ISBN 0 7340 2193 3

Continuous Compounding, Volatility and the Equity Premium

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RESEARCH PAPER NUMBER 98

June 2002

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Paper presented to the Joint Institute of Actuaries and Faculty of Actuaries'
Finance and Investment Conference 27th June 2002

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Abstract

This paper reviews some of the fundamental ideas of modern portfolio theory and the supporting empirical evidence that is relevant to actuarial modelling of equity markets or equities as an asset class.

There is a chasm between those who believe that modern portfolio theory is absurd and those who believe it to be so logical as to be almost self-evident. This paper identifies ambiguity in the meaning of the term *'mean rate of return'* as a partial cause of much of the controversy between advocates and critics of modern portfolio theory and share market efficiency.

Depending on the appropriate meaning of *'mean rate of return'*, equity markets may not be mean-variance efficient which weakens the argument that stock market volatility is responsible for the superior long term performance of shares over less volatile asset classes. An alternative model for the equity premium, based on return on equity (or corporate profitability) rather than stock market volatility is then suggested as an explanation for the equity premium and some of the apparently anomalous empirical evidence relating to market efficiency.

The implications of this explanation for the nature and size of the equity premium, and its relevance to actuarial valuation assumptions, are then explored.

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0 Executive summary

0.1 It seems well known that the linear relationship between beta values and return is a single period model. However this limitation is often ignored and empirical evidence which only supports a single period model is sometimes used, implicitly or unconsciously, in support of a multiple period or continuous time model.

0.2 Before the empirical evidence was available, a linear relationship between beta values and return was theoretically derived from a number of assumptions, including an equity market that is in mean-variance equilibrium. For this linear relationship to be positive, there must also be an equity premium caused by the volatility averse behaviour of investors. While it can be argued that these are models of expectations rather than outcomes, their adoption in practice would not have happened without supporting empirical evidence.

0.3 In considering both theoretical models and empirical evidence, there is an important ambiguity in the meaning of the term *'mean rate of return'* which could mean the arithmetic average of a sequence of discrete rates of return, a geometric average, or the arithmetic mean of a sequence of continuously compounded rates of return. Using geometric means, or continuous compounding, there may not be any empirical support for a relationship between β values and return even if there is ample evidence of a simple linear relationship between β values and arithmetic means of discrete rates of return.

0.4 Despite this ambiguity, there is still substantial historical evidence to support the existence of an equity premium, irrespective of the definition of *'mean rate of return'*. If, using geometric means or continuous compounding, there is an equity premium but no positive relationship between β values and return, then the equity premium may be completely unrelated to stock market volatility. If the equity premium does not result from volatility, then its future existence as a reward for accepting volatility or 'risk' cannot be assumed.

0.5 An alternative explanation for the existence and size of the equity premium is that it mainly arises as a consequence of individual companies seeking a return on equity which exceeds their cost of capital and possible government intervention if the corporate sector, as a whole, cannot achieve such a level of profitability. There are economic grounds for believing that profitability will fluctuate as a weakly stationary stochastic process.

0.6 Rather than pricing individual stocks on the assumption that return on equity will revert to some form of equilibrium, investor psychology seems to exhibit the opposite assumption. In mathematical terms, this manifests itself in price/earnings multiples that do not reflect a reversion to equilibrium levels of profitability, instead market multiples tend to rise with return on equity.

0.7 Finally, this paper briefly explores the consequences of the return on equity explanation for the equity premium. If the return on equity explanation is close to the truth, then the starting point for a general measure of share market investment risk is the relationship between aggregate stock market capitalisation and underlying shareholders' equity. In the long term the equity premium will depend on return on equity, but in the medium term the equity premium also depends inversely on the ratio of market capitalisation to shareholders' funds. In consequence, the medium term relationship between asset class return and risk, which fluctuates over time with stock market levels, may be inverse.

1 Introduction: Foundations and contradictions of modern portfolio theory

1.1 Although this paper is more concerned with the nature of the equity premium, the volatility or 'risk' explanation for the existence of the equity premium is related to the idea of a volatility or 'risk' related return within an equity market. This paper therefore begins with a brief history of the development of what is generally known as modern portfolio theory so that crucial empirical evidence can be placed in context.

Modern Portfolio Theory, sometimes called mean-variance portfolio theory, exhibits many of the standard attributes of a scientific theory - assumptions, predictions, supporting empirical evidence and contradictions which require explanation.

With one important exception, most of the assumptions and observations of modern portfolio theory are open to debate or alternative explanation. The important exception is the evidence of a positive linear relationship between β values and return. It is difficult to explain this phenomenon except as a result of a mean-variance efficient market.

1.2 An analysis of the relationship between β values and return is therefore an important part of this paper. It is widely believed that this relationship is a simple linear one. Also, that such a relationship should be positive is usually unstated, presumably because it is considered self-evident. To quote The Institute and Faculty of Actuaries (2000b), the 'core reading' for Subject 109:

> *'A powerful result that can be derived ... is that a linear relationship exists between the expected return on individual securities and their so-called "β factors".'*

1.3 Modern portfolio theory appears to have originated from the observation that stock price movements appeared random, at least in the short term. If stock price movements are random, then risk can be quantified in terms of the standard deviation or variance of stock price movements. If we know the future mean return and standard deviation of return of all stocks (and all cross-correlations of returns) then we can create an optimum portfolio which maximises our mean return for any given level of 'risk'. Alternatively we can find the lowest 'risk' portfolio for a given level of mean return. To quote Haugen (1995):

> *'Modern Finance was born in 1950 ... in the mind of a young graduate student named Harry Markowitz.... He was trying to figure out how to build portfolios of stocks with the highest expected return given their risk or the lowest possible risk given their rate of return.'*

However if everyone else is attempting the same optimisation exercise the market will be in mean-variance 'equilibrium' and it will not be possible to do any better than hold a linear combination of the 'market' portfolio and the 'riskless' asset.

Equities can be expected to outperform 'riskless' assets by an equity 'risk' premium. If we wish to reduce the 'risk' of our portfolio we should alter our asset allocation between equities and cash or gilts. If we seek to reduce our overall portfolio volatility by remaining fully invested in less volatile equities we can expect a lower return, for the same 'risk', than that offered by a combination of the benchmark equity portfolio and the 'riskless' asset.

1.4 In such a market, there will be a simple linear relationship between β values and return. Fund managers may be able to 'beat the market' but only by holding more 'risky' stocks or with an asset allocation which is geared. On a 'risk adjusted' basis, out-performance and underperformance are impossible.

In addition to Markowitz, these theoretical developments were essentially the work of Black, Sharpe and Lintner. To quote Fama and French (1992)

> *'The Asset-pricing model of Sharpe (1964), Lintner (1965) and Black (1972) has long shaped the way academics and practitioners think about average returns and risk.'*

1.5 The theoretical development emerged from a set of underlying assumptions described by Jensen (1972) as follows:

> *'[These models] all involve either explicitly or implicitly the following assumptions:*
>
> *1 All investors are single period expected utility of terminal wealth maximisers who choose among alternative portfolios on the basis of mean and variance (or standard deviation) of return. ...*
>
> *2 All investors can borrow or lend an unlimited amount at an exogenously driven risk-free rate of interest .. , and there are no restrictions on short sales of any asset.*
>
> *3 All investors have identical subjective estimates of the means, variances and covariances of returns among all assets.*
>
> *....'*

1.6 These assumptions are all open to debate. However, one of the key predictions of the theory was the existence of a simple ***positive*** linear relationship between β and 'excess return'. Once this simple ***positive*** linear relationship was supported with empirical evidence, it could be argued that the market behaved as if these assumptions were valid. To quote Fama and French (1992):

> *The central prediction of the model is that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz (1959). The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market βs (the slope in the regression of a security's return on the market's return) and (b) market βs suffice to describe the cross-section of expected returns.'*

While it could be argued that this simple linear relationship is one of expectations rather than outcomes, the widespread use of this assumption, as described by Fama and French, would not have occurred if empirical studies had not supported these theoretical developments.

