Welfare Consequences of Information Aggregation and Optimal Market Size*

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January 7th, 2015

Abstract

This paper studies a risk-sharing model where traders face endowment shocks and information asymmetries. We show that a negative participation externality arises due to the endogenous information aggregation by prices, and it creates a counter force to a standard positive externality of risk-sharing. As a result, the optimal market size that maximizes gains from trade per trader is finite. The model indicates that a collection of small markets can be a constrained efficient market structure. We also study a decentralized process of market formation, and show that multiple markets can survive because of the negative informational externality among traders.

Keywords: Asymmetric information, Imperfect competition, Information aggregation, Market fragmentation, Network externality puzzle.

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*I thank Antonio Bernardo, Bruno Biais, Bruce Carlin, Chris Edmond, Roger Farmer, Mark Garmaise, Shogo Hamasaki, Christian Hellwig, Ian King, Ichiro Obara, Lee Ohanian, Avanidhar Subrahmanyam, Lawrence Uren, Dimitri Vayanos, Venky Venkateswaran, Xavier Vives, Pierre-Olivier Weill, Mark Wright, and seminar participants at ANU, DBJ, LSE, TSE, University of Auckland, UCLA and UNSW for comments and suggestions. All errors are mine.

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1 Introduction

One of the propositions rarely questioned by economists is that increasing the size of a market benefits market participants by providing the better liquidity/depth/price discovery. In the market microstructure literature,[1] a popular view is that security trading *should* concentrate in a single venue due to positive liquidity externalities although intermediation costs (e.g. transaction costs, search costs etc) may prevent any market from becoming too large. From this perspective, however, the level of the market fragmentation observed today is puzzling if we believe that intermediation costs have been falling. For example, many financial securities are traded on multiple platforms, rather than in a single large market. A growing body of empirical work suggests that such market fragmentation is a common and robust phenomenon.[2] Borrowing a phrase from Madhavan (2000), this is best summarized as *the network externality puzzle*: why is trading for the same security split across multiple trading venues? What are the economic forces that work against the positive externalities?

We revisit this issue in a model of trading with finitely many risk averse traders, asymmetric information, and shocks to endowments. Traders are ex ante uncertain about both the size of their endowments and the unit value of the endowment. Once the traders participate in a market, the size of their endowment is realized and they observe private signals about the unit value of the endowment. They can submit orders contingent on the market-clearing price. After the market is cleared, the value of their endowments is realized. There is no additional supply beyond the initial endowments, and risk-sharing creates gains from trade.

We show that the optimal market size is finite, because the private information aggregated by prices at the interim trading stage creates a negative externality at the ex ante participation stage. The intuition is as follows. Trading is mutually beneficial because traders with different endowment realizations have different risk exposures. With symmetric information, prices do not aggregate any information and traders' equilibrium beliefs are independent of

[1] See Madhavan (2000), Biais, Glosten, and Spatt (2005) and Vives (2008) for surveys.

[2] Brown, Mulherin, and Weidenmier (2008) and Cantillon and Yin (2010, 2011) study competition between exchanges. Ready (2009) and O'Hara and Ye (2011) study the trade fragmentation across trading platforms.

the number of traders. The gains from risk-sharing increase with the market size, because endowment good positions become more identical in a larger market. This reallocation of risk exposures is a standard source of positive externalities. With asymmetric information, not only the endowment good positions but also beliefs change in equilibrium because prices aggregate the private information. This implies that prices are a more accurate predictor of the value of the endowment good in a larger market. The informative market price, however, becomes an aggregate risk which cannot be traded at the ex ante stage before endowment shocks are realized. In other words, the information aggregation at the interim stage endogenously increases the level of uninsurable risk at the ex ante stage — an endogenous version of the "Hirshleifer effect" (Hirshleifer 1971). Importantly, this is a negative externality: while it is rational to use both private information and the information revealed by prices at the interim stage, it hurts everyone ex ante. Therefore, unless traders can commit not to use their private information or not to internalize the information in market prices, a trade-off between the positive allocational force and the negative informational force exists. In a standard finance model we study, as the market size grows, the positive force dies out faster than the negative force does, leaving the latter dominant in a large market. This results in the hump-shaped gains from trade illustrated in **Figure 1**.

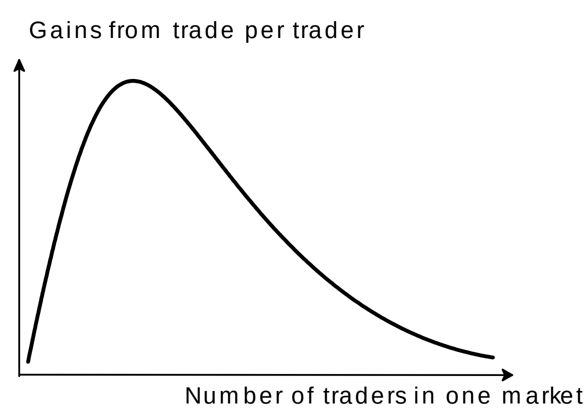

Figure 1. Hump-shaped gains from trade.

On the increasing part of the curve, the risk is shared among a small number of traders. Hence, prices do not aggregate much information and remain only a noisy indicator of the value of the endowment. Because the information aggregation grows only slowly with the market size, having an additional trader has a net positive effect. On the decreasing part of the curve, the positive externality still exists, but it is weaker than the negative externality. A smaller market is more desirable precisely because of *less informative* prices.

We first analyze the case where the value of the endowment good is fully revealed in a large market. We show that the gains from trade converge to zero, despite the active trading in the limit. We also study the case where the information aggregation is not perfect. We show that the endowment positions become identical in the limit as in the symmetric information case. Nevertheless, the gains from trade still decrease in the market size when the market becomes sufficiently large. Therefore, if we focused on the allocation of the endowment good, we could have drawn a false welfare implication. This apparent gap arises because even if the limit allocations of the endowment good are identical, the risk associated with the monetary payment could be completely different. The drastic change in the risk of monetary payment occurs when prices aggregate the large amount of information, which would not occur if the information were symmetric. A key message is that when the information aggregation occurs in a market setup, the risk can be inefficiently reallocated *intertemporally*.

In the final section, we propose a model of endogenous market formation. We study a market making game where each intermediary sets an entry fee for his market. Because traders are aware how the market size affects the allocation as well as the information aggregation, they might benefit more from a market with fewer participants. Therefore, the entry fee is not necessarily bid down to a zero-profit level and multiple markets can survive in equilibrium. Our analysis shows that a collection of small markets can be an endogenous reaction to the negative informational externality.

We present our main result using a price-taking equilibrium as a solution concept.[3] Because a single price equates marginal values of traders, a price-taking equilibrium is Pareto efficient *given* the equilibrium beliefs for a *given* market size. However, from an ex ante per-

[3] The main result also holds in a strategic equilibrium. See **Lemma A1** in the Appendix A.

spective, the volatility of the marginal value affects welfare. Therefore, how the market size *change* beliefs is relevant. Our result indicates that, taking the use of market-clearing prices and the associated belief updating as a constraint, the ex ante efficient trading arrangement may limit the market size to control the negative informational externality.

After the literature review, Section 2 describes the model environment, defines welfare measures, and presents solution concepts. Section 3 introduces a symmetric information benchmark where the optimal market size is infinite. Section 4 characterizes the information aggregation, the gains from trade, and the optimal market size. Section 5 proposes a model of endogenous market formation. Section 6 concludes. The Appendix A studies model extensions. The online Appendix B collects all proofs and the background analysis.

1.1 Related literature

O'Hara and Ye (2011) document the large variety of trading venues in US equity markets. A growing literature on competing exchanges provides explanations for this empirical observation by modelling the ways in which exchanges differentiate their service (e.g. collateral requirements, listing fees, trading speed).[4] We do not consider the service differentiation, but instead emphasize the information aggregation that creates negative externalities as a complementary rationale for the market fragmentation. From a theoretical point of view, a market with a large number of traders has been viewed as an ideal benchmark. For example, it is customary to consider the limit as the number of traders goes to infinity to obtain asset pricing implications.[5] When negative externalities exist, this limit may not be the ideal benchmark. Rostek and Weretka (2014) present a model in which welfare can be lower in a larger market, because the preference interdependence can change with the market size in a way that increases traders' market power. We present our main result assuming that traders

[4] See for example, Santos and Scheinkman (2001), Foucault and Parlour (2004), Pagnotta (2013) .

[5] For example, see Corollary 1 in Madhavan (1992).

are price-takers.[6] Therefore, the welfare loss in our model is driven by the distinct economic force. Finally, the Hirshleifer effect in an exchange economy has been extensively studied, but none of the preceding works studied the market size.[7] Our contribution is to show that the optimal market size can be finite when prices endogenously aggregate the information.

2 Model

2.1 Environment

There are $n+1$ traders indexed by $i \in \{1, ..., n+1\}$. Traders have identical preferences and trade endowments that have a random payoff v per unit. Before trading, each trader i receives (i) the endowment e_i, and (ii) a signal s_i about v. Both are privately known. There is no additional supply beyond the sum of endowments $\sum_{i=1}^{n+1} e_i$ in the market. The endowment $\{e_i\}_{i=1}^{n+1}$ are independently normally distributed with mean zero and variance τ_x^{-1}. The i.i.d. endowment assumption captures the standard positive externality from risk-sharing. The random payoff v has two components, v_0 and $\widetilde{v}$:

$$v = \sqrt{1-t}v_0 + \sqrt{t}\widetilde{v}, \quad t \in [0, 1], \tag{1}$$

where v_0 and $\widetilde{v}$ are independently normally distributed with mean zero and variance τ_v^{-1}. Each trader observes a signal about $\widetilde{v}$:

$$s_i = \widetilde{v} + \varepsilon_i,$$

[6] We also study the case where traders internalize their market power and show that in our model the market power cannot increase with the market size. See **Lemma A1**.

[7] Schlee (2001) analyses a general exchange economy, but he assumes a symmetric information environment. Pithyachariyakul (1986) compares the Walrasian system and the monopolistic market-making system and shows that the information revealed in the former system creates a welfare trade-off between the two systems. Naik, Neuberger, and Viswanathan (1999) show that trade disclosure can reduce welfare in a dealership environment. Marin and Rahi (2000) analyze a similar trade-off in a security design problem.

where ε_i is unobserved noise. Because no one is privately informed about $\sqrt{1-t}v_0$, a constant t in (1) measures *the scope* of private information. If t is zero, the signal s_i provides no information about the payoff $v = v_0$. As t increases, the larger fraction of the payoff is subject to information asymmetry. The signal noise ε_i can be cross-sectionally correlated:

$$\varepsilon_i = \sqrt{1-w}\epsilon_0 + \sqrt{w}\epsilon_i, \quad w \in [0,1], \tag{2}$$

where ϵ_0 and $\{\epsilon_i\}_{i=1}^{n+1}$ are independently normally distributed with mean zero and variance τ_ε^{-1}. A constant w in (2) measures *how differentially informed* traders are. If w is zero, the signals $\{s_i\}_{i=1}^{n+1}$ are identical and the information is symmetric. As w increases, the amount of information traders can share increases as they are more differentially informed. The parameters $(t, w) \in [0,1]^2$ determine the informational environment.[8] The parameterization (1) and (2) ensures that ex ante variances of v and s_i do not depend on (t, w), but the values of (t, w) make a difference at the interim trading stage. The larger they are, the higher is the degree of the information asymmetry.

