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**Modelling Behavioural Responses to Profit Taxation:
The Case of the UK Corporation Tax**

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Modelling Behavioural Responses to Profit Taxation: The Case of the UK Corporation Tax[1]

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Abstract

This paper examines behavioural responses by companies to changes in profit taxation in their home country. It argues that as well as distinguishing real from shifting responses for profits, it is important to separate the responses of gross profits from those for deductions (such as claims for past or current losses) where these are endogenously related to gross profits declared at home. This occurs in the UK and many other corporate tax regimes. This endogenous response can be expected to differ over the business cycle and, using a microsimulation model of the UK corporate tax regime, it is shown that this can be important for empirical estimates of firms' overall behavioural responses especially, but not exclusively, during cyclical downturns. It is shown also that endogenous responses of deductions to real or shifting responses for gross profits can be expected to be asymmetrical between periods of above- and below-trend growth.

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1 Introduction

This paper examines a number of different behavioural responses by companies to changes in the taxation of their profits in the home country, focusing in particular on the UK corporate tax regime. Such responses can take two forms. First, there are real responses, whereby activities are transferred to other tax jurisdictions. The second form of response involves income-shifting in which the location of economic activity is unchanged but the extent to which profits and deductions are declared in the home country changes. It is argued here that it is also important to distinguish between the responsiveness of gross profits and that of deductions allowable as profit off-sets. Where, as in the UK, these deductions are related to the size of companies' profits, it is found that allowing for an endogenous, or automatic, response may be important for empirical estimates of firms' overall behavioural responses.

In examining behavioural reponses to taxes, much use has been made of the notion of the elasticity of taxable income with respect to the retention, or net-of-tax, rate introduced by Feldstein (1995, 1999). The closely related concept of the elasticity of taxable profit with respect to the tax rate (rather than the retention rate) is a central focus of the present paper. Though this concept was initially proposed as a means of capturing real behavioural responses to tax reforms, Slemrod and Yitzhaki (2002) showed that the concept can be applied to any responses, including evasion and avoidance, which cause the tax base to respond to changes in exogenous tax parameters.[1] This can include changing the type of income declared by taxpayers, for example from self-employed to corporate, and the shifting of declared income, profits or deductions to a different tax jurisdiction.

In the context of company taxation, income shifting, as mentioned above, is the phenomenon whereby multinational companies can change the extent to which they declare their global profits in different countries in response

[1] The terms evasion and avoidance are used here to denote responses that have no counterpart in real economic changes. Avoidance might be described as tax planning activities which allow some flexibility in accounting for the financial flows arising from real activities, such that these can be arranged in a tax-minimising manner in accordance with tax laws.

to differences in international profits taxation, without changing their real activities. Empirical estimates suggest that these shifting responses could be substantial; see, for example, studies by Hines and Rice (1994), Grubert and Slemrod (1998), Bartelsman and Beetsma (2003) and Huizinga and Laeven (2007). In addition, as Markusen (2002) and Devereux and Hubbard (2003) have demonstrated, multinational firms' decisions regarding whether to locate real production facilities at home or abroad and trade between locations can be influenced by profit taxation and hence affect the locations where profits are earned, repatriated and declared for tax purposes. Real responses are not confined to multinational firms. They can also be expected for purely domestic firms because increases in tax rates reduce net-of-tax profits at the margin and so render some previously profitable production unprofitable. In some cases firms may change to non-corporate status where personal-corporate income tax regimes differ.

The remainder of the paper is organised as follows. Section 2 begins by defining and decomposing firms' behavioural responses. Section 3 considers the orders of magnitude of elasticities of tax paid with respect to the tax rate, for individual firms, using possible orders of magnitude of important components suggested by previous empirical studies. A major aim of the present paper is to examine the likely behaviour, particularly over the business cycle, of the aggregate tax revenue elasticity with respect to the tax rate. This requires information about the form of the distribution of profits and changes in firms' profits over time. Use is made of the simulation model, CorpSim, produced by Creedy and Gemmell (2007b, c); the features of the model required for the present analyses are described briefly in section 4 and in the appendix. An advantage of using this model is that it deals with the automatic responsiveness of deductions as a result of profit changes. Section 5 then reports resulting numerical values of aggregate elasticities over the business cycle. Section 6 reports a range of sensitivity analyses. Conclusions are in section 7.

2 Types of Behavioural Response

This section begins by defining alternative behavioural responses to corporate taxation, decomposing these into real responses, profit-shifting and deductions-shifting. The context is of a firm located in a home country, or tax jurisdiction, which may, at some cost, change its declared profits in that jurisdiction in response to a change in the home tax rate. This includes, but is not limited to, moving profits abroad which may or may not involve shifting some aspects of the firm's real economic activity abroad. For comparative static purposes, tax rates abroad are assumed throughout to be independent of the tax rate in the home country, so that responses to a change in the home tax rate can be interpeted as reponses to a change in the tax differential.

Subsection 2.1 begins by specifying the composition of taxable profits. Subsection 2.2 then decomposes the overall change in a firm's tax, in response to a change in the tax rate, into its various components. Subsection 2.3 considers the likely signs attached to the components, while subsections 2.4 and 2.5 examine changes in the deductions rate and differences among firms.

2.1 Taxable Profits

Consider a single company. Gross profits declared for tax are P^* and total deductions claimed against those profits are D, so that net taxable profits, P^T, are:

$$P^T = P^* - D \tag{1}$$

Deductions are assumed to be related to profits, hence $D(P^*)$, but the shorthand D is used here. Suppose, for simplicity, that there is a single tax rate of t. In the UK system there is in fact more than one rate, but the vast majority of corporation tax is raised at a single rate. When $P^T > 0$, the tax liability, $T(P^*)$, is thus:

$$T(P^*) = tP^T = t(P^* - D) \tag{2}$$

and when $P^T \leq 0$, $T(P^*) = 0$. This reflects the UK system of corporation tax in which losses (negative profits) do not attract an automatic tax rebate, but instead are deductable against current or future positive profits within the firm or group.[2]

This paper is concerned with revenue responses to a change in the tax rate. The notation uses the form $\eta_{x,y} = (dx/dy)(y/x)$ to denote the elasticity of x with respect to y. From equation (2) the elasticity of tax revenue with respect to the tax rate, in the simple case where profits and deductions do not respond to changes in tax rates, is:

$$\eta_{T,t} = \frac{dT}{dt}\frac{t}{T} = 1 \tag{3}$$

That is, for firms with $P^T > 0$, the percentage increase in tax paid is the same as the percentage increase in the tax rate. This is a simple consequence of the proportionality of the fax function in 2. For firms in aggregate the elasticity is also unity if all values of P^T are constant.

However, it might be expected that both gross declared profits, P^*, and deductions, D, would respond to changes in the tax rate. These responses may be real (in the sense described above) or they may arise from profit-shifting or deductions-shifting, where no real changes in economic activity are involved. Allowing for P^T to vary as the tax rate varies means that $dT/dt = P^T + tdP^T/dt$, giving the result that:

$$\eta_{T,t} = 1 + \eta_{P^T,t} \tag{4}$$

Thus the main elasticity of interest is the elasticity, $\eta_{P^T,t}$, of net taxable profit with respect to the tax rate. This elasticity is closely related to the Feldstein (1995) elasticity of taxable income with respect to the retention rate, $1-t$, using:

$$\begin{aligned} \eta_{P^T,t} &= -\left(\frac{t}{1-t}\right)\left(\frac{1-t}{P^T}\right)\frac{dP^T}{d(1-t)} \\ &= -\left(\frac{t}{1-t}\right)\eta_{P^T,1-t} \qquad (5) \end{aligned}$$

However, the following discussion is in terms of the elasticity with respect to the tax rate rather than the retention rate.

[2]For example, in the UK system, a current loss under one profit 'schedule' may be offset against a current profit under some, but not all, other 'schedules'. Thus a firm's ability to utilise its losses immediately can depend on the schedular characteristics of its profits and losses. Further conditions apply to firms which form part of a group.

2.2 Decomposing Behavioural Elasticities

Allowing for behavioural responses requires the extent to which profits and deductions are declared in the home tax jurisdiction to be specified. To simplify exposition of the following analysis, all other taxes, whether for alternative definitions of income at home (for example, unincorporated income) or for profits declared abroad, are assumed to be constant. At this stage the possibility of profits from different sources (for example, trading activity, property letting), being taxed using different 'schedules' is ignored. In this section time subscripts are also ignored for convenience.

Define θ_p as the proportion of total profits, P, which are declared at home. Profits declared for tax at home, P^*, are thus:

$$P^* = \theta_p P \tag{6}$$

Similarly, let θ_d denote the proportion of total deductions which are declared at home, and let E denote qualifying expenditures which are eligible as off-sets against declared profit. These include investment expenditures and accummulated losses. A proportion, s, of these qualifying expenditures can be deducted, so that declared deductions, D, are:

$$D = s\theta_d E \tag{7}$$

The deductions rate, s, is analogous to the term used by Devereux and Hubbard (2003, p. 473) to describe a 'factor which reflects the generosity of the provision for depreciation'. In the present paper, s represents the generosity of all qualifying expenditures, not just those on capital. To the extent that a firm's total profits or qualifying expenditures change in response to changes in taxes, whilst keeping constant the extent to which they are declared for tax at home, these may be regarded as real. Alternatively, where total profits or qualifying expenditures remain unchanged but the proportion declared at home alters, some profit or deductions shifting can be considered to have occurred.

Using the proportions described above, P^T can be written as:

$$\begin{aligned} P^T &= P^* - D \\ &= \theta_p P - s\theta_d E \end{aligned} \tag{8}$$

Let $\alpha = \theta_p P / P^T \geqslant 1$ denote the ratio of declared profits to the tax base, P^T; this is strictly greater than one as long as there are some declared deductions. Then, differentiating $T = t(\theta_p P - s\theta_d E)$ with respect to t, it can be shown that:

$$\eta_{P^T,t} = \alpha \left\{ \eta_{\theta_p,t} + \eta_{P,t} \right\} - (\alpha - 1) \left\{ \eta_{\theta_d,t} + \eta_{E,t} \right\} \tag{9}$$

In view of the fact that both weights α and $(\alpha - 1)$ can exceed unity, it is not appropriate to think of $\eta_{P^T,t}$ as a weighted average of the two terms in curly brackets in (9).

Equation (9) provides the basic decomposition of the elasticity of taxable profit with respect to the tax rate for a single firm. The first term in curly brackets, $\eta_{\theta_p,t} + \eta_{P,t}$, measures profit responses while the second term, $\eta_{\theta_d,t} + \eta_{E,t}$, measures deductions responses. The four component elasticities capture the four basic behavioural responses and are summarised in Table 1.

Table 1: Responses to a Tax Change

Income shifting		
Profit shifting:	$\theta_p = \theta_p(t)$	$d\theta_p/dt < 0$
Deductions shifting:	$\theta_d = \theta_d(t, s)$	$d\theta_d/dt > 0$
Real responses		
Real profit response:	$P = P(t)$	$dP/dt < 0$
Real deductions response:	$E = E(t, s)$	$dE/dt \lesseqgtr 0$

The willingness of firms to shift profits or deductions out of the tax net is likely to depend on the relative costs of each. For example, it may be easier to hide profits than to inflate deductions, depending on the specification of the tax code, the extent and form of enforcement activity, and the available evasion and avoidance facilities. However, consider the special case where, to the extent that such factors permit, a firm seeks to be indifferent at the margin between a £1 reduction in tax liability obtained via a reduction in declared profits, $P^* = \theta_p P$, and an increase in declared deductions, $D^* = s\theta_d E$. That is, firms would seek to set $dP^*/dt = -dD^*/dt$, implying that $\eta_{P^*,t} = -(D^*/P^*)\eta_{D^*,t}$. In this special case, the expression in (9) simplifies to:

$$\eta_{P^T,t} = 2\alpha \left\{ \eta_{\theta_p,t} + \eta_{P,t} \right\} \tag{10}$$

and knowledge of only the real profit and profit-shifting responses is required.

