Timing of investments, hold-up and total welfare

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Abstract

We explore hold-up when trading parties can make specific investments simultaneously or sequentially. With simultaneous investment both investors are held-up. With sequential investment contracting becomes possible after the project has commenced, so the second investor avoids being held-up. If the two investments are independent three effects are identified when comparing the total welfare of the two regimes: sequential investment increases the costs of delay; sequential investment reduces the incentive for the first player to invest; and the sequential regime increases the second player's incentive to invest. Given this, the (second-best) optimal regime will favour the more important investment. Similarly, if the choice of investment level of an investor is inelastic to the regime adopted, the timing regime adopted should maximise the incentive for the other party to invest. The paper also shows the timing of investment can act as an additional form of hold-up; if they have the option when to invest, a party may choose the regime that does not maximise total welfare.

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1 Introduction

It has been shown by Grout (1984) and Hart (1995), amongst others, that parties may not make efficient specific investments when contracts are incomplete.[1] These incomplete contract models typically have the following structure: the trading parties simultaneously make their investments that are sunk and, at least partially, specific; after these investments are made contracting on some relevant variable becomes possible; at this point the parties renegotiate and trade occurs according to the renegotiated contract. If, because of renegotiation, a party does not receive the full marginal return from their effort, investment will be inefficient.[2]

An alternative literature has considered how the hold-up problem can be overcome by allowing parties to stagger or sequence their investments. For example, Neher (1999) considered staged financing of a project when an entrepreneur is unable to commit not to renege on their contract with the financier. When the project is financed in stages, as the project matures, the alienable (contractible) element of the project, manifested in the accumulated physical assets, provides a better bargaining position during renegotiation for the financier in the event of default. As a consequence, the entrepreneur has less incentive to renege. De Fraja (1999) considered the Stackelberg-type sequencing of investments in the presence of hold-up. De Fraja's solution to the hold-up problem required the first party to make a general investment then make a take-it-or-leave-it offer to the other party that included him paying for

[1]Also see Grossman and Hart (1986) and Hart and Moore (1988).

[2]It has also been noted that the level of general investments can be effected in the presence of incomplete contracts: Malcomson (1997) noted that hold-up of general investment can occur when there are turnover costs.

the specific investment.[3] Given that this contract makes the first party the residual claimant she will invest efficiently. Admati and Perry (1991) showed two parties can overcome the free-rider problem by financing a public good in stages.

The model presented here develops a simple framework to contrast the simultaneous and staged (sequential) investment regimes. The essence of the model is that staging of the project allows some investment to be made after the point in time when a contract can be written. Here, the resolution of the incompleteness is facilitated by the completion of some aspect of the project. For example, in Grossman and Hart (1986) contracting became possible after the two parties made their initial investment. Similarly, Neher (1999) made the point that contracting becomes more feasible as a project progresses as more of the human capital invested is converted into physical assets.

The theoretical model incorporates this idea into the following structure. Two parties are required to make a specific investment in order to complete a project. Two distinct alternatives are possible. First, they can invest simultaneously at the start of the game. If they do so, both invest prior to when contracting is possible. After both investments are sunk the parties renegotiate and the payoffs are realised. Alternatively, one party can invest first while the other party waits. This first investment allows the project to take shape; as a result, contracting on the second investment becomes possible. At this stage, the parties will renegotiate and write a contract specifying the second party's investment. The final stage of investment will

[3] Although the investment may be industry-specific, it is not relationship-specific in the traditional sense. See Malcomson (1997).

then occur, completing the project and allowing the parties to receive their payoffs.

Several important results arise from this simultaneous versus sequential investment model. First, the paper investigates the relative efficiency of the two alternative investment regimes. When the investments are independent the model identifies three basic trade-offs between the regimes:

- The sequential system enlarges (relative to simultaneous investments) delay costs by increasing the length of time before the project matures.
- The sequential system reduces the first player's incentive to invest, vis-a-vis the simultaneous system, because of the longer time between when his investment is made and when the returns are realised.
- The sequential system enhances the incentive for the second player to invest efficiently as they do not suffer hold-up, which they injure with simultaneous investments.

The ultimate impact on total surplus is a combination of these trade-offs. We show that under different circumstances either regime of investment maximises welfare. Moreover, despite the simplicity of the model, no simple relationship between the welfare effects of the two regimes exists as there is no restriction on how these three trade-offs interact. However, given that the simultaneous regime encourages the first player to invest, if this player's contribution is relatively more important than the other player's contribution the simultaneous regime is preferred. In the same way, the sequential regime is preferred when the second investor's contribution is relatively more important, provided both players are sufficiently patient. Similarly, if a party is particularly responsive to the incentives provided by one timing regime, that regime is preferred. For example, when the first investor is very responsive to the additional incentive provided by the simultaneous regime, this timing generates a higher level of surplus than the alternative. On the other hand, if the second player is very responsive to the additional incentives to invest provided by the sequential regime, that timing regime is preferred. These predictions are similar in nature to the property rights predictions of Hart (1995), although the model is somewhat more general in that a player may voluntarily forgo the advantages of sequencing of investment (their property right) in order to encourage the other party to invest more. In this way the parties can opt for a (more) incomplete contract by choosing to invest simultaneously. The model is also extended in several other ways, for example by considering these trade-offs when the two investments are strategic complements and substitutes.

Second, we show that the possibility of investing sequentially does not always improve welfare. As it turns out, flexibility in the timing of investment can act as an additional form of hold-up. For want of a better expression we call this kind of hold-up 'follow-up'. This occurs when both parties should invest simultaneously at the start of the project in order to maximise surplus but there is an incentive for one party to wait until after the other player has sunk their effort before they follow-up with their own investment.[4] Consider the case when technology requires that one particular party must invest at the commencement of a project but that the other

[4]'Follow-up' can occur in addition to the regular hold-up of investment.

party can invest either at the same time or wait. The first party will anticipate that the second party will delay their investment - opt for the sequential regime - if it suits them. The first party will then adjust their investment accordingly. In the extreme this additional form of hold-up will prevent a potential surplus-enhancing project from proceeding. A similar result arises in the model presented in Smirnov and Wait (2001).

2 The model

There is a potentially profitable relationship between two parties that, for convenience, we label as a buyer and a seller. Specifically, if the buyer and seller invest I_1 and I_2 respectively the two parties share surplus R. The exact relationship between the investments and surplus is discussed below.

2.1 Timing

The timing of investment is the focus of this paper. Two alternatives are considered. First, both players invest simultaneously at time $t = 1$, as shown in Figure 1. At this stage, contracting on either investment is not possible; consequently renegotiation will occur after both investments are sunk.[5] Definition 1 reiterates this discussion.

Definition 1. *Simultaneous investment occurs when both parties invest at the same time, prior to renegotiation.*

[5]The renegotiation process is discussed below.

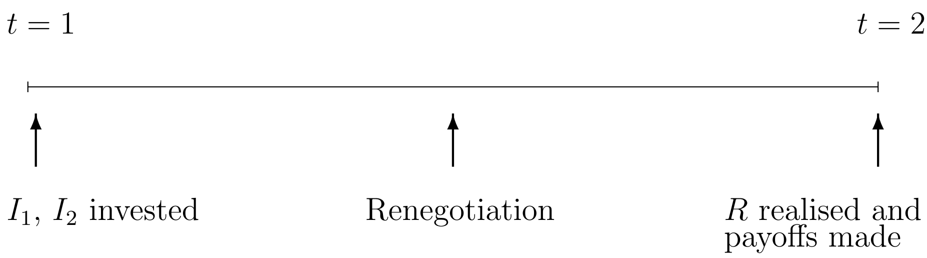

Figure 1: Simultaneous investment

Figure 2 outlines the timing of the alternative investment regime. In this regime the buyer invests I_1 at time $t = 1$ prior to when contracting is possible. However, this investment makes the contracting process possible, so having observed I_1 the two parties renegotiate and contract on I_2. It is only after this that the seller makes her investment I_2. This occurs at time $t = 2$. After both investments have been made, surplus is realised and the payoffs to each party are made. Definition 2 defines sequential investment.

