Non-linear Co-Movements in Output Growth: Evidence from the United States and Australia.

by

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Abstract

This paper investigates comovements between the United States and Australia. Our non-linear model allows the dynamic response to shocks to differ if countries are in recession. Generalised Impulse Response Functions highlight a significant asymmetric response to positive and negative shocks.

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1 Introduction

This paper examines the linkage between the rate of economic growth in the United States and the growth rate in Australia. The tendency for countries' outputs to co-move positively with the output of the United States is well known. Backus, Kehoe and Kydland (1992), for example, document correlation coefficients for a number of countries' respective outputs with that of the United States; with only one exception, these correlation coefficients are positive. Similar findings have been reported by other researchers (for example Gregory, Head and Raynauld 1997; and Ambler, Cardia and Zimmermann 1999).

This previous literature has derived measures of co-movement on the assumption that the relationship between United States' growth and growth in other countries is linear. An exception is the paper by Henry and Summers (2000) who document the existence of a threshold non-linearity in the relationship between Australian and US economic growth. In their paper, the size and sign of shocks influence the nature of the dynamic adjustments that follow.

Our approach allows for non-linearity in the comovement between the United States and Australia. In particular, the dynamic response to a shock may differ if one, or both, economies are in recession. Beaudry and Koop (1993), in the context of a single country's economic growth path, refer to this as a "bounce-back" effect, where the rate of growth accelerates in the aftermath of a recent recession. To date, the question of whether recessions impact on the international co-movement of output growth has not been investigated.

2 The Modelling Framework

Our starting point is the standard, linear, Vector Autoregressive Moving Average (VARMA) representation of output growth. This is given by,

$$\Theta\left(L\right)\Delta\mathbf{y}_t = \boldsymbol{\mu} + \Phi\left(L\right)\boldsymbol{\varepsilon}_t \tag{2.1}$$

where $\Delta\mathbf{y}_t$ is a vector of output growths (assumed to be stationary) so that $\Delta\mathbf{y}_t = (\Delta y_{US,t}\ \Delta y_{AUS,t})'$, $\Theta\left(L\right)$ and $\Phi\left(L\right)$ are 2×2 matrix polynomials in the lag operator L, with fixed 2×2 coefficient matrices Θ_k and Φ_k, with $\Theta\left(0\right) = \Phi$ and $(0) = I_2$. $\boldsymbol{\mu} = (\mu_{US}, \mu_{AUS})'$ is a vector of constants to capture any drift in growth and $\boldsymbol{\varepsilon}_t = (\varepsilon_{US,t}, \varepsilon_{AUS,t})'$ is a vector of i.i.d. error terms with a 2×2 variance-covariance matrix $\Sigma = \sigma_{ij}$, $i,j = US, AUS$. Assuming that stability and invertibility conditions hold, such a representation is general enough to directly accommodate the interdependencies in the determination of $\Delta y_{US,t}$ and $\Delta y_{AUS,t}$ through the lag filters $\Theta\left(L\right)$ and $\Phi\left(L\right)$ and indirectly through the covariance matrix Σ. Therefore the modelling framework is potentially able to incorporate the effects of shocks which influence both output growth in the US and output growth in Australia directly, and the feedbacks from output growth in the US to output growth in Australia, and vice versa. We introduce non-linearity by allowing for regime changes are governed by observable variables. This is achieved by augmenting (2.1) with a measure of the "current depth of recession" (CDR). The CDR term can be written as,

$$CDR_{i,t} = \max \left\{ y_{i,t-s} \right\}_{s=0}^{t} - y_{i,t}. \tag{2.2}$$

where i indexes the country. The CDR is defined as the gap between the current level of output and its historical maximum. That is, CDR will take non-zero values when output dips below its trend value due to a negative shock. The asymmetry implied by the CDR term is reflected in the "bounce-back" effect, the tendency for output growth to recover relatively strongly following a recent recession. Hence, the CDR approach treats the historical maximum level of output as an attractor which influences the dynamics of output growth when output falls below its previous peak. Beaudry and Koop (1993) hypothesise that there is a non-linearity in this "peak reversion"; the further output falls from its peak, the greater is the pressure that builds up for output to return to its historical maximum. As a result, the speed at which output recovers varies according to the severity of the recession.

