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EARLY RETIREMENT AND THE OPTIMAL RETIREMENT AGE

by

Angela Ryan

The University of Melbourne

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Centre for Actuarial Studies
Department of Economics
The University of Melbourne
Parkville, Victoria, 3052
Australia.

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1. INTRODUCTION

The purpose of this paper is to describe retirement patterns in Australia and a methodology for assessing an "optimal" retirement age when retirement is voluntary. In the first part of the paper definitional issues are discussed, together with background on current and prospective retirement patterns and the possible causes of those patterns. The second part of the paper develops an age of retirement model designed to assess an optimal retirement age. The model is based on the trade-off an individual must make between income and leisure in choosing a time to retire. The paper discusses possible implications for government policy which are suggested by the results.

1.1 Defining early retirement

The Australian Bureau of Statistics (ABS) defines "early retirement" as persons aged 45 and over who retire before the age of 60 years if female or 65 years if male (ABS Cat No. 6238.0).

In Australia, the labour force participation rates of males over the age of 55 has shown a large and steady decline over the last 20 years or so, although there has been less dramatic declines, and even a slight increase, in very recent years (see Table 1). These trends are consistent with retirement patterns documented in many other developed countries (see for example OECD, 1988 (p 77) and Kohli, 1991). The dramatic increase in the number of men retiring aged 60-64 since 1974 signified the beginning of early retirement on a large scale in Australia (Howe and Manning, 1987, p 313).

Table 1: Male labour force participation rates (%) by age, 1970-1994 (a)

Year	55-59	60-64	65 and over
1970	91.2	77.4	22.1
1975	87.8	68.6	16.7
1980	83.3	50.2	11.1
1985	76.4	42.6	8.9
1990	76.0	50.5	8.5
1991	71.8	49.6	9.2
1992	74.0	48.1	9.3
1993	70.1	46.6	8.3
1994	72.5	47.3	9.0

(a) Estimates from 1986 are based on a revised definition. Figures shown are for August.

Source: 1970 to 1980, ABS, *The Labour Force Australia*, cited in Merrilees (1982) p 82; from 1980, ABS, *The Labour Force Survey (unpublished data)*.

The participation rates have been more stable for women, and have even increased for the 55-59 year age group (see Table 2).

Table 2: Female labour force participation rates (%) by age, 1978-1994 (a)

Year	55-59	60-64	65 and over
1978	30.2	13.8	2.8
1980	29.1	13.5	2.9
1985	27.1	11.2	2.0
1990	33.8	16.3	2.3
1991	35.6	14.7	2.4
1992	36.7	12.4	2.2
1993	36.5	14.6	2.4
1994	37.6	14.3	2.3

(a) Estimates from 1986 are based on a revised definition. Figures shown are for August.

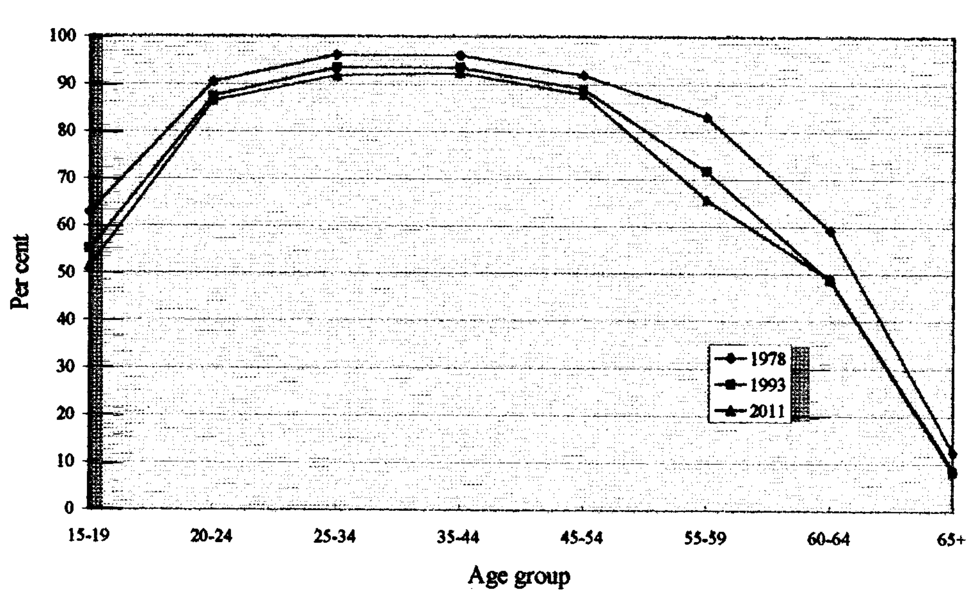
Source: ABS, *The Labour Force Survey (unpublished data).*

While there has been a clear trend to early retirement among males over the last two decades, there has been some weakening in this trend in recent years. For females, the strong increases in labour force participation observed over the last 15 years for all age groups have also begun to taper off. What, then, can we expect in terms of early retirement in the future in Australia?

The most comprehensive projections have been prepared by the ABS (Cat No. 6260), and cover the period to 2011, although projections have been prepared to 2051 by the Retirement Income Modelling Task Force. Figure 1 shows that the male participation rates are projected to fall for all age groups, with the greatly reduced participation by males aged 55-59 years most evident. Figure 2 shows that for females, participation rates are projected to increase for all age groups except those aged 15-19 and those over 65.

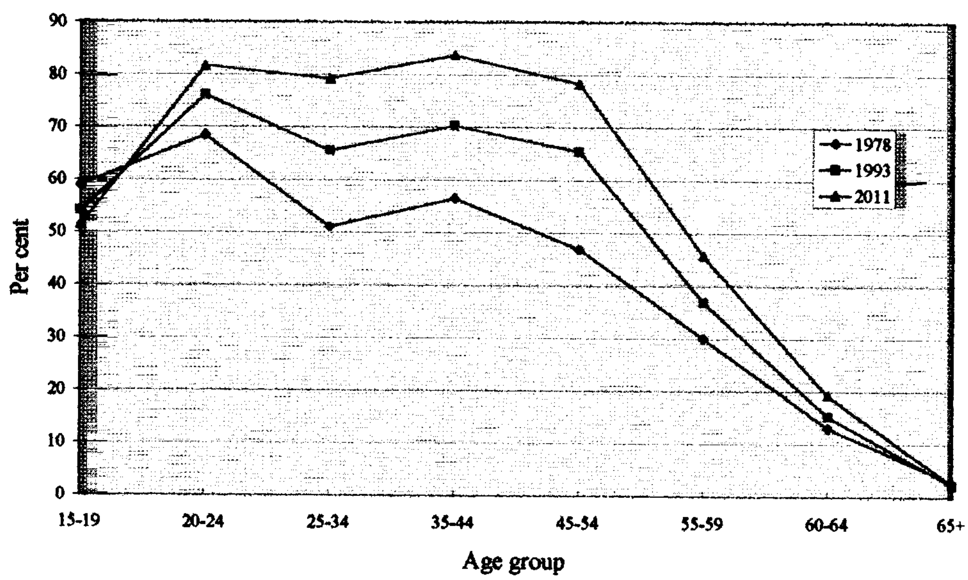
For the purposes of this paper, "early retirement" is defined as retirement before age pension age. In one sense this is an unusual definition, because it means that most retirees would be classified as having retired early (for males over age 45 who have retired from full-time work, 74 per cent did so before age 65, and for women, 87 per cent did so before age 60) (ABS Product No. 6238.0.40.001). However, it is the relationship of retirement to age pension age which best reflects the costs and benefits to society as a whole, since it is from age pension age that the potential costs to the government come fully into play.

