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Abstract

We develop an open economy general equilibrium model, with auction-based directed search unemployment, to study the interactions of trade and unemployment. The theory ascribes all outcomes purely to the fundamentals of technology and endowment. If countries differ by endowment, trade makes both the unemployment rate and the rental in the capital-(labor-) abundant country rise (decline), but does not lead to equalization. Furthermore, the expected wage is higher in the capital-abundant country. If countries differ by technology, trade increases (decreases) the unemployment rate in the country whose technology is relatively superior (inferior) for producing the capital-intensive good.
JEL-Classification: F16, J64.

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1 Introduction

The relationship between international trade and unemployment has been a major concern for economists for a long time. A clear understanding of this relationship requires a framework that integrates trade theory with a plausible theory of unemployment – preserving the insights of both, and illustrating their interaction. Trade theory has traditionally concerned itself with understanding how patterns of trade, and conditions within each country, are determined, in general equilibrium, by the fundamentals: preferences, endowments and technologies. Historically, attempts to introduce unemployment into this type of model have, to some degree, traded off this fidelity to fundamentals with the practicalities surrounding the imposition of some sort of friction to generate unemployment (for example, sticky wages) rather than concentrating on frictions that arise endogenously from the fundamentals themselves.[1]

Search-based theories of unemployment are natural candidates to consider in this context, due to the fact that they themselves emphasize the role of fundamentals in determining unemployment rates. Several authors have successfully explored the implications of introducing search-theoretic unemployment into trade models. One strand of the literature has considered the role of random matching in otherwise standard models of trade.[2] More recently, the role of random matching has been explored in trade models with firm and worker heterogeneity.[3] All of these papers rely on a particular version of search theory that, itself, introduces an extra technology into the economic model: a matching function, based on the work of Diamond (1982),

[1]See for example, Davis and Harrigan (2007) for a model with efficiency wages, and Egger and Kreickemeyer (2009) for a model with fair wages. Both papers are embedded in models of firm heterogeneity; see Melitz (2003) and Melitz and Ottaviano (2008) for the key papers of this literature.

[2]See for example Hosios (1990), Davidson, Martin and Matusz (1988, 1999), and, more recently, Davidson, Matusz and Shevchenko (2008) and Davidson, Matusz and Nelson (2010).

[3]See for example Felbermayr, Larch and Lechthaler (2009), Felbermayr, Prat and Schmerer (2008), Helpman and Itskhoki (2010) and Helpman, Itskhoki and Redding (2010).

Mortensen (1982) and Pissarides (1985).

The matching function approach has proved useful for studying many issues. One issue that this approach side-steps, though, is the fundamental source of the matching frictions themselves. That is, when introducing a matching technology into a model, while researchers typically envision that some sort of fundamental friction (such as heterogeneity, informational problems, etc.) underlies this technology, when they do so, researchers implicitly theorize. Technically, the matching function acts as a behavioral function in the model – which can, in principle, expose policy conclusions to the Lucas critique. Due to the fact that this function matches firms and workers randomly, without formally modeling their fundamental characteristics, it necessarily ignores how these characteristics themselves may influence matching rates and unemployment rates. The matching function approach itself also introduces additional parameters into the model, which are not directly linked to fundamentals: the parameters in the matching function and the relative bargaining power of agents in the model.

In this paper we take a different approach by considering a trade model where unemployment arises from the coordination problem inherent in "directed search".[4] In this framework, agents in the model are not matched randomly by a matching technology but, rather, choose whom to approach directly, in an economic game. This framework has several advantages. First, by allowing the characteristics of agents in the model to influence matching rates, it allows for an extra channel of causation, through which the fundamentals of the model can effect equilibrium outcomes. Second, due to the fact that the equilibrium matching process in the model is driven entirely by the fundamentals, no new parameters are introduced into the model to generate unemployment. This keeps the analysis relatively clean and free from subsidiary parameters. Third, in large markets, as is well known, directed search equilibria are (constrained) efficient. Thus, the introduction of this type of unemployment into a trade model does not bring with it any extra source of inefficiency. In the particular specification that we consider in this paper,

[4] See Shi (2008) for a survey on directed search theory.

firms approach workers, assigning probabilities to each one. Also, wages are determined according to the number of firms that approach workers, through an ex post auction. This allows for a very simple analysis of wages, and some wage dispersion in equilibrium.[5]

This model of unemployment is embedded in an otherwise standard general equilibrium trade model with two goods and two factors. One good is relatively capital-intensive, the other is labor-intensive. We characterize the equilibrium allocations, both with and without trade, the implied trade patterns and the comparative statics of the system. In particular, we identify conditions under which a cone of diversification exists, where both goods are produced in the domestic country. The comparative static effects we find are quite striking and, as far as we know, quite novel. For example, we show that an increase in the relative price of the capital-intensively produced good may have ambiguous effects on expected wages in a country, but will unambiguously increase its unemployment rate.[6] Moreover, opening up trade will increase both the unemployment rate and the return to capital in a capital-abundant country and have the opposite effects in a labor-abundant country. Furthermore, although factor prices move closer together with trade, in this environment with directed search frictions, they do not equalize in the way they would in the absence of frictions.

The remainder of the paper is organized as follows: Section 2 introduces the model, derives the production possibility frontier, identifies the cone of diversification, and demonstrates that the economy behaves as if it maximizes the value of production, despite labor market frictions. Section 3 considers price,

[5] Kennes (2008) and Ritter (2009) also introduce directed search into trade models. Kennes' paper focuses on frictions in the goods market, rather than the labor market as we do here. Ritter considers a considerably richer labor market than the one analyzed here, with two-sided heterogeneity, intermediate goods, skill-biased technologies, and wage posting by firms – but without the general equilibrium feature that we emphasize in this paper.

[6] Davidson, Martin and Matusz (1999) also find that trade will increase the unemployment rate of a large, capital-abundant country, but their result relies on the relative efficiencies of the assumed search technologies in the different countries.

endowment and technology changes as to develop the pattern of trade and the impact of trade on job creation, unemployment and factor prices. Section 4 concludes, and the Appendix provides proofs for most of the propositions in the paper.

2 The Model

We consider an economy with two sectors (1 and 2, respectively) producing distinct goods (goods 1 and 2, respectively), both drawing from a fixed pool of identical risk-neutral workers of size $L > 0$. Each (representative) worker has one unit of labor to sell, and owns k units of capital. Thus, the economy's capital endowment is equal to $K = Lk$. The prices of the goods (p_1 and p_2) are set on domestic or international markets (depending on the context) and, without loss of generality, we assume that $p_2 > p_1 > 0$. Of course, only the relative price matters, and without loss of generality, we can employ good 1 as numeraire and use $p \equiv p_2/p_1 > 1$ in what follows. However, as will be seen soon, it turns out to be easier to understand the model of directed search we are using when we continue to use the (nominal) prices p_1 and p_2 as well. In what follows, we shall denote nominal factor rewards by a tilde.

The market for capital is frictionless and can be regarded as a market for ownership rights or ownership assets. Capital is used to form an establishment which hires labor as to produce one unit of output. The production of one unit of good $i \in \{1, 2\}$ requires one worker and κ_i units of capital. We assume that $\kappa_2 > \kappa_1 > 0$. Thus, the creation of a job in industry 2 is more expensive than in industry 1. Furthermore, we assume that $\kappa_2 > k > \kappa_1$ so that, in principle, full employment would be possible because capital is sufficiently abundant such that all workers could work in industry 1. As is standard, we assume that labor and capital are intersectorally, but not internationally mobile. Furthermore, we assume that both κ_1 and κ_2 are small compared to K such that firms are perfectly competitive in the (frictionless) commodity market.

