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Optimal Sharing Strategies in Dynamic Games of Research and Development[1]

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Abstract

A question central to R&D policy making is the impact of competition on cooperation. This paper builds a theoretical foundation for the dynamics of knowledge sharing in private industry. We model an uncertain research process and ask how the incentives to license intermediate steps to rivals change over time as the research project approaches maturity. Such a dynamic approach allows us to analyze the interaction between how close the firms are to product market competition and how intense that competition is. We uncover a basic dynamic of sharing such that firms are less likely to share as they approach the product market. This dynamic is driven by a trade-off between three effects: the rivalry effect, the duplication effect and the speed effect. We show that this dynamic can be reversed when duopoly profits are sufficiently low. We also explore the implications of the model for patent policy and R&D subsidies, and discuss under what circumstances such policies should be directed towards early vs. later stage research.

JEL Codes: L24,O30,D81

Keywords: Multi-stage R&D; innovation; knowledge sharing; licensing; dynamic games; patent policy.

1 Introduction

This paper builds a theoretical foundation for the dynamics of knowledge sharing in private industry. The substantial evidence on licensing, research alliances and joint ventures reveals that knowledge sharing arrangements are a central way in which firms acquire technological knowledge. From a social welfare perspective, sharing of research outcomes is desirable because it results in less duplication. Since the 1980s, governments in the US and Europe have actively promoted joint R&D projects through subsidies, tolerant antitrust treatment, and government-industry partnerships.[1] At the same time, economics research has studied the private and social incentives to have knowledge sharing arrangements, focusing on issues of appropriability and spillovers. However, none of these studies has focused on the basic dynamics of private sharing incentives. Research projects in industries such as biotechnology, automobiles and computers can take years or even decades to complete. Over such long time horizons, firms may decide to share some intermediate steps, but not all of their research outcomes. Consider, for example, the collaboration between GM and Toyota to develop fuel cell technology for automobiles. In 2006, after more than 6 years of working together on fuel cell research, the two companies ended their collaboration because they could no longer agree to terms for sharing intellectual property.[2,3]

Focusing on the dynamics of research, we ask how the incentives to license research outcomes to rivals change over time as a research project approaches maturity. A question central to the policy debate, as well as the study of knowledge sharing arrangements, is the impact of

[1] For example, in the US, the National Cooperative Research and Production Act (NCRPA) of 1993 provides that research and production joint ventures be subject to a 'rule of reason' analysis instead of a per se prohibition in antitrust litigation. In the EU, the Commission Regulation (EC) No 2659/2000 (the EU Regulation) provides for a block exemption from antitrust laws for RJVs, provided that they satisfy certain market share restrictions and allow all joint venture participants to access the outcomes of the research.

[2] A spokesman for GM, Scott Fosgard, reportedly said that "the companies will no longer collaborate on fuel cells because that technology is moving out of the research stage and into the more proprietary development stage. But both companies remain open to other research projects in mutually beneficial areas." See "GM, Toyota end joint fuel cell research" at http://www.msnbc.msn.com/id/11654151/ns/business-oil_and_energy/t/gm-toyota-end-joint-fuel-cell-research/#.UNJvFMXNm0Y. See also "GM and Toyota end collaboration on fuel cells" at http://www.businessrespect.net/page.php?Story_ID=1537.

[3] As another example, consider alliances in biotechnology. Lerner and Merges (1998) find that while in a few cases the alliances covered technologies well along the way to regulatory approval, in most cases they were arranged at the earliest stages of research (prior to animal studies, clinical trials and regulatory approval).

competition on cooperation. This is because in many cases, the most suitable research partner for a firm may be one of its competitors. However, as in the case of GM and Toyota, such sharing poses especially difficult challenges because it may reduce the commercial value of the firms' R&D efforts.[4] A dynamic perspective allows us to analyze the impact of competition on cooperation in two different ways. We can analyze the impact of both how close the firms are to product market competition and how intense that competition is. Our results reveal an interesting interaction between these two factors.

From a dynamic perspective, the process of research is generally characterized by a high level of uncertainty in the beginning. For example, at the outset of research on a new medical drug, the expected success rate may be as low as 2% and the expected time to market may be more than a decade.[5] Similarly, fuel cell technology for automobiles has been in active development since the 1990s and is not expected to reach full commercial viability for another decade.[6] In such environments, progress in research can be described as a decrease in the level of uncertainty that researchers face. One of the novel aspects of this paper is to analyze how firms' incentives to share research outcomes change during a research process as the level of uncertainty they face decreases. We show that the impact of uncertainty on firms' sharing incentives depends on the intensity of product market competition.

We assume that research projects consist of two sequential steps. Researchers cannot earn any profits before completing both steps of the project. An important feature of the model is that the research steps are symmetric in all respects except in regards to how far away they are from the end of the project. We deliberately assume that there are no spillovers in research in order to focus on the role uncertainty plays in knowledge sharing. It has been stressed in the literature that firms may have higher spillover rates and bigger appropriability problems in earlier stages of research than in later stages of research. Although the rate of spillovers may

[4] Empirical evidence suggests that firms do take measures to avoid opportunistic behavior when they are collaborating with their competitors. For example, Oxley and Sampson (2004) show that direct competitors choose to limit the scope of alliance activities. Majewski (2004) shows that direct competitors are more likely to outsource their collaborative R&D.

[5] See Northrup (2005).

[6] For a timeline, see "Fuel Cell Electric Vehicles: The Road Ahead" at http://www.fuelcelltoday.com/media/1711108/fuel_cell_electric_vehicles_-_the_road_ahead.pdf.

shape the dynamics of sharing, our results show that this is not the only factor that matters.

We assume that firms are informed about the progress of their rivals and make joint sharing decisions after each success. The leading firm sets a licensing fee which is paid by the lagging firm. We identify three effects which shape the sharing decisions. Sharing has the benefits of avoiding the duplication of R&D costs (duplication effect) and bringing product market profits forward (speed effect). However, on the negative side, joint product market profits may be reduced as a result of sharing since sharing decreases the leader's ability to earn monopoly profits (rivalry effect).

Our main results for symmetric firms reveal a basic dynamic of sharing such that the firms are less likely to share as they approach the product market. That is, the incentives to share research outcomes decrease with progress. This is so, even though licensing fees may increase with progress. We show that this dynamic arises whenever duopoly profits are so high that a lagging firm stays in the race no matter how far behind it is. The reason for the decline in the sharing incentives is that earlier in the race, the speed effect is more important relative to the rivalry effect. We show that this is tied to the resolution of uncertainty. Since the leader's chance of finishing first is lower earlier in the race, the importance of the rivalry effect is diminished. Intuitively, there is less value in maintaining a lead when it is more likely to be lost in the future.

That sharing incentives decrease with progress makes intuitive sense because it shows that as the firms get closer to the end of the R&D race, the impending competition harms cooperation. However, we show that a different dynamic can arise when duopoly profits are too low relative to the costs of research to keep lagging firms in the race. When lagging firms can be induced to exit the race at some histories, sharing incentives may increase with progress rather than decrease. This is because a lagging firm is most likely to exit early in the race when it has not made much progress. Thus, the firms may decide against sharing early on to take advantage of this.

When duopoly profits are so low that a lagging firm exits even late in the race, the incentives to share again decrease with progress. However, the reason for this is different from above.

The firms are willing to share early stage research because they know that one of them will exit later on. We refer to such markets, where only one firm will profitably produce output, as winner-take-all markets. The late stage exit eliminates the rivalry effect, leaving only positive incentives to share (the duplication and speed effects) at the early stages.

Considering the case of asymmetric firms reveals an additional effect which may shape the sharing incentives. We show that if firms have different abilities to conduct different stages of research (such as in the biotechnology industry) and if one of the firms has increasing research costs over time, sharing may cause a firm to start working on a more costly research step. This progress effect may cause sharing incentives to increase over time.

These results have implications for policy making in innovation environments. They show that the design of optimal policies should be sensitive to the dynamic sharing patterns which would emerge in the absence of such policies. Our results emphasize that the dynamics of sharing may be driven by the degree of product market competition. Hence, whether policies should be optimally directed towards early stage or late stage research may depend on the particular industry because of differences in the intensity of competition.

To illustrate, we consider how patent policy and R&D subsidies can be used to change the sharing and investment incentives in our dynamic framework. Our discussion reveals that the two policy instruments have distinct roles to play in the economy. We first consider how patent policy can be used to eliminate the rivalry effect by inducing the lagging firm to exit when the leading firm is done with research. This protects monopoly profits and may enhance social welfare by encouraging efficient investment and sharing early on. We then consider how patent policy can be used to enhance social welfare by changing sharing decisions when investment incentives are adequate. Intuition suggests that broader patent protection should encourage sharing by increasing the cost of duplication.[7] We show that in a dynamic framework, this may not always be the case. Broader patent protection of late stage research may feed back to

[7] It has also been stressed in the literature that patents can facilitate the transfer of technologies through licensing because they increase the bargaining power of the licensor. See, for example, the discussion in Gallini (2002). Anand and Khanna (2000), and Arora and Fosfuri (2000) show empirically that licensing is more prevalent in industries where patent protection is more effective. See Anton and Yao (1994) on licensing without formal intellectual property rights.

discourage sharing of early stage research. This is due to a progress effect similar to the one discussed above for the case of asymmetric firms.

We also discuss under what circumstances policy makers should rely on R&D subsidies rather than patent policy. In so far as R&D subsidies generally encourage investment but discourage sharing, they should be used in industries where sharing incentives are adequate, but investment incentives are not. One example of this is when the level of product differentiation is so high that firms earn higher profits under duopoly than monopoly. In this case, because the firms are not rivalrous, the firms share their research regardless of the patent policy. Hence, we show that patent policy has no impact on investment decisions on the equilibrium path and so is ineffective. An R&D subsidy, by contrast, can be used to encourage investment.

The impact of competition on cooperation in R&D has been the focus of many papers in the economics literature. These papers have mainly studied firms' incentives to share research outcomes at one point in time, either before the start of research, as in the case of research joint ventures, or after the development of a technology, as in the case of licensing.[8] A general result in these papers is that as product market competition increases, incentives to cooperate decrease.[9] We contribute to this literature by focusing on the interaction between competition and cooperation in a dynamic setting.

Our paper is related to the literature which models R&D as a multi-stage process.[10] Grossman and Shapiro (1987) and Harris and Vickers (1987) analyze how firms vary their research efforts over the course of a research project.[11] A common assumption that has been made in this literature is that the different stages of R&D differ from each other in a fundamental way. For example, Reinganum (1985) considers a model with a research and a development stage,

[8] See, for example, Kamien (1992) on licensing, and D'Aspremont and Jacquemin (1988) on research joint ventures. Severinov (2001) studies the impact of product market competition on the incentives for informal sharing of R&D outcomes between employees.

[9] For example, Choi (1993) shows that competing firms will cooperate if the level of spillovers are sufficiently high. Wang (2002) shows that licensing between competitors will take place if they produce sufficiently differentiated products.

[10] An important issue in this literature is whether the different stages of R&D are carried out by the same player or by different players. If they are carried out by different players, this raises the question of how the rents should be distributed between the different generations of innovators. See, for example, Scotchmer (1991) and more recently, Hopenhayn (2006).

[11] See also Cabral (2003) and Judd (2003) who analyze the risk-taking behavior of firms.

and assumes that the findings in the first stage rapidly become public knowledge (see also Vonortas, 1994). Aghion, Dewatripont and Stein (2008), and Cozzi and Galli (2011) assume that different stages of research are conducted by different institutional players, namely academia and private firms, which is why there may be more dissemination of research outcomes early on (see also Hellmann and Perotti, 2011). Although declining sharing incentives is also one of our results, it happens for very different reasons in our model with symmetric firms.

Disclosure of intermediate research outcomes has also been considered in Scotchmer and Green (1990), d'Aspremont et al. (2000), Bar (2006), Bessen and Maskin (2009), and Fershtman and Markovich (2010). Scotchmer and Green (1990) consider disclosure through patenting while Bar (2006) studies disclosure through publishing. Similar to our paper, d'Aspremont et al. (2000), Bessen and Maskin (2009), and Fershtman and Markovich (2010) consider licensing of intermediate research outcomes. However, none of these papers focus on the dynamics of sharing incentives.

The paper proceeds as follows. In the next section, we describe the set-up and explain, as a benchmark, what happens if the firms are allowed to collude in the product market. In section 3, we analyze the effect of competition on the dynamic sharing incentives of symmetric firms. We conclude this section by discussing industry examples which are in line with our results. In section 4, we consider the case of asymmetric firms. We then analyze in section 5 how the investment and sharing incentives in our dynamic set-up can be shaped by patent policy and R&D subsidies. We conclude in section 6 with some observations on the robustness of our results.

2 Model

Since we are interested in the effect of competition on firms' incentives to share, we consider an environment with two firms, $i = 1, 2$, which invest in a research project. On completion of the project, a firm can produce output in a product market. We consider Markov Perfect Equilibria (MPE), where each firm maximizes its discounted expected continuation payoff given the Markov strategy of the other firm. Before describing the payoffs and the MPE, we first explain the research and production phases.

2.1 Research Environment

To capture the idea of progress, we consider a research project with 2 distinct steps. These steps may be thought of as early and late stage research. There is no difference between the steps in terms of the technology or the options available to the firms. This is because we seek to derive endogenous differences between the research steps that result from the dynamics in the decisions made by the firms. A firm cannot start to work on the next step before completing the prior step, and all steps of the project need to be completed successfully before a firm can produce output.

We assume that each firm operates an independent research facility. We model research activity using a Poisson discovery process. Time is continuous, and the firms share a common discount rate r. Following Lee and Wilde (1980), we assume that to conduct research, a firm must incur a flow cost c per unit of time.[12] Investment provides a stochastic time of success that is exponentially distributed with hazard rate α. A higher value of α corresponds to a shorter expected time to completion. For a firm which has not yet completed the project, a decision not to invest the flow cost c is assumed to be irreversible and equivalent to dropping out of the game.

When one firm successfully completes a stage of research before the other firm does, we assume that the leading firm can share this knowledge with the lagging firm and thereby save the lagging firm from having to continue to invest to complete the stage. There are a variety of ways to model the sharing process. We consider ex post sharing or licensing, where the leading firm shares its result with the lagging firm in exchange for a licensing fee.

Regarding the information structure, we assume that the lagging firm cannot observe the technical content of the rival's research without explicit sharing.[13] In this sense, there are

[12] In the concluding section of the paper, we discuss how the results extend to a model with continuous effort choices. The discrete effort assumption can be motivated by presuming a fixed amount of effort that each firm can exert, which is determined by the capacity of its R&D division. As an example, Khanna and Iansiti (1997) explain that given the highly specialized nature of the R&D involved in designing state-of-the-art mainframe computers, firms in this industry find it very expensive to increase the number of researchers available to them.