Definition 2. *Sequential investment occurs when one party (the buyer) invests at time $t = 1$, while the other party (the seller) waits and invests at time $t = 2$.*

2.2 Assumptions

As noted above, the investments of the buyer and seller (I_1 and I_2) combine together to generate surplus R. The investments of both parties are sunk and completely specific to the relationship in that they are worth zero outside the relationship. R is only available at the completion of the project and investment in the relationship is

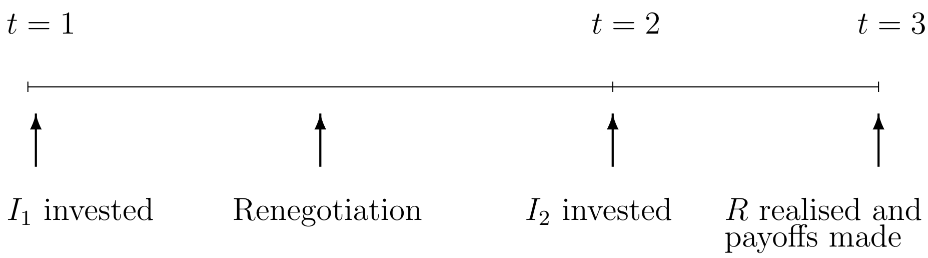

Figure 2: Sequential investment

always efficient. Further, it is assumed that each party's outside option is normalised to zero.

Although there is complete and symmetric information between the trading parties, the investments are unverifiable ex ante. However, as discussed above, once the buyer's investment has been sunk the project becomes tangible allowing subsequent investment to be verifiable. On the contrary, the surplus generated by the project is always unverifiable. This prevents the parties writing surplus sharing agreements.

As in Hart and Moore (1988) and MacLeod and Malcomson (1993), the two parties cannot vertically integrate to overcome their hold-up problem, due to specialisation, for example.[6]

Finally, both the parties discount future returns and costs with a constant discount factor δ.

When the parties renegotiate they must decide how to split the available surplus. We adopt a reduced-form bargaining solution in which each party receives one-half

[6] Williamson (1983) noted that if the parties can vertically integrate they can overcome hold-up and investment will be efficient.

of the available surplus.[7]

3 Simultaneous and sequential investments

This section explores the relative advantage of investing simultaneously or sequentially. Here we assume that total surplus is a function of both investments where $R(I_1, I_2)$ is two times differentiable, non-decreasing in both variables and concave; that is $R_i' = \partial R(I_1, I_2)/\partial I_i \geq 0$, $R_{ii}'' = \partial^2 R(I_1, I_2)/\partial I_i^2 \leq 0$ for $i = 1, 2$ and $R_{11}'' R_{22}'' - (R_{12})^2 \geq 0$, as summarised by Assumption 1.

Assumption 1. *$R_i' = \partial R(I_1, I_2)/\partial I_i \geq 0$, $R_{ii}'' = \partial^2 R(I_1, I_2)/\partial I_i^2 \leq 0$ for $i = 1, 2$ and $R_{11}'' R_{22}'' - (R_{12})^2 \geq 0$ where $R_{12} = \partial^2 R(I_1, I_2)/\partial I_1 \partial I_2$.*

We consider four different possibilities: (1) when investment is simultaneous and contracts are complete; (2) when investment is simultaneous and contracts are incomplete; (3) when investment is sequential and contracts are complete; and (4) sequential investment when contracts are incomplete. The main emphasis in this paper is on comparing the relative efficiency of (2) and (4).

[7] This reduced form bargaining solution can be thought of relating to an extensive form bargaining game, for example Rubinstein's (1982) alternating bargaining game or the Nash bargaining solution. For example, see Sutton (1986) and Muthoo (1999, pp. 15-16). Unlike many incomplete-contracts models the results in this paper are not sensitive to the bargaining solution used.

3.0.1 Simultaneous investment with no renegotiation

If investments are contractible, it is possible to achieve the first-best levels of outcome. If investments are made simultaneously the two parties will maximise

$$\max_{I_1, I_2} \delta R(I_1, I_2) - I_2 - I_1. \tag{1}$$

Surplus is discounted because the return from investment take one period in which to mature. The first-order conditions are:

$$R_1^{'} = 1/\delta; \tag{2}$$

and

$$R_2^{'} = 1/\delta. \tag{3}$$

Assumption 1 guarantees there is a unique solution for both I_1 and I_2. Let the first best level of investment be denoted as I_1^* and I_2^*.

3.0.2 Simultaneous investment with renegotiation

Second, when investments are made simultaneously but contracts are incomplete both parties know that renegotiation will occur. They adjust their investments from the first-best level accordingly. The buyer chooses I_1 in order to maximise

$$\max_{I_1} \frac{\delta}{2} R(I_1, I_2) - I_1. \tag{4}$$

Here, the returns are discounted by δ because they are only available after one period. Renegotiation occurs after both investments have been sunk; as a consequence each party anticipates receiving one half of the available surplus. The first-order condition for the buyer is

$$R_1' = \frac{2}{\delta}. \tag{5}$$

The seller faces a similar decision choosing her level of I_2. She will set I_2 to maximise

$$\max_{I_2} \frac{\delta}{2} R(I_1, I_2) - I_2, \tag{6}$$

which yields the first-order condition of

$$R_2' = \frac{2}{\delta}. \tag{7}$$

Let the buyer's and seller's choices when investments are set simultaneously and renegotiation occurs to be $\widehat{I}_1$ and $\widehat{I}_2$ respectively. These values solve system of equations 4 and 6. The solutions are unique because of Assumption 1.

3.0.3 Sequential investment with complete contracts (no renegotiation)

Alternatively, if investments are made sequentially, the buyer will invest I_1 in the first period and the seller will invest I_2 in the second period. As contracts are complete renegotiation will never occur. Both investments will be set so as to maximise

$$\max_{I_1,I_2} \delta^2 R(I_1, I_2) - \delta I_2 - I_1. \tag{8}$$

The first-best first-order conditions are:

$$\delta^2 R_1^{'} = 1; \tag{9}$$

and

$$\delta R_2^{'} = 1 \tag{10}$$

so that $R_1^{'} = 1/\delta^2$ and $R_2^{'} = 1/\delta$. Again, Assumption 1 ensures a unique solution for both investments. Let the first best level of investment in this case be I_1^{**} and I_2^{**}.

3.0.4 Sequential investment with incomplete contracts (renegotiation)

The final case is when the investments are made sequentially and contracts are incomplete. The buyer invests I_1 at time $t = 1$. Following renegotiation, at time $t = 2$ the seller chooses I_2. In this case the buyer sets I_1 to maximise

$$\max_{I_1} \frac{\delta}{2}[\delta R(I_1, I_2) - I_2] - I_1. \tag{11}$$

The first-order condition for this problem is

$$R_1' = \frac{2}{\delta^2}. \tag{12}$$

The seller, who sets her investment level after observing I_1 and renegotiating with the buyer will maximise

$$\max_{I_1} \frac{\delta}{2}[\delta R(I_1, I_2) - I_2]. \tag{13}$$

The first-order condition for this maximisation problem is

$$R_2' = \frac{1}{\delta}. \tag{14}$$

Let the buyer's and the seller's levels of investment be $\widetilde{I}_1$ and $\widetilde{I}_2$ when contracts are made sequentially and contracts are incomplete. These values are the solution to the system of equations 12 and 14. The solutions are unique because of Assumption 1.

3.1 Timing of investment and total welfare

As it turns out, very little can be said about the trade-off between simultaneous and sequential investments when functions are general and contracts are incomplete. To explore the issue further assume that each investment has no influence on the marginal productivity of other player's investment; that is, $R_{12} = 0$, as stated in the following assumption. [8]

Assumption 2. $R_{12}^{''} = 0$.

Remark If $R_{12}^{''} = 0$, it follows that $R = f_1(I_1) + f_2(I_2)$, where $f_i^{'} > 0$ and $f_i^{''} \leq 0$ for $i = 1, 2$.

[8] $R_{12}^{''} = 0$ could arise when an investment by the buyer increases his benefit from trade whereas investment by the seller reduces her costs. Although they do not affect one another, each investment increases the potential surplus available to be split upon renegotiation. A similar assumption is made by Hart and Moore (1988).

In this framework three separate effects can be isolated that, when combined, give the relative advantage of either timing regime. First, consider the costs of delay. Let the total surplus ex ante with simultaneous investment be S_2 and the total surplus ex ante when investment is sequential be S_4.[9] For two fixed levels of $\overline{I}_1$ and $\overline{I}_2$

$$S_2 = \delta R(\overline{I}_1, \overline{I}_2) - \overline{I}_1 - \overline{I}_2 > \delta^2 R(\overline{I}_1, \overline{I}_2) - \overline{I}_1 - \delta \overline{I}_2 = S_4. \tag{15}$$

As sequential investment delays the payoff an extra period, the surplus from simultaneous investment is greater than with sequential investments when I_1 and I_2 are fixed: the costs of delay always favour simultaneous investment. Further, the relative payoff of simultaneous investments is increasing as the players become more impatient. This effect is summarised below.

Effect 1. *The costs of delay reduce the surplus generated by sequential investment relative to the surplus with simultaneous investments.*

Second, consider the investment levels generated from each system. Examining the first-order conditions in equations 5 and 12, $\widehat{R}'_1 = \frac{2}{\delta} \leq \widetilde{R}'_1 = \frac{2}{\delta^2}$. Given the assumption of concavity and monotonicity of R:

$$\widehat{I}_1 > \widetilde{I}_1. \tag{16}$$

[9]Both S_2 and S_4 relate to when contracts are incomplete.

