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Initial Capital and Margins Required to Secure a Japanese Life Insurance Policy Portfolio Under Variable Interest Rates

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Initial capital and margins required to secure a Japanese life insurance policy portfolio under variable interest rates

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Abstract

In recent years, several large Japanese life insurance companies have failed due to deficits on interest earnings. These interest deficits were caused by actual investment returns falling below interest rates used for premium calculations and have also been damaging the financial condition of life insurance companies which have not failed. When a life insurance policy portfolio consists of a large number of similar policies on independent lives, random fluctuations in mortality will have a relatively minor overall effect in relation to the business in force. However, the same interest rate risks apply to all policies and consequently interest rate risk is far more important than random fluctuations in mortality. In the past, investment returns on the reserves of Japanese life insurance policy portfolios have been closely linked to market interest rates which are volatile. The projection of net asset values of a hypothetical Japanese life insurance policy portfolio were therefore simulated using stochastic interest rate models. For this purpose, the Heath-Jarrow-Morton approach was chosen, modified to accommodate the very low interest rate environment in Japan. The results suggest that current premium bases may not be sustainable because considerable initial capital or large profit margins in premiums will be required to provide a reasonable level of protection against future insolvency.

1. Introduction

In March 2001, the Tokyo Mutual Life Company became insolvent and filed an application under the Japanese Reorganization and Rehabilitation Act. It was the seventh successive life insurance company failure during the current lingering stagnant economy in Japan. The data in Table 1.1 illustrate the magnitude of these failures. The total gross assets (or policyholders' funds) held by the failed companies exceeded the Yen equivalent of AUD 200 billion.

Table 1.1 - Failed Japanese life insurance companies

Company Name	Failure Date	Gross Asset Value (Yen trillion)	(In AUD[1] billion)
Tokyo Life	March 2001	1.1	17
Kyoei Life	October 2000	4.6	73
Chiyoda Life	October 2000	3.5	56
Taisyo Life	August 2000	0.2	3
Daihyaku Life	May 2000	2.2	35
Toho Life	June 1999	2.8	44
Nissan Life	April 1997	Unavailable	-

Source: *the Financial Services Agency (FSA hereafter) (2001a)*

The Japanese economy has been stagnant during the last decade after the stock market declined sharply from its peak in 1989 – an event known as the "collapse of the bubble". The failed companies had been suffering from interest losses, which arise when the actual rates of return achieved on policyholders' reserves are lower than the interest rate used to calculate premiums (the "premium basis rate").

The surviving Japanese life insurance companies are far from being in a healthy financial condition. A recent investigation conducted by the FSA revealed that all life insurance companies were still being affected by severe interest losses. In The Daily Mainichi (2001), it was reported that:

> *"According to the summary by the FSA, the total interest loss of the whole life insurance industry amounted to Yen 1.4-2.7 trillion (AUD 22-43 billion) between the fiscal periods ending March 1998 and March 2000. However, that amount of loss was fully covered by the expense profit and mortality profit. The net operating profit amounted to Yen 1.2-2.2 trillion (AUD 19-35 billion)."*

[1] *Using the exchange rate 63 YEN/AUD*

These figures clearly show that the industry has been exposed to severe interest rate risk. Management of this risk is clearly an urgent task for the Japanese life insurance industry, regardless of the formula for calculating regulatory reserves.

2. The nature of the risk profile of life insurance policy portfolios

Under normal social and economic circumstances the major risks confronting life insurance companies are mortality, interest rates and expenses. We now consider the interest rate risk and mortality risk when a life insurance company has a large number of policies in force which share similar terms and conditions. In practice, Japanese life insurance companies have been marketing standardized products such as whole-life-with-term assurances.

There are two aspects to mortality risk, the emergence of changes or events which affect the underlying probabilities of mortality and random fluctuations. The risk of non-random changes to mortality rates, or events which have widespread effects, is a problem that must be borne by insurers. It cannot be eliminated by holding a large number of policies on independent lives, even though life insurance companies may seek to transfer some or all of this risk to reinsurers. However, insurers can usually mitigate this risk by building margins for adverse mortality into their premium calculations. On the assumption that margins for adverse mortality are adequate, we do not deal with this type of mortality risk in our study. We also ignore random fluctuations. This type of mortality risk is small. When there are large numbers of similar policies on independent lives, the observed mortality rate (or the number of deaths per unit of business in force) will be close to the underlying mortality rate.

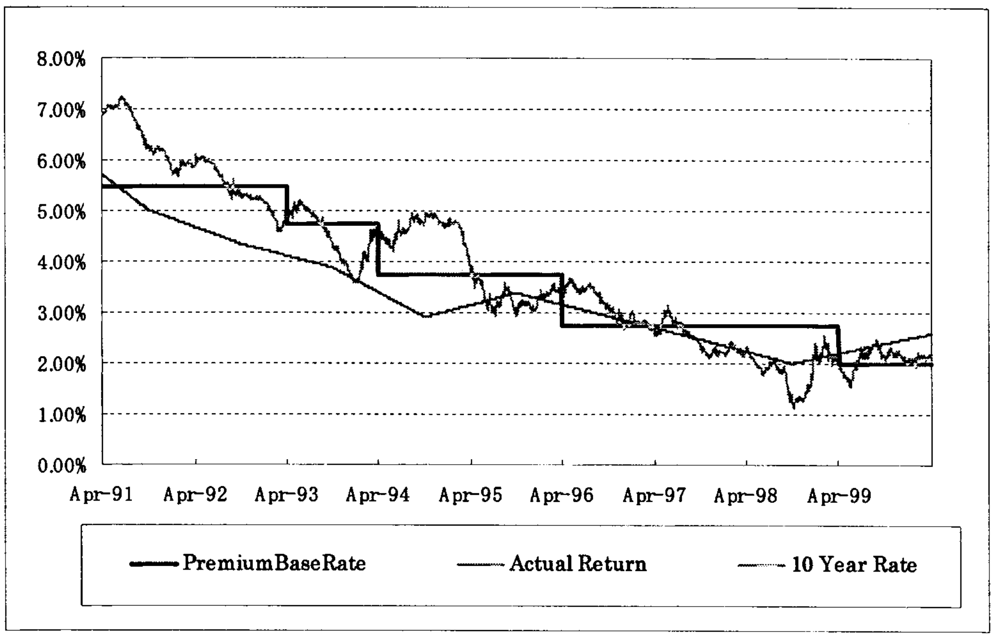
However, interest rate risk (per unit of business in force) cannot be reduced by increasing the number of policies on issue since every policy has the same or similar investment risk.

When similar types of policy are held by large numbers of independent policyholders, interest rate risk is far more important than the risk of random mortality fluctuation. Assuming an appropriate mortality table, a portfolio of such policies can effectively be regarded as a fixed income portfolio exposed to interest rate risk with random fluctuations in mortality ignored for the purpose of practical risk investigation provided the number of policies is large and policies are independent with respect to mortality risk.

3. Modelling framework

In April 1999, the FSA adopted a new rule[2] under which the premium basis rate is determined by the yield on 10-year government bonds. We observe in Figure 3.1 that the premium basis rate[3], which is used as the discount rate in premium calculation, has been close to the market long-term rate (10-year swap rate) for the period April 1991 to April 2000.

Figure 3.1 - Premium basis rates, actual returns and market rates



Source: *Bloomberg online data service, FSA (2001b)*

We also observe that the actual return is close to the market interest rates. In other words, the net asset value of a Japanese life insurance portfolio is exposed primarily to the risk of interest rate fluctuations. Therefore, we will use a stochastic interest rate model as the tool to analyse the variability of the net asset value of a Japanese life insurance portfolio.

We now introduce some notation which is common in financial economics. See, for example, Neftci (2000, Chapter 18). In the following definitions, the time unit is always one year.

Let $B(t,T)$ denote the price at time t of a zero coupon bond, that is redeemable at time T with unit redemption amount.

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2 *Ministry Of Finance Decree 48, article 4*

3 *The premium basis rate is determined by the FSA, according to Ministry Of Finance Decree 48.*

Let $Y(t,T)$ denote the continuously compounded spot yield at time t for duration $T-t$. It is given by

$$Y(t,T) = \frac{-\log B(t,T)}{T-t} \tag{3.1}$$

We use the word "yield" here in order to explicitly distinguish $Y(t,T)$ from the instantaneous spot rate which is defined below.

For $t \le S < T$, let $F(t,S,T)$ denote the continuously compounded forward rate at time t for the period $[S,T]$. It is given by

$$F(t,S,T) = -\frac{\log B(t,T) - \log B(t,S)}{T-S}.$$

Thus, $F(t,S,T)$ represents the yield of a zero coupon bond at time t, which will be issued at time S and which will mature at time T.