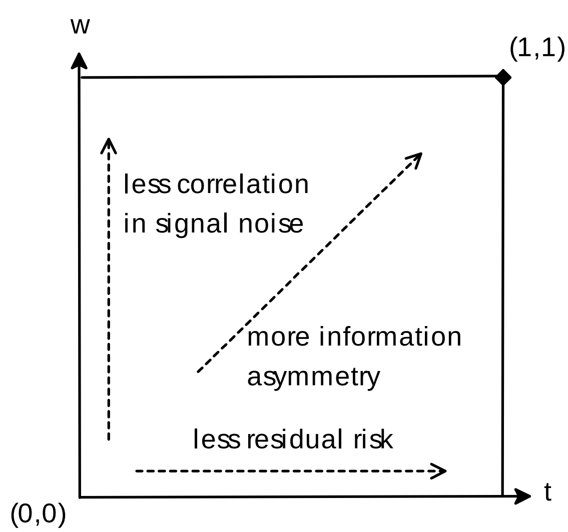

Figure 2. Trading environment parameterized by (t, w).
Note. $\sqrt{1-t}v_0$ measures the residual risk in v not subject to private information.
$\sqrt{1-w}\epsilon_0$ determines the correlation in signal noise ε_i.

[8] Our framework covers Diamond and Verrecchia (1981), Pagano (1989), and Madhavan (1992).

In **Figure 2**, a symmetric information benchmark corresponds to two axes. While we present $(t, w) = (1, 1)$ as our baseline case, our main result is that the optimal market size is finite for all (t, w) such that $tw > 0$. In sum, the random variables $\left(v_0, \widetilde{v}, \{e_i\}_{i=1}^{n+1}, \epsilon_0, \{\epsilon_i\}_{i=1}^{n+1}\right)$ are normally and independently distributed with zero means, and variances

$$Var[v_0] = Var[\widetilde{v}] = \tau_v^{-1}, Var[e_i] = \tau_x^{-1}, Var[\epsilon_0] = Var[\epsilon_i] = \tau_\varepsilon^{-1}.$$

Each trader has an exponential utility function with a risk-aversion coefficient $\rho > 0$

$$U(\pi_i) = -\exp(-\rho \pi_i), \ \pi_i = v(q_i + e_i) - pq_i. \tag{3}$$

The net payoff π_i consists of the gross payoff from the *allocation* $q_i + e_i$ and the *payment* pq_i for trading q_i units of the endowment good at price p. The price p is the only cost of trading, until we introduce a fixed fee in Section 5.[9]

Key parameters. We use the following "normalized" parameters for our analysis:

$$\alpha \equiv \frac{\rho^2}{\tau_v \tau_x}, \ d_\varepsilon \equiv \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_v}, \ \alpha_\varepsilon \equiv \frac{\rho^2}{\tau_\varepsilon \tau_x}.$$

First, α governs *the fundamental gains from trade*, because a large α is associated with (i) a large ex ante risk $\frac{1}{\tau_v \tau_x}$ and/or (ii) a high level of risk aversion ρ, both of which make risk-sharing more valuable. We maintain the following assumption throughout the paper:

$$\alpha < 1.$$

This ensures that the ex ante welfare is well-defined with our preference specification. The other parameters $(d_\varepsilon, \alpha_\varepsilon)$ affect equilibrium beliefs. In the model, there are three timings for beliefs: ex ante, interim, and posterior. First, $d_\varepsilon \in (0, 1)$ measures the distance between

[9] The Appendix A also presents an extension with a volume tax/discount $-\frac{c_q}{2} q_i^2$.

the prior information τ_v and the private information τ_ε (i.e., ex ante and interim beliefs). Second, we will show that α_ε affects the amount of information aggregated by prices, hence the distance between interim and posterior beliefs. We fix α and study comparative statics in $d_\varepsilon \in (0,1)$, which corresponds to $\tau_\varepsilon \in (0,\infty)$. By $\alpha_\varepsilon = \frac{1-d_\varepsilon}{d_\varepsilon}\alpha$, a change in d_ε simultaneously changes the interim and posterior beliefs.

2.2 Welfare measure

Given the ex ante symmetry of traders, we base our welfare analysis on the individual *ex ante gains from trade* (henceforth GFT).

Definition 1 (gains from trade)
The ex ante payoff is $\Pi \equiv -\frac{1}{\rho}\log\left(E\left[\exp\left(-\rho\pi_i\right)\right]\right)$.
The ex ante no-trade payoff is $\Pi^{nt} \equiv -\frac{1}{\rho}\log\left(E\left[\exp\left(-\rho v e_i\right)\right]\right)$.
The ex ante gains from trade is $G \equiv \Pi - \Pi^{nt}$.

Some intuition for the main result can be gained from (3) and **Definition 1**. If a market maker offers a fixed pair of price and trade (p, q_i), the only ex ante risk in (3) would be the variation in v and e_i. Clearly, such a rigid trade arrangement is neither efficient nor individually rational after the realization of endowment shocks.[10] In stead, we allow traders to submit demand functions contingent on *possible* values of p so that the market maker can choose a particular p to clear the market. Without the signals, the price and trade respond only to endowment shocks. This makes (p, q_i) variable, but because endowment shocks are idiosyncratic, the ex ante risk of $q_i + e_i$ and p decreases as the market size grows. As a result, the ex ante payoff and the GFT increase in the market size. With the signals, submitted orders depend on the signals, and so does the price to clear the market. When the price aggregates the information contained in many signals, it may become a very accurate

[10] Obviously, an offer of no-trade ($p = q_i = 0$) is weakly individually rational.

indicator of the value, i.e., $p \approx v$. This implies that π_i in (3) approaches the no-trade payoff, because $(v - p)\, q_i + ve_i \approx ve_i$. Thus, the ex ante gains from trade can approach zero even if $q_i \neq 0$. At the interim stage, traders do not care about the price risk because they submit price-contingent orders, but by speculating on the signals, they pass the risk of v on p, which is *uninsurable ex ante*. Therefore, more traders who make prices more informative at the interim stage can reduce welfare from the ex ante perspective.

To formalize the claim made above, we study the optimal market size defined as the number of traders that maximizes the GFT.

Definition 2 (optimal market size)

$$n^* \equiv \arg\max_{n \geq 1} G. \tag{4}$$

Remark 1. The definition (4) does not take into account costs of organizing a market. This reflects our emphasis on the role of trading rules and the associated endogenous information structure in shaping market structures. A popular view is that (4) is infinite due to positive externalities but intermediation costs constrain the market size. We do not necessarily disagree with this view. However, in this paper we argue that the optimal market size can be finite *without exogenously imposed intermediation costs.* Our definition makes it clear that negative externalities arise as an endogenous feature of the trading activity.

Remark 2. In general, $(n + 1)\, G$, the welfare of $n + 1$ traders *in a single market*, is not maximized by n^*. In fact, in Section 5 we show that $(n + 1)\, G$ is maximized at the size larger than n^*. However, if there is a large population, it is best to have multiple markets, each of the size n^*, rather than a single large market. For example, if the population size is $10n^*$, no market structure can improve on 10 markets of size n^*.

2.3 Solution concepts

We characterize an equilibrium in demand functions a la Kyle (1989). After observing (e_i, s_i), each trader chooses her order $q_i(p; e_i, s_i)$. Because orders can be explicitly conditioned on p, traders internalize information contents of the market-clearing price. This solution concept captures the idea that rational traders learn from the systematic relationship between market-clearing prices and their strategies. Importantly, because equilibrium beliefs depend on the market size, our definition of the optimal market size based on *the ex ante welfare* is appropriate. Market-clearing prices must satisfy

$$\sum_{i=1}^{n+1} q_i(p; e_i, s_i) = 0. \tag{5}$$

To make explicit the dependence of the market-clearing price and an allocation on the strategies, write $p = p(q)$ and $q_i = q_i(q)$, where $q = (q_1, ..., q_{n+1})$ is a vector of strategies. An equilibrium with imperfect competition is a strategy profile q that satisfies

$$\forall i \in \{1, ..., n+1\}, \; E[U((v - p(q)) q_i(q) + ve_i)] \geq E[U((v - p(q')) q_i(q') + ve_i)] \tag{6}$$

for any q' differing from q only in the i-th component. We call this equilibrium *a strategic equilibrium*, where traders realize that their orders affect p. An alternative solution concept, *a price-taking equilibrium*, is defined by replacing $p(q')$ with $p(q)$ in (6).

Price-taking v.s. strategic. A price-taking equilibrium is Pareto efficient with respect to equilibrium beliefs, while a strategic equilibrium is not. However, we show in the Appendix A that our main result holds for both solution concepts (**Lemma A1**), and also that GFT in a price-taking equilibrium can be implemented by a volume discount (**Lemma A2**). We use a price-taking equilibrium in the main text for the ease of the presentation.

Equilibrium beliefs. We use $E_i[\cdot]$ to denote trader i's conditional expectation $E[\cdot|e_i, s_i, p]$ based on his private information and the market-clearing price. Let $Var_i[\cdot] = Var[\cdot|e_i, s_i, p]$

be a conditional variance operator. We define trader i's *interim payoff* Π_i by

$$E_i[-\exp(-\rho\pi_i)] = -\exp(-\rho\Pi_i). \tag{7}$$

Given the normality of equilibrium beliefs that v follows $N(E_i[v], Var_i[v])$,

$$\Pi_i = E_i[v](q_i + e_i) - \frac{\rho}{2} Var_i[v](q_i + e_i)^2 - pq_i.$$

Given the price-taking behavior, the first-order condition is

$$E_i[v] - \rho Var_i[v](q_i + e_i) = p, \tag{8}$$

and the second-order condition $\rho Var_i[v] > 0$ is satisfied. From (8), the optimal order is

$$q_i(p; e_i, s_i) = \frac{E_i[v] - p - \rho Var_i[v] e_i}{\rho Var_i[v]}. \tag{9}$$

The value of $(t, w) \in [0,1]^2$ affects the optimal order (9) through the equilibrium belief $(E_i[v], Var_i[v])$. Due to the normality of distributions and the ex ante symmetry, the conditional variance depends neither on the realization of (e_i, s_i) nor the identity of traders. Therefore, we introduce the following notations:

$$\tau_1 \equiv (Var_i[\widetilde{v}])^{-1} \text{ and } \tau \equiv (Var_i[v])^{-1}.$$

From (1), $\tau = \left(\frac{1-t}{\tau_v} + \frac{t}{\tau_1}\right)^{-1}$. Overall, the belief updating is summarized by

$$\frac{\tau_v}{\tau} = 1 - t + t\frac{\tau_v}{\tau_1}. \tag{10}$$

The way the market size n affects (10) plays an important role in our analysis.