2.3 Expected Signs

The definitions above treat θ_p and θ_d as propensities to shift profits and deductions. Unchanged propensities are represented by $d\theta_p/dt = d\theta_d/dt = d\theta_d/ds = 0$.[3] However, in general the expected directions of change are indicated in the final column of Table 1. Total profits and the proportion declared for tax at home respond negatively to increases in t. Conversely total deductions and the proportion claimed at home respond positively to changes in t and s: increased tax or deductions rates increase the corporate tax deductions value of qualifying expenditures. These sign expectations assume that substitution effects dominate any income effects, an assumption that accords with Gruber and Saez's (2002) finding that compensated and uncompensated taxable income elasticities are similar. Furthermore, the overwhelming majority of taxable income elasticity studies since Feldstein (1995, 1999) find the overall elasticity with respect to the retention rate to be positive.

Therefore the two profit responses, $\eta_{\theta_p,t}$ and $\eta_{P,t}$, encourage a negative value of $\eta_{P^T,t}$ and, to the extent that tax rate increases attract additional declared deductions, when multiplied by $\alpha - 1 > 0$, $\eta_{\theta_d,t}$ generates further negative effects on $\eta_{P^T,t}$ which compound the negative effect from profit responses. This negative deductions effect is stronger, the larger is a firm's initial deductions claim. On the other hand, if there were no deductions of any kind, $\alpha = 1$ and $P^* = P^T$, implying that there would be no additional negative impact on the elasticity, $\eta_{P^T,t}$.

Even if the elasticity terms on the right hand side of (9) were to take similar values across firms, differences in α would ensure that $\eta_{P^T,t}$ varies. In particular, firms with a larger deductions base have a higher α, *ceteris*

[3] An alternative definition would be to regard a real-only profit response (that is, no shifting) as occuring when each marginal tax-induced £ of profit is declared for tax: $dP^*/dt = dP/dt$. It can be shown that for this condition to hold requires $d\theta_p/(1-\theta_p) = dP/P$. That is, an increase in θ_p, as a proportion of $(1-\theta_p)$, equal to the rate of total profit growth is required if there is to be no change in profit shifting.

paribus, and hence a larger absolute $\eta_{P^T,t}$.[4] As a result, profit-making firms with a recent history of losses (or profit-making members of a group with large losses elsewhere) and firms with large capital allowances can be expected, *ceteris paribus*, to have stronger negative responses to a tax change. For firms declaring a current loss or zero profit, $\eta_{P^T,t}$ is of course zero.

The sign of $\eta_{E,t}$ in (9) is less straightforward. It is complicated by the fact that, to the extent that some qualifying expenditures are related to profits, there may be some automatic response of deductions to tax-induced changes in profits declared at home. For example, consider the case where a firm transfers production abroad in response to a tax change. Some profits previously obtained at home are now earned abroad. The associated investment which shifts abroad, previously deductible from profits declared at home, are no longer deductible. The elasticity $\eta_{E,t}$ can therefore be decomposed as:

$$\eta_{E,t} = \eta_{E,t}\big|_{dP^*=0} + \left(\eta_{E,P^*}\right)\left(\eta_{P^*,t}\right) \tag{11}$$

where η_{E,P^*} represents this automatic response. To make this expression less cumbersome, write $\eta_{E,t}\big|_{dP^*=0} = \eta'_{E,t}$; this captures any tendency for firms to generate additional qualifying expenditures independently of declared profits. Hence:

$$\eta_{E,t} = \eta'_{E,t} + \left(\eta_{E,P^*}\right)\left(\eta_{P^*,t}\right) \tag{12}$$

For example, where enforcement of tax rules make it easier for firms to generate additional deductions, rather than shift profits or deductions abroad, $\eta'_{E,t}$ could be high relative to $\eta_{\theta_p,t}$ or $\eta_{\theta_d,t}$.

In general the sign of $\eta_{E,t}$ is ambiguous. Consider the components on the right hand side of (12). Qualifying expenditures, E, are likely to rise while profits declared at home, P^*, are expected to fall in response to an increase in the tax rate: thus $\eta'_{E,t} > 0$, $\eta_{P^*,t} < 0$. The sign of the automatic response, η_{E,P^*} is likely to depend on the type of qualifying expenditure and whether changes in P^* arise from changes in total profits, P, or changes in profit-shifting, θ_p. It might also be expected that where the tax code causes a greater automatic response, that is, a larger absolute value of $(\eta_{E,P^*})(\eta_{P^*,t})$,

[4] The term 'larger absolute' is preferred here to 'smaller (more negative)'. Similarly, the term 'smaller absolute' is preferred to 'larger (less negative)'.

firms may have a larger shifting response, $\eta'_{E,t}$, to compensate. Where, for example, a tax rise leads to more investment and the associated profits shifting abroad, firms may attempt to compensate for the loss of capital allowances at home by shifting other deductions into the home tax jurisdiction where they have a greater tax off-setting value.

The elasticity η_{E,P^*} captures the extent to which, for given s and θ_d, claimed deductions change as declared profits change. This is affected both by changes in firms' economic circumstances and by tax rules. In a situation of steady-state or trend growth, a value of η_{E,P^*} equal or close to unity might be expected, otherwise deductions would become a persistently increasing or declining fraction of declared profits over the long-run. However, away from the steady-state, η_{E,P^*} may be greater than unity, for example when, following a recession, deductions claimed rise faster than profits. Alternatively it may be less than unity during booms when past losses are exhausted and profits grow faster than deductions. In this latter case, η_{E,P^*} could even be negative.[5]

Substituting the expression for $\eta_{E,t}$ in (12) into (9) gives:

$$\eta_{P^T,t} = \alpha \left\{ \eta_{\theta_p,t} + \eta_{P,t} \right\} - (\alpha - 1) \left\{ \eta_{\theta_d,t} + \eta'_{E,t} + \left(\eta_{E,P^*} \right) \left(\eta_{P^*,t} \right) \right\} \tag{13}$$

This identifies all the components that determine the value of the elasticity, $\eta_{P^T,t}$. If, as argued above, in a steady-state, $\eta_{E,P^*} = 1$, then (13) simplifies to:

$$\eta_{P^T,t} = \left\{ \eta_{\theta_p,t} + \eta_{P,t} \right\} - (\alpha - 1) \left\{ \eta_{\theta_d,t} + \eta_{E',t} \right\} \tag{14}$$

where this uses the property that:

$$\eta_{P^*,t} = \eta_{\theta_p,t} + \eta_{P,t} \tag{15}$$

Equation (14) shows that in the steady state, the value of the overall elasticity, $\eta_{P^T,t}$, hinges on α and four elasticity components. These elasticities determine the real responses of profits, P, and qualifying expenditures, E (holding declared profits constant) and the shifting parameters, θ_p and θ_d.

[5]The nature of the tax code generally affects the magnitude of the elasticity η_{E,P^*}. For example, a condition that deductions can only be claimed against any currently declared positive profits effectively limits the scope of those qualifying expenditures.

2.4 Changes in the Deductions Rate

Governments can change the extent to which qualifying expenditures are deductible against profit for the purpose of calculating tax liability. This was summarised above by the deductions rate, s. The response of the tax base to changes in this deductions rate, can also be obtained by differentiating $P^T = \theta_p P - s\theta_d E$ with respect to s to give:

$$\frac{dP^T}{ds} = -\theta_d E - sE\frac{d\theta_d}{ds} - s\theta_d \frac{dE}{ds} \tag{16}$$

from which the elasticity, $\eta_{P^T,s}$, is obtained as:

$$\eta_{P^T,s} = \frac{dP^T}{ds}\frac{s}{P^T} = -(\alpha - 1)\left\{1 + \eta_{\theta_d,s} + \eta_{E,s}\right\} \tag{17}$$

where $\alpha - 1 = s\theta_d E/P^T$ is the ratio of deductions claimed to net taxable profit. Expected signs are: $\eta_{\theta_d,s}, \eta_{E,s} > 0$. Also $\alpha - 1 > 0$ if there are any declared deductions ($D > 0$). Furthermore $\alpha - 1$ exceeds unity when $D > P^*/2$ and tends to infinity as $D \to P^*$. [6]

The expression in equation (17) takes a similar form to the second term in curly brackets in (9) but with the addition of a unity term. This latter component reflects the fact that, unlike changes in t, changes in s have a direct effect on the tax base by altering the size of eligible deductions, sE. The remaining terms in (17) capture the responses of E and θ_d to changes in s. There is no automatic deductions response in this case so long as $dP^*/ds = 0$. As for $\eta_{P^T,t}$, even if the elasticities on the right hand side of (17) were similar across firms, those with larger deductions relative to taxable profits would have larger values of $\alpha - 1$ and hence larger $\eta_{P^T,s}$.

The elasticity expressions for changes in the tax rate, t, and deductions rate, s, in (9) and (17) are not symmetric. Thus an equal percentage increase in the tax and deductions rates have different effects because, whereas the tax rate change affects declared profits *and* deductions, a change in the deductions rate affects only the latter.

[6] For tax-paying firms, declared deductions cannot exceed declared profits

2.5 Differences in Behavioural Responses Across Firms

The net taxable profits elasticity in equation (13) can be expected to vary across firms, both due to inter-firm differences in the elasticity terms on the right hand side of (13) and differences in the weights, α and $\alpha - 1$.

As already discussed, the composition of the weights suggest that firms with high total profits relative to their available deductions (low α) would be expected, *ceteris paribus*, to display smaller absolute values of $\eta_{P^T,t}$. This would apply to firms with low levels of losses or capital allowances. Thus, firms in sectors with low capital expenditures (such as service industries) or sectors enjoying better economic fortunes where losses are low, *ceteris paribus* would tend to have smaller absolute revenue elasticities.

In addition, deductions rates, s, may vary across firms. For example, a firm's present discounted value of deductions for a given level of qualifying expenditure depends on its private discount rate, which contributes to different $\eta_{P^T,s}$ values across firms (and $\eta_{P^T,t}$, since s is a component of α). Deduction rates are also lower where more of a firm's losses are 'stranded', that is when they can only be used in future years rather than currently. Loss pools, unlike current losses, can only be used to offset profits in the firm and profit source where they arose.

In addition to any 'levels' effects from initial profit and deduction levels, profit-shifting or deductions-shifting abroad might be expected to be easier for large firms, especially multinationals, both because of their pre-existing foreign presence, and because they can more readily absorb any fixed costs of setting up avoidance schemes. On the other hand, shifting profit between alternative domestic income categories to avoid corporation tax, such as between self-employment and corporate income, might be expected to be easier for small firms which can more readily move between those classifications.

The ease with which different firms can pursue real profit or deductions responses when tax rates change is likely to depend on the types of real response possible. For example, where real responses involve shifting investment abroad or setting up overseas production facilities, this is again likely to be easier for large, multinational firms (and hence observed more in sectors where multinationals dominate). Opportunities for real profit responses within the tax jurisdiction are likely to be more limited, but could include changes to employment or corporate status to take advantage of different deductions available under the alternative tax codes. For example, some deductions which are available only to incorporated businesses may not be sufficient to persuade some self-employed to incur the costs of incorporation when corporate tax rates are high, but incorporation becomes attractive when corporate tax rates are reduced. These arguments again suggest that responses by small firms and self-employed individuals would be expected to dominate this type of real response. In aggregate – in terms of impacts on corporate tax revenues – these latter effects would be expected to be small, with revenues in the UK, as in most OECD countries, dominated by a relatively small number of large firms.

Finally, Grubert and Slemrod argue that firms which create opportunities for real profit responses, for example by setting up foreign subsidiaries, are likely to find it easier to enagage in profit-shifting; indeed the two may be joint decisions. As a result it might be expected that firms with larger values of $\eta_{P,t}$ are more likely to have larger values of $\eta_{\theta_p,t}$.

3 Illustrative Examples

To illustrate orders of magnitude for the elasticity, $\eta_{P^T,t}$, for a single firm or group, it is necessary to consider possible values for the components in equations (13) or (14). Subsection 3.2 first examines a number of empirical studies that can provide a guide regarding orders of magnitude. Based on these estimates, a set of benchmark parameters are described in subsection 3.2, after which subsection 3.3 presents numerical results.

