On Robust Estimation in Bühlmann-Straub's Credibility Model

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Abstract

The presence of outliers due to large claims or catastrophic events is a special problem in ratemaking and tariff calculations. In particular, excess claims lead to an unsatisfactory behavior of classical linear credibility estimators.

Our purpose is to robustify credibility premium calculations using robust mean and variance estimators. For instance, truncation of claims with a truncation point depending on the data is used for the mean estimator.

Our model is an extention of Künsch (1992) to the case where contract weights are allowed to vary in time. We re-visit the solutions proposed by Kremer (1991) and Gisler & Reinhard (1993) for that case by focusing on the bias-treatment alternatives available here.

Keywords: Robust credibility; M-estimators; Influence function.

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1 Introduction

Künsch (1992) is a first formal attempt to robustify the classical linear credibility estimator in Bühlmann's model [c.f. Bühlmann (1967)]. He considers K contracts and n years of experience, denoting the usual unobservable risk parameters by Θ_j and claims sizes by $X_{ij} \geq 0$, for $1 \leq i \leq n$ and $1 \leq j \leq K$. A derivation of a robust credibility estimator, based on M-estimation, is given. Empirical credibility estimators are briefly discussed.

Gisler & Reinhard (1993) propose a model that applies to Swiss law for premium calculations and is based on a method developed by Ammeter (1982). Their assumptions are as in the Bühlmann & Straub (1970) model, where the X_{ij} are assumed to represent loss ratios with corresponding natural weights w_{ij}. They separate the expected loss ratio into a term explaining ordinary losses and another for excess losses, estimating each component separately, i.e. $\mu(\Theta_j) = \mu_0(\Theta_j) + \mu_{xs}(\Theta_j)$ where $\mu(\Theta_j) = E(X_{ij}|\Theta_j)$. The ordinary part $\mu_0(\Theta_j)$ is the conditional expected loss ratio, given Θ_j, generated by ordinary losses, while the xs-part $\mu_{xs}(\Theta_j)$ is the additional conditional expected loss ratio generated by the extraordinarily large claims (for instance due to catastrofies). A robust M-estimator, T_j, is used for $\mu_0(\Theta_j)$. For the xs-part, $\mu_{xs}(\Theta_j)$, they assume that the actuary knows, a priori, to what extent the risks in the portfolio are exposed to outlier events and use this knowledge in establishing the rate.

Kremer (1991) also proposes a robustification of the linear credibility estimator of Bühlmann & Straub (1970) and of Hachemeister's (1975) regression credibility model.

We extend here the study of Künsch (1992) to robustify Bühlmann-Straub's linear credibility estimator, to the case where the weight of a contract is allowed to vary in time. We propose two approaches, Portfolio-Unbiased and Pure Robust credibility estimation. These differ from the method of Gisler & Reinhard (1993) as large losses are not treated separately in the estimation procedure. They also differ from the proposal of Kremer(1991) which uses M-estimation to robustify classical credibility estimators, but produces robust credibility estimators that are portfolio-biased.

2 Robust Portfolio-Unbiased Credibility Estimation

The model proposed here is an extention of Künsch (1992) model to the case where the weight of a contract is allowed to vary in time. Our approach is to robustify the individual premium estimation by using a robust location estimator instead of a weighted average.

Consider Künsch's assumptions:

(i) The contracts $j = 1, \ldots, K$ [i.e. the pairs $(\Theta_j, \underline{X}_j)$] are idependent, and the variables Θ_j's are identically distributed according to U,

(ii) Given Θ_j, the observations $X_{1j}, \ldots, X_{nj}$ are conditionally independent,

(iii) $E(X_{ij}|\Theta_j) = \mu(\Theta_j)$ for all $j = 1, \ldots, K$ and $i = 1, \ldots, n$.

(iv) $\mathrm{Cov}(X_{rj}, X_{ij}|\Theta_j) = \delta_{ri}\frac{1}{w_{ij}}\sigma^2(\Theta_j)$ where w_{ij} are known weights for $j = 1, \ldots, K$, $i = 1, \ldots, n$ and δ_{ri} denotes Kronecker's symbol.

He distinguishes the following two situations:

Case I : U and the conditional distribution of the X_{ij}'s, given Θ_j, are known. In this case a single contract is sufficient, and j can be dropped.

Case II : U and the conditional distribution of the X_{ij}'s, given Θ_j, are unknown.

To estimate $\mu(\Theta_j) = E(X_{ij}|\Theta_j)$ we follow Künsch's approach and propose the following portfolio-unbiased credibility estimator

$$\hat{\mu}_j = \mu + Z_j(T_{Wj} - \mu_T) \ , \tag{1}$$

where $\mu = E[\mu(\Theta_j)]$ and $\mu_T = E(T_{Wj})$ and the robust weighted estimator T_{Wj} is defined implicity as a solution of

$$\frac{1}{w_{.j}} \sum_{i=1}^{n} w_{ij}\chi\left(\frac{X_{ij}}{T_{Wj}}\right) = 0 \ , \tag{2}$$

with $\chi(z) = \max[-c_1, \min(z-1, c_2)]$, $0 < c_1 \leq 1, 0 < c_2$ and $w_{.j} = \sum_i w_{ij}$. The above definition assumes that the outlier contamination (here large claims), if any, only appears in the values X_{ij}. Otherwise, when the weights are also affected by outliers, then T_{Wj} could be defined implicity as a solution of

$$\sum_{i=1}^{n} \chi(\frac{w_{ij}}{w_{\cdot j}} \frac{X_{ij}}{T_{Wj}}) = 0 \ . \tag{3}$$

Now, an alternative formulation of (2) is obtained by rewriting it as

$$\frac{1}{w_{\cdot j}} \sum_{i=1}^{n} w_{ij} \tilde{\chi}(\frac{X_{ij}}{T_{Wj}}) = 1 \ , \tag{4}$$

with

$$\tilde{\chi}(z) = \max[1 - c_1, \min(z, 1 + c_2)] = \chi(z) + 1 \ . \tag{5}$$

This suggests an iterative algorithmic formulation similar to that of Huber (1981) [c.f. section 8.6]:

$$T_{Wj}^{(m+1)} = \left\{ \frac{1}{w_{\cdot j}} \sum_{i=1}^{n} w_{ij} \tilde{\chi}(\frac{X_{ij}}{T_{Wj}^{(m)}}) \right\}^{\frac{1}{2}} T_{Wj}^{(m)}, \ \text{ for } \ m \geq 0 \ , \tag{6}$$

with $T_{Wj}^{(0)} = \text{median}(X_{1j}, \ldots, X_{nj})$. A proof of convergence is given by Huber.