Figure 1: Participation rates, males



Source: ABS, *1995-2011 Labour Force Projections Australia*, Cat No. 6260.0

Figure 2: Participation rates, females



Source: ABS, *1995-2011 Labour Force Projections Australia*, Cat No. 6260.0

1.2 The costs and benefits of early retirement

Early retirement has implications for individual welfare, government policy, labour markets and financial markets. Early retirement can be seen in both a negative and a positive light. Wolozin (1981) has emphasised the negative aspects, suggesting that sweeping changes need to be implemented to reverse the trend to early retirement because of its high psychological and social costs, and the associated loss of valuable human capital and higher fiscal cost.

While recognising these issues, Maxwell (1989) has pointed out that early retirement can be seen as a desirable social goal, with earlier retirement providing the scope for a better quality of life through more leisure time. In looking at retirement across a range of countries Markides and Cooper (1987, p 7) concluded that "no longer can we keep on writing that the transition to retirement is particularly difficult because of lack of socialisation to the retired role. For better or for worse, more and more people are retiring and more of them are having fun being retired." Further, they found that there is increasing evidence that retirement is not detrimental to physical and mental health. Early retirement programs have come to be regarded as social achievements by governments and unions in many European countries (including Denmark, France, Sweden, West Germany and UK) and indicators point to continuing early labour-force withdrawal (Rix and Fisher, 1982, p 113).

In Australia, the most common reason given for those who intended to retire early was "decided not to work any more, more leisure time" (ABS, Cat No. 6238.0). From an individual perspective, therefore, it would appear that early retirement is seen as a positive development and one which is welcomed. If the aim of economic growth and development is to provide a better quality of life for individuals, and that quality is improved by increasing leisure time, a preference for earlier retirement can be seen as a natural outcome of continued growth. This paper focuses only on the individual, although it is acknowledged that there is a broader question of whether early retirement is necessarily beneficial for society as a whole; that is, will early retirement undermine the prosperity which is required to finance it and other social goals?

1.3 The causes of early retirement

Research suggests that in Australia, the trend to earlier retirement has been produced by a number of factors, including

i. the increasing use of early retirement policies in response to employers' needs to adjust labour inputs, particularly as technological change becomes more prevalent across industries and the pace of change increases
ii. increased preference by workers for early retirement (because of such factors as maturation of superannuation for a proportion of the workforce and higher average incomes), possibly associated with changed attitudes to early retirement
iii. increasing eligibility for, and take-up of service pensions by Second World War veterans (although this factor has become less important over time, as veterans pass the age of early retirement)
iv. increasing liberalisation and increased generosity of the age pension

v. at least in some of the time frame, unemployment among older workers - this and poor health would appear to be the major involuntary components associated with early retirement.

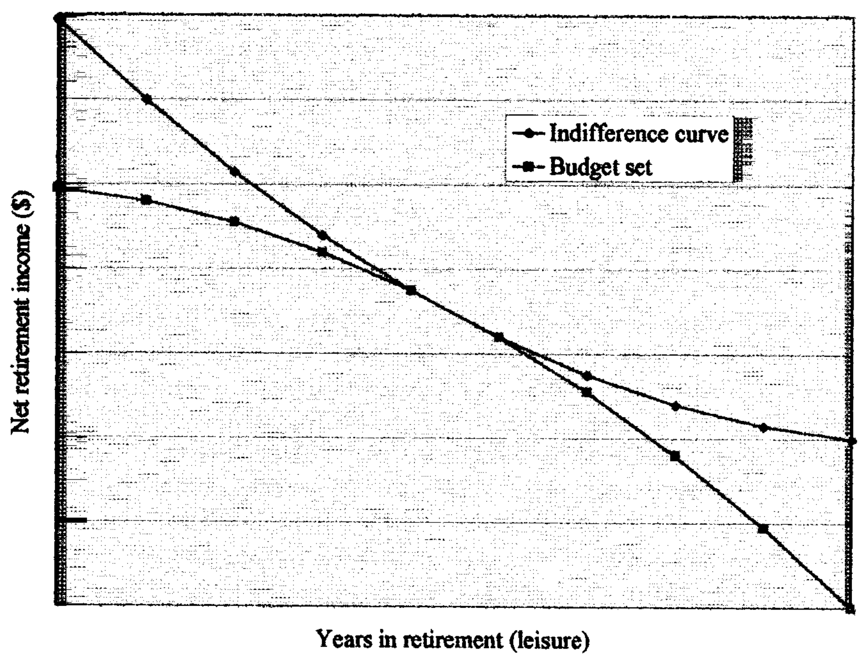
Older Australians are, on average, working less and retiring earlier. It would appear that most workers choose when to retire, and that that choice is sensitive to economic incentives. This paper develops a model to look at the role of some of those incentives.

2. DEVELOPING A THEORETICAL MODEL TO DEFINE AN OPTIMAL RETIREMENT AGE

Earlier models developed to deal with retirement looked at the retirement decision as affecting one year at a time (see Lazear (1986) pp 312-313 for a discussion). Ideally, it would be desirable to assess the choice of retirement age within a life-cycle framework that continues to be based around the work-leisure dichotomy used in the simplest models.

Using standard microeconomic principles, the age of retirement can be considered optimal from the individual's perspective if it coincides with the point in time where the marginal utility of consumption from an additional year's work is equal to the marginal utility of an additional year of leisure as shown in Figure 3.

Figure 3: Optimal retirement age



The budget set shows that as the years in retirement increase, the expected value of net income in retirement falls. The indifference curve shows that utility is a positive function of the net value of retirement income and leisure, so that for a given level of utility, as income falls leisure must rise and vice versa.

More formally, the development of a budget set requires comparison of income streams over time (discounted to show present values). The net present value of expected income (NPVY) for retirement at age R is given by:

$$NPVY = \sum_{t=0}^{R} . E_t e^{-\delta t} + \sum_{t=R}^{T} .(P_t + S_t) e^{-\delta t} \qquad (1)$$

where E_t are the earnings in the working years from year *0* (the first possible year of retirement) to *R*; *T* is the individual's planning horizon, though he or she may die before or after then; P_t and S_t are, respectively, the private pension and social security benefits in the retirement years from *R* to *T*; and δ_t is the discount rate.

Once the budget set has been determined, this can then be matched against a utility function showing consumption and leisure preferences to find the optimal retirement age. Theoretically, these preferences can be represented by a utility function, with utility (U) being a positive function of two factors - planned lifetime consumption (C) and leisure during retirement (L) - so that for an individual (i):

$$U_i = U(\overset{+}{C_i}, \overset{+}{L_i}) \qquad (2)$$

or, alternatively, it can be expressed as a positive function of consumption and a negative function of retirement age (R) - so that

$$U_i = U(\overset{+}{C_i}, \overset{-}{R_i}) \qquad (3)$$

If individuals plan to consume all of their income in their lifetime, or leave it as a bequest, then income (or NPVY) can be substituted in this utility function in place of consumption.

As shown in Figure 3, the optimal age of retirement is that which maximises utility subject to the budget set. As previously noted, retirement is optimal where the marginal utility of consumption from an extra year's work is equated to the marginal utility of an additional year's leisure. This can be shown as

$$\frac{\partial NPVY}{\partial R} = \frac{\partial U}{\partial L} \Big/ \frac{\partial U}{\partial C} \qquad (4)$$

3. BUILDING AN AGE OF RETIREMENT MODEL

Based on this theoretical analysis, we want to develop a model which is able to generate budget sets for net retirement income; and indifference curves showing the relative preference for leisure (or retirement years) and consumption which are relevant to the retirement decision facing Australians.

