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Non-monotone incentives in a model of coexisting hidden action and hidden information

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Summary. In this paper I consider a model of coexisting moral hazard and adverse selection, similar to one considered by Guesnerie, Picard, and Rey (1989). I provide an explicit solution for the optimal incentive scheme in the case, when the effort is observed with a normally distributed error. The main observation is that in this case the optimal incentive scheme often fails to be monotone. If the monotonicity constraint is imposed on the solution for economic reasons there would exist a region of profit realizations, such that the optimal compensation will be independent of on performance.

Keywords and Phrases: hidden action, hidden information, Fredholm integral equations of the first type, Hermit polynomials.

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1 Introduction

Many real life situations can be described as agency relationship. Often the agent has to provide and unobservable action on the behalf of the principal and also possesses a payoff relevant private information. Below, for concreteness, I will call the unobservable action *effort* and the hidden information the *type* of the agent. Models of coexisting hidden action and hidden information were pioneered by Laffont and Tirole (1986) and later developed by Picard (1987), Rogerson (1988), Guesnerie, Picard, and Rey (1989), Melumad and Reichelstein (1989), and Caillaud, Guesnerie, and Ray (1992).

I will use the following framework, rather common in this literature. Assume that both the principal and the agent are risk-neutral and the agent has to undertake a task on behalf of the principal. To do this, she has to exert some effort. The cost of effort is the private information of the agent. The principal cannot observe effort directly. However, a signal equal to effort plus a normally distributed white noise, which is uncorrelated with the type of the agent, is publicly observable.

The usual way to solve such problems is to proceed in two steps. First, assume that the effort is contractible and solve the adverse selection problem.

The result will be a wage schedule conditional on the effort level. To implement the same effort at the same cost when the effort is not contractible, the principal has to find a wage schedule, which depends only on the observable signal such that the expectation of this schedule conditional on the effort gives the schedule found at stage one.

Under an assumption that the solution to the first stage problem is analytical, I will find the explicit solution to the complete problem, expressed as a converging power series. From the explicit formula, one will be able to immediately deduce that for a wide range of the solutions of stage one, the solution to the complete problem is represented by a non-monotone wage schedule (this will happen, for example, if the solution to the first stage is given by a monomial of a power of at least two).

The possibility of non-monotone wage schedules in pure moral hazard situations was long recognized. The usual reason is the failure of the monotone likelihood ratio property (MLRP). Note, however, that failure of monotonicity here has noting to do with MLRP. Indeed, had the cost of effort been observable, the optimal contract would simply have sold the enterprise to the agent, making her the residual claimant on the stream of profits. Such a contract is clearly monotone in performance. The optimal wage schedule obtained on the first stage, on the other hand, is also always a monotone function of effort. Therefore, non-monotonicity here is a result of *interaction* of hidden action and hidden information components. To understand the intuition behind this result, recall that in pure hidden action situations, though the principal knows the agent's choice of effort in equilibrium, the optimal incentive scheme is best understood if one assumes that the principal uses the agent's performance to form a statistical inference about her effort. Similarly, to understand the features of the optimal incentive scheme in this case, let us assume that the principal uses the agent's performance to form a statistical inference about her effort and her type. Then a higher level of profit signals a higher effort, which is a reason to increase the reward, but it also signals a lower cost of effort, which is a reason to decrease the compensation. The interaction between these two inferences can easily lead to a non-monotone incentive schemes.

The paper is organized in the following way. In Section 2, I introduce the model and derive the master integral equation. In Section 3, I solve the equation explicitly and study the properties of the solution. Section 4 concludes.

2 The model

Consider a principal and an agent, both of whom are risk neutral with respect to money, who are engaged in a following type of a transaction. An agent undertakes an effort, z, that generates expected utility $\pi(z)$ for the principal. The effort is unobservable, however, one can observe a signal x, where

$$x = z + \varepsilon. \tag{1}$$

Here ε is a normally distributed variable with zero mean.