3.1 Estimates of Response Parameters

This subsection discusses various estimates available in the empirical literature. These can be used to guide choices in producing illustrative examples and simulations, where it is necessary to make assumptions regarding the key elasticities and parameters in (13), both on average, and allowing for differences across firms.

There are various estimates of the 'Feldstein elasticity' of taxable income with respect to the retention or net-of-tax rate. However, these generally relate to personal, rather than corporate, incomes. After reviewing various approaches, Gruber and Saez (2002) claim that best estimates for this are around 0.4. From (5), a value of $\eta_{P^T,(1-t)} = 0.4$ would imply $\eta_{P^T,t}$ in the range -0.13 to -0.27 for t in the range 0.25 to 0.4.

Estimates for various income shifting responses for samples of multinational corporations were reported by Bartelsman and Beetsma (2003), Grubert and Slemrod (1998) and Hines and Rice (1994). Using OECD country-level data on the share of labour income in value added, Bartelsman and Beetsma (2003) estimated pure profit-shifting for OECD countries on average. Their 'back-of-the-envelope' central estimate of profit-shifting is that about 65 per cent of additional revenue following a tax rate rise leaks abroad.

Thus the elasticity of declared revenue with respect to the tax rate is around 0.35. From (4), since $\eta_{P^T,t} = \eta_{T,t} - 1$, the implied tax base elasticity is -0.65. Bartelsman and Beetsma obtained UK parameter estimates close to the OECD average. This may be regarded as an estimate of the profit-shifting component, $\alpha\eta_{\theta_p,t} - (\alpha - 1)\eta_{\theta_d,t}$, in (13) rather than of the total real-plus-shifting response. By focussing only on shifting responses Bartelsman and Beetsma argued that their estimates could be regarded as lower bounds. More detailed recent estimates for European multinationals, from Huizinga and Laeven (2007), are somewhat smaller for the UK than those derived from the Bartelsman and Beetsma results. Their estimate of the semi-elasticity of reported profits with respect to the top statutory tax rate (of around 1.1 for the UK) implies an elasticity of -0.33, assuming a 30 per cent corporate tax rate.[7]

Grubert and Slemrod (1998) focused specifically on profit-shifting in 1987 by US multinationals to Puerto Rico, which has a unique tax status viz a viz

[7] However, the Huizinga and Laeven semi-elasticities are based on profits data in commercial accounts and are not necessarily equivalent to the elasticity measured here which relates to net taxable profits.

US tax rules, using a sample of over 200 firms. Their model allows for both real foreign investment and profit-shifting to tax havens. Though estimates of an elasticity are not readily derivable, their results confirm that substantial real plus profit-shifting responses by US multinationals was mainly motivated by the profit-shifting opportunities which the real foreign investment provides.

Hines and Rice (1994) examined aggregate 1982 country-level data for reported non-financial profits of US parents and affiliates with investments in tax havens and other foreign countries. They report that a 1 percentage point higher tax rate reduces reported profits by 3 per cent. Across such a wide-ranging sample of countries, the corporate tax rate is likely to vary. An average of around 30 per cent implies an elasticity around -1; a 15 per cent tax rate implies an elasticity around -0.5.

The Hines-Rice elasticity probably includes both real and profit-shifting responses and so approximates $\eta_{P^T,t}$. However, part of the large observed tax response arises because of the US system of taxing world-wide income, which makes this unrepresentative of the UK. The Bartelsman and Beetsma (2003) estimate, on the other hand, relates only to shifting to other OECD countries while the Gruber-Slemrod and Hines-Rice estimates relate to shifting to especially low-tax havens – hence the larger estimates for the latter.

3.2 Benchmark Parameters

Adopting a steady-state value of $\eta_{E,P^*} = 1$ allows equation (14) to be used. The illustrations below also set the deductions rate equal to unity; that is, $s = 1$ and qualifying expenditures are fully eligible as deductions (but there is no implicit subsidy). Table 2 shows the assumed values of the four elasticity components and the declared proportions, θ_p and θ_d, required to calculate $\alpha - 1$ in (14). It might be expected that these parameters cannot be chosen independently by firms. For example, as Slemrod and others have suggested, if it becomes more costly to shift further increments of profits abroad, then $\eta_{\theta_p,t}$ and $\eta_{\theta_d,t}$ may become smaller as θ_p and θ_d are reduced. However, the illustrations below examine individual parameter changes holding all others constant.

Table 2: Benchmark Parameter Values

	Elasticity	Benchmark	Alternatives
Profit shifting	$\eta_{\theta_p,t}$	-0.375	-0.625
Deductions shifting	$\eta_{\theta_d,t}$	0.25	0.5
Real profit response	$\eta_{P,t}$	-0.05	$-0.1, -0.2$
Real deductions response	$\eta_{E',t}$	0.05	$0.1, 0.2$
Proportion of P declared	θ_p	0.8	0.6
Proportion of D declared	θ_d	0.8	0.6
Deductions rate	s	1.0	$0.8, 0.6$

The benchmark case assumes a small real profit response to changing tax rates, of 5 per cent, but alternatives of 10 per cent and 20 per cent are also examined. Comparable positive values are used for the discretionary real deductions response, $\eta'_{E,t}$. With $s = 1$ and for a given θ_d, the response of qualifying expenditures is the same as that for declared deductions. This response is referred to as 'discretionary' to distinguish it from the automatic deductions response.

Possible values for the profit-shifting and deduction-shifting elasticities could be based, for example, on Bartelsman and Beetsma's (2003) estimate of an overall shifting elasticity around 0.65. The benchmark $\eta_{\theta_p,t} = -0.375$, for example, implies that a 10 per cent change in the tax rate (for example, from 30 per cent to 33 per cent) generates a 3.75 per cent reduction in the share of declared profits (for example, from 80 per cent to 77 per cent). The alternative of $\eta_{\theta_p,t} = -0.625$ implies a fall from 80 per cent to 75 per cent in the share of declared profits.[8] Deductions shifting is assumed to be slightly more difficult (smaller) with $\eta_{\theta_d,t} = 0.25, 0.5$. The total shifting response is given by $\eta_{\theta_p,t} - (\alpha - 1)\eta_{\theta_d,t}$. This effect depends on the value of α, which

[8] If profit-shifting is driven by changes in the tax rate differential between home and overseas tax jurisdictions, the assumed precentage change in the home tax rate will be small compared with the percentage change in the differential. For example if the home rate falls from 25 per cent to 23 per cent (a -8 per cent change) but the relevant overseas rate remains at, say, 35 per cent, the differential has changed by 20 per cent (from 10 per cent to 12 per cent). Thus a relatively large response to a relatively small change in the home tax rate may not be so surprising.

in turn depends on the value of E/P, in addition to the other benchmark parameters in Table 2. Using $E/P = 0.5$ (so that $\alpha = 2$), gives a benchmark total 'shifting elasticity' of $\eta_{\theta_p,t} + (1-\alpha)\eta_{\theta_d,t} = -0.625$.

These illustrative values should not be interpreted as representing 'average' responses, since many firms' responses could be expected to be very small or zero. However they serve to illustrate the responsiveness properties of those firms with more substantial behavioural responses to tax changes. (Possible average responses across the distribution of firms are illustrated in section 5).

3.3 Numerical Results

Some numerical results are shown in Figure 1 where each of the four quadrants shows the elasticity $\eta_{P^T,t}$ on the vertial axis and the size, expressed as a percentage, of qualifying expenditures relative to total profits, E/P, on the horizontal axis. Each quadrant shows a range of profiles for $\eta_{P^T,t}$, resulting from changes in one of the relevant parameters while leaving all others fixed at their benchmark values. The top left and right hand quadrants show respectively the effects of varying the proportions (of profits and deductions) declared and the degree of shifting (again of profits and deductions). The bottom left and right hand quadrants show respectively the effects of varying real profit responses and real deductions responses.

The ratio, E/P is not observable in UK taxpayer data, but the ratio of deductions to profits claimed, D/P^*, is observable, where $D = s\theta_d E$. HMRC data show this to be around the range 0.45 to 0.56, for companies in aggregate.[9] The benchmark values of $\theta_p = 0.8$ and $s = 1$ then yield E/P in the range 0.56 to 0.70.

To interpret the diagrams, it is useful to remember that the impact of the E/P ratio on $\eta_{P^T,t}$ operates via changes in α in (14). For example, with a central estimate of $E/P = 0.5$ and $\theta_d = 0.8$, then $\alpha = 2$, and (14) simplifies

[9] HMRC data on all companies (excluding Life Assurance and North Sea Oil companies) over 1997-98 to 2003-04 show that the ratio of all deductions (excluding a small amount of tax credits), to gross profits, ranges from a low of 0.46 in 1998-99 to a high of 0.56 in 2002-03. see http://www.hmrc.gov.uk/stats/corporate_tax/table11_2.pdf.

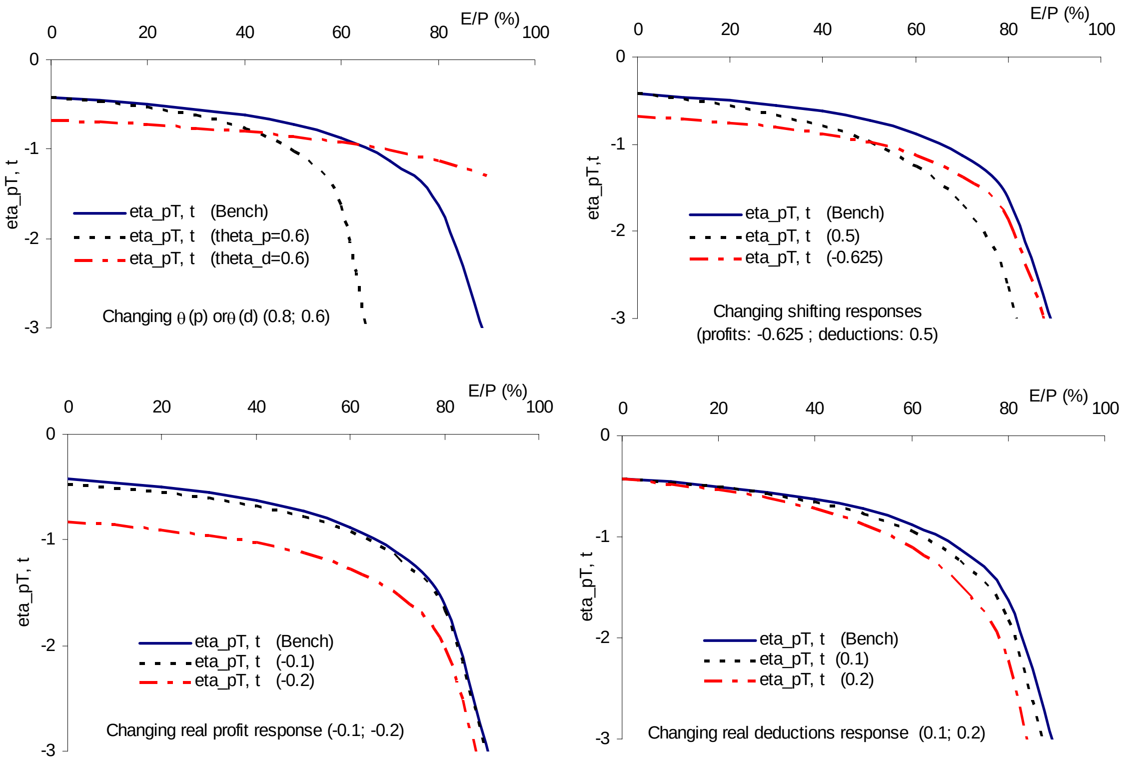

Figure 1: Relationship between $\eta_{P^T,t}$ and E/P: Individual Firms

to:

$$\eta_{P^T,t} = \eta_{\theta_p,t} + \eta_{P,t} - \eta_{\theta_d,t} + \eta_{E',t} \tag{18}$$

where (18) also uses $\eta_{E,P^*} = 1$.

At the extremes, as $E/P \rightarrow 1$, the weight $\alpha - 1 \rightarrow \infty$, so that the elasticity, $\eta_{P^T,t} \rightarrow -\infty$. And as $E/P \rightarrow 0$, the term $\alpha - 1 \rightarrow 0$ and the elasticity is determined solely by the first two profit-related terms in (18). In all the diagrams it is clear that E/P has important, non-linear effects on the overall elasticity, $\eta_{P^T,t}$.