[13] Alternatively, we could assume that research results can be copied, but successful firms win immediate patents. A leading firm could then prevent a lagging firm from copying its research by enforcing its patent. If

no technological spillovers. Everything else in the game is common knowledge. In particular, firms observe whether their rival is conducting research as well as whether the rival has a success. Third parties such as courts also observe this information and can enforce the licensing contracts.

2.2 Product Market Competition

We represent the product market competition in the following reduced form way. If both firms have completed the research project, they compete as duopolists and each earns a flow profit of $\pi^D \geq 0$ forever. If only one firm has completed the research project, the firm earns a monopoly flow profit of $\pi^M > 0$ as long as the other firm does not produce output. Here, $\pi^M > \pi^D$. As a benchmark, we will consider the case that the firms make production decisions to maximize their joint profits in the product market. This results in a joint flow profit of π^J. We assume that the magnitudes of π^D, π^M and π^J do not depend on the decisions taken during the research phase.

These payoffs are sufficiently flexible to capture various models of product competition.[14] For example, if the firms produce homogeneous products and compete as Bertrand or Cournot competitors, then $\pi^J = \pi^M > 2\pi^D$. If the firms produce differentiated products, then $\pi^J > \max\left\{\pi^M, 2\pi^D\right\}$ and the relationship between π^M and $2\pi^D$ will depend on the degree of product differentiation. For low levels of product differentiation, $\pi^M > 2\pi^D$; for high levels of product differentiation, $\pi^M \leq 2\pi^D$.[15]

Section 5 contains normative analysis about the impact of R&D policy on the investment and sharing decisions of firms. For this purpose, we let TS^M and TS^D denote the flow of total surplus in the product market under monopoly and duopoly, respectively. We assume that

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the patent does not prevent the rival from developing a non-infringing technology at the same flow cost c and with the same hazard rate, then the formal set up would be equivalent to ours. We consider the case when patenting changes the research cost of the lagging firm in section 5.

[14] We assume that the firms conduct the research to solve the same technical problem. However, unmodelled differences in production technologies can still lead them to produce differentiated products.

[15] As an example, consider a demand function of the type $q_i = \left(a\left(1-\gamma\right) - p_i + \gamma p_j\right) / \left(1-\gamma^2\right)$, where $0 < \gamma < 1$ so that the products are substitutes. The goods are less differentiated the higher is γ. It is possible to show that $\pi^M \leq 2\pi^D$ if and only if γ is sufficiently small. Singh and Vives (1984) show how these demand functions derive from particular consumer preferences. The Hotelling model provides another example of a differentiated duopoly.

$TS^M > TS^D$.

We have now introduced all of the technology and profit parameters used in the model. To denote the space for technology and profit parameters, we use $\Omega = \{\boldsymbol{\omega} = (\alpha, r, c, \pi^M, \pi^D)$ such that $0 < \alpha, 0 < r < 1, c > 0, \pi^M > \pi^D \geq 0\}$.

2.3 Equilibrium, Payoffs and Sharing Dynamics

Research Histories and Markov States To represent the progress made by the firms, we define a set of research histories. We use the notation (h_1, h_2) where h_i stands for the number of steps that firm i has completed. When firm i completes a research step, h_i increases by one. We refer to research histories where $h_1 = h_2$ as symmetric histories and to those where $h_1 \neq h_2$ as asymmetric histories. At asymmetric research histories, the firms have the opportunity to share research, as described below.

We also define a set of Markov states for the game. The research histories (h_1, h_2) are all Markov states. Some additional Markov states will also be defined. Below, we describe the available actions at each state and how transitions between states occur.

We first describe actions at the symmetric research histories. At histories (h, h) with $h = 0, 1$, the firms simultaneously decide whether to invest in the next step of research. If both firms invest, they each incur the flow cost c. When one of them, say firm 1, is successful, the state transitions to $(h + 1, h)$. At (h, h), if one of the firms, say firm 1, does not invest, the state transitions to (X, h) where X denotes that a firm has exited the game. If both firms drop out at (h, h), the state transitions to (X, X) and the game is over. At the symmetric history $(2, 2)$, the firms earn duopoly profits.

At asymmetric research histories (h_1, h_2), the firms decide whether to enter into a licensing agreement. This is a joint decision, not a strategic one. The firms agree to share if and only if doing so increases their joint continuation profits. Although this is not essential to our results, we assume that the leading firm claims the whole surplus from the agreement through a licensing fee paid by the lagging firm. Under this assumption, the leading firm has all of the bargaining power and makes a take-it-or-leave-it licensing fee offer to the lagging firm. If an agreement is concluded, the license fee is paid, the research is shared, and the history transitions to the research history (h_1+1,h_2) or (h_1,h_2+1) depending on which firm is the leader. If an agreement is not concluded, the history transitions to a Markov state which we denote by (h_1,h_2,NS). At these states, the firms do not have the option to share research.

At asymmetric states (h_1,h_2,NS), if neither firm has completed both steps of research, then the firms simultaneously decide whether to invest in the next step of research. The state transitions are the same as described above for the symmetric states. If one of the firms is done with research at (h_1,h_2,NS), that firm earns monopoly profits while the other firm decides whether or not to invest.

At the asymmetric states (X,h) and (h,X), there is only one firm in the game. If $h=2$, the firm is a monopolist. If $h<2$, the firm decides whether or not to invest.

The complete set of Markov states is

$$H=\left\{\begin{array}{c}(h_1,h_2),(h_1,X),(X,h_2)\text{ for }h_i=0,1,2\text{ and }i=1,2;\\(h_1,h_2,NS)\text{ for }h_i=0,1,2\text{ and }i=1,2\text{ such that }h_1\neq h_2;\ (X,X)\end{array}\right\}.$$

Markov Perfect Equilibrium A Markov strategy specifies an action for a firm at each $h\in H$ where an investment decision is made. The value function $V_i(h)$ is the expected continuation profits for firm i at $h\in H$ when the firms play equilibrium strategies.

Definition 1 *A pure strategy Markov Perfect Equilibrium (MPE) for the R&D game $\omega=\{\alpha,r,c,\pi^M,\pi^D\}$ consists of Markov strategies for $i=1,2$ and value functions $V_i(h)$ such that the strategy for firm i maximizes $V_i(h)$ at each h given the strategy of firm j.*

The value function $V_i(h)$ depends on the nature of the actions that are taken at h. At $(2,2)$, $(2,X)$, $(X,2)$, and (X,X), there are no actions. Active firms earn flow profits in the product market. The value functions for firm 1 are:

$$\begin{aligned}V_1(2,2)&=\int_0^\infty e^{-rt}\pi^D dt=\frac{\pi^D}{r}\equiv\widetilde{\pi}^D\\V_1(2,X)&=\int_0^\infty e^{-rt}\pi^M dt=\frac{\pi^M}{r}\equiv\widetilde{\pi}^M\\V_1(X,X)&=V_1(X,h_2)=0,\text{for }h_2=0,1,2\end{aligned}$$

At $(0,0)$, $(1,1)$, $(1,0,NS)$ and $(0,1,NS)$, both firms make investment decisions. If firm 2 invests, firm 1's value function is given by a Bellman equation:

$$\begin{aligned} V_1(0,0) &= \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(1,0)+\alpha V_1(0,1)-c\right)dt\right\} \\ V_1(1,1) &= \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(2,1)+\alpha V_1(1,2)-c\right)dt\right\} \\ V_1(1,0,NS) &= \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(2,0)+\alpha V_1(1,1)-c\right)dt\right\} \\ V_1(0,1,NS) &= \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(1,1)+\alpha V_1(0,2)-c\right)dt\right\}. \end{aligned}$$

If firm 2 does not invest, then there is a state transition to (X, X) if firm 1 does not invest either. If firm 1 invests, the state becomes either $(0, X)$ or $(1, X)$, depending on how many steps firm 1 has already completed. At $(0, X)$ and $(1, X)$, we have

$$V_1(h_1, X) = \max\left\{0, \int_0^\infty e^{-(\alpha+r)t}\left(\alpha V_1(h_1+1, X)-c\right)dt\right\} \text{ for } h_1 = 0, 1.$$

At $(0, 2, NS)$ and $(1, 2, NS)$, firm 1 has an investment decision and firm 2 has no action. The value function for firm 1 is

$$V_1(h_1, 2, NS) = \max\left\{0, \int_0^\infty e^{-(\alpha+r)t}\left(\alpha V_1(h_1+1, 2)-c\right)dt\right\} \text{ for } h_1 = 0, 1.$$

At $(2, 0, NS)$ and $(2, 1, NS)$, firm 1 has no action and firm 2 has an investment decision. If firm 2 invests, then

$$V_1(2, h_2, NS) = \int_0^\infty e^{-(\alpha+r)t}\left(\pi^M + \alpha V_1(2, h_2+1)\right)dt \text{ for } h_2 = 0, 1.$$

If firm 2 does not invest, then the value functions are the same as at $(2, X)$.

At states (h_1, h_2) with $h_1 \neq h_2$, the firms make a sharing decision through a licensing process that maximizes their joint profits. When firm 1 is the leader, the following sharing condition describes whether the firms enter into a licensing agreement:

$$V_J(h_1, h_2+1) > V_J(h_1, h_2, NS), \tag{1}$$

where $V_J = V_1 + V_2$ is the joint value function of the firms.

Although the licensing fee does not enter the sharing condition, it does affect the value function of each firm at (h_1, h_2). Because the leader is assumed to claim the entire surplus from the agreement, the licensing fee leaves the lagging firm just indifferent between accepting and rejecting the agreement. When firm 1 is the leader, the licensing fee is equal to

$$F(h_1, h_2) = V_2(h_1, h_2 + 1) - V_2(h_1, h_2, NS). \tag{2}$$

The value functions when the firms share are given by

$$\begin{aligned} V_1(h_1, h_2) &= F(h_1, h_2) + V_1(h_1, h_2 + 1) \\ V_2(h_1, h_2) &= V_2(h_1, h_2 + 1) - F(h_1, h_2). \end{aligned}$$

If the firms do not share at (h_1, h_2), the value functions are the same as at (h_1, h_2, NS).

Sharing Dynamics Our analysis focuses on the dynamics of sharing for rivalrous firms. That is, we want to know how the incentives to license research change over time. For ease of exposition, we restrict attention to symmetric MPE and consider sharing at histories such that firm 1 is the leader. To analyze the dynamics of sharing, we compare the sharing conditions of the firms at $(2, 1)$ and $(1, 0)$. At each of these histories, the leader is exactly one step ahead of the lagging firm. We say that the sharing incentives are decreasing over time if the firms have a stronger incentive to share at $(1, 0)$ than at $(2, 1)$. Conversely, sharing incentives are increasing over time if the firms have a stronger incentive to share at $(2, 1)$ than at $(1, 0)$.

The firms also decide whether to share at $(2, 0)$. Because the number of steps that the lagging firm is behind is a factor in the firms' sharing conditions, we do not analyze dynamics for this state. In a game with more than two periods, dynamics across histories where one firm is two steps behind could be analyzed.

In our discussion, we refer to sharing patterns. A sharing pattern simply describes the sharing decisions at $(1, 0)$ and $(2, 1)$. The possible sharing patterns are (S,S), (S,NS), (NS,NS), and (NS,S). When sharing incentives are decreasing over time, the pattern (NS,S) does not arise. Conversely, when sharing incentives are increasing over time, the pattern (S,NS) does not arise.

2.4 Joint Profit Maximization Benchmarks

We discuss two benchmarks. First, we consider what the firms would do if they could make all of their decisions (investment, sharing and production) jointly. We call this the *joint profit* *maximization benchmark*. In this benchmark, it is optimal for the firms to cooperate in the product market and earn flow profits of π^J. During the research process, it is always optimal for the firms to share research successes as soon as one of them is ahead. There are two distinct reasons for this. The first is that duplication of research is purely wasteful. We refer to this as the *duplication effect*. In our analysis, this benefit is captured by a savings of the flow costs c of research. It is present in the analysis of every sharing decision. The second reason is that sharing can allow the firms to reach the product market sooner. We refer to this as the *speed effect.* The speed effect is present only when neither firm has completed both steps of research, so at $(1, 0)$ and $(0, 1)$. By sharing at $(1, 0)$, the firms can both work on the second step of research, thereby speeding up the time until one of them completes the second step. When the firms make production decisions jointly, there is no downside to sharing to counterbalance these positive effects and it is always optimal for the firms to share.

In the joint profit maximization benchmark, the firms make investment decisions at $(0, 0)$ and $(1, 1)$. They invest provided the expected payoffs are positive, and if one firm invests, so does the other. The reason for this is that in the Poisson discovery process with identical firms, if it is optimal for one firm to invest in a step, then it is optimal for both to invest even if the firms could agree to have just one of them to invest. This speeds up the time to innovation, and the benefits of the time savings outweigh the costs of running simultaneous facilities.

In our model, firms are not allowed to make joint production decisions in the product market. This motivates a second benchmark, which we call the *constrained joint profit maximization benchmark*. The firms again make all of their investment and sharing decisions jointly, but they do not make joint production decisions. The best outcome they can achieve is flow profits of $2\pi^D$ or π^M, whichever is higher. When $\pi^M > 2\pi^D$, there is a downside to sharing that we refer to as the *rivalry effect.* Sharing can reduce the joint profits of the firms if it enables a lagging firm to enter the product market and disrupt the flow of monopoly profits to the leading firm. In this case, if one firm has completed both steps of research, it is optimal for the lagging firm to exit the race. This allows the firms to achieve the flow profits of π^M. If $\pi^M \leq 2\pi^D$, the firms would like to achieve $2\pi^D$ as the flow profits in the product market and they can do this by sharing once one firm has completed both steps of research. Since either the lagging firm exits or the firms share after one firm finishes both steps of research, there is no downside to sharing the first step at $(1, 0)$ and $(0, 1)$, so the firms share at these histories. At $(0, 0)$ and $(1, 1)$, the firms invest if their expected payoffs are positive. For the reasons stated above, if it is optimal for one firm to invest, then it is optimal for both firms to invest.

3 Equilibrium Dynamics

To explore the dynamics of sharing, we compare the firms' incentives to share early stage research at the history $(1, 0)$ to their incentives to share late stage research at the history $(2, 1)$. Because each of the research steps in our model is identical from a technology standpoint, a conclusion that sharing incentives must change over time is not obvious. As mentioned in section 2.4, the sharing decision depends on the balance of three effects. At both of the histories $(1, 0)$ and $(2, 1)$, sharing has the benefit of avoiding the duplication of one step of research (the duplication effect). As we will see, the importance of the speed and rivalry effects change over time. A basic intuition is that as firms approach the end of the research process, their decisions might increasingly reflect the impending rivalry. If so, then firms might be less likely to share late stage research. In this section, we test this intuition and analyze the mathematical reasons for it.

