such that location λ_B will be occupied prior to location ρ^m in any equilibrium.

Under the additional restriction that f is concave, we can also say something about the ordering of the profits of the firms locating in $[0, \lambda^{n-1}]$ when f is increasing and of those locating in $[\rho^{m-1}, 1]$ when f is decreasing.

Theorem 6 *(i) If f is increasing and weakly concave, then*

$$\pi(\lambda^{n-1}) > ... > \pi(\lambda_B)$$

holds, i.e the settlement of $\{\lambda_B, ..., \lambda^{n-1}\}$ occurs from the right to the left.

(ii) If f is decreasing and weakly concave, then

$$\pi(\rho^{m-1}) > ... > \pi(\rho_B)$$

holds, i.e. the settlement of $\{\rho^{m-1}, ..., \rho_B\}$ occurs from the left to the right.

Despite the fact that distances between neighboring firms become smaller as the density increases, firms still tend to prefer these locations to those in less densely populated segments of the product spectrum.[21] Note that concavity of f is only a sufficient condition. The result would also hold if f were somewhat convex, but not too much so. Equally or even more importantly, the theorem holds for linear densities. In particular, it holds for any density with slope $f'(x) = \varepsilon$, where $\mid \varepsilon \mid \neq 0$ is arbitrarily small. Consequently, for any such density the order of settlement will be generically unique (i.e. for almost every K). This contrasts sharply with the uniform case, where the sequence of settlement is indeterminate since all, but at most three, equilibrium locations are equally profitable (see also Section 7.1). Note also that an outside-in principle as claimed by PV (p. 385) does not apply for strictly monotone densities: as just shown, the sequence of settlement tends to be either from right to left (for f increasing) or from left to right (for f decreasing). Moreover, as shown in Theorem 4, the determination of the equilibrium locations themselves is driven by a sequence of λ- or ρ-functions. These are defined with respect to λ_B or ρ_B, depending on whether f is increasing or decreasing and so this process either works from left to right or from right to left, but not outside-in in the sense of PV.

[21] A potential exception is location λ^n, since its right-hand neighbor is at ρ_B and thus not determined by a λ-distance. For K large and n fixed, this distance, and the resulting profit to the right of λ^n become very small, in which case it is a rather unattractive location.

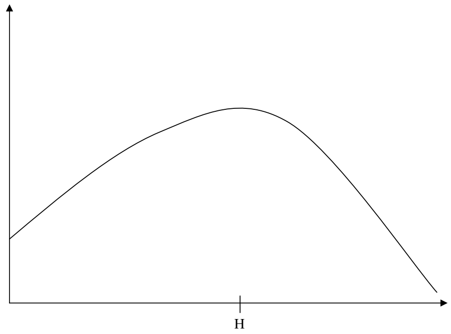

Figure 4: Hump shaped f

6 Non-monotone densities

6.1 Hump shaped densities

Consider now hump-shaped quasiconcave density functions such as depicted in Figure 4 which exhibit a single hump at location H, and are thus increasing for all $x < H$, and decreasing for all $x > H$. Moreover, let $r \geq 0$ be such that $f(\lambda^r)$ is still increasing while $f(\lambda^{r+1})$ is decreasing. Analogously, let $s \geq 0$ be such that $f(\rho^s)$ is decreasing and $f(\rho^{s+1})$ is increasing.

Two cases emerge, depending on whether or not profitable entry is possible in the interval (λ^r, ρ^s) (see Figures 5 and 6): In the first case, the locations λ^r and ρ^s are entry-deterring, while in the second case they are not.[22]

It turns that the second case is substantially more complicated, because the equilibrium locations and sequence of settlement can no longer be determined separately. Intuitively, for a given number of entrants, the location "under the hump" tends to be the more profitable, the lower K.[23] Therefore, when K is sufficiently low, the first firm will indeed optimally locate under the hump. As a result, since the density is increasing in the direction of the hump, future entrants (the neighbors of the first firm, in particular) will optimally locate as close as possible without inviting further entry to their other side (i.e. they choose some λ- or ρ-location as in

[22]From the definitions of $\lambda(\cdot)$, $\rho(\cdot)$, and part (ii) of Lemma 4, it follows that $\rho^{s+1} < \lambda^r \Leftrightarrow \rho^s < \lambda^{r+1}$ holds in the first case, and $\lambda^r < \rho^{s+1} \Leftrightarrow \lambda^{r+1} < \rho^s$ in the second.

[23]Of course, whether or not a given location is attractive or not is endogenous to the game, which only goes to further stress the complications that arise.

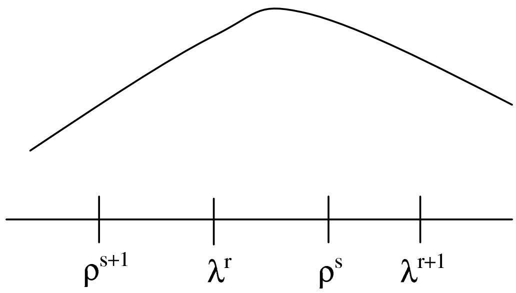

Figure 5: Interval (λ^r, ρ^s) entry-deterring (case I)

the case where f is monotone, see Theorem 4).

On the other hand, when K is large, the location under the hump will only be occupied at a later stage. In this case, earlier entrants might have an incentive to depart from these λ- and ρ-locations in order to push future entrants further away (including the one locating under the hump). As shown in Lemma 2, this is possible when optimal locations are given by a first-order condition. As a result, even when holding fixed the number of active firms, both, the equilibrium locations and the order of settlement may differ for different values of K.

In the present paper, we focus on the first case for which the equilibrium outcome can be characterized by combining our previous result for monotone densities:[24]

Theorem 7 *Let f be hump-shaped satisfying $\rho^{s+1} < \lambda^r$. Then the unique set of equilibrium locations is*

$$\{\lambda_B, .., \lambda^r, \rho^s, .., \rho_B\},$$

so that $r + s + 2$ firms enter.

If, in addition, f is symmetric in the sense that $f(x) = f(1-x)$ for any $x \in [0,1]$, the theorem yields the equilibrium locations whenever the number of entrants is even. The reason is that, for f symmetric, an even number of entrants means "no entry under the hump", i.e. $r = s \equiv n$ and $\rho^{n+1} < \lambda^n$, so that the number of entrants is $2n + 2$.

[24] We refer the interested reader to a companion paper (Loertscher and Muehlheusser, 2008a), which is exclusively devoted to the second case.