The sequential investment regime delays the collection of returns to the buyer: this reduces the incentive for the buyer to invest.[10]

Effect 2. *Relative to the sequential regime, the simultaneous investment regime increases the incentive for the buyer to invest in I_1.*

For the seller the relative incentives to invest with simultaneous and sequential investments are given by equations 7 and 14. Again, because of Assumption 1,

$$\widehat{I}_2 < \widetilde{I}_2. \tag{17}$$

With simultaneous investment the seller is held-up. With sequential investment, however, the seller invests after renegotiation, thus avoiding any hold-up problems. In fact, the sequential investment level chosen by the seller equals the first-best level, so that $\widetilde{I}_2 = I_2^{**}$; this is the advantage of the sequential regime over simultaneous investment. Effect 3 summarises this discussion.

Effect 3. *The sequential investment regime increases I_2 to its first-best level.*

Effect 2 states that the simultaneous regime increases I_1. Effect 3 suggests that the sequential regime increases I_2. To assess the impact of an increase in either investment on total welfare, isolated from the costs of delay, consider S_2 relative to an augmented S_4, termed U_4, that has the same discount structure as the simultaneous system. U_4

[10]Note that both $\widehat{I}_1$ and $\widetilde{I}_1$ are below the first-best level. With simultaneous investments $\widehat{R}_1' = 2/\delta > R_1^{'} = 1/\delta$, meaning that $\widehat{I}_1 < I_1^*$. Similarly, with sequential investment, $\widetilde{R}_1' = 2/\delta^2 > R_1^{'} = 1/\delta^2$, meaning that $\widetilde{I}_1 < I_1^{**}$.

ignores the additional discounting of R and of I_2 that occurs because of the additional period. In this case:

$$S_2 = \delta f_1(\widehat{I}_1) - \widehat{I}_1 + \delta f_2(\widehat{I}_2) - \widehat{I}_2. \tag{18}$$

where the level of investments are determined by equations 5 and 7. Similarly, using equations 12 and 14

$$U_4 = \delta f_1(\widetilde{I}_1) - \widetilde{I}_1 + \delta f_2(\widetilde{I}_2) - \widetilde{I}_2. \tag{19}$$

The relative incentives to invest for the seller and buyer are summarised in the following lemma.

Lemma 1. $\delta f_1(\widetilde{I}_1) - \widetilde{I}_1 < \delta f_1(\widehat{I}_1) - \widehat{I}_1$, *and* $\delta f_2(\widetilde{I}_2) - \widetilde{I}_2 > \delta f_2(\widehat{I}_2) - \widehat{I}_2$.

Proof. See the Appendix. $\square$

Lemma 1 indicates that increasing I_1 towards its first-best level always increases the surplus it generates. The same argument applies to I_2 . As a consequence of Lemma 1, we can say that the surplus generated by I_1 is greater with the simultaneous regime. Similarly, the surplus generated by I_2 is greater with the sequential regime.

In terms of total surplus, the ultimate trade off between simultaneous and sequential systems depends on these three effect: costs of delay incurred with the sequential regime favour simultaneous investments; delayed returns also amplify hold-up arising with sequential system and reduce the incentive for the buyer to invest, favouring the simultaneous system; and, finally, the sequential system increases the incentive for the seller to invest, increasing her contribution to total surplus. Two of these effects work in favour of the simultaneous system and one works in favour of the sequential system. Result 1 summarises this discussion.

Result 1. *There are three factors that affect the total surplus generated by the simultaneous system relative to the total surplus that will be generated by the sequential system: (Effect 1) costs of delay favour the simultaneous system; (Effect 2) the simultaneous regime increases the buyer's incentive to invest, increasing his contribution to total surplus; and (Effect 3) the sequential regime increases the seller's incentive to invest, relative to the simultaneous regime.*

The combined effect of these three effects can be complicated. Note, however, that the three effects each depend on δ: the costs of delaying the return of surplus another period directly relate to δ; the level of I_1 depends on δ as the two relevant first-order conditions are $\widetilde{R}'_1 = 2/\delta^2$ and $\widehat{R}'_1 = 2/\delta$; and the two first-order condition for the choice of I_2 are $\widetilde{R}'_2 = 1/\delta$ and $\widehat{R}'_2 = 2/\delta$. However, if $\delta = 1$ the first two of these effects disappear. The only remaining effect is that sequential investment allows the seller to avoid being held-up, increasing her incentive to invest. Thus, if $\delta = 1$, $S_2 < S_4$. As R is a continuous function it follows that there is a neighbourhood for δ close to 1 where the surplus from sequential investment exceeds the surplus generated with simultaneous investments. This is summarised in the following remark.

Remark 1. *There is a small enough ε such that for any $\delta \in (1-\varepsilon, 1]$, $S_2 < S_4$; that is, the surplus from sequential investments exceeds that produced with simultaneous investments.*

Example 1. *Consider the case when $R(I_1, I_2) = \alpha ln I_1 + \beta ln I_2$. Figure 3 shows the four different utilities for both simultaneous and sequential investments when contracts* *are both complete and incomplete.* [11] *First note that U_1, the utility when investment is contractible and simultaneous, and U_3, the total utility when both investments are contractible but made sequentially, are equal when $\delta = 1$ as there are no costs of delay. Second, consider the surplus generated when contracts are incomplete. U_2 represents the total surplus with simultaneous investment, while U_4 represents the total surplus with the sequential regime. With low values of δ, U_2 exceeds U_4. However, for values of δ greater than about 0.9, $U_4 > U_2$; that is, the total surplus from sequential investments exceeds the total surplus with the simultaneous regime.*

It is not possible, however, to establish that the relative difference between the surplus from sequential and simultaneous investments is monotonically increasing in δ. With general functions, the relationship between δ, I_1 and I_2 and total surplus, R can be complicated.

Remark 2. *No monotonic relationship between the surplus from the simultaneous and sequential systems as δ changes.*

[11] *With simultaneous investment and complete contracts the first-order conditions are $\frac{I_1}{\alpha} = \frac{I_2}{\beta} = 1/\delta$. When contracts are incomplete and investments are simultaneous the first-order conditions are $\frac{I_1}{\alpha} = \frac{I_2}{\beta} = 2/\delta$. When investments are sequential and contracts complete: $\frac{I_1}{\alpha} = 1/\delta^2$ and $\frac{I_2}{\beta} = 1/\delta$. Finally, when investments are sequential and contracts incomplete the first-order conditions are: $\frac{I_1}{\alpha} = 2/\delta^2$ and $\frac{I_2}{\beta} = 1/\delta$. The specific functions used assume $\alpha = \beta = 5$: that is $U_1(\delta) = 10\delta(\ln 5\delta - 1)$, $U_2(\delta) = 10\delta(\ln 5\delta - 0.5 - \ln 2)$, $U_3(\delta) = 5\delta^2(\ln 5\delta^2 - 1) + 5\delta^2(\ln 5\delta - 1)$ and $U_4(\delta) = 5\delta^2(\ln 5\delta^2 - 0.5 - \ln 2) + 5\delta^2(\ln 5\delta - 1)$.*