The cost of effort, $c(\cdot;\theta)$, is assumed to be increasing, convex, twice differentiable, and satisfy the Spence-Mirrlees condition, i.e. $c_{z\theta} < 0$.. It depends on both the effort level, z, and the type of the agent, θ. Parameter θ is private information of the agent. However, the principal knows that it comes from a distribution with a continuously differentiable and strictly positive on its support density $g(\cdot)$. The support of the distribution is assumed to be a segment $[\underline{\theta}, \overline{\theta}]$, where $0 \leq \underline{\theta} < \overline{\theta} \leq \infty$. Denote by $G(\cdot)$ is the corresponding cumulative distribution function.

Define function $V(z,\theta)$ by

$$V(z,\theta) = \pi(z) - c(z,\theta) + \frac{1-G(\theta)}{g(\theta)} c_\theta(z,\theta). \tag{2}$$

Assume that for any $\theta \in [\underline{\theta}, \overline{\theta}]$ the maximizer of $V(\cdot,\theta)$ is unique and that $V(\cdot,\cdot)$ supermodular in (z,θ). The last assumption, in particular, rules out the possibility of partial pooling, i.e. of several different types supplying the same effort in equilibrium. The lack of pooling is a necessary condition for the optimal wage schedule under observable effort to be analytical.

Let us first consider an auxiliary problem, in which the choice of effort is observable.

2.1 The case of the observable effort

In this subsection I will concentrate on the case of observable effort, i.e. I assume that the principal faces a pure hidden information problem. In that case the analysis is standard and the assumptions made above imply that the optimal effort schedule, $z(\theta)$, solves:

$$z(\theta) = \arg\max V(z,\theta). \tag{3}$$

For a discussion, see Mussa and Rosen (1978). Define the agent's surplus, $\xi(\cdot)$ as the unique solution to the following Cauchy problem

$$\begin{cases} \xi_\theta = -c_\theta(z(\theta), \theta) \\ \xi(\underline{\theta}) = 0 \end{cases} . \quad (4)$$

Then the wage schedule that implements effort levels $z(\theta)$ can be found as

$$v(z) = \min_\theta(c(z, \theta) + \xi(\theta)). \quad (5)$$

Intuitively, assume that the principal has to compensate the agent for the cost of effort and leave her information rents $\xi(\theta)$. If she wants to induce level of effort z, she selects the type for which the total cost of inducing this effort is minimal.

2.2 The case of the unobservable effort

In the case of unobservable effort the principal will be able achieve the same effort-type allocation at the same expected cost as in the observable effort case, if she chooses a wage schedule, $w(\cdot)$, to solve:

$$\int_{-\infty}^{+\infty} w(x)f(x-z)dx = v(z), \tag{6}$$

where

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{y^2}{2\sigma^2}) \tag{7}$$

is the normal density function and $v(\cdot)$ is defined by (5). Indeed, faced with such a wage profile the agent's expected payment conditional on exerting effort z is $v(z)$. Since the agent is risk neutral she ends up solving the same problem as in the case of the observable effort. The same is true about the principal's payoffs.

I will refer below to equation (6) as the *master equation*. It is a first type Fredholm integral equation. For an analysis of the existence of a solution for this class of equations under a general set of conditions see, for example, Pogorzelski (1966). In this paper I will be concerned with finding explicit solutions for a broad class of wage-effort schedules.

3 The properties of the explicit solution of the master equation

Let us assume, for simplicity, that $\sigma^2 = 1/2$. This can be done without loss of generality, since the general case can be reduced to it by an appropriate rescaling of variables x and z. Then equation (6) becomes

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} w(x) \exp(-(x-z)^2) dx = v(z). \tag{8}$$

Let us find the solution of equation (8) in the case when the right hand side is represented by a monomial, i.e.