Let $f(t,T)$ denote the instantaneous forward rate at time t with maturity time T. It is given by

$$f(t,T) = \lim_{S \to T} F(t,S,T).$$

Let r_t denote the instantaneous spot rate at time t, and be given by

$$r_t = \lim_{T \to t} f(t,T).$$

Lastly, a set $\mathbf{Z}_t$ whose elements are identified by non-negative real numbers denotes the yield curve at time t, so that $\mathbf{Z}_t = \{Y(t,T),\ T \in [t,\infty)\}$.

We model all these quantities as random variables.

If we know forward rates, $f(t,u)$ for $t \le u$, then by definition, the zero coupon bond price is given by

$$B(t,T) = \exp\left[-\int_t^T f(t,u)du\right].$$

This formula tells us that forward rates are what is known as the force of interest in actuarial circles. Alternatively, if we know both the current instantaneous spot rate, r_t, and all future random instantaneous spot rates r_u for $t < u$ then in the risk-neutral world

$$B(t,T) = E\left[\exp\left[-\int_t^T r_u du\right]\right].$$

See, for example, Neftci (2000, p391).

In general, the no arbitrage condition requires a variable called "the market price of risk", which represents the degree of compensation for taking the risk of interest rate fluctuations. We conduct our study under the risk-neutral assumption, i.e. the market price of risk is assumed to always be zero. This is the practical choice to deal with interest rate term structure in a variety of economic environments. In addition, this assumption is consistent with the expectations theory of interest rate term structure. See, for example, Hull (2000, p663).

Vasicek (1977) proposed one of the most fundamental models for the term structure of interest rates. In this model, the instantaneous spot rate is modelled as a mean reverting process with

$$dr_t = a\left(b - r_t\right)dt + \sigma dW_t,$$

where r_t is the instantaneous spot rate at time t, a, b and σ are constants, and $\{W_t\}$ is a standard Wiener process.

It is well known (see, for example, Rolski et al (1998, p562)) that, for a given r_0,

$$r_t = b + (r_0 - b)e^{-at} + \sigma\int_0^t e^{-a(t-s)}dW_s \ .$$

It then follows that:

$$E[r_t] = b + (r_0 - b)e^{-at} \ \text{ and } \ Var[r_t] = \sigma^2 \frac{1 - e^{-2at}}{2a}$$

and the volatility of the price of the zero coupon bond with duration t is given by

$$\sigma \frac{1 - e^{-at}}{a}.$$

See Vasicek (1977).

Heath et al (1992) introduced what is commonly known as the Heath-Jarrow-Morton (HJM) approach. This approach models forward rates as random variables that drive an arbitrage-free stochastic development of yield curves, as opposed to classical approaches where instantaneous spot rates are modelled. (See, for example, Neftci (2000, p435).) There are several ways to represent the HJM approach. Hull (2000) starts his treatment of the HJM approach with

$$dB(t,T)/B(t,T) = r_t dt + \sigma(t,T,B(t,T))dW_t^T \tag{3.2}$$

where $\{W_t^T\}$ is the standard Wiener process driving the bond maturing at time T and $\sigma(t,T,B(t,T))$ is the volatility of the price at time t of a bond maturing at time T when the bond price is $B(t,T)$ at time t.

In equation (3.2), the instantaneous spot rate is the drift coefficient. This choice is equivalent to assuming risk neutrality, where all bonds share the same instantaneous risk-free return, regardless of their maturities and associated price risks. It should also be noted that the Wiener process is indexed by T. This means that, in principle, bonds with different maturities are allowed to be influenced by different shocks.

In the HJM approach, the development of a yield curve starts from the known current yield curve. There is therefore no initial fitting problem. Another advantage of this approach is that there is great flexibility in specifying volatility parameters.

Cairns' (1998) model will be used to describe a yield curve. The model is

$$f(t,t+s) = b_0 + \sum_{i=1}^{4} b_i \exp\{-c_i s\} \tag{3.3}$$

where $s > 0$ and, $\{b_i\}_{i=0}^{4}$ and $\{c_i\}_{i=1}^{4}$ are constants.

By straightforward integration

$$B(t,T) = \exp\left[-\left[b_0(T-t) + \sum_{i=1}^{4} b_i(1-\exp\{-c_i(T-t)\})/c_i\right]\right]. \tag{3.4}$$

The choice of $(c_1, c_2, c_3, c_4) = (0.2, 0.4, 0.8, 1.6)$ was found to be appropriate for the UK gilt market for the period 1992 to 1996. See Cairns (1998) for details.

4. Yield curves in a very low interest rate environment

The Bank of Japan (BOJ) introduced a "zero rate policy" on 9 September 1998. Since then, Japanese short-term interest rates have been hovering just above zero as illustrated Figure 4-1.

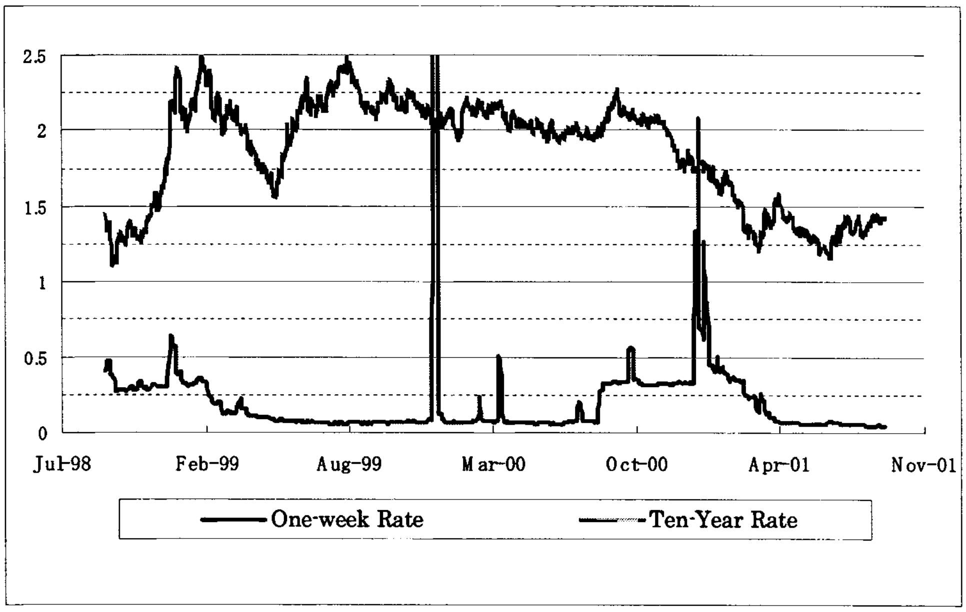
In reviewing the monetary policy framework of the BOJ, Oda and Okina (2001) applied a model under which various factors determine long-term interest rates. They modelled R_t, the long-term risk-free interest rate at time t, as

$$R_t = \sum_{j=0}^{n} \alpha_j E[SR_{t+j}] + \theta_t,$$

where n is the maximum maturity term, SR_t is the short-term risk-free interest rate at time t, θ_t is the risk premium, and $\{\alpha_j\}$ is a set of weights.

Under this model, long-term interest rates are determined as a weighted average of future expected short-term interest rates and a risk premium. Analysing likely options for further monetary policy, Oda and Okina (2001) concluded that the scope for any further reduction in expected short-term interest rates and/or the risk premium is limited. Therefore, we assume that there is a lower bound for long-term interest rates in a zero rate policy environment.

Figure 4.1 - Interest rates in Japan (September 1998 – September 2001)



Source: *Bloomberg market data online service*

5. The interest rate models used in our study

5.1 Models adopted

Cairns' model and the HJM approach were chosen for our study. Cairns' model was used to graduate the initial yield curve with the parameters $(c_1, c_2, c_3, c_4) = (0.2, 0.4, 0.8, 1.6)$ in formulae (3.3) and (3.4). The HJM approach (modified to handle zero interest rates) was used to obtain developments of the initial term structure using Monte Carlo simulation. We formulated the evolution of the initial term structure under the general one-factor HJM framework adapted to discrete time intervals of length Δt. Here, we assume that the volatility of both a bond price and the instantaneous spot rate remain constant during the period $(t, t+\Delta t)$ and that the fixed-income market operates in a risk-neutral environment. Then

$$\begin{aligned} B(t+\Delta t, T) &= B(t,T)\cdot \exp\left\{\left(r_t - \frac{1}{2}\sigma(t,T,B)^2\right)\Delta t + \sigma(t,T,B)\Delta W_t\right\} \\ &= B(t,T)\cdot \exp\left[r_t \cdot \Delta t\right]\cdot \varepsilon_t, \end{aligned} \tag{5.1}$$

where $\Delta W_t = W_{t+\Delta t} - W_t$ and ε_t is a random error:

$$\varepsilon_t = \exp\left[-\frac{1}{2}\sigma(t,T,B)^2 \cdot \Delta t + \sigma(t,T,B)\cdot \Delta W_t\right].$$

Suppose that $B(t,T)$ is given. Since $\Delta W_t \sim N(0,\Delta t)$, $E[\varepsilon_t]=1$. Then, taking expectation in (5.1), the expected value of the random bond price and its yield are calculated as:

$$E[B(t+\Delta t,T)] = B(t,T)\cdot \exp[r_t \cdot \Delta t],$$

and

$$\begin{aligned}\tilde{Y}(t+\Delta t,T) &= -\log(E[B(t+\Delta t,T)])/\{T-(t+\Delta t)\}\\ &= \{Y(t,T)\cdot(T-t)-r_t\cdot\Delta t\}/\{T-(t+\Delta t)\}.\end{aligned}$$

In particular, the instantaneous spot rate can be written as:

$$r_t = Y(t,t+\Delta t) = -\log(B(t,t+\Delta t))/\Delta t$$

by virtue of the assumption of a constant instantaneous spot rate for the period from t to $t+\Delta t$.