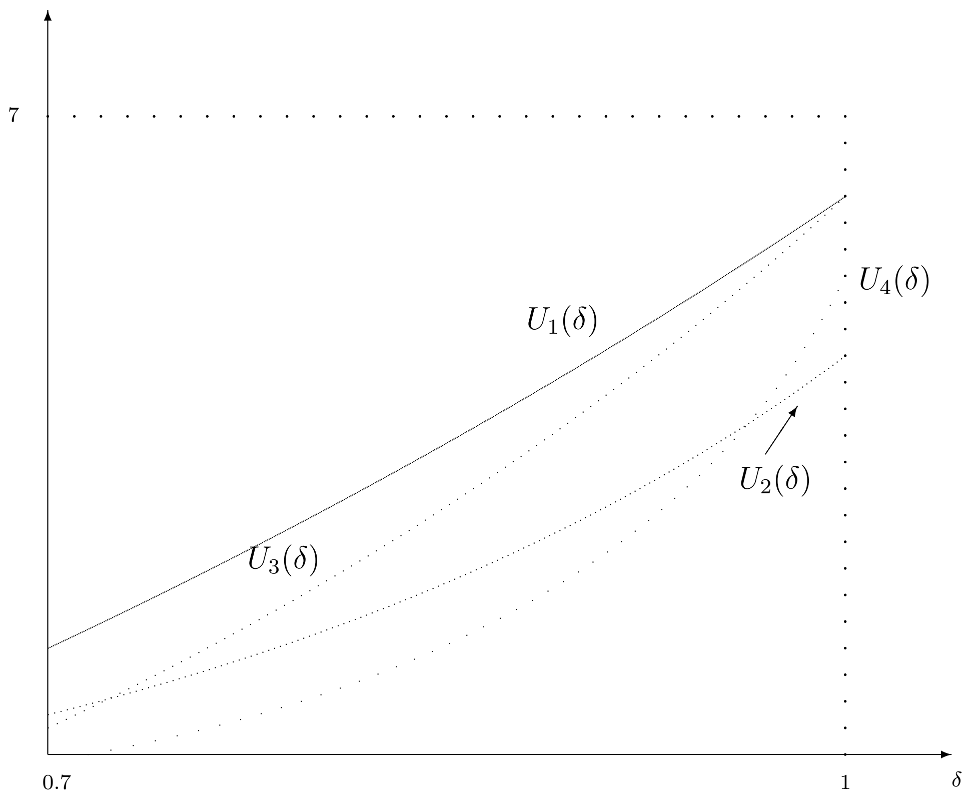

Figure 3: Illustration to example 1

Example 2. *As an example consider the following explicit function where:*

$$f_1 = aI_1^e \tag{20}$$

and

$$f_2 = bI_2^c. \tag{21}$$

Here, consider the case when $a = 11$, $b = 10$, $c = 0.3$ and $e = 0.7$. Using the explicit solutions to each party's first-order condition, the total utility generated with simultaneous investment can be written as a function of δ:

$$S_2(\delta) = a\delta\left(\frac{ea\delta}{2}\right)^{\frac{e}{1-e}} - \left(\frac{ea\delta}{2}\right)^{\frac{1}{1-e}} + b\delta\left(\frac{cb\delta}{2}\right)^{\frac{c}{1-c}} - \left(\frac{cb\delta}{2}\right)^{\frac{1}{1-c}}. \tag{22}$$

Similarly, the total surplus with sequential investment is:

$$S_4(\delta) = a\delta^2\left(\frac{ea\delta^2}{2}\right)^{\frac{e}{1-e}} - \left(\frac{ea\delta^2}{2}\right)^{\frac{1}{1-e}} + b\delta^2(cb\delta)^{\frac{c}{1-c}} - \delta(cb\delta)^{\frac{1}{1-c}}. \tag{23}$$

Figure 4 compares these two surpluses. First, there is clearly a non-monotonic relationship between δ and the difference between $S_2(\delta)$ and $S_4(\delta)$. Second, the two functions cross twice, once when δ is close to 0 and another time when δ is close to 1.

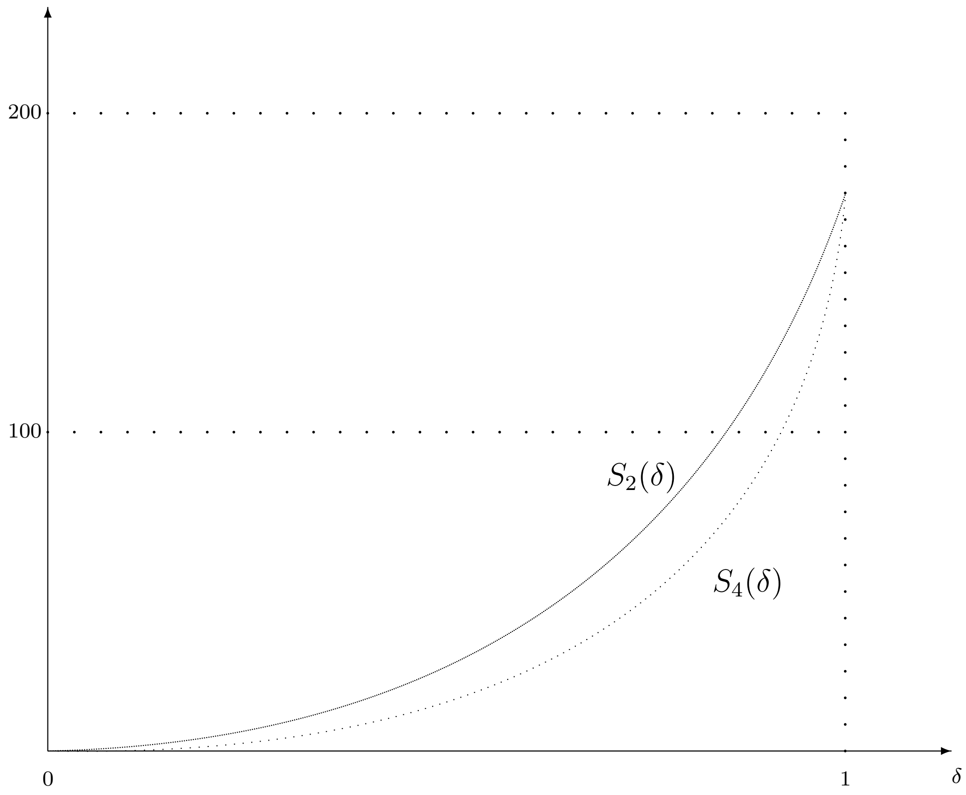

Figure 4: Illustration to example 2

4 Hold-up and the choice of investment regime

Thus far we have considered the relative merits of the various timing arrangements in terms of total welfare. The focus shifts here to explore the incentive for the seller, acting in self-interest, to choose the investment regime that does not maximise total surplus. Implicit in this discussion is the assumption that the buyer must invest at the beginning of the project. As a result, only the seller has the opportunity to delay her investment and follow-up the buyer.[12]

There is a trade-off for the seller when she chooses between the two regimes. As simultaneous system encourages the buyer to invest, this may allow the seller to capture more surplus during renegotiation. However, sequential investments allow the seller herself to invest without the fear of hold-up. The seller will choose the regime that maximises her welfare. Where her interests differ sufficiently from the first-best incentives the seller will adopt the 'wrong' system, reducing total welfare.

The seller may find it in her interests to adopt the sequential system when simultaneous investments maximise welfare. She will not, however, adopt a simultaneous system when the sequential regime maximises welfare. With inefficient simultaneous investment the seller will lose out on two fronts: first, she will incur hold-up with simultaneous investments; and second, she will be sharing a lower total surplus. Consequently, she will never have any incentive to opt for the simultaneous regime inefficiently.

To further investigate the incentives of the seller assume that $R_{12} = 0$ and that the buyer's level investment is invariant to the seller's choice of regime.[13] Consequently, I_1 can be suppressed, allowing all attention to revolve around the choice about the timing of I_2. The seller will then choose the system (and the level of investment) that maximises her surplus, regardless of the effect on total welfare.

[12]See Smirnov and Wait (2001) for a discussion relating to the case when each party can invest either first or last.

[13]The buyer's investment may be invariant, for example, because he has extreme beliefs about the seller's investment strategy: the buyer could be either naive or pessimistic as to whether the seller will opt for the simultaneous or sequential regime.

With simultaneous investments, total welfare can be written as

$$\delta\widehat{R}-\widehat{I}_2 \tag{24}$$

for the seller's choice of investment $I_2=\widehat{I}_2$, suppressing I_1 . The seller will set I_2 to maximise

$$\frac{\delta}{2}\widehat{R}-\widehat{I}_2. \tag{25}$$

Denote the seller's objective function under the simultaneous regime as v_1 ; that is, $v_1=\frac{\delta}{2}\widehat{R}-\widehat{I}_2$. This allows the total welfare generated with simultaneous investments to be written as $2v_1+\widehat{I}_2$.

With sequential investments total welfare is

$$\delta^2\widetilde{R}-\delta\widetilde{I}_2, \tag{26}$$

while the seller's objective function is

$$\frac{\delta^2}{2}\widetilde{R}-\frac{\delta}{2}\widetilde{I}_2. \tag{27}$$

Denote the seller's objective functions under sequential investment as v_2 : that is, $v_2=\frac{\delta^2}{2}\widetilde{R}-\frac{\delta}{2}\widetilde{I}_2$. This means that total surplus generated with sequential investments is $2v_2$.

Now assume that these potential payoffs for the seller are also equal: $v_1=v_2$. Given that $\widehat{I}_2>0$, simultaneous surplus will be greater than the surplus from sequential investments. It is possible, however, to perturb v_2 such that $v_2 > v_1$ while it remains true that simultaneous surplus exceeds the surplus with sequential investments, as $2v_1 + I_2 > 2v_2$. In this case the seller will opt for the sequential regime even though total surplus is maximised with the simultaneous regime. The above discussion is summarised in the result below.