1.7 Once risk was identified with variance of return, modern portfolio theory provided a relatively simple explanation for the superior long-term performance of equities compared to fixed interest securities or other less volatile assets. Indeed this phenomenon is often called the equity *risk* premium and sometimes just the *risk* premium.

This explanation has a major advantage for the actuarial profession in deciding discount rates for valuing liabilities. If the superior performance of equities is due to risk, or stock market volatility, then it will always be there. If the equity premium has nothing to do with stock market risk or volatility, then this assumption is built on quicksand.

1.8 A second advantage for the actuarial profession is the relative simplicity that equity market efficiency brings to some mean-variance asset/liability models. If a stock market is mean-variance efficient, then the optimum way of structuring a mixed portfolio of (say) bonds and shares is to vary the asset allocation. Attempting to reduce the overall portfolio 'risk' by selecting a low-beta equity portfolio will produce a lower return, for the same amount of 'risk', than a mixed portfolio of bonds and equities that has a lower asset allocation to equities, but with the equity component based on market benchmarks.

Even though the issue of market efficiency has no direct effect on actuaries using mean-variance asset class models, the underlying assumption is still there. If this were not the case, a separate model of the performance of a particular portfolio would be required.

1.9 A further reason why the question of mean-variance efficiency is important is its role in developing the idea that an equity premium exists because of stock market 'risk'. If investors behave as described in the assumptions listed by Jensen (1972), there should be an equity 'risk' premium' and it follows from these assumptions that there should be a ***positive*** linear relationship between β and expected return within the equity market, confirmed by empirical evidence of a positive linear relationship between β values and mean returns.

1.10 The empirical evidence in favour of mean-variance efficient equity markets appears to be considerable. In general, this evidence falls into four main categories:

(a) *Event studies.*

There is a large number of studies investigating the reaction of markets to the arrival of new information, generally known as 'event studies'. These studies show that, in general terms, information relating to earnings, dividends, stock splits etc tends to be reflected in market prices either before it is announced, or so quickly thereafter that profitable exploitation is impossible.

It is argued that this shows that the market is efficient. However studies by Shiller (1981) and others have tended to suggest that investors may over-react to information.

(b) *The performance of professional investors*

Fund managers, as a whole, seem to have difficulty in achieving their benchmarks. It is argued that this is because the market is efficient and it is impossible to beat a benchmark on a risk-adjusted basis. The marketing success of index funds is partially due to the public acceptance of this argument.

This argument ignores two alternative explanations. The first is that professional fund managers are so large they cannot readily buy and sell without affecting the market. In any event the market index should, by definition, match their average performance.

A second alternative explanation is that professional investors are more interested in managing their business risk than the investment risk to which they expose their clients. They know, from hard experience, that departure from benchmarks or accepted 'norms' can have catastrophic business consequences if they appear to be 'wrong' in the short term.

The observations of Keynes (1936),

'.. most of these persons are, in fact, largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life but ... what the market will value it, under the influence of mass psychology, three months to a year hence.'

and more recently Kohler (2001), suggest that this is a long-standing phenomenon:

'Instead of being invested for the long term, the world's (not just Australia's) retirement savings are invested in liquid, short-term assets that are designed to protect the business risks of those doing the investing - not to protect the investment risks of the customers, or even to maximize their long-term returns. ... the principal aim of institutional investment businesses is to reduce their tracking error to the index and so lower their chances of being sacked.'

(c) *The historical evidence of an equity premium*

There is extensive evidence that shows a consistently superior long-term performance of shares compared to either long-term bonds or cash securities in many countries including Australia, the USA and the UK. See for example Dimson, Marsh and Staunton (2002). It is often argued that the equity premium needs to exist because investors will only invest in more volatile equities if they can reasonably expect a 'risk premium' for doing so and the data from the 20th century supports the reasonableness of this argument.

On the other hand, the size of the equity premium has led to a view, argued by Mehra and Prescott (1985), that it is too high to be justified by rational economic behaviour - the so-called 'equity premium puzzle'. To add to this puzzle, there is the curious phenomenon, as Bernstein (1997) observes, that the long-term return from equities is more predictable than the long-term return from bonds.

'Stocks are fundamentally less risky than bonds, not only because their returns have been consistently higher than those of bonds over the long term, but also because less uncertainty surrounds the long-term return investors can expect on the basis of past history'.

Also, there are alternative explanations, other than stock market risk, for the superior long-term performance of equities compared to bonds or cash. These explanations, based on inflation, retained profits or a combination of these two factors are discussed in Section 3.

(d) *Empirical evidence of a simple linear and positive relationship between β values and return.*

In the final category of evidence are the painstaking studies of the observed relationship between β and return which almost invariably show a simple linear relationship. It is argued that this phenomenon is a direct result of rational 'risk' averse investors pricing individual stocks in accordance with their expected mean returns and variances of return. To quote Walsh (1976), in bringing modern portfolio theory to the attention of Australian actuaries:

'One significant fact appears consistently in nearly all empirical studies on asset pricing models - namely the linearity of the relationship between a portfolio's volatility (β_i) and its expected return.'

1.11 In this final category of empirical evidence, the study of Black, Jensen and Scholes (1972) seems to have been a watershed in gaining practitioner acceptance of modern portfolio theory and the linear relationship between β values and return. Event studies could be dismissed as being in the realm of the short-term speculator while ideas such as the rationality of investors, the use of volatility to measure risk and the mediocre performance of fund managers were open to argument and alternative explanations.

To quote Walsh (1976):

> *'This paper by Black Jensen and Scholes shows that **after exhaustive testing** a linear relationship does exist between market risk and the investment return of a security.'*

Unlike other ideas and empirical evidence, no alternative explanation appears to have been put forward for the phenomenon of a linear relationship between β values and 'excess return' other than a 'mean-variance' efficient market.

However, these important studies of the relationship between beta values and return may have been misinterpreted from the day they were published. This is due to the ambiguity in the term 'mean rate of return' explained in Section 2 of this paper.

Before considering the empirical evidence of a positive linear relationship between β values and return in more detail, evidence that contradicts modern portfolio theory also needs to be noted.

1.12 While there is considerable empirical support for some of its assumptions, there are several empirical contradictions to modern portfolio theory, including the following:

(a) *Mean reversion and autocorrelation in price movements*

In the medium to long term, the logarithms of stock prices do not seem to conform to a random walk. This phenomenon was clearly recognised by the Maturity Guarantees Working Party (1980). While stock and stock market index movements appear to be approximately uncorrelated, the stronger assumption of independent and identically distributed changes is not supported by the empirical evidence. For example, the Maturity Guarantees Working Party referred to "the unusually straight track of United States and United Kingdom indices" (p137) and also commented that "there appears to be a significant correlation between changes of the De Zoete Equity Index at intervals of seven or eight years".