Finally in terms of $\tilde{\chi}$ we re-write (4) by inserting (5)

$$\frac{1}{w_{\cdot j}} \sum_{i=1}^{n} w_{ij} \max[1 - c_1, \min(\frac{X_{ij}}{T_{Wj}}, 1 + c_2)] = 1 \ . \tag{7}$$

Implicitely this defines the estimator as

$$T_{Wj} = \frac{1}{w_{\cdot j}} \sum_{i=1}^{n} w_{ij} \max\{(1 - c_1) T_{Wj}, \min[X_{ij}, (1 + c_2) T_{Wj}]\} \ , \tag{8}$$

which suggests the notation $T_{Wj} = \frac{1}{w_{\cdot j}} \sum_{i=1}^{n} w_{ij} T_{ij}$, for

$$T_{ij} = \max\{(1 - c_1) T_{Wj}, \min[X_{ij}, (1 + c_2) T_{Wj}]\} \ . \tag{9}$$

Unlike X_{ij}, here the T_{ij} are not conditionally independent given Θ_j. But since $V(X_{ij}|\Theta_j) = \frac{1}{w_{ij}} \sigma^2(\Theta_j)$ we can use [see Gisler & Reinhard (1993)] that $V(T_{ij}|\Theta_j) \simeq \frac{1}{w_{ij}} \sigma_T^2(\Theta_j)$, where $\sigma_T^2(\Theta_j)$ is the asymptotic conditional variance of T_{ij} and $E(T_{ij}|\Theta_j) \simeq \mu_{T_j}(\Theta_j)$, for all i, j, is its asymptotic conditional expectation. We also need the following asymptotic notations

$$E(T_{ij}) = \mu_T \ , \tag{10}$$

$$a_T = V[E(T_{ij}|\Theta_j)] \ , \tag{11}$$

$$s_T^2 = E[\sigma_T^2(\Theta_j)] \ . \tag{12}$$

Lemma 2.1 *Asymptotic Covariance relations:* for $i, r = 1, \ldots, n$ and $j, l = 1, \ldots, K$

$$\begin{aligned} \mathrm{Cov}[\mu_T(\Theta_j), T_{il}] &= \delta_{jl} a_T \ , \quad (13) \\ \mathrm{Cov}[T_{rj}, T_{il}] &\simeq \begin{cases} 0 & \text{for } j \neq l \ , \\ a_T + \delta_{ri} \frac{s_T^2}{w_{rj}} & \text{for } j = l \ , \end{cases} \quad (14) \end{aligned}$$

and

$$a_{\mu T} = \mathrm{Cov}[\mu(\Theta_j), T_{ij}] = \mathrm{Cov}[\mu(\Theta_j), \mu_T(\Theta_j)] \ . \quad (15)$$

Proof: (13) follows immediately by writing

$$\mathrm{Cov}[\mu_T(\Theta_j), T_{il}] = E\{\mathrm{Cov}[\mu_T(\Theta_j), T_{il}|\Theta_j]\} + \mathrm{Cov}\{E[\mu_T(\Theta_j)|\Theta_j], E(T_{il}|\Theta_j)\} \ ,$$

and

$$E(T_{il}|\Theta_j) = \begin{cases} \mu_T(\Theta_j) & \text{if } l = j \ , \\ \mu_T & \text{otherwise.} \end{cases}$$

Now

$$\begin{aligned} \mathrm{Cov}(T_{rj}, T_{ij}) &= E\{\mathrm{Cov}(T_{rj}, T_{ij}|\Theta_j)\} + \mathrm{Cov}\{E(T_{rj}|\Theta_j), E(T_{ij}|\Theta_j)\} \quad (16) \\ &\begin{cases} \simeq E[\sigma_T^2(\Theta_j)] \frac{1}{w_{rj}} + a_T & \text{if } i = r \ , \\ = a_T & \text{if } i \neq r \ , \end{cases} \quad (17) \end{aligned}$$

which proves (14). Finally (15) is derived similarly:

$$\begin{aligned} \mathrm{Cov}[\mu(\Theta_j), T_{ij}] &= E\{\mathrm{Cov}[\mu(\Theta_j), T_{ij}|\Theta_j]\} + \mathrm{Cov}\{E[\mu(\Theta_j)|\Theta_j], E(T_{ij}|\Theta_j)\} \\ &= \mathrm{Cov}[\mu(\Theta_j), \mu_T(\Theta_j)] \ . \quad (18) \end{aligned}$$

□

Using a quadratic loss function

$$Q_j = E\{[\mu(\Theta_j) - c_0^j - \sum_{l=1}^{K} \sum_{i=1}^{n} c_{il}^j T_{il}]^2\} \ , \quad (19)$$

we can, as in Bühlmann-Straub's case, find the optimal $c_0^j, c_{11}^j, \ldots, c_{nK}^j$ that minimize Q_j.

Theorem 2.1 Under the above assumptions, the optimal linearized non-homogeneous robust portfolio-unbiased credibility estimator for $\mu(\Theta_j)$ is :

$$\hat{\mu}_j = \mu + Z_j(T_{Wj} - \mu_T) \tag{20}$$

where T_{Wj} is the individual robust estimator for $\mu(\Theta_j)$, and the corresponding credibility factor for contract j is given by

$$Z_j = \frac{a_{\mu T} w_{.j}}{(a_T w_{.j} + s_T^2)} \quad . \tag{21}$$

Proof: Taking the derivative with respect to c_0^j and $c_{r'i'}^j$ and setting to 0 gives the following normal equations:

$$c_0^j = \mu - \sum_{l=1}^{K} \sum_{i=1}^{n} c_{il}^j \mu_T \tag{22}$$

and for $i' = 1, \ldots, n$ and $l' = 1, \ldots, K$ fixed,

$$\mathrm{Cov}[\mu(\Theta_j), T_{i'l'}] = \sum_{l=1}^{K} \sum_{i=1}^{n} c_{il}^j \mathrm{Cov}(T_{il}, T_{i'l'}) \ . \tag{23}$$

Now for i' and l' fixed,

$$\begin{aligned} \sum_{l=1}^{K} \sum_{i=1}^{n} c_{il}^j \mathrm{Cov}(T_{il}, T_{i'l'}) &= \sum_{i=1}^{n} c_{il'}^j \mathrm{Cov}(T_{il'}, T_{i'l'}) \ , \\ &= a_T \sum_{i=1}^{n} c_{il'}^j + c_{i'l'}^j (\frac{s_T^2}{w_{i'j}}) \ , \end{aligned} \tag{24}$$

since $\mathrm{Cov}(T_{il}, T_{i'l'}) = 0$ if $l \neq l'$ and $\mathrm{Cov}(T_{il'}, T_{i'l'}) = 0$ if $i \neq i'$.