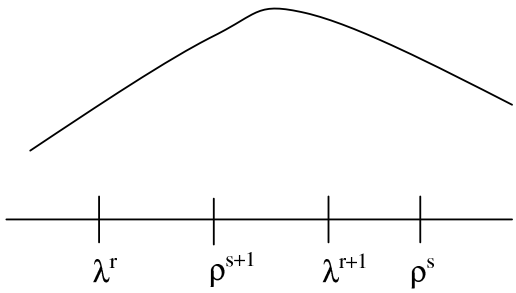

Figure 6: Interval (λ^r, ρ^s) not entry-deterring (case II)

Given Theorem 6, an albeit incomplete characterization of the sequence of settlement is also at hand:

Corollary 4 *Let f be hump-shaped quasiconcave satisfying $\rho^{s+1} < \lambda^r$. Then, the sequence of settlement of the locations $\{\lambda_B, .., \lambda^{r-1}\}$ occurs from the right to the left, and the sequence of settlement of the locations $\{\rho^{s-1}, .., \rho_B\}$ occurs from the left to the right.*

The result follows directly from Theorem 6 and requires no separate proof. The characterization is incomplete because without additional information on f it is not possible to (i) say anything about the profits at locations λ^r and ρ^m and (ii) rank the locations to the left relative to those on the right. Note that the sequence of settlement is again not "outside-in" in the sense of PV but, essentially, "inside-out".

6.2 U-shaped densities

Consider now densities which are U-shaped around some trough location M, i.e. f is decreasing for all $x < M$ and increasing for all $x > M$ (see Figure 7).

Lemma 7 *When f is U-shaped over the interval (L, R), then $x^*(L, R) \in \{L^+, R^-\}$.*

Lemma 7 has the following corollary:

Corollary 5 *The minimum point M is never occupied in equilibrium.*

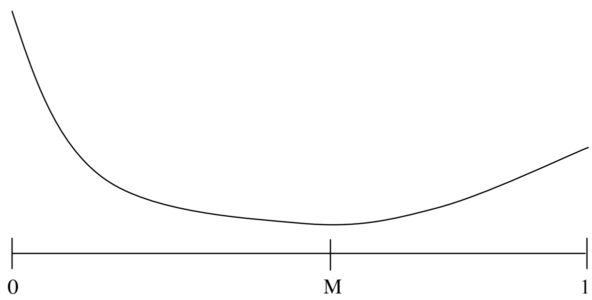

Figure 7: U-shaped f

Determining the full set of equilibrium locations when f is U-shaped is considerably simplified by imposing the following symmetry condition. Denote by ρ_M and λ_M the analogs to ρ_B and λ_B with respect to the trough M when M is not occupied. That is, $F(M) - F(\rho_M) = K$ and $F(\lambda_M) - F(M) = K$.

Definition 3 *The distribution F is **trough-symmetric** if*

$$F(\lambda_M) - F(M) = F(M) - F(2M - \lambda_M).$$

Observe that trough-symmetry implies that ρ_M and λ_M are at the same distance from M, i.e. $\lambda_M - M = M - \rho_M$. Notice also that any distribution that is symmetric around M is trough-symmetric. Next, let

$$\rho_M^1 \equiv \rho(M), \quad \rho_M^2 \equiv \rho(\rho_M^1), \quad \text{and} \quad \rho_M^{j+1} \equiv \rho(\rho_M^j)$$

and

$$\lambda_M^1 \equiv \lambda(M), \quad \lambda_M^2 \equiv \lambda(\lambda_M^1), \quad \text{and} \quad \lambda_M^{k+1} \equiv \lambda(\lambda_M^k).$$

Finally, let s and n be such that

$$\rho_M^{s+1} < \lambda_B < \rho_M^s \quad \text{and} \quad \lambda_M^n < \rho_B < \lambda_M^{n+1}.$$

Clearly, n and s will only be positive integers when the fixed cost K is sufficiently small, which we are assuming for the remainder of this section.

Theorem 8 *Let F be trough-symmetric. Then in equilibrium, $r+s+4$ firms enter at locations*

$$\{\lambda_B, \rho_M^s, .., \rho_M^1, \rho_M, \lambda_M, \lambda_M^1, .., \lambda_M^n, \rho_B\}.$$

Intuitively, trough-symmetry ensures that the midpoint between firms at ρ_M and λ_M is at the trough M. By definition of ρ_M and λ_M, each firm gets profit K in this interval. Moreover, from Lemma 7, $x^*(\rho_M, \lambda_M) = \{\rho_M^+, \lambda_M^-\}$ so that an entrant optimally locating in-between would also just reap K and thus prefers not to enter.

As for the sequence of settlement, it follows directly from Theorem 5 that location λ_B (ρ_B) will be occupied prior to location ρ_M^s (λ_M^n). Moreover, when in addition f is concave on each of its two branches, Theorem 6 also applies:

Corollary 6 *Let f be trough-symmetric and concave over the intervals $[0, M]$ and $[M, 1]$, respectively. Then, the sequence of settlement of the locations $\{\lambda_M^{n-1}, .., \lambda_M\}$ occurs from the right to the left, and the sequence of settlement of the locations $\{\rho_M^{s-1}, .., \rho_M\}$ occurs from the left to the right.*

Again, the relative profitability of the locations on each branch of f depends on the exact specification. Since the sequence of settlement is, broadly speaking, from the outside in, it is in accordance with PV's claim.

7 Discussion

7.1 Uniform distribution

PV analyze a special case of the present model where f is uniform. They focus on (subgame perfect) equilibria where $2n+2$ firms enter at locations $\{K, .., (2n+1)K, 1-(2n+1)K, .., 1-K\}$ for $n \geq 1$. Using our notation, this can be equivalently written as[25]

$$\{\lambda_B, \lambda^1, .., \lambda^n, \rho^n, .., \rho^1, \rho_B\}.$$

[25] Recall that for the uniform case, $\lambda_B = K$, $\rho_B = 1 - K$, $\lambda(y) = y + 2K$ and $\rho(y) = y - 2K$.

They argue that the equilibrium sequence of settlement for these locations is from outside in, and if the number of active firms is odd, a final entrant enters at 1/2. An interesting question is therefore whether these equilibrium locations and the sequence of settlement emerge as the limit case of the cases previously analyzed.[26]

Monotone densities Define

$$f_\varepsilon^m(x) \equiv 1 - \varepsilon/2 + \varepsilon x$$

for all $x \in [0,1]$ and $\varepsilon \in [-2,2]$. Note that $f_\varepsilon^m(x)$ is an affine function that increases in x if $\varepsilon > 0$ and decreases if $\varepsilon < 0$ and that converges to the uniform density as $\varepsilon \to 0$. Because $f_\varepsilon^m(x)$ is an affine function, it is also concave. Therefore, we know that for $\varepsilon > 0$, the equilibrium locations will be $\{\lambda_B, \lambda^1, .., \lambda^n, \rho_B\}$ and the sequence of settlement will tend to occur from right to left, while for $\varepsilon < 0$, the equilibrium locations are $\{\lambda_B, \rho^m, .., \rho^1, \rho_B\}$ and the sequence of settlement is from left to right.[27] So neither the equilibrium locations nor the equilibrium sequence of settlement correspond to those of PV.

Hump-shaped densities As discussed after Theorem 7, we can solve for the equilibrium locations with hump-shaped, symmetric densities when the number of entrants is even and thus equal to $2n+2$.[28] As will be shown, PV's equilibrium locations, though not their sequence of settlement, can be obtained as the limiting case of the following hump-shaped density: Let

[26]In the non-generic cases where $\frac{1}{K}$ is an even number, $\frac{K}{2}$ firms enter indeed at locations $\{K, 3K, ..., \frac{1}{2}, ..., 1-3K, 1-K\}$. But since all firms then earn a profit of $2K$, the order of sequence is indeterminate, and so PV's outside-in sequence is only one out of many possible sequences.