The labor market is subject to potential frictions – workers and firms have to be matched. Agents can spread a portfolio across several investment projects and sell and buy goods to/from different buyers/sellers at the same time, but workers can sell labor services to one firm only.[7] Therefore, there is a binding capacity constraint such that labor services can be offered to one firm only, and if more than one firm competes for the worker, at least one firm will not be able to fill its vacancy.

This would not be a problem if creating a job did not warrant an irrevocable investment. In this paper, we assume that employers potentially compete by wage bids for a worker who can supply labor services to one employer only.[8] Accordingly, the sequence of events is as follows. First, firms are established through the capital market such that M_i jobs of each type i are created by M_i firms. (Thus, we use the terms "firm" and "job" interchangeably – depending on the context.) Creating M_i jobs means that $\phi_i = M_i/L$ jobs of type i per capita are created. The rental cost of capital is denoted by r. For each job, firms make a capital commitment of κ_i and thus have a cost of $\tilde{r}\kappa_i$ for establishing the firm. Once firms (and, equivalently, the number of jobs) have been established, firms choose which candidate to approach for each job. Once firms have approached workers, wages are determined through a bidding game. That is, given the number of firms (of each type) that have approached the worker, the worker conducts an auction to determine where to work. The model is solved by backward induction.

2.1 Wage Determination

Once firms have approached workers, different workers face, in general, different ex post opportunities. Those that have not been approached by any firms

[7] In the real world there is a natural capacity constraint in the labor market: each worker has only 24 hours in the day, and cannot service the entire labor market. This makes the capacity-constrained equilibria studied by Peters (1984) relevant in this market – as emphasized in Julien, Kennes and King (2000) and others.

[8] Alternatively, workers could be assumed to post wage demands. However, the cost of creating a vacancy can be thought of as including the search cost, so we find wage bids more compelling in our setup. As will become clear, below, *ex post* bidding of this sort also creates wage dispersion among identical workers in equilibrium – something observed empirically and emphasized in, for example, Mortensen (2003).

are, of course, unemployed and receive only their outside option $\tilde{w}_0^0 = p_0$, which is equal to 0.[9] All other workers sell their labor to the highest bidder, as long as the highest bid at least matches the outside option. If only one firm approaches a worker, then the highest bid will be exactly the outside option. When at least two firms approach the worker, the bidding game determines that he will work at the job with the highest valuation for the worker, and is paid the amount of the second highest valuation from among the jobs that have approached him. If, for example, at least two firms from industry 2 approach the worker then he will work at one of these jobs (with equal probability) and will be paid p_2. If, alternatively, only one firm from industry 2 good job and at least one firm from industry 1 approaches the worker, he will work in industry 2 but will be paid only p_1. If no firm from industry 2, but at least two firms from industry 1 approach the worker then he will work in one of the industry 1 jobs (with equal probability) and will be paid p_1.

Thus, in equilibrium, the nominal wage $\tilde{w}_i^j$ of a worker who is employed in a job of type i, and whose second best offer was from a job of type j, is given by:

$$\tilde{w}_i^j = p_j \tag{1}$$

for all $i, j \in \{0, 1, 2\}$.

2.2 Firms Choosing Workers

We now consider the problem, facing firms, of which workers to approach. All firms understand the structure of wages, given in equation (1), that operates once firms are assigned. Firms also correctly anticipate the numbers of each type of job that have been created. Firms choose which worker to approach in a simultaneous move game. There are many asymmetric pure strategy equilibria, where firms are uniquely assigned to workers. However, in large economies such as this, coordination on any of these equilibria becomes practically impossible. Coordination would require firms to collude on

[9] Note that workers always receive a capital income.

the labor market in order to guarantee that each worker is approached by a single firm only.

Therefore, as is standard in directed search environments, here we restrict our attention to the unique symmetric mixed strategy equilibrium in which each firm of each type randomizes over the workers (see, for example, Julien, Kennes and King, 2006). Consequently, the probability ρ_i that a worker is approached by at least a firm producing the good with maximum price p_i is given by (see Appendix A.1 for details):

$$\begin{aligned} \rho_0 &= e^{-\phi_1}e^{-\phi_2}, \\ \rho_1 &= e^{-\phi_2}(1-e^{-\phi_1}), \\ \rho_2 &= (1-e^{-\phi_2}). \end{aligned} \tag{2}$$

2.3 The Wage Distribution

It also follows that, in a large market, from the pool of jobs producing good i, a worker obtains either (i) no offer, (ii) one offer, or (iii) multiple offers with probabilities $e^{-\phi_i}$, $\phi_i e^{-\phi_i}$ and $1-e^{-\phi_i}-\phi_i e^{-\phi_i}$, respectively. Therefore, the wage distribution, Ω_s, is given by the following matrix:

$$\Omega_s \equiv \left[\tilde{w}_i^j, \rho_i^j\right] = \begin{bmatrix} \tilde{w}_0^0 = 0, & \rho_0^0 = e^{-\phi_1}e^{-\phi_2} \\ \tilde{w}_1^0 = 0, & \rho_1^0 = \phi_1 e^{-\phi_1}e^{-\phi_2} \\ \tilde{w}_1^1 = p_1, & \rho_1^1 = (1-e^{-\phi_1}-\phi_1 e^{-\phi_1})e^{-\phi_2} \\ \tilde{w}_2^0 = 0, & \rho_2^0 = \phi_2 e^{-\phi_2}e^{-\phi_1} \\ \tilde{w}_2^1 = p_1, & \rho_2^1 = \phi_2 e^{-\phi_2}(1-e^{-\phi_1}) \\ \tilde{w}_2^2 = p_2, & \rho_2^2 = (1-e^{-\phi_2}-\phi_2 e^{-\phi_2}) \end{bmatrix} \tag{3}$$

where ρ_i^j denotes the probability that worker obtains a wage $\tilde{w}_i^j$. The wage distribution (3) shows the outcome of the auction. The worker gets his (zero) outside option if he is not approached at all or approached by only one firm, either from industry 1 or 2 (see $\tilde{w}_0^0, \tilde{w}_1^0, \tilde{w}_2^0$). He will receive p_1 if there are at least two industry 1 firms (see $\tilde{w}_1^1$) or only one industry 2 firm competing against at least one industry 1 firm (see $\tilde{w}_2^1$). If there are at least two industry 2 firms, the wage is equal to p_2 (see $\tilde{w}_2^2$).

If the numbers of jobs were given exogenously (i.e., if ϕ_1 and ϕ_2 were parameters) then (3) would represent the solution of the model. However, we will allow ϕ_1 and ϕ_2 to be determined endogenously. Before doing so, though, it is useful to consider the production possibility frontier in our model.