Adding CDR terms to (2.1) implies that the conditional expectation of future output is influenced by whether the current level of output is above, below or at its historical maximum. Further, these effects can impact across countries so that, for example, US output growth being above, below or at its historical maximum, may directly or indirectly, affect Australian output growth (and vice versa).

With the introduction of the CDR term, equation (2.1) becomes

$$\Theta(L)\,\Delta\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\Xi}(L-1)\,\mathbf{CDR}_t + \Phi(L)\,\boldsymbol{\varepsilon}_t \tag{2.3}$$

where $\mathbf{CDR}_t = (CDR_{US,t}, CDR_{AUS,t})'$ and $\boldsymbol{\Xi}$ is a 2×2 matrix of lag polynomials with fixed 2×2 coefficient matrices Ξ_k. This parameterisation for $\Delta\mathbf{y}_t$ nests the VARMA model (2.1) while capturing the possibility of asymmetric responses to positive and negative shocks to growth in both economies.

Other approaches to capture regime switching behaviour, such as Markov, or Threshold switching models, have been employed in the literature. Our approach has a major advantage over such models. Tests of the null of linearity based on (2.3) are not subject to unidentified parameters under the null (the Davies 1987 problem) and, consequently, asymptotic inference is valid.

To investigate the effect of shocks on the future values of output growth in either the linear VARMA model in (2.1), and the non-linear VARMA-CDR model in (2.3), we use Generalised Impulse Response Functions (GIRFs) introduced by Koop *et al* (1996). Whereas the VARMA specification easily admits analytical solutions for the impulse responses, simulation methods must be used to trace the impact of a shock to the elements of the state vector, or to the entire system, for the non-linear VARMA-CDR case.

If Y_t is a random vector, the GIRF for a specific shock υ_t and history ω_{t-1} is defined as,

$$GIRF_Y(n, \upsilon_t, \omega_{t-1}) = E\left[Y_{t+n} | \upsilon_t, \omega_{t-1}\right] - E\left[Y_{t+n} | \omega_{t-1}\right], \tag{2.4}$$

for $n = 0, 1, 2, ...$ Hence, the GIRF is conditioned on υ_t and ω_{t-1} and constructs the response by averaging out future shocks given the past and present. Given this, a natural reference point for the impulse response function is the conditional expectation of Y_{t+n} given only the history ω_{t-1}, and, in this benchmark response, the current shock is also averaged out. Assuming that υ_t and ω_{t-1} are realisations of the random variables V_t and Ω_{t-1} that generate realisations of $\{Y_t\}$, then, following Koop *et al* (1996), the GIRF defined in (2.4) can be considered to be a realisation of a random variable given by,

$$GIRF_Y(n, V_t, \Omega_{t-1}) = E\left[Y_{t+n} | V_t, \Omega_{t-1}\right] - E\left[Y_{t+n} | \Omega_{t-1}\right]. \tag{2.5}$$

The computation of the GIRF for the linear VARMA model in equation (2.1) is relatively straightforward, where the distribution of the GIRF is given by $GIRF_Y(n, V_t, \Omega_{t-1}) \sim N(0, \Psi_n \Sigma \Psi_n')$ for the case where $V_t \sim N(0, \Sigma)$. Ψ_n are 2×2 matrices of coefficients assumed to be square-summable, where

$$\Psi(L) = \Theta(L)^{-1} \Phi(L) = \sum_{n=0}^{\infty} \Psi_n L^n, \tag{2.6}$$

$\Psi_0 = I_2$, and the (i, j)th element of $\Psi(L)$ is denoted by lag polynomials $\psi_{i,j}(L)$. The GIRF is characterised by the variance-covariance matrix $\Psi_n \Sigma \Psi_n'$, for $n = 0, 1, 2....$[1]. If we scale the GIRF by a unit shock, (defined by one standard error $(\sqrt{\sigma_{ii}})$), then the effect of a unit shock to the ith equation in the model on the jth variable at time horizon n is given by the jth element of $\Psi_n \Sigma e_i' / \sqrt{\sigma_{ii}}$, or equivalently, as $e_j \Psi_n \Sigma e_i' / \sqrt{\sigma_{ii}}$. e_i (e_j) is a 1×2 selection vector, with unity in the ith (jth) element and zero in the other element and Σ is the 2×2 variance-covariance matrix of ε_t with individual elements σ_{ij}, $i, j = US, AUS$.