3 Symmetric Information Benchmark

We use the notation $\bar{e} \equiv \frac{1}{1+n}\sum_{i=1}^{n+1} e_i$ etc for the market average. When $t = 0$, no one has a useful signal and (10) becomes $\frac{\tau_v}{\tau} = 1$. When $w = 0$, all traders have an identical signal $s_i = \widetilde{v} + \epsilon_0$. The symmetric belief is given by $E_i[\widetilde{v}] = \frac{\tau_\varepsilon s_i}{\tau}$ and $\tau_1 = \tau_v + \tau_\varepsilon$, and (10) becomes

$$\frac{\tau_v}{\tau} = 1 - td_\varepsilon \leq 1. \tag{11}$$

Lemma 1 (symmetric information benchmark) *Assume $t = 0$ or $w = 0$. Equilibrium trade, price, and the GFT are*

$$q_i^* = \bar{e} - e_i, \;\; p^* = \sqrt{t}d_\varepsilon\left(\widetilde{v} + \epsilon_0\right) - \frac{\rho}{\tau_v}\frac{\tau_v}{\tau}\bar{e},$$

$$G = \frac{1}{2\rho}\log\left\{1 + \alpha\frac{\tau_v}{\tau}\frac{n}{1+n}\left(1 + \frac{\alpha}{1-\alpha}\frac{n}{1+n}\right)\right\}, \tag{12}$$

where $\frac{\tau_v}{\tau}$ is given by (11)*. The GFT* (12) *increase in n with the limit*

$$\lim_{n\to\infty} G = \frac{1}{2\rho}\log\left\{1 + \frac{\alpha}{1-\alpha}\frac{\tau_v}{\tau}\right\}. \tag{13}$$

Because endowment shocks are iid, as n increases, the allocation $q_i^* + e_i = \bar{e}$ become less volatile. This positive externality is captured by $\frac{n}{1+n}$ in (12).[11] For a finite n, the allocation and the price respond to the endowment shocks. However, as $n \to \infty$ the average endowment $\bar{e}$ converges to the ex ante mean of the endowment shock (which are normalized to zero) in probability. Therefore, as $n \to \infty$, $q_i^* + e_i \to 0$ and the ex ante price risk monotonically decreases to $p^* = \sqrt{t}d_\varepsilon\left(\widetilde{v} + \epsilon_0\right)$.

Hirshleifer discount. We call $\frac{\tau_v}{\tau} = 1 - td_\varepsilon$ *the Hirshleifer discount* of the GFT. The

[11] The GFT also increase in $\alpha = \frac{\rho^2}{\tau_v\tau_x}$. As noted in Section 2, we need $\alpha < 1$ for the ex ante welfare to be well-defined. As $\alpha \to 1$, G goes to positive infinity, while as $\alpha \to 0$, G goes to its minimum value, zero. This comparative statics also holds with information asymmetries.

term $\frac{\tau_v}{\tau}$ captures the *quality* effect working through the assessment of the risk. It appears in the GFT and the price but not in the trade and the allocation. Given $t > 0$, τ is larger than τ_v and it reduces *the perceived value of risk sharing.* This can be seen in (12) where $\frac{\tau_v}{\tau}$ replaces $\alpha = \frac{\rho^2}{\tau_v \tau_x}$ with a smaller value $\frac{\rho^2}{\tau \tau_x}$. Also, an increase in t or d_ε reduces $\frac{\tau_v}{\tau} = 1 - td_\varepsilon$, hence it leads to more discounting (see **Figure 3**). However, the magnitude of the Hirshleifer discount is unrelated to the market size, because *beliefs are independent of* n. In the next section, we allow for asymmetric information and study how the market size affects welfare through its impact on the information aggregation.

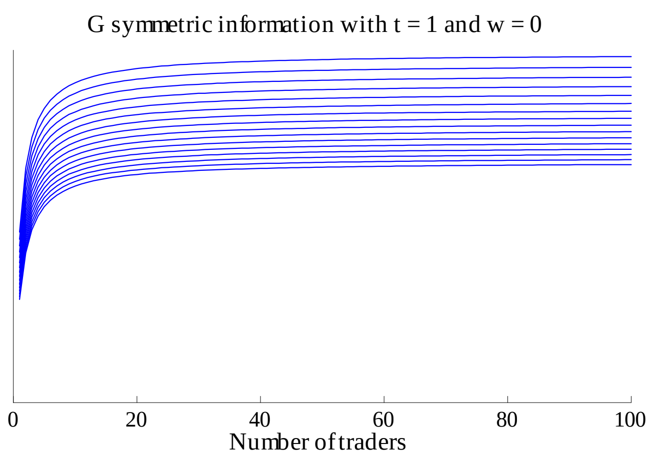

Figure 3. Symmetric information benchmark for different values of d_ε.
Note. The gains from trade as a function of the market size when $t = 1$ and $w = 0$. The lower lines correspond to higher values of d_ε.

4 Optimal Market Size with Asymmetric Information

In this section, we assume $t > 0$ and $w > 0$, i.e. traders are differentially informed. Therefore, equilibrium beliefs are affected by the market size n. Because $\frac{\tau_v}{\tau}$ decreases in n as shown below, the market size has two opposing effects: it increases risk-sharing gains for a fixed belief (quantity effect), while it leads to the larger Hirshleifer discount (quality effect). We need to compare these two forces to determine the optimal market size. We characterize an equilibrium by a guess-and-verify method. First, conjecture a strategy

$$q_i\left(p; e_i, s_i\right) = \beta_s s_i - \beta_e e_i - \beta_p p. \tag{14}$$

We derive the best response to (14) and solve a fixed point problem in $(\beta_s, \beta_e, \beta_p)$.[12] To focus our attention on welfare consequences of the information aggregation, details of the equilibrium characterization are gathered in the Appendix B. The next subsection provides an intuitive explanation of the key mechanism, followed by the two subsections which characterize equilibrium beliefs and the GFT. Subsection 4.4 contains our main result.

4.1 Information aggregation and trading motives

We explain (i) how the information aggregation and traders' motives are jointly determined, and (ii) how the market size affects this joint determination. Combining (14) with the market-clearing condition (5), trader i can construct

$$h_i \equiv \frac{n\beta_p p - q_i}{n\beta_s} = \widetilde{v} + \left(\overline{\varepsilon}_{-i} - \frac{\beta_e}{\beta_s}\overline{e}_{-i}\right), \tag{15}$$

where $\overline{\varepsilon}_{-i}$ and $\overline{e}_{-i}$ denote the average except trader i, i.e. $\overline{e}_{-i} \equiv \frac{1}{n}\sum_{j\neq i} e_j$ etc. The informativeness of (15) depends on two factors:

(i) Traders' motives $\frac{\beta_e}{\beta_s}$ (the intensive margin),

(ii) Market size n (the extensive margin).

For the intensive margin, suppose that traders ignore their signals. Trading would be driven by risk-sharing and the market-clearing price will not aggregate any information. This corresponds to setting $\beta_s = 0$, which destroys the information in (15). At the opposite extreme, if

[12] The linearity is not an exogenous constraint on equilibria because the best response is not constrained to be linear. We follow the market microstructure literature by focusing on this class of equilibria. To the best of our knowlege, the existence (or non-existence) of other types of equilibria is an open question.

traders ignore the risk of their endowment value and trade only for the informational reason, then the price would aggregate private information effectively. This corresponds to setting $\beta_e = 0$, which makes (15) equivalent to $\bar{s}_{-i}$. In general, the information aggregated by (15) depends on the balance between the two motives $\frac{\beta_e}{\beta_s}$, chosen by traders. For the extensive margin, note that (15) is the information contained in the *average* of submitted orders.[13] When many idiosyncratic shocks are averaged, they cancel out each other. A larger market can aggregate more information by removing idiosyncratic noise more effectively.

Conversely, trading motives depend on the information aggregation via rational expectations. Information in prices affects marginal valuations through $(E_i[v], \tau)$. When traders equate the marginal value of the endowment good $E_i[v] - \frac{\rho}{\tau}(q_i + e_i)$ to its marginal cost p, the balance of the two motives is endogenously determined. In general, the impact of reduced noise in prices on $\frac{\beta_e}{\beta_s}$ is ambiguous: while β_s becomes smaller as the informational weight on prices relative to signals becomes larger in the Bayesian learning, β_e also becomes smaller as the perceived risk of the endowment value decreases.

In sum, the market size affects the two-way interaction between the information aggregation and trading motives by *changing the amount of idiosyncratic noise in prices*. This mechanism is assumed away in a model with a continuum of traders, because, by construction, the Law of Large Number purges all the idiosyncratic noise from the model.[14]

4.2 Information aggregation

This subsection characterizes the information aggregation in three steps:

Step 1. Given a conjectured strategy (14), characterize beliefs $E_i[v]$, τ_1, and τ.

Step 2. Derive the optimal order $q_i(p; e_i, s_i)$ from (9).

Step 3. Characterize $\frac{\beta_e}{\beta_s}$ as a solution to a fixed point problem.

[13] The market clearing condition is equivalent to $\frac{1}{n}q_i = -\frac{1}{n}\sum_{j\neq i} q_j(p) = \beta_s \bar{s}_{-i} - \beta_e \bar{e}_{-i} - \beta_p p$.

[14] In such a model, one can add another unobservable random variable in the market-clearing condition, and call it "aggregate supply/demand" or noise traders. For our purpose, this is not satisfactory because it obscures the meaning of the market size and welfare.

Step 1. We define

$$\varphi \equiv \left\{1+\frac{\tau_\varepsilon}{\tau_x}\left(\frac{\beta_e}{\beta_s}\right)^2\right\}^{-1} \in (0,1) \tag{16}$$

to write the variance of the noise in (15) as

$$Var\left[\overline{\varepsilon}_{-i}-\frac{\beta_e}{\beta_s}\overline{e}_{-i}\right]=\frac{1}{n\tau_\varepsilon}\left\{((1-w)\,n+w)+\left(\frac{\beta_e}{\beta_s}\right)^2\frac{\tau_\varepsilon}{\tau_x}\right\}=\frac{1}{n\tau_\varepsilon}\left\{\frac{1}{\varphi}+(1-w)\,(n-1)\right\}.$$

Let Σ be the covariance matrix of $[s_i,e_i,h_i]^\top$. It is easily verified that $Cov\left[\widetilde{v},[s_i,e_i,h_i]\right]=\left[\frac{1}{\tau_v},0,\frac{1}{\tau_v}\right]$. By the Bayes' rule, $E_i[\widetilde{v}]=\left[\frac{1}{\tau_v},0,\frac{1}{\tau_v}\right]\Sigma^{-1}\left[s_i,e_i,h_i\right]^\top$ and

$$\begin{aligned}\tau_1 &= \left(\tau_v^{-1}-\left[\frac{1}{\tau_v},0,\frac{1}{\tau_v}\right]\Sigma^{-1}\left[\frac{1}{\tau_v},0,\frac{1}{\tau_v}\right]^\top\right)^{-1} \\ &= \tau_v+\tau_\varepsilon\frac{1-\varphi+(1+n)\,w\varphi}{1-\varphi+\{1+(1-w)\,n\}\,w\varphi}.\end{aligned} \tag{17}$$

Using $E_i[v]=\sqrt{t}E_i[\widetilde{v}]$ and $\tau=\left(\frac{1-t}{\tau_v}+\frac{t}{\tau_1}\right)^{-1}$, we obtain beliefs as a function of $\frac{\beta_e}{\beta_s}$. Thus, an informational impact of the strategy (14) is summarized by (16). To gain some intuition for (16) and (17), note that τ_1 is bounded from below by $\tau_v+\tau_\varepsilon$: a case where traders ignore the information in prices. Also it is bounded from above by a case where each trader observes all signals $(s_1,..,s_{n+1})$. By the Bayes rule, this upper bound is

$$\left(Var_i\left[\widetilde{v}|s_1,..,s_{n+1}\right]\right)^{-1}=\tau_v+\tau_\varepsilon\frac{1+n}{1+(1-w)\,n}. \tag{18}$$

These two bounds for τ_1 are attained by (17) when $\varphi=0$ and when $\varphi=1$, respectively.[15] Therefore, (16) measures *the fraction* of the information contained in one signal that is revealed by prices. If φ is zero, equilibrium prices do not reveal any information about

[15] With $w=1$, signals are conditionally independent and (17) becomes $\tau_1=\tau_v+\tau_\varepsilon(1+n\varphi)$ while the upper bound (18) is $\tau_v+\tau_\varepsilon(1+n)$. With $w=0$, the upper bound (18) is equivalent to the lower bound $\tau_v+\tau_\varepsilon$, because signals are identical. With $t=0$, $\tau=\tau_v$ and τ_1 is irrelevant.