2.4 The Production Possibility Frontier

Per-capita production y_i is equal to the matching rate in each industry and given by

$$\begin{aligned} y_1 &= e^{-\phi_2}\left(1-e^{-\phi_1}\right), \\ y_2 &= 1-e^{-\phi_2}. \end{aligned} \tag{4}$$

Capital market clearing requires:

$$\phi_1\kappa_1 + \phi_2\kappa_2 - k = 0. \tag{5}$$

Eq. (5) follows from $M_1\kappa_1 + M_2\kappa_2 = Lk$ since $M_i/L = \phi_i$. Note that $\phi_1 > 1$ cannot be excluded so the number of jobs created in industry 1 may be larger than the aggregate number of available workers. However, $\phi_2 > 1$ can be excluded because it would require that $k > \kappa_2$.

Interestingly, although our input requirements have a Ricardian flavor, the production possibility frontier exhibits the same shape as models which employ concave and linear-homogeneous production functions.

Lemma 1 *The production possibility frontier is strictly concave.*

Proof: See Appendix A.2.

Note carefully that our notion of the production possibility frontier differs to some extent from the notion in the literature. In our model, firms find workers only through an auction, and the outcome is not certain. Hence, our production possibility frontier is derived as an outcome of the auctions as described above, and strictly speaking the auctions determine the expected production levels in both industries. Thus, it is not just technology and

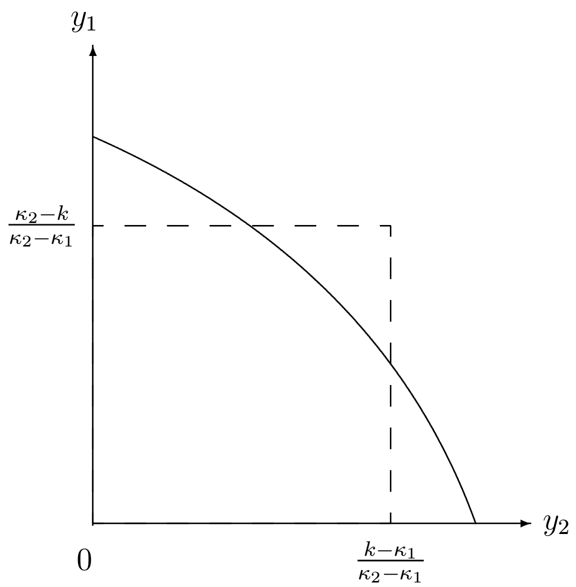

Figure 1: Production possibility frontier

endowment, but also the coordination problem which determines the production possibility frontier in our model.

It is useful to contrast our results to an economy without any frictions. Appendix A.3 has the details of this case and shows that this simple model behaves like a classic model without factor intensity reversal. In a frictionsless economy, per-capita outputs are given by

$$y_1 = \frac{\kappa_2 - k}{\kappa_2 - \kappa_1}, y_2 = \frac{k - \kappa_1}{\kappa_2 - \kappa_1}. \tag{6}$$

Since there is full employment of both capital and labor, the output pattern of expression (6) is not feasible with frictions on the labor market. Figure 1 summarizes these findings and plots the production possibility frontier with frictions and (6). Frictions imply that the chances of filling a vacancy are relatively small if a lot of vacancies have already been established in this industry, and this is the reason why the production possibility frontier is concave.

2.5 Job Creation in General Equilibrium

We now turn to the determination of ϕ_1 and ϕ_2 – the number of each type of job that is created in equilibrium. In equilibrium, the expected profit of a firm is equal to its revenues, given by its price, minus its capital costs and the wage it pays to the worker. Therefore, the (nominal) profit $\tilde{\Pi}_i^j$ of a job of type i that makes an offer to a worker who has a best rival offer of type j is given by:

$$\tilde{\Pi}_i^j = \max\{p_i - p_j, 0\} - \tilde{r}\kappa_i. \tag{7}$$

Let q_i^j be the probability that a firm earns a profit equal to $\tilde{\Pi}_i^j$. Assuming that both types of firms co-exist, the expected (nominal) profit $\tilde{\Pi}_i$ of a type i firm is given by:

$$\begin{aligned} \tilde{\Pi}_1 &= q_1^0 p_1 - \tilde{r}\kappa_1, \\ \tilde{\Pi}_2 &= q_2^0 p_2 + q_2^1(p_2 - p_1) - \tilde{r}\kappa_2 \end{aligned} \tag{8}$$

The probability that a firm does not face competition from a rival firm of type i is given by $e^{-\phi_i}$. Therefore $q_1^0 = q_2^0 = e^{-\phi_1}e^{-\phi_2}$ is the probability that a firm does not face a rival firm of either type, and $q_2^1 = (1 - e^{-\phi_1})e^{-\phi_2}$ is the probability that an industry 2 firm faces a rival from industry 1 but not from industry 2.

In equilibrium, the supply of jobs of each type and the number of firms are determined by free entry, so the expected profit $\tilde{\Pi}_i$ of offering a vacant job of type i is equal to zero if both types of jobs are operated in equilibrium:

$$\tilde{\Pi}_1 = \tilde{\Pi}_2 = 0. \tag{9}$$

The assumption that the price of a particular type of job is greater than the capital cost of the job does not guarantee that the number of firms of that type is positive. (For example, it is easy to see that $q_1^0 p_1 - \tilde{r}\kappa_1$ can be negative if ϕ_2 is sufficiently large - making q_1^0 sufficiently small.) Therefore, at this stage we do not know, based on our present assumptions, whether or not the two different types of firms will exist in equilibrium, or, in terms of trade theory, whether there is a cone of diversification.

2.6 The Cone of Diversification

Let us now use the definition $p \equiv p_2/p_1$ and express variables in real terms. In a closed economy setting, the model can be closed by assuming a specific preference structure, and since we want to focus on the role that directed search plays in trade, we assume that preferences are homothetic and identical across countries, implying identical expenditure shares for the same relative prices. Let relative demand d_2/d_1 be denoted by $d(p)$ with $d' < 0$. Commodity markets clear in a closed economy for a relative price p such that

$$d(p) = \frac{1 - e^{-\phi_2}}{e^{-\phi_2}(1 - e^{-\phi_1})} \tag{10}$$

where the RHS follows from (4). This can be easily extended to a free trade situation in which two countries, the one under consideration here which we label domestic, and a foreign country, labeled with an asterix, trade with each other. World markets clear if

$$d(p) = d^*(p) = \frac{(1 - e^{-\phi_2}) + (1 - e^{-\phi_2^*})}{(e^{-\phi_2}(1 - e^{-\phi_1})) + (e^{-\phi_2^*}(1 - e^{-\phi_1^*}))}. \tag{11}$$

Both (10) and (11) assume that both countries diversify their production. Let us now scrutinize under which condition both industries are active in creating job vacancies in the domestic country. Assuming interior solutions, we have three equations with three unknowns $\phi_1, \phi_2, r,$ *i.e.*, the two zero profit conditions,

$$\Pi_1 = e^{-\phi_1} e^{-\phi_2} - r\kappa_1 = 0, \tag{12}$$

and

$$\Pi_2 = e^{-\phi_1} e^{-\phi_2} p + (1 - e^{-\phi_1}) e^{-\phi_2} (p - 1) - r\kappa_2 = 0, \tag{13}$$

and the market clearing condition for capital (5). Although we have established that the production possibility frontier is strictly concave, we must still determine under which circumstances this economy will be diversified in equilibrium.

Proposition 1 *A cone of diversification exists where the country creates job vacancies in both sectors.*

Proof: See Appendix A.4.