If the firms decide against sharing, this protects the leader's monopoly profits in two principle ways. First, if the lagging firm continues to research, it will take longer to finish the project, allowing a longer expected period of monopoly profits for the leading firm. Second, if the lagging firm exits the game, the leader can expect to earn monopoly profits forever upon finishing. It turns out that these two forms of the rivalry effect can lead to different dynamics over time. We consider environments with and without exit separately.

3.1 Dynamics of Sharing When Firms do not Exit the Game

The incentive to share depends on the joint profits of the two firms, not their individual profits. We first present our main result on the dynamics of sharing incentives, and then discuss the dynamics of the individual profits and licensing fees.

We start by distinguishing parameter regions in Ω with and without exit.

Definition 2 *Region A consists of those parameter values such that in every Markov perfect equilibrium of the game, firms do not exit at any history either on the equilibrium path or off the equilibrium path. Region B consists of all other parameter values.*

Region A is given as follows:

Lemma 1 *Region A consists of all parameters such that* $\pi^D \geq c\frac{r}{\alpha}\left(2+\frac{r}{\alpha}\right)$.

The proof of Lemma 1, in section A of the appendix, focuses on a firm that is as far behind the leader as possible when the leader has not shared its research. Because the lagging firm does not have any bargaining power, its payoff if there is no sharing at $(2,0)$ is the payoff it would get by conducting the two steps of research on its own and then producing in the output market as a duopolist. Intuitively, this is the worst possible position for a firm. We show that the lagging firm stays in at these histories if and only if the inequality $\pi^D \geq c\frac{r}{\alpha}\left(2+\frac{r}{\alpha}\right)$ holds. This inequality implies that environments without exit arise when competition in the product market is relatively soft so that duopoly profits are high by comparison to the costs of research (in terms of time and money).

Because the firms never exit in Region A, solving the game for its equilibria reduces to determining the sharing decisions of the firms.

Proposition 1 *Consider parameters π^D, r, c and α satisfying Lemma 1. We have:*

(i) When $\pi^D < \pi^M < 2\pi^D + c$, there is a unique MPE. The sharing pattern is (S,S).

(ii) When $2\pi^D + c < \pi^M < \frac{(3\alpha^2+2\alpha r)}{(2\alpha^2-r^2)}2\pi^D + \frac{(2\alpha+r)^2}{(2\alpha^2-r^2)}c$, there is a unique MPE. The sharing pattern is (S,NS).

(iii) When $\pi^M > \frac{(3\alpha^2+2\alpha r)}{(2\alpha^2-r^2)}2\pi^D + \frac{(2\alpha+r)^2}{(2\alpha^2-r^2)}c$, there is a unique MPE. The sharing pattern is (NS,NS).

The proposition is proved in section B of the appendix.[16] A first observation is that the sharing pattern (NS,S) does not arise in Region A. We would expect to see only the sharing patterns (S,S), (S,NS) and (NS,NS). Sharing may break down as the firms approach the product market, as in (ii) where the firms share at $(1,0)$ but not at $(2,1)$. However, the reverse dynamic is not possible. A second observation is that when monopoly profits are higher, the firms share less often. As π^M increases so that we move from (i) to (ii), sharing breaks down at $(2,1)$. As π^M increases further so that we move from (ii) to (iii), sharing breaks down at $(1,0)$ as well. Hence, when sharing breaks down, it breaks down at the later history first. These observations express the idea that sharing incentives decrease over time.

We explain the reasons behind Proposition 1 by discussing the sharing conditions at $(2,1)$ and $(1,0)$. Formally, firms share at $(2,1)$ if $V_J(2,2) > V_J(2,1,NS)$. If the firms share, they compete as duopolists in the product market. Their continuation profits are $V_J(2,2) = 2\widetilde{\pi}^D$. If the firms do not share, the leading firm earns a flow profit of π^M and the lagging firm invests c until the lagging firm finishes. Their joint continuation profits are $V_J(2,1,NS) = \int_0^\infty e^{-(\alpha+r)t}\left(\pi^M - c + \alpha V_J(2,2)\right)dt = \frac{\pi^M - c + \alpha 2\widetilde{\pi}^D}{\alpha+r}$. The sharing condition simplifies to

$$2\pi^D - \pi^M + c > 0. \tag{3}$$

The term $2\pi^D - \pi^M$ in condition (3) captures the rivalry effect. This term must be negative for there to be a downside to sharing. The term c is the duplication effect.[17]

Consider first the case when condition (3) fails, so the firms do not share at $(2,1)$. As shown in the appendix, the sharing condition at $(2,0)$ is also given by condition (3). Thus, the firms do not share at $(2,0)$ either. At the earlier history $(1,0)$, the sharing condition is $V_J(1,1) > V_J(1,0,NS)$, which simplifies to condition (19) in the appendix. At $(1,0)$, there is a new benefit of sharing that did not exist at $(2,1)$. The lagging firm now has a chance of finishing first. If the firms knew that firm 2 would finish first, they would want to share at $(1,0)$ so as to realize monopoly profits sooner. This is the speed effect. In contrast, if the firms

[16] On the boundaries between (i) and (ii), and between (ii) and (iii), the firms are indifferent between sharing and not sharing at one or more histories. As a result, there are multiple equilibria that are payoff equivalent.

[17] For an alternative interpretation, condition (3) can also be written as $\pi^M - \pi^D < c + \pi^D$. The LHS represents the per-period loss of the leader due to sharing while the RHS represents the per-period gain of the lagging firm due to sharing.

knew that firm 1 would finish first, then they would not want to share at $(1,0)$ because this shortens the duration of monopoly profits. We can re-write the sharing condition (19) in the following way:

$$\beta(\pi^M + c) + (1-\beta)(2\pi^D - \pi^M + c) > 0, \tag{4}$$

where $\beta = \frac{(1+\frac{r}{\alpha})^2}{(2+\frac{r}{\alpha})^2}$. The second term in (4) is the net loss in joint flow profits when the leading firm finishes first. This is the same as condition (3) and is negative. The first term in (4) is the increase in joint flow profits when the lagging firm finishes first. Here, the firms jointly benefit from replacing the lagging firm's R&D costs $-c$ with monopoly profits π^M. The net benefit, $\pi^M + c$, which is due both to the speed effect (monopoly profits π^M are earned sooner) and the duplication effect (flow costs c are saved), is positive.

The β and $(1-\beta)$ can be interpreted as weighted probabilities. There is a weighted probability β that the lagging firm finishes first and a weighted probability $(1-\beta)$ that the leading firm finishes first. Since $\beta > 0$, condition (4) is easier to satisfy than (3), and hence sharing incentives are decreasing. At $(2,1)$, $\beta = 0$ because the leading firm is already done. The duplication effect c has the same weight in both sharing conditions (4) and (3) in so far as the coefficient on c is simply 1. In this sense, it is the changing importance of the rivalry and speed effects that drives the sharing dynamics, not the duplication effect.

When β is larger, there is more weight on the speed effect and less weight on the rivalry effect, so there is more incentive to share. The magnitude of β depends on how impatient the firms are. The ratio $\frac{r}{\alpha}$ in the expression for β can be interpreted as a discount factor. The underlying interest rate r is adjusted by the effectiveness α of the research technology. The probability β is increasing in $\frac{r}{\alpha}$ so that when the firms are more impatient, the incentives to share are stronger.

When condition (4) holds, there is a unique MPE with the sharing pattern (S,NS). When it fails, there is a unique MPE with the sharing pattern (NS,NS). When the condition holds with equality, there are two MPEs, one for each sharing pattern.

Next, consider the case when condition (3) holds, so the firms share at $(2,1)$. As shown in the appendix, the sharing condition at $(2,0)$ is again given by condition (3). Thus, the firms share at $(2,0)$ also. The sharing condition at $(1,0)$ simplifies to

$$\pi^D + c > 0, \tag{5}$$

which holds trivially and implies that the equilibrium sharing pattern is (S,S). Since the firms share at both $(2,1)$ and $(2,0)$, neither firm can ever earn monopoly profits and, thus, there is no cost to sharing at $(1,0)$. That is, there is no rivalry effect. Sharing merely reduces the expected time to market (the speed effect) and expected R&D costs (the duplication effect) by enabling the lagging firm to finish sooner. The sharing condition captures the change in joint flow profits when this happens. When the lagging firm reaches the history $(1,2)$, the firms share so that both firms enter the product market. As shown in (5), the joint flow profits increase from $-2c$ to $2\pi^D$ for a net benefit of $2(\pi^D + c) > 0$. Sharing at $(1,0)$ creates this benefit by enabling the firms to reach the history $(1,2)$ sooner and with a higher probability. Clearly, condition (5) is easier to satisfy than condition (3) in so far as it holds for more parameter values. Hence, the incentives to share are decreasing.

In summary, there are two explanations for why sharing patterns are decreasing over time. The first explanation is that if the firms do not share at $(2,1)$, sharing at $(1,0)$ may still be beneficial because it reduces the time to market by enabling the lagging firm to finish first. This speed effect is not present at the histories $(2,0)$ and $(2,1)$ where one firm has already reached the market. The second explanation is that if the firms share at $(2,1)$ and $(2,0)$, this eliminates the only cost of sharing which is the rivalry effect. As a result, at $(1,0)$, neither firm expects to earn monopoly profits in the future and the sharing condition at $(1,0)$, given by (5), holds trivially. It is interesting to note that the dynamics described above continue to hold when research costs c are zero. Hence, savings of duplicated R&D costs are not the only reason the firms find it optimal to share. Firms are also motivated to share by the speed effect.[18]

[18] The results for Region A extend to a model with three research steps. A proof is available on request. With a three-step research process, we can compare histories where the leader is one step ahead of the lagging firm (i.e., $(1,0)$, $(2,1)$, and $(3,2)$), and histories where the leader is two steps ahead of the lagging firm (i.e., $(2,0)$ and $(3,1)$). The sharing conditions in Region A have the form (4) at all histories and in every equilibrium, where $\beta \in [0,1)$ depends on the history and the future sharing decisions. In each equilibrium, when we compare the sharing conditions at two histories with the same gap, we find that the value of β is higher at the earlier

To finish our discussion of Region A, we briefly discuss individual payoffs and licensing fees. Since sharing decisions are made jointly, they do not depend on this analysis. However, it is still interesting to consider whether the licensing fees have the same dynamics as the sharing incentives. In section C of the appendix, we analyze an MPE in which the firms share at all histories. We find that both firms have a higher payoff at $(2,1)$ than at $(1,0)$. Essentially, this is because costs are invested upfront while profits are earned later and are discounted. Hence, as the game progresses, individual payoffs rise. This is in contrast with the sharing incentives which decrease over time. The dynamics of the licensing fees depend on the magnitude of the discount factor $\frac{r}{\alpha}$. When $\frac{r}{\alpha}$ is sufficiently high, the payoffs increase significantly over time, and the licensing fees increase along with them so that $F(2,1) > F(1,0)$. When $\frac{r}{\alpha}$ is low, however, the payoffs do not increase as much over time. The licensing fees now have the same dynamics as the sharing incentives. They decrease over time so that $F(2,1) < F(1,0)$.

Although the licensing fees are not relevant for the analysis of sharing, they do impact the investment decisions which are based on individual payoffs. Our assumption that the leading firm has all the bargaining power acts to discourage investment by the lagging firm. If the lagging firm had more bargaining power, then the regions of parameters associated with equilibria would change. There would be more parameters in Region A and fewer in Region B.

3.2 Dynamics of Sharing When Firms Exit the Game

We next consider Region B, where duopoly profits are too low to keep the lagging firm in the race at all histories. This introduces an important strategic motive for a leading firm to refuse to share. A leading firm can now prevent the erosion of its monopoly profits (the rivalry effect) by causing the lagging firm to exit the game. Our question is whether, in light of this, the sharing incentives continue to be decreasing. We find that this is not the case. A lagging firm may be more likely to drop out earlier in the game when it has more research left to complete. Given this, the firms may be less likely to share (i.e., the rivalry effect may be stronger) earlier

history. This means that the speed effect is more important relative to the rivalry effect, and it gives us the result that sharing incentives are decreasing.

in the game.

Proposition 2 *In Region B, for an open set of parameters, there is a MPE such that the sharing pattern is (NS,S). The histories* $(1,0)$ *and* $(2,1)$ *both arise on the equilibrium path.*

The proposition is proved in section D of the appendix. For a region of parameter values, we demonstrate a unique equilibrium in which the sharing pattern (NS,S) arises on the equilibrium path. In the equilibrium, $(2,0,NS)$ is the only history at which the lagging firm drops out. At $(2,1,NS)$ and $(1,0,NS)$, the lagging firm stays in the race. Thus, the firms have a strong incentive to forego sharing at $(1,0)$ in order to reach $(2,0)$. At $(2,0)$, the firms do not share, the lagging firm drops out, and the leading firm then earns monopoly profits forever. The sharing pattern (NS,S) arises on the equilibrium path when, after choosing not to share at $(1,0)$, the firms next reach the history $(1,1)$ rather than $(2,0)$. The game then proceeds to $(2,1)$ or $(1,2)$, at which point the firms share step 2.

We briefly discuss the sharing conditions at $(1,0)$ and $(2,1)$. Since the lagging firm does not drop out at $(2,1,NS)$, the sharing condition at $(2,1)$ is the same as condition (3) for Region A. The rivalry effect $2\pi^D - \pi^M$ is negative in the equilibrium, but the duplication effect c more than offsets it and the sharing condition holds. Hence, the firms share at $(2,1)$. The sharing condition at $(1,0)$ is given by (28) in the appendix. It can be rewritten as

$$\beta\left(2\pi^D + 2c\right) + (1-\beta)\left(2\pi^D - \pi^M\right) > 0, \tag{6}$$

where $\beta = \frac{r}{(2\alpha+2r)}$. As before, the first term in (6) is the increase in joint profits when the lagging firm finishes first. It is due to the speed and duplication effects. By comparison to the first term in (4), the speed effect involves duopoly profits rather than monopoly profits and the duplication effect is doubled. This is because when the lagging firm finishes, the firms share step 2. When this happens, the flow $2c$ of research costs is replaced by the flow $2\pi^D$ of duopoly profits. The second term in (6) is the net loss in joint flow profits when the leading firm finishes first. By comparison to the second term in (4), there is a rivalry effect, but no duplication effect. The reason there is no duplication effect is that if the firms do not share at $(1,0)$ and the leading firm then has the next research success, the firms reach $(2,0)$ where the lagging firm exits the race and so does not invest in wasteful duplication of R&D. Since the weight β is increasing in the discount factor $\frac{r}{\alpha}$, when the discount factor is sufficiently small, the rivalry effect more than offsets both the speed and duplication effects, and the firms choose not to share. Intuitively, when the firms are sufficiently patient, they prefer to protect the monopoly profits that are available at $(2, X)$ even though this slows down their progress in research.