Fields and Mitchell (1984) have developed a life-cycle model which looks at optimising the age of retirement. Drawing on their approach, an age-of-retirement model has been developed which has five key features:

i. the decision about when to retire involves the individual deciding whether to work this year, whether to work next year, and so on up to the end of the planning horizon
ii. the decision is how many years to work before retirement. Each period's decision is dichotomous: whether to work or not to work
iii. optimal retirement involves choosing the retirement age that maximises utility (assumed to depend positively on consumption of goods and years of leisure)
iv. the choice is constrained by economic opportunities. Earnings, private pensions and social security enter into the retirement decision
v. the retirement decision is affected by economic opportunities in the past, the present and the future.

3.1 Constructing budget sets

The development of budget sets alone will allow us to examine changes in income as retirement is postponed. To do this, we need to compare income streams over time. The questions we want to answer in constructing budget sets are:

i. what would a typical worker retiring at, say, age 55 anticipate receiving in earnings, private pension and social security benefits
ii. how would these income streams vary if retirement were deferred.

The Lifetime Income, Taxation, Expenditure and Superannuation (LITES) model developed by Atkinson et al (1994a) can be used to generate simulations of life-time working and retirement profiles. Full details of the LITES model are described in Atkinson et al (1994a). Only the features of the model required to generate results for this paper are summarised here. LITES is used to generate values for each of the variables in the first equation shown above - that is, net earnings, private pensions and age pension payments.

Specifically, the model was run using the assumptions set out in Appendix 1 for three males with hypothetical earnings profiles, where each person is assumed respectively to obtain in each year of working life the earnings equivalent to the lower quartile, median and upper quartile of his age group. Using these profiles, earnings rise during early working life, flatten out, and then decline with advancing age. (The choice of single males rather than females reflects the difficulty of constructing an earnings profile for women given the evolving patterns of female participation rates.) It was assumed that these men enter the work force at age 20 and have a choice of retiring between the ages of 55 and 70. Given the focus of this paper on early retirement the results for ages 55 to 65 only are reported, although features of interest in later years are also discussed where appropriate.

The men are assumed to reach the age of retirement, after which the age at death of each man is calculated with regard to their effective earnings experience. The age at death reflects the tendency for individuals of relatively higher total lifetime earnings to experience lower mortality rates. Such mortality rates cannot be constructed accurately from available data, so the sensitivity of the results to various mortality assumptions are tested. Full retirement is assumed to be the only means of leaving the work force.

It is assumed that the Superannuation Guarantee Charge (SGC) is fully mature, so that compulsory contributions are made to a superannuation scheme at the rate of 9 per cent of gross earnings from the employer and 3 per cent of gross earnings from the individual from the year of entry to the work force. The proposed Government co-contribution announced in the 1995-96 Budget has not been included. It is assumed that the superannuation fund pays an effective tax rate of 7.5 per cent on the accumulated superannuation. It is also assumed that each individual saves a further 5 per cent of his net salary each year (that is, after tax and superannuation contributions), and that this is accumulated in some other savings vehicle (with the earnings not being attributable to the contributor for tax purposes until the point of retirement). The effective tax rate on the earnings on this saving is assumed to be 25 per cent. The sensitivity of the results to the assumed additional savings level is also tested.

Income is taxed at the rates applicable for the 1994-95 income year, with the income tax thresholds indexed at 5.5 per cent, with average weekly ordinary time earnings (AWOTE) assumed to be growing at 6 per cent and inflation at 5 per cent. During working life, the income tax liability faced includes possible eligibility for the rebate on superannuation contributions and the low income earner rebate (with the rebate and thresholds indexed to AWOTE). In retirement, the tax rate faced also takes the age pension rebate into account. The level of age pension and the associated rebate and thresholds are all indexed to AWOTE, with the base amounts being those applying in July 1994. For the purposes of the income and assets tests, it is assumed that all of the men are single home owners.

Upon reaching retirement, two scenarios are tested, both of which involve purchase of an annuity. This is assumed to be a whole life annuity, escalating at 5 per cent per annum. The annuity rates used are shown in Appendix 1. The scenarios are as follows:

i. the decisions in route A involve taking all savings at retirement as a lump sum, with lump sum tax paid. All cash available is used to purchase an annuity (so that there is no money available for any other form of saving)
ii. the decisions in route B involve taking all savings at retirement as an annuity, with no lump sum tax paid (so that there is no money available for any other form of saving).

These "decision routes" are very similar - the tax treatment of superannuation at and after retirement is the only difference between them, although this in turn impacts on age pension entitlements. These routes were chosen as they correspond to the highest ranked options of those tested by Atkinson et al (1994b) in terms of income and consumption in retirement (other options included non-annuity and partial annuity purchase).

The results obtained are set out in Table 3 to Table 8. All figures are net present values. For example, from Table 3, if a man with earnings at the 25th percentile were to retire at age 60, he could expect to receive

i. a gross annuity of $119,220, which would be paid over the period from age 60 to age 72 (his year of death - this is lower than the average life expectancy of a man aged 60, but reflects the higher mortality assumed for those with lower earnings). This annuity would be financed from his lifetime savings, generated by his superannuation and personal savings
ii. an age pension of $83,988, which would be paid, subject to the income and assets tests, from age 65
iii. gross earnings of $57,861. These are his earnings between age 55 and age 60. That is, the gross earnings in the table show the additional earnings received as a result of postponing retirement from age 55.

Tax is then deducted to obtain a net income figure. That net figure of $249,064 shows the total income which the man will earn after reaching age 55, assuming he retired at age 60 and died at age 72. Reading across the table, the net income figures show the net present value of income that would be received from age 55 if retirement occurred between age 55 and 65.

Table 3: Results route A*, earnings at 25th percentile

RETIREMENT AGE	55	56	57	58	59	60	61	62	63	64	65
(1) Gross annuity	123066	123368	123209	122378	121122	119220	116592	113150	108788	103230	96583
(2) Gross pension	84599	84306	84075	83972	83916	83988	84192	84468	84844	85315	85770
(3) Gross earnings	0	12289	24227	35805	47018	57861	68328	78419	88131	97463	106417
(4) Gross income	**207664**	**219963**	**231511**	**242155**	**252056**	**261068**	**269113**	**276037**	**281763**	**286009**	**288770**
(1+2+3)											
(5) Tax on earnings	0	979	1884	2713	3465	4140	4736	5252	5689	6046	6323
(6) Tax in retirement	3991	4780	5569	6338	7111	7865	8597	9280	9924	10502	11084
(include lump sum tax)											
(7) Net income	**203674**	**214204**	**224058**	**233104**	**241480**	**249064**	**255780**	**261505**	**266151**	**269461**	**271362**
(4-(5+6))											
(8) Annual change	**na**	**10531**	**9853**	**9046**	**8376**	**7584**	**6716**	**5725**	**4646**	**3310**	**1901**

*Route A: All savings at retirement are taken as a lump sum, with lump sum tax paid. An annuity is purchased so that there is no money available for any other form of saving.