Appendix A.4 shows that our assumption $\kappa_2 > \kappa_1$ is essential. If, instead, $\kappa_2 < \kappa_1$ while $p > 1$, the cone of diversification would be empty. In what follows, we shall focus on a diversified economy as the results for a completely specialized economy are relatively straightforward.

The production possibility frontier has been shown to have the same features as those which would arise in classic models using well-behaved production functions. We have also shown the equilibrium conditions in this economy and, surprisingly, the economy behaves in the same way as in classic models. This leads us, naturally, to a discussion of efficiency.

2.7 Efficiency

Proposition 2 *The economy behaves such that it maximizes the value of production subject to the production possibility frontier.*

Proof: See Appendix A.5.

This is a remarkable result as it shows that the coordination problem can be completely incorporated into the production possibility frontier. It is by no means a trivial conclusion since there is a friction in the labor market, and hence nothing guarantees efficiency in this setup. Hence, Proposition 2 shows that the result from the directed search literature (that directed search is constrained efficient) also carries over to a general equilibrium setup.

It also has a clear implication for the aggregate welfare effects of trade. Since trade will allow consumptions patterns not feasible under autarky, aggregate welfare will unambiguously rise. We summarize this result in

Corollary 1 *Trade in this model has the same qualitative aggregate welfare effects as in standard models of trade.*

However, as well known from standard models of trade, this does not mean that each individual will benefit from trade. Furthermore, we do not yet know how trade patterns will depend on endowment and technology, and how trade changes job creation and unemployment. We deal with these effects in the next section.

3 The pattern of trade

In this section, we assume that $\phi_1, \phi_2 > 0$, that is, that the economy is not completely specialized but in the cone of diversification. We are first interested how unemployment, expected wages and the rental rate change with prices – a Stolper-Samuelson-like exercise. We then use these results to determine the pattern of trade. As a first step, we are interested in the impact of changing prices on the number of vacancies created in both sectors. Hence, we will deal with these variables as functions of the relative price. The expected wage is given by

$$\begin{aligned} \hat{w} &= [(1-e^{-\phi_1}-\phi_1 e^{-\phi_1})e^{-\phi_2} \\ &\quad +\phi_2 e^{-\phi_2}(1-e^{-\phi_1})] \\ &+ p[(1-e^{-\phi_2}-\phi_2 e^{-\phi_2})] \end{aligned} \tag{14}$$

and follows from (3). The first part of (14) is the probability that at least two firms of industry 1 approach the worker and outbid each other such that the worker receives a unity real wage; and the second term is the probability that only one industry 2 firm outbids an industry 1 firm, leading to the same unity wage. The last term is the probability of the best outcome from the worker's perspective as at least two industry 2 firms outbid each other, leading to a real wage of size p. The unemployment rate is equal to

$$u[\phi_1, \phi_2] = \pi_0^0 = e^{-\phi_1}e^{-\phi_2} \tag{15}$$

and follows immediately from (3). Note that both the unemployment rate and the expected wage depend on the number of vacancies created in both industries. The following lemma summarizes the effects of an increase in the relative price on vacancies and the rental rate.

Lemma 2 *An increase in p leads to less (more) vacancies in sector 1 (2) and an increase in the rental rate.*

Proof: See Appendix A.6.

The change in vacancies is no surprise, and the increase in the rental follows from increasing vacancies in the relatively capital-intensive industry which requires more capital than is given up by the decline in capital needed by the relatively labor-intensive industry. A direct consequence of Lemma 2 is, of course, that the per-capita output of the capital-intensively produced commodity expands, and the per-capita output of the labor-intensively produced commodity shrinks. We shall now consider what a relative price increase means for labor income and the unemployment rate.

Proposition 3 *(i) An increase in p increases unemployment. (ii) The effect of an increase in p on the expected wage is ambiguous.*

Proof: Differentiating (15) yields

$$\frac{\partial u}{\partial p} = e^{-\phi_1} e^{-\phi_2} \frac{e^{-\phi_2}}{r\kappa_2} > 0.$$

To demonstrate the ambiguity of the impact on the expected wage, all that is needed is an example, which is provided in Table 1. □

The intuition for part (i) of Proposition 3 is straightforward. An increase in the relative price makes production in industry 2 more profitable, and consequently more job vacancies are created in industry 2 at the expense of industry 1. However, it is more costly to create vacancies in the expanding industry, and hence, the overall number of vacancies declines, leading to a higher unemployment rate.[10]

In part (ii) of Proposition 3, there are two opposite effects at work: on the one hand, an increase in p will increase the payoff of a (lucky) worker who is

[10] It also follows from eq. (12) that $u = e^{-\phi_1} e^{-\phi_2} = r\kappa_1$ in equilibrium, and hence an increase in the rental rate must go along with an increase in u.

approached by two (or more) firms out of industry 2. In this case, his wage will go up with an increase in p. On the other hand, as can be seen from the change in the unemployment rate, fewer jobs are created overall, so there is a higher unemployment risk. Consequently, the change in the expected wage could go either way. Table 1 summarizes the result of a simulation which clearly demonstrates that the behavior of the expected wage can be non-monotonic. The expected wage $\hat{w}$ declines first with a price increase because the unemployment effect dominates the price effect, but when the relative price increases further, the price effect starts to dominate the unemployment effect.

Table 1: The effect of an increase in p on job creation rates, the unemployment rate, the expected wage and the variance of wages

p	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
ϕ_1	1.6094	1.204	0.9163	0.6931	0.5108	0.3567	0.2231	0.1054
ϕ_2	0.0953	0.298	0.442	0.5534	0.6446	0.7217	0.7884	0.8473
u	0.1818	0.2227	0.2571	0.2875	0.3149	0.3402	0.3636	0.3857
$\hat{w}$	0.5091	0.4538	0.4229	0.4075	0.4033	0.4074	0.4181	0.4343
$Var(w)$	0.2509	0.2621	0.285	0.3216	0.372	0.4358	0.5127	0.6019

Simulation uses $\kappa_1 = 1, \kappa_2 = 2, k = 1.8$.

Table 1 also shows the job creation rates in both industries, and as predicted, ϕ_1 (ϕ_2) decreases (increases) with p. Furthermore, for a sufficiently low p, industry 1 creates more vacancies than workers are on the labor market because production is this industry is very profitable and establishment costs are so low. The numerical example also allows us to consider the degree of wage dispersion as measured by the variance of the wage.[11] We observe

[11] Due to the non-monotonic behavior of the expected wage, we cannot arrive at a general result for the behavior of the wage variance.

in our example that the wage variance goes up with the relative price. The reason is twofold: first, both the probability of receiving a zero wage and of receiving the maximum surplus p increases with p in our simulation, while the probability of receiving a unity wage declines with p. Thus, the "riskier" outcomes become more likely. Second, the relative price increases which increases the variance further. This result indicates that an increase in the relative price may imply more vulnerability with respect to wages. Although it plays no role in our model which assumes risk-neutral agents, we report this result here as an increase in variance would definitely have an impact on welfare if workers were risk-averse instead.

3.1 Differences in Endowment

We also want to trace back labor market tightness to the fundamentals of general equilibrium models. The attraction of this simple model is that we can directly consider how a change in relative capital endowment will affect factor prices and unemployment without having to assume a specific matching function with extra parameters. Proposition 4 shows how the key variables respond to endowment differences.