Result 2. *There exists a range of parameters for which the seller chooses sequential investments when the simultaneous regime maximises total surplus.*

Example 3. *Consider the case when $R = 10lnI_1 + 8lnI_2$. Figure 5 plots the surplus of the seller with different investment regimes (on the Y-axis) against δ (on the X-axis). U_{22} shows two times the seller's surplus when investments are made simultaneously. U_4 shows two times the surplus of the seller - this equals the total surplus - when investments are made sequentially. U_2 shows the total surplus of both parties with simultaneous investments. It can be seen that for $\delta > 0.8$ (approximately) the seller will opt for the sequential system over the simultaneous option. However, from U_2 and U_4 it is only when $\delta > 0.95$ (approximately) that the sequential system produces more surplus than simultaneous regime. Thus, for $\delta \in (0.8, 0.95)$ the seller opts for the regime that does not maximise total welfare. Also note, in this example the buyer's investment is assumed fixed at $\widehat{I}_1$ for all of the functions. The specific functions used are $U_2(\delta) = \delta(10 \ln 5\delta + 8 \ln 4\delta) - 4\delta$, $U_{22}(\delta) = \delta(10 \ln 5\delta + 8 \ln 4\delta) - 8\delta$ and $U_4(\delta) = \delta^2(10 \ln 5\delta + 8 \ln 8\delta) - 8\delta$.*

If the seller opts for the inefficient investment regime the buyer's share of surplus is necessarily reduced. Herein lies how the choice of timing of investment can act as

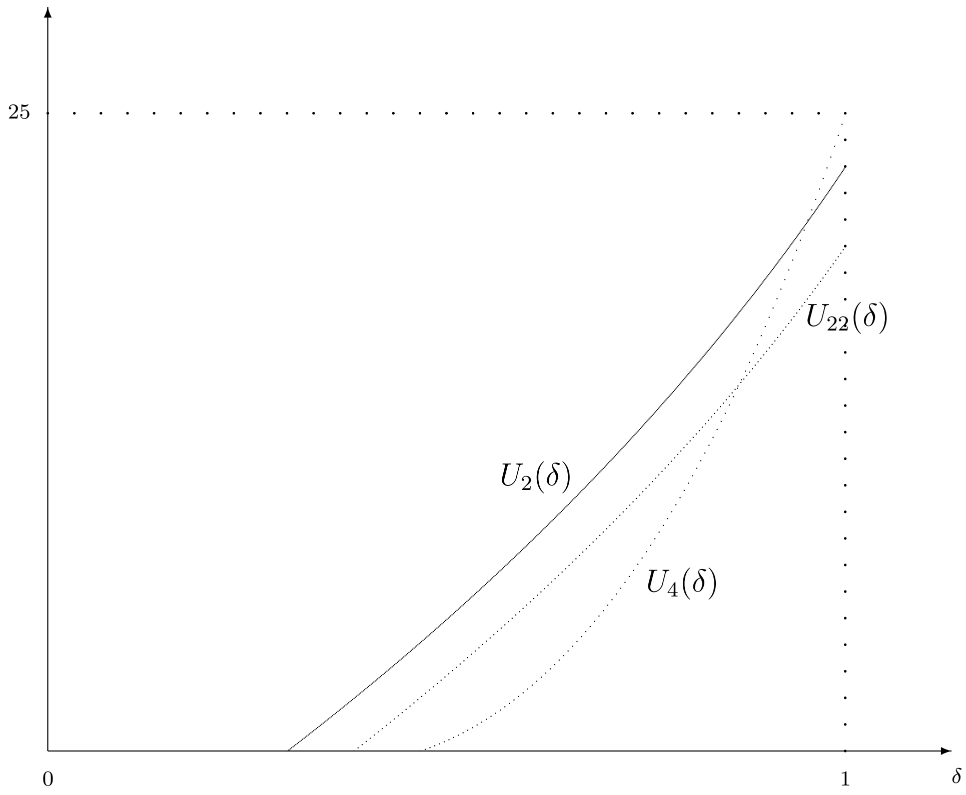

Figure 5: Illustration to example 3

an additional form of hold-up; we term this new form of hold-up 'follow-up'. In the extreme the reduction in surplus may lower the buyer's utility inside the relationship below his outside option: that is $\frac{\delta}{2}[\delta R(\widetilde{I}_1, \widetilde{I}_2) - \widetilde{I}_2] - \widetilde{I}_1 < 0$. In this case the option of the sequential regime prevents trade from occurring;[14] the inability to commit to a particular timing regime (the simultaneous regime) hurts the seller as well as the

[14]Note that provided $\frac{\delta}{2}R(\widehat{I}_1, \widehat{I}_2) - \widehat{I}_1 > 0$, the buyer would have opted into the relationship if only the simultaneous regime were available.

buyer.[15]

5 Extensions

This section makes several extensions to the model presented above. First, we explore the relationship between the two systems when I_1 and I_2 are strategic complement or substitute investments. This allows the relative efficiency of each system to be examined when one player's investment decision is highly sensitive with respect to which regime is adopted. Second, the section explores the relative efficiency of each regime when one investment is very important in terms of its contribution to overall surplus. Third, we examine the situation when one party's investment is invariant to the regime adopted. To conclude the section, we investigate the implications for total welfare when there is a lack of commitment so that either party can trigger renegotiation at any point in time.

5.1 Strategic substitute and complementary investments

When investment are strategic complements or substitutes $R_{12} \neq 0$. As this can significantly complicate matters, assume that $\delta = 1$, as summarised in Assumption 3.

Assumption 3. $\delta = 1$

[15]Similar analysis could be used to show that the sequential regime can create trading possibilities not available with only the simultaneous regime. For example, the seller may not be willing to invest simultaneously because the hold-up that occurs during the subsequent renegotiation may leave them with negative utility. On the other hand, sequential investment gives her the opportunity to delay their investment until when contracts are complete. This encourages the seller to invest and allows trade to proceed. This result is similar to the results of other authors, for example Neher (1999) and Admati and Perry (1991), albeit in a different context.

When Assumption 3 holds the total surplus is

$$S = R(I_1, I_2) - I_1 - I_2 \tag{28}$$

with both regimes. For the two regimes each player will choose their level of investment given their respective first-order conditions, shown in the Appendix. As $R_{12} \neq 0$, an adjustment in one investment will alter the marginal productivity of the other player's investment; this will affect each player's incentive to invest.

If the cross derivative of the investments is positive ($R_{12} > 0$) the investments are strategic complements as an increase in I_1 enhances the marginal productivity of I_2. This is summarised in Definition 3.

Definition 3. *If $R_{12} > 0$, I_1 and I_2 are strategic complements.*

Complementary investments may arise between trading parties, for example, when investment in a particular location enhances the value of the other party's investment. Effort in learning about the specific requirements of the trading partner can also enhances the productivity of the other player's investment. Similarly, investing in machinery or retooling in such a way to fit the requirements of the trading partner can help increase the marginal product of the other investment.

First, consider the case when investments are simultaneous. As one of the parties shades their investment, this encourages the other party to also shade their investment. The overall effect is that both parties reduce their investments even further below their levels when $R_{12} = 0$. This is the familiar underinvestment of the hold-up literature when there are externalities.[16] As a consequence, when investments are simultaneous and the investments are complementary there is underinvestment in both I_1 and I_2. This is summarised in Result 3.

Result 3. *When the investments are strategic complements and made simultaneously, there is underinvestment in both I_1 and I_2.*

Now consider when $R_{12} < 0$.

Definition 4. *If $R_{12} < 0$, I_1 and I_2 are strategic substitutes.*

An example of strategic substitute investments is when the two parties both require the use of a third asset, such as a particular location or venue, the supply of which is fixed or severely limited.[17] In this case, the buyer using the asset reduces the seller's return on any investment because their use of the asset is subsequently limited.

It is shown in the Appendix that the overall impact on I_1 and I_2 is ambiguous when investments are made simultaneously. For example, if the seller shades her investment the buyer has an incentive to increase I_1. Provided that the substitutability of the investments, as measured by the absolute size of $R_{12}^{''}$, exceeds the effect of diminishing returns to investment, as measured by the absolute value of $R_{22}^{''}$, the buyer will have an incentive to increase his investment above the first-best level. Similarly, there can be over-investment in I_2 provided the substitutability of the investments outweighs

[16] For example, see De Fraja (1999).

[17] Another example could be a negative externality between the parties. See, for example, Pitchford and Snyder (1999). Alternatively, if the two parties both produce a byproduct or pollutant, the output of which is limited by government regulation, an increase in output by one party limits the permissible output by the other.

the negative effect of the diminishing returns of investment ($\mid R''_{12} \mid > \mid R''_{11} \mid$). It follows from the assumption of concavity, however, that there will be underinvestment in at least one of the investments, even if there is over-investment in one of the investments.[18] The above discussion is summarised in the following result.