$(s_1, .., s_{n+1})$. If φ is one, prices allow traders to share the information in $(s_1, .., s_{n+1})$ perfectly. With an endogenously determined $\varphi \in (0,1)$, $n\varphi$ measures *the total amount* of information each trader learns from prices. In the following analysis, φ plays a central role.

Step 2. Given the characterized belief $(E_i[v], \tau)$, (9) determines the best response. This can be shown to be linear in (s_i, e_i, p). By equating this with (14), we obtain a fixed point problem in $\left(\beta_s, \beta_e, \beta_p\right)$. To characterize equilibrium beliefs, it suffices to look at the balance of the two motives $\frac{\beta_e}{\beta_s}$. In the Appendix B, we show

$$\sqrt{t}\frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s} = \frac{\tau_1}{\tau}\frac{1+(1-w)(nw-1)\varphi}{1-(1-w)\varphi}. \tag{19}$$

This defines a fixed point problem in $\frac{\beta_e}{\beta_s}$, because φ is a function of conjectured $\frac{\beta_e}{\beta_s}$ as in (16).

Step 3. It is convenient to define $k \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$ so that (16) and (19) become

$$\varphi = \left(1+\alpha_\varepsilon k^2\right)^{-1}, \tag{20}$$

$$k = \frac{\tau_1}{\sqrt{t}\tau}\frac{1+(1-w)(nw-1)\varphi}{1-(1-w)\varphi}. \tag{21}$$

The two conditions (20) and (21) represent the two-way interaction discussed in the previous subsection: (20) describes how the information aggregation depends on the balance of two motives k, while (21) describes how the balance responds to the degree of the information aggregation φ. By combining (20) and (21), we obtain the equation that determines $k = \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$:

$$F(k) \equiv \left(\alpha_\varepsilon k^2 + w\right)\left(\sqrt{t}k - \frac{1-td_\varepsilon}{1-d_\varepsilon}\right) - \ w\left(\frac{1-td_\varepsilon}{1-d_\varepsilon} - w\right)n = 0. \tag{22}$$

This has a unique solution $k \geq \frac{1}{\sqrt{t}}\frac{1-td_\varepsilon}{1-d_\varepsilon}$. By investigating (20) and (22), we characterize how $(k, \varphi, n\varphi)$ depens on n. Importantly, $k=1$ if $t=w=1$, but otherwise k increases in n. When $k=1$, from (20) φ is independent of n and $n\varphi$ linearly increases in n. When k increases in n, traders' motive tilts toward risk-sharing as the market size grows. This makes prices more noisy and lowers φ. However, whether the total amount of the aggregated information $n\varphi$ goes up or down depends on the speed at which φ decreases in n.

Lemma 2 (information aggregation)

If $t = w = 1$, then $\varphi = (1+\alpha_\varepsilon)^{-1}$ and $\tau = \tau_v + \tau_\varepsilon(1+n\varphi)$.

Otherwise, φ decreases in n at the rate $n^{-\frac{2}{3}}$ and τ increases in n but $\lim_{n\to\infty} \frac{\tau_v}{\tau} > 0$.

When $t = w = 1$, (17) simplifies to $\tau_1 = \tau_v + \tau_\varepsilon(1+n\varphi)$. Because φ is independent of n, $\tau = \tau_1$ increases in n linearly. In this case, the Hirshleifer discount $\frac{\tau_v}{\tau}$ has a strong impact on the GFT as n increases. When $t < 1$ or $w < 1$, φ decreases in n but $n\varphi$ still increases in n without a bound, although at a much slower rate of $n^{\frac{1}{3}}$. In this case, τ increases in n but has a finite upper bound. We conclude that with information asymmetries, $\frac{\tau_v}{\tau}$ decreases in the market size, i.e. the Hirshleifer discount is a source of *negative externalities.*

4.3 Gains From Trade

We explicitly derive the GFT to identify different channels through which the market size affects welfare. Readers not interested in details can go directly to **Proposition 1**.[16] Fist, we characterize the interim payoff Π_i defined by (7) and the interim no-trade payoff Π_i^{nt} defined by $E_i[-\exp(-\rho v e_i)] = -\exp(-\rho \Pi_i^{nt})$. We also define the *interim* gains from trade $G_i \equiv \Pi_i - \Pi_i^{nt}$, and its certainty equivalent value $\widetilde{G}$ by $E[-\exp(-\rho G_i)] = -\exp\left(-\rho \widetilde{G}\right)$. Note that $\widetilde{G}$ and $G \equiv \Pi - \Pi^{nt}$ are *not* equivalent.

Lemma 3 (interim characterization)

$$\Pi_i = \tfrac{\tau}{2\rho}\left(a_i^2 + b_i^2 - c_i^2\right),\ \Pi_i^{nt} = \tfrac{\tau}{2\rho}\left(b_i^2 - c_i^2\right),\ \textit{and}\ G_i = \tfrac{\tau}{2\rho} a_i^2,$$
$$\textit{where}\ a_i \equiv E_i[v] - \left(p + \tfrac{\rho}{\tau} e_i\right),\ b_i \equiv E_i[v],\ c_i \equiv E_i[v] - \tfrac{\rho}{\tau} e_i.$$

[16] A technical contribution lies in **Lemma 3**, which derives a representation of interim payoffs that do not contain cross-products of different random variables. This greatly simplifies the algebras.

Lemma 3 presents a useful representation of (Π_i, Π_i^{nt}, G_i), because (a_i, b_i, c_i) are jointly normally distributed. From the definition of the ex ante and interim payoffs,

$$\begin{aligned} -\exp(-\rho\Pi) &= E\left[-\exp(-\rho\Pi_i)\right], \\ -\exp\left(-\rho\Pi^{nt}\right) &= E\left[-\exp\left(-\rho\Pi_i^{nt}\right)\right]. \end{aligned} \tag{23}$$

To characterize $\left(\Pi, \Pi^{nt}, \widetilde{G}\right)$, we apply the following fact to (Π_i, Π_i^{nt}, G_i):

Fact 1. *Given the n-dimensional random vector z that is normally distributed with mean zero and variance-covariance matrix Σ, $E[-\exp(-\rho(zCz^\top))] = -\{\det(I_n + 2\rho\Sigma C)\}^{-\frac{1}{2}}$, where I_n is the n-dimensional identity matrix and C is an n-by-n matrix.*

By **Lemma 3**, **Fact 1** and (23), we obtain $\exp(2\rho\Pi) = \det(I_3 + 2\rho\Sigma_{abc}C)$, where

$$C \equiv \frac{\tau}{2\rho}\begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix} \text{ and } \Sigma_{abc} \equiv Var\left[[a_i, b_i, c_i]\right] = \begin{bmatrix} V_a & V_{ab} & V_{ac} \\ & V_b & V_{bc} \\ & & V_c \end{bmatrix}. \tag{24}$$

Similarly $\exp(2\rho\Pi^{nt})$ and $\exp\left(2\rho\widetilde{G}\right)$ can be characterized. We define

$$\Delta \equiv \tau^2\left(V_{ac}^2 - V_{ab}^2\right) + \tau^3\left(V_{ac}^2 V_b + V_{ab}^2 V_c - 2V_{ab}V_{bc}V_{ac}\right).$$

Lemma 4 (ex ante characterization)

$$\exp(2\rho G) = \exp\left(2\rho\widetilde{G}\right) + \Delta\exp\left(-2\rho\Pi^{nt}\right),$$

where $\exp\left(2\rho\widetilde{G}\right) = 1 + \tau V_a$ *and* $\exp(2\rho\Pi^{nt}) = (1 + \tau V_b)(1 - \tau V_c) + (\tau V_{bc})^2$.

The expressions of G and Π^{nt} are obtained by evaluating (24).[17],[18]

[17] $V_b = Var[E_i[v]] = Var[v] - V_i[v] = \frac{1}{\tau_v} - \frac{1}{\tau}$ is straightforward. Other results such as $V_{ab} = 0$ and $V_a = V_{ac}$ are less intuitive, but follow from the orthogonality of $b_i = E_i[v]$ with respect to endowment, i.e., $Cov[b_i, e_i] = 0$. See the Appendix B for more details.

[18] The ex ante no-trade payoff $\Pi^{nt} = \frac{1}{2\rho}\log(1-\alpha) < 0$ goes to negative infinity as $\alpha \to 1$, while it goes to its maximum value 0 as $\alpha \to 0$. We fix $\alpha \in (0,1)$ to isolate the impact of α on the GFT through Π^{nt}.

Proposition 1 (gains from trade)

$$G = \frac{1}{2\rho} \log\left\{1 + \alpha\frac{\tau_v}{\tau}\frac{n}{1+n}L(\varphi)\left(1 + \frac{\alpha}{1-\alpha}\frac{n}{1+n}L(\varphi)\right)\right\}, \tag{25}$$

$$\textit{where } \frac{\tau_v}{\tau} = \frac{1 - td_\varepsilon + \frac{w}{1-(1-w)\varphi}n\varphi\left(1 - td_\varepsilon - w\left(1 - d_\varepsilon\right)\right)}{1 + \frac{w}{1-(1-w)\varphi}n\varphi\left(1 - w\left(1 - d_\varepsilon\right)\right)} < 1 - td_\varepsilon, \tag{26}$$

$$\textit{and } L(\varphi) \equiv \frac{1-\varphi}{1-(1-w)\varphi} < 1. \tag{27}$$

It is helpful to compare $\exp(2\rho G)$ for (25) and for the symmetric information case (12).

Table 1: Comparison of $\exp(2\rho G)$.