For interested readers, we solve for all of the equilibria of the model in a companion appendix.[19] There, we demonstrate another equilibrium where sharing incentives are increasing. In that equilibrium, the firms choose not to share at $(1, 0)$ because then the lagging firm immediately drops out. Because of this, the firms never reach the history $(2, 1)$ on the equilibrium path. They do share at $(2, 1)$ off the equilibrium path however.[20]

Although we have demonstrated how the possibility of drop out may result in no sharing at $(1, 0)$, it is important to note that it may also increase the incentives to share. An interesting example of this arises where the firms share at $(1, 0)$ in order to keep the lagging firm in the race.[21] In equilibrium, the firms share at $(1, 0)$ despite the fact that the lagging firm would drop out at $(1, 0, NS)$ and cannot otherwise be induced to drop out in the future. Sharing enhances joint profits because the lagging firm may finish the race faster than the leading firm (the speed effect) so that duopoly profits are earned sooner. Although the rivalry effect is still present, it is dominated by the speed and duplication effects. In this region, there are also multiple equilibria at $(0, 0)$, one where both firms invest and another one where neither firm invests. When both firms invest, the firms benefit from each other's presence because of future sharing at $(1, 0)$ and $(0, 1)$. But for this sharing, neither would have wanted to invest at $(0, 0)$.

In the examples discussed so far, duopoly profits are such that the lagging firm drops out at $(2, 0, NS)$, but it invests at $(2, 1, NS)$. This happens when duopoly profits satisfy $\frac{cr}{\alpha} \leq \pi^D < c\frac{r}{\alpha}\left(2 + \frac{r}{\alpha}\right)$. When $\pi^D < \frac{cr}{\alpha}$, the lagging firm exits at $(2, 1, NS)$. We refer to such

[19] The companion appendix is available at http://www.economics.unimelb.edu.au/staff/nisvan/research.htm.

[20] The equilibrium is demonstrated in section 5.8 of the companion appendix. The sharing pattern (NS,S) also arises off the equilibrium path in equilibria where the payoffs are so low that both firms drop out at $(0, 0)$. Such equilibria are demonstrated in sections 5.14 and 5.15.

[21] The equilibrium is demonstrated in section 5.9 of the companion appendix.

games as winner-take-all games. When the lagging firm exits at $(2, 1, NS)$ and the firms are rivalrous ($\pi^M > 2\pi^D$), there is an MPE with the decreasing sharing pattern (S,NS). At $(2, 1)$, the firms choose NS because monopoly profits are higher than duopoly profits. The lagging firm then drops out. Because the firms will never compete as duopolists, there is no rivalry effect at $(1, 0)$ and $(2, 0)$. The duplication effect induces the firms to share at both histories, with the speed effect also contributing at $(1, 0)$. Along the equilibrium path, the firms achieve the constrained joint profit maximization benchmark that was introduced in section 2.4.[22]

3.3 Discussion and Industry Examples

The results in sections 3.1 and 3.2 relate to a fundamental question in the economics of R&D on how competition affects the incentives for cooperation in R&D. They reveal that the dynamic impact of competition on cooperation is complex. The sharing incentives may increase or decrease throughout a research process. In industries where duopoly profits are high relative to the costs of research (as in Region A) so that the lagging firm pursues duopoly profits rather than exiting, the firms have decreasing incentives to share. In industries where duopoly profits are low relative to the cost of research (as in Region B) so that the lagging firm exits at some histories, the firms may have either decreasing or increasing incentives to share. A natural question is whether particular industries are best described by the dynamics of Region A or Region B.

The computer software industry is often described as a winner-take-all industry. Research costs are high and firms race to reach the market for fear of losing out. Microsoft and Apple, for example, have each dominated the market for particular products for long stretches of time. As is well-known, this is partly explained by network effects that increase the value of the dominant brand. Our results for Region B suggest that software firms may be willing to share research at early stages, given that the rivalry effect will be resolved eventually by the winner-take-all nature of the market and one firm will become dominant. One of the most famous stories of sharing in Silicon Valley, that of engineers at Xerox Parc showing their graphical

[22] Such equilibria are demonstrated in sections 5.11, 5.12, and 5.16 of the companion appendix. Section 5.12 provides another example of an MPE in which sharing at $(1, 0)$ acts to keep the lagging firm in the race.

user interface technology to Steve Jobs at Apple, arguably fits this description.[23]

In the automobile industry, by contrast, while research costs are again high, product differentiation relaxes competition and many models of cars profitably coexist in the product market. In this case, our model again predicts that sharing of early stage research will be more likely, but for a different reason than in the software example. In Region A, sharing breaks down as the firms approach the product market because the rivalry effect becomes more important relative to the speed and duplication effects. As discussed in the introduction, a collaboration between GM and Toyota on fuel cell technology for automobiles lasted for more than 6 years before their competitive rivalry made cooperation too difficult. A collaboration on fuel cell technology between Daimler and Ford has proved longer lasting, perhaps because the rivalry between these two firms is less intense.[24]

4 Asymmetric Firms

So far we have assumed that firms are symmetric in their research capabilities to focus on the impact of uncertainty and progress on the firms' sharing decisions. In this section, we analyze to what extent our conclusions in section 3 apply when firms have different abilities to conduct different stages of research. For example, in the biotechnology industry, alliances often involve a firm which has developed expertise in research on a particular biotechnology and a large pharmaceutical which may be better able to bring the product through the clinical testing and regulatory approval process to the market (Lerner and Merges, 1998).

We consider an environment where one of the firms is better at one step of research than the other step of research. Assume that firm 1 has a cost of c_1 for both steps of research, but firm 2 has a cost of c_2^1 for the first step and c_2^2 for the second step. This is the simplest

[23] In 1979, Steve Jobs allowed the venture capital division of Xerox Parc to invest \$1 million in the second round of Apple financing, in exchange for a demonstration of its graphical user interface(GUI) technology based on a bitmap system. When Apple went public one year later, the Xerox Parc investment was worth \$17.6 million, but Apple got the better end of the deal, because it went on to develop and commercialize personal computers with bitmapped screens. See chapter 8 in Isaacson (2011).

[24] In 1997, Daimler, Ford Motor, and Ballard Power Systems formed an alliance to develop Ballard's fuel cell technology. In 2008, Daimler and Ford formed a new company, Automotive Fuel Cell Cooperation (AFCC), to continue the development of Ballard's fuel cell technology. See Steinemann (1999) and http://cafcp.org/about-us/members/automotive-fuel-cell-cooperation

way to model two firms with different relative advantages. The model is now defined for $\Omega = \{(\alpha, r, c_1, c_2^1, c_2^2, \pi^M, \pi^D)$ such that $0 < \alpha$, $0 < r < 1$, $c_1 > 0$, $c_2^1 > 0$, $c_2^2 > 0$, $\pi^M > \pi^D \geq 0\}$. We restrict attention to Region A, where both firms invest at every history.[25]

Due to the cost asymmetry, the dynamics of sharing incentives depend on which firm is the leader. Preceding in the same way as we did in the symmetric case, we have the following result.

Proposition 3 *In Region A, if the firms share at* $(1,2)$ *in an MPE, they also share at* $(0,1)$*. If* $c_2^1 \geqslant c_2^2$*, then if the firms share at* $(2,1)$ *in an MPE, they also share at* $(1,0)$*. For some values of* $c_2^1 < c_2^2$*, however, there exists an MPE such that the firms share at* $(2,1)$*, but they do not share at* $(1,0)$*.*

The novel result in Proposition 3 is that the sharing pattern (NS,S) may arise when firm 1 is the leader. This means if the firms share at $(2,1)$, they do not necessarily share at $(1,0)$. To explore this, we derive an equilibrium in section E of the appendix where the firms share at the four histories $(2,1)$, $(1,2)$, $(2,0)$ and $(0,2)$.[26] As in the symmetric model, this future sharing eliminates the rivalry effect at $(1,0)$. However, the firms may still decide not to share. From (31) in the appendix, the sharing condition at $(1,0)$ is

$$\beta\left(2\pi^D + c_1 + c_2^2\right) + (1-\beta)(c_2^1 - c_2^2) > 0, \tag{7}$$

where $\beta = \frac{\alpha}{(3\alpha + r)}$. The new term $c_2^1 - c_2^2$ captures the change in investment costs when the lagging firm stops research on step 1 and begins research on step 2. We refer to this as the *progress effect.* If $c_2^1 - c_2^2 < 0$, this is a loss and (7) does not always hold. When (7) fails, the firms share at $(2,1)$ but not at $(1,0)$. By not sharing at $(1,0)$, the firms prevent firm 2 from starting to work on step 2, where it would incur higher research costs. If firm 1 subsequently completes step 2, firm 2 will never have to work on it.[27]

[25] Region A is the set of parameters such that $\pi^D \geq \max\left\{\frac{c_1 r}{\alpha}\left(2 + \frac{r}{\alpha}\right), \frac{c_2^1 r}{\alpha}\left(1 + \frac{r}{\alpha}\right) + \frac{c_2^2 r}{\alpha}\right\}$. The proof is a straightforward generalization of Lemma 1.

[26] The full proof of Proposition 3 is a generalization of our results for Region A of the basic model and is available on request.

[27] The firms would attain even higher joint profits if firm 2 were simply to refrain from conducting further research at $(1,0)$. However, by assumption, the firms cannot agree to this.

The first term in (7) captures the speed effect (because the firms earn duopoly profits $2\pi^D$ sooner) and the duplication effect (because the firms save research costs $c_1 + c_2^2$ by finishing the project sooner). These benefits are both realized at the point that the firms enter the product market. By contrast, the progress effect is realized immediately when the firms share at $(1, 0)$. Hence, when firms are more impatient (i.e., when the discount factor $\frac{r}{\alpha}$ is high so that β is low), they put more weight on the progress effect and may decide against sharing.

Proposition 3 shows that when research costs increase over time, the firms may have a stronger incentive to share as the race progresses in Region A. However, if research costs decrease or do not change over time, the sharing incentives continue to be decreasing. Proposition 3 implies that it is the asymmetry of costs for a single firm that changes the sharing dynamics, not the asymmetry between the firms. That is, in the special case where one firm is better at both stages of research, but where the research costs of each firm do not change over time, the sharing pattern (NS,S) does not arise.

In the benchmark model with symmetric costs, an increase in c always makes sharing more attractive. Here, while an increase in the first-stage research cost of firm 2, c_2^1, makes sharing at $(1, 0)$ more attractive, an increase in the second-stage research cost, c_2^2, makes sharing at $(1, 0)$ less attractive.

5 R&D Policy

In this section, we consider how R&D policy can be used to change the investment and sharing incentives of firms to increase social welfare. Our main focus is on patent policy, but for comparison, we also consider subsidies for R&D. We consider a social planner maximizing expected social welfare at $(0, 0)$, defined as the expected value of the flow of total surplus in the product market net of the firms' expected flow costs of research in the R&D phase.[28]

[28] Recall from section 2.2 that total surplus under duopoly and monopoly are TS^D and TS^M, respectively, where $TS^D > TS^M$. The expected social welfare at $(0, 0)$ is derived in the same way as the value functions for the firms. Following the calculation of V^J at $(0, 0)$, and replacing π^M with TS^M and π^D with TS^D would yield the expected social welfare function at $(0, 0)$.

5.1 Patent Policy

Patent policy, at its core, is about protecting an innovator's expected profits from competition as a way to encourage investment in R&D. Because a social planner values consumer surplus and firms do not, a social planner may prefer more investment than that arises in the market outcome. In the analysis so far, we have assumed that firms face the same research cost c regardless of whether the step they are working on has been patented. In this section, we relax this assumption and assume that patenting increases the research cost faced by a lagging firm by forcing it to invent around (Gallini, 1992). This in turn can protect the profits of leading firms, encouraging investment and sharing.

Let c_p^1 and c_p^2 stand for the research cost of the lagging firm for the first and second research steps, respectively. We assume that $c_p^1 \geq c$ and $c_p^2 \geq c$. The gap $c_p^i - c$ is the extra cost of inventing around the leading firm's patent, and is a measure of the breadth of the patent policy. An early stage policy c_p^1 changes the flow cost of research for the lagging firm at the histories $(1, 0)$ and $(2, 0)$. A late stage policy c_p^2 changes the flow cost of research for the lagging firm at the history $(2, 1)$. Otherwise, the model is unchanged.[29]

Our first result shows how patent policy can be used to enhance social welfare by eliminating the rivalry effect and protecting monopoly profits. In Proposition 4, a patent policy protecting late stage research feeds back to encourage investment and sharing of early stage research.

Proposition 4 *Consider an industry with parameters π^D, π^M, r, c and α such that $2\pi^D < \pi^M$. For any late stage patent policy c_p^2 such that $c_p^2 > \frac{\alpha}{r}\pi^D$, the constrained joint profit maximizing outcome is realized along the equilibrium path of an MPE.*

In Proposition 4, the social planner offers such strong protection c_p^2 for the second step of research that the lagging firm drops out at $(2, 1, NS)$. The leading firm is then a monopolist in the product market. Anticipating this, the firms do not share at $(2, 1)$. These decisions eliminate the rivalry effect at all earlier histories. The firms benefit from sharing early research

[29] It is possible that a firm holding a patent may exit the race. The remaining firm might then have to invent around the patent, even though the firm holding the patent has exited. Such situations would arise off the equilibrium path and are not very interesting. Rather than modeling them, we assume them away so that when a firm exits the race, if it has not shared its research, then its research and its patent disappear.

due to the speed and duplication effects. In this outcome, the joint profits of the firms from investing at $(0,0)$ are as high as possible without collusion in the product market. This is the constrained joint profit maximizing outcome discussed in section 2.4.[30] It is straightforward to construct examples where firms invest at $(0,0)$ with such patent protection, but they do not invest without it (i.e., when $c_p^2 = c$).[31] Since social welfare is zero when the firms do not invest, such a policy obviously enhances social welfare.

We next explore how patent policy can be used to enhance social welfare by changing sharing decisions alone. For this, we focus on Region A where firms invest at all possible histories. Region A is defined by

$$\pi^D \geq \frac{r}{\alpha}\left[\left(1+\frac{r}{\alpha}\right)c_p^1 + c_p^2\right]. \tag{8}$$

When (8) holds, because investment incentives are adequate, the only role of patent policy is to modify sharing decisions.[32] From the social planner's perspective, sharing is always desirable in Region A. Sharing increases social welfare by eliminating duplication and speeding up the time for the first firm to reach the product market. Sharing also helps the lagging firm to reach the product market sooner. Since total surplus is higher under duopoly than monopoly, the social planner also values this effect.

Intuitively, one would expect broader patent policy to encourage firms to share in Region A by increasing the cost to the lagging firm of working around the leading firm's patent. If this is the case, then broader patent protection increases social welfare. Surprisingly, we find that there are cases when this basic intuition does not hold.