Proposition 4 *An increase in k (i) reduces the rental r, (ii) has no effect on the job vacancies created in industry 1, (iii) increases the job vacancies in industry 2 and thus (iv) decreases unemployment and increases the expected wage. (iv) Furthermore, the per-capita output of the capital-(labor-) intensively produced commodity increases (declines).*

Proof: See Appendix A.7.

When countries differ by factor endowments only, not by production technologies, this case is similar to the Heckscher-Ohlin analysis of trade patterns, and there are several messages to take away from Proposition 4. First, parts (i) and (iv) clearly demonstrate that this model does not imply factor price equalization. If p is the relative price under free trade and countries do not differ by technology, a capital-abundant country will have a lower rental and a higher expected wage in a free trade equilibrium. Furthermore, it is remarkable that the higher expected wage will not go along with a higher, but a lower unemployment rate. Although the number of vacancies created in industry 1 does not change, its per-capita output changes. The reason is that a larger k increases the number of vacancies in industry 2, and thus decreases the chances of an industry 1 firm to successfully hire a worker.

Proposition 4 allows us to make clear prediction about the pattern of trade. Without loss of generality, let us assume that the domestic country is relatively capital-abundant, so the foreign country is relatively labor-abundant, that is, $k > k^*$. Proposition 4 clearly indicates that the domestic country will be a net exporter of the capital-intensively produced commodity and a net importer of the labor-intensively produced commodity. This prediction is completely in line with classic trade theory results.

Furthermore, it must be true that the domestic country will have a lower relative price under autarky: if the free trade price leads to exports of commodity 2 and imports of commodity 1, the same price under autarky would lead to excess supply (demand) on the market for commodity 2 (1), and thus condition (10) must warrant a lower relative price. Similarly, the foreign country will have a larger relative price under autarky as it is a net exporter (importer) of commodity 1 (2). Again, this result is perfectly in line with the results produced by standard trade models.

Proposition 4 shows us that factor prices do not equalize in equilibrium even if countries do not differ by technologies; hence, trade is not a perfect substitute for factor mobility. The domestic country will have the lower rental, the lower unemployment rate and the higher expected wage. More importantly, we can make a clear prediction about the change in the unemployment rate and the rental when countries move from autarky to trade: according to Lemma 2 and Proposition 3, the capital-abundant country (which experiences a price increase) will face an increase in both the unemployment rate and the rental, whereas the labor-abundant country (which experiences a price decrease) will face a drop in the unemployment rate and a decrease in the rental. However, we know from Proposition 4 that the domestic country will still have the lower unemployment rate and the lower rental; thus there is convergence, but not equalization. As for the expected wage, all we can say is that the expected wage will be larger in the capital-abundant country, but due to Proposition 3 we cannot conclude without making further specific assumptions whether trade will increase or decrease the expected wage. We summarize our findings in the following

Theorem 1 *If countries differ by endowment only, in a directed search environment, free trade leads to: (i) a rise (decline) in both the unemployment rate and the rental in the capital-(labor-) abundant country, but not to an equalization, (ii) a lower unemployment rate and rental and a higher expected wage in the capital-abundant country, (iii) an ambiguous change of the expected wage in each country.*

Theorem 1 is one of our key results, and it clearly demonstrates that trade may have an adverse effect on employment in a capital-abundant country. This may seem surprising, but the comparative advantage of this country makes it specialize towards creating vacancies in the capital-intensive industry, and since these vacancies are more costly, it comes at the price of a higher unemployment rate. However, this rate is still lower than in the other country.

3.2 Differences in Technology

Let us now consider the implications of technological differences on trade patterns, a Ricardian analysis. For this case, we assume that per-capita capital endowments do not differ across countries, that is $k = k^*$. As a first step, we are interested how an increase in capital input requirements, that is a decline in productivity, affects the rental, job creation and unemployment.

Lemma 3 *An increase in κ_1 (i) increases the rental r, (ii) decreases the job vacancies in industry 1, (iii) increases the job vacancies created in industry 2 if $\phi_1 \leq 1$, but has an ambiguous effect on these job vacancies otherwise, and (iv) increases the unemployment rate. An increase in κ_2 (i) decreases the rental r, (ii) decreases the job vacancies in industry 2, (iii) increases the job vacancies created in industry 1 and (iv) decreases the unemployment rate.*

Proof: See Appendix A.8.

Increasing the capital input requirement of industry 1 (2) makes industry 2 (1) relatively more profitable, and since this is the industry requiring more (less) capital, the rental will increase (decrease). As for job creation, the effects of κ_1 and κ_2 are not symmetric: while an increase in κ_i decreases the number of vacancies in industry i, it does not necessarily increase them in the other industry. This is true only for κ_2, but an increase in κ_1 may also reduce the number of vacancies in industry 2. Appendix A.8 shows that this can happen only if industry 1 is a dominant employer such that $\phi_1 > 1$, because it will then reduce job vacancies underproportionately compared to the increase in κ_1.

Interestingly, the effects of these changes on the unemployment rate are unambiguous: it increases if either κ_1 increases or κ_2 declines. Hence, technological progress in industry 2 leads to more unemployment. Of course, this is not a fully-fledged equilibrium analysis as we have kept the relative price constant. We now explore how the relative price is related to the relative efficiency of capital in both industries.

Proposition 5 *If countries differ by technologies only then countries with identical relative capital input requirements κ_2/κ_1 have the same relative autarky price.*

Proof: See Appendix A.8.

Proposition 5 demonstrates that only relative capital efficiencies matter for trade. This is a remarkable result as it is not derived in a Ricardian framework with only one factor of production, but in a two-factor-framework which gives rise to a strictly concave production possibility frontier. Proposition 5 clearly states that countries will trade only if $\kappa_2/\kappa_1 \neq \kappa_2^*/\kappa_1^*$. However, it does not mean that countries with identical relative capital efficiencies are identical in all respects.

Proposition 6 *If countries differ by technologies only and $\kappa_2/\kappa_1 = \kappa_2^*/\kappa_1^*$ but $\kappa_i < \kappa_i^*$, then the domestic country has a lower unemployment rate than the foreign country.*

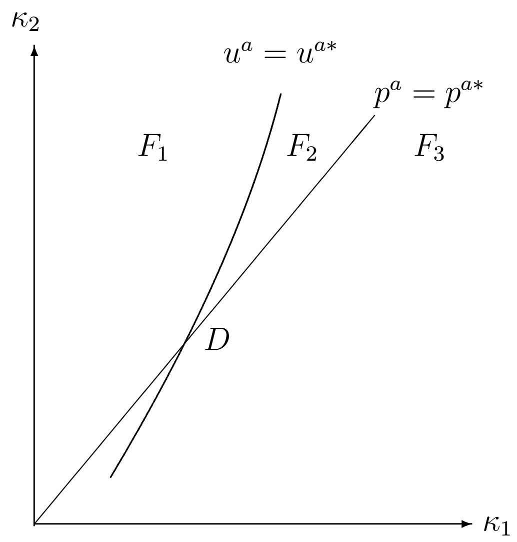

Figure 2: Technological differences

Proof: See Appendix A.8.

Thus, absolute productivity differences are the key to explaining differences in unemployment, and higher productivities imply lower unemployment rates when relative productivities do not differ.