The top left and bottom right hand side quadrants of Figure 1 reveal that changing θ_p, θ_d or $\eta'_{E,t}$ causes the benchmark profile to rotate (around a value at $E/P = 0$), whilst changes in $\eta_{P,t}$ cause the benchmark profile to shift (the bottom left hand quadrant). The top right hand quadrant also reveals that changes in the 'shifting elasticities' have differing effects on the overall elasticity, with an increase in the absolute value of $\eta_{\theta_p,t}$ causing the profile to shift downwards whilst an increase in $\eta_{\theta_d,t}$ causes the profile to rotate clockwise. This difference reflects the fact that the impact of $\eta_{\theta_d,t}$ on the overall elasticity is mediated via $(\alpha - 1)$, whereas this is irrelevant to the impact of changes in $\eta_{\theta_p,t}$.

These illustrations show how differences in α can affect observed profit and deductions responses. However, by maintaining $\eta_{E,P^*} = 1$, they cannot demonstrate the endogenous impact of changes in declared profits on those deductions. As is shown in section 5, this aspect is likely to be important when behavioural responses are estimated at different points in the economic cycle. In order to examine aggregate elasticities, it is necessary to have information about the distribution of profits: this is therefore discussed in the following section.

4 Simulating Profit Dynamics

The distribution of profits, and changes over the business cycle, along with the endogenous variations in firms' deductions in response to profits changes, can be examined using the corporation tax microsimulation model, **CorpSim**, developed by Creedy and Gemmell (2007c). The relevant features of this model are described briefly in this section and in the appendix. This model was designed to examine the behaviour of net taxable profits, deductions and tax revenues in response to changes in gross taxable profits. It is based on HMRC tax and profit data for the UK.

For present purposes, the model provides estimates of the responsiveness of deductions to declared profits by UK firms and the weights, α_i. However, the use of HMRC data necessarily means that the initial input data used by **CorpSim** relate to firms' declared profits, $P_i^* = \theta_{p_i} P_i$, and declared deductions used as profit off-sets, $D_i = s\theta_{d_i} E_i$, rather than the P_i and E_i which were treated as exogenous in the illustrations reported above. Simulations therefore use a modified version of equation (13) for each firm, given by:

$$\begin{aligned}\eta_{P^T,t} &= \alpha\left\{\eta_{\theta_p,t}+\eta_{P,t}\right\}-(\alpha-1)\left\{\eta_{\theta_d,t}+\eta'_{D,t}+\left(\eta_{D,P^*}\right)\left(\eta_{P^*,t}\right)\right\} \\ &= \left[\alpha-(\alpha-1)\,\eta_{D,P^*}\right]\eta_{P^*,t}-(\alpha-1)\,\eta_{D,t} \end{aligned} \tag{19}$$

where α for each firm is calculated as $\alpha_i = P_i^*/P_i^T$, $\eta_{P^*,t} = \eta_{\theta_p,t} + \eta_{P,t}$, and $\eta_{D,t} = \eta_{\theta_d,t} + \eta'_{D,t}$. Initial values of $P_{i,0}^T$ are obtained, given an initial distribution of gross taxable profits $P_{i,0}^*$ (discussed below), by subtracting the initial value of deductions claimed, $D_{i,0}$, from the relevant firm's initial $P_{i,0}^*$. This relationship includes $\eta'_{D,t} > 0$, the elasticity of declared deductions, for given declared profits, with respect to the tax rate. This captures any tendency for higher tax rates to encourage increased spending on qualifying expenditures in order to increase firms' deductions and hence reduce their tax liabilities, *ceteris paribus*. The tendency for tax-induced deduction-shifting is again captured by $\eta_{\theta_d,t}$.

4.1 Profit Distributions

The microsimulation model builds a simplified version of the actual UK corporation tax regime which captures its essential characteristics. Using data on the two main profit sources – trading profits and interest income – for around 150,000 UK firms, **CorpSim** generates a simulated distribution for each profit source across firms. These are fitted to the actual distributions using a mixture of lognormal distributions, suitably adjusted to produce negative profit values, as described in the appendix. The actual trading profit distribution, for a sample of firms in 2003-04, is shown in Figure 2. Important features of profit distributions are, first, the extent of losses (shown by the long left-hand tail) and, second, the fact that the bulk of firms lie below the £1.5 million threshold at which the main corporate tax rate of 30 per cent applies. Third, the vast majority of corporation tax revenues are paid by a small number of large firms in the right-hand tail of the distribution.[10]

[10] For example, in 2003-04, the largest 7 per cent of corporate taxpayers contributed 87 per cent of total corporation tax revenues. The small numbers of firms in the tails of the distribution in Figure 2 are too small to be visible.

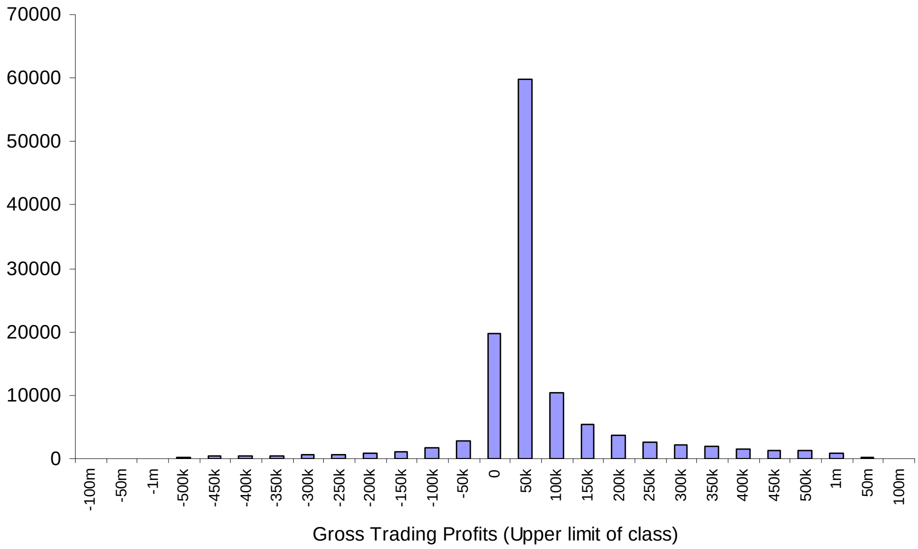

Figure 2: The Distribution of Trading Profits, 2003-04

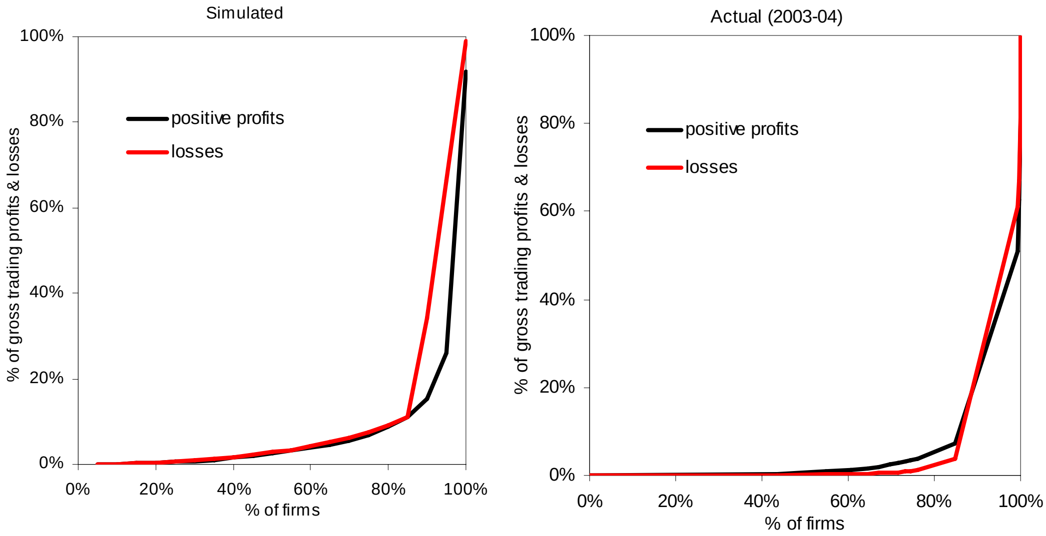

Figure 3: Simulated and Empirical Lorenz Curves for Profits and Losses

The ability of **CorpSim** to match actual data on firms' profits and losses can be seen in Figure 3. This shows Lorenz curves for observed (positive) trading profits and losses, for the year 2003-04, and those obtained by simulation. It can be seen that the actual distributions of both profits and losses are highly unequal, and this feature is well captured by the simulated equivalents, with the latter displaying slightly less concentration. As a further check, the aggregate ratio of all deductions to gross declared profits, D/P^*, can be compared with actual data from HMRC. The most recent (2002-03 and 2003-04) observed ratios are 0.56 and 0.53; equivalent values produced by **CorpSim** are around 0.52.

4.2 Deductions

Most large UK firms form part of larger groups which are allowed to share some deductions under the tax code, though corporation tax is levied at the unconsolidated firm level. The profit distribution data reveal a small number of firms with very large losses, almost certainly belonging to groups in which other members make large positive profits. In fact, it is common practice for some large conglomorates to arrange their group losses within one large loss-making company, with these losses then allocated as profit offsets to profit-making group members. Hence, an assumption is made within **CorpSim** that firms form groups consisting of pairs of firms, in order to take advantage of the regulations allowing losses to be deducted from gross profits both within and between firms in a group. **CorpSim** does not attempt to endogenise group formation. Instead, firms are allocated randomly to groups of two, except for those large profit-making firms generated from an upper tail of a lognormal distribution, which are matched randomly with firms making large losses.[11]

In addition to losses being deductible from firm or group profits, firms in **CorpSim** undertake investment which determines the value of their capital

[11] That is, though most firms are grouped randomly so that two profit-making, two loss-making or loss-profit coombinations may arise, large profit-makers are grouped only with large loss-makers. The matching of large profit and loss makers accounts for 12 per cent of the total firm pairs in the sample.

allowances, with investment assumed to be a positive function of past and current profits (both trading profit and interest income).[12] Though investment is not affected directly by tax parameters, the extent to which capital allowances are declared at home is affected by t and s as described earlier. In addition, any tax-induced profit-shifting affects capital allowance claims automatically through the effect of profits on investment.

CorpSim employs an range of algorithms to ensure that, given the configuration of group members' profits and losses, deductions are allocated within the group in a tax-minimising way, subject to tax code restrictions. With two profit sources and two firms in a group there are ten different resulting profit-loss combinations possible. Allowing for more than two group members would considerably complicate the tax-minimising procedure.

As discussed in section 3, though the elasticity of deductions with respect to declared profits, $\eta_{D,P*}$, can be expected to be unity in a steady-state, at different stages in an economic cycle $\eta_{D,P*}$ can be expected to vary, depending on how capital allowances and losses respond to changing profit growth over the cycle. This potentially affects the value of $\eta_{P^T,t}$ for each firm at different points in the cycle. The aggregate equivalent, $\Omega_{P^T,t}$, is further affected by changes in the numbers of, and tax payments by, taxpaying firms over the cycle. To investigate these aspects within **CorpSim,** firms are subjected to a dynamic process involving a trend rate of profit growth on which a cycle is superimposed. In addition, firms may experience relative profit movements as a result of stochastic changes in profits.[13]

5 Behavioural Responses in Aggregate

This section uses the simulation model **CorpSim**, allied with alternative assumptions regarding behavioural elasticities, to examine the aggregate elas-

[12] A more sophisticated investment function could be adopted here but the present assumption is designed to capture a positive correlation between investment and profits. One rationale for such a correlation is that imperfect capital markets lead firms to prefer internal sources of finance for investment projects.