In the special case where $j = l'$ then (24) becomes

$$\sum_{l=1}^{K} \sum_{i=1}^{n} c_{il}^j \mathrm{Cov}(T_{il}, T_{i'l'}) = a_T \sum_{i=1}^{n} c_{ij}^j + c_{i'j}^j (\frac{s_T^2}{w_{i'j}}) \ , \tag{25}$$

and together with (15) this simplifies (23) into

$$a_{\mu T} = a_T \sum_{i=1}^{n} c_{ij}^j + c_{i'j}^j (\frac{s_T^2}{w_{i'j}}) \ , \tag{26}$$

or equivalently,

$$a_{\mu T} - a_T c^j_{.j} - c^j_{i'j}(\frac{s_T^2}{w_{i'j}}) = 0 \ . \tag{27}$$

Multiplying (27) by $w_{i'j}$ and taking summations

$$c^j_{.j} = \frac{a_{\mu T} w_{.j}}{a_T w_{.j} + s_T^2} = Z_j \ . \tag{28}$$

Now by (27) and (28) we can solve for c^j_{ij} to get

$$\begin{aligned} c^j_{i'j} &= \frac{a_{\mu T} w_{i'j} - a_T \left(\frac{a_{\mu T} w_{.j}}{a_T w_{.j} + s_T^2} \right) w_{i'j}}{s_T^2} \ , \\ &= \left(\frac{a_{\mu T}}{a_T w_{.j} + s_T^2} \right) w_{i'j} = \frac{w_{i'j}}{w_{.j}} Z_j \ . \end{aligned} \tag{29}$$

If $j \neq l'$ then from (24) we have

$$a_T c^j_{.l'} + c^j_{i'l'}(\frac{s_T^2}{w_{i'j}}) = 0 \ , \tag{30}$$

which holds if and only if $c^j_{.l'} = 0$. Therefore the credibility estimator of $\mu(\Theta_j)$ is

$$\begin{aligned} \hat{\mu}_j &= \mu - \sum_{l=1}^{K} \sum_{i=1}^{n} c^j_{il} \mu_T + \sum_{l=1}^{K} \sum_{i=1}^{n} c^j_{il} T_{il}; \ , \\ &= \mu - \mu_T Z_j + \sum_{i=1}^{n} \frac{w_{ij}}{w_{.j}} Z_j T_{ij} = \mu + Z_j (T_{Wj} - \mu_T) \end{aligned} \tag{31}$$

□

Now by (15) we have $\mathrm{Cov}[\mu(\Theta_j), T_{ij}] = \mathrm{Cov}[\mu(\Theta_j), \mu_T(\Theta_j)]$ and

$$\mathrm{Cov}(X_{Wj}, T_{Wj}) = E[\mathrm{Cov}(X_{Wj}, T_{Wj} | \Theta_j)] + \mathrm{Cov}[\mu(\Theta_j), \mu_T(\Theta_j)] \ . \tag{32}$$

This result and Von Mises linearization of M-estimators [see Huber (1981)], suggests the following estimator for $a_{\mu T} = \mathrm{Cov}[\mu(\Theta_j), \mu_T(\Theta_j)]$

$$\begin{aligned} \hat{a}_{\mu T} &= \frac{w_{..}}{w_{..}^2 - \sum_{j=1}^{K} w_{.j}^2} \{ \sum_{j=1}^{K} w_{.j} (T_{Wj} - T_{WW})(X_{Wj} - X_{WW}) - \\ &\quad \frac{(K-1)}{K(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{K} w_{ij}^{1/2} \widehat{IF}(X_{ij}, T_{Wj})(X_{ij} - X_{Wj}) \} \ . \end{aligned} \tag{33}$$

with the weighted empirical influence function defined as

$$\widehat{IF}(X_{ij},T_{Wj}) = \frac{w_{ij}^{1/2}\chi(\frac{X_{ij}}{T_{Wj}})}{\frac{1}{T_{Wj}^2}\sum_{i=1}^n \frac{w_{ij}}{w_{.j}} X_{ij}\mathbf{1}_{(1-c_1<\frac{X_{ij}}{T_{Wj}}<1+c_2)}}\ , \tag{34}$$

Then estimated asymptotic variance becomes

$$\widehat{V}(T_{Wj}) = \frac{1}{n}\sum_{i=1}^n w_{ij}\frac{\chi^2(\frac{X_{ij}}{T_{Wj}})}{[\frac{1}{T_{Wj}^2}\sum_{i=1}^n \frac{w_{ij}}{w_{.j}} X_{ij}\mathbf{1}_{(1-c_1<\frac{X_{ij}}{T_{Wj}}<1+c_2)}]^2}\ . \tag{35}$$

and the estimator of $E[V(T_{Wj})]$

$$\hat{s}_T^2 \simeq \frac{1}{K(n-1)}\sum_{i=1}^n\sum_{j=1}^K w_{ij}\frac{\chi^2(\frac{X_{ij}}{T_{Wj}})}{[\frac{1}{T_j^2}\sum_{i=1}^n \frac{w_{ij}}{w_{.j}} X_{ij}\mathbf{1}_{(1-c_1<\frac{X_{ij}}{T_{Wj}}<1+c_2)}]^2}\ , \tag{36}$$

where $\frac{1}{n}$ has been changed to $\frac{1}{(n-1)}$ for asymptotic unbiasedness. If $c_1 = 1$, $c_2 = \infty$ then T_{Wj} reproduces X_{Wj} and we obtain the usual within-variance estimator, $\hat{s}^2$. Similarly in (33) under the same modification we obtain the usual heterogeneity estimator $\hat{a}$ [see Bühlmann & Sraub (1970)].

Consider $\underline{\mathrm{T}}_j$, the vector of truncated values in $\underline{\mathrm{X}}_j$, referred to as pseudo-observations and defined as

$$T_{ij} = T_{Wj} + \frac{\chi(\frac{X_{ij}}{T_{Wj}})}{\frac{1}{T_{Wj}^2}\sum_{i=1}^n \frac{w_{ij}}{w_{.j}} X_{ij}1_{(1-c_1<\frac{X_{ij}}{T_{Wj}}<1+c_2)}}\ . \tag{37}$$

The T_{ij} are asymptotically independent [see Gisler & Reinhard (1993) and the asymptotic covariance relations in Lemma 1.] Then (33) becomes

$$\begin{aligned}\hat{a}_{\mu T} \ &= \ \frac{w_{..}}{w_{..}^2 - \sum_{j=1}^K w_{.j}^2}\{\sum_{j=1}^K w_{.j}(T_{Wj} - T_{WW})(X_{Wj} - X_{WW}) - \\ &\quad \frac{(K-1)}{K(n-1)}\sum_{j=1}^K\sum_{i=1}^n w_{ij}(T_{ij} - T_{Wj})(X_{ij} - X_{Wj})\}\ .\end{aligned} \tag{38}$$

and

$$\hat{s}_T^2 = \frac{1}{K(n-1)}\sum_{j=1}^K\sum_{i=1}^n w_{ij}(T_{ij} - T_{Wj})^2 \tag{39}$$

An estimator of $V[E(T_{Wj}|\Theta_j)]$, is given by

$$\hat{a}_T = \frac{w_{..}}{w_{..}^2 - \sum_{j=1}^K w_{.j}^2}\{\sum_{j=1}^K w_{.j}(T_{Wj} - T_{WW})^2 - (K-1)\hat{s}_T^2\} \ . \qquad (40)$$

The estimators in (38) and (40) are unbiased estimators of $a_{\mu T}$ and a_T, respectively. The proof of unbiasednesss is analogue to the proof of $\hat{a}$ in De Vylder (1978).