[30] We note that an individual firm's profits are higher in this outcome than in an outcome such that the firm is the only active firm at $(0,0)$. In the constrained joint profit maximizing outcome, each firm pays half of the joint research costs in return for getting the monopoly profits half of the time. Since the research process is more efficient, this yields a higher profit than paying all of the research costs in order to get the monopoly profits all the time.

[31] In the companion appendix, section 5.7 demonstrates parameters in Region B such that there is a unique MPE in which neither firm invests at $(0,0)$. The late stage patent policy in Proposition 4 can be shown to induce investment at $(0,0)$ for a subset of these parameters.

[32] When the expression holds, it is profitable for the lagging firm to invest at $(2,0)$ despite having to pay c_p^1 to research the first step and c_p^2 to research the second step. When $c_p^1 = c_p^2 = c$, the expression is the same one that defines Region A in the basic model. Otherwise, the expression shows that increases in the costs of first-step vs. second-step research will affect Region A differently. Specifically, increases in the patent protection c_p^1 of early research reduces the continuation profits of the lagging firm at $(2,0)$ more than do equivalent increases in c_p^2. The expression holds more easily as $\frac{r}{\alpha} \to 0$.

Our first result considers broader patent protection of early research outcomes. We focus on environments where sharing incentives are weak. That is, we consider parameters π^D, π^M, r, c, and α such that the firms do not share at any history when $c_p^1 = c_p^2 = c$. We consider how the MPE changes as c_p^1 is increased so that $c_p^1 > c_p^2 = c$. Proposition 5 confirms the basic intuition that broad patent policy can enhance social welfare by encouraging firms to share their research.

Proposition 5 *(Early Stage Patent Policy) Consider an industry with parameters* π^M, π^D, r, α *and* c *such that when* $c_p^1 = c$ *we are in Region A, and the equilibrium research outcome does not involve sharing at any of the research histories (*$(1,0)$*,* $(2,0)$*, and* $(2,1)$*). There exist threshold levels* $\bar{\bar{c}}_p^1 > \bar{c}_p^1 > c$ *such that:*

(i) when $\bar{c}_p^1 > c_p^1 > c$*, the firms do not share at any history;*

(ii) when $\bar{\bar{c}}_p^1 > c_p^1 > \bar{c}_p^1$*, the firms share at* $(1,0)$*, but not at* $(2,0)$ *or* $(2,1)$*;*

(iii) when $c_p^1 > \bar{\bar{c}}_p^1$*, the firms share at* $(1,0)$ *and* $(2,0)$*, but not at* $(2,1)$*.*

Notice that in Proposition 5, broader patent protection of early research has no impact on the decision to share the second step. This is because first-stage research costs are irrelevant to the sharing decision at $(2,1)$. Notice also that in Proposition 5, as c_p^1 increases, the firms start to share at $(1,0)$ before they start to share at $(2,0)$. At both histories, the firms are deciding whether to share the first research step as it is subjected to broader patent protection. The difference is that at $(2,0)$, the leader earns monopoly profits while at $(1,0)$, the leader is engaged in research, and there is still uncertainty about which firm will finish first. The fact that the leading firm is making monopoly profits at $(2,0)$ makes the rivalry effect very strong, and the firms benefit by delaying the lagging firm's entry into the market.

Patent policy increases social welfare in parts (ii) and (iii) of Proposition 5. This is because sharing at $(1,0)$ eliminates duplication of research and speeds the time to market. In part (i), the patent policy reduces social welfare. Here, the policy is not strong enough to induce sharing and acts only to increase the lagging firm's research costs at $(1,0)$ and $(2,0)$.

In the next proposition, we consider the impact of broader patent protection of late research. For this, we assume $c_p^2 > c_p^1 = c$ and analyze what happens as c_p^2 increases. Although policies that affect costs of early research do not have an impact on the incentives to share late research outcomes, the converse is not true. This is because the firms' payoffs at $(2,1)$ are incorporated into their continuation payoffs at earlier histories. As c_p^2 increases, one would expect to see more sharing of the second research step. Since this eliminates the rivalry effect at $(2,1)$, it may also be reasonable to expect more sharing earlier in the game. However, we show that this is not necessarily the case due to the progress effect that we introduced in our discussion of asymmetric firms.

To illustrate, we consider a region of parameters such that when $c_p^2 = c$, the equilibrium research outcome involves sharing at $(1,0)$ but no sharing at $(2,0)$ or $(2,1)$.

Proposition 6 *(Late Stage Patent Policy) Consider an industry with parameters* π^M, π^D, r, α *and* c *such that when* $c_p^2 = c$, *we are in Region A and the equilibrium research outcome involves sharing at* $(1,0)$ *but no sharing at* $(2,0)$ *and* $(2,1)$. *In addition, assume that* $c < \overline{c} = \left(\pi^M - 2\pi^D\right) - \left(\frac{2\alpha+2r}{4\alpha+3r}\right)\left(\pi^M - \pi^D\right)$. *There exist threshold levels* $\overline{c} > \overline{\overline{c}}_p^2 > \overline{c}_p^2 > c$ *such that:*

(i) when $\overline{c}_p^2 > c_p^2 > c$, *the firms share at* $(1,0)$ *but not at* $(2,0)$ *or* $(2,1)$*;*

(ii) when $\overline{\overline{c}}_p^2 > c_p^2 > \overline{c}_p^2$, *the firms do not share at any history;*

(iii) when $c_p^2 > \overline{\overline{c}}_p^2$, *the firms share at* $(2,1)$ *but not at* $(2,0)$ *or* $(1,0)$.

In Proposition 6, the policy targets sharing of late research and such sharing is achieved, but at the cost of discouraging sharing at $(1,0)$.[33] As patent protection of late research broadens, it becomes more expensive for firm 2 to conduct research at $(2,1)$. This introduces a progress effect similar to the one we discussed in section 4. The progress effect discourages sharing at earlier histories because a decision not to share step 1 delays the lagging firm from reaching step 2 and, hence, delays the research expense that the lagging firm incurs at step 2. This explains why sharing breaks down at $(1,0)$ in part (ii) of Proposition 6. In part (iii), c_p^2 is so high that the duplication effect leads the firms to share at $(2,1)$. As a result, c_p^2 no longer impacts the

[33] Sharing at all histories can be achieved with sufficiently broad patent protection of both early and late research. Suppose the lagging firm's research cost is $c_p^1 = c_p^2 = c_p$, where c_p is greater than c. If we start with an environment where there is no sharing at any of the histories when $c_p = c$, then as c_p increases within Region A, we first get sharing at $(1,0)$. With further increases in c_p, the firms simultaneously switch from not sharing to sharing at both $(2,0)$ and $(2,1)$. Hence, a social planner can achieve sharing at all histories using a policy which applies the same degree of patent protection to all research.

sharing incentives at earlier histories. With the rivalry and progress effects eliminated at $(2,1)$, it might be reasonable to expect that the firms would share at $(1,0)$. This does not happen, however, because the rivalry effect is still present at $(2,0)$.[34] The sharing condition at $(2,0)$ is $\pi^M - 2\pi^D < c$, which fails to hold for the selected parameters so that the firms do not share. This decision then feeds back to deter sharing at $(1,0)$. Because the firms will share at $(2,1)$, the only way firm 1 can expect to earn π^M is by reaching $(2,0)$, and this can only be achieved if the firms do not share at $(1,0)$.

Social welfare is reduced in parts (i) and (ii) of Proposition H as patent protection c_p^2 increases. In part (i), this is because the lagging firm incurs higher costs to invent around the patent at $(2,1)$ while the sharing and investment decisions of the firms are unchanged. In part (ii), social welfare is further reduced when sharing breaks down at $(1,0)$. In part (iii), as c_p^2 increases further, the firms begin to share at $(2,1)$. This increases social welfare by converting monopoly profits to duopoly profits. It also eliminates the costly duplication of second stage research by the lagging firm, which enhances social welfare.

An interesting question is whether welfare is higher under the patent policy in part (iii) than in the basic model where $c_p^1 = c_p^2 = c$. In part (iii), the firms share late stage research at $(2,1)$, but they do not share early stage research at $(1,0)$. In the basic model, we have the opposite sharing pattern. The firms share early stage research at $(1,0)$, but they do not share late stage research at $(2,1)$. It is not obvious which sharing pattern is preferred by the social planner. Sharing of late stage research encourages competition in the product market, but sharing of early stage research allows the firms to reach the product market earlier (the speed effect). Considering the expected social welfare under both scenarios, it is straightforward to show that the social planner would prefer sharing of early stage research under a relatively mild condition, given by $TS^M > \frac{1}{2}TS^D$.[35]

[34] In section 3.1, when the rivalry effect was eliminated at $(2,1)$, it was also eliminated at $(2,0)$. This does not happen here because the patent policy introduces an asymmetry in the costs for the two steps of research.

[35] To see this, ignore the flow costs of R&D for the moment. Sharing at $(2,1)$ increases the flow of social surplus from TS^M to TS^D so that the change in surplus is $TS^D - TS^M > 0$. By contrast, sharing at $(1,0)$ reduces the time until the first firm reaches the product market. When this happens, social surplus increases from 0 to TS^M so that the change in surplus is $TS^M > 0$. Hence, if $TS^M > TS^D - TS^M$ the social planner prefers the sharing pattern in which the firms share at $(1,0)$ to the sharing pattern in which they share at $(2,1)$. This is also true after taking the flow costs of R&D into account.

So far in our discussion of Region A, we have considered how patent policy can be used to increase the duplication effect and hence encourage sharing. Duplication can also be avoided by having the lagging firm drop out. In this case, the policy shifts the game from Region A to Region B by reducing the investment incentives of the lagging firm such that the lagging firm exits. This may not always harm social welfare. To see this, consider the region of the parameter space where, when $c_p^1 = c_p^2 = c$, the firms are in Region A, sharing at $(1, 0)$ but not sharing at $(2, 1)$. Suppose the social planner adopts the policy in Proposition 4. This results in the same sharing pattern as in the basic model, but now the lagging firm exits at $(2, 1, NS)$ whereas it continues to invest at $(2, 1, NS)$ in the basic model. Such a policy involves a trade-off between avoiding duplication and reducing competition. The elimination of the flow cost c of research has a positive impact on social welfare, but the elimination of future competition in the product market affects it negatively. Other things equal, an impatient social planner with a sufficiently high discount factor $\frac{r}{\alpha}$ will prefer the immediate savings of R&D costs to the future harm on competition.[36]

We conclude this section by observing a basic limitation of patent policy, namely that it is ineffective in improving social welfare when firms are not rivalrous (that is, when $2\pi^D > \pi^M$). When there is enough differentiation between the products of the two firms, they are jointly better off when both produce in the product market than when only one of them produces. It is therefore always in their best interest to share the outcomes of R&D, regardless of the cost of duplication under the patent policy. When the firms share, patent policy does have an impact on the licensing fees. Under broader patent protection, a lagging firm pays a higher licensing fee to the leading firm. This, however, is simply a transfer from one firm to the other. The policy does not impact the decision to share, and the firms share at every asymmetric history. Given this sharing, investment decisions arise on the equilibrium path only at $(0, 0)$ and $(1, 1)$. Since the firms are equally likely to lead or lag in the future at these histories, patent policy

[36] Comparing the social welfare of these two outcomes, it is straightforward to show that the social planner prefers the patent policy to the basic model if and only if $c > \frac{\alpha}{r}(TS^D - TS^M)$. Here, the left hand side of the inequality captures the flow cost of research that is saved if the lagging firm drops out of the game at $(2, 1, NS)$. The right hand side is the expected increase in social surplus that arises if the lagging firm stays in the game at $(2, 1, NS)$, and is successful at completing the second step of research.

does not change their expected payoffs. Hence, if the firms are not willing to invest at $(0, 0)$ when $c_p^1 = c_p^2 = c$, a social planner cannot use patent policy to improve on the outcome.[37] As we discuss next, this scenario is where R&D subsidies may be most called for.

5.2 R&D Subsidies

Another type of policy intervention which may affect the investment incentives of firms is R&D subsidies. R&D subsidies are commonly used in many countries throughout the world and take many forms.[38] R&D subsidies are in many respects the opposite of patent policy. While patent policy makes research more expensive, R&D subsidies make research less expensive.

In Region A, research subsidies do not typically enhance social welfare. Because the firms are already investing, the social planner has no need to strengthen the investment incentives. The social planner can improve welfare in Region A by encouraging sharing. R&D subsidies, however, tend to discourage sharing for similar reasons to why patent policy encourages it. Since the way R&D subsidies change sharing decisions is analogous to the way patent policy changes sharing decisions, we do not repeat the full analysis here.[39] The main intuition can be understood by considering the sharing condition (3) in our basic model. If a subsidy policy reduces the flow cost c of the lagging firm at $(2, 1)$, this inequality becomes harder to satisfy. From the firms' perspective, the duplication effect is lessened. As a result, there are fewer parameters such that the firms share step 2.

Since R&D subsidies tend to encourage investment but discourage sharing, they should primarily be used in industries where investment incentives are inadequate. In Proposition 4, we analyzed a patent policy that provides the strongest possible investment incentives at

[37] For some parameters, in addition to the MPE that maximizes the firms' joint profits, there is an MPE in which the firms fail to coordinate on investment at $(0, 0)$ even though investment is jointly profitable. Patent policy, however, does not solve the coordination problem, so the two MPE continue to exist.

[38] In many programs, firms apply to get funding for a specific project and there is a selection process which takes place based on different criteria. Depending on the program, the goal may be to support specific types of industries, technological problems, or early vs. late stage research. In some countries, tax policy, which is less targeted than programs designed to support specific projects, is also used to subsidize R&D (Hall and Van Reenen, 2000).

[39] R&D subsidies targeted at early stage research discourage the firms from sharing step 1. As with early stage patent policy, such subsidies do not have any impact on the sharing of step 2. R&D subsidies targeted at late stage research discourage the firms from sharing step 2. As with late stage patent policy, such subsidies may feed back to increase the incentives to share step 1.

$(0, 0)$. Even this policy however is not sufficient to ensure investment at $(0, 0)$ for all parameters. R&D subsidies, by contrast, can be set high enough to ensure investment at $(0, 0)$ since a full subsidization of the flow cost c is possible. Since subsidies require government funding, a social planner may want to limit their use to industries where patent policy is ineffective. An insight from our model is that patent policy is least effective when the firms are not rivalrous (when $2\pi^D > \pi^M$). As stated above, patent policy impacts the terms of licensing fees in such cases, but it has no impact on the investment decisions (which take place at $(0, 0)$ and $(1, 1)$). Hence, a social planner who would like to increase investment incentives may want to rely on R&D subsidies in industries with a high level of product differentiation.