Since $B(t+\Delta t,t+\Delta t)$ is always 1, the price movement from time t to time $t+\Delta t$ of the bond with the maturity $t+\Delta t$ is deterministic.

5.2 The volatility function

In our study, we used the following form of yield volatility function:

$$\sigma_Y(y,t) = \sigma_0\cdot\left(1-\exp\{-\alpha y\}\right)\cdot(t+0.01)^{\beta}\left(1-\exp\{-\beta t\}\right)/\beta t, \tag{5.2}$$

where y is the yield to maturity, t is the bond duration, σ_0 is the maximum volatility, i.e. $\sigma_Y(\infty,\infty)$, and α and β are positive constants.

The term $\left(1-\exp\{-\beta t\}\right)/\beta t$ creates a pattern of decay of volatilities similar to that seen in the Vasicek model. The term $1-\exp\{-\alpha y\}$ captures the feature that the yield volatility decreases dramatically when the yield to maturity becomes low. The term $(t+0.01)^{\beta}$, together with the exponential term, creates the well known "humped" shape in the volatility curve observed in the market (see Hull (2000, p603)).

Following Hull (2000, p536) we convert from yield volatility to price volatility, denoted by σ_p, by multiplying σ_Y by the bond duration. Thus

$$\sigma_p = \sigma_p(t,T,B) = \sigma_Y\cdot(T-t).$$

5.3 Special treatment of very low interest rates

Once the entire yield curve becomes flat around zero (we refer to this as a "zero flat curve"), the yield curve tends to keep this shape in developments thereafter. This shape of yield curve is unlikely in practice, so we modified the general HJM approach in order to prevent the zero flat curve from emerging.

First, we define a "very low interest environment" mathematically, using three threshold constants, r_1, r_2 and r_3. If all of the following three conditions are satisfied, the yield curve is defined to be in a very low rate interest rate environment. The conditions are:

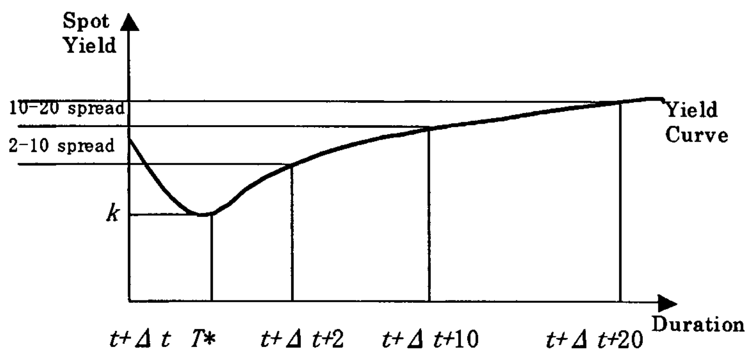
(1) The spot yield on any bond with any maturity is below r_1.

(2) The difference between the spot yield on a bond with a two-year duration and the spot yield on a bond with a ten-year duration (we refer to this as 2-10 spread) is below r_2.

(3) The 10-20 spread is below r_3.

Figure 5.1 illustrates the yield curve expected at time $t+\Delta t$, the 2-10 spread, the 10-20 spread, k and T^*, where k, the lowest spot yield on the yield curve, is the yield of the bond with maturity T^*.

Figure 5.1 - Illustration of k, T^* and spreads



Second, we assume that yield curves move in parallel in a very low interest rate environment. The minimum random spot yield, i.e. $Y(t+\Delta t, T^*)$, is assumed to follow a discrete stochastic movement over the time period $(t, t+\Delta t)$, namely

$$Y(t+\Delta t, T^*) = \begin{cases} k \cdot 0.5^{\Delta t} & \text{with probability} \quad p \\ k & \text{with probability} \quad 0.5 \\ k \cdot 2.0^{\Delta t} & \text{with probability} \quad 0.5-p \end{cases}$$

where $0 < p < 0.5$.

The probability p that the spot yield goes down (bond price goes up) can be calculated by equating expected bond prices:

$$\exp\{-k \cdot \theta\} = p \cdot \exp\left\{-k \cdot 0.5^{\Delta t} \cdot \theta\right\} + 0.5 \cdot \exp\{-k \cdot \theta\} + (0.5-p) \cdot \exp\left\{-k \cdot 2^{\Delta t} \cdot \theta\right\}$$

where $\theta = T^* - (t+\Delta t)$.

Now define $\Delta d = k(0.5^{\Delta t} - 1)$, and $\Delta u = k(2^{\Delta t} - 1)$. These values represent the possible upside and downside movement amounts of the yield curve. For bonds with maturity T_i other than maturity T^*, the stochastic movements of the spot yields are described by:

$$Y(t+\Delta t, T_i) = \begin{cases} \tilde{Y}(t+\Delta t, T_i) + \Delta d + \xi(t, T_i) & \text{with probability } p \\ \tilde{Y}(t+\Delta t, T_i) & \text{with probability } 0.5 \\ \tilde{Y}(t+\Delta t, T_i) + \Delta u + \xi(t, T_i) & \text{with probability } 0.5 - p \end{cases}$$

where $\xi(t, T_i)$ is an adjustment term which allows a correct expected bond price to be obtained. The adjustment term is also calculated by equating expected bond prices:

$$\exp\{-\tilde{Y}_i \cdot \theta_i\} = p \cdot \exp\{-[\tilde{Y}_i + \Delta d + \xi(t,T_i)] \cdot \theta_i\} + 0.5 \cdot \exp\{-\tilde{Y}_i \cdot \theta_i\} + (0.5 - p) \cdot \exp\{-[\tilde{Y}_i + \Delta u + \xi(t,T_i)] \cdot \theta_i\}$$

where $\tilde{Y}_i = \tilde{Y}(t+\Delta t, T_i)$ and $\theta_i = T_i - (t + \Delta t)$ so that

$$\xi(t, T_i) = -\log\left[0.5 / \left[p \cdot \exp\{-\Delta d \cdot \theta_i\} + (0.5 - p) \cdot \exp\{-\Delta u \cdot \theta_i\}\right]\right] / \theta_i .$$

Note the probability p is independent of i.

Once spot yields are given, bond prices can be calculated using formula (3.1).

6. Concepts for simulation

Our study is based on 1,000 simulations of the policy portfolio over time. We chose 1,000 as a compromise between computer run time on a personal computer and a number large number to produce meaningful results. We note that the HJM approach is computationally intensive. (See, for example, Hull (2000, p607).) In this section, we outline some of the key aspects of our simulation procedure.

6.1 Bonds

We simulated bond prices at times $\Delta t, 2\Delta t, 3\Delta t, \ldots, M\Delta t$. We chose $\Delta t = 1/8$ year. We can think of this as two events occurring each quarter which influence interest rates. Consider bonds whose maturity dates are $T_j = j\Delta t, j = 0,1,2,\ldots,M$, and let the bonds be denoted by $BOND_0$, $BOND_1$,..., $BOND_M$. Let $B(i;t,T_j)$ denote the present value of bond $BOND_j$ at time t in the i-th simulation. We need to simulate values of $B(i;t,T_j)$ for $j = 0,1,2,\ldots,M$ and $t = t_0, t_1, t_2, \ldots, t_M$. At time 0, i.e. t_0, we know the prices of all bonds since the initial yield curve is given. Note that $B(i;t_0,T_0) = 1$, and in general $B(i;t_j,T_j)=1$ since $BOND_j$ whose maturity date is $T_j(=t_j)$ is redeemable at par at time t_j.