We are now ready to explore the patterns and effects of trade based on productivity differences. These effects can be best demonstrated by Figure 2 which plots two graphs in the $\kappa_2 - \kappa_1$–space. The line $p^a = p^{a*}$ is the line where both autarky prices coincide. The other upward-sloping curve labelled $u^a = u^{a*}$ gives all combinations of κ_2 and κ_1 which leave the domestic unemployment rate unchanged. Points to the left of this curve have lower unemployment rates, and points to the right have higher rates. In autarky, the domestic country D is located exactly at the intersection of these two graphs. In Appendix A.8 we show that the $u^a = u^{a*}$–curve must have a steeper slope at point D which also confirms Proposition 6. If the foreign firm were also located on the $p^a = p^{a*}$–line under autarky, there would be no trade, but differences in unemployment rates.

Without loss of generality, we assume that the foreign country has a lower capital productivity in industry 2 and is thus located to the north of D.

Three different cases can emerge: if located at F_1, the foreign country has the higher relative autarky price but the lower autarky unemployment rate; if located at F_2, it has both the higher relative autarky price and the higher autarky unemployment rate; and if located at F_3, it has the lower relative autarky price but the higher autarky unemployment rate.

Trade creates an integrated market and equalizes the relative price. If D trades with F_1 or F_2, the relative price will increase for the domestic country and decrease for the foreign country,[12] and the domestic country becomes a net importer (exporter) of good 1 (2). If D trades with F_3, the relative price will decrease for the domestic country and increase for the foreign country, and the domestic country becomes a net exporter (importer) of good 1 (2). Of course, the effect on the expected wage is not clear in general due to Proposition 3. However, we have a clear result on how trade changes job creation, unemployment and the rental. It follows immediately from Lemma 2 and Proposition 3.

Theorem 2 *If the foreign country's absolute capital productivity in industry 2 is smaller ($\kappa_2^* > \kappa_2$) and $\kappa_2^*/\kappa_1^* > \kappa_2/\kappa_1$, free trade will: (i) increase (decrease) the domestic (foreign) country's unemployment rate, (ii) lead to less (more) vacancies in sector 1 and more (less) vacancies in sector 2 in the domestic (foreign) country, (iii) increase (decrease) the rental rate in the domestic (foreign) country. Vice versa if $\kappa_2^*/\kappa_1^* < \kappa_2/\kappa_1$.*

Theorem 2 is the other key result of this paper. Countries with a relative capital productivity advantage for producing (the high-price) good 2 experience an increase in the unemployment rate. They shift resources to the high-price sector whereas the other country shifts resources to the low-price sector. Note carefully that we cannot unambiguously determine which country will have the lower unemployment rate if the foreign country was located at F_2 in Figure 2 under autarky. In this case, the foreign country had the higher unemployment rate under autarky. If the foreign country is located at

[12] Price changes can easily be demonstrated by assuming that the world market price is equal to $p^a = p^{a*}$ which creates excess supply and demand.

F_1 in Figure 2 under autarky, however, it had the lower unemployment rate which trade will reduce further, and the opposite is true for the domestic country such that unemployment rates will further diverge with trade.

4 Concluding remarks

This paper has developed a simple and basic model of unemployment and trade in a general equilibrium model. Although Ricardian in terms of input requirements, diversification may occur in our model. Furthermore, we could clearly work out a prediction how trade affects unemployment rates. If countries differ by endowment, the capital-abundant country will still have the lower rate, but it comes closer to the lowered rate of the labor-abundant country. The change in the rental is similar to the Stolper-Samuelson prediction, but without further assumptions, we cannot tell whether trade will increase or decrease the expected (average) wage. However, the capital-abundant country will enjoy the higher wage. If countries differ by technology, the country with a larger relative productivity for producing the capital-intensive good will experience an increase in unemployment.

The model in this paper is, intentionally, very simple. As such, its properties are relatively easy to characterize, and it delivers relatively stark results. How do our results compare to trade models which employ a matching technology? First, we do not have to rely on a certain functional specification, but our results are the outcome of a micro-founded auction game. Therefore, we can trace back all results to the fundamentals of standard models of trade. Second, our model can produce wage dispersion although workers are all homogeneous in ability. There is an ongoing debate about the cause of wage inequality caused by trade both in the theoretical and the empirical literature (see, for example, Menehes-Filho, Muendler and Ramey, 2008). Our model makes clear that part of the wage dispersion simply follows from frictions in the labor market, and different abilities among workers are not necessary to observe wage dispersion. Third, we feel that it is changes in the patterns of intersectoral trade rather than intra-industry trade which are at the heart of the policy debate. Workers in Western countries are worried about countries like China and India integrating into the world economy, so intra-industry trade models assuming similar or even identical countries may not be able to address their issues. Our paper, however, offers a missing link to rationalize the effects of fundamental changes in intersectoral trade on wages and unemployment.

Of course, our model is still too simple to draw many policy conclusions. However, we could show that the aggregate welfare effects of trade are qualitatively the same as in standard models of trade. Furthermore, we have shown that trade does not equalize factor prices. Therefore, a labor-abundant country will still have a higher rental. If it considers the introduction of a tariff on capital-intensive imports, the expected wage may go up, along with a decrease in the wage dispersion as shown by our simulation. Also, the rental rate will increase, and this effect may be welcome from a dynamic perspective: it will increase the country's capital stock which may be a long-run development goal.[13] However, the incentive to restrict imports is weaker in our model compared to models which tend to lead to more convergence of factor prices. Even without technological differences, a larger per-capita investment can be expected in the labor-abundant country. Hence, there may be no conflict between trade liberalization and development goals.

Of course, a thorough analysis of changes in capital stocks requires an extension of the model. Given this background, several straightforward extensions come immediately to mind. First, a dynamic version of the model, where workers and firms have opportunities to match beyond the single period considered here, would allow us to generate more plausible unemployment rates, and bring the model closer to empirical relevance more generally. Second, it would be interesting to allow savings and growth in this model, to bring the model closer to macroeconomic models of dynamic general equilibrium. Overall, however, we expect that the key results in this paper will remain pertinent in any of these extensions.

[13] Starting with the famous Singer-Prebisch thesis, there is a large literature on the role import protection and substitution can or cannot play for development goals. For a summary, see Bruton (1998).

Appendix

A.1 Derivation of eq. (2)

The derivation and proof of existence of the mixed strategy equilibrium follows Julien, Kennes and King (2000, 2006). Suppose that there are M_i firms out of industry i creating vacancies and L workers, then the number of matches, denoted by x_i, when each firm approaches each worker with probability ρ, is given by

$$x_i = \sum_{i=1}^{L} \left(1 - (1-\rho)^{M_i}\right) = L\left(1 - (1-\rho)^{M_i}\right) \tag{A.1}$$

Note that in a symmetric equilibrium, with identical workers, the probability ρ is the same across workers, and thus $\rho = 1/L$.[14] Division by L and replacing M_i by $\phi_i L$ leads to

$$\frac{x_i}{L} = 1 - (1-\rho)^{\phi_i L} = 1 - \left(\left(1 - \frac{1}{L}\right)^L\right)^{\phi_i}. \tag{A.2}$$

Note that x_i/L is number of matches per capita, and thus it gives the probability of a match with a firm of industry i. We are interested in the case in which the number of workers is large, and thus we consider the limit of L becoming infinitely large:

$$\begin{aligned} \lim_{L\to\infty} \frac{x_i}{L} &= \lim_{L\to\infty} \left(1 - \left(\left(1 - \frac{1}{L}\right)^L\right)^{\phi_i}\right) \\ &= 1 - \left(\lim_{N\to\infty} \left(1 - \frac{1}{L}\right)^L\right)^{\phi_i} \\ &= 1 - e^{\phi_i}. \end{aligned} \tag{A.3}$$

Eq. (2) follows from combining all possibilities for the two industries.