Result 4. *When the investments are substitutes and made simultaneously there can be under or over-investment in I_1 and I_2, however, there will be underinvestment in at least one of the investments.*

Now consider when investments are made sequentially. The buyer will underinvest regardless as to whether the investments are strategic complements or substitutes, as was the case when $R_{12} = 0$. In regards to I_2, when the investments are strategic complements the seller also underinvests.[19] This is because, unlike when $R_{12} = 0$, the underinvestment in I_1 reduces the incentive for the seller to invest in I_2.

In contrast, when the investments are substitutes the underinvestment in I_1 by the buyer provides an incentive to the seller to overinvest in I_2 . The following result summarises the above discussion.

Result 5. *When investment is sequential, there is underinvestment in I_1. When investments are strategic complements there is also underinvestment I_2, while if investments are strategic substitutes there is overinvestment in I_2.*

This subsection has explored the situation when investment by one party affects the marginal productivity of the other's investment, either in a negative or positive

[18] For details see the Appendix.

[19] Note that when $R_{12} = 0$ the sequential regime encouraged the seller to set I_2 at the first-best level.

manner. It was shown previously that when $R_{12} = 0$ the relative welfare of the two systems depended on the interaction of three effects. When the investments are either complements or substitutes these three effects are complicated somewhat by the impact each investment can have on the other investor's incentives. The next subsection extends this analysis further, notably by relaxing the assumption that $\delta = 1$.

5.2 Substitutes and complements with hyper-incentives

To further explore this issue consider the following specific functional form:

$$R = f_1(I_1) + f_2(I_2) + \varepsilon I_1 I_2 \tag{29}$$

such that total surplus with simultaneous investment is

$$\delta R - I_2 - I_1 \tag{30}$$

and total surplus with sequential investment is

$$\delta^2 R - I_1 - \delta I_2. \tag{31}$$

With this function, when $\varepsilon < 0$ the investments are strategic substitutes and when $\varepsilon > 0$ they are strategic complements.

With the simultaneous regime, the first-order conditions for each party are:

$$\widehat{f}_1' = \frac{2}{\delta} - \varepsilon \widehat{I}_2; \tag{32}$$

and

$$\widehat{f}_2' = \frac{2}{\delta} - \varepsilon \widehat{I}_1. \tag{33}$$

When investment is sequential the relevant first-order conditions are:

$$\widetilde{f}_1' = \frac{2}{\delta^2} - \varepsilon \widetilde{I}_2; \tag{34}$$

and

$$\widetilde{f}_2' = \frac{1}{\delta} - \varepsilon \widetilde{I}_1. \tag{35}$$

If $|\varepsilon|$ is small the complementarity or substitutability between I_1 and I_2 will be outweighed by effects 1, 2 and 3, outlined when the investments are independent ($\varepsilon = 0$). As the impact of ε is relatively small it remains the case that $\widehat{I}_1 > \widetilde{I}_1$ and $\widehat{I}_2 < \widetilde{I}_2$, in a similar manner as to when $R_{12} = 0$. Further, there are the same welfare trade-offs between the regimes, namely that simultaneous investment increases the contribution to total welfare from I_1 while the surplus generated by I_2 is enhanced with sequential investment. Note that, however, when $R_{12} \neq 0$ the sequential regime will merely encourage the seller to invest at the surplus maximising level given the buyer's investment; this will not necessarily be the first-best level. This discussion is summarised in the following remark.

Remark 3. *When $R = f_1(I_1) + f_2(I_2) + \varepsilon I_1 I_2$, provided the investments are not strong strategic complements or substitutes the same three effects outlined in Result 1 determine the relative welfare of the simultaneous and sequential regimes. Note, the directions of these three effects remain unchanged, although the values may be different.*

Proof. See the Appendix. □

When $|\varepsilon|$ is large the effects arising from the interaction between investments can lead to other possibilities. For example, if $\varepsilon > 0$ it is possible for the any one of the relevant first-order conditions to be less than zero. This provides that party with the incentive to invest ∞; given the complementarity between investments, the other party will also invest ∞, and the first-best will be achieved (ignoring the costs of delay). Another interpretation is that the first party will invest as much as they can, given their budget constraint. Again, this will encourage the other party to increase their investment. When a party's derivative is negative, this produces a 'hyper-incentive' for that party to invest. This term is defined below.

Definition 5. *A hyper-incentive is created when the first-order condition for a party is negative.*

Interestingly, one regime may produce a negative first-order condition while the other may not. For example, the simultaneous regime may produce a negative first-order condition for the buyer while the sequential system remains positive. In this case, the simultaneous regime produces a hyper-incentive for the buyer to invest - this means that this regime is favoured over the alternative. On the other hand, the sequential regime may produce a hyper-incentive for the seller, while her first-order condition with the simultaneous may still be positive. It is not the case that the sequential regime is always preferred, however, as the sequential regime involves additional costs of delay. For sequential investment to be favoured these costs of delay must be outweighed by the extra surplus generated from the hyper-incentive. The above discussion is summarised in the following result.

Result 6. *When the simultaneous regime creates a hyper-incentive for the buyer it is favoured over the sequential regime. When sequential investments generates a hyper-incentive for the seller it is favoured over the simultaneous investment regime, provided the players are sufficiently patient.*

Of course, when both systems generate hyper-incentives for a particular party simultaneous investment is preferred as it avoids some costs of delay.

In this subsection we have relaxed the assumption that $\delta = 1$ when the investments are either complements or substitutes. When the complementarity or substitutability between I_1 and I_2 is sufficiently small the same welfare trade-offs apply as when $R_{12} = 0$: the simultaneous regime encourages investment in I_1 and lowers costs of delay while the sequential regime encourages investment in I_2. With significant interaction between the investments the matter is further complicated so that other outcomes are possible.

5.3 Important investments and timing

From effects 2 and 3 above, sequential investments favour I_2 while simultaneous investments favour I_1. As a consequence, when I_1 is very important relative to I_2 the simultaneous investment system is preferred over sequential investments. Using similar reasoning, when I_2 is very important relative to the unimportant I_1 the sequential system is favoured over the simultaneous investment system.

To see this, we adopt a variant of Hart's (1995) definition of an unimportant investment.[20] For simplicity we assume $f_1(0) = f_2(0) = 0$.

Definition 6. *I_1 is unimportant if: $R(\widetilde{I}_1, I_2) = \delta^2 f_1(\widetilde{I}_1) + \delta^2 f_2(I_2) - \widetilde{I}_1 - \delta I_2$ is close to $R(0, I_2) = \delta^2 f_2(I_2) - \delta I_2$; and $R(\widehat{I}_1, I_2) = \delta f_1(\widehat{I}_1) + \delta f_2(I_2) - \widehat{I}_1 - I_2$ is close to $R(0, I_2) = \delta f_2(I_2) - I_2$. Similarly, I_2 is unimportant if: $R(I_1, \widetilde{I}_2) = \delta^2 f_1(I_1) + \delta^2 f_2(\widetilde{I}_2) - I_1 - \delta \widetilde{I}_2$ is close to $R(I_1, 0) = \delta^2 f_1(I_1) - I_1$; and $R(I_1, \widehat{I}_2) = \delta f_1(I_1) + \delta f_2(\widehat{I}_2) - I_1 - \widehat{I}_2$ is close to $R(I_1, 0) = \delta f_1(I_1) - I_1$.*

The key element here is that when a particular investment is unimportant it contributes relatively little to total surplus, although the marginal incentive to invest for the relevant player is unchanged.[21] The term 'close to' in Definition 6 can be considered as equivalent to the statement that A is close to B iff $A \gg A - B$.

First consider when I_2 is unimportant. Using the definition above, if I_2 is unimportant total surplus with simultaneous investment, $\delta f_1(I_1) + \delta f_2(I_2) - I_1 - I_2$, can

[20] See Hart (1995), p. 44.

[21] The first-order conditions for both players are unchanged from the initial problem. With simultaneous investments $f_i'(I_i) = \frac{2}{\delta}$ for $i = 1, 2$. With sequential investments the first-order condition for the buyer is $f_1'(I_1) = \frac{2}{\delta^2}$ and the seller's first-order condition is $f_2'(I_2) = \frac{1}{\delta}$.

be replaced by

$$\delta f_1(I_1) - I_1. \tag{36}$$

As a result all that matters to overall welfare is I_1. Surplus is then maximised by the system that promotes the highest level of I_1. As noted above, the level of I_1 with simultaneous investments, $\widehat{I}_1$, is closer to the first-best level than $\widetilde{I}_1$. Following from Definition 6:

$$R(\widehat{I}_1, \widehat{I}_2) \cong \delta f(\widehat{I}_1) - \widehat{I}_1 > R(\widetilde{I}_1, \widetilde{I}_2) \cong \delta^2 f(\widetilde{I}_1) - \widetilde{I}_1. \tag{37}$$

A similar argument can be made when I_1 is unimportant. In this case the sequential regime provides the seller with greater incentive to invest efficiently. There is, however, additional costs of delay with the sequential regime as compared with the simultaneous regime. The sequential regime will only be preferred if the benefits from the seller's additional investment outweigh these delay costs. From Definition 6 the total surplus with simultaneous investments is $\delta f(\widehat{I}_2) - \widehat{I}_2$, whereas the total surplus with sequential investments is given by $\delta^2 f(\widetilde{I}_2) - \delta \widetilde{I}_2$. The following result summarises this discussion.