Table 4: Results route A*, earnings at 50th percentile

RETIREMENT AGE	55	56	57	58	59	60	61	62	63	64	65
(1) Gross annuity	334783	344473	353727	362000	370096	377479	384034	389630	394117	396711	398096
(2) Gross pension	142666	140261	137983	136076	134232	132737	131606	129317	124526	116853	105831
(3) Gross earnings	0	38265	76001	113150	149660	185480	220561	254862	288340	320960	352690
(4) Gross income	**477449**	**522999**	**567710**	**611226**	**653988**	**695695**	**736201**	**773809**	**806983**	**834525**	**856617**
(1+2+3)											
(5) Tax on earnings	0	9213	18180	26899	35347	43506	51357	58883	66069	72901	79367
(6) Tax in retirement	33790	36260	38741	41157	43607	46000	48333	50074	51588	52953	54294
(include lump sum tax)											
(7) Net income	**443659**	**477526**	**510789**	**543171**	**575034**	**606190**	**636512**	**664851**	**689326**	**708671**	**722956**
(4-(5+6))											
(8) Annual change	**na**	**33868**	**33262**	**32382**	**31864**	**31155**	**30322**	**28340**	**24474**	**19345**	**14285**

*Route A: All savings at retirement are taken as a lump sum, with lump sum tax paid. An annuity is purchased so that there is no money available for any other form of saving.

Table 5: Results route A*, earnings at 75th percentile

RETIREMENT AGE	55	56	57	58	59	60	61	62	63	64	65
(1) Gross annuity	895100	938532	982523	1025674	1070388	1102514	1133285	1163707	1193663	1221077	1248433
(2) Gross pension	108887	97385	85891	75136	61256	48006	35955	27218	20953	16520	12645
(3) Gross earnings	0	119148	238433	357628	476505	594841	712414	829010	944419	1058439	1170875
(4) Gross income	**1003988**	**1155065**	**1306847**	**1458438**	**1608149**	**1745361**	**1881654**	**2019935**	**2159035**	**2296036**	**2431953**
(1+2+3)											
(5) Tax on earnings	0	47071	94163	141168	187979	234489	280595	326194	371187	415479	458978
(6) Tax in retirement	123934	133059	142338	151494	160157	175933	193264	210525	227981	245529	262836
(include lump sum tax)											
(7) Net income	**880054**	**974935**	**1070346**	**1165777**	**1260014**	**1334939**	**1407795**	**1483216**	**1559866**	**1635028**	**1710139**
(4-(5+6))											
(8) Annual change	**na**	**94881**	**95411**	**95430**	**94238**	**74925**	**72856**	**75421**	**76650**	**75162**	**75112**

*Route A: All savings at retirement are taken as a lump sum, with lump sum tax paid. An annuity is purchased so that there is no money available for any other form of saving.

Table 6: Results route B*, earnings at 25th percentile

RETIREMENT AGE	55	56	57	58	59	60	61	62	63	64	65
(1) Gross annuity	124755	125655	126061	125754	124980	123507	121248	118102	113953	108500	101849
(2) Gross pension	76325	74809	73199	71528	69708	67768	65695	63472	61089	58596	55875
(3) Gross earnings	0	12289	24227	35805	47018	57861	68328	78419	88131	97463	106417
(4) Gross income	**201080**	**212753**	**223487**	**233087**	**241706**	**249136**	**255271**	**259992**	**263172**	**264560**	**264141**
(1+2+3)											
(5) Tax on earnings	0	979	1884	2713	3465	4140	4736	5252	5689	6046	6323
(6) Tax in retirement	0	0	0	0	0	0	0	0	0	0	0
(7) Net income	**201080**	**211774**	**221603**	**230374**	**238240**	**244996**	**250535**	**254741**	**257484**	**258514**	**257818**
(4-(5+6))											
(8) Annual change	**na**	**10695**	**9829**	**8771**	**7866**	**6756**	**5539**	**4205**	**2743**	**1031**	**-696**

*Route B: The decisions in route B involve taking all savings at retirement as an annuity, with no lump sum tax paid (so that there is no money available for any other form of saving).

Table 7: Results route B*, earnings at 50th percentile

RETIREMENT AGE	55	56	57	58	59	60	61	62	63	64	65
(1) Gross annuity	360094	371711	382864	392961	402872	412004	420228	427397	433336	437176	439661
(2) Gross pension	109636	102611	95116	87291	78760	69643	59870	49375	38095	26247	13312
(3) Gross earnings	0	38265	76001	113150	149660	185480	220561	254862	288340	320960	352690
(4) Gross income (1+2+3)	**469730**	**512587**	**553980**	**593403**	**631291**	**667126**	**700660**	**731633**	**759771**	**784384**	**805663**
(5) Tax on earnings	0	9213	18180	26899	35347	43506	51357	58883	66069	72901	79367
(6) Tax in retirement	1015	1274	1539	1789	2050	2305	2555	3342	5754	8512	11322
(7) Net income (4-(5+6))	**468716**	**502100**	**534261**	**564715**	**593895**	**621316**	**646748**	**669408**	**687948**	**702971**	**714974**
(8) Annual change	**na**	**33385**	**32161**	**30454**	**29179**	**27421**	**25432**	**22660**	**18540**	**15023**	**12003**

*Route B: The decisions in route B involve taking all savings at retirement as an annuity, with no lump sum tax paid (so that there is no money available for any other form of saving).

Table 8: Results route B*, earnings at 75th percentile

RETIREMENT AGE	55	56	57	58	59	60	61	62	63	64	65
(1) Gross annuity	994654	1045176	1096425	1146830	1199076	1251672	1304468	1357294	1409955	1459972	1510231
(2) Gross pension	8522	33	0	0	0	0	0	0	0	0	0
(3) Gross earnings	0	119148	238433	357628	476505	594841	712414	829010	944419	1058439	1170875
(4) Gross income (1+2+3)	**1003175**	**1164357**	**1334858**	**1504458**	**1675581**	**1846513**	**2016882**	**2186304**	**2354374**	**2518411**	**2681106**
(5) Tax on earnings	0	47071	94163	141168	187979	234489	280595	326194	371187	415479	458978
(6) Tax in retirement	46568	63155	79525	95599	112122	128651	148348	164063	195187	217689	240501
(7) Net income (4-(5+6))	**956607**	**1054131**	**1161171**	**1267691**	**1375480**	**1483373**	**1587939**	**1696047**	**1788000**	**1885243**	**1981628**
(8) Annual change	**na**	**97524**	**107039**	**106520**	**107789**	**107893**	**104566**	**108108**	**91953**	**97243**	**96384**

*Route B: The decisions in route B involve taking all savings at retirement as an annuity, with no lump sum tax paid (so that there is no money available for any other form of saving).

3.1.1 Examining the budget set results

A number of hypotheses can be tested against the generated results, which show the changes in income as retirement is postponed.

First, as retirement is postponed, income is expected to increase because additional earnings from work (together with the associated boost this provides to financing a private pension) will outweigh the net pension forgone. This trend may not hold for those on lower incomes because there is no actuarial adjustment to social security payments if retirement is delayed (such adjustment is common in other countries) and for those on very low incomes the age pension may be in excess of the salary which could be earned, especially in old age.

Second, it is expected that where there are positive gains from deferring retirement, these gains may not be uniform. This is because of the combined impact of additional earnings on eligibility for social security and the non-linear (that is, progressive rather than proportional) tax rates that apply to those earnings.

Third, because both routes A and B involve the purchase of a lifetime annuity financed from the savings made until the point of retirement, if the tax system is neutral the net income received in retirement would not be influenced by the route chosen.

We can look at these hypotheses in respect of each of the earnings profiles we have considered.