[14] To be clear, symmetry of the equilibrium implies that all vacancies apply the same strategy which, in the case of heterogenous workers, does not imply that all workers are assigned the same probability. In general, workers with higher productivities are assigned higher probabilities by vacancies. In this model, however, all workers are identical, and the equilibrium probabilities are therefore the same (see Basov, King and Uren, 2010, for details).

A.2 Proof of Lemma 1

The per-capita production levels are equal to $y_1 = e^{-\phi_2}(1 - e^{-\phi_1})$ and $y_2 = 1 - e^{-\phi_2}$, respectively. Using (5), we can write y_1 as a function of y_2:

$$y_1(y_2) = \left(1 - e^{-\frac{k+\ln(1-y_2)\kappa_2}{\kappa_1}}\right)(1 - y_2), \tag{A.4}$$

and the next two derivatives are

$$\frac{dy_1}{dy_2} = -1 - \frac{\kappa_2 - \kappa_1}{\kappa_1} e^{-\frac{k}{\kappa_1}} (1 - y_2)^{-\frac{\kappa_2}{\kappa_1}} < 0, \tag{A.5}$$

$$\frac{d^2 y_1}{dy_2^2} = -\frac{\kappa_2 - \kappa_1}{\kappa_1^2} e^{-\frac{k}{\kappa_1}} (1 - y_2)^{-1-\frac{\kappa_2}{\kappa_1}} < 0. \tag{A.6}$$

A.3 The economy without frictions

If the labor market is frictionless, and therefore there is no unemployment and a common wage determined by labor supply and demand only, we have two zero profit conditions $\Pi_1 = p_1 - \tilde{w} - \tilde{r}\kappa_1 = 0, \Pi_2 = p_2 - \tilde{w} - \tilde{r}\kappa_2 = 0$ and – since $\phi_i = y_i$ without unemployment – two factor market clearance conditions $y_1\kappa_1 + y_2\kappa_2 k, y_1 + y_2 = 1$. Solving the zero-profit conditions and the two factor market clearing conditions yield (6) and factor prices which are determined by commodity prices only:

$$w = \frac{\kappa_2 - p\kappa_1}{\kappa_2 - \kappa_1}, \qquad r = \frac{p - 1}{\kappa_2 - \kappa_1},$$
$$\frac{\partial w}{\partial p} = -\frac{\kappa_1}{\kappa_2 - \kappa_1} < 0, \qquad \frac{\partial r}{\partial p} = \frac{1}{\kappa_2 - \kappa_1} > 0.$$

The derivatives show Stolper-Samuelson-like results; the wage (rental) decreases (increases) with an increase in the relative price of the capital-intensively produced commodity. Diversification requires that $p < \kappa_2/\kappa_1 \equiv \overline{p}$. If this were not true, and hence the price of commodity 2 is relatively large, the economy would produce commodity 2 only and factor prices would be determined only by the technology of industry 2. In case of diversification, production structures depend only on endowment as can be seen from the differentiating per-capita production patters (6) w.r.t. per-capita factor endowment:

$$\frac{\partial y_1}{\partial k} = -\frac{1}{\kappa_2 - \kappa_1} < 0, \frac{\partial y_2}{\partial k} = \frac{1}{\kappa_2 - \kappa_1} > 0. \tag{A.7}$$

The derivatives show Rybczynski-like results: the per-capita-output of the capital-(labor-)intensively produced commodity increases (decreases) with the per-capita capital endowment. Note that diversification requires $\kappa_2 > k > \kappa_1$, which is fulfilled due to our assumption that capital is so abundant that full employment is technically feasible.

A.4 Proof of Proposition 1

Eqs. (12) and (13) hold only true if vacancies are created in both sectors. If $\Pi_1 < 0$ for $\phi_1 = 0$, only jobs in sector 2 will be created. In this case, the equilibrium number of vacancies in sector 2 and the equilibrium rental solve

$$\Pi_2 = e^{-\phi_2} p - r\kappa_2 = 0 \tag{A.8}$$

and

$$\phi_2 \kappa_2 - k = 0. \tag{A.9}$$

The equilibrium values are

$$\phi_2 = \frac{k}{\kappa_2} < 1, r = \frac{e^{-\frac{k}{\kappa_2}}}{\kappa_2}. \tag{A.10}$$

Expression (A.10) is true only if

$$\Pi_1 = e^{-\phi_2} - r\kappa_1 = \frac{e^{-\frac{k}{\kappa_2}}}{\kappa_2}(\kappa_2 - \kappa_1 p) < 0$$

which shows that the country will specialize completely in the production of commodity 2 iff

$$p > \frac{\kappa_2}{\kappa_1} \equiv \overline{p}.$$

If $\Pi_2 < 0$ for $\phi_2 = 0$, only jobs in sector 1 will be created. The equilibrium number of vacancies and the equilibrium rental solve

$$\Pi_1 = e^{-\phi_1} - r\kappa_1 = 0 \tag{A.11}$$

and

$$\phi_1 \kappa_1 - k = 0. \tag{A.12}$$

The equilibrium values are

$$\phi_1 = \frac{k}{\kappa_1} > 1, r = \frac{e^{-\frac{k}{\kappa_1}}}{\kappa_1}. \tag{A.13}$$

Expression (A.13) is true only if

$$\Pi_2 = e^{-\phi_1} p + (1 - e^{-\phi_1})(p - 1) - r\kappa_2 = p - \left(1 + \frac{e^{-\frac{k}{\kappa_1}}(\kappa_2 - \kappa_1)}{\kappa_1}\right) < 0$$

which shows that the country will specialize completely in the production of commodity 1 iff

$$p < 1 + \frac{e^{-\frac{k}{\kappa_1}}(\kappa_2 - \kappa_1)}{\kappa_1} \equiv \underline{p}.$$

Given these results, we can now identify the conditions under which diversification is obtained in equilibrium. It happens if $p \in [\underline{p}, \overline{p}]$, and the cone of diversification is non-empty if $\overline{p} - \underline{p} > 0$ which is true because $\kappa_2 > \kappa_1$:

$$\overline{p} - \underline{p} = \frac{e^{-\frac{k}{\kappa_1}}\left(e^{\frac{k}{\kappa_1}} - 1\right)(\kappa_2 - \kappa_1)}{\kappa_1} > 0.$$

A.5 Proof of Proposition 2

We can rewrite the two zero profit conditions (12) and (13) such that

$$\begin{aligned} e^{-\phi_1} e^{-\phi_2} &= r\kappa_1, \\ e^{-\phi_2} p - e^{-\phi_2} + e^{-\phi_1} e^{-\phi_2} &= r\kappa_2, \end{aligned}$$

and division of these two equations eliminates the rental and leads us to

$$\frac{e^{-\phi_2} p - e^{-\phi_2} + e^{-\phi_1} e^{-\phi_2}}{e^{-\phi_1} e^{-\phi_2}} = \frac{\kappa_2}{\kappa_1} \Leftrightarrow p = 1 - e^{-\phi_1} + e^{-\phi_1} \frac{\kappa_2}{\kappa_1}. \qquad \text{(A.14)}$$

Furthermore, we can rewrite (A.5) such that

$$\begin{aligned} \frac{dy_1}{dy_2} &= -1 - \frac{\kappa_2 - \kappa_1}{\kappa_1} e^{-\frac{k}{\kappa_1}} \left(e^{-\phi_2}\right)^{-\frac{\kappa_2}{\kappa_1}} \\ &= -1 - \frac{\kappa_2 - \kappa_1}{\kappa_1} \underbrace{e^{-\frac{k}{\kappa_1} + \phi_2 \frac{\kappa_2}{\kappa_1}}}_{= e^{-\phi_1}} \\ &= -1 + e^{-\phi_1} - e^{-\phi_1} \frac{\kappa_2}{\kappa_1} = -p \end{aligned}$$

according to (A.14) which proves that the production structure is determined such that the slope of the production possibility frontier is tangential to the relative price. Therefore, the economy behaves such that it maximizes real production $y_1 + py_2$ subject to the production possibility frontier.