Symmetric information ($t = 0$ or $w = 0$)	$1 + \alpha \times \frac{\tau_v}{\tau} \times \frac{n}{1+n}\left(1 + \frac{\alpha}{1-\alpha}\frac{n}{1+n}\right)$	$\frac{\tau_v}{\tau} = 1 - td_\varepsilon$
Asymmetric information ($tw > 0$)	$1 + \alpha \times \frac{\tau_v}{\tau} \times \frac{n}{1+n}L(\varphi)\left(1 + \frac{\alpha}{1-\alpha}\frac{n}{1+n}L(\varphi)\right)$	$\frac{\tau_v}{\tau} = (26)$ $L(\varphi) = (27)$

The standard quantity effect $\frac{n}{1+n}$ exists in both cases. The Hirshleifer discount also appears in both, but as (26) shows, it becomes *more significant and dependent on n.*

Learning discount. We call $L(\varphi)$ *the learning discount.* While the Hirshleifer discount $\frac{\tau_v}{\tau}$ is about *how much trades learn* about v, the learning discount is about *how much traders substitute trading with learning* and works as a *quantity* discount. Recall that trader i isolates the new information in prices from his own information by subtracting q_i in (15). In the Appendix B, we show that this subtraction leads to

$$E_i[\widetilde{v}] = \frac{\tau_\varepsilon}{\tau_1}\frac{s_i - \varphi\left\{s_i - w\frac{\beta_e}{\beta_s}e_i - w\frac{\beta_p}{\beta_s}(n+1)p\right\}}{1 + (1-w)(nw-1)\varphi}.$$

Thus, the *informational* impact $\varphi > 0$ results in smaller weights on (s_i, e_i, p) in the optimal order (9). This leads to the discounted trading activity.[19]

[19] Because we are assuming a price-taking behavior, the learning discount is orthogonal to a standard "price

For the optimal market size, what is important is *how the various discounts change with the market size.* **Lemma 2** shows that φ weakly decreases in n, i.e., each trader's informational impact becomes smaller as n increases. Because $L(\varphi)$ decreases in φ, the learning discount is a source of *positive externalities* in our model.[20] In sum, with symmetric information, only the standard quantity effect is responsive to the market size, while the information aggregation creates two more channels through which the market size affects welfare. The next subsection characterizes the combined effect of the three channels.

4.4 Welfare consequences of the information aggregation

When traders use strategies (14), prices aggregate the information, but the quantity traded does not. To see this, note that (14) and the market-clearing condition (5) imply[21]

$$p^* = \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e} \text{ and } q_i^* = \beta_s (s_i - \overline{s}) - \beta_e (e_i - \overline{e}), \tag{28}$$

and that $s_i - \overline{s}$ does not depend on $\widetilde{v}$. The next lemma suggests that welfare consequences of the information aggregation cannot be identified if one focused on the allocation of the endowment good or on the informational efficiency of prices.

Lemma 5 (trade and price in the large market)

As $n \to \infty$,

(a) *trade q_i^* converges to a non-degenerate random variable.*

If $t < 1$ or *$w < 1$, it converges to $-e_i$.*

(b) *price p^* converges to $\frac{\sqrt{t} d_\varepsilon}{(1-w)(1-d_\varepsilon)+d_\varepsilon}\left(\widetilde{v} + \sqrt{1-w}\epsilon_0\right)$.*

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impact" in a strategic equilibrium. In Diamond and Verrecchia (1981) a price-taking behavior is assumed and only the learning discount exists. In Pagano (1989) symmetric information is assumed and only the price impact exists. Kyle (1989) studies an environment with both. The Appendix A studies a strategic equilibrium, and shows our main result is robust to the price impact.

[20] In a previous version of the paper, we present an extension in which the learning discount can become a source of negative externalities. This strengthens our main result that the optimal market size is finite.

[21] From (5) and (14), the equilibrium price is $p^* = \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e}$. Substituting this back into (14) yields q_i^*.

Part (a) shows that there is trading in the limit $n \to \infty$. When $t < 1$ or $w < 1$, a risk-sharing motive becomes more dominant as n increases, and in the limit the allocation of the endowment good is the same as in the symmetric information benchmark. Therefore, one may conclude that the information asymmetry is not a big issue in a sufficiently large market. We show below that such a conclusion is false, because welfare decreases in n for sufficiently large n when information asymmetries exist. Moreover, when $t = w = 1$, the GFT converge to zero even though there is trading in the limit. The key difference from the symmetric information case is that as n increases prices reveal more information *to traders*.[22] In fact, **Part (b)** shows $p^* \to \widetilde{v}$ when $t = w = 1$. In this case, the equilibrium *payoff* approaches the no-trade payoff, i.e. $\pi_i = (\widetilde{v} - p)\, q_i + \widetilde{v} e_i \to \widetilde{v} e_i$, even though the equilibrium trade q_i does *not* approach zero. This indicates that the equilibrium trading activity and allocation are not necessarily a good welfare measure as they do not capture the welfare impact of the aggregate risk endogenously absorbed into prices. Obviously, focusing on the informational efficiency (i.e. how informative p^* is) does not lead to a proper welfare analysis either.

Welfare analysis. We first analyze the case with $t = w = 1$, which gives us a clear intuition. This is a boundary case where reduced noise in prices has exactly offsetting impacts on the two motives. As a result, the balance of two trading motives $k = \frac{\tau_\varepsilon}{\rho} \frac{\beta_e}{\beta_s}$ is independent of n, and so is $\varphi = (1 + \alpha_\varepsilon)^{-1}$. The two types of discounts (26) and (27) simplify to

$$\frac{\tau_v}{\tau} = \frac{1 - d_\varepsilon}{1 + n\varphi d_\varepsilon}, \tag{29}$$

$$L(\varphi) = 1 - \varphi, \tag{30}$$

and only (29) depends on n. From **Proposition 1**, the optimal market size solves

$$\max_n \frac{1}{1 + n\varphi d_\varepsilon} \frac{n}{1 + n} \left(1 + \frac{\alpha}{1 - \alpha} \frac{(1 - \varphi)\, n}{1 + n} \right).$$

[22] In a symmetric information case with $t > 0$ and $w = 0$, as n increases prices reveal more information about the identical signal to *those who only observe prices*, because the noise due to endowment shocks is reduced. However, this does not affect traders' beliefs because they already observed the signal itself.

The first term $\frac{1}{1+n\varphi d_\varepsilon}\frac{n}{1+n}$ is uniquely maximized at $n=\sqrt{\frac{1}{\varphi d_\varepsilon}}$, while $1+\frac{\alpha}{1-\alpha}\frac{(1-\varphi)n}{1+n}$ increases in n with a bound. Hence, the optimal market size is greater than $\sqrt{\frac{1}{\varphi d_\varepsilon}}$ but it is finite.

Proposition 2 (optimal market size with $t=w=1$)

The optimal market size n^* is finite, and $\lim_{n\to\infty} G=0$.

As n increases, the risk-sharing externality $\frac{n}{1+n}$ increases while the Hirshleifer discount (29) decreases. The latter force eventually "wins", because when $\frac{\tau_v}{\tau}$ approaches zero, prices fully absorb the risk of v, making it non-diversifiable. With $t=w=1$, the information aggregation becomes arbitrarily effective in the limit (i.e. $\lim_{n\to\infty}\tau=\infty$), which eliminates the GFT. While an analytical expression is specific to our model, its implication is general: when the information aggregation decreases the amount of the diversifiable risk, the GFT from risk-sharing is reduced.

Remark on the efficiency. Because a single price equates marginal values across traders, a price-taking equilibrium is Pareto efficient *given n and the associated equilibrium beliefs*. However, from an ex ante perspective, the variance of the marginal value also matters for the welfare. Our result indicates that, taking the rational belief updating as a constraint (i.e. the planner cannot prevent traders from learning from the equated marginal value), the ex ante efficient trading arrangement may limit the market size to control the negative informational externality. Therefore, a collection of small markets can be a constrained-efficient market structure. **Figure 4** shows the GFT and n^* for different values of $d_\varepsilon\in(0,1)$.

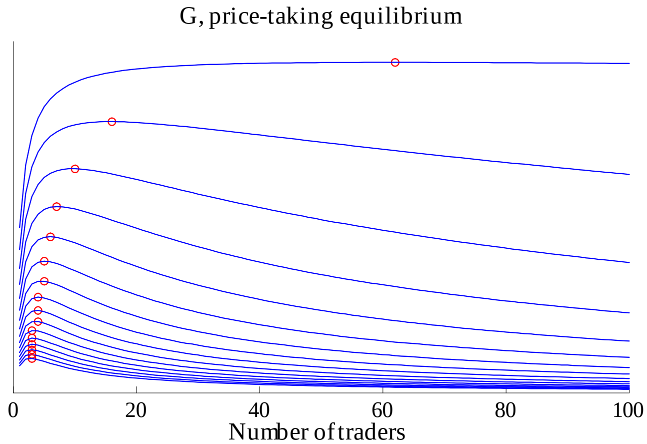

Figure 4. Optimal market size with $t = w = 1$ for different d_ε.
Note. Circle markers indicate the optimal market size.
The lower lines correspond to higher values of d_ε.

A general case with residual risk. When $t < 1$ or $w < 1$, the diversifiable risk never disappears (i.e. $\lim_{n\to\infty} \tau < \infty$). Nevertheless, we show below that the GFT still decrease despite the fact that the allocation converges to that of the symmetric information case. With $t < 1$ or $w < 1$, neither $k = \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$ nor φ is independent of n. We show in the Appendix B that the balance of trading motives *slowly* tilts toward risk-sharing in a larger market, i.e., k increases in n at the rate $n^{\frac{1}{3}}$. Using (20) in (26), the Hirshleifer discount is

$$\frac{\tau_v}{\tau} = \frac{\frac{1-td_\varepsilon}{1-d_\varepsilon} + \frac{wn}{\alpha_\varepsilon k^2+w}\left(\frac{1-td_\varepsilon}{1-d_\varepsilon} - w\right)}{\frac{1}{1-d_\varepsilon} + \frac{wn}{\alpha_\varepsilon k^2+w}\left(\frac{1}{1-d_\varepsilon} - w\right)} \in \left(\frac{\frac{1-td_\varepsilon}{1-d_\varepsilon} - w}{\frac{1}{1-d_\varepsilon} - w}, 1 - td_\varepsilon\right). \tag{31}$$

The market size affects (31) through $\frac{wn}{\alpha_\varepsilon k^2+w}$, which increases in n at the rate $n^{\frac{1}{3}}$. Thus, (31) decreases in n to its lower bound and it is still a source of negative externalities. Next, we denote by B the combined quantity effect of risk-sharing $\frac{n}{1+n}$ and the learning discount (27). Using (20) in (27), the positive externality is summarized by

$$B = \frac{1}{1+n^{-1}}\frac{\alpha_\varepsilon}{\alpha_\varepsilon + wk^{-2}} \in (0,1), \tag{32}$$

which increases in n and approaches 1. Because both (31) and (32) have positive limits, whether the GFT decrease in n for sufficiently large n depends on *which force dies out faster*. The speed at which the negative force (31) dies out is the rate at which $\frac{\alpha_\varepsilon k^2+w}{wn}$ decreases to zero, i.e. $n^{-\frac{1}{3}}$. The speed at which the positive force (32) dies out is the rate at which k^{-2} decreases to zero, i.e. $n^{-\frac{2}{3}}$. Hence, the positive force dies out twice as fast as the negative force does. This creates a hump shape.