[13] As discussed in the appendix, even without stochastic profit variations, some firms experience relative profit movements as a result of the nature of the profit generating process.

ticity of taxable profits and hence corporation tax with respect to the tax rate. The above notation must therefore be modified to denote aggregates. Let $\Omega_{P^T,t}$ represent the elasticity of the tax base with respect to the tax rate for all firms in aggregate; this equivalent to $\eta_{P^T,t}$ (or strictly, $\eta_{P_i^T,t}$) for an individual firm. It can be shown that the individual and aggregate elasticities are related according to:

$$\Omega_{P^T,t} = \sum_i \eta_{P_i^T,t} \left(\frac{P_i^T}{P^T} \right) \tag{20}$$

That is, the aggregate elasticity is a taxable profit share-weighted average of the individual elasticities. An aggregate result corresponding to equation (4) is also available, so that:

$$\Omega_{T,t} = 1 + \Omega_{P^T,t} \tag{21}$$

To consider the responsiveness of firms in aggregate requires information on the decomposition of $\eta_{P_i^T,t}$ in equation (13). Firms could vary in their behavioural responses (the elasticities, $\eta_{P_i,t}$, $\eta_{\theta_{p_i},t}$ and so on) and in the size of their profits and deductions, which affects D_i/P_i^* and hence the α_is of each firm, where $D_i/P_i^* = (\alpha_i - 1)/\alpha_i$. Further, automatic effects operating via the elasticity of deductions with respect to declared profits, η_{D,P^*}, in (19) can be expected to vary across firms depending on the extent and type (whether losses or capital allowances) of their deductions.

As argued above, the behavioural response parameters, $\eta_{P,t}$, $\eta_{\theta_p,t}$ and so on, might be expected to be greater for larger and/or multinational firms who can absorb the fixed costs of profit-shifting or deduction-shifting more easily. Without information on firms' domestic or multinational status and size (other than their annual profits), the simulations treat each behavioural response as common across firms. This means that the aggregate elasticity is not computed using equation (20), but is based directly on changes in the relevant aggregate values. However, in obtaining these results the values of η_{D,P^*}, are effectively allowed to vary endogenously across firms, since each firm's deductions are evaluated in a tax minimising manner, as discussed above.

5.1 Simulation Parameters and Results

To investigate the impact of profit-shifting and deduction-shifting parameters, and differences in firm behaviour, on the aggregate elasticity, $\Omega_{PT,t}$, **CorpSim** was run over a ten-period profit cycle described by a sine wave. In the benchmark simulation this involves a trend growth rate of 2 per cent per period and a low cycle (with growth in the range 0.4 to 3.6 per cent), with trend growth observed in years 1, 6, 11, and so on. Benchmark values for the key exogenous parameters are as used previously, and given in Table 3. This benchmark also suppresses stochastics, so that the trend and cycle in profit growth rates affect all firms similarly.[14]

Table 3: Benchmark Values for Aggregate Simulations

	Parameter	Benchmark	Comment
Profit shifting	$\eta_{\theta_p,t}$	-0.375	
Deductions shifting	$\eta_{\theta_d,t}$	0.25	
Real profit response	$\eta_{P,t}$	-0.05	
Real deductions response	$\eta'_{D,t}$	0.05	
Deduction-profit response	η_{D,P^*}	endogenous:	from **CorpSim**
Initial declared profit	$P^*_{i,0}$	exogenous:	initial profit distrib
Initial declared deductions	$D_{i,0}$	endogenous:	from **CorpSim**
Trend profit growth		2 per cent	
Profit growth cycle		$0.4 - 3.6$ pc	Range (low cycle)
(sine wave)		10	Wavelength

The key difference from the individual firm illustrations above is that both α_i and the elasticity, η_{D,P^*}, are determined endogenously. To see how this affects results, simulations also examine the case where $\eta_{D,P^*} = 1$ is imposed. In addition, though benchmark simulations assume all firms have the same behavioural response elasticities ($\eta_{\theta_p,t}$, $\eta_{\theta_d,t}$, $\eta_{P,t}$, $\eta'_{D,t}$), differences in the size of firms' profits and deductions, generate inter-firm differences in the weights α_i.

[14]Nevertheless, the dynamic specification of the model implies that only firms with positive profits experience similar growth rates when there is no stochastic component of profit changes. For firms making large losses, growth rates are closer to the growth rate of the maximim possible loss. The precise relationship is described in further detail at the end of the appendix.

The role of the aggregate automatic response of deductions to profits, Ω_{D,P^*}, can best be seen by re-writing (19), for all firms in aggregate, as

$$\Omega_{P^T,t} = \left[\alpha - (\alpha - 1)\,\Omega_{D,P^*}\right]\Omega_{P^*,t} - (\alpha - 1)\Omega_{D,t} \tag{22}$$

where $\Omega_{P^*,t}$ ($= \Omega_{\theta_p,t} + \Omega_{P,t}$) and $\Omega_{D,t}$ ($= \Omega_{\theta_d,t} + \Omega_{D',t}$) are the combined 'discretionary' real and shifting responses by firms and Ω_{D,P^*} is the endogenous automatic response.

Table 4: Three Effects on Tax Base Elasticity

Variable	Effect on $\Omega_{P^T,t}$	Magnitude depends on:
$\Omega_{P^*,t}$	$-ve$ direct effect	Relative size of deductions, α
	$+ve$ indirect effect	Relative size of deductions, α, and endog. response to profits, Ω_{D,P^*}
$\Omega_{D,t}$	$-ve$ direct effect	Relative size of deductions, α

Equation (22) reveals three effects on the tax base elasticity, $\Omega_{P^T,t}$. These are shown in Table 4. Both profits and deductions have direct negative effects on $\Omega_{P^T,t}$. That is, the responses of both to increases in tax rates (profit outflow, deductions inflow) serve to increase the absolute value of $\Omega_{P^T,t}$. However, there is an additional indirect effect of a profit outflow, namely the loss of some deductions that otherwise could be claimed against declared profit: this reduces $\Omega_{P^T,t}$. It can be seen from (22) that the direct effect dominates as long as condition (23) holds:

$$\Omega_{D,P^*} < \frac{\alpha}{\alpha - 1} \tag{23}$$

This inequality identifies the conditions under which a reduction in declared profits in response to a tax increase (whether via real or shifting effects) raises or lowers tax liabilities, relative to the case where $\Omega_{P^*,t} = 0$. Where condition (23) holds, a negative profit response to the increased tax rate has a lower tax liability than when there is no response. However, where condition (23) does not hold, the loss of deductions which could be used to off-set profits, when declared profits are driven down by a tax rate rise, would have a net effect of increasing firms' tax liabilities.

In general, there is no reason to expect (23) to hold since it depends on how the endogenous response of deductions to profit *changes* compares to the relative *size* of deductions with profits. Both could be determined by different characteristics of a corporate tax system.[15] In the UK, in aggregate $\alpha \approx 2$ and, as the simulations below show, Ω_{D,P^*} is always less than this. Hence, the inequality in (23) seems to hold in the UK and the net effect on $\Omega_{P^T,t}$ of $\Omega_{P^*,t}$ is expected to be to increase $\Omega_{P^T,t}$. Nevertheless, larger values of Ω_{D,P^*} increase the indirect effect and thus reduce $\Omega_{P^T,t}$. To the extent that losses dominate deductions, larger values of Ω_{D,P^*} can be expected to be associated with cyclical downturns. Of particular interest among the simulation results are the on-trend values of the aggregate elasticity, $\Omega_{P^T,t}$ (where $\Omega_{D,P^*} = 1$) for different profit-shifting assumptions, and the sensitivity of elasticity values to cyclical changes. For example, it is useful to examine how $\Omega_{P^T,t}$ changes over business cycles of different magnitudes, compared with values involving trend profit growth. Furthermore, the model can be used to examine whether alternative shifting assumptions yield differences in $\Omega_{P^T,t}$ which are of similar magnitude at all points in the cycle.

5.2 Profit Growth Cycles

Before examining elasticity results it is useful to consider the different growth cycles associated with different profit definitions. Previous sections have highlighted three profit definitions: gross profit including losses, P; gross profits declared for tax, P^*; and net taxable profits (after deductions), P^T. Though official UK data only measure declared profits P^*, rather than P, this can include losses, as with National Accounting definitions, or treat all losses as zero profits, as with HMRC's gross taxable profit data. HMRC also measure net taxable profits, P^T, after losses and capital allowances have been used as off-sets.

Figure 4 shows the exogenously set benchmark growth cycle for gross

[15] For example, the use of past and current losses as profits off-sets tends to generate a relationship between deductions and profits. However, the introduction of other deductions which may be unrelated to profits, or changes in qualifying expenditures, can raise the level of total deductions allowable against profits.

profits, *including losses*, dP^*/P^*, varying between 0.4 per cent and 3.6 per cent around the trend rate of 2 per cent. Gross taxable profit (where losses are set to zero) can be seen to follow a smoothed cycle compared with dP^*/P^*, ranging between 1.5 per cent and 2.5 per cent. The effect of deductions is to generate a profile for net taxable profit growth which lies between the other two profiles – with slower deductions growth in above-trend years causing net taxable profits to grow faster than the gross equivalent, and vice versa in below-trend years. The deductions profile in Figure 4 does not display the same cyclical pattern as profits. This is due to the opposing counter-cyclical and pro-cyclical effects of capital allowances and losses used within the deductions total. Setting capital allowances to zero, would yield a regular counter-cyclical profile for deductions growth in Figure 4.[16]

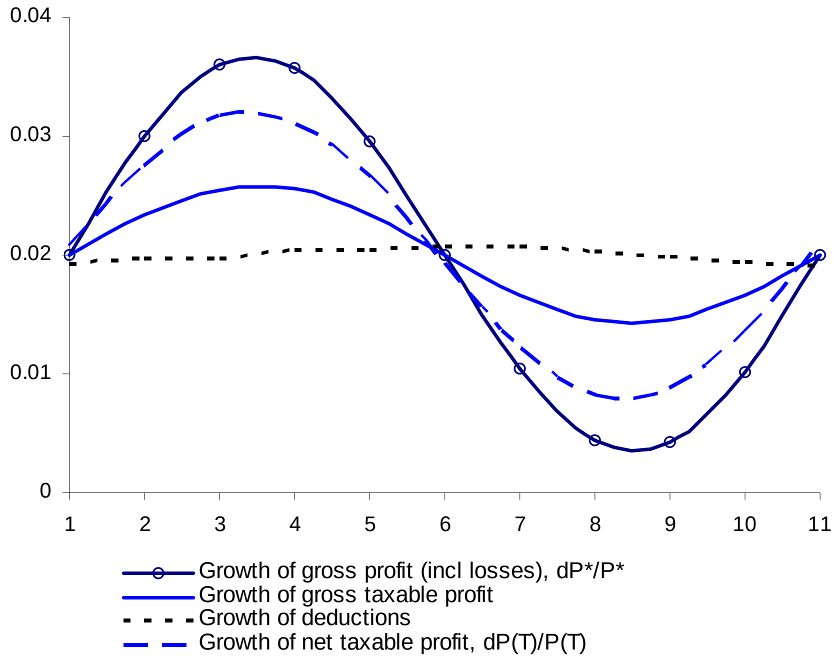

Figure 4: Profit Growth Rates Over the Cycle

[16]It can be shown that $\Omega_{P^T,P^*} = \sum_{i=1}^{N} \left(\eta_{P_i^T,P_i^*}\right)\left(\eta_{P_i^*,P^*}\right)\left(\frac{P_i^T}{P^T}\right)$ and therefore depends, among other things, on the way in which individual firms' values of P_i^* varies with the aggregate P^*.

5.3 Benchmark Results

In estimating aggregate behavioural elasticities, CorpSim is used to provide estimates of αacross all firms, together with estimates for profit growth and the aggregate endogenous deductions response, Ω_{D,P^*}.Together with assumed values for the behavioural responses in (22) this allows $\Omega_{P^T,t}$ to be calculated. Figure 5 shows elasticity profiles for $\Omega_{P^T,t}$ over eleven periods for two cases. In case A parameters are set at the benchmark values in Table 3. In case B, $\Omega_{D,P^*} = 1$ in all periods, so that automatic deductions change endogenously in proportion to declared profits. This allows the contribution of the endogenous response to be identified. In both cases, α in (22) is allowed to vary over the business cycle. The growth of dP^*/P^*, for the benchmark cycle is also shown for comparison in Figure 5, on the right-hand axis.