A.6 Proof of Lemma 2

Totally differentiating (12), (13) and (5) yields

$$\underbrace{\begin{bmatrix} -r\kappa_1 & -r\kappa_1 & -\kappa_1 \\ -r\kappa_1 & -r\kappa_2 & -\kappa_2 \\ \kappa_1 & \kappa_2 & 0 \end{bmatrix}}_{A} \begin{bmatrix} d\phi_1 \\ d\phi_2 \\ dr \end{bmatrix} = \begin{bmatrix} 0 \\ -e^{-\phi_2} \\ 0 \end{bmatrix} dp$$

where

$$\det A = -r\kappa_1\kappa_2(\kappa_2 - \kappa_1) < 0. \tag{A.15}$$

Thus, we find:

$$\frac{\partial \phi_1}{\partial p} = -\frac{e^{-\phi_2}}{r(\kappa_2 - \kappa_1)} < 0, \tag{A.16}$$

$$\frac{\partial \phi_2}{\partial p} = \frac{e^{-\phi_2}\kappa_1}{r\kappa_2(\kappa_2 - \kappa_1)} > 0, \tag{A.17}$$

$$\frac{\partial r}{\partial p} = \frac{e^{-\phi_2}}{\kappa_2} > 0. \tag{A.18}$$

A.7 Proof of Proposition 4

We find that

$$\frac{\partial \phi_1}{\partial k} = 0, \tag{A.19}$$

$$\frac{\partial \phi_2}{\partial k} = \frac{1}{\kappa_2} > 0, \tag{A.20}$$

$$\frac{\partial r}{\partial k} = -\frac{r}{\kappa_2} < 0, \tag{A.21}$$

and all other results follow from these derivatives because $\partial w/\partial \phi_2 > 0$ and $\partial u/\partial \phi_2 < 0$.

A.8 Proof of Propositions 5 and 6

Using the change in vacancies for the change in the unemployment rate, we find that

$$\begin{aligned}
\frac{\partial \phi_1}{\partial \kappa_1} &= -\frac{\kappa_2}{\kappa_1(\kappa_2-\kappa_1)} < 0, && \text{(A.22)}\\
\frac{\partial \phi_2}{\partial \kappa_1} &= \frac{1}{\kappa_2-\kappa_1} - \frac{\phi_1}{\kappa_2}, && \text{(A.23)}\\
\frac{\partial r}{\partial \kappa_1} &= \frac{r\phi_1}{\kappa_2} > 0, && \text{(A.24)}\\
\frac{\partial u}{\partial \kappa_1} &= \left(\frac{1}{\kappa_1} + \frac{\phi_1}{\kappa_2}\right) u > 0 && \text{(A.25)}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \phi_1}{\partial \kappa_2} &= \frac{1}{\kappa_2-\kappa_1} > 0, && \text{(A.26)}\\
\frac{\partial \phi_2}{\partial \kappa_2} &= -\frac{\kappa_1}{\kappa_2(\kappa_2-\kappa_1)} - \frac{\phi_2}{\kappa_2} < 0, && \text{(A.27)}\\
\frac{\partial r}{\partial \kappa_2} &= -\frac{(1-\phi_2)r}{\kappa_2} < 0, && \text{(A.28)}\\
\frac{\partial u}{\partial \kappa_2} &= \left(\frac{1-\phi_2}{\kappa_2}\right) u > 0 && \text{(A.29)}
\end{aligned}$$

because $\phi_2 < 1$.

Consider an economy which produces at a point tangential to the production possibility frontier according to (A.14). Let us consider how

$$1 - e^{-\phi_1} + e^{-\phi_1}\frac{\kappa_2}{\kappa_1} \tag{A.30}$$

changes with a technology change which keeps κ_2/κ_1 constant. A constant κ_2/κ_1 warrants

$$\frac{d\kappa_2}{d\kappa_1} = \frac{\kappa_2}{\kappa_1}$$

Since expression (A.30) depends on ϕ_1 only and κ_2/κ_1 is constant, we only have consider the change of ϕ_1 when both κ_1 and κ_2 change but keep κ_2/κ_1 constant. We find that

$$\begin{aligned}
\frac{d\phi_1}{d\kappa_1} &= \frac{\partial \phi_1}{\partial \kappa_1} + \frac{\partial \phi_1}{\partial \kappa_2}\frac{d\kappa_2}{d\kappa_1}\\
&= -\frac{\kappa_2}{\kappa_1(\kappa_2-\kappa_1)} + \frac{1}{\kappa_2-\kappa_1}\frac{\kappa_2}{\kappa_1} = 0,
\end{aligned}$$

and thus expression (A.30) does not change when κ_2/κ_1 stay constant. Therefore, countries with an identical κ_2/κ_1–ratio must have the same autarky relative price. From (A.30), we can also observe that any larger (but constant) κ_2/κ_1 must imply a higher relative price.

Totally differentiating the unemployment rate leads to

$$\frac{du}{d\kappa_1} = -u\left(\frac{d\phi_1}{d\kappa_1} + \frac{d\phi_2}{d\kappa_1}\right)$$

where

$$\begin{aligned}\frac{d\phi_1}{d\kappa_1} &= \frac{\partial\phi_1}{\partial\kappa_1} + \frac{\partial\phi_2}{\partial\kappa_2}\frac{d\kappa_2}{d\kappa_1} \\ \frac{d\phi_2}{d\kappa_1} &= \frac{\partial\phi_2}{\partial\kappa_1} + \frac{\partial\phi_2}{\partial\kappa_2}\frac{d\kappa_2}{d\kappa_1}.\end{aligned}$$

We now ask how $d\kappa_2/d\kappa_1$ must behave such that $du/d\kappa_1 = 0$ which warrants $d\phi_1/d\kappa_1 + d\phi_2/d\kappa_1 = 0$ and implies

$$\frac{d\kappa_2}{d\kappa_1} = \frac{\kappa_2}{\kappa_1}\frac{\kappa_2}{(\kappa_2-\kappa_1)(1-\phi_2)} + \frac{\phi_1}{1-\phi_2} \tag{A.31}$$

Expression (A.31) is clearly larger than κ_2/κ_1. This proves that the $u^a = u^{a*}$–curve has a steeper slope than the κ_2/κ_1–line at point D in Figure 2.

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