$$v(z) = az^n, \tag{9}$$

where $n \in N$. A solution for the stage one problem is given by (9) with $a = 1/2$ if, for example,

$$\pi(z) = z^n \tag{10}$$

$$c(z,\theta) = \frac{z^{2n}}{2\theta} \tag{11}$$

$$g(\theta) = 1/\theta^2, \ \theta \in [1, \infty). \tag{12}$$

Let us look for a solution of equation (8) in a form:

$$w(x) = cH_n(x), \tag{13}$$

where Hermite polynomials $H_n(\cdot)$ are defined by:

$$H_n(x) = (-1)^n \exp(x^2) \frac{d^n(\exp(-x^2))}{dx^n} \tag{14}$$

(see, Pugachev and Sinitsyn, 1999). Substituting (13) into (8) one obtains:

$$cI_n(z) = z^n, \tag{15}$$

where $I_n(z)$ is defined by:

$$I_n(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} H_n(x) \exp(-(x-z)^2) dx. \tag{16}$$

Using definition (14) and integrating (16) by parts one can prove that $I_n(z)$ solves:

$$I_n(z) = 2zI_{n-1}(z), \; I_0(z) = 1. \tag{17}$$

Therefore,

$$I_n(z) = 2^n z^n \tag{18}$$

and $c = \frac{1}{2^n}$, i.e. the solution is given by:

$$w(x) = \frac{aH_n(x)}{2^n}. \tag{19}$$

Finally, if function $v(\cdot)$ is analytical at zero, i.e. in some neighborhood of $z = 0$ it can be represented as a sum of a convergent series:

$$v(z) = \sum_{n=0}^{\infty} a_n z^n, \tag{20}$$

then according to formula (18) and the superposition principle for linear equations:

$$w(x) = \sum_{n=0}^{\infty} \frac{a_n}{2^n} H_n(x). \tag{21}$$

Let me write down explicit formulae for the wage schedule for small values of n. For $n = 1$ one obtains

$$w(x) = ax, \tag{22}$$

for $n = 2$:

$$w(x) = a(x^2 - \frac{1}{2}), \tag{23}$$

for $n = 3$:

$$w(x) = ax(x^2 - 1). \tag{24}$$

Observe, that for $n > 1$ the optimal wage profile is not monotone. One can see it explicitly, for $n = 2$ and 3. In general, it follows from the well-known property of the Hermitian polynomials that all their roots are distinct and real (see, Pugachev and Sinitsyn, 1999). Therefore, there are n different profit realizations for which the wage will be zero. For example, if $n = 1$ the wage is zero if and only if the profit is zero, if $n = 2$ the wage is zero when the profit realization is either $1/\sqrt{2}$ or $-1/\sqrt{2}$, when $n = 3$ the wage is zero when the profit is in the set $\{-1, 0, 1\}$, etc.

One can see from the above analysis that the optimal wage is often non-monotone. If the monotonicity constraint has to be imposed for some economic reasons, for example, because the agent can costlessly destroy the output, the optimal wage schedule will have a flat region, i.e. there will exist an open set of profit levels over which the wage is constant. Such contracts can be approximated by a fixed wage contract with a bonus for an outstanding performance and a penalty for a dismal one. It is straightforward to observe that if the monotonicity constraint binds hidden action leads to the additional welfare losses comparatively to the hidden information case. Note that the reason here is different from the example in Bolton and Dewatripont (2005), where the additional welfare losses arise from the fact that the production noise is correlated with type. For another example along similar lines see Basov and Bardsley (2005).

4 Conclusions

In this paper I consider a model of coexisting moral hazard and adverse selection, similar to one considered by Guesnerie, Picard, and Rey (1989). I provide an explicit solution for the optimal incentive scheme in the case, when the effort is observed with a normally distributed error. The main observation is that in this case the optimal incentive scheme often fails to be monotone. As explained in the Introduction, the intuition for this result is rather intricate and has to do with a subtle interaction between the hidden action and hidden information dimensions of the problem. The formal derivation, on the other hand is rather straightforward.

If the monotonicity constraint is imposed on the solution for economic reasons there would exist a region of profit realizations, such that the optimal compensation not dependent on performance. The optimal compensation scheme in that case can be found using the techniques developed in Basov and Bardsley (2005).

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