Note that when $j>k$, $B(i;t_j,T_k)$ denotes the accumulated value of the bond with maturity time T_k at time t_j in the i-th scenario. As $BOND_k$ is redeemable at par at time T_k, we assume that the redemption money is always reinvested at instantaneous spot rates. For example, the accumulated values of $BOND_2$ at times t_4 and t_M are respectively

$$B(i;t_4,T_2)=B(i;t_3,T_2)\cdot\exp\{r(i;t_3)\cdot\Delta t\}=\exp\{[r(i;t_2)+r(i;t_3)]\cdot\Delta t\}$$

and

$$B(i;t_M,T_2)=\exp\{[r(i;t_2)+r(i;t_3)+...+r(i;t_{M-1})]\cdot\Delta t\}$$

where $r(i;t)$ is the realised sample instantaneous spot rate at time t in the i-th simulation.

6.2 Rate of return and discount factors

Throughout we assume that all cashflows derived from our policy portfolio will be invested in a single fund managed by the insurance company. We also assume that the expected rate of return on the fund up to time t is the sum of the then-prevailing spot yield for duration t and a constant ϕ, which we refer to as the extra return. Then the discount factor for $BOND_k$ at time t_j in the i-th simulation, denoted by $v(i;t_j,T_k)$, is obtained as

$$v(i;t_j,T_k)=\exp\{-(Y(i;t_j,T_k)+\phi)(T_k-t_j)\}=B(i;t_j,T_k)\cdot\exp\{-\phi(T_k-t_j)\} \tag{6.1}$$

where

$$Y(i;t_j,T_k)=\frac{1}{k-j}\sum_{s=j}^{k-1}r(i;t_s).$$

Further, we assume that the rate of return for the immediate future is realised as expected, i.e. the rate of return from time t to $t+\Delta t$ in the i-th simulation is $r(i;t)+\phi$. Under this assumption, formula (6.1) is valid for accumulation, i.e. when $j>k$.

6.3 Expected cashflows

For our study we considered a life insurance policy portfolio comprising a large number of identical policies, each of which combines a T year term assurance with a whole life assurance. We assume that benefits are payable at the end of the year of death, that premiums are payable annually in advance as long as the term assurance is in force, and that all policyholders are males who were aged exactly x at issue. Let ST, SW, I, e and P denote the sum assured under the term assurance, the sum assured under the whole-life assurance, the initial expenses, the renewal expenses and the gross annual premium per policy. Now define $P(t)$ and $CF(t)$ to be the expected premium income and the expected cashflow at time t per policy issued. Then for integer $t \geq 0$,

$$P(t) = \begin{cases} P \cdot l_{x+t} / l_x & \text{for } t < T \\ 0 & \text{for } t \geq T, \end{cases}$$

and

$$CF(t) = \begin{cases} P - I, & \text{for } t = 0 \\ -(ST + SW) \cdot (l_{x+t-1} - l_{x+t}) / l_x + P(t) - e \cdot l_{x+t} / l_x, & \text{for } t \leq T - 1 \\ -SW \cdot (l_{x+t-1} - l_{x+t}) / l_x - e \cdot l_{x+t} / l_x, & \text{for } t \geq T \\ 0 & \text{if } x + t > w, \end{cases}$$

where w is the maximum age in our mortality table. Since we have assumed a large number of policies, we further assume that cashflows are realised as expected as time elapses.

6.4 Policy values

Let $PVB(i;t)$ denote the expected present value of future cashflows at time t in the i-th simulation. Using the expected cashflows and the discount factors, we have

$$PVB(i;t) = \sum_{j=1}^{w-t} v(i;t,t+j) \cdot CF(t+j)$$

Note that under this definition, cashflows which occur at time t are not included in $PVB(i;t)$.

We define the net policy value of our policy portfolio as the expected present value of net future benefits arising from the policies in force using the then prevailing yield curve. Let $V(i;t)$ denote the net policy value at time t in the i-th simulation. Then

$$V(i;t) = -\sum_{j=1}^{w-t} v(i;t,t+j) \cdot CF(t+j) - P(t) = -PVB(i;t) - P(t).$$

This is different from the conventional policy value in Japan, whose calculation is based on the premium basis rate. These conventional policy values do not allow for any changes in interest rates after a policy has been issued. In contrast, our net policy values always reflect the market conditions at the date of calculation.

6.5 The surplus process

We define the surplus at time t to be the accumulated realised cash amount arising from the policy portfolio less the net policy value of the portfolio at time t. Let $SUR(i;t)$ and $AC(i;t)$ denote the surplus and the accumulated cash amount of the portfolio at time t in the i-th simulation, so that

$$AC(i;t) = \sum_{j=0}^{t} CF(j)v(i;t,j) - P(t)$$

and

$$SUR(i;t) = AC(i;t) - V(i;t).$$

A simple calculation then gives

$$SUR(i;t) = \sum_{j=0}^{w-x} CF(j) \cdot v(i;t,j).$$

For the i-th simulation, $SUR(i;t)$, $t = 0,1,2,...,\omega - x$, gives a stochastic development of the surplus process. Repeating this process N times gives N simulated realisations of the surplus process.

We also consider a discounted surplus process. For the i-th simulation, we use $DSUR(i;t)$ to denote the surplus at time t discounted to time 0, and define

$$DSUR(i;t) = SUR(i,t) / v(i;t,0).$$

For $t > 0$, the quantity $v(i;t,0)$ represents the accumulated value at time t of a payment of 1 at time 0 under the i-th simulation. Its inverse, $1/v(i;t,0)$, is thus the discount factor at time 0 for duration t. It can be shown that the process $\{DSUR(i;t)\}$ is a martingale. Therefore, the expected value of the discounted surplus is always the initial surplus amount, i.e.

$$E\left[DSUR(i;t)\right] = DSUR(i;0) = SUR(i;0).$$

In particular, when the premium is set so that the initial surplus is zero, the expected value of the discounted surplus is zero for any t.

We use the discounted surplus process as a measure to track the development of the surplus of the policy portfolio. The discounted surplus is always expressed as a value at time 0, and this standardisation enables us to track developments of the surplus process consistently.

6.6 Capital requirements

We now define initial capital as the amount of capital which should be injected into the portfolio at time 0 to prevent the surplus from becoming negative at any stage over the lifetime of the portfolio with some given (small) probability. If the surplus ever becomes negative, we say that "portfolio ruin" has occurred. We assume that this capital earns interest as described above.

Let $MLD(i)$ be defined as the minimum value of $DSUR(i;t)$ in the i-th simulation, so that $|MLD(i)|$ is the maximum discounted loss. The set of simulated values of $\{MLD(i)\}$ can be used to create an empirical distribution for the minimum value of the discounted surplus. Let the random variable MLD denote the maximum discounted loss for the portfolio, and let C be such that

$$\Pr(MLD > -C) = \Pr(MLD + C > 0) = 1 - \varepsilon$$

where $0 < \varepsilon < 1$. Then the initial capital requirement under the given probability ε is C.

Now suppose that initial capital of C is required. Instead of injecting initial capital at time 0, we may obtain a similar effect by levying profit margins on the gross premiums whose present value at time 0 is C. However, in this case the cashflows change, and the distribution of the minimum value of the discounted surplus with profit margins in the premiums is different to that of MLD.

7 Initial settings for simulation

7.1 Financial settings

For Cairns' model, we obtained the parameters

$$(b_1, \ldots, b_5) = (0.0283, 0.0044, -0.1216, 0.1386, -0.0525)$$

by numerically minimising the square error sum between model values and market observations given in the Appendix.

For the volatility function given by (5.2), we chose $\alpha = 45$, and using the same minimisation technique as for Cairns' model, we obtained $\sigma_0 = 1.53\%$, and $\beta = 0.061$.

To deal with the very low interest rate environment, we set r_1, r_2 and r_3 to 0.25%, 0.95% and 0.80% respectively based on historical swap rates from February 1999 to September 2001. We remark that in practice swap rates are par rates, and not yields on zero coupon bonds. However, we can ignore this difference for practical purposes.

Finally, we set the extra return, ϕ, introduced in Section 6.2, as 1.5%.

7.2 Mortality basis

We applied the 1999 male mortality table published by the Ministry of Health, Labor and Welfare in Japan. In this table, the last published mortality rate is at age 100, so we set a limiting age of 105 and extrapolated assuming Gompertz's Law.

7.3 Policy terms

According to statistics published by the Life Insurance Association of Japan (2000), whole-life with term assurance products account for 63% of the total amount of in force life products for individuals. Based on product illustrations in that publication, we have selected a whole-life with term assurance policy with the following terms.