3.1.2 Earnings at the 50th percentile

Looking first at the man with median earnings, income always rises with the postponement of retirement under routes A and B (see Figure 4). Under route A the benefits from working an additional year decline as age increases. Under route B there is a "kink" after age pension age (not shown in the graph) due to the impact of the age pension income and assets test.

In general, route B generates a higher net income for the individual than route A, although over the "kink" in the budget set for route B (age 64 to 68), route A provides a higher income. This is due to the differential tax treatment of the superannuation savings, which affects age pension entitlements.

Figure 4: Net income for alternative retirement ages: earnings at 50th percentile

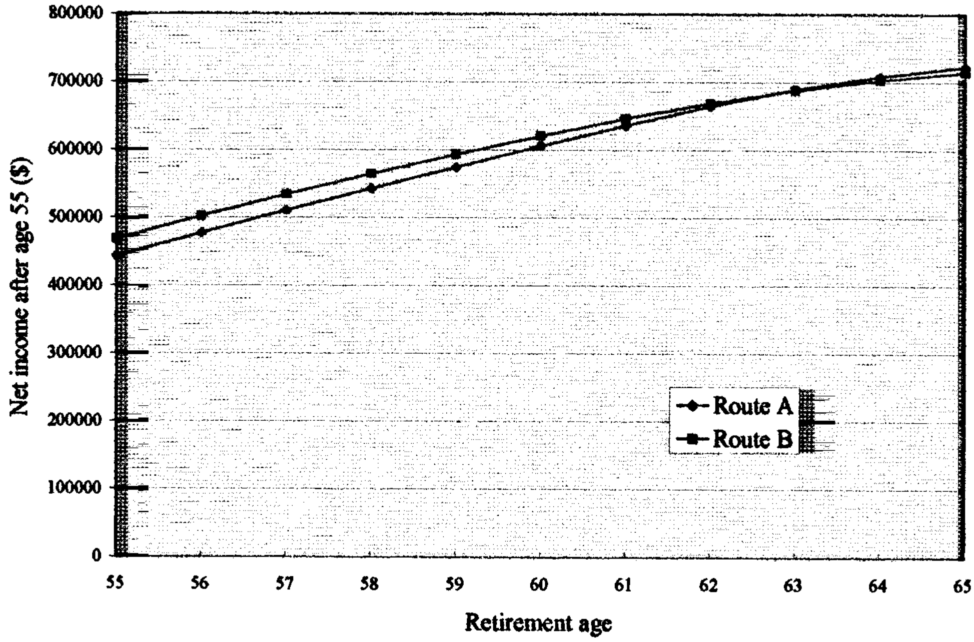
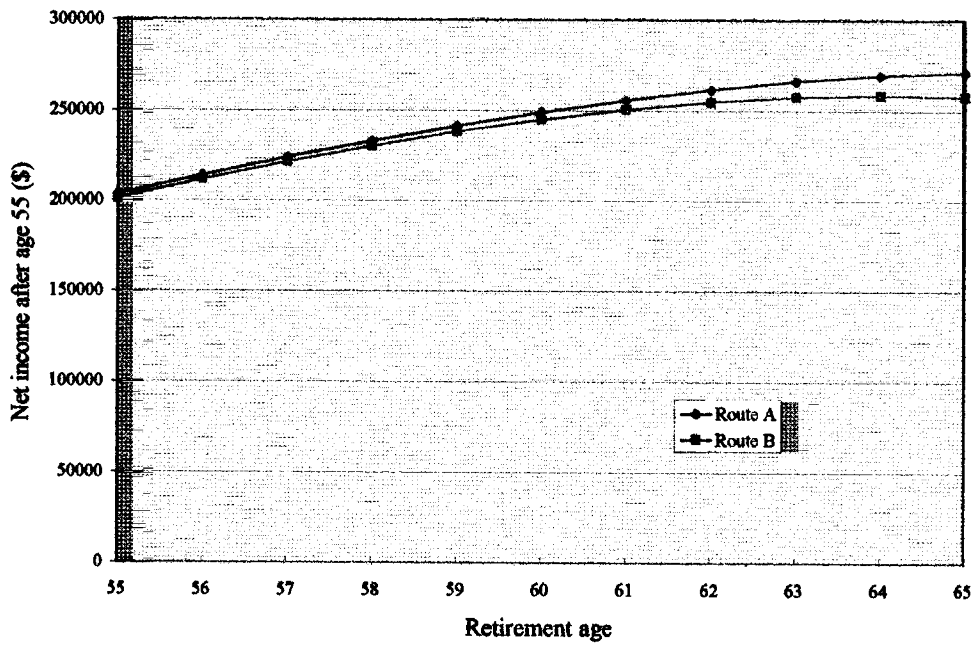


Figure 5: Net income for alternative retirement ages: earnings at 25th percentile



3.1.3 Earnings at the 25th percentile

The main feature of the results generated for the individual at the 25th percentile is that postponing retirement beyond age 65 produces a net loss of income (see Figure 5). That is, income is maximised if the person retires at age 65 for route A and age 64 for route B. This is due to the fact that, while total earnings continue to rise as retirement is postponed, the rise in earnings is not as large as the loss of age pension receipts, and the size of the annual annuity generated from the savings accumulated by age 65 exceeds the annual earnings which could be earned beyond that age. This is partly due to the assumed earnings profile for this group, which sees annual earnings declining later in life.

The loss of the age pension may not be as large as suggested by the reported results because the model assumes that if the individual is working he is not eligible for a part pension. It is likely that earnings at this level would not rule out payment of a part pension.

Net income increases as retirement is delayed between age 55 and age 65 (route A) or age 64 (route B). Nevertheless, the net benefits from working beyond age 55 are relatively moderate (averaging around $6,000 per annum under both routes), and the net gains get progressively smaller as the age of retirement rises. In fact, the size of the annual annuity payment generated by the savings accumulated is almost equal to possible earnings from age 60. This would suggest that if there is even a moderate preference for leisure over work, early retirement is relatively attractive.

The attractiveness of early retirement is enhanced still further if other factors which are not incorporated into the model are taken into account. In particular, receipt of an age pension provides eligibility for a range of fringe benefits, including pharmaceutical allowance and rent assistance (if renting) and telephone concessions. Further, State and local governments commonly provide transport and rates concessions to age pensioners. These benefits are very difficult to estimate, but Gruen (1985) estimated that in 1981-82, fringe benefits added 17-31 per cent to the value of the age pension for the "average" pensioner.

Possible eligibility for the mature age allowance (MAA) from age 60 could also make earlier retirement attractive for people on lower incomes. The MAA is available to people over the age of 60 provided they have been in receipt of a social security payment for at least 12 months and have been registered with the Commonwealth Employment Service as looking for work for that period. The income and assets tests are the same as apply to the age pension, and the payments are of the same amount as the age pension.

The results for a male with earnings at the 25th percentile also show that net income is higher under route A (where lump sum tax is paid and the remaining cash is used to purchase an annuity) than route B (where no lump sum tax is paid, and the entire savings are used to purchase an annuity), even though there is no tax paid in retirement under route B. This is because of the difference in the tax treatment of the superannuation savings under routes A and B, and the impact this has on age pension entitlements.