Proposition 3 (optimal market size with $tw \in (0,1)$)

The optimal market size n^* is finite, and

$$\lim_{n\to\infty} G = \frac{1}{2\rho}\log\left(1+\frac{\alpha}{1-\alpha}\frac{1-w+(1-t)\frac{d_\varepsilon}{1-d_\varepsilon}}{1-w+\frac{d_\varepsilon}{1-d_\varepsilon}}\right). \tag{33}$$

Proposition 3 extends our main result to all $(t,w) \in (0,1]^2$. The existence of the information asymmetry, no matter how small, is enough to make the optimal market size finite. The limit value of the GFT (33) shows a continuity in t and w. As $t \to 1$ and $w \to 1$, (33) approaches zero, which is the value of $\lim_{n\to\infty} G$ with $tw = 1$. As $t \to 0$ or $w \to 0$, (33) approaches the symmetric information benchmark (13). Thus, (33) applies for all $(t,w) \in [0,1]^2$. However, the impact of the information aggregation *on the optimal market size* is discontinuous in (t,w). As soon as $tw > 0$, no matter how small, the information aggregation in equilibrium creates hump-shaped GFT. Also, when $tw < 1$, no matter how close to 1, $n\varphi$ grows at the much slower rate of $n^{\frac{1}{3}}$ than the rate of n when $tw = 1$.

4.5 Comparative statics in d_ε

We present comparative statics in $d_\varepsilon = \frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_v} \in (0,1)$, which measures the distance between prior and interim beliefs. Using $\alpha_\varepsilon = \frac{1-d_\varepsilon}{d_\varepsilon}\alpha$, we can express (20) as

$$\varphi = \frac{d_\varepsilon}{d_\varepsilon + (1-d_\varepsilon)\,\alpha k^2}. \tag{34}$$

When $t = w = 1$, there is a tight connection between d_ε and φ because $k = 1$. Also, the parameter restriction $\alpha < 1$ implies $d_\varepsilon < \varphi$. An increase in d_ε results in the lower GFT for any given n through the Hirshleifer discount *and* the learning discount.

Lemma 6 (comparative statics in d_ε)

(a) *If* $t = w = 1$, *then* n^* *decreases in* d_ε. *Moreover,* $n^* \in (n^-, n^+)$, *where* $n^\pm$ *defined in the proof satisfy* $1 < n^-$, $\lim_{d_\varepsilon \to 0} n^\pm = \infty$, and $\lim_{d_\varepsilon \to 1} n^\pm = 1$.

(b) $\lim_{d_\varepsilon \to 0} \varphi = 0$ *and* $\lim_{d_\varepsilon \to 0} n^* = \infty$.

$$\lim_{d_\varepsilon \to 1} \varphi = \begin{cases} 1 & \text{if } t = 1, \\ 0 & \text{if } t < 1, \end{cases} \quad \text{and} \quad \lim_{d_\varepsilon \to 1} n^* = \begin{cases} 1 & \text{if } tw = 1, \\ \infty & \text{if } tw < 1. \end{cases}$$

Part (a) is illustrated in **Figure 4**. As $d_\varepsilon \to 0$, φ approaches zero, and the equilibrium belief approaches that of the symmetric information case. Therefore, the negative externality disappears as $d_\varepsilon \to 0$ and the optimal market size increases without a bound. As **Part (b)** shows, this result extends to general $(t, w) \in (0, 1]$. To see why n^* decreases in d_ε, consider $n^*(d_\varepsilon)$ for a given $d_\varepsilon \in (0, 1)$. Because n^* is determined by the trade-off between $\frac{1}{1+n\varphi d_\varepsilon}$ and $\frac{n}{1+n}$, two forces are marginally balanced at $n^*(d_\varepsilon)$. Now suppose d_ε is increased to $d'_\varepsilon > d_\varepsilon$. This raises the negative impact of the marginal trader on welfare through increased φd_ε, while not affecting his allocational impact $\frac{n}{1+n}$. Therefore, the GFT must be *decreasing* in n at $n^*(d_\varepsilon)$, implying $n^*(d'_\varepsilon) < n^*(d_\varepsilon)$. Finally, **Part (b)** indicates the non-monotonicity of the optimal market size with respect to d_ε for $tw < 1$.

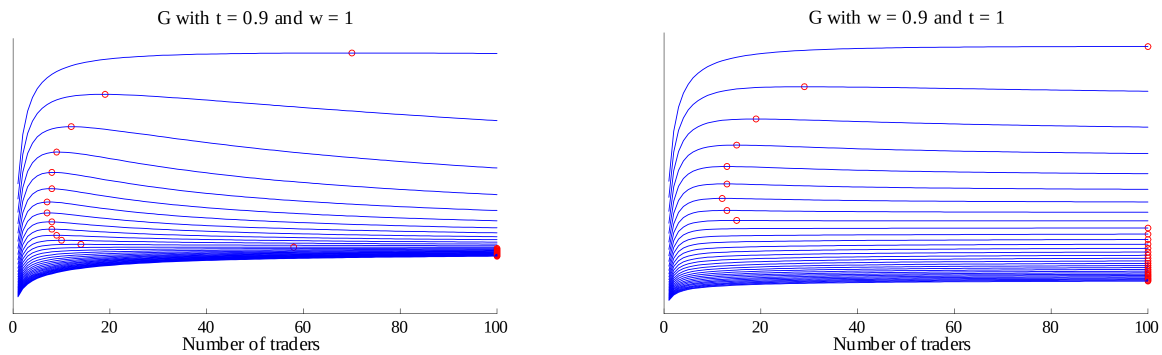

Figure 5. Optimal market size with $tw < 1$ for different d_ε.
Note. Circle markers indicate the optimal market size (markers at 100 only indicate that the optimal market size is above 100). The lower lines correspond to higher values of d_ε. The left panel is for $t = 0.9$, $w = 1$, the right panel is for $w = 0.9$, $t = 1$.

In sum, fine details matter for the behavior of the optimal market size. When the information asymmetry affects the entire payoff risk and signals are conditionally independent (i.e. $t = w = 1$), n^* monotonically decreases in d_ε (**Figure 4**). When a part of the payoff risk is not subject to any private information, or signals have correlated errors (i.e. $tw < 1$), n^* is infinite in both limits $d_\varepsilon \to 0$ and $d_\varepsilon \to 1$ (**Figure 5**).

5 Endogenous Market Structure

This section presents a model of endogenous market structure. We study a two-stage game played by traders and intermediaries at the ex ante stage. As we believe that market formation would take much more time compared to trading, we assume that traders and intermediaries can commit to a market structure determined before the realization of private information to avoid the cost of redesigning it in every information state.

Let $N = \{1, .., \overline{n}\}$ be the set of potential traders and $J = \left\{0, 1, .., \overline{j}\right\}$ be the set of markets. *A market structure* for a given N is a partition of N such that $N = \underset{j \in J}{\cup} N_j$ and $N_j \cap N_k = \emptyset$ for any $j \neq k$. We use N_0 for the set of traders who do not participate in any markets. A lowercase letter n_j denotes the number of traders in N_j. The GFT for each trader in N_j are denoted by $G(n_j)$, while $C(n_j)$ is the total operational cost of a market j with n_j traders. To focus on the implication of $G(\cdot)$, we assume $C(n) = cn$. This assures that market fragmentation is not driven by an assumption on the cost function.[23] Traders decide whether or not to participate in any market based on the GFT G and entry fees for each market. The game proceeds in two steps. First, intermediaries simultaneously set the fees $\{\phi_j\}_{j\in J}$, where $\phi_0 \equiv 0$.[24] Second, traders simultaneously decide which market to participate in, taking the fees as given. After we analyze traders' participation decision for given fees, we let intermediaries compete to determine fees. Trader i's problem is:

$$\max_{r_i=\{r_{i,j}\}_{j\in J}} \sum_{j\in J\backslash\{0\}} r_{i,j} \left\{ G\left(\sum_{i\in N} r_{i,j}\right) - \phi_j \right\} \tag{35}$$

$$\text{s.t. } r_{i,j} \in \{0,1\} \text{ for all } j \in J \text{ and } \sum_{j\in J} r_{i,j} = 1. \tag{36}$$

A participation equilibrium for given $\{\phi_j\}_{j\in J}$ is $\{r_i^*\}_{i\in N}$ such that for all $i \in N$, r_i^* solves (35) subject to (36) given r_{-i}^*. Note that (36) constrains each trader to participate in at most one market. The equilibrium $\{r_i^*\}_{i\in N}$ determines a market size for each j:

$$n_j(\phi_j, \phi_{-j}) = \sum_{i\in N} r_{i,j}^* \tag{37}$$

From intermediary j's perspective, (37) is a demand function. Given (37), intermediaries compete by setting fees $\{\phi_j\}_{j\in J}$. Intermediary j's problem is

$$\max_{\phi_j} (\phi_j - c)n_j, \text{ s.t. } n_j = n_j(\phi_j, \phi_{-j}). \tag{38}$$

[23] Other things equal, a rapidly increasing cost function would trivially make each market small. Qualitative results hold for more general cost functions, as long as $G(n) - \frac{C(n)}{n}$ has a single peak.

[24] There is no room for horizontal differentiation because all traders are ex ante identical. This contrasts our analysis to that of Economides and Siow (1988), which is a model of horizontal differentiation.

A fee-setting equilibrium is $\{\phi_j^*\}_{j\in J}$ such that for all $j \in J\backslash\{0\}$, ϕ_j^* solves (38) given ϕ_{-j}^*.

Equilibrium selection. As traders' participation decisions are complementary, there are many Nash equilibria. To select a non-trivial equilibrium, we use a cooperative notion of stability: no subset of traders in one market has an incentive to deviate.

Definition 3 (stable participation)

A Nash equilibrium $\{\phi_j, n_j\}_{j\in J}$ *is not stable if it satisfies either*

(i) $\exists j,k \in J\backslash\{0\}$ *and* $n \in \{1,..,n_j\}$ *s.t.* $G(n_j) - \phi_j < G(n+n_k) - \phi_k$, *or*

(ii) $\exists j \in J\backslash\{0\}$ *and* $n \in \{1,..,n_0\}$ *s.t.* $0 < G(n+n_j) - \phi_j$.

Definition 3 stipulates that any subsets of traders in each market are free to move if there is a better alternative. By focusing on stable equilibria, we exclude a trivial equilibrium where no trader participates in any markets, as well as a mixed-strategy equilibrium where traders randomly choose a market. Let $0 \leq \underline{\phi} \equiv \lim_{n\to\infty} G(n) < \overline{\phi} \equiv G(n^*)$, where $n^* \equiv \arg\max_n G(n)$. By setting $\phi_j > \overline{\phi}$, intermediary j does not attract any traders, while setting $\phi_j < c$ would only cause a loss. Hence, we can focus on $\phi_j \in [c, \overline{\phi}]$. **Lemma 7** is a necessary condition for the existence of the stable equilibrium.