A policy is issued to a male aged exactly 27 with a 30-year term assurance with sum assured Yen 35,000,000, and a whole-life assurance with sum assured of Yen 5,000,000. The benefits are paid at the end of the year of death and premiums are paid in advance for 30 years. We assume that initial expenses are 1% of the sum assured for the term assurance and 3% of the sum assured for the whole-life assurance, and that renewal expenses are 0.01% of the sum assured for the term assurance, 0.01% of the sum assured for the whole-life assurance and 3% of annual net premium. These renewal expenses are incurred at the start of each policy year, excluding the first.

7.4 Premium calculation

Using formula (6.1), we can calculate all discount factors at time 0, and hence expected present values of premium income and benefit outgo. By the principle of equivalence, i.e. by equating the expected present value of the premium income with that of benefits, we obtained a net annual premium, Π, of Yen 103,963. The expected present values of expenses are shown in Table 7.1.

Table 7.1 - Expected present values of expenses

Initial	Term Assurance	-350
	Whole-Life Assurance	-150
Ongoing	Term Assurance	-66
	Whole-Life Assurance	-12
	Premium Income	-59
	Total	-637

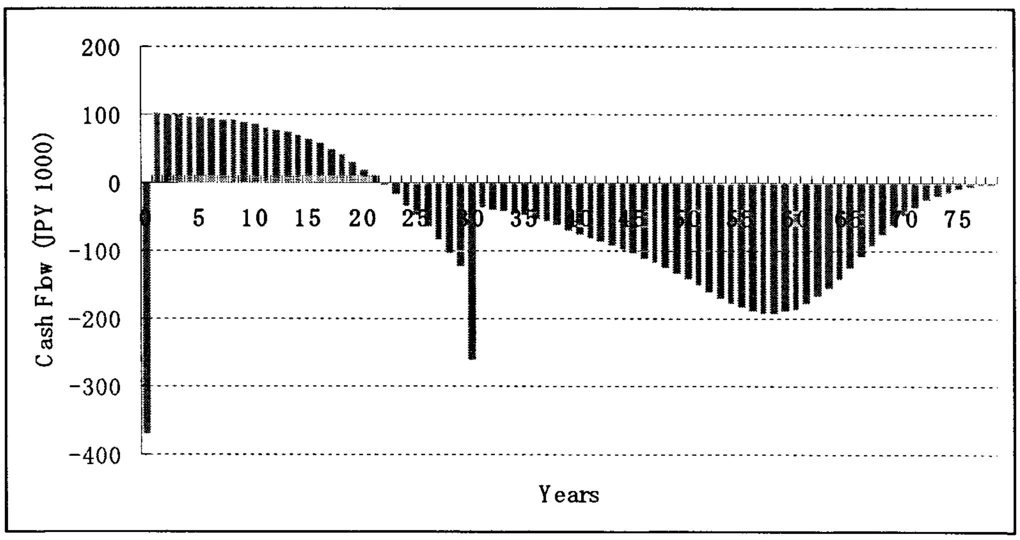
We next define the margin ratio $m(u)$ such that we earn a profit whose expected present value is u at time 0 by levying the margin, whose amount is $m(u)$ multiplied by the net annual premium, on top of annual net premiums. The expected present value of margin income is calculated as:

$$\Pi \cdot m(u) \cdot \ddot{a}_{27:\overline{30}|}.$$

In order to calculate the margin ratio to cover expenses, we set $u = 0$ in the above, then equate this with the expected present value of expenses. This yields a margin ratio of 32%, i.e. the gross premium is 1.32Π or Yen 137,596.

Figure 7.1 shows expected cashflows per policy issued including expected benefits, expenses and gross premiums. For policy years 2 to 22, the expected cashflows are positive due to low mortality rates. The first expected cashflow is negative due to initial expenses, while the pattern of negative expected cashflows changes after the 30th policy year due to the cessation of term assurance benefits.

Figure 7.1 - Expected cashflows



8 Simulation results

8.1 Individual realisations

Figures 8.1 to 8.5 illustrate the outcomes of five of the 1,000 simulations, which we label as Simulations A to E.

In Simulation A, interest rates fluctuate considerably. It can be seen that the surplus process evolves largely in line with the development of short rates in the early years of the policy portfolio. The low interest rates that occurred from 30 to 40 years caused the final surplus to be negative.

In Simulation B, the yield curves stayed at low levels for the entire lifetime of the policy portfolio. As a result, realised short rates were low (less than 0.80%) throughout. These low interest rates resulted in negative levels of surplus over virtually the entire lifetime of the policy portfolio.

In the case of Simulation C, interest rates started to rise after a 45-year period of low rates due to random fluctuations even though the yield curve at time 40 years did not suggest any future increase of interest rates. The level of the discounted surplus process therefore started to increase after 45 years. However, after that time, the surplus did not rise above zero to a level of profit.

Simulation D has some similarities with Simulation C. However, the low interest rate period ends before 40 years in this case. The discounted surplus process recovers around time 35 years, and as a result of this earlier end to the low interest rate period, the final discounted surplus (loss) turned out to be half of the loss in Simulation C.

Figure 8.1 - Simulation A

Figure A - Half-year forward rates

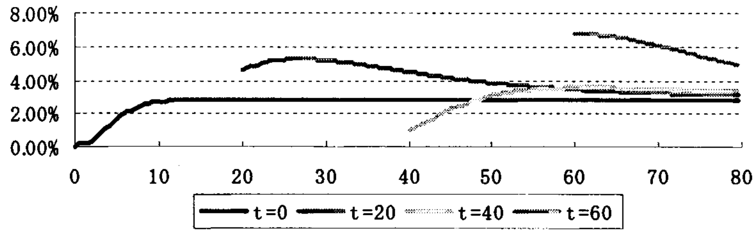


Figure B - Realized Short Rates

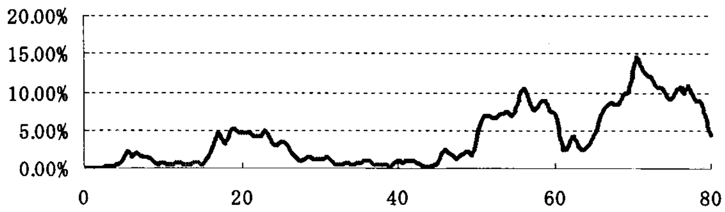


Figure C - Accumulated Surplus (in Yen 1,000)

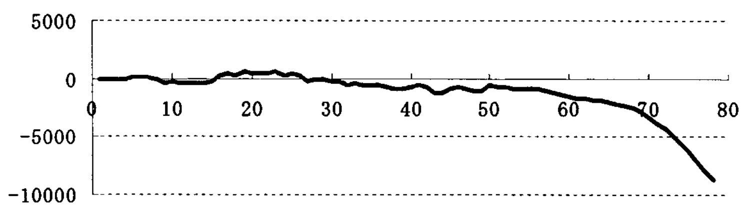


Figure D - Discounted Accumulated Surplus (in Yen 1,000)

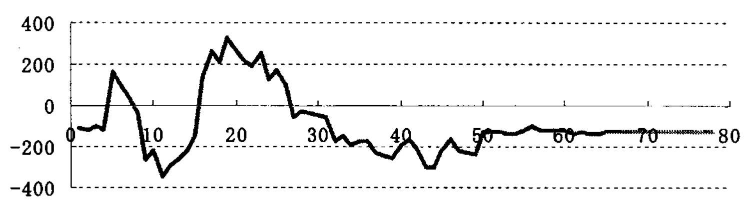


Figure 8.2 - Simulation B

Figure A - Half-year forward rates

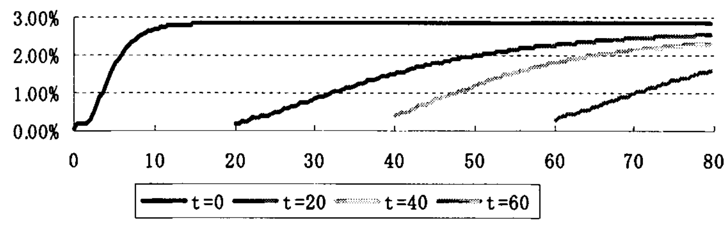


Figure B - Realized Short Rates

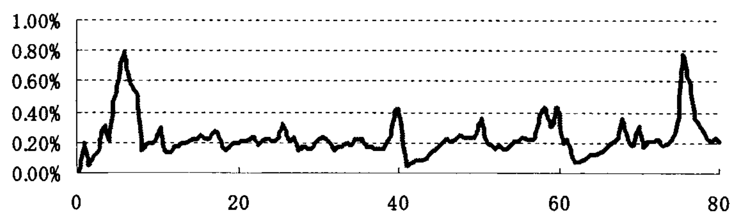


Figure C - Accumulated Surplus (in Yen 1,000)