[27]It follows from Theorem 4 that n and m are (weakly) increasing respectively decreasing in ε. Thus, for ε sufficiently large (in absolute terms), there will be more entry compared to the uniform case. However for ε sufficiently close to zero (which is the case of interest in our context) the number of active firms is the same.

[28]The two conditions for the number of active firms being equal to $2n+2$ are (i) $\lambda^n < \rho^n$ and (ii) $\rho^{n+1} < \lambda^n$. This leads to the following condition with respect to K:

$$\frac{1}{4n+4} < K < \frac{1}{4n+2},$$

where the first inequality follows from (ii) and the second from (i).

$f^h_\varepsilon(x) \equiv 1 - \varepsilon/4 + \varepsilon x$ for $x \in [0, 1/2]$ and $f^h_\varepsilon(x) \equiv 1 + 3\varepsilon/4 - \varepsilon x$ for $x \in (1/2, 1]$ and $\varepsilon \in [0, 4]$ be the symmetric hump-shaped density with constant slopes.[29]

Before we can state the limit result as $\varepsilon \to 0$, we must make sure this limit is well defined insofar as the number of entrants remains at $2n + 2$. Theorem 9 below implies that no fewer firms will enter under f^h_ε if $2n + 2$ firms enter under the uniform. However, we also need to verify that $2n + 2$ firms enter for all $\varepsilon \in [0, \varepsilon_0]$ if $2n + 2$ firms enter for some ε_0:

Lemma 8 *If $\rho^{n+1} < \lambda^n$ holds for some $\varepsilon_0 > 0$, then it also holds for any $\varepsilon \in [0, \varepsilon_0]$.*

Lemma 8 implies a monotonicity property. As ε increases from 0 to some positive number ε_0, the equilibrium number of entrants increases monotonically (and weakly). So if the number of active firms under the uniform is $2n + 2$, this will also be true for some ε_0. This allows us to take the limit $\varepsilon \to 0$, starting from ε_0. From Theorem 7, as ε approaches zero, the equilibrium locations will indeed be the ones derived by PV. This partially corroborates the equilibrium PV focus on. It does so only partially because the equilibrium sequence of settlement is, broadly speaking, from inside out, rather than from outside in.[30]

U-shaped densities Last, consider the density $f^h_\varepsilon(x)$ for $\varepsilon \in [-4, 0]$. This is a two sided triangle distribution that is symmetric around 1/2 and hence trough-symmetric. Therefore, from Theorem 8, the equilibrium locations are determined from the minimum of $f^h_\varepsilon(x)$, i.e. from 1/2. Since each branch of the density is again a concave function, it follows from Corollary 6 that the sequence of settlement is, broadly speaking, from the outside in, which is in accordance with PV, but the equilibrium locations are not.

We conclude our discussion of the uniform case with the following result on the number of active firms in equilibrium for the uniform case:

[29] Note also that for $\varepsilon = 4$, $f^h_\varepsilon(x)$ is the triangle distribution. If $\varepsilon < 0$, $f^h_\varepsilon(x)$ is U-shaped (or V-shaped) and has a trough at 1/2. For $\varepsilon = -4$ it is the (trough symmetric) two-sided triangle.

[30] To be precise, the relative profitability of the locations λ^n and ρ^n will depend on ε, and so the sequence of settlement may not be strictly from inside out. However, as shown in Corollary 4, all equilibrium locations to the left (right) of λ^n (ρ^n) are the more profitable the closer they are to λ^n (ρ^n).

Theorem 9 *The number of active firms in equilibrium is minimum when F is uniform.*

For an intuition, consider the uniform density as the limit case of a monotonically increasing density $f_\varepsilon^m(x)$ which makes sure that all best replies are uniquely pinned down. Now consider a firm locating at some R to the right of a firm located at L, where R is interior in the sense that its righthand neighbor will not be ρ_B. Since R^- is a best location in the interval (L, R), and indeed for any $\varepsilon > 0$ it will be the unique best location, it follows that when locating at $R = L + 2K$ a firm can get exactly K to its left without attracting further entry in (L, R). Moreover, and perhaps more importantly, by doing so, which is in its very best interest, the firm locating at R generates profits of also exactly K to the right of L for the firm at L. Iterating the argument once more, it follows that the firm at R will get K to its right as well and so gets $2K$ in total, which is the upper bound on the equilibrium profit (part (iii) of Lemma 4). Thus, in the equilibrium of the uniform case, all active firms except the ones at λ^n and ρ_B reap the maximum share of the overall industry profit. Therefore, the smallest number of active firms is supported in equilibrium.

7.2 Multiple outlet firms

Consider a variant of our model where firms can operate multiple outlets so that, when given the move, each firm can choose to occupy as many locations as it wants to. Does this modification have any effect on the set of equilibrium locations and therefore on the available product variety consumers can enjoy?

Assume for simplicity that the density is monotone and that the cost per outlet is K for each firm. Then, the first firm will optimally occupy the *same* set of locations as derived in Theorem 4 for the single-outlet case, thereby monopolizing the market. Intuitively, these locations are essentially determined by firms' concern of deterring further entry; whether a firm is a single- or a multi-product firm does not affect this concern. Since this set of locations forestalls further entry with the minimal number of outlets, it is the optimal choice for the first entrant.

Using this insight, our analysis contributes to a classic antitrust debate, where Bonanno (1987) argued against Schmalensee (1978) that entry deterrence will not necessarily lead to product proliferation, but may rather involve product locations that differ from those absent the concern of deterring entry. In our framework, whether locations are chosen competitively or collusively does not affect the set of locations, provided there is a threat of entry. However, since absent the threat of entry a monopoly would open only one outlet because of the fixed setup cost and the full market coverage assumption (i.e. all consumers would consume the good if it is the only one supplied), the model entails product proliferation in the sense of Schmalensee (1978).

The extended model with multi-product firms can also capture the effects of horizontal mergers. Our model's predictions are consistent with the hypothesis that the equilibrium locations of two outlets do not depend on whether or not they are owned by the same firm.[31] In this respect, our model is in line with empirical findings in the context of mergers of radio stations presented by Federal Communications Commission (2001), but contrasts with Berry and Waldfogel (2001). However, as noted in Federal Communications Commission (2001, Footnote 15), empirical results seem to crucially depend on how music formats are defined (see also Sweeting, 2008).

8 Conclusion

In this paper, we study dynamic location games, in which firms enter sequentially and pay a fixed cost upon entry. Our analysis focuses on the impact of the underlying distribution of consumer preferences on the subgame perfect equilibrium outcome, i.e. the number of active

[31] This statement needs to be slightly qualified as equilibrium locations need no longer be unique with multi-product firms. For example, whenever three neighboring locations are occupied with outlets owned by the same firm, this firm is indifferent with respect to the location of the outlet in the middle (provided it still deters entry) since the firm's total profit does not depend on the location of the middle outlet. Similarly, if, say, the two rightmost locations are occupied by outlets of the same firm, the location of the rightmost outlet is indeterminate as long as it forestalls further entry.

firms, their locations and the sequence of settlement.