6 Conclusion

This paper considers how the incentives to share knowledge change over time as a research project reaches maturity in the context of technological competition. For symmetric firms, sharing dynamics is shown to be driven by three effects: the duplication effect, speed effect and rivalry effect. While the first two effects work in favor of sharing, the last one works against it. Hence, whether the firms share or not depends on how strong the rivalry effect is relative to the other two effects. When the rivalry effect is eliminated, the firms always share.

Our results reveal that both how close the firms are to product market competition and how intense that competition is shape the firms' sharing behavior. We show that when duopoly profits are high relative to the costs of research so that the lagging firm is never expected to drop out of the race, the incentives to share always decrease with progress. However, when duopoly profits are lower so that the lagging firm is expected to drop out at some histories, the incentives to share may increase with progress. In this case, the reason for the reluctance to share early on is the desire to force the lagging firm to drop out. This is easiest to do early on when the lagging firm has not made much progress. When duopoly profits are even lower so that a lagging firm can be forced to exit late in the race, the incentives to share again decrease with progress. This is because if the firms can force the lagging firm to drop out near the end of the race, the rivalry effect is removed at earlier histories so that there is no downside to sharing early on. These results imply that the prevalence of sharing in early stages of research in certain industries, often attributed to efficiencies of internalizing spillovers, could be due to competitive dynamics. Thus, to the extent that the competitive dynamics matter, the propensity to share in early stages would not indicate its higher social value.

These results have implications for policy making in innovation environments. One general implication of our study is that policy makers may want to distinguish between markets with different levels of duopoly profits and research costs in their policy design, since the sharing dynamics as well as the investment incentives are shaped by them. Our discussion of optimal R&D policy, focusing on the instruments of patent policy and R&D subsidies, reveals that these two policy instruments have distinct roles to play in innovative environments. When duopoly profits are high so that lagging firms always invest in R&D, our results imply that a social planner aiming to increase sharing incentives should mainly rely on patent policy. However, when investment incentives are a concern, R&D subsidies may sometimes be preferred to patent policy. R&D subsidies increase investment incentives by decreasing the cost of investment through public funding of R&D. This tends to discourage sharing, but it encourages competition in the product market. Patent policy, by contrast, can increase investment incentives by eliminating competition in the product market. With the rivalry effect removed, investment and sharing are both encouraged at early stages of R&D. However, if firms are not rivalrous to begin with (as in industries characterized by a high level of product differentiation), then patent policy will be ineffective and R&D subsidies should be preferred.

Our results suggest new directions for empirical research on innovation.[40] Although there is a large literature on research alliances, there has been little empirical research focusing on the dynamics of these alliances. Our theoretical work focuses on the dynamics of sharing where the intensity of product market competition, the difficulty of research, and the impatience of firms are the key factors. Future research could address whether these dynamics can be identified and empirically distinguished from the impact of other dynamic variables, such as the intensity of spillovers, financing issues, and the degree of antitrust risk, which are also likely shape the

[40] Recent work by Deck and Erkal (2013) using laboratory experiments finds confirmation of our monotonicity result in a two-stage game.

patterns of sharing. The role played by each factor may depend on the industry and the nature of the research.[41]

We conclude by mentioning two ways in which the current analysis can be extended. First, we allowed the firms to use fixed-fee contracts only. Suppose the firms can use contingent-fee licensing where the payment for sharing is structured as a share of profits earned in the product market. Such contracts can reduce the rivalry effect by inducing exit. As an example, we have analyzed whether there are parameter values where the firms find it profitable to enter into a profit-sharing contract at $(1,0)$ that induces the lagging firm to exit at $(2,1)$ and $(1,2)$. The leader at $(1,0)$ may prefer to drop out at $(1,2)$ if it has a sufficient contractual stake in the leading firm's profits since its exit implies that the leader at $(1,2)$ can earn monopoly profits. Such a contract implements the constrained joint profit-maximization benchmark of section 2.4. Under fixed-fee licensing, the firms are never able to achieve this outcome in Region A, where the lagging firm always invests. With contingent-fee licensing, we find that the firms are able to implement the profit-sharing contract for some, but not all, parameters in Region A. The contract would always be implemented if it were legal for the leading firm to make a lump-sum payment to the lagging firm at $(1,0)$ to accept the contract.

Second, we have assumed the effort choices of the firms to be discrete. Suppose the effort choices are continuous so that higher effort levels have both higher costs and higher success rates. In a continuous model, firms may have a stronger incentive not to share early on in the R&D race because it is easiest to get the lagging firm to reduce its research intensity early on. Thus, the sharing pattern (NS,S) might arise more often, in Region A as well as in Region B. We have examined this using a numerical approach with various functional forms relating effort to the hazard rate and flow cost. The equilibria and their relationship to the underlying parameter space Ω did not change significantly. We did find some examples where (NS,S) arose in Region A for parameters very close to the border with Region B. That is, the transition between the two regions became smoother.

[41] For example, Lerner and Merges (1998) find that in the biotechnology industry, it is the R&D firms' need for financing which may cause alliances to form at the earlier stages of research.

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Appendix

A Proof of Lemma 1

We must show that Region A consists of all parameters such that $\pi^D \geq c\frac{r}{\alpha}\left(2+\frac{r}{\alpha}\right)$. Consider the continuation payoff that a firm would receive by conducting two steps of research on its own and then earning duopoly profits in the output market. Because a firm can always achieve this payoff on its own, it is necessarily a lower bound on any firm's payoff at any history and in any equilibrium. In Region A, where by definition there is no exit, the payoff of the lagging firm at $(2,0)$ always equals this payoff. To see this, note that if the firms decide not to share at $(2,0)$ and $(2,1)$, this is clearly the case. If the firms decide to share at $(2,0)$ or $(2,1)$, then because the lagging firm has no bargaining power, its payoff is the same as if they do not share.

We compute this payoff by working backwards. After completing the two steps of research, the firm produces output as a duopolist to earn $\widetilde{\pi}^D = \frac{\pi^D}{r}$. To complete the second step of research, the firm invests a flow cost of c. The firm's expected payoff is

$$\int_0^\infty e^{-(\alpha+r)t}\left(\alpha\widetilde{\pi}^D - c\right)dt = \frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}.$$

To complete the first step of research, the firm again invests a flow cost of c and the hazard rate is again α. The firm's expected payoff is

$$\int_0^\infty e^{-(\alpha+r)t}\left(\alpha\left(\frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}\right) - c\right)dt = \frac{\alpha\left(\frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}\right) - c}{\alpha+r}.$$

This payoff is strictly positive if and only if

$$\pi^D > c\frac{r}{\alpha}\left(2+\frac{r}{\alpha}\right), \tag{9}$$

which is the inequality that defines Region A.

B Proof of Proposition 1

In Region A, by definition, no firm ever drops out of the game. To solve for the MPE, we only need to determine whether firms share at the six asymmetric histories. We derive the equilibrium sharing conditions for $(1,0)$, $(2,0)$ and $(2,1)$. The three mirror histories $(0,1), (0,2)$, and

$(1,2)$ have the same analysis. To derive the sharing conditions, we use backwards induction to solve for the MPE. To prove the proposition, we compare the equilibrium sharing conditions at $(1,0)$ and $(2,1)$ for every MPE.

The last history is $(2,2)$. At $(2,2)$, each firm produces output and earns discounted duopoly profits of

$$V_1(2,2) = V_2(2,2) = \widetilde{\pi}^D. \tag{10}$$

Working backwards, the next history is $(2,1)$. The firms are willing to share at $(2,1)$ iff this maximizes their joint profits. The sharing condition (1) is

$$V_J(2,2) > V_J(2,1,NS). \tag{11}$$

Joint profits under sharing are $V_J(2,2) = V_1(2,2) + V_2(2,2) = 2\widetilde{\pi}^D$. Joint profits under no sharing are

$$V_J(2,1,NS) = V_1(2,1,NS) + V_2(2,1,NS) = \frac{\pi^M + 2\alpha\widetilde{\pi}^D - c}{\alpha + r}, \tag{12}$$

where

$$V_1(2,1,NS) = \frac{\pi^M + \alpha V_1(2,2)}{\alpha + r} = \frac{\pi^M + \alpha\widetilde{\pi}^D}{\alpha + r}$$

since firm 1 earns monopoly profits until firm 2 completes the second step and

$$V_2(2,1,NS) = \frac{\alpha V_2(2,2) - c}{\alpha + r} = \frac{\alpha\widetilde{\pi}^D - c}{\alpha + r} \tag{13}$$

since firm 2 invests until it completes the second step.

The sharing condition (11) simplifies to $2\widetilde{\pi}^D(\alpha + r) > \pi^M + 2\alpha\widetilde{\pi}^D - c$ or

$$c > \pi^M - 2\pi^D \tag{14}$$

This condition holds, strictly fails, or holds as an equality. We consider each possibility in turn.

Case 1: The sharing condition at (2,1) holds. For parameter values such that the sharing condition (14) holds, the firms share step 2 at $(2,1)$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share step 1 at $(2,0)$. The sharing condition

(1) is $V_J(2,1) > V_J(2,0,NS)$. Joint profits under sharing are $V_J(2,1) = V_J(2,2) = 2\widetilde{\pi}^D$ since the firms share at $(2,1)$ after sharing at $(2,0)$. Joint profits under no sharing are

$$\begin{aligned} V_J(2,0,NS) &= V_1(2,0,NS) + V_2(2,0,NS) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c}{\alpha + r} = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r} = \frac{\pi^M + \alpha 2\widetilde{\pi}^D - c}{\alpha + r}. \end{aligned}$$

The sharing condition $V_J(2,1) > V_J(2,0,NS)$ simplifies to

$$\begin{aligned} 2\widetilde{\pi}^D(\alpha + r) &> \pi^M + 2\alpha\widetilde{\pi}^D - c \\ c &> \pi^M - 2\pi^D. \end{aligned}$$

This is condition (14), which we have assumed to hold. Hence, the firms share step 1 at $(2,0)$. The joint payoffs are $2\widetilde{\pi}^D$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. Joint profits under sharing are $V_J(1,1) = 2V_1(1,1)$ where

$$V_1(1,1) = \frac{\alpha V_1(1,2) + \alpha V_1(2,1) - c}{2\alpha + r} = \frac{\alpha V_J(2,1) - c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D - c}{2\alpha + r}. \tag{15}$$

Joint profits under no sharing are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D + \alpha V_J(1,1) - 2c}{2\alpha + r}.$$

The sharing condition simplifies to

$$(2\alpha + r)V_J(1,1) > 2\alpha\widetilde{\pi}^D + \alpha V_J(1,1) - 2c. \tag{16}$$

Substituting for $V_J(1,1)$ in (16) we get

$$\pi^D + c > 0, \tag{17}$$

which is trivially true. This proves that sharing incentives are decreasing for all parameter values for which the sharing condition (14) holds. In the unique MPE for these parameter values, the firms share at $(2,1)$ and $(1,0)$. The sharing pattern is (S,S).

Case 2. The sharing condition at (2,1) fails. For parameter values such that the sharing condition (14) strictly fails, the firms do not share at $(2,1)$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition (1) is $V_J(2,1) > V_J(2,0,NS)$. Joint profits under no sharing are

$$\begin{aligned} V_J(2,0,NS) &= V_1(2,0,NS) + V_2(2,0,NS) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c}{\alpha + r} = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}. \end{aligned}$$

The sharing condition simplifies to

$$V_J(2,1) > \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}$$

Since the firms do not share at $(2,1)$, we can substitute for $V_J(2,1)$ from (12). Simplifying, we get $c > \pi^M - 2\pi^D$. This is the same as condition (14) which does not hold. Hence, the firms do not share step 1 at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. Joint profits under no sharing are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r}. \tag{18}$$

The sharing condition simplifies to $(\alpha + r) V_J(1,1) > \alpha V_J(2,0) - 2c$. We can substitute for $V_J(1,1) = 2V_2(1,1)$. We have

$$V_J(1,1) = 2\frac{\alpha V_2(1,2) + \alpha V_2(2,1) - c}{2\alpha + r} = 2\frac{\alpha V_J(2,1) - c}{2\alpha + r} = 2\frac{\alpha(\pi^M + 2\alpha\widetilde{\pi}^D) - c(2\alpha + r)}{(2\alpha + r)(\alpha + r)},$$

where the last equality follows from (12). Since there is no sharing at either $(2,0)$ or $(2,1)$, we use (12) to get

$$V_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r} = \frac{(2\alpha + r)\pi^M + 2\alpha^2\widetilde{\pi}^D - c(2\alpha + r)}{(\alpha + r)^2}.$$

Substituting for $V_J(1,1)$ and $V_J(2,0)$, the sharing condition simplifies to

$$c > (\pi^M - 2\pi^D) - \frac{2(\alpha + r)^2}{(2\alpha + r)^2}(\pi^M - \pi^D). \tag{19}$$

Since $\pi^M > \pi^D$, this condition is easier to satisfy than (14). Hence, sharing incentives are decreasing. Solving for π^M, (14) can be written as

$$\pi^M < \frac{(3\alpha^2 + 2\alpha r)}{(2\alpha^2 - r^2)} 2\pi^D + \frac{(2\alpha + r)^2}{(2\alpha^2 - r^2)} c.$$

This condition defines the boundary between parts (ii) and (iii) in the statement of the proposition.

For parameter values such that the sharing condition (19) holds, there is a unique MPE such that the firms share at $(1,0)$ but not at $(2,1)$. The sharing pattern is (S,NS). For parameter values such that the sharing condition (19) strictly fails, there is a unique MPE such that the firms do not share at either $(1,0)$ or $(2,1)$. The sharing pattern is (NS,NS).

C Calculation of the licensing fees

Consider an MPE such that the firms share at every sharing history. The leading firm sets the licensing fee according to equation (2), so that the lagging firm is just indifferent between sharing and not sharing. At $(2,1)$, the licensing fee is

$$F(2,1) = V_2(2,2) - V_2(2,1,NS) = \frac{\pi^D + c}{\alpha + r} \tag{20}$$

where the last equality makes use of (10) and (13). At $(1,0)$, the licensing fee is

$$F(1,0) = V_2(1,1) - V_2(1,0,NS).$$

We can substitute for $V_2(1,1)$ from (15). $V_2(1,0,NS)$ is given by

$$V_2(1,0,NS) = \frac{\alpha V_2(1,1) + \alpha V_2(2,0) - c}{2\alpha + r}. \tag{21}$$

Since the lagging firm has no bargaining power at $(2,0)$, its profit is $V_2(2,0,NS)$ even though the firms share at $(2,0)$. Similarly, its profit at $(2,1)$ is $V_2(2,1,NS)$ even though the firms share at $(2,1)$. Using (13), we have

$$V_2(2,0) = V_2(2,0,NS) = \frac{\alpha V_2(2,1) - c}{\alpha + r} = \frac{\alpha^2 \widetilde{\pi}^D - c(2\alpha + r)}{(\alpha + r)^2}$$

Hence, $F(1,0)$ simplifies to

$$F(1,0) = \left(\frac{\pi^D + c}{\alpha + r}\right)\left(\frac{5 + 6\frac{r}{\alpha} + 2(\frac{r}{\alpha})^2}{4 + 8\frac{r}{\alpha} + 5(\frac{r}{\alpha})^2 + (\frac{r}{\alpha})^3}\right).$$

Comparing the fees $F(1,0)$ and $F(2,1)$, we find that $F(2,1) > F(1,0)$ iff $\frac{r}{\alpha}$ is above a cut-off of approximately $\frac{r}{\alpha} \cong 0.325$.