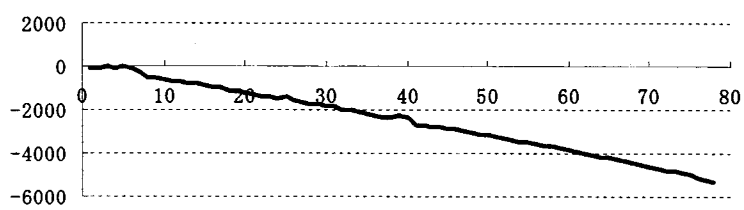


Figure D - Discounted Accumulated Surplus (in Yen 1,000)

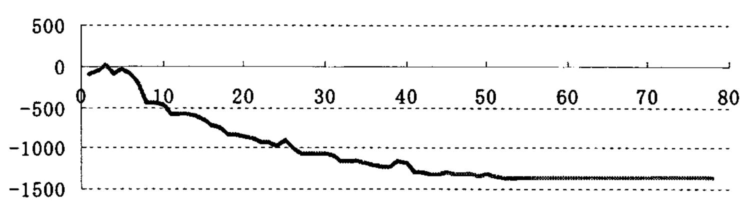


Figure 8.3 – Simulation C

Figure A - Half-year forward rates

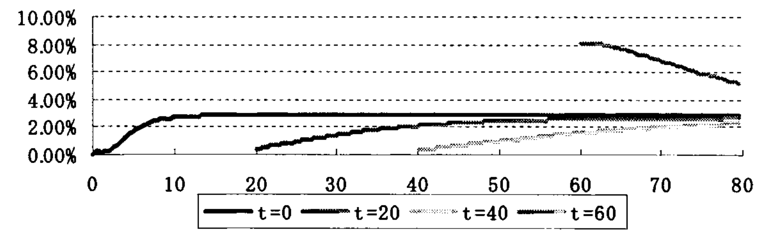


Figure B - Realized Short Rates

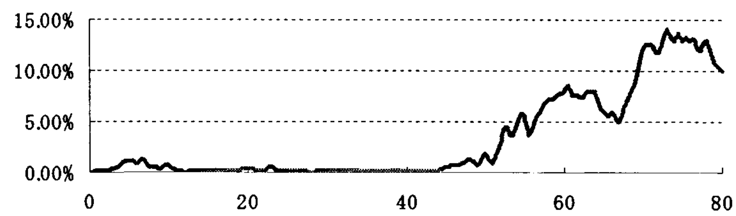


Figure C - Accumulated Surplus (in Yen 1,000)

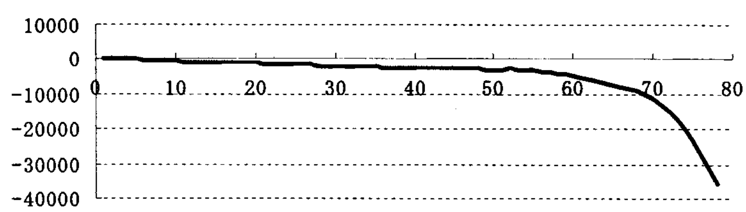


Figure D - Discounted Accumulated Surplus (in Yen 1,000)

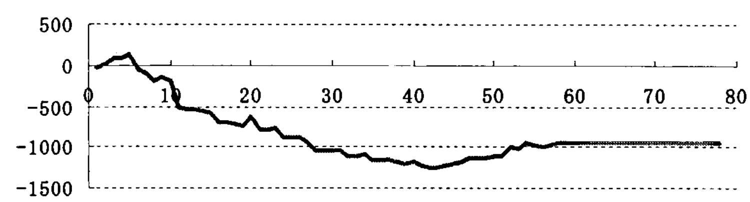


Figure 8.4 – Simulation D

Figure A - Half-year forward rates

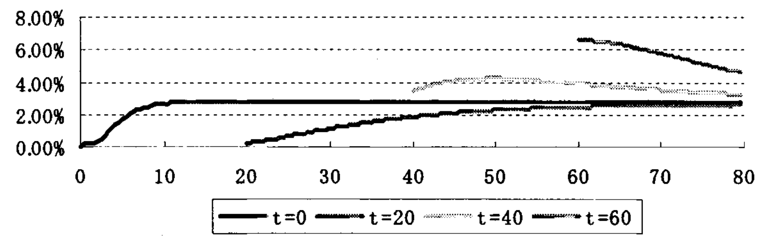


Figure B - Realized Short Rates

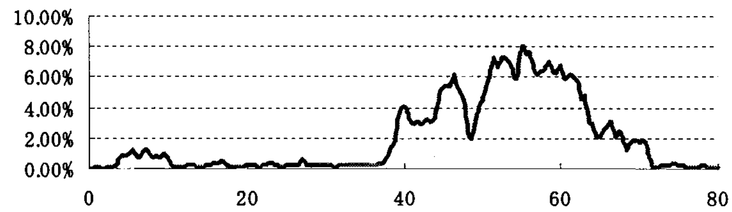


Figure C - Accumulated Surplus (in Yen 1,000)

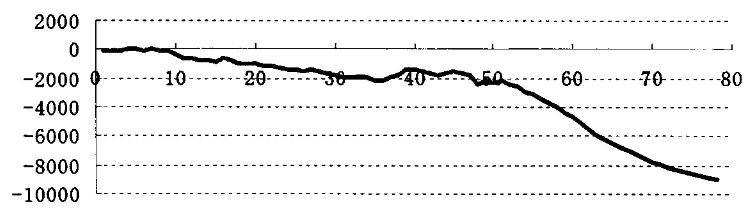


Figure D - Discounted Accumulated Surplus (in Yen 1,000)

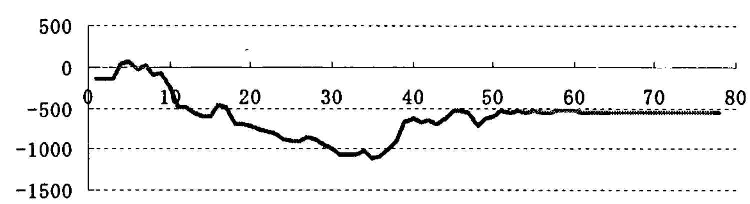


Figure 8.5 – Simulation E

Figure A - Half-year forward rates

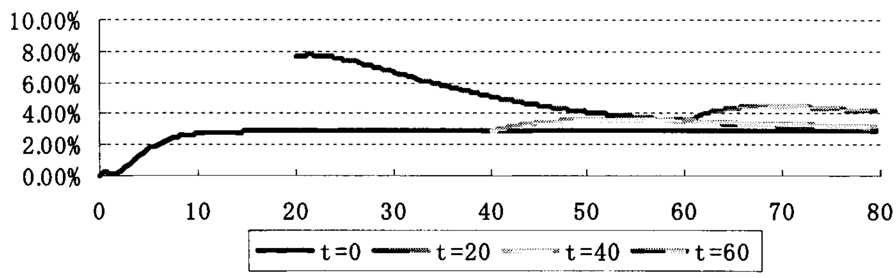


Figure B - Realized Short Rates

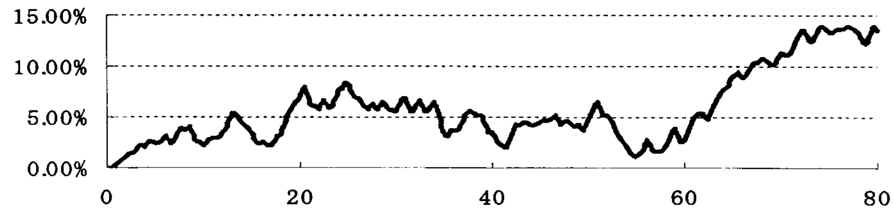


Figure C - Accumulated Surplus (in Yen 1000)

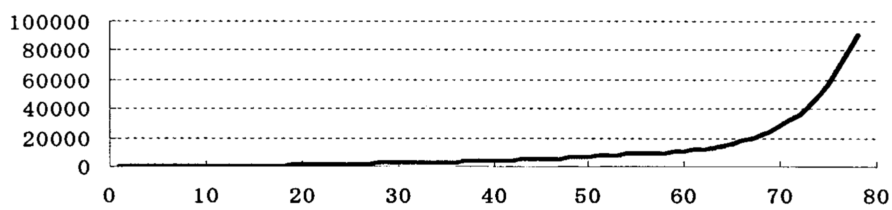
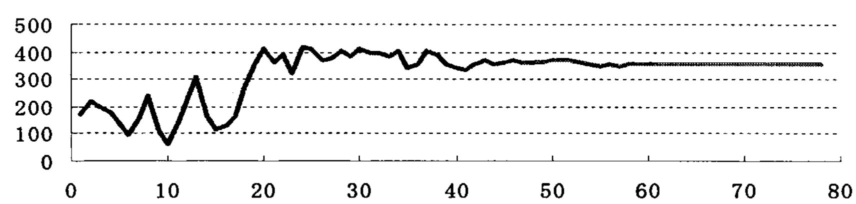


Figure D - Discounted Accumulated Surplus (in Yen 1000)



Interest rates were more than 4% during most of the portfolio's lifetime in Simulation E. The yield curve at time 20 years suggests such a future development. These higher interest rates result in the surplus process always being positive.