The estimated credibility factor then becomes,

$$\hat{Z}_j = \frac{\hat{a}_{\mu T} w_{.j}}{\hat{s}_T^2 + \hat{a}_T w_{.j}} \ . \qquad (41)$$

The estimator of μ is $\hat{\mu} = X_{WW} = \sum_{j=1}^K \frac{w_{.j}}{w_{..}} X_{Wj} = \sum_{j=1}^K \sum_{i=1}^n \frac{w_{ij}}{w_{..}} X_{ij}$ and that of μ_T is $\hat{\mu}_T = T_{WW} = \sum_{j=1}^K \frac{w_{.j}}{w_{..}} T_{Wj} = \sum_{j=1}^K \sum_{i=1}^n \frac{w_{ij}}{w_{..}} T_{ij}$. Finally the empirical credibility estimator is

$$\hat{\hat{\mu}}_j = X_{WW} + \frac{\hat{a}_{\mu T} w_{.j}}{\hat{s}_T^2 + \hat{a}_T w_{.j}}(T_{Wj} - T_{WW}) \ . \qquad (42)$$

3 Pure Robust Credibility Estimation

Alternatively, we now look at the credibility problem from a pure robustness point of view, without separating the risk premium into an ordinary and extraordinary part. The categorization of claims as ordinary and extraordinary yields good results under the assumption that all risks in the portfolio are equally exposed to large claims. However, when dealing with tail observations in practice, we are unable to distinguish easily whether or not all risks in the portfolio are equally exposed to outliers. Even in cases where the information is available, the amount of time required to calculate the different exposure levels becomes an issue. Applying credibility estimation with a pure robustness objective, we ignore temporarily the bias that appears in each contract estimator. To obtain portfolio (global) unbiasedness we add the excess of claims to all contracts premiums. This approach is simpler and produces a fair contribution from all contracts to the cost of large claims. In this case the robust estimator of the conditional expected loss ratio is

$$\hat{\mu}_j^\star = \mu_T + Z_j^\star (T_{Wj} - \mu_T) \ . \qquad (43)$$

The corresponding credibility factor becomes,

$$Z_j^\star = \frac{a_T w_{.j}}{s_T^2 + a_T w_{.j}} \tag{44}$$

and the empirical credibility estimator

$$\hat{\hat{\mu}}_j^\star = T_{W\hat{Z}} + \frac{\hat{a}_T w_{.j}}{\hat{s}_T^2 + \hat{a}_T w_{.j}} (T_{Wj} - T_{W\hat{Z}}) \ , \tag{45}$$

where $\hat{\mu}_T = T_{W\hat{Z}} = \sum_{j=1}^K \frac{\hat{Z}_j^\star}{\hat{Z}_.^\star} T_{Wj}$, where $\hat{Z}_.^\star = \sum_{j=1}^K \hat{Z}_j^\star$, and the estimators $\hat{a}_T$ and $\hat{s}_T^2$ are as in the previous section.

In insurance, unbiasedness is particularly important. Therefore to obtain the final $\hat{\hat{\mu}}_j$ we add back the excess $X_{W\hat{Z}} - \frac{1}{w_{..}} \sum_{j=1}^K w_{.j} \hat{\hat{\mu}}_j^\star = X_{W\hat{Z}} - T_{W\hat{Z}}$ to all contracts, hence

$$\hat{\hat{\mu}}_j = X_{W\hat{Z}} + \frac{\hat{a}_T w_{.j}}{\hat{s}_T^2 + \hat{a}_T w_{.j}} (T_{Wj} - T_{W\hat{Z}}) \tag{46}$$

with $X_{W\hat{Z}} = \sum_{j=1}^K \frac{\hat{Z}_j^\star}{\hat{Z}_.^\star} X_{Wj}$.

Theorem 3.1 Under the above assumptions, the optimal linearized non homogeneous pure robust credibility estimator is :

$$\hat{\mu}_j^\star = \mu_T + Z_j^\star (T_{Wj} - \mu_T) \tag{47}$$

where T_{Wj} is the individual robust estimator for $\mu(\Theta_j)$, and the corresponding credibility factor for contract j is given by

$$Z_j^\star = \frac{a_T w_{.j}}{(a_T w_{.j} + s_T^2)} \tag{48}$$

with $a_T = V[E(T_{ij}|\Theta_j)]$, $s_T^2 = E[\sigma_T^2(\Theta_j)]$, $\mu_T = E(T_{ij})$, $\mu_{T_j}(\Theta_j) = E(T_{ij}|\Theta_j)$ as defined above.

Proof: The proof is similar to the proof in Theorem 2.1. □

4 Numerical Illustrations

To illustrate the models described above, we consider Hachemeister's (1975) data set. The advantage of using this data set is that is well known and extensively used in the actuarial literature. The weakness, however, is that it should be fitted a regression credibility model, as claims show a clear inflation trend through time.

Hachemeister considered five different U.S. states (contracts) i.e. $K = 5$ and twelve quarters of claims experience (periods) i.e. $n = 12$. The first five columns of Table 1 represent the average claim amounts X_{ij} per 3 month period for total private passenger bodily injury insurance from July 1970 until June 1973. Here X_{ij}, $i = 1, \ldots, n$ and $j = 1, \ldots K$ are expressed in U.S.\$. The last five columns of Table 2 represent the weights w_{ij}, which reflect the number of claims that correspond to X_{ij}.

To show the sensitivity to large claims of the credibility estimators in Bühlmann's and Bühlmann & Straub's models, we replaced the last claim of the 5th contract \$1690 in Table 1 with different outlier values. Table 2 reports the three cases $X_{12,5} =$ \$5000, \$6000 and \$7000. It shows how, as the value of the outlier becomes larger, the credibility factor decreases to zero.