Result 7. *When I_2 is unimportant the simultaneous investment system maximises total welfare. When I_1 is unimportant either regime may maximise total welfare.*

This result parallels Proposition 2(B) in Hart (1995). Hart argued that when one investment was unproductive asset ownership would be organised as to give the other party as much incentive to invest as possible. The model presented here suggests that when one investment is relatively unimportant the timing of investment should provide as much incentive as possible to the other party (ignoring the costs of delay). As in Hart (1995) there is no need to worry about the loss of surplus from reducing the other player's investment because it contributes relatively little to investment.

5.4 Inelastic investments

A party's level of investment may be invariant to the timing regime adopted. This may result, for example, because of a binding wealth constraint. This inelasticity can be utilised by concentrating on maximising the incentive for the other party to invest.

To facilitate the discussion consider the following definition.

Definition 7. *The buyer's (seller's) investment decision is inelastic when his (her) level of investment I_1 (I_2) is the same for both the simultaneous and the sequential regimes.*

Definition 7 is the analogue of Definition 1 in Hart (1995, p. 44).

If the buyer will invest $\overline{I}_1$ with either regimes, the sequential regime enhances the seller's incentive to invest. There is, however, an additional cost of delay. In terms of maximising welfare, these two factors work against each other. As a result, either regime could maximise welfare when the buyer's investment is inelastic. Alternatively, when the seller's investment is inelastic - that is, she always invests $\overline{I}_2$ regardless of the regime adopted - the simultaneous regime both encourages greater investment by the buyer and reduces the cost of delay. In this case the simultaneous regime is unambiguously superior. This discussion is summarised in the following result.

Result 8. *When the buyer's investment is inelastic there is an ambiguous relationship* *between regime type and total welfare. If the seller's investment is inelastic, the simultaneous regime unambiguously superior maximises total surplus.*

This result is similar to Proposition 2(A) in Hart (1995, p. 45). There, if one party's incentive to invest is invariant to asset ownership the other party should own the assets in order to encourage more efficient investment. Similarly here, when one party's incentive to invest is inelastic to the regime adopted, the regime chosen should maximise the incentive for the other party to invest. The only complication here is that the cost of delay also need to be taken into account. For example, if the generation of additional surplus from more efficient investment by the seller with the sequential regime does not outweigh the costs of delay, the simultaneous system should still be adopted.

5.5 Renegotiation

Hart and Moore (1999) assumed it was not possible for trading parties to make a credible commitment not to renegotiate. Grout (1984) also noted that industrial relations contracts are often not binding. Similarly, in an 'at-will' contracting environment either party can unilaterally trigger renegotiation or terminate the contract if they wish.[22] In this section we assume that either party can trigger renegotiation at any point in time.

[22]See the discussion of 'at-will' contracts in Malcomson (1997). The contracts in this subsection are slightly different from a typical 'at-will' contract environment. Usually in an 'at-will' environment there is an asymmetry in the bargaining power between the buyer and the seller. For example, if the buyer (firm) starts negotiations and proposes a new lower price, the seller (worker) is taken to have accepted this new proposed contract if she continues to supply her services (labour). On the other hand, if the seller attempts to raise price the default price takes precedence, unless the buyer explicitly accepts the new contract.

When this is the case, only the final renegotiation affects the distribution of surplus (and hence the incentive to invest). The last opportunity to renegotiate occurs after the last investment has been made, that is, once I_2 has been completed. Renegotiation will always occur at this stage because the buyer is better off with a new distribution of surplus after I_2 is sunk.

First, consider when investment is simultaneous. As before, renegotiation will occur after both investments have been made. Consequently, the first-order conditions for both players are the same as described above. With sequential investment renegotiation will always occur after the seller has invested. As both investments are sunk the parties will split the surplus 50-50. The buyer will set his investment to maximise:

$$\frac{\delta^2}{2}[R(I_1, I_2)] - I_1. \tag{38}$$

His first-order condition under these circumstances will be

$$R_1^{'} = \frac{2}{\delta^2} \tag{39}$$

which is unchanged from when there is no subsequent renegotiation. Label the level of the buyer's investment when commitment is not possible with sequential investment as $\widetilde{\widetilde{I}}_1$. From this it can be seen that

$$\widetilde{\widetilde{I}}_1 = \widetilde{I}_1 < \widehat{I}_1. \tag{40}$$

On the other hand, the seller will maximise:

$$\frac{\delta^2}{2}[R(I_1, I_2)] - \delta I_2 \tag{41}$$

which yields the first-order condition

$$R_2^{'} = \frac{2}{\delta}. \tag{42}$$

Label the seller's choice of her investment when commitment is not possible at any stage and investment is sequential as $\widetilde{\widetilde{I}}_2$. Comparing the first-order condition for the seller when there is simultaneous investment ($R_2^{'} = \frac{2}{\delta}$) and when investment is sequential but there is no after-investment renegotiation ($R_2^{'} = \frac{1}{\delta}$) it can be seen that

$$\widetilde{\widetilde{I}}_2 = \widehat{I}_2 < \widetilde{I}_2. \tag{43}$$

If there is ex post renegotiation it does not matter that the investments were initially made sequentially as both parties suffer from hold-up. As the buyer is always held-up, assuming $R_{12}^{''} = 0$, his incentive to invest is unchanged from the usual sequential regime discussed above. Now, however, any potential advantage of the sequential regime is eliminated: the seller also suffers from hold-up with the sequential regime reducing her incentive to invest. As the sequential system involves more costs of delay, the simultaneous system produces higher total surplus than the sequential regime. Consequently, if commitment is not possible, simultaneous investment is strictly preferred to sequential investment. Moreover the ability of either party to trigger renegotiation at any time effectively renders the possibility of sequential investment (or its attractiveness) redundant. This is summarised in the following result.

Result 9. *If the parties cannot commit not to renegotiate after both investments have been made, the simultaneous system strictly dominates sequential investment for $\delta < 1$ in terms of total welfare as well as welfare of the seller. Consequently, if the parties are unable to commit not to renegotiate, the sequential regime is never adopted.*

This lack of commitment may be advantageous, however, if the seller would like to commit not to adopt the sequential regime, as it provides the buyer with a lower level of surplus than his outside option (as discussed in section 4). The knowledge that the buyer will trigger renegotiation acts as a credible commitment by the seller to invest simultaneously. This may in turn encourage the buyer to invest.

6 Conclusion

This paper develops a model in which two parties can invest in a mutually beneficial project at the same time (simultaneous investment) or they can choose to invest one after the other (sequential investment). It is assumed that contracting on any future investment becomes possible after some investment has been made as it allows the project to become more clearly defined. Consequently, the advantage of the sequencing of investments is it allows the party that has delayed making their investment to avoid being held-up. The disadvantage of staging is that it reduces the incentive to invest of the first-mover. This can also have feed-back effects on the second party's investment depending on the relationship between the two investments. In addition, sequencing of investment lengthens the time from the start of the project until the returns are realised, reducing the ex ante value of total surplus when parties discount future returns. The relative advantage of the sequential versus the simultaneous investment regime depends on the precise nature of these trade-offs. Two principles apply, however, provided the parties are sufficiently patient: first, the regime that favours the most important investment in terms of its contribution to total surplus is preferred; and, second, if one investment is invariant to the regime adopted, the optimal timing of investment will be the regime that maximises the incentive for the other party to invest.

Much of the emphasis in the existing literature has focused on how staging investments can improve welfare when there are incomplete contracts or when parties are unable to commit. In the model presented in this paper it is demonstrated that, in some cases, the option of sequencing investments can reduce welfare. It is shown that under certain conditions a party will opportunistically opt for the sequential regime, reducing total surplus. We interpret this possibility as a new form of hold-up and term it 'follow-up'.

7 Appendix

Lemma 1 $\delta f_1(\widetilde{I}_1) - \widetilde{I}_1 < \delta f_1(\widehat{I}_1) - \widehat{I}_1$, *and* $\delta f_2(\widetilde{I}_2) - \widetilde{I}_2 > \delta f_2(\widehat{I}_2) - \widehat{I}_2$.