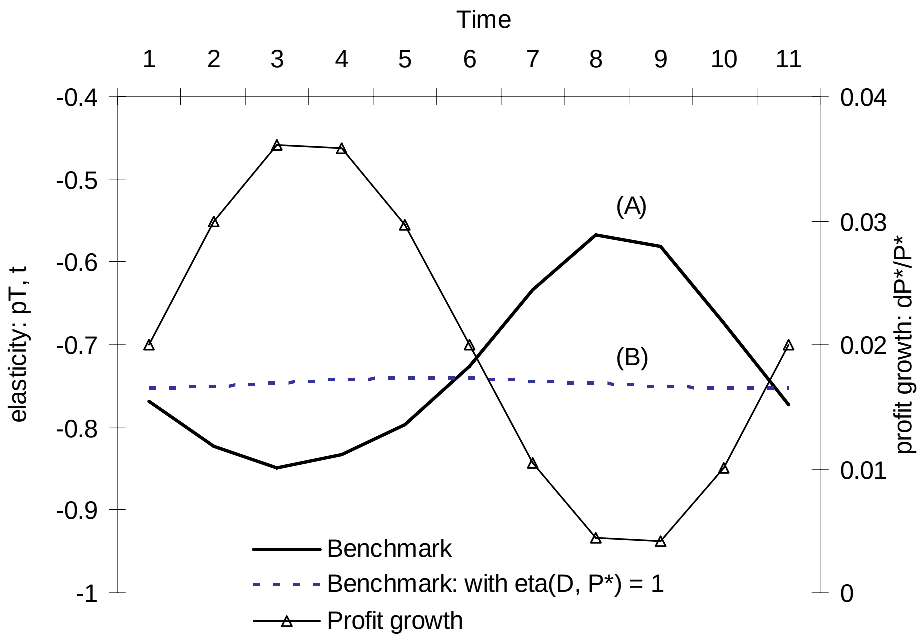

Figure 5: Elasticities Over the Cycle

The aggregate elasticities in Figure 5 are obtained from simulations of 18,000 paired firms drawn from the initial simulated profit distributions described in subsection 4. This yields firms in either positive profit or loss which, via the methods for allocating firms to groups and the tax-minimising algorithm within groups, determine firms' deductions and the extent to which they are claimed currently or carried forward to future years. The trend and cyclical growth components then cause firms' profits and losses to change over the years of the simulation.[17]

First, it can be seen that the elasticity in profile B is approximately constant over the cycle, implying that the variation observed in profile A is almost entirely due to cyclical variations in Ω_{D,P^*}; that is, variations in α over the cycle have minimal effect. In these benchark cases α is around 2 on average across all firms. The trend level of $\Omega_{PT,t}$ is approximately -0.77. This can be seen using the benchmark values of parameters in Table 3 and equation (22), in which the aggregate value of α obtained from **CorpSim** is around 2 (that is, around half of all gross taxable profits are tax-relieved via deductions) and $\Omega_{D,P^*} \approx 1$ at mid-points in the cycle.[18]

Comparing profiles A and B with profit growth rates shows that during above-trend growth, the effect of differences in Ω_{D,P^*} are relatively small at the top of the cycle. The aggregate elasticity, $\Omega_{PT,t}$, reaches around -0.85 (or about 110 per cent of its trend value) in the benchmark case compared with -0.77 when $\Omega_{D,P^*} = 1$. This larger absolute value compared with the benchmark $\Omega_{PT,t}$ reflects the fact that $\Omega_{D,P^*} < 1$ during above-trend growth when there are fewer losses available to be used as profit off-sets. However, Figure 5 shows that in recessionary years, when profit growth on average is low, $\Omega_{PT,t}$ in profile A deviates noticeably more from profile B, becoming -0.56, or around 73 per cent of its trend value, at the bottom of the cycle. This largely reflects the impact of especially large increases in Ω_{D,P^*} in association with cyclical downturns. The behaviour of η_{D,P^*} over the cycle reflects the procyclical charateristics of capital allowance deductions and the countercyclical characteristics of loss-based deductions. The latter

[17] These initial simulations use no stochastics; see the sensitivity analyses in the next section. However, growth rates differ as discussed above. All simulations reported follow an inital twenty year simulation period which is necessary to prevent 'initial conditions' influencing the trend and cyclical components (for example, as loss pools build up from zero to trend levels.

[18] The elasticity produced by **CorpSim** is not exactly unity at 2 per cent growth because of lagged effects as annual growth rates transit across the 2 per cent average rate.

tends to dominate.

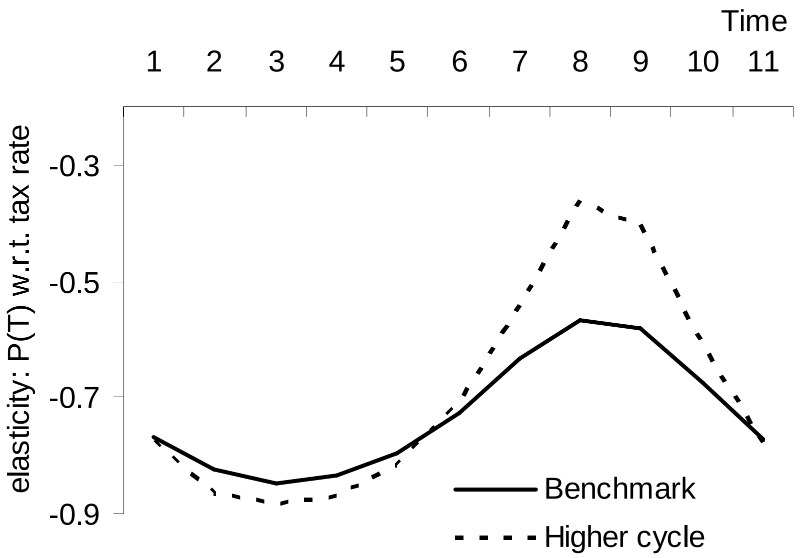

Figure 6: Elasticities with a High Profit Cycle

The asymmetry between booms and recessions arises because the use of losses as deductions is relatively unimportant in above-trend growth (when aggregate losses are relatively small) but becomes particularly important in below-trend growth when losses are larger on average. Because the taxable profit distribution is effectively truncated below zero, large losses both generate additional deductions and limit the ability of firms to claim them until positive profits return (or they can be shared with group partners in profit). The asymmetry becomes more pronounced for larger profit cycles, as shown in Figure 6. The time profile for $\Omega_{PT,t}$ using a higher cycle (range: -0.05 per cent to 4.5 per cent around a 2 per cent trend) can be seen to be quite similar to the benchmark case during above-trend growth but substantially lower in absolute terms during below-trend growth. These profit growth rates are well within the range of rates observed in the UK in practice, with HMRC data showing annual growth rates for gross taxable profits as high as 18 per cent and as low as -4 per cent since the early 1990s.

These results suggest an important conclusion for empirical methodologies testing for behavioural responses of profits or deductions to tax rate changes. Namely, in circumstances of trend or above-trend growth, recognising the impact of automatic changes in deductions may be less important. However, behavioural responses in recessionary periods could be substantially affected by the extent to which firms are constrained by the tying of deductions such as past losses to profits claimed in the home jurisdiction.

6 Sensitivity Analyses

This section considers the effects on the cyclical pattern of the aggregate elasticity, $\eta_{P^T,t}$, of varying a number of the parameters from their benchmark values. Subsection 6.1 examines the effects of different assumptions regarding profit and deductions shifting while subsection 6.2 discusses the role of stochastic changes in relative profit movements. Cyclical changes in the shifting parameters are examined in subsection 6.3.

6.1 Alternative Shifting Assumptions

Figure 7 shows how the elasticity profiles change under different assumptions regarding the extent of shifting. The upper part of the figure considers different values of profit-shifting, $\Omega_{\theta_p,t}$, while the lower part examines the effects of different deductions-shifting elasticities, $\Omega_{\theta_d,t}$. Other benchmark assumptions are maintained. It can be seen that, for profit-shifting, changing the values of the elasticity $\Omega_{\theta_p,t}$ causes the $\Omega_{P^T,t}$ profiles to shift non-uniformly. At the depression part of the cycle, when absolute values of $\Omega_{P^T,t}$ are relatively small, the shifting due to changed $\Omega_{\theta_p,t}$ values is small. The shift is larger when absolute values of $\Omega_{P^T,t}$ are larger during the boom periods. This reflects the fact that, since $\Omega_{\theta_p,t}$ is a component of $\Omega_{P^*,t}$, and is multiplied by Ω_{D,P^*} in (22), this magifies the impact on the tax base elasticity, $\Omega_{P^T,t}$. However, for deductions-shifting there is no such multiplier effect via Ω_{D,P^*} and therefore the profiles shift uniformly when $\Omega_{\theta_d,t}$ is changed, regardless of the point in the cycle.

In general, benchmark or lower response assumptions (for example, $\Omega_{\theta_p,t} = -0.225$; $\Omega_{\theta_d,t} = 0.125$) yield values for $\Omega_{P^T,t}$ which are smaller than -0.8.

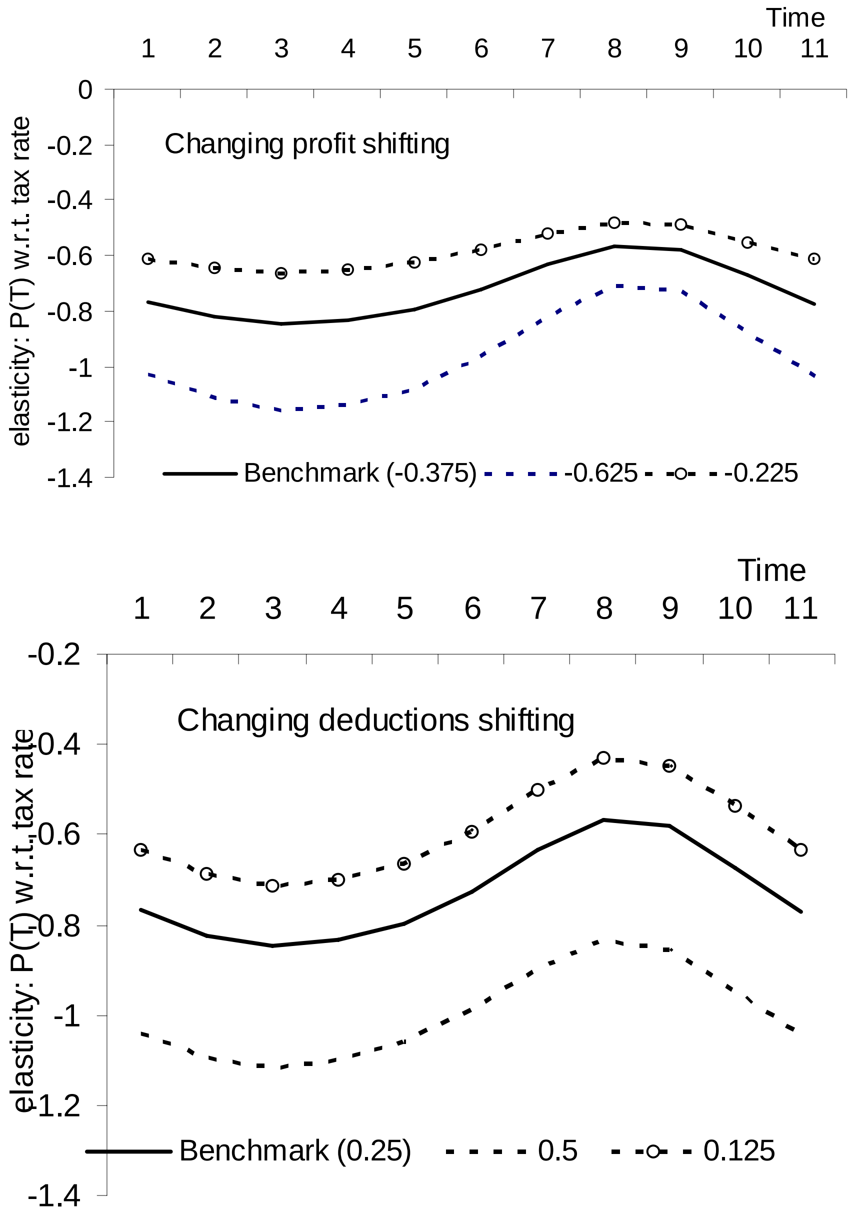

Figure 7: Changing Profit-shifting and Deductions-shifting Assumptions

However, the higher response assumptions (for example, $\Omega_{\theta_p,t} = -0.625$; $\Omega_{\theta_d,t} = 0.5$) can yield values of $\Omega_{PT,t}$ in excess of -1. In view of the relationship in equation (21), this implies a negative tax elasticity, $\Omega_{T,t} < 0$, which in turn implies that tax rates are set on the negatively sloped portion of the corporate tax Laffer curve. Such elasticity values are unlikely to hold for the current UK tax regime. This finding therefore suggests either that the lower behavioural responses examined are more likely in practice, or when automatic effects raise the value of $\Omega_{PT,t}$, *ceteris paribus*, this may induce changes in firms' discretionary responses, $\eta_{\theta_p,t}$, $\eta_{P,t}$, and so on. Subsection 6.3 considers possible trade-offs between automatic and discretionary responses.