Lemma 7 *In a stable equilibrium given* $\{\phi_j\}_{j\in J}$, *for all* $j \in J\backslash\{0\}$,

$$G(n_j) - \phi_j \geq \max\{G(n_k+1) - \phi_k, 0\} \textit{ for all } k \in J\backslash\{0,j\}. \tag{39}$$

Additionally, either one of the following two conditions holds:

$$\text{(i) } \textit{For all } j \in J\backslash\{0\},\ G(n_j) \geq G(n_j+1). \tag{40}$$

(ii) $n_0 = 0$ *and there exists one* $j \in J\backslash\{0\}$ *s.t.* $G(n_j) < G(n_j+1)$ *and*

$$G(n_k+1) - \phi_k \leq G(n_j) - \phi_j < \overline{\phi} - \phi_j \leq G(n_k) - \phi_k \text{ for all } k \in J\backslash\{0,j\}. \tag{41}$$

Lemma 7 shows that there are two types of stable equilibria. The first type satisfies (40), i.e., all markets operate on the decreasing part of G. The second type satisfies (41), i.e., all but one market operates on the decreasing part of G. The second type of equilibrium is asymmetric in that the smallest market offers the smallest net benefits to traders compared to all other markets.[25] While the possibility of the asymmetric equilibrium is interesting, we focus on a symmetric equilibrium characterized by (39) and (40) for two reasons. First, given symmetric intermediaries, it seems natural to expect they operate in a symmetric way, if there is such an equilibrium. Second, the asymmetric equilibrium requires that G should be quickly decreasing so that it is not worth switching to larger markets. The numerical evaluation of G shows that it is unlikely to be satisfied (see **Figure 4, 5**).

For the rest of our analysis, we use an approximation to avoid issues associated with the integer restriction.[26] The conditions (39) and (40) are approximated by

$$G(n_j) - \phi_j = G(n_k) - \phi_k \geq 0 \text{ for all } j, k \in J \backslash \{0\}, \tag{42}$$

$$n^* \leq n_j \text{ for all } j \in J \backslash \{0\}. \tag{43}$$

In a participation equilibrium characterized by (42) and (43), all markets operate at the decreasing part of G and provide the same net benefit to traders. The indifference condition (42) implicitly defines demand functions for intermediaries.

Monopoly. When there is one intermediary ($\overline{j} = 1$), the nature of the demand function depends on the relative size of $\overline{n}$ and n^*. If $\overline{n} \leq n^*$, the market size is always $\overline{n}$ as long as $G(\overline{n}) > c$. The demand is completely elastic: setting $\phi \leq G(\overline{n})$ attracts all traders $\overline{n}$ while setting $\phi > G(\overline{n})$ attracts no trader. If $n^* < \overline{n}$, the demand function has an inelastic part: setting $\phi \in (G(\overline{n}), \overline{\phi}]$ would attract only some traders $n(\phi) = G^{-1}(\phi) \in [n^*, \overline{n})$. In this part of the demand function, raising a fee only slightly reduces participating traders.

[25] Traders in the smallest market do not want to move to other markets because if they do, the larger markets will be too "congested".

[26] This approximation was also used by Economides and Siow (1988).

Lemma 8 *If $\overline{n} > n^*$, then the monopoly market size n^m is larger than n^*.*

To see why the monopoly market is larger than n^*, define a marginal surplus function

$$H(n) \equiv \frac{\partial}{\partial n} \{(G(n) - c)\, n\} = G(n) - c + G'(n)n.$$

This is an increase in the social surplus by adding a marginal trader in a market of size n. The term $G(n) - c$ captures the net surplus for the marginal trader, while $G'(n)n$ represents the externality imposed on the others. At n^*, there is no externality ($G'(n^*) = 0$), but the profit is still increasing in the market size because $H(n^*) = G(n^*) - c > 0$. Therefore, it is in the monopoly market's interest to have more than n^* traders.

Competition. We characterize a fee-setting equilibrium assuming that $\overline{j} \geq 2$ competing intermediaries rationally anticipate that a stable participation equilibrium will be played once they set fees. The indifference condition (42) implies that each intermediary offers the same level of net benefit to traders in a participation equilibrium. We assume that intermediaries treat this value as a parameter denoted by U.[27]

$$G\left(n_j\right) - \phi_j = U \geq 0. \tag{44}$$

Because G is invertible at n_j by (43), (44) determines a demand function $n_j(\phi_j) = G^{-1}(U + \phi_j)$. By substituting (44) into (38), intermediary j solves

$$\max_{\phi_j} \{G\left(n_j(\phi_j)\right) - c - U\} n_j(\phi_j). \tag{45}$$

Because $G - c$ is a surplus per trader, U determines the share of the surplus left for each trader. There are two possibilities. First, if $U = 0$, then some traders must be excluded from

[27] This is the market utility approach commonly used in the competitive search literature (Galenianos and Kircher 2012). Alternatively, we could analyze each intermediary's strategic influence on the value of U. A previous version of the paper considers this and obtains a qualitatively similar result.

any markets ($n_0 > 0$). Second, if $U > 0$, then all traders must be participating in markets ($n_0 = 0$). **Lemma 9** summarizes competition between intermediaries.

Lemma 9 *In a fee-setting equilibrium, either*

(i) $U = 0$ *and for all* $j \in J\backslash\{0\}$, $(G(n_j) - c)\, n_j$ *is maximized at* $n_j = n^m \in \left(n^*, \frac{\overline{n}}{\overline{j}}\right)$,

or

(ii) $U = H\left(\frac{\overline{n}}{\overline{j}}\right) > 0$ *and for all* $j \in J\backslash\{0\}$, $(G(n_j) - c)\, n_j$ *is increasing at* $n_j = \frac{\overline{n}}{\overline{j}}$.

In the latter, $\phi_j^* = c - G'\left(\frac{\overline{n}}{\overline{j}}\right)\frac{\overline{n}}{\overline{j}} \geq c$.

Lemma 9 shows that competing intermediaries either (i) behave as if each of them is a monopoly, leaving some traders excluded from markets and extracting all the surplus from participating traders, or (ii) accommodate all traders and extract some, but not all, surplus from traders. In both cases, intermediaries' negotiation power comes from the negative externality among traders. However, for the latter case, each trader receives a surplus that is equal to *his marginal contribution to the surplus generated by one market.* **Figure 6** illustrates the latter case with two markets.

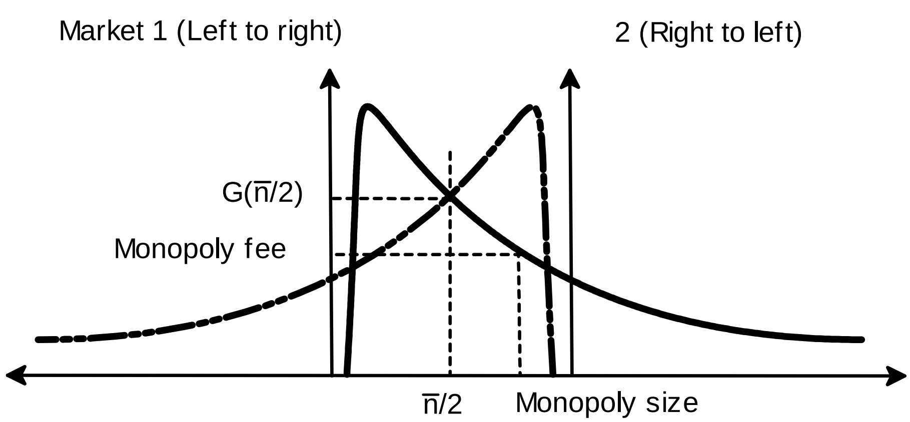

Figure 6. Two markets.

Note. The length between the vertical axes represents the number of potential traders $\overline{n}$. The size of market 1 is measured from the left axis to the right, while the size of market 2 is measured in the opposite direction.

In **Figure 6**, the shared market size is smaller than the monopoly size (i.e., $n^* < \frac{\overline{n}}{2} < n^m$). The equilibrium fee is $\phi^* = c - G'\left(\frac{\overline{n}}{2}\right)\frac{\overline{n}}{2} < G\left(\frac{\overline{n}}{2}\right)$. At the shared market size, the total profit is increasing in the market size, which creates an incentive to set the fee lower than $G\left(\frac{\overline{n}}{2}\right)$. However, such an incentive is not strong enough because the slope of G at the shared market size is negative, i.e. the demand is inelastic. Therefore, the equilibrium fee is higher than marginal costs and intermediaries earn positive profits.[28] From the expression of the fee, zero profit is attained if and only if $\frac{\overline{n}}{2} = n^*$. More generally, for a given population size $\overline{n}$, the free entry into market making should result in approximately $\frac{\overline{n}}{n^*}$ markets.

6 Conclusion

Large and small markets are different because they aggregate the information differently. When traders understand this difference, their ex ante welfare is affected by the market size through its informational implication. We showed that negative externalities arise in a standard risk-sharing model subject to information asymmetries. The result implies that negative externalities in financial markets may be more relevant than usually believed. In related works, Spiegel and Subrahmanyam (1992) show that when traders are limited to using market orders, negative externalities can arise due to price volatility. Foucault and Menkveld (2008) argue that order fragmentation enhances liquidity by reducing limit order congestion. Identifying and quantifying different sources of negative externalities are important tasks for the design of market structures.

In this paper we focused on the welfare of traders. If market prices provide the useful information outside financial markets (e.g. for guiding real investment decisions), the socially optimal market structure should reflect the benefit of the price discovery outside the financial markets. Our analysis of market structures should be taken only as a starting point for a more general analysis of the socially optimal market structure.

[28] This result is related to Kreps and Scheinkman (1983), who show that Bertrand competition combined with capacity constraints yields Cournot outcomes. Intermediaries in our model are not subject to physical capacity constraints, but they use *a trading rule*, which creates negative externalities among traders. This works as an *endogenous* capacity constraint.

7 Appendix A

We present three extensions: (i) a strategic equilibrium, (ii) the transaction fee and the implementation of a price-taking equilibrium, and (iii) the market-size dependent signal quality τ_ε.