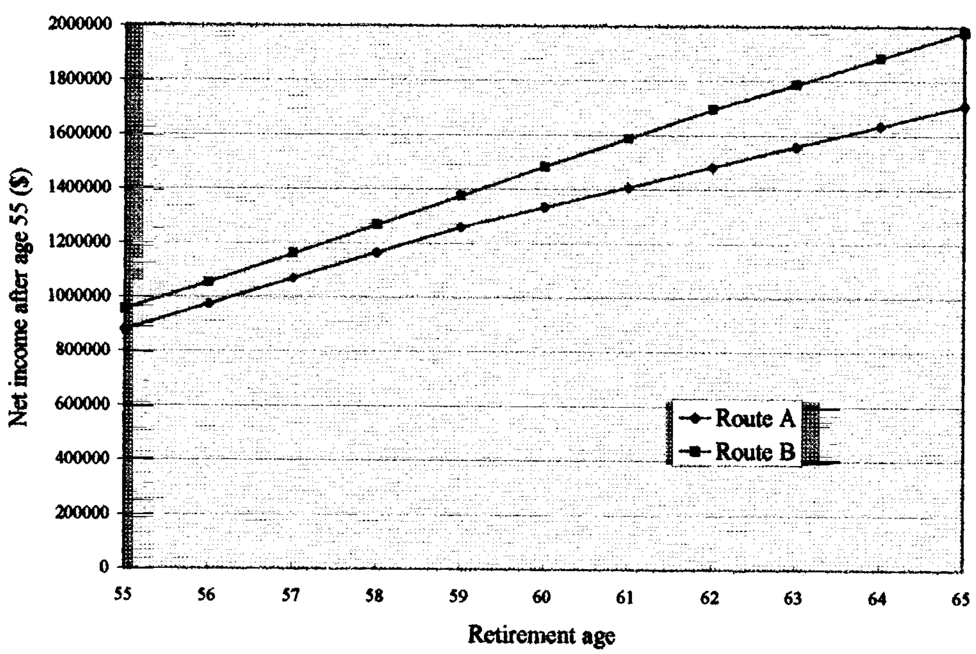
3.1.4 Earnings at the 75th percentile

Like the 50th percentile case, the net income of the male with earnings at the 75th percentile always rises with the postponement of retirement under routes A and B (see Figure 6).

As with the other earnings profiles there is a large variation in the benefits from working for a particular year, with the net addition to income tending to fall as retirement age rises. The dominant (and in some instances, only) source of income in retirement for retirees with this earnings profile is the annuity. Such results are due to the flat rate, means-tested, age pension. The dominance of the annuity reflects the assumed higher than average life expectancy of those on higher earnings and the larger annuity which is generated because of the higher earnings.

The results for a male with earnings at the 75th percentile also show that at whatever age he retires, net income is always higher under route B than route A, even though there is no age pension received under route B unless retirement occurs at age 55 or 56. Again, this is a reflection of the tax treatment of the superannuation and the interaction with the age pension system.

Figure 6: Net income for alternative retirement ages: earnings at 75th percentile



3.1.5 Summary of budget set results

The results generally confirmed the first two hypotheses, but not the third hypothesis

i. as a rule, net income increases as retirement is postponed. The exception to this is for low income earners beyond age pension age, when net income falls as retirement is deferred
ii. there is variation in the income generated from working for any particular year, with net income tending to increase at a decreasing rate - this reflects the assumed earnings profiles and the impact of the income and assets tests on the age pension
iii. the route chosen has some influence on the expected income in retirement. In terms of generating the highest net income in retirement, the preferable route varies with retirement age.

3.2 Generating utility functions

The next step involves constructing a utility function to solve for the optimal retirement age for any given budget constraint. As discussed below, the optimisation rule is relatively straightforward for an individual, but is far more complex if decisions are to be made jointly by a couple. This is one of the reasons for limiting the analysis to single males, rather than the more common couple situation. Sensitivity analysis of the retirement age solution can then be conducted for a range of parameters underlying the utility function. Such analysis provides an indication of behavioural changes as a result of policy changes impacting on the budget constraint.

As with the budget sets, there are no empirical data on which to draw to specify the relative utility from additional earnings compared to more leisure (longer retirement). The methodology employed here uses a parametric approach to specifying a utility function based on the Cobb-Douglas function:

$$U_i = NPVY_i^{\alpha_i} \cdot L_i^{1-\alpha_i} \qquad (5)$$

where U is utility, NPVY is net present value of income in retirement, and L is leisure (or period of retirement).

The indifference curve implied by the equation has a slope defined by

$$\frac{dNPVY_i}{dL_i} = \left(\frac{1-\alpha_i}{\alpha_i}\right)\frac{NPVY_i}{L_i} \qquad (6)$$

If empirical data were available, it would be possible to solve for α_i:

$$\alpha_i = 1/\left[1 + \frac{(dNPVY_i)}{NPVY_i} / \frac{(dL_i)}{L_i}\right] \qquad (7)$$

If α_i could be determined empirically, this could be substituted back into the equation for any (NPVY, L) pair, so that indifference curves could be mapped for each person.

To determine the optimal retirement age we can draw on the NPVY and L pairs generated for the budget sets and substitute those back into the equation. A range of α_i can be examined. Retirement age is optimal when utility is maximised.

For example, Table 4 shows the net present value of retirement income for a man on median earnings retiring from age 55 to age 65. We can take the net income figures and pair these with the assumed period of leisure (years in retirement). It is assumed that a man on median earnings will survive for 14 years at age 65. For each year that he retires later or earlier than this, a year is subtracted or added to give us the years of leisure.

At age 57, for example, his net income is $510,789 and leisure is 22 years. Putting these figures into the Cobb-Douglas equation, and using an α of 0.4 gives a utility of 1227. When this is ranked with the utility generated by the other income/leisure pairs in Table 4, the highest utility corresponds to retirement at age 57. This means that age 57 is the optimal retirement age, given an α of 0.4.

To construct the indifference curve, it is then a matter of using the fixed utility of 1227, while varying the years of leisure (in our example, the years of leisure range from 24 years if retiring at age 55 to 9 years if retiring at age 70). This allows the generation of net income figures for this given utility. Figure 7, Figure 8 and Figure 9 show some results.

3.2.1 Choosing a value for α

Given the use of a Cobb-Douglas utility function, the choice of a value for the single parameter α is crucial to determining the optimal retirement age. In the formulation of the Cobb-Douglas function used in this paper, when in equilibrium the parameter α can be interpreted as the proportion of total utility comprised of income (and conversely, the share of total utility from leisure is $1-\alpha$). Given this, a reasonable range of values for α would appear to be of the order of 0.3 to 0.7.

Empirical data from the US, cited in Fields and Mitchell (1984, pp 90-91), suggests an α value of around 0.4. That is, the average individual valued a percentage increase in income relatively less than the same percentage increase in leisure years during retirement. While this mean value was consistent across two very detailed data sets, it also indicated a wide degree of variation around the mean - older persons differed greatly in the relative weights they assigned to income and leisure.

For the purposes of our model, utility rankings were generated using an α value of 0.4. The income and leisure parameters developed when generating the budget sets for route A were used. The results appeared reasonable and are consistent with retirement being taken early by the majority of the population. For this reason, the following charts illustrate the results generated by an α of 0.4. Sensitivity to this choice is tested.

4. OPTIMAL RETIREMENT AGE - RESULTS

4.1 Model results

4.1.1 Earnings at 50th percentile

The results for a man with earnings at the 50th percentile are shown in Figure 7. It uses the net income figures from Table 4 (the budget set from route A) and a utility function based on an α of 0.4. As shown, the optimal retirement age is 57 years of age for a man with median earnings.

4.1.2 Earnings at 25th percentile

Figure 8 plots the budget set (net income) from route A shown in Table 3 and the utility function using an α of 0.4. Figure 8 shows that the optimal retirement age is 55 for someone with earnings at the 25th percentile.