6.2 Allowing for Stochastic Profit Growth

So far it has been assumed that all firms making positive gross profits grow at the same rate.[19] To capture the possibility of a range of firm profit growth rates, **CorpSim** incorporates a random growth component.[20] Simulations reported below use a variance of profit growth of 0.0001 around mean growth as determind by the trend and cycle components. This implies, for example, that with 2 per cent trend growth on average, around half of all firms experience profit growth outside a 1 to 3 per cent range. This has the effect of generating an additional source of difference in $\eta_{D,P*}$ across firms, with resulting differences in the aggregate equivalent $\Omega_{D,P*}$.

The resulting time-profile for $\Omega_{PT,t}$ is shown in Figure 8 where it is compared with the (zero variance) benchmark case. It can be seen that elasticity values can vary quite considerably especially, but not exclusively, associated with cyclical downturns during years 6 to 11, when negative profit/loss growth is more prevalent.[21] These results suggest that automatic responses of deductions to changes in profits declared at home can be quite volatile.

[19] As explained earlier, and in detail at the end of the appendix, the dynamic process implies that firms with losses experience different growth rates.

[20] The model can also include the possibility of serial correlation where, for example, 'success breeds success', but this phenomenon is not considered here.

[21] Creedy and Gemmell (2007a, c) show that the built-in flexibility of corporation tax (the aggregate elasticity of tax revenue with respect to aggregate profits) is also inherently variable over the business cycle, and this variability increases when stochastic elements are introduced.

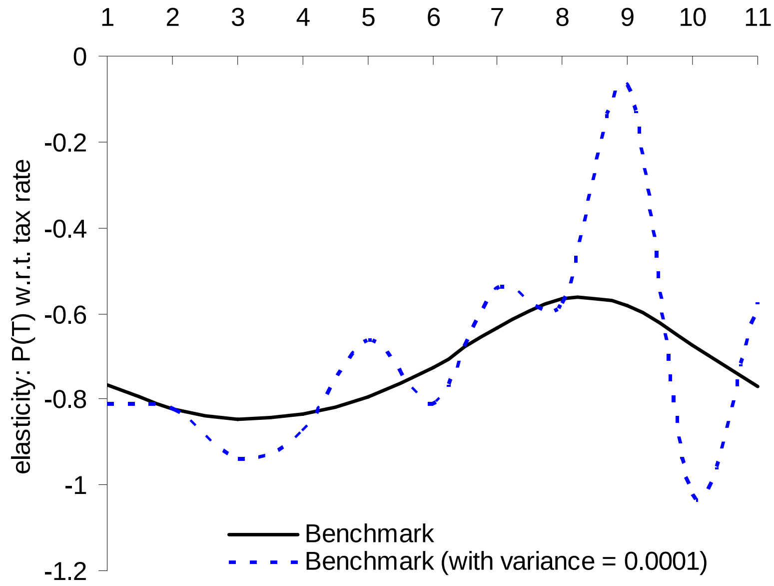

Figure 8: Allowing for Stochastic Component in Profit Growth

Thus, the ability or willingness of firms to shift profits abroad may be quite different, depending on the size and growth of their deductions, since these can only be claimed against profits declared at home. The simulations suggest that these factors may be important at the aggregate level. This is perhaps not surprising when it is recalled that the vast bulk of corporate tax revenues in the UK are paid by a small fraction of (large) taxpaying firms. Volatility in their profit performances can have a large influence on aggregate taxable profit outcomes.

6.3 Cyclical Changes in Real and Shifting Responses

Results reported so far assume that the four aggregate behavioural elasticities ($\Omega_{\theta_p,t}$, $\Omega_{P,t}$, $\Omega_{\theta_d,t}$, $\Omega'_{D,t}$) remain unchanged in the face of induced changes in firms' abilities to claim deductions due to cyclical factors. Clearly, when recessionary forces reduce firms' deductions claiming (or increase the required profits declared at home to qualify for those deductions), they may be expected to react. In particular, they may seek to mitigate cyclical effects by increasing or reducing any of the relevant behavioural elasticities.

This could be investigated in the current context by specifying a relationship between automatic and discretionary responses. However, in the absense of empirical evidence on the nature and extent of firms' behavioural responses, such modelled relationships would be arbitrary and of unknown empirical relevance. As an alternative, this subsection considers the changes in each behavioural response that would be required to keep the tax base elasticity, $\Omega_{P^T,t}$ constant, given the automatic changes induced by cyclical factors; that is, if behavioural changes were aimed to neutralise fully the automatic cyclical changes. Since **CorpSim** identifies declared profits, P^* (rather than total profits, P, and the declared fraction, θ_p) and similarly for declared deductions, it is more relevant below to combine real and shifting responses to consider values of $\Omega_{P^*,t}$ and $\Omega_{D,t}$.[22]

The two discretionary elasticities, $\Omega_{P^*,t}$ and $\Omega_{D,t}$, can be obtained in this case by rearranging equation (22) in aggregate terms. Letting $\lambda = \alpha - (\alpha - 1)\Omega_{D,P^*}$, this gives:

$$\Omega_{P^*,t} = \frac{1}{\lambda}\left\{(\alpha - 1)\Omega_{D,t} + \Omega_{P^T,t}\right\} \tag{24}$$

and

$$\Omega_{D,t} = \frac{1}{(\alpha - 1)}\left\{\lambda\Omega_{P^*,t} - \Omega_{P^T,t}\right\} \tag{25}$$

where, in each case, the elasticities on the right hand side are held constant at their benchmark values; the variations are determined by variations in α and Ω_{D,P^*}.

Results are reported in Table 5 for a complete cycle. These examples use the period 1 benchmark tax base elasticity of $\Omega_{P^T,t} = -0.77$, as shown in Figure 5, yielding the benchmark behavioural elasticities $\Omega_{P^*,t} = -0.43$ and $\Omega_{D,t} = 0.30$ in period 1 when the induced response, $\Omega_{D,P^*} \approx 1$.[23] The two columns of Table 5 show the values of the discretionary total profit or deductions responses, using (24) and (25) respectively, which maintain

[22] Since **CorpSim** produces values for Ω_{D,P^*}, rather than separate values for Ω_{D,θ_p} and $\Omega_{D,P}$, it is not meaningful in this context to consider separate discretionary responses for θ_p and P.

[23] That is $\Omega_{P^*,t} = -(0.375 + 0.05)$, and $\Omega_{D,t} = 0.25 + 0.05$.

Table 5: Compensatory Shifting and Real Responses

year	total profit response $\Omega_{P*,t}$	total deductions response $\Omega_{D,t}$
1	-0.43	0.30
2	-0.38	0.25
3	-0.36	0.23
4	-0.37	0.24
5	-0.40	0.28
6	-0.47	0.34
7	-0.61	0.43
8	-0.78	0.49
9	-0.73	0.48
10	-0.57	0.39
11	-0.43	0.30

$\Omega_{P^T,t} = -0.77$. It can be seen in column 1, for example, that if $\Omega_{P*,t}$ is adjusted to compensate fully for cyclical effects, this has to rise (in absolute terms) to -0.78 at the cyclical low point in period 8 (from -0.43 in period 1). That is, combined real and shifting profit responses would need to increase by around 80 per cent. To put this in prespective, recall that a profit response of -0.43 implies that a 10 *per cent* increase in the tax rate would cause declared profits, on average, to fall from, say, 80 to 77. The value of -0.78 implies a fall from 80 to around 74. Column 2 shows that the equivalent response for deductions involves a deductions elasticity, $\Omega_{D,t}$, in period 8 more than 60 per cent higher than its value when growth in on-trend, increasing from 0.30 to 0.49.

The changes in the profit and deductions responses in Table 5 are shown in Figure 9, as precentages of trend values (period 1). This shows that both the required profit and deduction responses (needed to keep the overall behavioural response, $\Omega_{P^T,t}$, constant) are larger in recessions, reflecting the fact that the positive indirect effect from $\Omega_{D,P*}$ is greater in recessions and hence needs a larger discretionary reponse to counteract it. Figure 9 also shows however that the required change in the profit response is less than that for deductions in above-trend growth, and vice versa in below-trend growth.

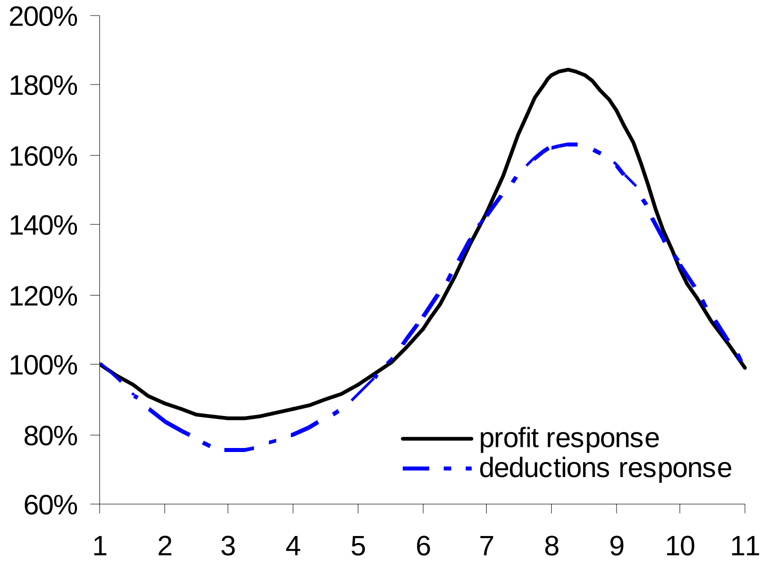

Figure 9: Percentage Profit and Deductions Responses

This arises because the indirect effect Ω_{D,P^*}, operating through $\Omega_{P^*,t}$, is less than unity in above-trend growth but is greater than unity in below-trend growth. Thus the effect on the profit response is respectively dampened then magnified, over the cycle, compared with the deductions response.

It is perhaps not surprising that, when recession restricts a firm's ability to shift profits out (due to the greater simultaneous loss of automatic deductions at home), more discretionary deductions should be shifted into the home jurisdiction. That declared profits should be reduced even more in this situation is less clear. However this result follows from the condition in (23) that $\Omega_{D,P^*} < \frac{\alpha}{\alpha-1}$. Since this condition holds here, reducing declared profits reduces automatic deductions claiming but by less than the reduction in profits. Hence, reducing declared gross profits also reduces declared net profits, despite the loss of some automatic deductions. In a recession, shifting profits out (which reduces tax liability) simultaneously shifts a greater amount of deductions (which increases tax liability) compared with non-recessionary periods. Hence more profits must be shifted ($\Omega_{P^*,t}$ must be higher) in order to stop tax liability rising, and keep the overall response, $\Omega_{P^T,t}$, constant. Nevertheless, Table 5 and Figure 9 show that, in a recession, a given overall behavioural response can be achieved via a smaller change in the deductions response compared with the profit response.