Whether it is still true that the weak-form of market efficiency holds remains a matter of conjecture. However, this is principally an issue for technical analysts and short-term traders.

(b) *The relationship between traditional 'value' indicators such as price/earnings ratios, dividend yields or price/book ratios and subsequent rates of return.*

Calculations performed by practicing security analysts such as SF Nicholson (1968), JD McWilliams (1966) and Dreman (1982) have frequently demonstrated an inverse relationship between price/earnings ratios and subsequent stock market performance. Initially it was suggested that these results could be explained by the higher performing low price/earnings ratio securities having higher β values until Basu (1977) showed that, if anything, the low price/earnings ratio phenomenon was accompanied with lower, and not higher, β values.

These analyses are supported by Wilkie (1993) who showed that, in addition to price/earnings ratios affecting intra-market performance (the cross-section of returns), aggregate dividend yields were correlated with overall stock market performance over the following several years - justifying an autoregressive term based on dividend yields in all versions of his stochastic model. See, for example, Wilkie (1995 p292).

More recently, there has been the study of Fama and French (1992) identifying price/book ratios and capitalisation as valuable indicators of subsequent relative stock market performance.

(c) *Behavioural aspects of decision making by investors, including the influence of 'business risk' on the psychology of fund managers*

A series of investigations of a behavioural nature have raised questions concerning the rationality and risk-averse focus of professional fund managers and other market participants. See, for example Hodgson et al (2000) and Thaler (1993).

In addition, the business risk of fund managers, and their need to conform to benchmarks may prevent them from exploiting any market inefficiencies. On the other hand, there seem to be a number of investors who are able to consistently exploit discrepancies between price and value. These are discussed in Train (1980 and 1989) as well as Cottle et al (1988).

(d) *The Australian 'Resources anomaly'*

In Australia, the market can be divided into two quite separate sectors, Industrials and Resources, both of which have accumulation indices based at 1,000 points at the end of 1979. In 1979, the resources component of the national index exceeded the capitalisation of the industrials component. Since that time, the relative performance of these two sectors has been ***opposite*** to that predicted by the supposedly positive relationship between beta values and return, as shown in the table below.

Table 1.1

Australian Index portfolios December 1979 - April 2002

Accumulation Indices	All Industrials	All Resources
31st December 1979	1000	1000
30th April 2002	32887	6375
Beta value	0.8	1.2
Average rate of return (% pa) (continuously compounded)	15.6	8.3

(e) *Fundamental philosophical issues*

While practitioners may have been ready to embrace the simple linear relationship between 'risk' and return, some members of the academic community have been more circumspect for two main reasons. The first is that the empirically derived linear beta/return graphs indicated an expected return for a zero-beta security which was significantly higher that the 'riskless' rate of return. The second issue is whether the market itself is a mean-variance 'efficient' asset. To quote Haugen (1990):

'The single, independent prediction of the capital asset pricing model is that the market portfolio is positioned on the efficient set. Several other conditions follow automatically, given this prediction, including a linear, positively sloped relationship between beta and expected rates of return. ***Unfortunately, this relationship between beta and expected return is a necessary, but not a sufficient condition for the efficiency of the market portfolio.'***

1.13 Despite these contradictions, the idea of 'risk' related return has considerable intuitive appeal and this intuitive appeal may have been a factor in gaining widespread acceptance. Also, it seems that once a theory becomes entrenched it will be difficult to dislodge especially when it provides the terminology and methods of measurement by which alternative hypotheses are judged. In consequence, perhaps this contradictory evidence is being assessed by prejudiced minds.

Keynes (1936 p viii) once commented: -

> *'The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those of us brought up as most of us have been, into every corner of our minds.'*

Examples of the entrenched position of modern portfolio theory include the following:

(a) The use of the term anomaly to describe any deviation from market efficiency or apparent contradiction of the linear β return relationship envisaged in the Capital Asset Pricing Model, even when describing a phenomenon representing a majority of the market capitalisation of a national index.

(b) A mean-variance framework is usually used to measure alternative ideas such as the predictive power of price/earnings ratios, with the implicit assumption that unless a statistically significant deviation is found, market efficiency prevails. Even so, a deviation becomes just another 'anomaly' and the theory remains intact. As Peters (1991) pointed out, most of these tests have underlying assumptions involving normal or lognormal distributions, which are only approximately correct.

(c) There seems to be a view that a model which does not work, or which is questionable, is preferable to an environment where there is no model. To quote Dumas and Allaz (1996):

> *'In the absence of a convincing alternative, the CAPM remains an extremely common paradigm in modern financial thought.'*

Even when their own studies identify serious errors in the concept of market efficiency, some researchers still argue for the retention of the doctrine of market efficiency. For example, Beechey et al (2000):

> *"The 1987 stock market crash, and the unprecedented run-up in US stock prices in the 1990s are hard to understand except in terms of markets which have moved some distance from levels consistent with fundamentals.*
>
> .....
>
> *The efficient market hypothesis is almost certainly the right place to start when thinking about asset price formation."*

It is difficult to understand the scientific integrity of this view or its logic. In essence, it is being argued that a flawed model is preferred to no model, no matter how serious the errors in the flawed model. Furthermore, it is implicit in the use of the term 'anomaly' that the assumptions of the flawed model are used to pass judgement on any suggested contradictions.

1.14 To summarise this section: -

(a) The positive linear relationship between β values and expected return within an equity market can be theoretically derived from certain assumptions. The most important is market dominance by volatility or 'risk' averse investors which also leads to a 'risk' premium for equities over bonds.

(b) There is extensive empirical evidence supporting a historical positive linear relationship between β values and actual return within an equity market and a historical premium for equities over bonds.

(c) While much of the evidence of market efficiency is subject to debate, there seems to be no explanation, other than a mean-variance efficient market, for a simple positive linear relationship between β values and past return within an equity market.

In the next section it is argued, not that a positive linear relationship between β and mean return does not exist, but that the empirical evidence has been misinterpreted as a result of ambiguity in the meaning of the term *mean rate of return*. The relationship may rely on an inappropriate definition of mean rate of return and, in consequence, the relationship may have no practical value.

In view of the possibility that the extension of this argument to challenge the relationship between stock market volatility and the equity premium will amount to nothing if an alternative model is not put forward, section three of this paper tentatively suggests an alternative explanation of the equity premium mainly based on economic and accounting principles.

2 The ambiguity of 'mean rate of return'

2.1 The actuarial profession has a variety of ways of measuring mean rate of return such as time-weighted rate of return, money weighted rate of return or geometric mean rate of return. The geometric mean of n successive rates of return is given by:

$$\left[(1+r_1)\times(1+r_2)\times\cdots\times(1+r_n)\right]^{\frac{1}{n}}-1$$

where r_i is the rate of return for the i^{th} time interval. The term discrete rate of return is sometimes used to refer individual values of r_i while $\log_e(1+r_i)$ is known as the continuously compounded rate of return for the i^{th} time interval.

As far as an individual investor is concerned, the mean rate of return that determines the success of an investment portfolio is the money weighted rate of return because it allows for the combined effects of cash flow and the performance of the investments. Given differing cash flows and the same investment performance, money weighted rates of return will differ from one investor to another.

As far as monitoring fund managers or investigating investment theories are concerned, actuaries tend to use the time-weighted rate of return, a geometric mean return or an approximation thereto because this allows for compounding but is not affected by the individual investor's cash flow over which a fund manager has no control. Further, fund managers tend to benchmark themselves against the performance of an appropriate index, or a combination of indices, calculated on a 'geometric mean' or compound total return basis.