The indifference curve rises relatively steeply for this group because life expectancy is lower than for the other earnings profiles. This is because life expectancy is linked to the level of lifetime earnings. The indifference curve therefore reflects that the opportunity cost of foregone leisure is very high later in life.

In fact, as was discussed with the budget set results alone, it is never optimal for a person with this earnings profile to retire beyond the age of 65 (that is, even with an α value of 1, it is optimal to retire at age 65).

4.1.3 Earnings at 75th percentile

Figure 9 shows the results for a man with earnings at the 75th percentile (Table 5). As shown, the optimal retirement age with an α of 0.4 is 59 years of age.

Figure 7: Optimal retirement age, earnings at the 50th percentile, $\alpha = 0.4$

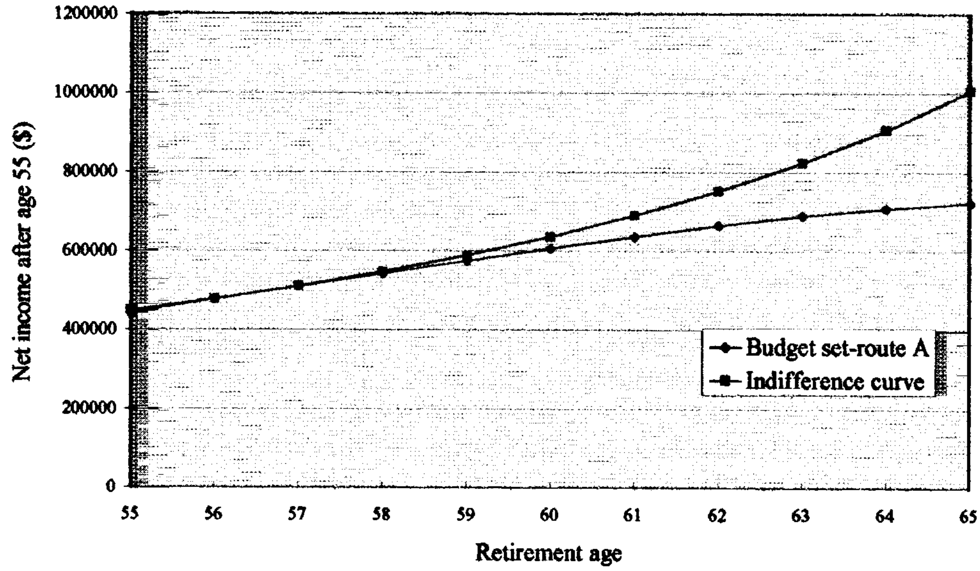


Figure 8: Optimal retirement age, earnings at 25th percentile, $\alpha = 0.4$

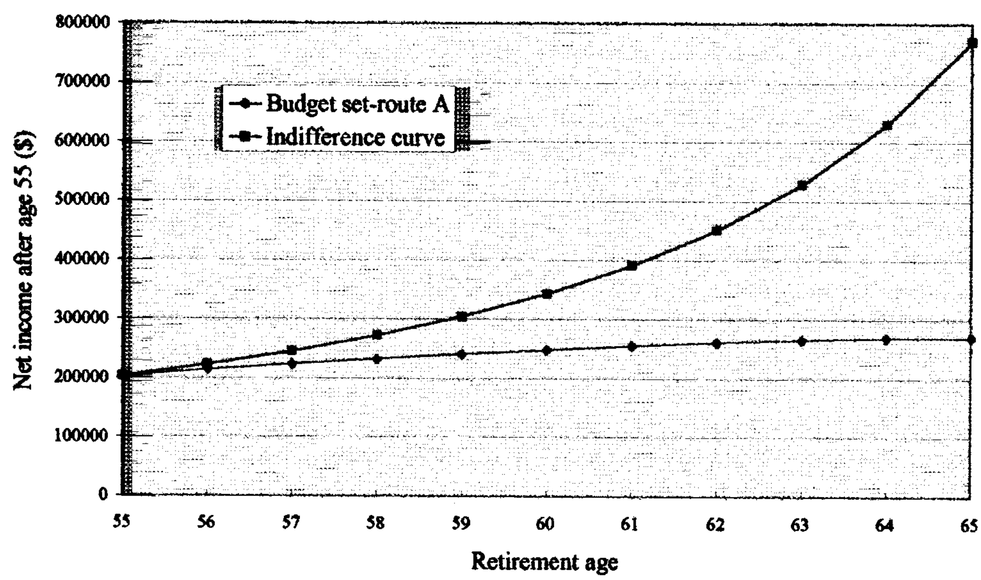
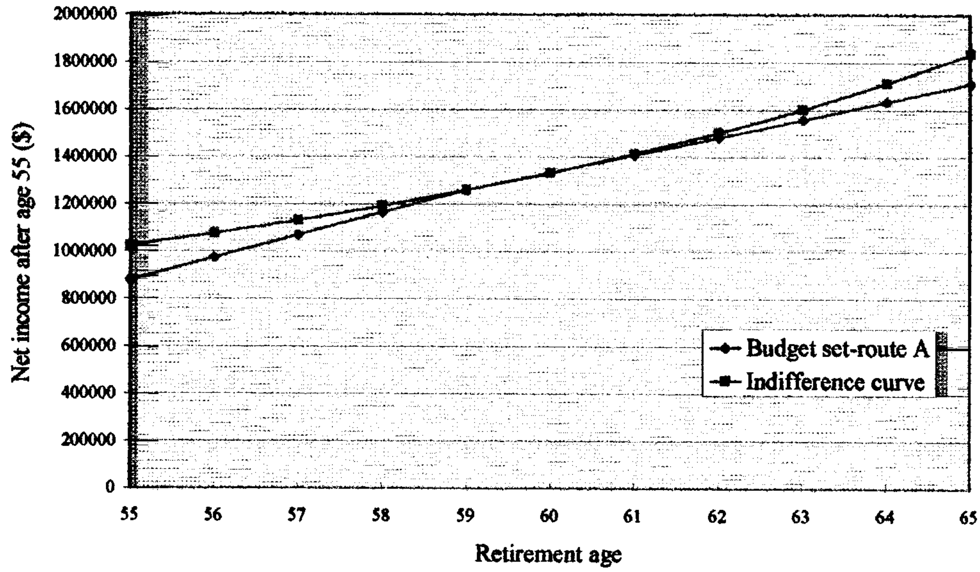


Figure 9: Optimal retirement age, earnings at 75th percentile, α = 0.4



4.2 Sensitivity of results

4.2.1 Sensitivity of the results to α

The sensitivity of the results to the choice of α is evident in 9 and Table 10. High α values are required to lift the retirement age to 65, even for the man with earnings at the 75th percentile. Comparing 9 and Table 10 also shows that route A tends to produce marginally higher retirement ages for the 25th and 50th percentile earnings profiles, while route B provides the higher retirement ages for the 75th percentile. This is expected, given that these are the respective routes which provide the higher retirement income for each profile, while the period in retirement is consistent across the routes.

Table 9: Optimal retirement age, route A - sensitivity to the value of α

Value of alpha	Optimal retirement age - route A		
	25th percentile	50th percentile	75th percentile
0	(a)	(a)	(a)
0.1	(a)	(a)	(a)
0.2	(a)	(a)	55
0.3	(a)	55	58
0.4	(a)	57	59
0.5	55	60	64
0.6	57	62	68
0.7	59	64	70
0.8	61	65	(a)
0.9	63	68	(a)
1	65	70	(a)

(a) The ages in these cells could fall outside the tested range (age 55 to 70).