The rate of decrease is different in the weigthed case. The reason is the lack of robustness of Bühlmann's and Bühlmann & Straub's premium estimators, due to the sensitivity of the between-contracts variance, $V[\mu(\Theta_j)]$, and the expected value of the within-contract variance, $E[\sigma^2(\Theta_j)]$, to a single large claim. Tables 2.a and 7a illustrate the increase in s^2 due to the increase of the within-contract variance. The decrease of the between-contracts variance $\hat{a}$ is a specific result of the data used for this case.

Using Künsch's and our robust-portfolio unbiased estimators, we note that as the outlier becomes larger, we have a decrease in the value of credibility factor. This decrease is usually smaller than in the case of Bülmann's linear credibility estimator but it still affects the stability of credibility factor estimation giving an unfair contribution to the excess claims.

Using truncation points $c_1 = 1$, $c_2 = 0.$ in Tables 3, 6 and 7 we obtain values of the credibility factor and empirical credibility estimators. The credibility premium for the 5th contract, $\hat{\hat{\mu}}_5$, increases due to the large claim. However $\hat{\hat{\mu}}_2$ and $\hat{\hat{\mu}}_4$ also increase substantially, even though only the 5th contract is contaminated. This not only indicates that a single outlier affects the empirical credibility estimator of the contract within which it appears, but that it also affects contracts that only have small claims (here the 2nd and 4th contracts). At this point we can say that Künsch's premiums, as well as ours that include weights, do not equitably redistribute the cost of large claims over contracts.

Pure robust credibility estimation provides very good premiums. Tables 4, 8 and 9 for $c_1 = 1$, $c_2 = 0.5$ and $c_1 = 1$, $c_2 = 0.2$, respectively, show the improvement in $\hat{\hat{\mu}}_j^\star$ as the value of c_2 becomes smaller than 0.5. For $c_2 = 0.2$, $\hat{\hat{\mu}}_j^\star$ resembles results obtained in the case where no outlier is present, i.e. the values of $\hat{\hat{\mu}}_j^\star$ in column 2 to column 4 (with outlier) tend towards the values of column 1 (no outlier). To obtain $\hat{\hat{\mu}}_j$ we uniformly distribute the excess claims to all contracts. We can now say that from a pure robust point of view there is equity between contacts in premium sharing. Tables 8a and 9a show the values of $\hat{s}_T^2$ and $\hat{a}_T$ for different values of c_2.

In Table 10 we use Gisler & Reinhard's (1993) $\psi(z) = \min(z-1, 1)$ function in the unweighted case. This corresponds to our $c_2 = 1$, fixed independently of the sample size. We see that, the empirical credibility estimator of the 5th contract, $\hat{\hat{\mu}}_5$, increases due to the large claim. However $\hat{\hat{\mu}}_2$ and $\hat{\hat{\mu}}_4$ also increase substantially, even though only the 5th contract is contaminated. We also see a decrease of the credibility premium of contracts with large claims (for instance the 1st and 3rd contracts). Again here, a single outlier affects the credibility premium of the contaminated contract and of all other contracts. In part, this is due to the fact that our contract sample size ($n = 12$) and the value of $c_2 = 1$ which does not truncate enough to guarantee equity between contract premiums.

In Table 11 we apply Gisler & Reinhard's (1993) algorithm in our pure robust weighted case with the ψ function dependent on the volume w_{ij}, i.e. $\psi(z, w_{ij}) = \min[z-1, f(w_{ij})]$, where $f(w_{ij}) = c w_{ij}^{-1/2}$ with $c = \sqrt{\bar{w}}$ and $\bar{w} = \frac{1}{nK}\sum_{j=1}^{K}\sum_{i=1}^{n} w_{ij}$. Similarly as in the unweighted case, we see that the credibility premium $\hat{\hat{\mu}}_2^\star$ and $\hat{\hat{\mu}}_4^\star$ increase substantially, even though only the 5th contract is contaminated. We also see a decrease of the empirical credibility estimators for contracts with large claims. Here we can say that from a pure robust point of view, Gisler & Reinhard's (1993) weighted algorithm does not robustify enough to guarantee equity between contracts.

Remark 1 The values of $c_2 = 1$, $c_2 = 0.5$ and $c_2 = 0.2$ correspond to truncation points of 2, 1.5 and 1.2, respectively.

Table 1: Hachemeister's Data Set ($n = 12$ periods)										
	Average Claims per Period					No. of claims per Period				
i	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	w_{i1}	w_{i2}	w_{i3}	w_{i4}	w_{i5}
1	1738	1364	1759	1223	1456	7861	1622	1147	407	2902
2	1642	1408	1685	1146	1499	9251	1742	1357	396	3172
3	1794	1597	1479	1010	1609	8706	1523	1329	348	3046
4	2051	1444	1763	1257	1741	8575	1515	1204	341	3068
5	2079	1342	1674	1426	1482	7917	1622	998	315	2693
6	2234	1675	2103	1532	1572	8263	1602	1077	328	2910
7	2032	1470	1502	1953	1606	9456	1964	1277	352	3275
8	2035	1448	1622	1123	1735	8003	1515	1218	331	2697
9	2115	1464	1828	1343	1607	7365	1527	896	287	2663
10	2262	1831	2155	1243	1573	7832	1748	1003	384	3017
11	2267	1612	2233	1762	1613	7849	1654	1108	321	3242
12	2517	1471	2059	1306	1690	9077	1861	1121	342	3425

Table: 2 Bülmann's Premiums				
$X_{12,5}$	1690	5000	6000	7000
$\bar{X}_{.1}$	2063.83	2063.83	2063.83	2063.83
$\bar{X}_{.2}$	1510.50	1510.50	1510.5	1510.50
$\bar{X}_{.3}$	1821.33	1821.33	1821.33	1821.33
$\bar{X}_{.4}$	1360.33	1360.33	1360.33	1360.33
$\bar{X}_{.5}$	1598.58	1874.41	1957.75	2041.08
$\hat{Z}$	0.95	0.7545837	0.653349	0.5520951
$\hat{\hat{\mu}}_1$	2044.04	1980.97	1952.56	1927.52
$\hat{\hat{\mu}}_2$	1518.58	1563.43	1591.044	1622.03
$\hat{\hat{\mu}}_3$	1814.23	1798.36	1794.45	1793.92
$\hat{\hat{\mu}}_4$	1375.98	1450.12	1492.93	1539.13
$\hat{\hat{\mu}}_5$	1602.23	1838.37	1883.25	1914.96
$\bar{X}_{..}$	1671.02	1726.18	1742.85	1759.51

Table 2a: Variance Components				
$X_{12,5}$	1690	5000	6000	7000
$\hat{s}^2$	46,040	239,645.38	369,969.62	533,627.2
$\hat{a}$	72,310	61,403.32	58,108.24	54,813.17