2.2 With few exceptions, financial economists tend to use arithmetic means of discrete rates of return as Wilkie (1995) pointed out. Although some stock market models involving continuously compounded rates of return are used, the evidence supporting the linear relationship between β values and rates of return seems to be based on arithmetic averages.

Wilkie's discussion of the choice between arithmetic and geometric mean rates of return accounted for a whole section of his 1995 paper - but elsewhere the issue has received little attention. For example, a standard finance text such as Elton and Gruber (1995) devotes three in 700 pages to this question and, when discussing studies such as Basu (1977 and 1983) which do not use arithmetic means, such texts usually do not mention the way in which mean rates of return were calculated.

Wilkie (1995 p284) argues in favour of the use of arithmetic means to determine a discount rate for assessing capital projects and also to assess the risk premium between different investment classes, provided successive rates of return are independent.

However, the evidence - including that of Wilkie (1993) - suggests that independence is a false assumption, and one cannot escape the simple fact that when there is no net cash flow into an investment portfolio, *it is the geometric mean rate of return, or its equivalent, which determines the multi-period outcome of an investment, not the arithmetic mean rate of return*.

This is fairly obvious from the example below which shows that knowing the arithmetic mean rate of return does not permit us to calculate the terminal value of a portfolio at the end of a number of successive periods.

Table 2.1

Termination values and arithmetic means

Portfolio	A	B
Market value time 0	100	100
Market value time 1	40	110
Market value time 2	80	121
Rate of return period 1	-60%	+10%
Rate of return period 2	+100%	+10%
Arithmetic mean rate of return (% per period)	+20%	+10%

In this example, a stock which shows a net fall over successive periods can still have a positive arithmetic mean return.

2.3 Such examples are sometimes dismissed on the grounds that they are extreme. However, the termination values of some of the more volatile portfolios of the Black, Jensen and Scholes (1972) study would be overstated by a factor of 20, as shown in Table 2.2, if compounded arithmetic means are used to estimate termination values.

If we take the arithmetic mean excess rate of return of a high β portfolio, a low β portfolio and the market portfolio from the Black, Jensen and Scholes study and add 0.1% per month (being the average riskless rate of return over the period of their study), then we can calculate the mean monthly return.

If it is correct to use arithmetic means to estimate expected returns and we had known, in 1931, what the average return was going to be over the next 35 years, then we should have been able to predict the value of an initial portfolio of (say) $US100,000 35 years later with all dividends re-invested. However, if we use the technique described in para 2.8 to estimate the continuously compounded mean return, and then use continuous compounding to estimate the termination value, we arrive at completely different answers.

Table 2.2

Termination value in 1965 of an initial portfolio of $100,000 in 1931

Black Jensen and Scholes portfolio	#2	#6	'market'
Arithmetic mean return (% per month)	1.87	1.47	1.52
β value	1.37	0.92	1.00
Termination value based on arithmetic mean ($ million)	240	46	56
Termination value based on continuous compounding ($ million)	9.1	10.6	10.6

Which estimate is correct? This is discussed below, but we do know, from independently compiled index data, discussed in paragraph 2.7, that the answer for the 'market' portfolio should be somewhere between \$4 and \$14 million.

2.4 It seems well accepted that stock price changes and index movements are better represented by a lognormal distribution than a normal distribution. Alternatively we can say the continuously compounded return is approximately normally distributed.

If the rate of return for the i^{th} time interval is r_i and $log_e(1+r_i)$ is normally distributed with mean μ and variance σ^2 then the expected value of $1+r_i$ will be $e^{\mu+\frac{1}{2}\sigma^2}$. Consequently if arithmetic means of returns over periods such as one month are used as estimates of either the parameter μ or the geometric mean return $e^{\mu}-1$, the result will be biased by a term that depends on the variance.

2.5 The way in which the results of an empirical study are affected by the choice of arithmetic mean, geometric mean or continuous compounding will therefore depend on the way in which portfolios are selected and/or sorted.

If portfolios are selected on volatility, as was effectively the case with the Black, Jensen and Scholes (1972) study, then the volatility factor will have a major bearing on the comparative results between portfolios.

If stocks are initially grouped into portfolios on some other basis such as price/book ratio, as was the case with the Fama and French (1992) study, and there is no significant difference in volatility between the groups, then a comparison based on arithmetic means will be consistently biased and cross-sectional differences will be unaffected even though the absolute values will be exaggerated as estimates of long-term returns.

2.6 Given the level of mathematical sophistication which is evident in many papers on this topic, it seems strange that this aspect of the empirical evidence has not been given more attention.

Consider for example the Basu (1977) study, which grouped stocks into portfolios based on their price/earnings ratios and then calculated their mean return, using continuous compounding, and beta values. The results of this study are shown below.

Table 2.3

Basu's study of US Stocks 1956-1971

P/E quintile	Mean return (% pa)	Beta value
Highest	9.3	1.1121
Highest *	9.6	1.0579
2	9.3	1.0387
3	11.7	0.9678
4	13.6	0.9401
5	16.3	0.9886

* excluding companies with negative earnings

A review of Basu's paper by Ball (1978) argued that showing that the superior performance of low price/earnings ratio portfolios did not depend on higher β values was not enough to demonstrate market inefficiency. This was because low price/earnings ratios encapsulate some other form of (as yet unidentified) risk and there therefore remains a relationship between risk and return. It is interesting that neither Ball, nor a standard text such as Elton and Gruber (1995), mention the possibility that Basu's use of continuous compounding rather than arithmetic averages of discrete returns may have had a bearing on the results.

In their well known paper, Fama and French (1992) used arithmetic mean excess rates of return. Given that they were considering evidence that appeared to contradict the single-period Capital Asset Pricing Model, this may have been appropriate. However, in their paper, there was no discussion of the merits of the various definitions of return, even though they cited the results of Basu (1977), who had used continuous compounding.

2.7 The Black, Jensen and Scholes (1972) study involved 500-1000 securities over a 35 year period beginning in 1931. It involved 10 portfolios, with stocks reassigned on a regular basis according to β values and demonstrated a clear linear relationship between β values and arithmetic mean returns. Yet a simple order of magnitude check by reference to well known indices should have prompted some questions about the interpretation of these results. Such a check could have been conducted at the time along the lines that follow.

As well as the 10 portfolios, the Black, Jensen and Scholes study showed an 'excess mean return' of 1.42% per month for the market portfolio. From data published by Ibbotson and Sinquefield (1996), we know that the 'riskless return' over this period was approximately 0.1% per month, so the arithmetic mean rate of return should have been approximately 1.52% per month.

If it is correct to argue that an arithmetic mean is the correct estimate of expected returns, and we actually know what the arithmetic mean is, then we should be able to accurately estimate what an initial portfolio of (say) $100,000 would amount to 35 years later with all dividends re-invested. The accumulated value of this portfolio 35 years later by compounding at 1.52% per month would be $56 million - but this probably overstates the true position by a factor of five.

The Black Jensen and Scholes 'market' portfolio was an equally weighted portfolio of all eligible stocks comprising companies with large and small capitalisations. It could be argued that the performance of such a market portfolio would be somewhere between that of a market weighted portfolio of large companies such as the Standard and Poors' 500 (and its predecessor) and a small companies portfolio.