We show that for certain classes of densities, the equilibrium locations and the equilibrium sequence of settlement are independent. In particular, this is true for monotone densities for which case the equilibrium outcome is characterized. Under some further conditions, independence is shown to hold also for non-monotone densities (hump- and U-shaped), in which case the equilibrium number of firms and their locations are readily determined by combining our results for monotone densities.

Our framework exhibits the intuitive features that larger markets attract more entry and that areas with higher density attract more firms. Thereby, it gives rise to what has become known as preference externalities in a natural and concise way. If densities are monotone and concave, the equilibrium sequence of settlement tends to begin with the locations with high density despite the fiercer competition these locations entail. By and large, equilibrium locations with little density are occupied last.

We also show that the optimal (entry-deterring) locations that are chosen by single-outlet firms are also optimal locations chosen by a monopolistic firm that operates multiple outlets and faces the threat of entry. Last, but not least, the uniform distribution, on which the previous theoretical literature has almost exclusively focused, has the special feature of inducing the minimum numbers of firms to enter in equilibrium.

One avenue for further research seems particularly promising: We have assumed throughout that though entry occurs sequentially, the only cost of late entry is that profitable locations are occupied first. An interesting modification would be to consider a multi-period model, where in every period one firm may enter, but payoffs accrue to every active firm in every period. This would add a trade-off between short-term and long-term profits, which seems empirically relevant. From a short-term perspective, the most attractive locations are those where the closest neighbors are far away or where the density of consumers is large. However, these are also the locations that are prone to attract additional entrants in the future.

Appendix

A Proofs

A.1 Proof of Lemma 2

The total differential of the first order condition $f\left(\frac{x^*+L}{2}\right) = f\left(\frac{x^*+R}{2}\right)$ with respect to L is

$$\frac{1}{2}f'\left(\frac{x^*+L}{2}\right)\left(\frac{\partial x^*}{\partial L}+1\right) = \frac{1}{2}f'\left(\frac{x^*+R}{2}\right)\frac{\partial x^*}{\partial L}.$$

This is equivalent to

$$\frac{\partial x^*}{\partial L} = \frac{-f'\left(\frac{x^*+L}{2}\right)}{f'\left(\frac{x^*+L}{2}\right) - f'\left(\frac{x^*+R}{2}\right)} < 0$$

since $f'\left(\frac{x^*+L}{2}\right) > 0 > f'\left(\frac{x^*+R}{2}\right)$. That it is larger than minus one follows from the fact that $-f'\left(\frac{x^*+R}{2}\right) > 0$. Analogously, the total differential with respect to R is

$$\frac{1}{2}f'\left(\frac{x^*+L}{2}\right)\frac{\partial x^*}{\partial R} = \frac{1}{2}f'\left(\frac{x^*+R}{2}\right)\left(\frac{\partial x^*}{\partial R}+1\right)$$

which is equivalent to

$$\frac{\partial x^*}{\partial R} = \frac{f'\left(\frac{x^*+R}{2}\right)}{f'\left(\frac{x^*+L}{2}\right) - f'\left(\frac{x^*+R}{2}\right)} < 0.$$

That it is larger than minus one follows from the fact that $f'\left(\frac{x^*+L}{2}\right) > 0$. ■

A.2 Proof of Lemma 3

In the proof, we confine attention to π^* as the arguments regarding $\hat{\pi}$ are completely analogous. The proof for the reaction of $\pi^*(L,R)$ to changes in L and R relies on a revealed preference argument: Fix some $x^*(L,R) \in X^*(L,R)$ and suppose that the competitor to the left moves to some $L' > L$. We have to consider two cases.

Case 1: $x^*(L,R) \in X^*(L',R)$. From Lemma 1, it follows directly that $\pi(x^*(L,R),L',R) < \pi^*(L,R)$.

Case 2: $x^*(L,R) \notin X^*(L',R)$. To see that $\pi^*(L',R) < \pi^*(L,R)$ holds, suppose otherwise that $\pi^*(L',R) \geq \pi^*(L,R)$. By definition of $x^*(L,R)$, however, $\pi^*(L,R) \geq \pi(x^*(L',R),L,R)$.

Therefore, if the first inequality holds, then so does

$$\pi^*(L', R) \geq \pi(x^*(L', R), L, R).$$

But this is a contradiction to Lemma 1. Completely analogous arguments apply to changes of R. ■

A.3 Proof of Lemma 4

Part (i) Suppose, for notational simplicity, that optimal locations are unique. By definition, when locating at $x^*(y, \lambda(y))$, an entrant gets K. When the firm to the left is instead located at some $y' > y$, $\pi^*(y', \lambda(y)) < K$ follows from Lemma 3. This Lemma also implies that $\pi^*(y', \lambda(y')) = K$ can hold only if $\lambda(y') > \lambda(y)$. A completely analogous argument establishes that $\rho(\cdot)$ is also increasing in y.

Part (ii) By construction $\lambda(\rho(R)) = R$ and by part (i) $\lambda(y)$ increases in y. Hence, $\rho(R) < L$ implies $\lambda(L) > R$. That this implies $\pi^*(L, R) < K$ follows from Definition 2.

Part (iii) $F(\lambda(y)) - F(y) > K$ and $F(y) - F(\rho(y)) > K$ follows trivially from the definition of $\lambda(\cdot)$ and $\rho(\cdot)$. The remainder of the proof for the statement with respect to $\lambda(y)$ relies on the fact that

$$F(\frac{\lambda(y)+y}{2}) - F(y) \leq K \quad \text{and} \quad F(\lambda(y)) - F(\frac{\lambda(y)+y}{2}) \leq K. \tag{3}$$

To see this, suppose to the contrary that $F(\frac{\lambda(y)+y}{2}) - F(y) > K$. Then an entrant could locate at y^+ and get $\pi(y^+, y, \lambda(y)) = F(\frac{\lambda(y)+y}{2}) - F(y) > K$ which contradicts the definition of $\lambda(\cdot)$. An analogous argument establishes the second part of (3). But now (3) implies $F(\frac{\lambda(y)+y}{2}) - F(y) + F(\lambda(y)) - F(\frac{\lambda(y)+y}{2}) = F(\lambda(y)) - F(y) \leq 2K$. The proof for the statement with respect to $\rho(y)$ is completely analogous.

Part (iv) By definition, when locating at $x^*(y, \lambda(y))$, an entrant gets K. When K increases to $K' > K$, the set of optimal locations does not change, and thus $\pi^*(y, \lambda(y)) < T'$ holds. Thus, by Lemma 3, for a given y, $\pi^*(y, \lambda(y)) = K'$ can hold only if $\lambda(y)$ increases. A completely analogous argument establishes that $\rho(y)$ decreases in K. ■

A.4 Proof of Theorem 1

Recall first that because of the symmetry of $\lambda(\cdot)$ and $\rho(\cdot)$, the cases $\rho(R) < L < \lambda(L) < R$ and $L < \rho(R) < R < \lambda(L)$ cannot occur. So only the three cases stated in the theorem need to be considered.