D Proof of Proposition 2

We solve the game in the following region of parameters: $c\frac{r}{\alpha}\left(\frac{3}{2}+\frac{r}{2\alpha}\right) < \pi^D < c\frac{r}{\alpha}\left(2+\frac{r}{\alpha}\right)$ and $2\pi^D\left(\frac{2\alpha+2r}{2\alpha+r}\right)+c\left(\frac{2r}{2\alpha+r}\right) < \pi^M < 2\pi^D + c$. This is a subregion of Region B. A straightforward calculation shows that the region is non-empty if and only if $\frac{r}{\alpha} < \frac{1}{2}\left(\sqrt{5}-1\right)$ where $\frac{1}{2}\left(\sqrt{5}-1\right) \simeq 0.62$.

To find an equilibrium, we work backwards from the end of the game. We derive the continuation profits at each history and solve for the equilibrium actions. For symmetric histories such as $(2,1)$ and $(1,2)$, we analyze only one of the histories as the analysis is the same for both.

The last history is the history $(2,2)$. At this history, the firms produce output and each earns discounted duopoly profits of $V_i(2,2) = \widetilde{\pi}^D$. Working backwards, the next history is $(2,1)$. The firms are willing to share at $(2,1)$ iff this maximizes their joint profits. The sharing condition (1) is $V_J(2,2) > V_J(2,1,NS)$. The payoff $V_J(2,1,NS)$ depends on whether firm 2 invests. If firm 2 invests at $(2,1,NS)$, its payoff is

$$\frac{\alpha\widetilde{\pi}^D - c}{\alpha + r}. \tag{22}$$

This payoff is positive because by assumption $\pi^D > c\frac{r}{\alpha}$. Hence, firm 2 invests at $(2,1,NS)$.

Since firm 2 invests at $(2,1,NS)$, the analysis of the sharing condition is the same as the one in section (B), we do not repeat here. The firms share at $(2,1)$ iff (14) holds. This condition holds in this region and the firms share step 2 at $(2,1)$. Their joint payoffs are $V_J(2,1) = 2\widetilde{\pi}^D$.

At the history $(1,1)$, each firm has one success. There is no sharing decision to be made. The firms must, however, decide whether to invest to develop the second step. Assuming firm 1 invests, if firm 2 also invests, its payoff is

$$\frac{\alpha V_2(2,1) + \alpha V_2(1,2) - c}{2\alpha + r} = \frac{\alpha V_J(2,1) - c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D - c}{2\alpha + r}.$$

From this, we see that firm 2 invests if $\pi^D > \frac{cr}{2\alpha}$. Since this condition holds by assumption in this region, firm 2 invests. Hence, each firm invests at $(1,1)$ if the other does.

If firm 1 does not invest at $(1,1)$, firm 2 faces the same optimization problem as at the history $(X,1)$. At $(X,1)$, if firm 2 invests, its payoff is

$$\frac{\alpha V_2(X,2)-c}{\alpha+r}=\frac{\alpha\widetilde{\pi}^M-c}{\alpha+r}. \tag{23}$$

Firm 2 invests at $(X,1)$ if $\pi^M > c\frac{r}{\alpha}$. This condition holds because $\pi^M > \pi^D$and in this region $\pi^D > c\frac{r}{\alpha}$. Hence, firm 2 invests at $(X,1)$, which means firm 2 invests at $(1,1)$ when firm 1 does not. Since each firm invests regardless of whether the other firm invests, it follows that both firms invest at $(1,1)$ and we have

$$V_2(1,1)=\frac{2\alpha\widetilde{\pi}^D-c}{2\alpha+r}. \tag{24}$$

At the history $(2,0)$, the sharing condition (1) is $V_J(2,1) > V_J(2,0,NS)$. The payoff $V_J(2,0,NS)$ depends on whether firm 2 invests. If firm 2 invests, its payoff is

$$\frac{\alpha V_2(2,1)-c}{\alpha+r}.$$

We know from Lemma 1 that this payoff is positive if

$$\pi^D > c\frac{r}{\alpha}\left(2+\frac{r}{\alpha}\right).$$

This condition fails in this region, so firm 2 drops out at $(2,0,NS)$, and $V_J(2,0,NS) = V_1(2,X)=\widetilde{\pi}^M$.

Joint profits under sharing are $V_J(2,1)=2\widetilde{\pi}^D$ since if the firms share, the game reaches the history $(2,1)$ and the firms share step 2. Thus, the sharing condition at $(2,0)$ simplifies to

$$2\pi^D-\pi^M>0 \tag{25}$$

In this region, we have that $\pi^M > 2\pi^D$. Hence, the firms do not share at $(2,0)$. The lagging firm then drops out of the game.

Working backwards, we next consider the history $(1,0)$. At this history, firm 1 has one success and firm 2 has no successes. The sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. The payoff $V_J(1,0,NS)$ depends on whether each firm invests. Assuming firm 1 invests, firm 2 invests if

$$\frac{\alpha V_2(1,1)+\alpha V_2(2,0)-c}{2\alpha+r}>0 \tag{26}$$

We can substitute for $V_2(1,1)$ from (24). Moreover, $V_2(2,0) = 0$ since the firms do not share at $(2,0)$ and the lagging firm drops out. After substituting and simplifying, (26) becomes

$$\pi^D > c\frac{r}{\alpha}\left(\frac{3}{2} + \frac{r}{2\alpha}\right).$$

This holds in the region, so the lagging firm 2 invests at $(1,0,NS)$ if firm 1 does. It is straightforward to show that the leading firm 1 also invests at $(1,0,NS)$ if firm 2 invests. Hence, each firm invests at $(1,0,NS)$ if the other firm does. If firm 2 does not invest, firm 1's payoff is the same as at the history $(1,X)$, and firm 1 invests as shown above. Since the leading firm invests whether or not the lagging firm invests, it follows that both firms invest at $(1,0,NS)$. The joint payoffs are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r} = \frac{\alpha\widetilde{\pi}^M + \alpha V_J(1,1) - 2c}{2\alpha + r}. \tag{27}$$

The sharing condition, $V_J(1,1) > V_J(1,0,NS)$, simplifies to

$$\alpha\widetilde{\pi}^M + \alpha V_J(1,1) - 2c < (2\alpha + r)V_J(1,1).$$

Substituting for $V_J(1,1) = 2V_2(1,1)$ from (24) and simplifying, the sharing condition at $(1,0)$ is

$$\pi^M < 2\pi^D\left(\frac{2\alpha + 2r}{2\alpha + r}\right) + c\left(\frac{2r}{2\alpha + r}\right) \tag{28}$$

Since this inequality fails in this region, sharing incentives are increasing, rather than decreasing. That is, the firms do not share at $(1,0)$ even though they do share at $(2,1)$.

At the history $(0,0)$, assuming firm 2 invests, firm 1 will also invest if

$$\frac{\alpha V_1(1,0,NS) + \alpha V_1(0,1,NS) - c}{2\alpha + r} = \frac{\alpha V_J(1,0,NS) - c}{2\alpha + r} > 0.$$

Substituting from (27) and (24), we get

$$4\alpha\pi^D + (2\alpha + r)\pi^M > (4\alpha + r)(2\alpha + r)\frac{r}{\alpha^2}c + 2cr.$$

Since $\pi^M > 2\pi^D$ in this region, the condition holds if

$$(8\alpha + 2r)\pi^D > (4\alpha + r)(2\alpha + r)\frac{r}{\alpha^2}c + 2cr.$$

Since $\pi^D > c\frac{r}{\alpha}\left(\frac{3}{2}+\frac{r}{2\alpha}\right)$ in this region, the condition holds if

$$(8\alpha + 2r)\left(\frac{3}{2}+\frac{r}{2\alpha}\right)\frac{r}{\alpha}c > (4\alpha + r)(2\alpha + r)\frac{r}{\alpha^2}c + 2cr.$$

This simplifies to $2\alpha(2\alpha + r) > 0$, which always holds. Hence, firm 1 invests at $(0,0)$ if firm 2 invests.

Assuming firm 2 does not invest, if firm 1 invests, its payoff is the same as at $(0, X)$. Firm 1 invests if

$$\frac{\alpha V_2(1,X) - c}{\alpha + r} = \frac{\alpha\left(\frac{\alpha\widetilde{\pi}^M - c}{\alpha + r}\right) - c}{\alpha + r} > 0$$

where we have substituted for $V_2(1,X)$ from (23). Simplifying we get

$$\pi^M > c\frac{r}{\alpha}(2 + \frac{r}{\alpha}). \tag{29}$$

In this region, we have that $\pi^M > 2\pi^D$ and $\pi^D > c\frac{r}{\alpha}\left(\frac{3}{2}+\frac{r}{2\alpha}\right)$. These two conditions together imply that (29) holds. Hence, firm 1 invests at $(0,X)$, which means firm 1 invests at $(0,0)$ if firm 2 does not. It follows that both firms invest at $(0,0)$. This completes the derivation of the equilibrium. The equilibrium is unique.

E Proof of Proposition 3

To save space, we do not present a complete proof. Instead, we show why the sharing incentives are increasing in Region A for some parameters when firm 1 is the leader.

We solve the game by working backwards through the histories. At $(2,1)$, the sharing condition (1) is

$$V_J(2,2) > V_J(2,1,NS) = \frac{\pi^M + \alpha 2\widetilde{\pi}^D - c_2^2}{\alpha + r}.$$

Using $V_J(2,2) = 2\widetilde{\pi}^D$, this simplifies to

$$2\pi^D - (\pi^M - c_2^2) > 0. \tag{30}$$

Similarly, the firms share at $(1,2)$ if and only if $2\pi^D - (\pi^M - c_1) > 0$. From now on, we consider the subregion of Region A where both of these conditions hold, so the firms share at $(2,1)$ and $(1,2)$. We have $V_J(2,1) = V_J(1,2) = 2\widetilde{\pi}^D$.

At $(2,0)$, the sharing condition (1) is

$$V_J(2,1) > V_J(2,0,NS) = \frac{\pi^M + \alpha V_J(2,1) - c_2^1}{\alpha + r}.$$

Using $V_J(2,1) = 2\widetilde{\pi}^D$, this simplifies to $2\pi^D - (\pi^M - c_2^1) > 0$. From now on, we assume this also holds so that the firms share at $(2,0)$. Similarly, the firms share at $(0,2)$ iff $2\pi^D - (\pi^M - c_1) > 0$. This is the same condition as the condition for sharing at $(1,2)$, so it holds. So the firms share at $(2,0)$ and $(0,2)$. We have $V_J(2,0) = V_J(0,2) = 2\widetilde{\pi}^D$.

At (1,0), the sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. The joint payoff at $(1,1)$ is

$$V_J(1,1) = \frac{\alpha V_J(2,1) + \alpha V_J(1,2) - c_2^2 - c_1}{2\alpha + r} = \frac{4\alpha\widetilde{\pi}^D - c_2^2 - c_1}{2\alpha + r}.$$

The joint payoff at $(1,0,NS)$ is

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - c_2^1 - c_1}{2\alpha + r}.$$

Substituting for $V_J(2,0) = 2\widetilde{\pi}^D$ and $V_J(1,1)$, the sharing condition simplifies to

$$(2\pi^D + c_1 + c_2^2) + (c_2^1 - c_2^2)(\frac{2\alpha + r}{\alpha}) > 0. \tag{31}$$

When $c_2^1 - c_2^2 > 0$, the sharing condition (31) at (1,0) holds trivially, and so it is clearly easier to satisfy than (30). However, there are parameters in this subregion such that $c_2^1 - c_2^2 < 0$ and the sharing condition fails. The firms do not share at $(1,0)$ and hence the sharing pattern is increasing.

F Proof of Proposition 4

We show that the game has an MPE that achieves the same outcome as the constrained joint profit maximization benchmark, which we will refer to as the "benchmark." By this we mean that three conditions are satisfied along the equilibrium path in the MPE: (i) the firms do not share at $(2,1)$ and $(1,2)$ and the lagging firm then drops out; (ii) the firms share at $(1,0)$ and $(0,1)$; and (iii) the firms invest at $(0,0)$ and $(1,1)$ according to whether they do so in the benchmark. To demonstrate this MPE, we solve the game by working backwards through the histories. For mirror histories such as $(2,1)$ and $(1,2)$, we analyze only one of them since the analysis is the same for both.

At $(2,2)$, the firms are duopolists with joint profits of $V_J(2,2) = 2\widetilde{\pi}^D$. At $(2,X)$, firm 1 is a monopolist and joint profits are $V_J(2,X) = \widetilde{\pi}^M$. At $(2,1,NS)$, the lagging firm invests if

$$\frac{\alpha V_2(2,2) - c_p^2}{\alpha + r} = \frac{\alpha\widetilde{\pi}^D - c_p^2}{\alpha + r} > 0.$$

This condition does not hold because the patent policy satisfies $c_p^2 > \frac{\alpha}{r}\pi^D$. Hence, firm 2 does not invest at $(2,1,NS)$ and joint profits are $V_J(2,1,NS) = V_J(2,X) = \widetilde{\pi}^M$.

At $(2,1)$, the sharing condition (1) is $V_J(2,2) > V_J(2,1,NS)$, which simplifies to $2\pi^D > \pi^M$. This condition does not hold, so the firms do not share at $(2,1)$. The joint payoffs are $V_J(2,1) = V_J(2,1,NS) = \widetilde{\pi}^M$ and condition (i) is satisfied.

At $(2,0,NS)$, the lagging firm would earn a negative payoff from investing because at $(2,1)$ the firms do not share and firm 2 drops out. It follows that firm 2 does not invest at $(2,0,NS)$ and $V_J(2,0,NS) = V_J(2,X) = \widetilde{\pi}^M$. The sharing condition at $(2,0)$ is $V_J(2,1) > V_J(2,0,NS)$. This simplifies to $\pi^M > \pi^M$, reflecting the fact that the firms are indifferent between sharing and not sharing at $(2,0)$. The joint payoffs at $(2,0)$ are $V_J(2,0) = \widetilde{\pi}^M$.