Using the Ibbotson and Sinquefield data, $100,000 invested in the Black, Jensen and Scholes 'market' portfolio should therefore amount to something between $3.7 million (Standard and Poors' 500) and $14.1 million (Ibbotson and Sinquefield small company stocks) over the 35 year period.

Alternatively the geometric mean return should be between 10.9% per annum and 14.1% per annum compared to 19.8% per annum that is implicit in the Black Jensen and Scholes figure of an excess return of 1.42% per month.

2.8 Where an empirical study quotes both arithmetic mean rates of return and standard deviations or variances of rates of return, it is possible to estimate the continuously compounded mean return $\hat{\mu}$, and hence the geometric mean rate of return, without knowing the individual rates of return for each month.

Let r_i be the effective (or discrete) rate of return in month i, for $i = 1, 2, \ldots n$. Then

$$e^{n\hat{\mu}} = \prod_{i=1}^{n}(1+r_i)$$

and hence

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \log_e(1+r_i) \approx \frac{1}{n}\sum_{i=1}^{n} r_i - \frac{1}{2n}\sum_{i=1}^{n} r_i^2$$

Using this approach, the estimated mean continuously compounded rate of return for the Black, Jensen and Scholes 'market' portfolio was 13.3% per annum which is consistent with 10.3% per annum for the Standard and Poors' 500 and 14.1% per annum for the Ibbotson and Sinquefield index of small company stocks over the same period.

This estimate ignores terms which are non-central moments of a distribution with a positive mean. Consequently this method should, if the next (cubic) term were included, underestimate the continuously compounded mean. However, experiments with Australian data over the last 40 years where the results are known tend to suggest that this approximation tends to over-estimate the continuously compounded mean return by approximately 0.25% per annum. See Fitzherbert (2001, p710). This suggests that there may be some features of rates of return, in the real world, which make the assumption of independent and identically distributed returns invalid. The use of theoretical models to estimate the error in this conversion process is consequently unreliable if the models employ this underlying assumption.

2.9 If we now examine two of the portfolios of the Black, Jensen and Scholes (1972) study and compare their β values with both their arithmetic mean rates of return and their estimated continuously compounded mean rates of return it can be seen that there is a considerable difference in the relationship between β value and mean rate of return, depending on the interpretation of 'mean rate of return'.

Table 2.4

Arithmetic means, β values and continuously compounded means

Black, Jensen and Scholes portfolio	#2	#6	'Market'
Arithmetic mean excess return (% per month)	1.77	1.37	1.42
β value	1.37	0.92	1.00
Estimated continuously compounded mean return (% per month)	1.11	1.12	1.12

When it is claimed, for example by Walsh (1976), that this study demonstrates a clear linear relationship between β values and investment return, these calculations show that the results of this study only show a positive relationship between beta values and arithmetic mean returns. If such arithmetic means are used to estimate portfolio accumulations, or long term returns with dividends reinvested, the results are wildly inaccurate as shown in Table 2.2.

2.10 The observation that there may be no empirical relationship between β values and continuously compounded mean returns (and therefore geometric mean returns) is not new. Jensen himself (1972), in an editorial to the compendium which published the Black, Jensen and Scholes (1972) study, noted that he had attempted to fit a continuously compounded version of a linear β/return model to the data and it did not fit their data. Similar comments appeared in the discussion of Walsh (1976) as well as warnings about the single-period limitations of a linear β/return model.

A superficial glance at the Black, Jensen and Scholes (1972) study suggests a strong relationship between β values and mean return amounting to a difference of 1% per month between portfolios with β values of 0.5 and 1.5. In terms of a long term return from equities of the order of 10% per annum, such a relationship of 12% per annum for a difference in β values of 1 is substantial. However both the example above, and the estimates of continuously compounded returns for two of the portfolios of the Black, Jensen and Scholes study demonstrate that, for multi-period use, arithmetic means can be quite misleading. If the results are adjusted to a definition of mean return that is suitable for multi-period use, such as a model based on continuous compounding or geometric means, then studies based on arithmetic mean rates of return cannot be cited as evidence in favour of any relationship between β values and multi-period return.

Perhaps also it might be reasonable to expect that if there was a valid and meaningful multi-period relationship between β values and return, there would now be numerous high-β equity funds available for public subscription. They would be attractive to investors who were prepared to tolerate high volatility. After all, specialist small-company funds have emerged, notwithstanding the fact that it is usually argued that small company funds have above average 'risk'.

2.11 Failure to recognise the subtle difference in definitions of rate of return and their impact on empirical results is quite widespread. Completely different interpretations of the Black, Jensen and Scholes (1972) study would have followed if continuous compounding (or an equivalent traditional actuarial measure such as geometric means) had been used rather than arithmetic means of discrete rates of return.

The use of arithmetic means, as an estimate of multi-period expected returns, relies on serial independence which is only approximately true in the short-term. Also, we cannot escape from the simple fact that it is the geometric mean rate of return, or an equivalent such as the continuously compounded mean, which determines the multi-period total return of an investment. The arithmetic mean always over-estimates the final result, and the greater the volatility, the greater the exaggeration.

3 The return on equity explanation for the equity premium

3.1 If a capital market is mean-variance efficient then using theoretical arguments, there should be: -

(a) a positive linear β/return relationship within equities as an asset class,

(b) an equity 'risk' premium (otherwise the β /return relationship will not be positive).

If arithmetic mean returns are used to assess the empirical evidence, then both propositions are generally supported by the empirical evidence. However, if continuous compounding is used, the empirical evidence is contradictory because it supports the existence of a continuously compounded equity premium, but not a positive linear β/return relationship.

The fact that there is an equity premium when calculations are based on continuous compounding but that there may not be any linear relationship between β values and continuously compounded stock returns therefore calls into question at least one of the underlying assumptions of mean-variance efficient markets.

One such possibility is that the equity premium, whether measured using continuous compounding or arithmetic means, does not depend on risk-averse pricing by portfolio investors. This possibility is supported by the arguments of Mehra and Prescott (1985) that the risk premium is too high to be justified by rational economic behaviour. It is also supported by the analysis of Bernstein (1997) who suggests that long-term equity returns are more predictable than those of bonds. In consequence equities have a lower long term risk than fixed interest securities despite the higher short-term volatility.

3.2 The *return-on-equity* explanation presented here for the existence of the equity premium depends on two propositions:

(a) a company's profitability, or return on equity, fluctuates - probably as a weakly stationary stochastic process, and

(b) the mean, or long term equilibrium level, of this stochastic process needs to exceed a company's cost of capital which is dependent on interest rates.

3.3 The argument for return on equity, or profitability, being a weakly stationary stochastic process is an economic one at two levels.

At the micro-economic level, the directors and senior management of a company which is achieving what they, or the market, regard as an inadequate return on its shareholders funds will try hard to do something about it. Failure, as well as leading to corporate decline, is likely to invite a takeover or lead to eventual liquidation. These consequences provide a powerful motivating force for directors and senior management to turn the company around, possibly by moving into more promising areas of business.

Also at the micro-economic level, a company which is achieving a high return on its shareholders funds will attract the attention of others who might consider expanding into the same business leading to greater competition and a lower return on equity. There is also the prospect that the directors of companies making high returns on shareholders' funds are unable to utilise all their retained profits in businesses with which they are familiar. This might lead to unsuccessful expansion into apparently promising, but unfamiliar areas, the dilution of shareholders' interests through aggressive takeovers or simply unprofitable expansion of existing lines of business.