7.1 Strategic equilibrium

In a strategic equilibrium, traders are aware that their orders move prices. From strategies (14) and the market-clearing condition (5),

$$-q_i = \sum_{j\neq i} q_j = \beta_s \sum_{j\neq i} s_j - \beta_e \sum_{j\neq i} e_j - n\beta_p p.$$

Solving for the price isolating the impact of q_i,

$$\begin{aligned} p &= p_i + \lambda q_i, \\ \text{where } p_i &\equiv \frac{\beta_s}{\beta_p}\overline{s}_{-i} - \frac{\beta_e}{\beta_p}\overline{e}_{-i} \text{ and } \lambda \equiv \frac{1}{n\beta_p}. \end{aligned} \tag{46}$$

From trader i's point of view, p is stochastic due to p_i, but he is aware how q_i affects the mean of p through (46). The first- and second-order conditions of the trader i's problem are

$$E_i[v] - \frac{\rho}{\tau}(q_i + e_i) = p + \lambda q_i, \tag{47}$$

$$2\lambda + \frac{\rho}{\tau} > 0. \tag{48}$$

From (47), we obtain the optimal order

$$q_i^*(p; e_i, s_i) = \frac{E_i[v] - p - \rho Var_i[v]\, e_i}{\lambda + \rho Var_i[v]}. \tag{49}$$

Comparing (49) with (9) in a price-taking equilibrium, the only difference is a constant λ. Because this does not change the linearity of the best response as well as the balance between two trading motives, all the *informational* properties of a price-taking equilibrium carry over to a strategic equilibrium, given that it exists. Therefore, conditional on the additional parameter restriction imposed by (48), our main result holds. In the Appendix B, we derive the following expression

$$\exp(2\rho G) = 1 + c_v \frac{\tau_v}{\tau}\widetilde{B}\left(1 + \frac{c_v}{1-c_v}B\right),$$

$$\text{where } \frac{\widetilde{B}}{B} = 1 - \left(\frac{\left(w - \frac{1-w}{n}\right)\varphi + \frac{1}{n}}{1-\varphi}\right)^2 < 1. \tag{50}$$

The only difference form the price-taking case is $\widetilde{B}$, which is discounted from $B = \frac{n}{1+n}\frac{1-\varphi}{1-(1-w)\varphi}$ by *the strategic discount* (50). This shows that for a given n the GFT in a strategic equilibrium are smaller than in a price-taking equilibrium. Because φ weakly decreases in n, (50) strictly increases in n with a finite upper bound. Therefore, the strategic discount is a source of *positive externalities* in our model. Because a strategic equilibrium closes a gap from a price-taking equilibrium as the market size grows, the optimal market size is larger in a strategic equilibrium than in a price-taking equilibrium.

Lemma A1 (strategic equilibrium)
(a) *A strategic equilibrium exists if and only if* $1 < n$ *and*

$$\frac{n+1}{n-1} < \frac{1-\varphi}{\varphi} + 1 - w. \tag{51}$$

If $t < 1$*, then there is* $\underline{n} > 1$ *such that* (51) *is satisfied for all* $n > \underline{n}$*.*
If $t = 1$*, then the same holds if* $\alpha_\varepsilon > 1$*.*
(b) *The optimal market size in a strategic equilibrium is finite, and larger than the one in a price-taking equilibrium. Also, the price impact* λ *decreases in* n*.*

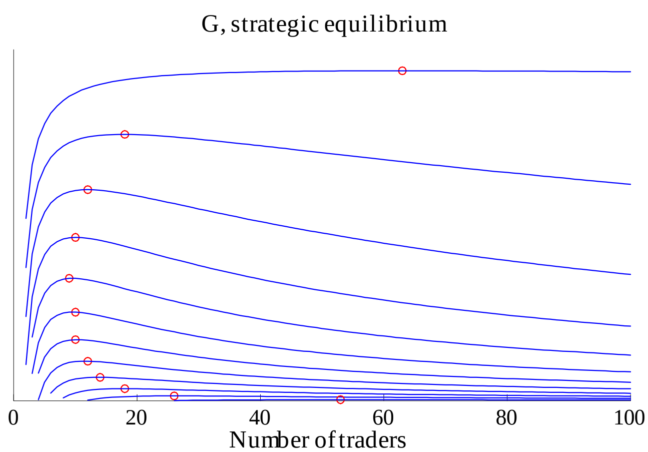

Figure A1. Strategic equilibrium for $t = w = 1$.
Note. Circle markers indicate the optimal market size.
The lower lines correspond to higher values of d_ε.

7.2 Implementing a price-taking welfare by the volume discount

Suppose that traders face the quadratic transaction cost or subsidy:

$$-\frac{c_q}{2}q_i^2,\ c_q \in \mathbb{R}$$

on top of the payment of pq_i. Traders face a non-linear schedule: large volume is taxed if $c_q > 0$, while there is volume discount if $c_q < 0$. From an intermediary's perspective, the baseline model $c_q = 0$ is a budget balance rule, while $c_q > 0$ presents a profit opportunity. With this transaction fee, the first and the second order conditions are

$$E_i[v] - \frac{\rho}{\tau}(q_i + e_i) = p_i + 2\lambda^c q_i + c_q q_i = p + (\lambda^c + c_q) q_i,$$

$$2\lambda^c + c_q + \frac{\rho}{\tau} > 0, \text{ where } \lambda^c \equiv \frac{1}{n\beta_p^c}.$$

The optimal order is

$$q_i^c(p; e_i, s_i) = \frac{E_i[v] - p - \frac{\rho}{\tau}e_i}{\lambda^c + c_q + \frac{\rho}{\tau}}. \tag{52}$$

The analysis goes through as before by replacing λ with $\lambda^c + c_q$. Again, because (52) does not affect the balance of the two motives, we have $\frac{\beta_e^c}{\beta_s^c} = \frac{\beta_e}{\beta_s}$. Therefore, the informational property of equilibrium is not affected by the value of c_q.

Lemma A2 (volume tax/discount)
(a) *The GFT are* $G^c = \frac{1}{2\rho}\log\left\{1 + \alpha\frac{\tau_v}{\tau}\widetilde{B}\left(1 + \frac{\alpha}{1-\alpha}B\right)\frac{\rho}{\rho + c_q\tau}\right\}$.
(b) *Tax revenue is maximized by* $c_q = c_q^m = \frac{\rho}{\tau}$. *With this tax level,*

$$G^{c_q^m} = \frac{1}{2\rho}\log\left\{1 + \frac{\alpha}{2}\frac{\tau_v}{\tau}\widetilde{B}\left(1 + \frac{\alpha}{1-\alpha}B\right)\right\}. \tag{53}$$

(c) *There exists* $c_q^* < 0$ *which implements the GFT in a price-taking equilibrium.*

(a) shows that, for $c_q > 0$, *the tax discount* $\frac{\rho}{\rho + c_q\tau} < 1$ arises in the GFT. Because $\frac{\rho}{\rho + c_q\tau}$ decreases in n, the tax discount is more significant in a larger market. This means that if $c_q > 0$ is fixed independent of n, the optimal market size (from traders' perspective) becomes smaller. However, **(b)** shows that at $c_q = c_q^m$, which maximizes the tax revenue for given n, the tax discount is $\frac{1}{2}$ independent of n. Therefore, if intermediaries choose $c_q = c_q^m$ after observing n, the relevant ex ante GFT is (53). Thus, the level of the GFT is reduced for each n relative to the strategic equilibrium, but the optimal market size is the same as the one in the strategic equilibrium. Finally, **(c)** shows that the appropriately chosen $c_q < 0$ can achieve the welfare level in a price-taking equilibrium. This requires subsidy at the interim stage, and hence commitment on the intermediaries' side. If this commitment is possible, intermediaries are willing to do so, as they can internalize the impact of this subsidy when they set the entry fee. Therefore, the analysis of market formation should be applied to the GFT curve in a price-taking equilibrium.

7.3 Market-size dependent signal quality

We assumed that the informativeness of the individual signal τ_ε is independent of the market size, because it is not clear a priori how the quality of *individual* private information is related to the market size. However, the assumption imposes a restriction that the total amount of information $n\tau_\varepsilon$ *linearly* increases in the market size. To study the robustness of our main result, we replace τ_ε with

$$\widetilde{\tau}_\varepsilon = \tau_\varepsilon n^{\delta_\varepsilon},\ \delta_\varepsilon \in \mathbb{R}. \tag{54}$$

The baseline model has $\delta_\varepsilon = 0$. The parameter δ_ε describes how the individual information changes as n increases. For example, if $\delta_\varepsilon < 0$ (> 0), then traders have less (more) precise signals as n increases. In the Appendix B, we show

$$\begin{aligned}
\frac{\tau_v}{\tau} &= \frac{1 - w + \frac{\alpha_\varepsilon k^2 + w n^{\delta_\varepsilon}}{w n^{\delta_\varepsilon + 1}} + (1-t)\frac{d_\varepsilon}{1-d_\varepsilon}\left(n^{\delta_\varepsilon} + \frac{\alpha_\varepsilon k^2 + w n^{\delta_\varepsilon}}{wn}\right)}{1 - w + \frac{\alpha_\varepsilon k^2 + w n^{\delta_\varepsilon}}{w n^{\delta_\varepsilon + 1}} + \frac{d_\varepsilon}{1-d_\varepsilon}\left(n^{\delta_\varepsilon} + \frac{\alpha_\varepsilon k^2 + w n^{\delta_\varepsilon}}{wn}\right)} \in (1-t, 1)\,, \\
B &= \frac{1}{1+n^{-1}}\frac{\alpha_\varepsilon k^2 n^{-\delta_\varepsilon}}{\alpha_\varepsilon k^2 n^{-\delta_\varepsilon} + w} \in (0,1)\,,
\end{aligned}$$

where k is now determined by

$$F(k) = \left(\alpha_\varepsilon k^2 n^{-\delta_\varepsilon} + w\right)\left(\sqrt{t}k - 1 - \frac{(1-t)\,d_\varepsilon}{1-d_\varepsilon} n^{\delta_\varepsilon}\right) - w\left(1 - w + \frac{(1-t)\,d_\varepsilon}{1-d_\varepsilon} n^{\delta_\varepsilon}\right) n = 0.$$

It boils down to characterizing the relative speed at which positive and negative forces change as the market size grows, for a given value of δ_ε.

Lemma A3 (market-size dependent signal quality)
Assume (54).
(a) *The optimal market size is finite for:*

$$\begin{array}{ll}
\delta_\varepsilon > -\frac{1}{2} & \text{if } t = w = 1, \\
\delta_\varepsilon \geq 0 & \text{if } 0 < w < t = 1, \\
\delta_\varepsilon \in \left(-\frac{1}{2}, 1\right) & \text{if } 0 < t < w = 1, \\
\delta_\varepsilon \in [0, 1) & \text{if } 0 < t, w < 1.
\end{array}$$

(b) *Suppose* $t = 1$. $\lim_{n\to\infty} G = 0$ *for* $\delta_\varepsilon > -\frac{1}{2}$ *if* $w = 1$, *and for* $\delta_\varepsilon > 0$ *if* $w \in (0,1)$.

For each (t, w), our main result extends to a range of values of δ_ε. For the main scenario $t = w = 1$, $\lim_{n\to\infty} G = 0$ also continues to hold for $\delta_\varepsilon > -\frac{1}{2}$. Therefore, even if the quality of the individual private information *deteriorates* with the market size, the information aggregation can have a strong negative welfare implication.

Relative to $t = w = 1$, assuming $t < 1$ puts an upper bound $\delta_\varepsilon < 1$, while assuming $w < 1$ puts a tighter lower bound $\delta_\varepsilon \geq 0$, for the value of δ_ε for which the main result holds. For $t < 1$, recall that we obtained $\lim_{d_\varepsilon\to 1} n^* = \infty$ with $\delta_\varepsilon = 0$. When δ_ε becomes sufficiently large, the limit $n \to \infty$ becomes qualitatively similar to $d_\varepsilon \to 1$. For $w < 1$, $\delta_\varepsilon < 0$ implies that the common signal noise ϵ_0 becomes infinitely volatile as $n \to \infty$.

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