8.2 Distributions

Figure 8.6 shows a histogram of the surplus after 78 years discounted to time 0, while Table 8.1 shows some summary statistics in units of Yen 1,000.

Figure 8.6 - Histogram of the surplus after 78 years discounted to time 0

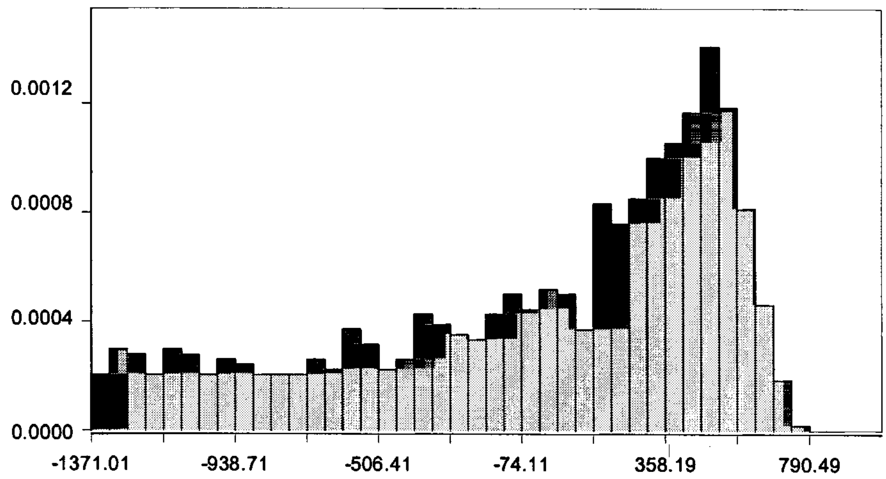


Table 8.1 - Summary statistics for Figure 8.6

Mean	Median	Minimum	Maximum	Standard Deviation
-22.4	177.4	-1,371.0	790.5	565.7

In each simulation, the policy portfolio has a surplus of 0 at time 0. Hence the expected value of the discounted surplus is also zero (see Section 6.5). Indeed we see that the sample mean is almost zero. The distribution is positively skewed with a median of 177.4. This means that we are more likely to obtain a positive surplus at time 78 years than a negative one. However, when negative discounted surpluses occur, they are larger than positive discounted surpluses in absolute value. This can be inferred from the minimum discounted surplus of –1,371, whose absolute amount is larger than the maximum of 790.5.

Figure 8.7 shows a histogram of the 1,000 simulated values of MLD, and summary statistics in are shown in Table 8.2 in units of Yen 1,000.

Figure 8.7 - Histogram of simulated values of MLD

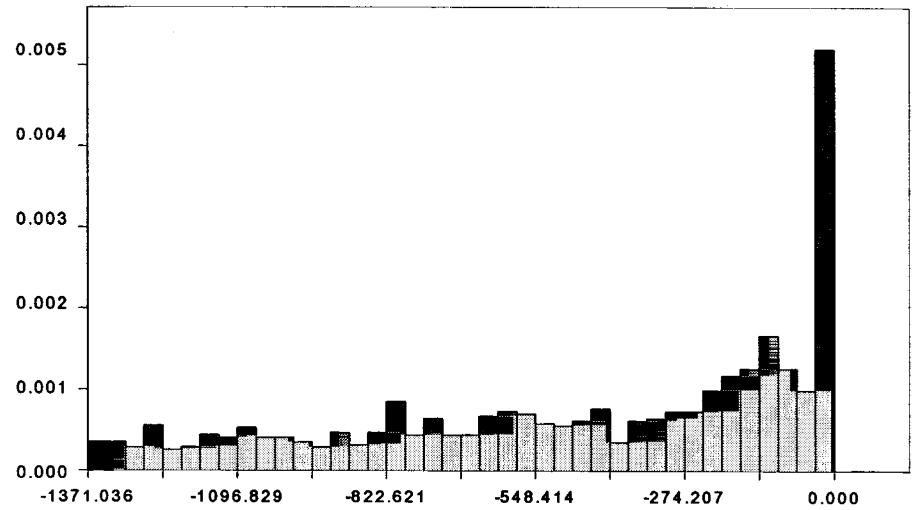


Table 8.2 – Summary statistics for Figure 8.7

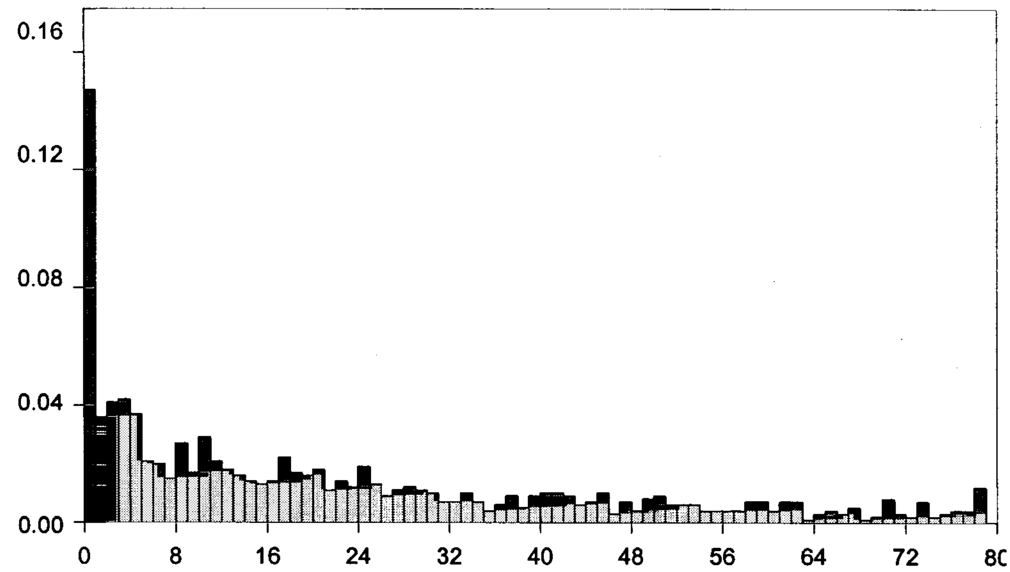
Mean	Median	Minimum	Maximum	Standard Deviation
-454.7	-340.8	-1,371.0	0	407.7

The empirical distribution of MLD has a probability mass of 0.147 at 0. As each simulation starts from a surplus of 0, the simulated value of MLD is 0 when the surplus is positive over the whole lifetime of the policy portfolio. In other words, with probability 0.853, the surplus falls below 0 at some point during the lifetime of the policy portfolio. Mainly due to the probability mass at 0, the empirical distribution is positively skewed with a median of –340.8, which is larger than the mean of –454.7. This mean indicates that the portfolio is in deficit at some stage during the lifetime of the policy portfolio by 454.7 on average, an amount which is 3.3 times the annual gross premium. The maximum simulated value of $|MLD|$ of 1,371 is approximately 10 times the annual gross premium.

8.3 Distribution of the time when the discounted minimum surplus occurs

Figure 8.8 shows a histogram of the times when the minimum values of the discounted surplus process occur.