Part (i) By definition of $\lambda(L)$ and $\rho(R)$, and from Corollary 1, profitable entry in the interval (L, R) is not possible in this case, and thus no firm will enter in equilibrium.

Part (ii) Label subsequent entrants by $i, i+1, i+2, \ldots$. We show that a) at most two firms enter in equilibrium, and b) at least one enters.

a) At most two firms enter

If the first entrant i enters at some $x_i \in [\rho(R), \lambda(L)]$, then by definition of $\lambda(\cdot)$ and $\rho(\cdot)$, there will be no further entry in the interval $[L, R]$. So consider the case where $x_i \notin [\rho(R), \lambda(L)]$, and suppose $x_i \in (L, \rho(R))$. The case $x_i \in (\lambda(L), R)$ is completely analogous and thus omitted. By Corollary 1, if subsequently $i+1$ enters, it must enter at some $x_{i+1} > \lambda(L)$: For $x_{i+1} \in (L, x_i]$, firm $i+1$ itself would incur a loss, for $x_{i+1} \in (x_i, \lambda(L)]$, firm i would do so. For two firms to enter, it therefore has to be the case that one, say i, locates at $x_i < \rho(R)$ and the other one at $x_{i+1} > \lambda(L)$. But now a third firm cannot profitably enter because at least one of the firms would not break even. This follows again from Corollary 1: For $x_{i+2} \in (L, x_i)$ or $x_{i+2} \in (x_{i+1}, R)$, firm $i+2$ does not break even, for $x_{i+2} \in (L, \lambda(L))$, i does not break even, and for $x_{i+2} \in (\lambda(L), x_{i+2})$, firm $i+1$ does not break even.

b) At least one firm enters

Three cases have to be considered:

Case 1: There is a $x^*(L, R) \in [\rho(R), \lambda(L)]$. In this case, the first entrant chooses this location, thereby preventing further entry. Moreover $\pi^*(L, R) > K$ since $L < \rho(R) < \lambda(L) < R$.

Case 2: There is no $x^*(L,R) \in [\rho(R), \lambda(L)]$ but $\hat{\pi}(\rho(R), \lambda(L), L, R) > K$. In this case, at least one firm will enter since $\hat{x}(\rho(R), \lambda(L), L, R)$ is a profitable and entry-deterring location. Whether one or two firms enter depends on whether the first firm i prefers an alternative location, thereby inducing subsequent entry, to $\hat{x}(\rho(R), \lambda(L), L, R)$ and thereby deterring entry.

Case 3: $\hat{\pi}(\rho(R), \lambda(L), L, R) \leq K$. Observe first that this implies $x^*(L, \lambda(L)) < \rho(R)$ and $x^*(\rho(R), R) > \lambda(L)$. We need to show that at least one firm enters, assuming equilibrium behavior by firms moving subsequently. That is, we have to show that there exists some $x_i \in (L, R)$ such that i's profit at x_i exceeds K if all subsequent firms play optimally. Let i occupy the location $x^*(L, \lambda(L))$. Observe first that there will be no subsequent entry to the left of firm i, because by Corollary 1, for any location $y \in (L, x_i)$, $\pi(y, L, x_i) < K$ holds. A necessary condition for i not to break even at $x^*(L, \lambda(L))$ is therefore that (at least) one other firm, say, $i+1$ enters to its right at some $x_{i+1} \leq \lambda(L)$. Only in this situation will i be "trapped" inside the $[L, \lambda(L)]$ interval (Corollary 1). So assume $x_{i+1} \leq \lambda(L)$. But for $i+1$ to enter at x_{i+1} in equilibrium, it must be the case that $i+1$ earns more than K either by deterring further entry or by "pushing" any subsequent entrant far enough to the right. But if $i+1$ earns more than K at x_{i+1} with $x_i > L$ to its left, then i could have chosen the location x_{i+1} itself, whereby it would have earned strictly more than $i+1$ now does. Therefore, i can guarantee itself a profit that is larger than K. Consequently, at least on firm will enter in equilibrium.

Part (iii) The proof relies on the validity of the following claim:

Claim: At least one firm can profitably enter either in the interval $(L, \lambda(L))$ or in the interval $(\rho(R), R)$.

We prove the claim for the case where the first entrant i enters in the interval $(L, \lambda(L))$, for the other one it is completely analogous. Suppose the first entrant i locates at $x_i = x^*(L, \lambda(L))$. Since $x^*(L, \lambda(L)) < \lambda(L)$, there will be no more entry to the left of x_i (by Corollary 1). Let the closest firm to the right of firm i be firm $i+1$ at some location x^0_{i+1}: If $x^0_{i+1} > \lambda(L)$, then $\pi(x^*(L, \lambda(L)), L, x^0_{i+1}) > K$.

Thus, as above, the critical case is $x_{i+1}^0 \leq \lambda(L)$ such that firm i would not break even (again by Corollary 1). Note that firm $i+1$ would choose such a position only if $\pi(x_{i+1}^0, x_i, x_{i+2}) > K$ where $x_{i+2} > \lambda(x_i)$ is the closest firm to the right of firm $i+1$. But then, firm i could itself locate at $x_i = x_{i+1}^0$ and earn $\pi(x_{i+1}^0, L, x_{i+2}) > \pi(x_{i+1}^0, x_i, x_{i+2}) > K$ since there will be no further entry in the interval (L, x_{i+1}). Consequently, there always exists a location in the interval $(L, \lambda(L))$ such that entry is profitable for at least one firm.

How many more firms enter depends on the location of $\lambda(x_i)$. If $\lambda(x_i) > \rho(R)$, we are in part (ii), where it was shown that at least one more firm enters. If $\lambda(x_i) < \rho(R)$, then we are again in part (iii) in which case at least two more firms enter. ■

A.5 Proof of Theorem 2

From Corollary 1, if $\rho(R) \leq L$, or if $R \leq \lambda(L)$, the firm at x could profitably deviate by staying out. Moreover from Corollary 2, when the distance between the firm at x and its neighbors exceeds $x - \rho(x)$ and $\lambda(x) - x$, respectively, then there will be entry in between, contradicting that x and L (respectively x and R) are neighbors. ■

A.6 Proof of Corollary 3

As shown in Theorem 2, in any equilibrium the maximum distance between a firm at location x and its neighbors to the left and right is $x - \rho(x)$ and $\lambda(x) - x$, respectively. Moreover, as shown in the proof of part (iii) of Lemma 4, $F(\frac{\lambda(x)+x}{2}) - F(x) \leq K$ and $F(x) - F(\frac{\rho(x)+x}{2}) \leq K$, so that for the *total* profit generated at location x, $F(\frac{\lambda(x)+x}{2}) - F(\frac{\rho(x)+x}{2}) \leq 2K$ holds. Therefore, independent of how this profit is shared between the firms located at x, at most one can break even. ■

A.7 Proof of Theorem 3

The proof is straightforward and by contradiction: Assume not, i.e. assume, say, $a > \lambda_B$. Then a firm could profitably enter at λ_B and get K to its left (without attracting further entry there) and earn strictly positive profit to its right. ■

A.8 Proof of Lemma 5

Consider without loss of generality part (i). If f is monotonously decreasing over the interval (L, R), then $x^*(L, R)$ is unique and equal to L^+: Recall that when locating in (L, R), the reach of a firm's customer base is $\Delta(L, R) = \frac{R-L}{2}$, and thus independent of the firm's location. However, the firm's profit strictly increases as it moves closer to x^* since the density is strictly higher to the left.