In some cases the perception of high returns on equity will lead to new companies being floated with the specific intention of entering an area of business that appears to offer high returns.

3.4 At the macro-economic level, a high level of profitability will probably lead to many new equity offerings, possibly creating inflationary pressure and a 'bubble' economy. In these circumstances a responsible government may seek to dampen public enthusiasm by various measures at its disposal which depress profitability and/or expectations thereof. Alternatively, the history of such episodes suggests that, if they are unchecked, bubbles eventually collapse. See for example, Kindleberger (1978).

On the other hand low profitability at the macro-economic level, if sustained for any length of time, will lead to an economic contraction with unpopular unemployment consequences. To some extent this was the central idea of Keynes (1936). The solution, from a government perspective, is to create an environment where there is a modest margin between corporate profitability and the cost of capital using the various mechanisms at its disposal, such as official interest rates.

3.5 Some time ago, Hemsted (1962) showed that if return on equity and the proportion of profits distributed in dividends is constant (the payout ratio), then the total return achieved by a long term equity investor will be equal to the dividend yield at the time of purchase plus retained profits expressed as a percentage of shareholders' funds.

Let S_t be shareholders' funds at time t, then company earnings in the following year will be

$$S_t \times [\textit{return on equity}]$$

and dividends paid and/or payable will be

$$S_t \times [\textit{return on equity}] \times [\textit{payout ratio}].$$

Hence shareholders' funds at the end of the year will be

$$S_{t+1} = S_t \times [\textit{return on equity}] \times [1 - \textit{payout ratio}] + S_t \ .$$

Using this straightforward accounting model, the dividend stream will grow at a constant rate equal to

$$[\textit{\textbf{return on equity}}] \times [\textit{\textbf{1 - payout ratio}}]$$

and the total return achieved by a 'buy-and-hold' investor will be equal to

$$[\textit{\textbf{dividend yield at purchase}}] + [\textit{\textbf{return on equity}}] \times [\textit{\textbf{1 - payout ratio}}] \ .$$

While not constant, overall return on equity, or profitability, is a great deal more stable than stock prices. In Australia, the standard deviation of annual return on equity (using the definition used in company reports) is approximately 1% for the industrial sector and 2% for the resources sector. See Fitzherbert (1998, p15-16). As dividends tend to be very stable, this variability also flows through to retained profits as a proportion of shareholders' funds.

Thus the long term return on equities compared to bonds and/or cash depends, in the long run, on the return on equity exceeding interest rates by a comfortable margin. According to this explanation the equity premium depends on product pricing and the selection of projects and businesses by company directors to achieve an adequate rate of return on shareholders' funds. According to this accounting model, the equity premium has no direct link with stock market volatility, except to the extent that it may influence 'hurdle' rates of return laid down by company directors and senior management.

If we use the formula,

$$[\textit{\textbf{growth from reinvested profits}}] = [\textit{\textbf{return on equity}}] \times [\textit{\textbf{1 - payout ratio}}]$$

then we should be able to estimate the contribution from this source when considering historical data.

Unfortunately, reliable data relating to profitability and payout ratios is not readily available. From data published by the Reserve Bank of Australia (1963 et seq) and Value Line Investment Survey (1990) we can make the following estimates:-

Table 3.1

Internally generated growth from retained profits

	Profitability (% pa)	Payout ratio	Growth (% pa)
US (Dow Jones Industrial Averages) 1920-89	11.6	65%	4.1
Australian non-mining stocks 1957-86	8.9	55%	4.0

These figures both indicate retained profits of 4% per annum of shareholders' funds. However, profitability is usually measured as a percentage of average capital. As a percentage of start of year capital, these figures could be increased by approximately 0.5% per annum.

3.6 One immediate objection to the return on equity, or accounting, explanation for the long term growth of dividends and share prices is that this explanation makes no allowance for inflation. It could be argued that company profits will broadly grow with Gross Domestic Profit which moves with inflation and also grows in real terms; in the long run this broad movement with Gross Domestic Product will be reflected in dividend streams and, subject to significant short term fluctuations, share prices as well. This argument seems to be behind the official 'core reading' for subject 102; to quote the Institute and Faculty of Actuaries (2000a):-

> *'.. dividends should increase with inflation and real growth in a company's earnings.'*

3.7 This argument would be valid if company shareholders were in the same position as a national tax collector. But company shareholders are periodically asked to supply additional capital to finance growth and expansion. If a company's business is growing at a faster rate than its shareholders' funds then, sooner or later, it will need to raise more equity capital. An investor who wishes to maintain a pro-rata interest in a dividend stream will therefore be required to outlay additional capital and this must be taken into account.

When inflation was significantly higher than it is today, there was a significant difference between companies' 'real' and reported profits which was quite obvious. The reason for this difference is under the historical cost accounting convention, it is only the money value and not the real value of a company's shareholders' funds which are preserved before profit is determined. When inflation is in excess of 10% per annum these distortions are very significant. To quote Parker and Gibbs (1974):

> *"Not only is the answer [from traditional historic cost accounts] without real meaning, it's also dangerously misleading."*

However with return on equity of 10% per annum and inflation of 2%, real profits - in aggregate - are still overstated by approximately 0.02 / (0.10 - 0.02) or 25%.

The so-called benefits of inflation (ie the difference between real and inflation adjusted profits) appear as illusory profits in the profit and loss account instead of being retained in corporate balance sheets. To quote Parker and Gibbs again:-

> *" amounts based on historic cost which set aside for depreciation of plant and machinery will, in a period of rapid inflation, be totally inadequate either to provide funds for the eventual replacement of those assets or to maintain the real value of shareholders' original capital investment. Similarly profits are overstated by the inclusion of profits on stock which arise solely from a general increase in price levels."*

There are some exceptions to the principle of historic cost accounts preserving the money value rather than the real value of shareholders' funds. These exceptions are permanent real assets such as property, licences, patents, newspaper mastheads and some intangible assets such as brand names. Where it is used, last-in-first-out accounting tends to defer (but not eliminate) the illusory effect of stock inflation.

Consequently, any monetary benefit from inflation only applies to a small proportion of corporate balance sheets. In addition, there may be an indirect benefit if return on equity is higher in inflationary periods than when inflation is negligible.

3.8 A similar argument to the above, based on his own observations, was expressed by Graham (1973):

> *"The cold figures demonstrate that all the large gain in the earnings of the [Dow Jones Industrial Averages] unit in the past 20 years was due to a proportionately large growth of invested capital coming from reinvested profits. If inflation had operated as a separate favourable factor, its effect would have been to increase the 'value' of previously existing capital; this in turn should increase the earnings on such old capital and therefore on the old and new capital combined. But nothing of the kind actually happened in the last 20 years, during which the wholesale price level has advanced by nearly 40%."*

3.9 In the 20th century, the long term growth in share prices in the US, the UK and Australia was between 5% per annum and 6% per annum, continuously compounded, as shown in Table 3.2.

Table 3.2

Share Price index growth 1900-2000

	Australia (% pa)	United Kingdom (% pa)	United States (%pa)
Capital growth	5.8	5.0	5.3
Inflation	4.0	4.0	3.1

The UK and US capital growth and inflation figures in this table were obtained from Dimson Marsh and Staunton (2002). These figures are consistent with the argument that company earnings, and therefore dividends, and therefore the long term trend of share prices will more than match inflation.