Table 3: Künsch's Premiums

$c_1 = 1,\ c_2 = 0.5$				
$X_{12,5}$	1690	5000	6000	7000
$\bar{T}_1$	2063.83	2063.83	2063.83	2063.83
$\bar{T}_2$	1510.5	1510.5	1510.5	1510.5
$\bar{T}_3$	1821.83	1821.83	1821.83	1821.83
$\bar{T}_4$	1360.33	1360.33	1360.33	1360.33
$\bar{T}_5$	1598.58	1666.00	1666.00	1666.00
$\hat{Z}$	0.95	0.86969	0.845223	0.831793
$\hat{\hat{\mu}}_1$	2044.04	2056.08	2067.00	2070.83
$\hat{\hat{\mu}}_2$	1518.58	1574.86	1594.19	1616.72
$\hat{\hat{\mu}}_3$	1814.23	1845.62	1860.21	1872.22
$\hat{\hat{\mu}}_4$	1375.98	1444.26	1465.87	1493.47
$\hat{\hat{\mu}}_5$	1602.23	1710.10	1727.06	1744.34
$\bar{X}_{..}$	1671.02	1726.18	1742.85	1759.51
$\bar{T}_{..}$	1671.02	1684.48	1684.48	1684.48
$c_1 = 1,\ c_2 = 0.2$				
$X_{12,5}$	1690	5000	6000	7000
$\bar{T}_1$	2063.83	2063.83	2063.83	2063.83
$\bar{T}_2$	1510.5	1510.5	1510.5	1510.5
$\bar{T}_3$	1821.83	1821.83	1821.83	1821.83
$\bar{T}_4$	1360.33	1360.33	1360.33	1360.33
$\bar{T}_5$	1598.58	1619.00	1619.00	1619.00
$\hat{Z}$	0.95	0.877163	0.854478	0.820729
$\hat{\hat{\mu}}_1$	2044.04	2067.16	2075.01	2082.86
$\hat{\hat{\mu}}_2$	1518.58	1581.8	1602.2	1622.61
$\hat{\hat{\mu}}_3$	1814.23	1854.89	1868.23	1881.57
$\hat{\hat{\mu}}_4$	1375.98	1450.08	1473.89	1497.69
$\hat{\hat{\mu}}_5$	1602.23	1676.98	1694.91	1712.85
$\bar{X}_{..}$	1671.02	1726.18	1742.85	1759.51
$\bar{T}_{..}$	1675.10	1675.10	1675.10	1675.10

Table 4: Pure Robust Premiums				
(no weights) $c_1 = 1$, $c_2 = 0.5$				
$X_{12,5}$	1690	5000	6000	7000
$\bar{T}_1$	2063.83	2063.83	2063.83	2063.83
$\bar{T}_2$	1510.50	1510.50	1510.50	1510.50
$\bar{T}_3$	1821.83	1821.83	1821.83	1821.83
$\bar{T}_4$	1360.33	1360.33	1360.33	1360.33
$\bar{T}_5$	1598.58	1666.00	1666.00	1666.00
$\hat{Z}^\star$	0.95	0.9339	0.9339	0.9339
$\hat{\hat{\mu}}_1^\star$	2044.04	2038.56	2038.56	2038.56
$\hat{\hat{\mu}}_2^\star$	1518.58	1522.09	1522.09	1522.09
$\hat{\hat{\mu}}_3^\star$	1814.23	1812.68	1812.68	1812.68
$\hat{\hat{\mu}}_4^\star$	1375.98	1381.92	1381.92	1381.92
$\hat{\hat{\mu}}_5^\star$	1602.23	1666.00	1666.00	1666.00
$\hat{\hat{\mu}}_1$	2044.04	2080.25	2096.91	2113.57
$\hat{\hat{\mu}}_2$	1518.58	1536.72	1580.43	1597.10
$\hat{\hat{\mu}}_3$	1814.23	1854.368	1871.03	1887.69
$\hat{\hat{\mu}}_4$	1375.98	1423.60	1440.27	1456.01
$\hat{\hat{\mu}}_5$	1602.23	1708.96	1725.58	1742.24
$\bar{X}_{..}$	1671.02	1726.18	1742.85	1759.51
$\bar{T}_{..}$	1671.02	1684.50	1684.50	1684.50
$c_1 = 1$, $c_2 = 0.2$				
$X_{12,5}$	1690	5000	6000	7000
$\bar{T}_1$	2063.83	2063.83	2063.83	2063.83
$\bar{T}_2$	1510.50	1510.50	1510.50	1510.50
$\bar{T}_3$	1821.83	1821.83	1821.83	1821.83
$\bar{T}_4$	1360.33	1360.33	1360.33	1360.33
$\bar{T}_5$	1598.58	1619.65	1619.65	1619.65
$\hat{Z}^\star$	0.95	0.947115	0.947115	0.947115
$\hat{\hat{\mu}}_1^\star$	2044.04	2043.26	2043.26	2043.26
$\hat{\hat{\mu}}_2^\star$	1519.22	1519.22	1519.22	1519.22
$\hat{\hat{\mu}}_3^\star$	1814.23	1814.07	1814.07	1814.07
$\hat{\hat{\mu}}_4^\star$	1377.00	1377.00	1377.00	1377.00
$\hat{\hat{\mu}}_5^\star$	1602.23	1622.59	1622.59	1622.59
$\hat{\hat{\mu}}_1$	2044.04	2098.42	2110.88	2127.54
$\hat{\hat{\mu}}_2$	1518.58	1574.38	1586.84	1603.5
$\hat{\hat{\mu}}_3$	1814.23	1869.23	1881.69	1898.35
$\hat{\hat{\mu}}_4$	1375.98	1432.16	1444.62	1461.28
$\hat{\hat{\mu}}_5$	1602.23	1677.75	1690.21	1706.87
$\bar{X}_{..}$	1671.02	1726.18	1742.85	1759.51
$\bar{T}_{..}$	1671.02	1675.23	1675.23	1675.23