The computation of GIRFs for non-linear models is less straightforward since analytical expressions for the conditional expectations in the GIRFs are not usually obtainable.

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[1] See also Lee and Pesaran (1993) who propose a scaled version of this measure, namely, 'persistence profiles', and applied by Pesaran and Shin (1995) and Lee and Shields (2000).

Monte Carlo methods of stochastic simulation need to be used to compute the conditional expectations, see Granger and Teräsvirta (1993, Ch. 8), and Koop *et al* (1996) for detailed descriptions of the various methods that can be used.

One advantage of the VARMA-CDR model is that there is no assumption of symmetric output response to positive and negative shocks. GIRFs can be used to measure the extent to which negative shocks are more persistent than positive shocks as well as to assess the potential diversity in the dynamics in the effects of positive and negative shocks on output.

Let $GIRF_Y(n, V_t^+, \Omega_{t-1})$ denote the GIRF from conditioning on the set of all possible positive shocks, where $V_t^+ = \{\upsilon_t | \upsilon_t > 0\}$ and $GIRF_Y(n, -V_t^+, \Omega_{t-1})$ be the GIRF from conditioning on the set of all possible negative shocks. These GIRFs will be referred to 'asymmetric' GIRFs. Following van Dijk *et al* (2000), the distribution of the random asymmetry measure given by,

$$ASY_Y(n, V_t^+, \Omega_{t-1}) = GIRF_Y(n, V_t^+, \Omega_{t-1}) + GIRF_Y(n, -V_t^+, \Omega_{t-1}), \qquad (2.7)$$

will be insignificantly different from zero if positive and negative shocks have the same effect, i.e. a positive unit shock will have exactly the same effect as a negative unit shock. The distribution of this measure can provide an indication of the asymmetric effects of positive and negative shocks.

3 Modelling US and Australian Output Growth

We employ data on real GDP for the USA and Australia, sampled at a quarterly frequency over the period 1959 III to 2001 I. Given the non-stationary nature of the GDP series and our focus on the transmission of growth shocks we transform the data into difference stationary growth rates as:

$$\Delta y_{i,t} = \log\left(Y_{i,t}/Y_{i,t-1}\right) \qquad (3.8)$$

where Δ represents the first difference operator and $Y_{i,t}$ represents the level of output in period t for $i = US,\ AUS$. Figure 1 presents time series plots of the data. It is not possible to reject the null of a unit root in the logarithm of GDP for either country at the 5% level ($ADF_{AUS} = -1.9829$, $ADF_{US} = -3.1359$, critical value $= -3.4381$). However the respective growth rates are clearly stationary ($ADF_{AUS} = -11.7027$, $ADF_{US} = -6.0528$).[2]

Table 1 displays the parameter estimates of the unrestricted VARMA and the VARMA-CDR models. On the basis of the Swartz (1978) information criterion a two lag VAR was chosen for both models. The models pass the usual diagnostic tests for serial correlation

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[2] Using the Johansen approach there was no evidence of cointegration between the logarithms of GDP. The results are available upon request.

and heteroscedasticity[3]. As the VAR-CDR nests the VAR we perform a LR test for the joint exclusion of the CDR terms. The results of the LR test ($LR = 11.5438$, $p-$ value $=0.0211$) suggest that the VAR-CDR provides a superior characterisation of the data to the linear VAR.

On the basis of a system based likelihood ratio test we were able to exclude the Australian CDR variable ($LR = 3.7553$, $p-$value 0.1530) but unable to exclude the corresponding US variable ($LR = 7.9201$, $p-$value $= 0.0191$). We were further able to exclude lags of Australian growth from the US equation of the VAR-CDR model.