3.10 Unfortunately the historical record of share price indices over the 20th century is also consistent with the proposition that retained profits lead to growth in earnings, dividends and, in the long term, share prices. In addition, there has been a significant equity premium for the long term return of shares over cash securities and bonds whether measured using arithmetic mean rates of return, geometric means or using continuous compounding.

The crucial issue is to identify the cause of long-term capital appreciation in ordinary shares, and then add a dividend factor to estimate total return. If the cause is retained profits, of the order of 4% per annum, then this will also apply in periods of low or negligible inflation. Suitable data is difficult to find; but we do know that the predecessor to the Australian All Ordinaries Index appreciated by 4.1% per annum over the period 1875-1933 while the general price level only rose by 0.5% per annum.

It is difficult to deny the argument that retained profits will augment shareholders' funds, thereby increasing a company's capacity to borrow; this will enable total capital employed also to expand, leading to higher profits and dividends. This factor may well account for almost all of the long term capital appreciation in all three countries. For reasons argued above, historic cost accounts allow a small proportion of shareholders' funds to be maintained in real terms - which may account for the remaining 1-2% per annum of capital appreciation not explained by retained profits.

If retained profits have averaged 4% per annum of shareholders' funds, then equity capital appreciation in the Australia, the US and the UK is explained mainly by the retention of profits, plus a minor benefit from inflation. This minor benefit is considerably less than the full rate of inflation.

3.11 It follows that the last 100 years of total return from ordinary shares, comprising both dividends and capital appreciation, may well be accounted for by retained profits with a small contribution from inflation. There is no additional room for a factor dependent on 'risk averse' behaviour by portfolio investors, except indirectly. The indirect effect occurs when stock market volatility is used as part of a formula to determine acceptable levels of corporate profits.

Also, if volatility averse stock market pricing is a direct cause of the equity premium, it is difficult to see how this could be the case unless there was also a positive relationship between β values and geometric or continuously compounded mean rates of return within equity markets.

For these reasons it is suggested that the ***cause*** of capital appreciation in the past has been mainly retained profits and hence the equity premium has arisen because profitability, or return on equity, exceeded interest rates. The prospect of unemployment, and possible government intervention, are good reasons for believing that this will continue.

Inflation seems to have had only a minor influence on long term equity returns once allowance is made for retained profits. This seems likely to continue to be true unless inflation adjusted accounts replace historic cost accounting conventions or a higher return on equity is sought in times of high inflation. Volatility has had no discernable influence on the long term trend of equity returns and hence the equity premium. This, too, seems likely to continue unless volatility has an indirect effect through any influence on profitability.

4 Discussion

4.1 *A standard meaning for 'rate of return'*

It is well known that an arithmetic mean is greater than a geometric mean. Consequently different investigations of the same data will give different results depending on which 'mean rate of return' is used. When comparing two alternative investment strategies, an arithmetic mean will tend to favour the more volatile assets or asset class.

For this reason, it is important that the method of calculating the mean is appropriate for the purpose to which a subsequent model will be put. It is not valid, for example, to rely on empirical evidence based on arithmetic averages to support a model of returns based on geometric means or continuous compounding.

While actuaries tend to use geometric means and financial economists use arithmetic means, both parties may benefit from the adoption of continuous compounding as their standard definition of rate of return for mathematical models and data analysis. There are areas where this has already occurred.

Apart from enjoying a direct relationship with geometric means, there are some theoretical advantages of continuous compounding.

(a) The expected value of the product of two random variables is usually only equal to the product of the expected values if they are independent. On the other hand the expected value of the sum of two dependent random variables is still equal to the sum of the expected values. Mathematically, if r_i is the rate of return in the i^{th} time interval then $E\{(1+r_1)\times(1+r_2)\}$ is not equal to $E\{1+r_1\}\times E\{1+r_2\}$ unless r_1 and r_2 are independent. However when $\delta_i = log_e(1+r_i)$

$$E\{\delta_1+\delta_2\} = E\{\delta_1\}+E\{\delta_2\}$$

irrespective of whether r_1 and r_2 are independent or not.

Given that there is a considerable evidence to suggest that rates of return in successive periods are not independent, continuous compounding avoids the need to use this assumption in many cases.

(b) Professionally managed portfolios are rebalanced, often to fixed percentage asset allocations, on a regular basis. In these circumstances, the continuously compounded returns of the component sectors can be combined to determine the continuously compounded return of the whole portfolio. Discrete rates of return can only be used for this purpose if no rebalancing takes place during the period under study.

(c) There may be a natural tendency, when confronted with a sequence of figures, for people to calculate simple averages for comparative purposes. However, when the data is a sequence of discrete rates of return, geometric means may be more appropriate. Unless this more sophisticated calculation is performed, arithmetic averages may be misinterpreted because of the approximately lognormal distribution of the data. If rates of return were presented in continuously compounded form, this potential source of misinterpretation could be avoided.

Another advantage of continuous compounding is that it is already acknowledged, in some sections of the non-actuarial finance community, as being theoretically superior. To quote a standard Australian text, Brailsford and Heaney (1998):

> *"While continuously compounded returns are preferred in theory, discrete returns tend to be used in practice.....*
>
> *.. discrete returns are easy to understand, whereas the concept of continuous time and continuous returns can be difficult to understand and explain to a layperson."*

Given the considerable differences that can emerge from data analysis depending on whether arithmetic means, geometric means or continuously compounded means are used, this is an issue which warrants careful thought by the actuarial profession. Perhaps some thought also needs to be given to quantifying the effect of the assumption of independence of successive rates of return, which is important when using arithmetic averages of discrete rates of return.

4.2 *The return on equity explanation of the equity premium.*

There are three potential explanations of the nature and extent of the equity premium:-

(a) the accounting, or return on equity, model of capital appreciation and its consequences, covered in section 3,

(b) the economic, or inflation and growth link model, which argues that profits and therefore dividends should broadly keep pace with Gross Domestic Product, and

(c) the modern portfolio theory model which argues that shares should be priced in such a way that they are expected to outperform less volatile assets.

If, as argued in section 3, the long term performance of ordinary shares is almost entirely determined by profitability or return on equity, then the level of the equity premium will depend on the relationship between corporate profitability and interest rates.

It is hard to claim that retained profits do not lead to increased profits and dividends. If inflation and/or economic growth are additional favourable factors then the rate of capital growth in Australia, the United Kingdom and the United States would have been at least 8% per annum over the 20th century compared to the actual outcome of only 5% to 6% per annum.

Consequently, the equity premium has only a little to do with inflation. Retained profits and inflation account for all of the capital appreciation over the 20th century. The long term total return from equities will only be affected by 'risk averse' market pricing if stock market volatility affects return on equity. In this respect the sophisticated methods now used to calculate cost of capital are too new to have had a major impact on 100 years of past data.

4.3 *The phenomenon of less than linear stock market volatility.*

There are a number of manifestations of the phenomenon of stock market volatility being less than predicted under the assumption of identical and independent successive rates of return. These observations include the apparently linear long term trend of stock market indices identified by the Maturity Guarantees Working party (1980) and the predictability of long term stock returns identified by Bernstein (1997).

Let P_t denote the value of a share price index at time t and let B_t denote the underlying book value or shareholders' funds per share according to the company's books at time t. Then we can express the relationship between the logarithms of the indices at times 0 and n in two ways.