Table 5: Bülmann-Straub's Premiums				
$X_{12,5}$	1690	5000	6000	7000
$\bar{X}_{W1}$	2060.87	2060.87	2060.87	2060.87
$\bar{X}_{W2}$	1511.17	1511.17	1511.17	1511.17
$\bar{X}_{W3}$	1805.81	1805.81	1805.81	1805.81
$\bar{X}_{W4}$	1352.98	1352.98	1352.98	1352.98
$\bar{X}_{W5}$	1599.00	1913.78	2008.63	2103.48
$\hat{Z}_1$	0.984740	0.8130	0.6077	0.1954
$\hat{Z}_2$	0.927635	0.4634	0.2353	0.0460
$\hat{Z}_3$	0.898475	0.3740	0.1752	0.0322
$\hat{Z}_4$	0.727909	0.1527	0.0603	0.00996
$\hat{Z}_5$	0.958791	0.6105	0.3583	0.0805
$\hat{\mu}_1$	2055.16	2018.45	1996.62	1978.83
$\hat{\mu}_2$	1523.70	1684.35	1806.25	1938.29
$\hat{\mu}_3$	1793.44	1823.39	1881.05	1953.96
$\hat{\mu}_4$	1442.96	1760.42	1864.20	1952.85
$\hat{\mu}_5$	1603.28	1882.65	1937.02	1970.54
$\bar{X}_{WZ}$	1683.71	1833.85	1897.03	1958.89

Table 5a: B-S Variance Components				
$X_{12,5}$	1690	5000	6000	7000
$\hat{s}^2$	1.39E8	7.94E8	1.23E9	1.79E9
$\hat{a}$	89,639	34,457	19,093	4336.28

Table 6: Portfolio-Unbiased Premiums				
(with weights) $c_1 = 1$, $c_2 = 0.5$				
$X_{12,5}$	1690	5000	6000	7000
T_{W1}	2060.87	2060.87	2060.87	2060.87
T_{W2}	1511.17	1511.17	1511.17	1511.17
T_{W3}	1805.81	1805.81	1805.81	1805.81
T_{W4}	1352.98	1352.98	1352.98	1352.98
T_{W5}	1599.00	1678.30	1678.30	1678.30
$\hat{Z}_1$	0.984740	0.684366	0.568734	0.453102
$\hat{Z}_2$	0.927635	0.623448	0.518108	0.41277
$\hat{Z}_3$	0.898475	0.593863	0.493522	0.393182
$\hat{Z}_4$	0.727909	0.438684	0.364563	0.290442
$\hat{Z}_5$	0.958791	0.656178	0.545308	0.434439
$\hat{\mu}_1$	2055.16	2053.19	2052.15	2051.1
$\hat{\mu}_2$	1523.70	1699.57	1758.27	1816.98
$\hat{\mu}_3$	1793.44	1885.5	1912.79	1940.08
$\hat{\mu}_4$	1442.97	1698.62	1757.49	1816.35
$\hat{\mu}_5$	1603.28	1797.11	1839.33	1881.56

Table 6a: Variance Components				
(with weights) $c_1 = 1$, $c_2 = 0.5$				
$X_{12,5}$	1690	5000	6000	7000
$\hat{s}_T^2$	1.39E8	1.87E8	1.87E8	1.87E8
$\hat{a}_T$	89,639	75,220.96	75,220.96	75,220.96
$\hat{a}_{\mu T}$	89,639	52,756.5	43,842.6	34,928.8

Table 7: Portfolio Unbiased Premiums				
(with weights) $c_1 = 1$, $c_2 = 0.2$				
$X_{12,5}$	1690	5000	6000	7000
T_{W1}	2060.87	2060.87	2060.87	2060.87
T_{W2}	1511.17	1511.17	1511.17	1511.17
T_{W3}	1805.81	1805.81	1805.81	1805.81
T_{W4}	1352.98	1352.98	1352.98	1352.98
T_{W5}	1599.00	1642.20	1642.20	1642.20
$\hat{Z}_1$	0.984740	0.670226	0.577874	0.465524
$\hat{Z}_2$	0.927635	0.627624	0.541142	0.435934
$\hat{Z}_3$	0.898475	0.606064	0.522553	0.420959
$\hat{Z}_4$	0.727909	0.482508	0.416022	0.33514
$\hat{Z}_5$	0.958791	0.650804	0.561128	0.452035
$\hat{\hat{\mu}}_1$	2055.16	2058.15	2060.25	2058.54
$\hat{\hat{\mu}}_2$	1523.70	1705.03	1755.79	1813.27
$\hat{\hat{\mu}}_3$	1793.44	1891.35	1916.43	1942.68
$\hat{\hat{\mu}}_4$	1442.97	1680.85	1734.93	1796.47
$\hat{\hat{\mu}}_5$	1603.28	1770.41	1812.15	1858.68

Table 7a: Variance Components				
(with weights) $c_1 = 1$, $c_2 = 0.2$				
$X_{12,5}$	1690	5000	6000	7000
$\hat{s}_T^2$	1.39E8	1.46E8	1.46E8	1.46E8
$\hat{a}_T$	89,639	85,075.904	85,075.904	85,075.904
$\hat{a}_{\mu T}$	89,639	57,995.9	50,004.5	40,282.7

Table 8: Pure Robust Premiums				
(with weights) $c_1 = 1$, $c_2 = 0.5$				
$X_{12,5}$	1690	5000	6000	7000
T_{W1}	2060.87	2060.87	2060.87	2060.87
T_{W2}	1511.17	1511.17	1511.17	1511.17
T_{W3}	1805.81	1805.81	1805.81	1805.81
T_{W4}	1352.98	1352.98	1352.98	1352.98
T_{W5}	1599.00	1678.30	1678.30	1678.30
$\hat{Z}_1^\star$	0.984740	0.975779	0.975779	0.975779
$\hat{Z}_2^\star$	0.927635	0.888920	0.888920	0.888920
$\hat{Z}_3^\star$	0.898475	0.846737	0.846737	0.846737
$\hat{Z}_4^\star$	0.727909	0.625482	0.625482	0.625482
$\hat{Z}_5^\star$	0.958791	0.935587	0.935587	0.935587
$\hat{\hat{\mu}}_1^\star$	2055.16	2052.38	2052.38	2052.38
$\hat{\hat{\mu}}_2^\star$	1523.70	1533.14	1533.14	1533.14
$\hat{\hat{\mu}}_3^\star$	1793.44	1790.93	1790.93	1790.93
$\hat{\hat{\mu}}_4^\star$	1443.97	1486.15	1486.15	1486.15
$\hat{\hat{\mu}}_5^\star$	1603.28	1680.26	1680.26	1680.26
$\hat{\hat{\mu}}_1$	2055.16	2107.04	2126.72	2146.4
$\hat{\hat{\mu}}_2$	1523.70	1753.92	1773.6	1793.28
$\hat{\hat{\mu}}_3$	1793.44	1940.24	1959.92	1979.6
$\hat{\hat{\mu}}_4$	1442.97	1729.74	1749.42	1769.1
$\hat{\hat{\mu}}_5$	1603.28	1819.30	1838.98	1858.66

Table 8a: Variance Components				
(with weights) $c_1 = 1$, $c_2 = 0.5$				
$X_{12,5}$	1690	5000	6000	7000
$\hat{s}_T^2$	1.39E8	1.87E8	1.87E8	1.87E8
$\hat{a}_T$	89639	75220.96	75220.96	75220.96