Taken in isolation the parameter estimates themselves are uninteresting, save for the sign and size of the CDR terms in the US and AUS equations. The positive sign of $CDR_{US,t-1}$ in the US equation is consistent with a bounce-back effect. In contrast the negative sign of $CDR_{US,t-1}$ in the Australian equation is consistent with the widely held view that "when the US sneezes, Australia catches the 'flu".

4 Generalised Impulse Responses

Figure 2 presents the cumulative generalised impulse response functions. We report GIRFs from both the estimated VAR and VAR-CDR models for comparative purposes. We report the GIRFs from the best fitting VAR to guage the economic importance of allowing for non-linearity.

The upper panel of Figure 2 highlights the dynamic time profile for Australian output of a shock that causes Australian growth to rise by one standard error on impact and suggests that the implied dynamic response to a domestic shock to Australian output is largely comparable across the models over the short and long-run, despite the presence of the US CDR term in the Australian equation. This similarity occurs becuse the US CDR term never turns on in response to an Australian shock.

The middle panel of Figure 2 plots the response of Australian growth to an external shock, in this case a shock that causes US growth to rise by one standard error on impact. The short-run dynamics of the impulse response differ markedly across models; for instance, the response function from the VAR model predicts that there will be a short and medium term increase in the level of output. In comparison, the VAR-CDR model predicts that there will be more volatility in the periods immediately after the shock (up to 6 quarters). Unlike the VAR which predicts a uniformly positive reaction, the response of the VAR-CDR becomes negative after roughly four quarters following the shock. As the time horizon increases, the dynamics of the responses fall into line with each other.

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[3] Results of the diagnostic tests are available upon request.

However, the difference between the responses remains markedly distinct, indicating the importance of the role of the US CDR term.

The impact of a one standard deviation domestic shock to US growth is displayed in the bottom panel of Figure 2. Here the VAR predicts a more persistent response to the shock than is implied by the VAR-CDR. This is consistent with a bounce-back effect following a negative shock.

Are the dynamic responses for the VAR-CDR model to positive and negative shocks significantly different from one another? The linear VAR explicitly assumes that the sign of a shock is of no importance. The asymmetry measures suggest the following. First, there is a statistically significant difference in the Australian response to positive and negative domestic and overseas shocks. The asymmetry measure for domestic shocks is small in magnitude but the measure itself, -0.0021, with a t-ratio of -15.2139, indicates significance at any level of confidence. The Australian response to an overseas shock is -0.00138, with a t-ratio of 1.6635 This is significant at the 10% level. The asymmetry measure for the US response to US shocks proved insignificant.

5 Concluding Comments

This paper compares the predicted responses from a linear and non-linear multivariate model of Australian and US economic growth. The linear model is rejected on the basis of a series of LR tests. Further exclusion restrictions suggest that Australian growth responds to lagged US growth and a measure of the depth of the US recession. The simulation experiments suggest that Australia responds significantly differently to positive and negative domestic and overseas shocks, although the magnitude of this response is small in an economic sense.

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Tables and Figures

Table 1: Parameter Estimates - VAR and VAR-CDR Models

	VAR Model		Restricted VAR-CDR Model	
	$\Delta y_{US,t}$	$\Delta y_{AUS,t}$	$\Delta y_{US,t}$	$\Delta y_{AUS,t}$
μ	0.0047 (0.0011)	0.0079 (0.0017)	0.0033 (0.0014)	0.0119 (0.0025)
$\Delta y_{US,t-1}$	0.2499 (0.0778)	0.1212 (0.1204)	0.3046 (0.0875)	-0.0225 (0.1337)
$\Delta y_{US,t-2}$	0.1444 (0.0768)	0.4215 (0.1189)	0.2201 (0.0863)	0.2827 (0.1313)
$\Delta y_{AUS,t-1}$	0.0857 (0.0479)	-0.1089 (0.0742)	*	-0.1350 (0.0738)
$\Delta y_{AUS,t-2}$	-0.0529 (0.0476)	-0.2368 (0.0737)	*	-0.2594 (0.0730)
$CDR_{US,t-1}$	*	*	0.2033 (0.1325)	-0.4763 (0.2052)
$CDR_{AUS,t-1}$	*	*	*	*
LR ($\chi^2(2)$)	*	*	7.760 [0.95]	2.945 [0.91]

Notes: Standard Errors are in parentheses and p-values are in square brackets.