If f is hump-shaped, then x^* is interior, satisfying $f(\frac{x^*+L}{2}) = f(\frac{x^*+R}{2})$. Consequently, for any $x > x^*$, $f(\frac{x+L}{2}) \geq f(\frac{x+R}{2})$ holds. Thus, when moving left from x the marginal gain on the left will be (weakly) larger than the marginal loss on the right. Hence, $\pi(y, L, R) \geq \pi(z, L, R)$ for any $z > y > x^*$ follows. Note that the inequality statements in the Lemma become strict when f is either monotone, or when $f' = 0$ for at most one location. ■

A.9 Proof of Lemma 6

That at least one additional firm enters follows from Theorem 1. So we are left to show that further entry deterrence is optimal for the first entrant. There are two cases: Either f is monotone or it is hump shaped as in Figure 4. We first consider the monotone case, focusing without loss of generality on increasing functions.

Let i be this entrant and suppose to the contrary that i accommodates further entry either by choosing $x_i \in (\lambda(L), R)$ or $x_i \in (L, \rho(R))$. If $x_i \in (L, \rho(R))$ is an equilibrium outcome, then the next entrant will deter entry (by Theorem 1). He optimally does so by choosing $\hat{x}(\rho(x_i), \lambda(L), L, x_i) = \lambda(L)$. But in this case, x_i is in between two neighbors who are in the interval $[\rho(R), R]$. By Corollary 1, i cannot break even. Hence, this cannot be optimal for i.

If, on the other hand, i chooses $x_i \in (L, \rho(R))$, the subsequently entering firm will, again, deter entry. It optimally does so by choosing $\hat{x}(\rho(R), \lambda(x_i), x_i, R) = \lambda(x_i)$. In this case, i earns strictly less than he would had he located at $\lambda(L)$ and thereby deterred entry: Both the length of the interval he captures is now $\frac{\lambda(x_i)-L}{2}$ instead of $\frac{R-L}{2}$ which he would cover when deterring entry, *and* the density over this interval is smaller than the density he would get when deterring entry. Hence, i will optimally deter entry. He optimally does so by locating at $\lambda(L)$. Note that i's profit will also be strictly higher than K, since $x^*(L,\lambda(L)) = \lambda(L)^-$ so that $\pi^*(L,\lambda(L)) = K$ and $R > \lambda(L)$ (by Lemma 3).

Consider now the case where f is hump-shaped. If there exist $x^*(L,R) \in [\rho(R),\lambda(L)]$ the claim is established since these optimal locations in (L,R) are themselves entry-deterring. So assume there is no $x^*(L,R) \in [\rho(R),\lambda(L)]$, and without loss of generality consider some $x^*(L,R) < \rho(R)$ (the case $x^*(L,R) > \lambda(L)$ is completely analogous). By Lemma 5, $\hat{x}(\rho(R),\lambda(L),L,R) = \rho(R)$ and $\hat{\pi}(\rho(R),\lambda(L),L,R) > \pi(x,L,R)$ for any $x > \lambda(L)$. Hence, the optimal entry deterring location yields a larger profit than locating to the right of $\lambda(L)$. Hence, i will never choose $x_i \in [\lambda(L),L)$. So assume $x_i \in (L,\rho(R))$. Then, either (a) the subsequent (and, by Theorem 1, last) entrant's unconstrained optimal location in (x_i,R), $x^*(x_i,R)$, will be given by the first order condition. In this case, though, by Lemma 2, $x^*(x_i,R) < x^*(L,R)$ and by Lemma 5, the last entrant's optimal entry deterring location will be $\rho(R)$. Or, (b) $x^*(x_i,R)$ will be a corner solution, in which case it will be x_i^+. Hence, again, the last entrant's optimal entry deterring location is $\rho(R)$. Both in case (a) and (b) x_i will be "trapped" inside $[L,\lambda(L)]$, where it cannot break even due to Corollary 1.

Observe also that by deterring entry, i nets a profit that is strictly larger than K. This is obvious for $x^*(L,R) \in [\rho(R),\lambda(L)]$. If such locations do not exist, then, by Lemma 5, for any $x^*(L,R) \in (L,\rho(R))$, we have $\hat{x}(\rho(R),\lambda(L),L,R) = \rho(R)$. Hence, $\pi(\rho(R)^+,\rho(R),R) = K$, but $\pi(\rho(R),L,R) > \pi(\rho(R)^+,\rho(R),R) = K$. Similarly, for any $x^*(L,R) > \lambda(L)$, $\hat{x}(\rho(R),\lambda(L),L,R) = \lambda(L)$ and consequently $\pi(\lambda(L),L,R) > K$. ■

A.10 Proof of Theorem 4

We only prove part (i), the proof for part (ii) is completely analogous.

Existence. We first show that $\{\lambda_B, \lambda^1, .., \lambda^n, \rho_B\}$ are equilibrium locations. To that end, assume for the moment that locations λ_B and ρ_B are occupied and that all remaining firms play the following strategy: "Enter to the right of some location x_i only if its closest righthand neighbor, $i+1$, is at some $x_{i+1} > \lambda(x_i)$ and when $\lambda(x_i) < \rho_B$. If you enter to the right of x_i, enter at $\lambda(x_i)$." Call this the λ-strategy.

To see that these strategies are mutual best responses, notice first that not to enter in $[x_i, x_{i+1}]$ if $x_{i+1} \leq \lambda(x_i)$ is, obviously, a best response. Second, if x_{i+1} is the future righthand neighbor of the entering firm and if $x_{i+1} > \lambda(x_i)$, then entry at $\lambda(x_i)$ is optimal within the interval $[x_i, x_{i+1}]$, as we know from Lemma 6. (And since all better options are taken before by other firms in case there are better options, at some point some firm will enter here.) If the righthand neighbor x_{i+1} has not taken its location yet but plays the λ-strategy, then a fortiori $\lambda(x_i)$ is optimal for the entrant: Not only is it the largest location that deters entry to its left, but it will also push its righthand neighbor $i+1$ as far to the right as possible.

Moreover, under the λ-strategy, the locations λ_B and ρ_B will also be occupied: As for the leftmost location, since the density is increasing, it is optimal to move right as far as possible without inviting further entry to the left which, by definition, is at λ_B. Moreover, under the λ-strategy, the rightmost location will not affect any of the location choices of the other firms. It follows that it is optimal to locate as far left as possible without inviting further entry to the right which, again by definition, is at ρ_B.