At $(1,1)$, the joint payoffs if both firms invest are the same as in the benchmark (that is, they equal the payoffs if the firms invest at $(1,1)$ and follow the benchmark thereafter) because condition (i) holds. In the benchmark, the firms invest if and only if this payoff is positive. First, assume this joint payoff is positive. If firm 2 invests at $(1,1)$, then firm 1's payoff from investing is equal to half of the joint payoff. Since this is positive, firm 1 invests. It follows that there is an MPE such that both firms invest at $(1,1)$ and condition (iii) is satisfied. Next, assume that the joint payoff when both firms invest at $(1,1)$ is negative. If firm 2 invests at $(1,1)$, firm 1's payoff from investing is half of the joint payoff. This is negative, so firm 1 does not invest. If firm 2 does not invest at $(1,1)$, then if firm 1 invests, the joint payoff is less than the joint payoff when both firms invest at $(1,1)$. This follows from the fact that the Poisson research technology is more efficient when both firms invest than when just one firm invests. This implies that the joint payoff is negative if one firm invests at $(1,1)$. The investing firm's payoff is equal to this joint payoff, so the firm does not invest. It follows that neither firm invests at $(1,1)$ and condition (iii) is met. That is, the investment decisions at $(1,1)$ are the same as in the benchmark.

At $(1,0)$, the sharing condition is $V_J(1,1) > V_J(1,0,NS)$. Intuitively, it is easy to see why this condition holds. With the rivalry effect removed by condition (i), there is no downside to sharing. Sharing merely moves the time up until one firm reaches the product market (the speed effect) and eliminates wasteful duplication of R&D. The analysis involves several cases, depending on whether firms invest or not in different scenarios. Here, we limit our discussion to the case that firms invest at all histories. For this case, we can express the sharing condition $V_J(1,1) > V_J(1,0,NS)$ as

$$\frac{\alpha V_J(2,1) + \alpha V_J(1,2) - 2c}{2\alpha + r} > \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r}.$$

Using the fact that $V_J(2,1) = V_J(1,2)$, this simplifies to

$$2V_J(2,1) > V_J(2,0) + V_J(1,1).$$

Using $V_J(2,1) = V_J(2,0) = \widetilde{\pi}^M$, this simplifies to $\widetilde{\pi}^M > V_J(1,1)$, which holds trivially. Hence, condition (ii) is met.

At $(0,0)$, the joint payoffs if both firms invest are the same as in the benchmark (that is, they equal the payoffs if the firms invest at $(0,0)$ and follow the benchmark thereafter) because conditions (i) and (ii) hold. Following the same argument given above for $(1,1)$, there is an MPE such that the firms take the same investment actions at $(0,0)$ as in the benchmark. That is, condition (iii) is met.

G Proof of Proposition 5

The proposition considers parameters π^M, π^D, r, α, and c such that when $c_p^1 = c$, we are in Region A and the firms do not share at any history. From the proof of Proposition (1), this is the subregion of Region A in which condition (19) does not hold. That is:

$$c < (\pi^M - 2\pi^D) - \frac{2(\alpha + r)^2}{(2\alpha + r)^2}(\pi^M - \pi^D). \tag{32}$$

Condition (32) implies that (14) does not hold.

In Region A, by definition, no firm ever drops out of the game. Because the firms never drop out, to solve for the MPE, we only need to determine whether the firms share at the six asymmetric histories. To derive the sharing conditions for $(1,0)$, $(2,0)$ and $(2,1)$, we work backwards to solve for the MPE.

At $(2,1)$, the sharing condition (1) is $V_J(2,2) > V_J(2,1,NS)$. As in the proof of Proposition 1, this condition simplifies to (14). The parameter for early stage patent protection, c_p^1, does not impact this condition. Since (14) fails, the firms do not share at $(2,1)$.

Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition (1) is $V_J(2,1) > V_J(2,0,NS)$. Joint profits under no sharing are

$$\begin{aligned} V_J(2,0,NS) &= V_1(2,0,NS) + V_2(2,0,NS) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c_p^1}{\alpha + r} = \frac{\pi^M + \alpha V_J(2,1) - c_p^1}{\alpha + r}. \end{aligned}$$

Substituting for $V_J(2,0,NS)$, the sharing condition simplifies to $c_p^1 > \pi^M - rV_J(2,1)$. Since the firms do not share at $(2,1)$, we can substitute for $V_J(2,1)$ from (12). Simplifying, we get

$$c_p^1 > \frac{\alpha}{\alpha + r}(\pi^M - 2\pi^D) + \frac{r}{\alpha + r}c. \tag{33}$$

For $c_p^1 = c$, this condition simplifies to the sharing condition (14) which does not hold. For c_p^1 sufficiently large, (33) holds. Let the threshold level $\bar{\bar{c}}_p^1$ be the patent policy such that the firms are indifferent between sharing and not sharing at $(2,0)$.

Case 1 We assume that the patent policy c_p^1 is too weak to induce sharing at $(2,0)$. That is, $c_p^1 < \bar{\bar{c}}_p^1$. At $(1,0)$, the sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. Joint profits under no sharing are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - c - c_p^1}{2\alpha + r}.$$

The sharing condition simplifies to $(\alpha + r)V_J(1,1) > \alpha V_J(2,0) - c - c_p^1$. Substituting for $V_J(1,1) = 2V_2(1,1)$, we have

$$V_J(1,1) = 2\frac{\alpha V_2(1,2) + \alpha V_2(2,1) - c}{2\alpha + r} = 2\frac{\alpha V_J(2,1) - c}{2\alpha + r}. \tag{34}$$

Since the firms do not share at $(2,0)$, we have

$$V_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c_p^1}{\alpha + r}.$$

Substituting for $V_J(1,1)$ and $V_J(2,0)$, the sharing condition at $(1,0)$ simplifies to

$$\alpha r(3\alpha+2r)V_J(2,1) > r(\alpha+r)c + \alpha(2\alpha+r)\pi^M - (2\alpha+r)^2 c_p^1.$$

Substituting for $V_J(2,1)$ from (12), the sharing condition at $(1,0)$ simplifies to

$$c_p^1 > \frac{\alpha}{(\alpha+r)}\frac{(2\alpha^2-r^2)}{(2\alpha+r)^2}(\pi^M - 2\pi^D) - \frac{\alpha(\alpha+r)}{(2\alpha+r)^2}2\pi^D + \frac{r}{(\alpha+r)}c.$$

We can rewrite this as:

$$c_p^1 > \frac{\alpha}{(\alpha+r)}(\pi^M - 2\pi^D) - \frac{2\alpha(\alpha+r)}{(2\alpha+r)^2}(\pi^M - \pi^D) + \frac{r}{(\alpha+r)}c. \tag{35}$$

For $c_p^1 = c$, condition (35) simplifies to the sharing condition (19) which does not hold. However, as the patent policy is strengthened to $c_p^1 > c$, (35) may start to hold. We define the threshold level $\bar{c}_p^1 > c$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(1,0)$. Comparing (35) with (33), it is clear that $\bar{c}_p^1 < \bar{\bar{c}}_p^1$. Therefore, as the policy maker increases the strength of early patent protection, the firms start to share at $(1,0)$ before they start to share at $(2,0)$.

We have shown that for a patent policy c_p^1 with $\bar{c}_p^1 > c_p^1 > c$, there is a unique MPE such that the firms do not share at any history. For a patent policy c_p^1 with $\bar{\bar{c}}_p^1 > c_p^1 > \bar{c}_p^1$, there is a unique MPE such that the firms share at $(1,0)$, but they do not share at either $(2,0)$ or $(2,1)$.

Case 2 We assume that the patent policy c_p^1 is strong enough to induce sharing at $(2,0)$. That is, $c_p^1 > \bar{\bar{c}}_p^1$. The firms share at $(2,0)$, but they do not share at $(2,1)$. At $(1,0)$, the sharing condition is $V_J(1,1) > V_J(1,0,NS)$. Since the firms share at $(2,0)$, we have that $V_J(2,0) = V_J(2,1)$. Joint profits under no sharing are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,1) + \alpha V_J(1,1) - c - c_p^1}{2\alpha+r}.$$

Substituting for $V_J(1,0,NS)$, the sharing condition simplifies to $(\alpha+r)V_J(1,1) > \alpha V_J(2,1) - c - c_p^1$. Substituting for $V_J(1,1)$ from (34), the sharing condition simplifies to $\alpha r V_J(2,1) > rc - (2\alpha+r)c_p^1$. Since the firms do not share $(2,1)$, we can substitute for $V_J(2,1)$ from (12). The sharing condition at $(1,0)$ then simplifies to

$$c_p^1 > \frac{r}{(\alpha+r)}c - \frac{\alpha r}{(\alpha+r)(2\alpha+r)}\pi^M - \frac{\alpha^2}{(\alpha+r)(2\alpha+r)}2\pi^D. \tag{36}$$

This holds because $c_p^1 > c$. We have shown that when $c_p^1 > \bar{\bar{c}}_p^1$, there is a unique MPE such that the firms share at $(1,0)$ and $(2,0)$, but they do not share at $(2,1)$.

H Proof of Proposition 6

The proposition considers parameters π^M, π^D, r, α, and c in Region A such that when $c_p^2 = c$ the firms share at $(1,0)$, but they do not share at $(2,0)$ and $(2,1)$. The proposition also assume that $c < \bar{c}$ where $\bar{c} = (\pi^M - 2\pi^D) - \frac{2(\alpha+r)}{(4\alpha+3r)}(\pi^M - \pi^D)$. This subregion of parameters is described by the following condition:

$$(\pi^M - 2\pi^D) - \frac{2(\alpha+r)^2}{(2\alpha+r)^2}(\pi^M - \pi^D) < c < (\pi^M - 2\pi^D) - \frac{2(\alpha+r)}{(4\alpha+3r)}(\pi^M - \pi^D) = \bar{c}. \tag{37}$$

The first inequality in (37) is condition (19) from the proof of Proposition 1. When $c_p^1 = c$, this inequality defines the subregion of Region A such that the firms share at $(1,0)$, but not at $(2,1)$ or $(2,0)$. The second inequality in (37) is an assumption that will be used below.

In Region A, by definition, no firm ever drops out of the game. Because the firms never drop out, to solve for the MPE, we only need to determine whether the firms share at the six asymmetric histories. To derive the sharing conditions for $(1,0)$, $(2,0)$ and $(2,1)$, we use backwards induction and solve for the MPE.

At $(2,1)$, the sharing condition is $V_J(2,2) > V_J(2,1,NS)$. Joint profits under sharing are $V_J(2,2) = 2\widetilde{\pi}^D$. Joint profits under no sharing are

$$V_J(2,1,NS) = V_1(2,1,NS) + V_2(2,1,NS) = \frac{\pi^M + 2\alpha\widetilde{\pi}^D - c_p^2}{\alpha + r}. \tag{38}$$

Substituting for $V_J(2,2)$ and $V_J(2,1,NS)$, the sharing simplifies to

$$c_p^2 > \pi^M - 2\pi^D. \tag{39}$$

We define the threshold level $\bar{\bar{c}}_p^2 = \pi^M - 2\pi^D$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(2,1)$.

Case 1 We first consider patent policies that are not strong enough to induce sharing at $(2,1)$. That is, $c_p^2 < \bar{\bar{c}}_p^2 = \pi^M - 2\pi^D$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition at $(2,0)$ is $V_J(2,1) > V_J(2,0,NS)$. Joint profits are under no sharing are

$$V_J(2,0,NS) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

Substituting for $V_J(2,0,NS)$, the sharing condition at $(2,0)$ simplifies to $rV_J(2,1) > \pi^M - c$. Substituting for $V_J(2,1)$ from (38), the sharing condition at $(2,0)$ simplifies to

$$(1+\frac{r}{\alpha})c - \frac{r}{\alpha}c_p^2 > \pi^M - 2\pi^D$$

This condition fails because $c \leq c_p^2$ and $c_p^2 < \pi^M - 2\pi^D$. Thus, the firms do not share at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. Joint profits under no sharing are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r}.$$

The sharing condition simplifies to $(\alpha + r) V_J(1,1) > \alpha V_J(2,0) - 2c$. On the LHS, the expression for $V_J(1,1)$ is given by (34). On the RHS, since the firms do not share at $(2,0)$, we have

$$V_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

The sharing condition at $(1,0)$ simplifies to

$$\alpha r(3\alpha + 2r)V_J(2,1) > \alpha(2\alpha + r)\pi^M - \alpha(4\alpha + 3r)c. \tag{40}$$

Substituting for $V_J(2,1)$ from (38), the sharing condition at $(1,0)$ simplifies to

$$c_p^2 < c\frac{(\alpha + r)(4\alpha + 3r)}{r(3\alpha + 2r)} + 2\pi^D\frac{\alpha}{r} - \pi^M\frac{(2\alpha^2 - r^2)}{r(3\alpha + 2r)}. \tag{41}$$

We define the threshold level $\bar{c}_p^2$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(1,0)$. It is straightforward, but tedious, to show that when (37) holds, the threshold $\bar{c}_p^2$ satisfies

$$c < \bar{c}_p^2 < (\pi^M - 2\pi^D).$$

where $\bar{\bar{c}}_p^2 = (\pi^M - 2\pi^D)$. We have shown that for patent policies c_p^2 with $c < c_p^2 < \bar{c}_p^2$, there is a unique MPE in which the firms share at $(1,0)$, but not at $(2,0)$ or $(2,1)$. For stronger patent policies c_p^2 with $\bar{c}_p^2 < c_p^2 < \bar{\bar{c}}_p^2$, the firms do not share at any history.

Case 2 We next consider patent policies that are strong enough to induce sharing at $(2,1)$. That is, $c_p^2 > \overline{\overline{c}}_p^2 = \pi^M - 2\pi^D$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition at $(2,0)$ is $V_J(2,1) > V_J(2,0,NS)$. Joint profits are under no sharing are

$$V_J(2,0,NS) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

Substituting for $V_J(2,0,NS)$, the sharing condition at $(2,0)$ simplifies to $rV_J(2,1) > \pi^M - c$. Since the firms share at $(2,1)$, we have $V_J(2,1) = 2\widetilde{\pi}^D$. Substituting for $V_J(2,1)$, the sharing condition at $(2,0)$ simplifies to

$$c > \pi^M - 2\pi^D.$$

From (37), this condition does not hold so the firms do not share at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > V_J(1,0,NS)$. Following the analysis in Case 1, the sharing condition simplifies to (40). Substituting for $V_J(2,1) = 2\widetilde{\pi}^D$, the sharing condition at $(1,0)$ simplifies to

$$c > (\pi^M - 2\pi^D) - \frac{2(\alpha + r)}{(4\alpha + 3r)}(\pi^M - \pi^D).$$

From (37), this does not hold, so the firms do not share at $(1,0)$. We have shown that when $c_p^2 > \overline{\overline{c}}_p^2$, there is a unique MPE in which the firms share at $(2,1)$, but they do not share at $(2,0)$ or $(1,0)$.