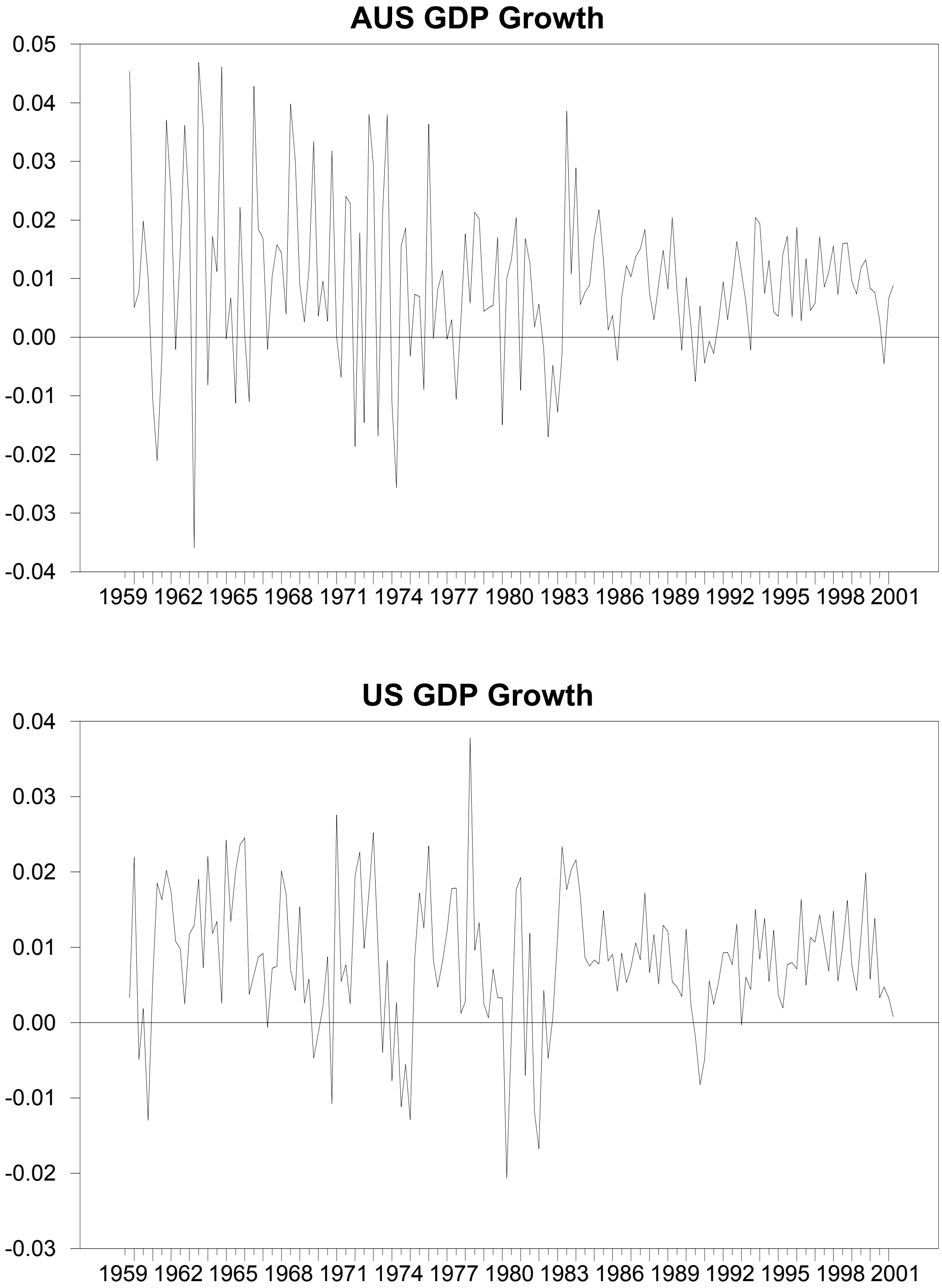
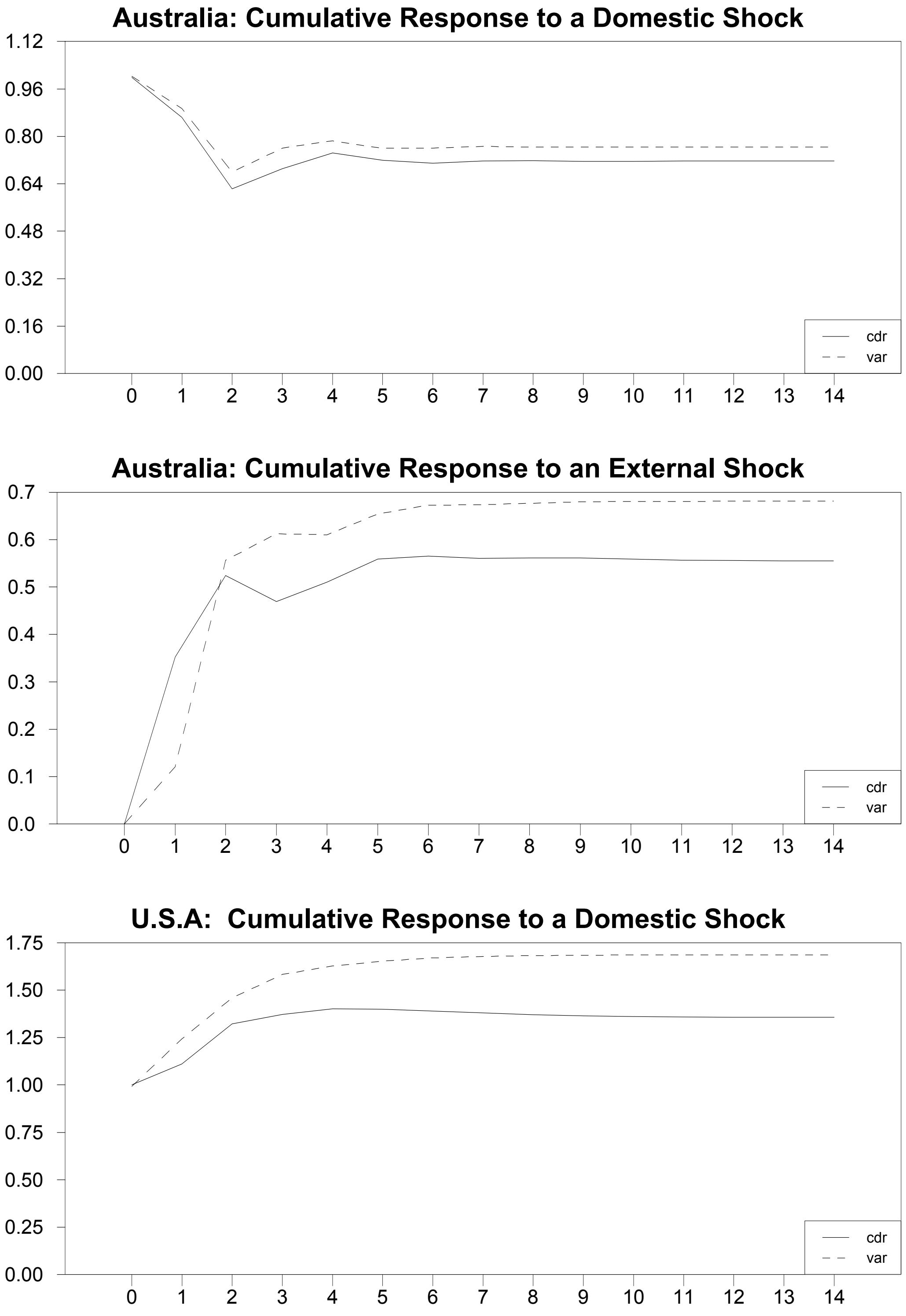


Figure 1: The growth rate data



Australia: Cumulative Response to a Domestic Shock

Australia: Cumulative Response to an External Shock

U.S.A: Cumulative Response to a Domestic Shock

Figure 2: The Generalised Impulse Responses.