Proof. The first-best investment level of I_1, derived from $\delta f_1(I_1) - I_1$, occurs when $f_1' = \frac{1}{\delta}$. This level of investment is termed I_1^*. For $I_1 < I_1^*$, $f_1'(I_1) \geq \frac{1}{\delta}$ because $f_1''(I_1) \leq 0$. For $I_1 < I_1^*$, $[\delta f_1(I_1) - I_1]' \geq 0$, hence $\delta f_1(I_1) - I_1$ is a non-decreasing function $\forall\ I_1 \in [0, I_1^*)$, which means $\delta f_1(\widetilde{I}_1) - \widetilde{I}_1 < \delta f_1(\widehat{I}_1) - \widehat{I}_1$. A similar argument applies to I_2. $\square$

Result 3 *When the investments are complements and made simultaneously, there is underinvestment in both I_1 and I_2.*

Proof. When Assumption 3 holds the total surplus is

$$S = R(I_1, I_2) - I_1 - I_2 \tag{44}$$

for the levels of investment chosen in the different systems. The first-order conditions are

$$\widehat{R}_1' = 2 \tag{45}$$

$$\widehat{R}_2' = 2 \tag{46}$$

for the simultaneous investment system, and

$$\widetilde{R}_1' = 2 \tag{47}$$

$$\widetilde{R}_1' = 1 \tag{48}$$

with sequential investments.

To investigate this further, replace substitute $a \in [1, 2]$ for 2 in each of the equations, so that

$$\widehat{R}_1' = a \tag{49}$$

$$\widehat{R}_2' = a \tag{50}$$

for the simultaneous investment equations, and

$$\widetilde{R}_1' = a \tag{51}$$

$$\widetilde{R}_1' = 1 \tag{52}$$

for the sequential system. This allows the buyer and seller's investment levels to be represented as functions of a: from equations 49 and 50 the relevant investment levels become $\widehat{I}_1(a)$ and $\widehat{I}_2(a)$; and from equations 51 and 52 $\widetilde{I}_1(a)$ and $\widetilde{I}_2(a)$ are the relevant investment levels. Totally differentiating equations 49 and 50 with respect to a yields

$$R_{11}'' I_1'(a) + R_{12}'' I_2'(a) = 1 \tag{53}$$

$$R_{21}'' I_1'(a) + R_{22}'' I_2'(a) = 1. \tag{54}$$

Solving this system of equations using Cramer's rule yields solutions

$$\widehat{I}_1'(a) = \frac{R_{22}'' - R_{12}''}{R_{11}'' R_{22}'' - (R_{12}'')^2} \tag{55}$$

$$\widehat{I}_2'(a) = \frac{R_{11}'' - R_{12}''}{R_{11}'' R_{22}'' - (R_{12}'')^2}. \tag{56}$$

Note that given the assumption of concavity the denominator is always negative.

When $R_{12} > 0$,

$$\widehat{I}_1'(a) < 0 \tag{57}$$

$$\widehat{I}_2'(a) < 0. \tag{58}$$

The overall effect of moving from the first-best level of investment (when $R_i' = 1$) to the second best solutions given by equations 45 and 46, must consider the integral of the marginal changes over the entire range of $a \in [1, 2]$. However, as the marginal change is always of the same sign we can discern that when the investments are complements there is underinvestment of both investments. $\square$

Result 4 *When the investments are substitutes and made simultaneously there can be under or overinvestment in I_1 and I_2, however, there will be underinvestment in at least one of the investments.*

Proof. From equations 55 and 56, when $R_{12} < 0$,

$$\widehat{I}_1'(a) \gtrless 0 \tag{59}$$

$$\widehat{I}_2'(a) \gtrless 0. \tag{60}$$

For I_1, the derivative is positive if $\mid R_{12}'' \mid > \mid R_{22}'' \mid$. Likewise, the derivative for I_2 is positive if $\mid R_{12}'' \mid > \mid R_{11}'' \mid$. In addition, it follows from the assumption of concavity that $I_1'(a) + I_2'(a) = \frac{R_{22}'' + R_{11}'' - 2R_{12}''}{R_{11}'' R_{22}'' - (R_{12}'')^2} < 0$. This suggests that there will be underinvestment in at least one of the investments, even if there is over-investment in one of the investments. □

Result 5 *When investment is sequential, there is underinvestment in I_1. When investments are complements there is also underinvestment I_2 while if investments are substitutes there is overinvestment in I_2.*

Proof. As above, totally differentiating the equations 51 and 52 yields

$$R^{''}_{11} I^{'}_1(a) + R^{''}_{12} I^{'}_2(a) = 1 \tag{61}$$

$$R^{''}_{21} I^{'}_1(a) + R^{''}_{22} I^{'}_2(a) = 0. \tag{62}$$

Solving using Cramer's rule shows that

$$\widetilde{I}^{'}_1(a) = \frac{R^{''}_{22}}{R^{''}_{11} R^{''}_{22} - (R^{''}_{12})^2} \tag{63}$$

$$\widetilde{I}^{'}_2(a) = \frac{-R^{''}_{12}}{R^{''}_{11} R^{''}_{22} - (R^{''}_{12})^2}. \tag{64}$$

Regardless of the sign of R_{12},

$$\widetilde{I}^{'}_1 < 0. \tag{65}$$

This indicates that there will be underinvestment in I_1.

For I_2, when the investments are complements - that is when $R^{''}_{12} > 0$ - there is underinvestment in I_2 as

$$\widetilde{I}^{'}_2 < 0. \tag{66}$$

When $R''_{12} < 0$,

$$\widetilde{I}'_2 > 0 \tag{67}$$

indicating that there will be over-investment in I_2. □

Remark 3 *When $R = f_1(I_1) + f_2(I_2) + \varepsilon I_1 I_2$, provided the investments are not strong complements or substitutes the same three effects outlined in section 3 determine the relative welfare of the simultaneous and sequential regimes. Note, the directions of these three effects remain unchanged, although the values may be different.*

Proof. Let us consider the following parameterised first-order conditions

$$f'_1(I_1) = a - \varepsilon I_2 \text{ and } f'_2(I_2) = b - \varepsilon I_1. \tag{68}$$

The following equation on the optimal level of I_1 can be derived from the above system:

$$f'_2\left(\frac{a - f'_1(I_1)}{\varepsilon}\right) = b - \varepsilon I_1. \tag{69}$$

Differentiating this equation with respect to I_1 when $b =$ constant and $a = a(I_1)$ gives

$$f''_2(\cdot)\frac{a' - f''_1(\cdot)}{\varepsilon} = -\varepsilon, \tag{70}$$

which means

$$\frac{\partial I_1}{\partial a} = \frac{1}{a'} = \frac{f''_2(\cdot)}{f''_2(\cdot)f''_1(\cdot) - \varepsilon^2} < 0. \tag{71}$$

Similarly when $a =$ constant and $b = b(I_1)$ differentiating of equation 69 with respect to I_1 gives

$$f_2''(\cdot)\frac{-f_1''(\cdot)}{\varepsilon} = b' - \varepsilon, \tag{72}$$

from which it follows

$$\frac{\partial I_1}{\partial b} = \frac{1}{b'} = \frac{\varepsilon}{\varepsilon^2 - f_2''(\cdot)f_1''(\cdot)} > 0. \tag{73}$$

When $|\varepsilon|$ is small the effect outlined in equation 73 can be ignored. Consequently, equation 71 has the dominant effect. From this we know that I_1 is higher with simultaneous investment than with the sequential regime, and that I_1 is greater still with complete contracts (first-best I_1). Further, in a similar manner as outlined in Lemma 1, higher levels of I_1 translate to a greater contribution to total surplus. We can rank the regimes in terms of the contribution I_1 makes to welfare: the simultaneous regime dominates the sequential regime.

We now derive the equation on the optimal level of I_2 from the parameterised system

$$f_1'\left(\frac{b - f_2'(I_1)}{\varepsilon}\right) = a - \varepsilon I_2. \tag{74}$$

Differentiating this equation with respect to I_1 when: $b =$ constant and $a = a(I_1)$; and when $a =$ constant and $b = b(I_1)$ gives

$$\frac{\partial I_1}{\partial a} = \frac{1}{a'} = \frac{\varepsilon}{\varepsilon^2 - f_2''(\cdot)f_1''(\cdot)} > 0 \tag{75}$$

and

$$\frac{\partial I_1}{\partial b} = \frac{1}{b'} = \frac{f_1''(\cdot)}{f_2''(\cdot)f_1''(\cdot) - \varepsilon^2} < 0 \tag{76}$$

respectively.

In a similar manner as described with I_1 above, when $|\varepsilon|$ is small the effect outlined in equation 76 has the dominant influence on I_2. This suggests I_2 is greater with the sequential regime than with simultaneous investments, although it is still lower than its first-best level. The levels of I_2 also directly translate into its contribution to total welfare: I_2 contributes more to total welfare with sequential investment than with the simultaneous regime. $\square$

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