Uniqueness. If the future left-hand and righthand neighbor to some entrant are given at, again, x_i and x_{i+1}, respectively, then $\lambda(x_i)$ is still the best response of the entrant in $[x_i, x_{i+1}]$. Observe also that it is the unique best response. So one way $\lambda(x_i)$ could not be the best response of the entrant with neighbors at x_i and x_{i+1} is that $i+1$ has not taken his location *and* threatens to locate the closer to the entrant the closer the entrant's location to $\lambda(x_i)$. Assume that the entrant believes this threat and that his best response would be to locate at some $y < \lambda(x_i)$.

To see that this threat is empty in equilibrium (i.e. even if i played his best response to this threat, the threat would in turn not be a best reply), consider the last entrant, say l, to the right of our entrant. Clearly, l's best response will be to locate at $\lambda(x_{l-1})$, where x_{l-1} is the last entrant's left-hand neighbor (which may or may not be $i+1$). Anticipating this, $l-1$ recognizes that l's best responses increases in his own location, and thus he chooses the largest location which allows him to deter entry to his left. By iteration, we see that $i+1$'s best response is to locate at the largest location that deters entry to the left. Thus, the threat is empty and the best response is unique. ■

A.11 Proof of Theorem 5

As for part (i), note first that the firm at ρ_B earns K to its right by definition of ρ_B. As for the firm at λ^n, note that when f is increasing, $x^*(y, \lambda(y)) = \lambda(y)^-$ so that $\pi(\lambda(y)^-, y, \lambda(y)) = K$. Thus, the firm at λ^n earns K to its left so that differences in their profits can only accrue from differences in earnings between λ^n and ρ_B. In terms of distances, both grasp exactly $\frac{\rho_B+\lambda^n}{2}$. However, the density over the share grasped by the firm at ρ_B being larger than for the share catered by the firm at λ^n, it follows that the firm at ρ_B earns strictly more. The proof for part (ii) is completely analogous. ■

A.12 Proof of Theorem 6

We only prove the result for the case where f is increasing, the case where it is decreasing being completely analogous. Consider Figure 8 to see that the equilibrium profit of the firm at location x_i is equal to the sum of two areas: To the left, it gets an area of size K, and to the

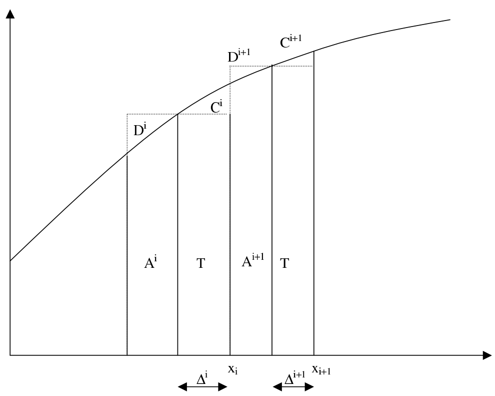

Figure 8: $\pi(x_i) = T + A^{i+1}$.

right an area of size A^{i+1}, which is smaller than K. So

$$\pi(x_i) = T + A^{i+1}.$$

Let $\Delta^i := \Delta(x_{i-1}, x_1)$, i.e. half of the distance between the equilibrium locations x_{i-1} and x_i. Because $f(x)$ increases in x, $\Delta^{i+1} < \Delta^i$ holds. We are now going to show that for the areas A^i the following holds: $A^i < A^{i+1}$ for any $i \geq 1$. Since $\pi(x_i) = T + A^{i+1}$, this will then complete the proof.

Observe first that $A^i = T - C^i - D^i$. So $A^i < A^{i+1}$ will hold if we can show that $D^{i+1} < D^i$ and $C^{i+1} < C^i$ holds. Define $\tilde{f}_i(x) \equiv f(x) - f(x_i - \Delta^i)$ for $x \in [x_i - \Delta^i, x_i]$. Clearly, $\tilde{f}_i' > 0$ and $\tilde{f}_i'' \leq 0$ holds. For any $y < \Delta^{i+1}$ this implies

$$\tilde{f}_i(x_i - \Delta^i + y) \geq \tilde{f}_{i+1}(x_{i+1} - \Delta^{i+1} + y). \tag{4}$$

Observe next that

$$C^i = \int_{x_i - \Delta^i}^{x_i} \tilde{f}_i dx > \int_{x_i - \Delta^i}^{x_i - \Delta^i + \Delta^{i+1}} \tilde{f}_i dx \geq \int_{x_{i+1} - \Delta^{i+1}}^{x_{i+1}} \tilde{f}_{i+1} dx = C^{i+1}.$$

The first and last equality are identities. The first inequality is due to the fact that $\Delta^i > \Delta^{i+1}$, and the weak inequality follows from (4). Thus, $C^{i+1} < C^i$ is established.

Similarly, define $\hat{f}_i(x) \equiv f(x_i) - f(x)$ for $x \in [x_i - 2\Delta^i, x_i - 2\Delta^i]$. For any $y < \Delta^{i+1}$,

$$\hat{f}_i(x_i - 2\Delta^i + y) \geq \hat{f}_{i+1}(x_{i+1} - 2\Delta^{i+1} + y) \tag{5}$$

holds. Observe then that

$$D^i = \int_{x_i - 2\Delta^i}^{x_i - \Delta^i} \hat{f}_i dx > \int_{x_i - 2\Delta^i}^{x_i - 2\Delta^i + \Delta^{i+1}} \hat{f}_i dx \geq \int_{x_{i+1} - 2\Delta^{i+1}}^{x_{i+1} - \Delta^{i+1}} \hat{f}_{i+1} dx = D^{i+1}.$$

The proof for part (ii) is completely analogous. ■

A.13 Proof of Theorem 7

The proof is based on Theorem 4 and is straightforward. On the increasing part, all best responses are independent of the location of the right-hand neighbor. Hence $\{\lambda_B, .., \lambda^r\}$ follows. On the decreasing part, best responses are independent of the left-hand neighbor's location, whence $\{\rho^s, .., \rho_B\}$ follows. ■

A.14 Proof of Lemma 7

The length of the interval captured by the entrant is always $\frac{R-L}{2}$ independent of his location. Now let the entrant locate at one end of the interval (say, at L^+) and let him contemplate moving marginally towards the middle. Either his profit increases immediately. In this case, however, his profit will keep increasing as he moves further to the right since he keeps losing less on the left and gaining more on the right. Thus, the optimal location will be R^- in this situation. Or, the move towards the right will initially involve losses. If this is the case for all positions to the right of L^+, then he optimally locates at L^+. If eventually the profit starts increasing by moving further right, then it will increase monotonically from there onwards. Hence, the optimal location in this case will be either L^+ or R^-. ■