Figure 8.8 - Histogram of the times when the minimum discounted surplus occurs



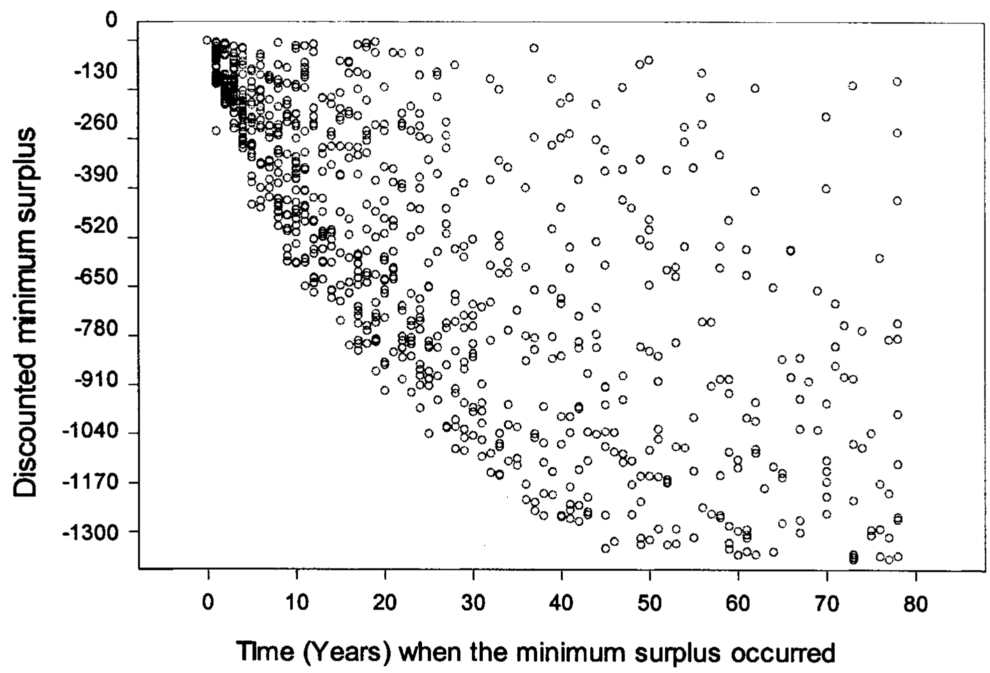
As our empirical probability of a minimum surplus of 0 is 0.147, the empirical probability that the time of the minimum discounted surplus is 0 is also 0.147. We see from Figure 8.8 that the discounted surplus is more likely to reach its minimum value in the early stages of the lifetime of the policy portfolio. Figure 8.8 also illustrates that the minimum can occur at any time, although the probabilities associated with later times are much smaller than those associated with earlier times. In Figure 8.9, the relationship between the simulated values of MLD and the time when the discounted surplus reached its minimum is shown.

Generally, the later the discounted surplus reached its minimum, the larger the expected present value of the loss. This result conforms with the fact that the fund is exposed to a greater interest rate risk as time passes and premium income is accumulated. In addition, the more time elapses, the greater the chance that actual returns will differ from those initially expected.

8.4 Initial capital and margins

In Section 6.6, we defined initial capital as the amount which should be injected at time 0 to prevent the surplus of the policy portfolio from becoming negative over its lifetime with some given probability. Now let us choose that probability to be 0.01, i.e. we are setting a portfolio ruin probability of 1%. In our simulations, 990 of our 1,000 simulated values of MLD are more than $-1{,}351$. Thus, an initial capital injection of 1,351, which is almost ten times the gross annual premium, shifts the distribution to the right so that 1% of our simulated values of MLD are less than 0. In other words, an initial capital requirement of 1,351 is required to achieve a 1% portfolio ruin probability according to the empirical distribution.

Figure 8.9 – Times and amounts of minimum discounted surplus



The estimated initial capital requirements for different levels of portfolio ruin probability are shown in Table 8.3 as amounts and as multiples of the annual gross premium.

Table 8.3 - Empirical ruin probability, required capital and gross premiums

Empirical ruin probability	Estimated required capital	Required Capital / Annual Gross Premium
1%	1,351	9.8
5%	1,246	9.1
10%	1,105	8.0

Let us now consider profit margins, in particular the situation corresponding to an initial capital injection of 1,105. A similar effect to this initial capital injection may be obtained by imposing a profit margin whose expected present value is 1,105 at time 0. In Section 7.4 we defined a margin ratio $m(u)$ as the amount that is paid on top of the net premium which gives an expected present value of u at time 0. Given u, $m(u)$ is calculated by the formula:

$$m(u) = (u + E)/(\Pi \ddot{a}_{27:\overline{30}|})$$

where E is the expected present value of expenses at time 0. Setting $u = 1{,}105$ in the above formula gives $m(u) = 88\%$. This margin ratio includes the portion allocated to expenses, which is 32%. Thus, the margin ratio to provide a profit whose expected present value is 1,105 is 56% of the net premium or 42% of the gross premium.

An initial capital injection of 1,105 gives a 10% empirical portfolio ruin probability. Thus, we might expect that an 88% margin ratio would also result in a 10% empirical ruin probability. However, if we include this margin with premium income and re-run our simulations, the result is only a 1% portfolio ruin probability. Unlike the case when capital is injected at time 0, the addition of the profit margin alters the shape of cashflows in a way in which interest rate risk is reduced. The benefit payment amounts increase during the later stages of the lifetime of the portfolio. Therefore, overall cashflows level off more when the margin is added as an annuity stream than when the initial capital is injected. Having more level cashflows reduces the interest rate risk.

We re-created our simulations for various values of $m(u)$, in order to estimate portfolio ruin probabilities associated with new cashflows redefined by $m(u)$. Table 8.4 shows empirical portfolio ruin probabilities for a range of values of $m(u)$, where the monetary unit is again Yen 1,000.

In Table 8.4, profit margin is defined as the margin ratio levied on top of the gross premium as a percentage of gross premium. Comparing Tables 8.3 and 8.4, we see that the same empirical ruin probability can be obtained by choosing a profit margin that results in an expected present value of profit smaller than the initial capital injection. For example, we can achieve a 10% empirical ruin probability by levying a profit margin of 36% or by an initial capital injection of 1,105. However, the profit margin of 36% results in an expected present value of profit of 938.

We can estimate the profit margin required to achieve a given portfolio ruin probability by locating the probability in the last column in Table 8.4. Table 8.5 shows the required profit margin for each probability.

Table 8.4 - Profit margins and empirical ruin probabilities

$m(u)$	u	Profit Margin	Empirical Ruin Probability
70%	741	31%	20%
80%	938	36%	10%
84%	1,017	39%	8%
86%	1,056	41%	4%
88%	1,105	42%	1%
90%	1,135	44%	0%

Table 8.5 - Required ruin probabilities and estimated profit margins

Ruin Probability	Estimated Profit Margin
10%	36%
5%	40%
1%	42%

9 Concluding remarks

According to the empirical distribution we obtained, without any profit margin, the initial capital required for a 10% probability of ruin is approximately eight times the annual gross premium. Setting a 1% portfolio ruin probability requires initial capital approximately ten times as much as the annual gross premium. When a 36% profit margin is added to the gross premium, the empirical ruin probability is also 10%. The empirical ruin probability became 1% by charging a profit margin of 42%.

Note that the introduction of a margin or an initial capital requirement is to deal with interest rate risk only. In practice, further margins may be needed for asset price fluctuation not attributable to interest rate fluctuations, mortality experience different from the mortality basis used for premium calculations, and fluctuations in expenses. Then there may be the additional question of dividend payments by the insurer.

In a competitive environment where information about financial products and valuation tools is publicly available, high profit margins may not be sustainable. Therefore, interest rate risks need to be correctly measured, managed and then reduced. There may be many ways to achieve this, including asset-liability matching, shortening policy durations, linking premiums to interest rates, or linking policy values to asset values. These measures are especially important as a country's entire life insurance industry shares the same interest rate risk, i.e. low interest rates would damage portfolios of all life insurance companies. This is an important problem for Japan in general, because the soundness of the Japanese life insurance industry has social implications that extend well beyond the financial status of individual companies.

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APPENDIX

Market observations

Table A – Interest rate market quotations (1 May 2001)

Term	Deposit Rates Bid	Deposit Rates Offer	Swap Rates Bid	Swap Rates Offer	Government Bond Yield
3 (Months)	0.035%	0.05%			0.028%
6					0.031%
2 (Years)			0.14%	0.18%	
3			0.24%	0.28%	
4			0.39%	0.43%	
5			0.57%	0.61%	
7			0.95%	0.99%	
10			1.40%	1.44%	1.361%
20					1.970%

Source: *Reuter, Tokyo Market Summary dated 1 May 2001, and Bloomberg.co.jp*

Table B - Base yields obtained from the data in Table A

Term	Base Yield
6 (Months)	0.04%
2 (Years)	0.16%
3	0.26%
4	0.41%
5	0.59%
7	0.97%
10	1.42%
20	2.03%

Table C - Lowest 2-10 spreads observed in the market (for the period 12 February 1999 to 11 September 2001)

	2-year rate	10-year rate	20-year rate	2-10 spread	10-20 spread
22 March 2001	0.17%	1.29%	2.16%	1.11%	0.87%
27 June 2001	0.13%	1.19%	2.44%	1.06%	1.24%

The 20-year rate was estimated by Cairns' model.

Table D - Volatility market quotations (19 April 2001)

	2 Year	5 Year	10 Year	20 Year
Strike Rate	0.22%	0.75%	1.60%	2.35%
Market Quotation[4]	0.16%	0.45%	0.65%	0.71%
Model Volatility[5]	0.14%	0.43%	0.68%	0.69%

[4] *At-the-money volatility for options whose maturities are 3 months.*
[5] *For reference purpose, using estimated parameters*

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