Table 10: Optimal retirement age, route B - sensitivity to the value of α

Value of alpha	Optimal retirement age - route B		
	25th percentile	50th percentile	75th percentile
0	(a)	(a)	(a)
0.1	(a)	(a)	(a)
0.2	(a)	(a)	55
0.3	(a)	55	58
0.4	(a)	56	62
0.5	55	59	65
0.6	57	61	68
0.7	59	63	70
0.8	61	64	(a)
0.9	62	69	(a)
1	64	70	(a)

(a) The ages in these cells could fall outside the tested range (age 55 to 70).

4.2.2 Sensitivity of the results to the mortality assumptions

The results shown have been based on differential mortality, with the age at death calculated with regard to the individual's effective earnings experience. The age at death reflects the tendency for individuals of relatively higher total lifetime earnings to experience lower mortality rates.

To test the sensitivity of the results to this differential mortality, the scenarios for route A were rerun using common mortality (the mortality used for the man on median earnings). The results proved very robust to the mortality assumptions used.

For example, for the man with earnings at the 25th percentile, the same optimal retirement age of 55 resulted for α values of 0.1 to 0.5, whichever mortality assumption was used. For higher α values, common mortality produced slightly later optimal retirement ages than differential mortality.

For the 75th percentile, common mortality tends to produce slightly lower retirement ages (for example, common mortality produces retirement at age 70 at an α of 0.8, but this is reached at an α of 0.7 under differential mortality). At an α of 0.4 the optimal retirement age was 59 regardless of whether common or differential mortality was used.

The other feature of the mortality assumed above is that life expectancy at age 65 was taken as the "pivot". For example, for a man on median earnings, life expectancy at age 65 was assumed to be 14 years (based on the 1985-87 Australian Life Tables), so if retirement were taken at age 65, it was assumed that there were 14 years in retirement. For earlier or later retirement, the expected years of retirement were adjusted so that, for this man, if retirement was taken at age 60, then the years in retirement were assumed to be 19 years (that is, the age of retirement is reduced by 5 years, so the years of retirement are also adjusted by 5 years). Similarly, if retirement was taken at age 70 then the years in retirement were assumed to be 9 years. This was a simplifying assumption because the LITES model requires an integer for life expectancy to operate. However, if a person lives beyond the age of 65, then life expectancy beyond that age increases relative to that expected at age 65 (and vice versa for younger ages). Using our example again, based on the Australian Life Tables life expectancy at age 60 is 18 years and at age 70 is 11 years, compared with the 19 and 9 years respectively used in the model. Because 65 is close to the middle of the retirement ages tested (being 55 to 70), using 65 years minimises the extent to which the actual life expectancy varies from that used in the model.

The sensitivity of the results to mortality assumptions make was not expected to be significant, since the much larger change from differential to common mortality did not alter the results significantly. Nevertheless, to test the sensitivity of the results to this assumption of a 65 year "pivot", route A was rerun using the life expectancy at age 55 as the "pivot". Age 55 was used because it represented one extreme, being the youngest possible retirement age in the model, and because many of the results clustered close to younger ages. The results showed that, when using 55 rather than

65 as the pivot, the optimal retirement age for a man on median earnings did not vary by any more than one year for α values up to 0.8, and never exceeded two years.

In summary, the results proved very robust to assumptions about mortality.

4.2.3 Sensitivity of the results to the savings rate in addition to compulsory superannuation

The results presented for both routes A and B assumed that savings of 5 per cent of net salary would be made in addition to the compulsory saving. This was added to the accumulated compulsory saving at the point of retirement to fund the annuity. For lower and middle income earners in particular, this may be regarded as a high level of additional saving - especially as the runs assume that the man arrives at retirement as a home owner (although the only impact this has on the results relates to the assets test used).

To test the sensitivity of the results to the assumed additional savings, route A was rerun with voluntary saving reduced from 5 per cent of net salary to 0 per cent. Using this new assumption, the optimal retirement ages did not change for the 25th and 50th percentile if α was 0.4. At that α value, the optimal retirement age increased by 1 year for the man with earnings at the 75th percentile (from age 59 to age 60). That is, the results proved to be relatively insensitive to the assumed voluntary savings rate. The sensitivity at lower α values for those on higher incomes was of little concern given the likely higher propensity to save of higher income earners - that is, the assumption of additional savings of 5 per cent is more likely to hold in the case of higher income earners.

5. CONCLUSIONS

The aim of this paper was to describe retirement patterns in Australia, and to develop a model to test the likely impact of retirement incomes policy, including a fully mature SGC, on voluntary retirement ages.

The results suggest it is unlikely that, under current policy settings, we would see a return to most men working to the "standard" retirement age of 65. An α of 0.8 would be required before all of those earning above the median income would be expected to work beyond age 65 - an extreme value. It will never be optimal for those on earnings at the lower quartile to retire later than age 65 in the absence of rewards other than additional income.

These findings appear consistent with current trends. At present, it is common for men to retire before age 65, even though their level of financial assets is, on average, much lower at the point of retirement than will be the case once the SGC matures (see Gallagher et al 1993 (pp 9-13) for a discussion of estimates of asset holdings). The model developed in this paper suggests that a fully mature SGC is likely to exacerbate the trend to early retirement since it is deliberately structured to force a higher level of income in retirement than would otherwise be the case. Access to SGC entitlements (including preservation rules and rules relating to the form in which benefits may be taken) will be crucial in determining the extent to which those entitlements influence the choice of retirement age.

Another interesting feature of the results is the non-neutral impact of the tax system in relation to the use of financial assets in retirement. While both routes A and B are consistent with the stated Government goal of encouraging people to take annuities rather than lump sums at retirement, it is apparent that the operation of the tax rules is not neutral between these choices - but whether one route is preferable to the other will depend on expected future income and the age of retirement. There would not appear to be any policy reason for preferring route A over route B or vice versa. The fact that the choice the individual makes produces different outcomes suggests that anomalies could arise. It highlights that, despite its complexity, the tax system fails to produce the desired neutral results.

APPENDIX 1: ASSUMPTIONS USED IN THE LITES MODEL

A1. ECONOMIC ASSUMPTIONS

Annual inflation rate	5%
Annual increase in AWOTE	6%
Annual increase in income tax thresholds	5.5%
Gross annual investment rate of return on superannuation accumulation	9%
Gross annual investment rate of return on savings accumulation	7%
Tax on superannuation fund investment income	7.5%
Tax on savings fund investment income	25%

A2. ANNUITY RATES AND LIFE EXPECTANCY

Male Age	Annuity rate (a)	Adjusted annuity rate (b)	Expectation of life (c)
55	15.38	17.15	22.04
56	14.96	16.68	21.23
57	14.54	16.21	20.43
58	14.13	15.76	19.65
59	13.71	15.29	18.88
60	13.29	14.82	18.13
61	12.87	14.35	17.39
62	12.45	13.88	16.67
63	12.03	13.41	15.96
64	11.62	12.96	15.27
65	11.21	12.50	14.60
66	10.79	12.03	13.93
67	10.39	11.59	13.29
68	9.98	11.13	12.66
69	9.58	10.68	12.05
70	9.19	10.25	11.46

(a) Annuity rates generated from Advance model.
(b) Adjusts the figures in (a) in proportion to the market rate current during the 12 months to June 1994 as reported in the Rice/Kachor Research Rollover/Annuity League Table for a man aged 65.
(c) Life expectation as set out in the 1985-87 Australian Life Tables.

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59	MAY 1998	THE MULTIVARIATE DE PRIL TRANSFORM	Bjørn Sundt
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