Table 9: Pure Robust Premiums				
(with weights) $c_1 = 1, c_2 = 0.2$				
$X_{12,5}$	1690	5000	6000	7000
T_{W1}	2060.87	2060.87	2060.87	2060.87
T_{W2}	1511.17	1511.17	1511.17	1511.17
T_{W3}	1805.81	1805.81	1805.81	1805.81
T_{W4}	1352.98	1352.98	1352.98	1352.98
T_{W5}	1599.00	1642.20	1642.20	1642.20
$\hat{Z}_1^\star$	0.984740	0.983174	0.983174	0.983174
$\hat{Z}_2^\star$	0.927635	0.920680	0.920680	0.920680
$\hat{Z}_3^\star$	0.898475	0.889053	0.889053	0.889053
$\hat{Z}_4^\star$	0.727909	0.707805	0.707805	0.707805
$\hat{Z}_5^\star$	0.958791	0.954684	0.954684	0.954684
$\hat{\hat{\mu}}_1^\star$	2055.16	2054.68	2054.68	2054.68
$\hat{\hat{\mu}}_2^\star$	1523.70	1525.43	1525.43	1525.43
$\hat{\hat{\mu}}_3^\star$	1793.44	1793.03	1793.03	1793.03
$\hat{\hat{\mu}}_4^\star$	1443.97	1451.57	1451.57	1451.57
$\hat{\hat{\mu}}_5^\star$	1603.28	1627.41	1627.41	1627.41
$\hat{\hat{\mu}}_1$	2055.16	2118.22	2137.9	2157.58
$\hat{\hat{\mu}}_2$	1523.70	1765.1	1784.78	1804.46
$\hat{\hat{\mu}}_3$	1793.44	1951.42	1971.10	1990.78
$\hat{\hat{\mu}}_4$	1442.97	1740.92	1760.60	1780.28
$\hat{\hat{\mu}}_5$	1603.28	1830.48	1850.16	1869.84

Table 9a: Variance Components				
(with weights) $c_1 = 1,\ c_2 = 0.2$				
$X_{12,5}$	1690	5000	6000	7000
$\hat{s}_T^2$	1.39E8	1.46E8	1.46E8	1.46E8
$\hat{a}_T$	89639	85075.904	85075.904	85075.904

Table 10: Gisler-Reinhard's Premiums				
(no weights) $c_1 = 1$, $c_2 = 1$				
$X_{12,5}$	1690	5000	6000	7000
$\bar{T}_1$	2063.83	2063.83	2063.83	2063.83
$\bar{T}_2$	1510.50	1510.50	1510.50	1510.50
$\bar{T}_3$	1821.83	1821.83	1821.83	1821.83
$\bar{T}_4$	1360.33	1360.33	1360.33	1360.33
$\bar{T}_5$	1598.58	1748.16	1748.16	1748.16
$\hat{Z}^\star$	0.95	0.857	0.857	0.857
$\hat{\mu}_1^\star$	2044.04	2012.80	2012.80	2012.80
$\hat{\mu}_2^\star$	1518.58	1538.59	1538.59	1538.59
$\hat{\mu}_3^\star$	1814.23	1805.40	1805.40	1805.40
$\hat{\mu}_4^\star$	1375.98	1409.90	1409.90	1409.90
$\hat{\mu}_5^\star$	1602.23	1767.95	1767.95	1767.95
$\hat{\mu}_1$	2044.04	2036.01	2052.68	2069.34
$\hat{\mu}_2$	1518.58	1561.80	1578.47	1595.13
$\hat{\mu}_3$	1814.23	1828.61	1845.28	1861.94
$\hat{\mu}_4$	1375.98	1433.11	1449.78	1466.44
$\hat{\mu}_5$	1602.23	1791.16	1807.83	1824.49
$X_{..}$	1671.02	1726.18	1742.85	1759.51
$\bar{T}_{..}$	1671.02	1702.97	1702.97	1702.97

Table 11: Gisler-Reinhard's Premiums (with weights)				
$X_{12,5}$	1690	5000	6000	7000
T_{W1}	2060.87	2060.87	2060.87	2060.87
T_{W2}	1511.17	1511.17	1511.17	1511.17
T_{W3}	1805.81	1805.81	1805.81	1805.81
T_{W4}	1352.98	1352.98	1352.98	1352.98
T_{W5}	1599.00	1760.12	1760.12	1760.12
$\hat{Z}_1^\star$	0.984740	0.950079	0.950079	0.950079
$\hat{Z}_2^\star$	0.927635	0.790817	0.790817	0.790817
$\hat{Z}_3^\star$	0.898475	0.722988	0.722988	0.722988
$\hat{Z}_4^\star$	0.727909	0.441020	0.441020	0.441020
$\hat{Z}_5^\star$	0.958791	0.872801	0.872801	0.872801
$\hat{\hat{\mu}}_1^\star$	2055.16	2045.14	2045.14	2045.14
$\hat{\hat{\mu}}_2^\star$	1523.70	1560.10	1560.10	1560.10
$\hat{\hat{\mu}}_3^\star$	1793.44	1788.95	1788.95	1788.95
$\hat{\hat{\mu}}_4^\star$	1443.97	1572.04	1572.04	1572.04
$\hat{\hat{\mu}}_5^\star$	1603.28	1758.18	1758.18	1758.18
$\hat{\hat{\mu}}_1$	2055.16	2118.87	2182.05	2243.91
$\hat{\hat{\mu}}_2$	1523.70	1633.83	1697.01	1758.87
$\hat{\hat{\mu}}_3$	1793.44	1862.68	1925.86	1987.72
$\hat{\hat{\mu}}_4$	1442.97	1645.77	1708.95	1770.81
$\hat{\hat{\mu}}_5$	1603.28	1831.91	1895.09	1956.95

5 Conclusions

Applying robust estimation techniques to the classical Bühlmann (1967) and Bühlmann & Straub (1970) credibility models, we compared two methods of premium estimation in a heterogeneous portfolio; the robust portfolio-unbiased credibility estimation and the pure robust credibility estimation. The robust portfolio-unbiased estimation based on Künsch's model and its extention to include weights, constrain the estimators to reproduce the total cost of claims, but lack equity in redistributing the total premium over contracts. This results in an unfair premium estimation, as the presence of a large claim not only increases the premium of the contaminated contract, but also modifies substantially the premiums of other contracts.

Pure robust credibility, together with the correction factor for portfolio-unbiasedness (and an appropriate value of c_2 depending on the data set), guarantees the recovery of total claims as well as keeping the credibility estimation equitable.

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