A.15 Proof of Corollary 5

Suppose to the contrary that in equilibrium M is occupied by, say, firm k. From Theorem 2, it follows that its left- and right-hand neighbors will be at some locations $x_L \geq \rho(M)$ and $x_R \leq \lambda(M)$, respectively. Without loss of generality, assume $f\left(\frac{x_R+M}{2}\right) \geq f\left(\frac{x_L+M}{2}\right)$. By moving marginally to the right, the profit of k increases: It gains more on the right than it loses on the left. Note that this is true independently of whether the right-hand neighbor is already there or not: If it is not there, by moving right to some $x > M$, k pushes its future right-hand neighbor to $\lambda(x)$. Even if the move to the right will attract entry to the left at $\rho(x)$, the loss due to this entry will be smaller than the gain to the left. Thus, M cannot be optimal. If $f\left(\frac{x_R+M}{2}\right) < f\left(\frac{x_L+M}{2}\right)$, analogous arguments apply for the opposite direction. ■

A.16 Proof of Theorem 8

We are going to argue that for trough-symmetric functions, ρ_M and λ_M are mutually best responses. Once this is shown, the Theorem follows immediately from the previous results on monotone densities. So, suppose some firm is located at λ_M. Observe then that any location $y \in [\rho_M, \lambda_M]$ will deter entry in between: For $y > \rho_M$, $x^*(y, \lambda_M) = \lambda_M^-$. But $\pi^* < K$ since $\frac{y+\lambda_M}{2} > M$ because of trough-symmetry. Since moving away from the middle without attracting entry is always beneficial, ρ_M dominates any interior location. Notice then that at $y = \rho_M$ the firm nets exactly K to the right, again because of trough-symmetry. Thus, this is a best response. Mutuality of best responses follows from trough-symmetry.

To see that these equilibrium locations are unique assume to the contrary that $y < \lambda_M$. Then the best replying left-hand neighbor will locate at some $z < \rho_M$. But then $y < \lambda_M$ cannot have been optimal in the first place. ■

A.17 Proof of Lemma 8

All we need to show is that an decrease in ε leads to an increase in the mass to the left of λ^n and to the right of ρ^n. Observe that both λ^n and ρ^n depend on ε and that because of symmetry there is no loss of generality if we focus on λ^n and the mass to its left. To see that the mass between two locations λ^i and λ^{i+1} increases as ε decreases, assume to the contrary that it does not. But then, because $f^h_{\varepsilon'}$ is flatter than f^h_{ε} for $\varepsilon' < \varepsilon$ it follows that the firm at λ^i attracts to its right more consumers for ε' than for ε while the one at λ^{i+1} attracts to its left less than for ε and thus less than K, which is a contradiction. Therefore, the mass between any two neighboring firms increases as ε decreases. Thus, if initially at ε_0 λ^n and ρ^n deter further entry in between, they will do so a fortiori for any $\varepsilon < \varepsilon_0$. ■

A.18 Proof of Theorem 9

In the uniform case, the left- and rightmost locations are λ_B and ρ_B, respectively. Moreover, the remaining interval (λ_B, ρ_B) has a mass of consumers of $F(\rho_B) - F(\lambda_B) = 1 - 2K$. As long as firms enter at λ- or ρ-distances from each other, each additional entrant reduces the remaining mass of consumers by the maximum amount $2K$ (see part (iii) of Lemma 4 and the discussion following that Lemma). Moreover, should a last, and smaller interval exist, one more firm will enter. This interval will satisfy the condition $L < \rho(R) < \lambda(L) < R$. As was shown, in part (ii) of Theorem 1, this is equal to the minimum number of firms entering in such an interval for any distribution.

For general distributions, as shown in Theorem 3, the left- and rightmost locations are $a \leq \lambda_B$ and $b \geq \rho_B$, respectively, so that the interval (a, b) has a mass of consumers of $F(b) - F(a) \geq 1 - 2K$. Moreover, from Theorem 2, firms cannot be located further away from each other than λ- or ρ-distances, so that each additional entrant will reduce the remaining mass of consumers by (weakly) less than the maximum amount $2K$.

Taken together, in the uniform case, the size of the relevant interval is minimum, and firms are located at maximum distance from each other in this interval, so that the number of active firms cannot be larger than under any other distribution. ■

B Price setting for advertisement revenue by media firms

In media markets, while consumer price competition is often either non-existing or of minor importance, media firms do compete in prices for advertisement revenue. Examples are free newspapers and private on-air TV and radio broadcasting in Europe, where there exists a typically binding restriction on how much broadcasters may advertise. In this appendix we allow media firms to set prices for advertisement and show that, under quite general conditions, the overall profit of a media firm is proportional to its market share. As a result, the model we have considered so far can be seen as a reduced form of an augmented setting where firms choose both, locations and prices for advertisement (thereby maintaining the assumption that do not set prices for consumers).

Consider a finite number of media firms located in the $[0, 1]$-interval. The market share of media firm i is denoted $\theta_i > 0$, and it can set a price p_i for an advertisement slot.

Moreover, there exists a unit mass of advertisement firms which differ with respect to their willingness to pay for placing ads. In particular, assume that the payoff of an advertisement firm of type s when placing in ad in media firm i is $s\theta_i - p_i$. Assume that s is private information to each advertisement firm, and distributed on [0,1] according to c.d.f. G with density $g > 0$, satisfying $(1 - G)/g \leq 0$. Thus, the model framework corresponds to that of Anderson and Coate (2005) except that consumers here are not assumed to dislike ads.

Clearly, each advertisement firm will place an ad in media outlet i if $s\theta_i - p_i \geq 0 \Leftrightarrow s \geq p_i/\theta_i$. Note that, consistent with reality, we allow advertisement firms to "multi-home", i.e. to place ads in more than one media outlet. Furthermore, each consumer "single-homes", i.e. patronizes only one media outlet so that the sets of consumers which advertisement firms address through different medias are disjoint.[32] As a result, each advertisement firm separately decides for each media outlets whether or not to place an ad.

Given the above decision rule, the number of advertisement firms who accept a given price p_i is $1 - G(p_i/\theta_i)$ so that media firm i's total net profit is equal to its advertisement revenue:

$$\max_{p_i} p_i(1 - G(p_i/\theta_i)),$$

[32]See Kim and Serfes (2006) for a standard Hotelling model with prices where consumers "multi-home".

yielding the first-order condition

$$0 = 1 - G - g\frac{p_i}{\theta_i} \quad \Leftrightarrow \quad \frac{p_i^*(\theta_i)}{\theta_i} = \frac{1 - G(p_i^*(\theta_i)/\theta_i)}{g(p_i^*(\theta_i)/\theta_i)}.$$

Observe that $\frac{p_i^*(\theta_i)}{\theta_i}$ is unique and independent of θ_i. Consequently, neither $G(p_i^*(\theta_i)/\theta_i)$ nor $g(p_i^*(\theta_i)/\theta_i)$ will vary with θ_i. The equilibrium profit of firm i with market share θ_i is then

$$\Pi^*(\theta_i) = p_i^*(\theta_i)(1 - G(p_i^*(\theta_i)/\theta_i)).$$

Invoking the Envelope Theorem, we get

$$\frac{d\Pi^*(\theta_i)}{d\theta_i} = \left(\frac{p_i^*(\theta_i)}{\theta_i}\right)^2 g(p_i^*(\theta_i)/\theta_i) > 0,$$

which is constant. Thus, each media firm's total profit is proportional to its market share, which corresponds to the framework we have analyzed in this paper.

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