7 Conclusions

This aim of this paper has been to examine behavioural responses by companies to changes in the taxation of their profits in the home country, and the possible pattern of such responses over the business cycle. Emphasis has been on the determinants of the elasticity of corporation tax paid, by individual firms and in aggregate, in response to a change in the corporation tax rate. This elasticity in turn depends crucially on the elasticity of net or taxable profits with respect to a change in the tax rate. In this respect the paper may be seen as following the broad agenda set by Feldstein (1995) when he emphasised the importance of the elasticity of taxable income with respect to the retention, or net-of-tax, rate.

Firms' responses to tax rate changes can take the form of real responses, whereby activities are transferred to other tax jurisdictions, and income-shifting responses in which the location of economic activity is unchanged but the extent to which profits and deductions are declared in the home country changes. The present paper has shown that it is also important to distinguish separate responses of gross profits and of deductions allowable as profit off-sets. Where, as in the UK, these deductions are related to the size of companies' profits, it is found that allowing for an endogenous, or automatic, response may be important for empirical estimates of firms' overall behavioural responses.

Behavioural responses of corporations to tax changes were therefore first decomposed into four main constituent components, consisting of real and income-shifting responses of both profits and deductions. The elasticity of taxable profits with respect to the tax rate was shown to depend on the four elasticities relating to these responses, along with the ratio of declared profits to taxable profits. It is also shown how the elasticity of qualifying expenditure with respect to the tax rate depends on the elasticity of qualifying expenditure with respect to declared profits. This last elasticity, reflecting an endogenous or automatic adjustment of deductions to profit changes, is shown to play an important role. In producing aggregate measures of the elasticity of taxable profits with respect to the tax rate over the business cycle, the automatic responses of deductions to profit changes were obtained using a range of algorithms designed to ensure that firms minimise taxation. These algorithms are contained in the microsimulation model **CorpSim**, which generates the changing distribution of profits for a large number of corporations over the business cycle.

The elasticity of aggregate taxable profits with respect to the tax rate was shown to be pro-cyclical, being in absolute terms at a maximum when aggregate profit growth is at a maximum, and an absolute minimum in the depths of the depression part of the business cycle. Importantly, the variation found in the elasticity of taxable profits with respect to the tax rate was found to be almost entirely due to cyclical variations in the endogenous or automatic component mentioned above, regarding the elasticity of deductions with respect to declared profits.

The variation in the elasticity of taxable profits with respect to the tax rate was not, however, symmetric, being greater in periods of depression. This asymmetry between booms and recessions arises because the use of losses as deductions is relatively unimportant in above-trend growth (when aggregate losses are relatively small) but becomes particularly important in below-trend growth when losses are larger on average. Because the taxable profit distribution is effectively truncated below zero, large losses both generate additional deductions and limit the ability of firms to claim them until positive profits return (or they can be shared with group partners in profit). The asymmetry increases as the amplitude of the profit cycle increases.

The implication of these findings for empirical attempts to measure behavioural responses of profits or deductions to tax rate changes is that in circumstances of trend or above-trend growth, recognising the impact of automatic changes in deductions may be relatively less important. However, behavioural responses in recessionary periods could be substantially affected by the extent to which firms are constrained by the tying of deductions such as past losses to profits claimed in the home jurisdiction.

When a stochastic component of profit changes for individual firms was introduced in the simulations, the resulting time-profile for the elasticity of taxable profits with respect to the tax rate was found to vary quite considerably. This variability was especially, but not exclusively, associated with cyclical downturns. These results suggest that automatic responses of deductions to changes in profits declared at home can be quite volatile. Thus, the ability or willingness of firms to shift profits abroad may be quite different, depending on the size and growth of their deductions, since these can only be claimed against profits declared at home. The simulations suggest that these factors may be important at the aggregate level.

The results therefore emphasis the need to allow for automatic changes in deductions in response to changes in declared profits, and the value of a microsimulation model which is capable of generating a changing distribution of profits over time and, importantly, can allow for the complex way in which firms, as members of groups for tax purposes, arrange their deductions in order to minimise tax paid.

Appendix: Simulating Profit Distributions

This appendix provides further details of the **CorpSim** model. As mentioned above, the model deals with declared profits, equivalent to P^* values, but for convenience the $*$ supercript is omitted from the following. To avoid the problems of dealing with negative values, the approach is first to convert profits into a positive variable, x_t, where t is a time subscript:

$$x_t = P_t + d_t \tag{26}$$

and a choice must therefore be made regarding the shift parameter, d_t.

Profit Distributions

In view of the form of the empirical distribution of trading profits, a functional form such as the lognormal distribution, which is widely used in analyses of incomes, is unable to capture the shape of the distribution of $P^A + d$.[24] The approach taken here is thus to use a mixture distribution, defined as follows. In general, a mixture distribution, $M(x)$, is defined on the random variable, x, as a linear combination of H independent distributions, $f_i(x)$, such that:

$$M(x) = \sum_{i=1}^{H} \beta_i f_i(x) \tag{27}$$

where β_i defines the proportion of density mass associated with the ith distribution. The use of a mixture distribution, in contrast to the search for a much more complex functional form of a single distribution that can handle the observed characteristics, has several advantages. First, relatively simple distributions can be combined intuitively in order to match particular features of an empirical distribution. Second, relatively straightforward analytical results can be derived for summary measures, despite the overall complexity of the form of the mixture, where well-established analytical results exist for the constituent distributions.[25]

[24] The lognormal is defined only for positive values, so that P^A+d is the relevant variable, rather than P^A.

[25] On the use of (conditional) mixture distributions to handle observed bimodality of the personal income distribution, and changes over time, see Bakker and Creedy (1999).

The following specification, involving a mixture of four distributions, is adopted here for trading profits. A proportion β_1 of the density of $P^A + d$ is modelled using a lognormal distribution $\Lambda(\mu_1, \sigma_1^2)$, in which σ_1^2 is relatively large to capture a platykurtic or flat feature. This has relatively low kurtosis and thus captures the more central portion of the distribution. To capture the leptokurtic, or peaked, feature, a proportion β_2 is modelled using a lognormal distribution $\Lambda(\mu_2, \sigma_2^2)$ in which σ_2^2 is relatively small. In the following analysis, $\mu_1 = \mu_2$. However, the use of just these two distributions does not capture the very long tails of the distribution. Hence a further two component distributions are used. The first forms a proportion, β_3, of the density and consists of the upper tail of yet another lognormal distribution, $\Lambda\left(P^A + d > \xi \,|\mu_3, \sigma_3^2\right)$, where μ_3 and σ_3^2 are both relatively large. Hence this applies only to values of $P^A + d$ above the threshold, ξ. The importance of this third distribution lies in the fact that the upper tail of the profit distribution is responsible for the bulk of corporation tax payments. These same high profit firms are then matched (in groups for tax purposes) with large loss making firms, where values are obtained from the left hand tail of another lognormal distribution where a much higher value of d is used.

Analysis of the distribution of loan-relationship profits reveals that it does not have the long tails of the distribution of trading (source A) profits. Hence a mixture distribution involving just two lognormal distributions is used, with proportions of the densities set at β_1 and $\beta_2 + \beta_3$ for the platykurtic and leptokurtic components respectively.

Changes in Profits Over the Cycle

Given, as before, $x_t = P_t + d_t$, the basic assumption regarding growth is that x_t is subject to a growth rate made up of a systematic component, g_t, and a random component, u_t. The t subscript on g allows the systematic growth of profits to vary in some way over time, along with the minimum profit. Thus x is specified to change according to:

$$x_t = x_{t-1}(1 + g_t + u_t) \tag{28}$$

Furthermore, serial correlation implies that:

$$u_t = \gamma u_{t-1} + v_t \tag{29}$$

and v is assumed to be Normally distributed as $N(0, \sigma_v^2)$. In terms of P_t, (28) becomes:

$$P_t + d_t = (P_{t-1} + d_{t-1})(1 + g_t + u_t) \tag{30}$$

and rearrangement gives:

$$P_t = P_{t-1}(1 + g_t + u_t) - (d_t - d_{t-1}) + d_{t-1}(g_t + u_t) \tag{31}$$

This is the basic equation describing the systematic (g_t, d_t) and stochastic (u_t) processes generating the changing profit level of each firm, and hence the changing distribution of profits over time. It is completed by the specification of the time-profiles of g_t and d_t.

The growth rate, g_t, is composed of a constant component, g^*, representing trend real growth, and a real cyclical component, g_t^c. This cyclical aspect can be described by an amplitude of a_g and a wavelength of w_g. Using a sine wave to represent the cycle, then:

$$\begin{aligned} g_t &= g^* + g_t^c \\ &= g^* + a_g \sin\left(\frac{2\pi t}{w_g}\right) \end{aligned} \tag{32}$$

Similarly, suppose that the proportional rate of change in d (the maximum loss) from one period to the next consists of a fixed term, $\dot{d}^*$, and a cyclical component, $\dot{d}_t^c$. Thus:

$$d_t = d_{t-1}\left(1 + \dot{d}^* + \dot{d}_t^c\right) \tag{33}$$

The cyclical component similarly has an amplitude of a_d and a wavelength of w_d, so that:

$$d_t = d_{t-1}\left(1 + \dot{d}^* + a_d \sin\left(\frac{2\pi t}{w_d}\right)\right) \tag{34}$$

This captures the notion that the extent of maximum losses can also behave cyclically; for example, in a recession when profit growth is lower on average, maximum losses are likely to become larger.

The above model then needs to be extended to deal with the fact that firms obtain profits from two sources, A and B. These two income sources give rise to profits of P^A and P^B, with corresponding values of $x_A = P^A + d^A$ and $x_B = P^B + d^B$. Starting from a given initial joint distribution of profits, such that there is some correlation, ρ, between A and B profits, it is necessary to generate profit flows in subsequent periods. The following sequence is used.

First, the random component of proportional changes for the A source is given, where $v_{u,i}^A$ is a random draw from an $N(0,1)$ distribution, by:

$$u_{i,t}^A = \gamma u_{i,t-1}^A + \sigma_{u_A} v_{u,i}^A \tag{35}$$

To allow for the possibility that stochastic shocks to A and B may be correlated, assume that u^A and u^B are jointly Normally distributed as

$$N\left(u^A, u^B \middle| 0, 0, \sigma_{u_A}, \sigma_{u_B}, \rho\right). \tag{36}$$

A value of u^B is then given by:

$$u_{i,t}^B = \rho\left(\frac{\sigma_{u_A}}{\sigma_{u_B}}\right) u_{i,t}^A + \left\{\sigma_{u_B}\sqrt{1-\rho^2}\right\} v_{u,i}^B \tag{37}$$

Thus, the two profit sources are generated using:

$$P_{i,t}^A = P_{i,t-1}^A\left(1 + g_t^A + u_{i,t}^A\right) - \left(d_t^A - d_{t-1}^A\right) + d_{t-1}^A\left(g_t^A + u_{i,t}^A\right) \tag{38}$$

and:

$$P_{i,t}^B = P_{i,t-1}^B\left(1 + g_t^B + u_{i,t}^B\right) - \left(d_t^B - d_{t-1}^B\right) + d_{t-1}^B\left(g_t^B + u_{i,t}^B\right) \tag{39}$$

Separate growth cycles, corresponding to (32) and (34) are then be specified for each of the terms g_t^A, d_t^A, and so on.

Profit Growth Rates

In the above model it is important to recognise that the absence of a stochastic component of proportionate changes in profits does not imply that all profits grow at the same rate. Consider a single profit source where, as above, $P_t = x_t - d_t$, with x and d growing at rates δ and θ respectively. These rates differ because the growth cycles of x and d are expected to be out of phase – in boom periods with relatively high δ it is likely that θ is relatively low. Combining these assumptions gives the result that:

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \delta - \frac{d_{t-1}}{P_{t-1}} (\delta - \theta) \tag{40}$$

For large profit makers the term d_{t-1}/P_{t-1} is low and hence the growth rate of profits is similar at δ. However, for loss-makers, $-1 < d_{t-1}/P_{t-1} < \infty$. For the largest loss makers d_{t-1}/P_{t-1} is close to unity and the growth rate of profits is close to θ.

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