The traditional way is to say that:

$$P_n = P_0 \times \frac{P_1}{P_0} \times \frac{P_2}{P_1} \times \cdots \times \frac{P_n}{P_{n-1}} \text{ and hence}$$

$$log\left(\frac{P_n}{P_0}\right) = log\left(\frac{P_1}{P_0}\right) + log\left(\frac{P_2}{P_1}\right) + \cdots + log\left(\frac{P_n}{P_{n-1}}\right)$$

which is the sum of a number of variables. By choosing months, or days, as our time unit we can make n as large as we like. Given the central limit theorem, the logarithms of (one plus) price changes then become normally distributed, provided the price (or index) movements are identically and independently distributed variables. Under these assumptions the variance of $log(P_n/P_0)$ will be a linear function of n. The available evidence therefore contradicts these assumptions.

One possibility for the observed phenomena is that profitability is a weakly stationary stochastic process which determines stock market pricing as well as company profits.

If we express price changes in terms of changes in price/book ratios and changes in book value per share we have:

$$\frac{P_n}{P_0} = \frac{P_n/B_n}{P_0/B_0} \times \frac{B_1}{B_0} \times \frac{B_2}{B_1} \times \cdots \times \frac{B_n}{B_{n-1}} \text{ and hence}$$

$$log\left(\frac{P_n}{P_0}\right) = \frac{log\{P_n/B_n\}}{log\{P_0/B_0\}} + log\left(\frac{B_1}{B_0}\right) + log\left(\frac{B_2}{B_1}\right) + \cdots + log\left(\frac{B_n}{B_{n-1}}\right)$$

and hence the price change depends on

(a) the price/book value ratio at time n (which we do not know),

(b) the price/book value ratio at time 0 (which we do know) and

(c) changes in book value over the time interval 0 to n.

This may sound like substituting two unknowns for one, until we recognise that changes in book value per share are due to retained profits and the fluctuations in this factor for the stock market as a whole (standard deviation of the order of 1% per annum) are comparatively minor compared to changes in stock market indices (standard deviation of the order of 20% per annum).

The yardstick most widely used by investors to compare shares, both within national markets and internationally, is the price/earnings ratio - usually related to future estimates rather than historical figures. A number of experienced observers such as Graham (1958) have suggested that the price/earnings ratio tends to rise with profitability - a company achieving a return on equity of (say) 15% per annum will tend to trade at a higher price/earnings ratio than a less profitable company achieving a return on equity of 8% per annum unless a substantial improvement in profitability (ie return on shareholders' funds) is anticipated.

Price/book ratios, profitability and price/earnings ratios are related by the straightforward equation:

$$\frac{\textit{Share price}}{\textit{Book value / share}} = \frac{\textit{Share price}}{\textit{Earnings / share}} \times \frac{\textit{Earnings / share}}{\textit{Book value / share}}$$

$$= [\textit{P / E ratio}] \times \frac{\textit{Earnings}}{\textit{Book value}}$$

$$= [\textit{P / E ratio}] \times [\textit{return on equity}]$$

It follows that if price/earnings ratios rise with return on equity, price/book ratios will rise with return on equity by more than a power of one.

To quote Graham (1958):-

> *"If, as many tests show, the earnings multiplier tends to increase with profitability - ie as the rate of return on book value increases - then the arithmetical consequence of this feature is that [market] value tends to increase directly as the square of earnings."*

If return on equity, or profitability, is a weakly stationary process and price/earnings multiples depend on profitability, then price/book value ratios will also tend to be mean reverting. Consequently a stock price index would fluctuate about an equilibrium that is directly related to underlying shareholders' funds. As the shareholders' funds underlying a stock market index (adjusted for new issues) mainly increase from retained profits, the underlying long term trend of a stock market index will be the same as the underlying trend of shareholders' funds (adjusted for new issues). If return on equity is mean reverting, this trend will be approximately exponential. The long term trend will therefore look like a straight line when drawn on a semi-logarithmic scale.

4.4 *The measurement of equity portfolio investment risk*

There is however, a second aspect of this explanation for non-linear stock market volatility that deserves attention.

If return on equity is a weakly stationary process, then investors should be pricing stocks on the assumption that high profitability will revert to an equilibrium level with time. Investor behaviour, however, seems to be exactly the opposite. At the micro-level this is a possible underlying cause of anomalous evidence relating the cross-section of returns to price/book ratios, dividend yields and price/earnings ratios. However, at the macro level, this phenomenon could be important for the actuarial profession, because it might establish a possible way of identifying and measuring equity asset class risk.

Let us suppose that return on equity is a weakly stationary process and that the tendency of investors to increase price/earnings ratios with profitability creates a relationship that also applies at a macro, or index level. If this happens, high return on equity will be accompanied by high price/earnings ratios, and an assessment of medium term risk should allow for return on equity reverting to an equilibrium position with a corresponding change in the price/earnings ratio. A simple way of combining these two effects into one calculation is to calculate an aggregate price/book ratio. An appropriate general measure of stock market risk should therefore be based on the current price/book ratio (ie total market capitalisation divided by underlying shareholders' funds) not volatility.

Under modern portfolio theory, volatility is used extensively as a general measure of risk and, in many circles, is regarded as risk. However, according to the analysis of this paper, volatility is caused (at least in part) by fluctuations in return on equity, exacerbated by investor behaviour which should anticipate that return on equity will revert to some sort of equilibrium, but which makes the opposite assumption.

Other factors such as interest rates are important and a model of price/book ratios would need to take these into account, but interest rates also affect profitability and investors' expectations. As far as return on equity and stock market volatility are concerned by themselves, ***volatility is mainly a symptom*** of fluctuating profitability, and its perverse effect on investor psychology ***rather than a cause*** of rational pricing. Models of equity class returns based on volatility are therefore unlikely to be reliable - even as a method of measuring risk.

4.5 *Discount rate for the valuation of liabilities*

Using the formula of paragraph 4.3 relating an ***n-period*** return to book values, the most likely equity investment return over a period of several years depends on the mean return on equity achieved in that time and the change in price/book ratios over the same time span. As we cannot predict the terminal price/book ratio, likely returns are inversely proportional to the starting price/book value ratio. The likelihood of unsatisfactory returns from a general market index over the medium to long-term (say 5-10 years), therefore depends on the price/book ratio at the beginning.

According to a recent survey reported by Head et al. (2001), it appears that the UK actuarial profession is moving toward a method of valuing pension liabilities that determines a valuation rate of interest by adding an 'equity premium' to a riskless rate of return. This valuation of liabilities is then compared with a market valuation of assets.

This method assumes that such a premium will always be present and that it will be positive. If, as suggested by this analysis, the future equity premium in the medium to long term depends on the current aggregate price/book ratio, then such an assumption may be unreliable. There may also be some occasions when volatility, as a general measure of stock market risk, is grossly inaccurate.

A sounder alternative might be to estimate the duration of the liabilities and to assume that the average return on equity and aggregate price/book ratio of the equity portfolio revert to empirically derived equilibrium levels. The equity portfolio is then deemed to be sold at a time such that the duration of assets and liabilities is the same and this data is then used as the starting point for determining an appropriate valuation assumption.

4.6 *Acknowledgements.*

I would like to acknowledge the assistance of several people in preparing this paper. In particular Michael Barker, Geoffrey George, Richard Mitchell and John Salamito read drafts of the paper at different stages and provided valuable comments. However, I must accept responsibility for the views expressed.

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