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A SIMPLE MODEL OF
UNEMPLOYMENT RATE DYNAMICS

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A simple model of unemployment rate dynamics

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The paper seeks to ‘explain’ certain stylised facts in relation to flows into and out of Unemployment and especially to identify the ‘proximate’ determinants of the amplitude and the frequency of fluctuations in the Unemployment Rate over the course of the business cycle. Since the evolution of the Unemployment Rate must be seen as dependent on the relative size of the Inflow and Outflow Rates, a multiple equation model has to be employed. It is found that the transition probability (ϕ) is a determinant of the amplitude, and thus the mean value of, the Unemployment Rate but not the frequency of fluctuations in the Unemployment Rate. However, it is found that the Inflow Rate is a determinant of both the amplitude and the frequency of fluctuations in the Unemployment Rate. We are also able to explain why it is that researchers find that the long-run effect of a change in the Inflow Rate upon the Outflow Rate is to change the Outflow Rate by exactly the same amount as the Inflow Rate has changed.

Key words: Worker flows, Unemployment, Business cycle

JEL classifications: E32, J64

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1. Introduction

This paper looks at the dynamics of the unemployment rate in a gross (worker) flows context using a simple aggregate Keynesian model to account for the stylized facts. In particular we look at the relationships which must obtain between the *Unemployment Rate* (defined as the number unemployed in any period expressed as a proportion of the labour force) on the one hand and the Inflow and Outflow Rates (the *Inflow Rate* is defined as the sum of the flows from Employment and Not in the labour force into Unemployment over any period expressed as a proportion of the Labour Force while the *Outflow Rate* is defined as the sum of the flows from Unemployment to Employment and to Not in the labour force over any period expressed as a proportion of the Labour Force) on the other. Since the evolution of the Unemployment Rate must be seen as dependent on the relative size of the Inflow and Outflow Rates, we need a multiple equation model which allows us to make statement about the evolution of Inflow and Outflow over time, whilst making any interdependencies (and especially endogeneities) between the variables explicit. In doing this, we will look at models of differing realism and thus complexity.

Adopting a Keynesian approach in this context means (inter alia): the theory is grounded in reality (by which I mean that it attempts explicitly to address important stylised facts); the approach is Macroeconomic and recognises that there is a business cycle and in particular that there are fluctuations in unemployment which are not 'voluntary', that these fluctuations are associated with aggregate demand shocks and thus that the Inflow into unemployment can usefully be regarded as exogenous (determined by fluctuations in aggregate demand).

The paper has two main aims. First, to 'explain' certain stylised facts and, in particular, to show why the empirical evidence shows that the long-run effect of a change

in the Inflow Rate upon the Outflow Rate is to change the Outflow Rate by exactly the same amount as the Inflow Rate has changed. Second, to identify the 'proximate' determinants of the amplitude and the frequency of fluctuations in the Unemployment Rate over the course of the business cycle.

2. Some stylised facts

It may be useful at this point to state some 'stylised facts' about how the Unemployment Rate and the Inflow and Outflow Rates behave over the course of the business cycle. They are: (1) The Unemployment Rate is cyclical as are the Inflow Rate and the Outflow Rate. (2) Both the Inflow Rate and the Outflow Rate are 'procyclical' where by that term we mean they both go up and down with the Unemployment Rate.¹ This means that both the Inflow Rate and the Outflow Rate rise in recessions and both fall in recoveries/booms. (3) The probability that an individual unemployed person will exit unemployment varies over the business cycle, falling in recessions and rising in recoveries. In other words, it is inversely related to the Unemployment Rate. (4) During a recession the Inflow Rate (which is rising) exceeds the Outflow Rate (which is also rising) and as a result the Unemployment Rate rises while the opposite occurs in recoveries/booms. (5) Outflow lags behind Inflow. (6) The Inflow Rate and the Outflow Rate are cointegrated with a cointegrating vector of (1, -1), implying that the long-run effect of a change in the Inflow Rate upon the Outflow Rate is to bring about a change in the Outflow Rate of exactly the same amount as the Inflow Rate has changed.

This last item is very important and needs elaboration. It is to do with the relationship (and, in particular the long-run relationship) between the Inflow Rate and the Outflow Rate. Using Australian worker flows data Dixon, Freebairn and Lim (2002) find

that the Inflow Rate and the Outflow Rate are cointegrated with a cointegrating vector of (1, -1). Balakrishnan & Michelacci (2001, p 142 and 143f) find that Inflow and Outflow Rates for the US, UK, Germany, France and Spain also have cointegrating vectors of (1, -1). This finding, that a change in the Inflow Rate will, in the long run, lead to a change the Outflow Rate by the same amount, has enormous implications for our understanding of the behavior not only of the relationship between the Inflow Rate and the Outflow Rate over time but also of the relationship between the Inflow Rate and the Outflow Rates on the one hand and the Unemployment Rate on the other. For example, we have noted that one stylized fact is that the Outflow Rate goes up and down with the Unemployment Rate.² For many, this is a surprising and puzzling or paradoxical finding. But, given that the Unemployment Rate is cyclical, given also the finding that the Inflow Rate and the Outflow Rate are cointegrated and that the cointegrating vector is (1, -1), it is not surprising to find that the Outflow Rate rises (with the Inflow Rate) in recessions and that the Outflow Rate falls (with the Inflow Rate) in recoveries.³ Indeed, it would be odd if it were not so. This is why one aim, indeed a most important aim, of the present paper is to explain why the Inflow Rate and the Outflow Rate are cointegrated and that the cointegrating vector is (1, -1).

3. The basic elements of a model

To construct even the most basic model to address these issues, we need expressions for the Inflow Rate, the Outflow Rate and the Unemployment Rate. It is convenient to begin with the Outflow Rate.

3.1 The Outflow Rate (*OUTR*)

It is common to regard any flow measured in terms of numbers of persons per period moving between any two states to be determined by the relevant transition probability in conjunction with the size of the relevant pool at the beginning of the period. Applying this idea, the flow measured in terms of numbers of persons per period moving out of Unemployment (*OUT*) is equal to the product of the (transition) probability of any one unemployed person moving out of Unemployment over any period (ϕ) and the number unemployed (*U*) at the beginning of the period. So that:

$$OUT_t = \phi_t \times U_t$$

where $0 < \phi < 1$

Dividing both sides by the size of the Labour Force gives an expression for the Outflow Rate (*OUTR*) in terms of the transition probability (ϕ) and the Unemployment Rate (*UR*).

$$OUTR_t = \phi_t \times UR_t \quad (1)$$

3.2 The Unemployment Rate (*UR*)

In modelling the extent to which the Unemployment Rate changes between any two periods, we will work with the approximation:⁴

$$d(UR) = \left(\frac{IN - OUT}{LF} \right) = \frac{IN}{LF} - \frac{OUT}{LF} = INR - OUTR \quad (2)$$

where *IN* and *OUT* are the absolute number of persons flowing in to and out of Unemployment over the period respectively and *LF* is the absolute size of the Labour Force at the beginning of the period. *INR* and *OUTR* are the Inflow and Outflow Rates, respectively.

Clearly, if the Inflow Rate exceeds the Outflow Rate, the Unemployment Rate will rise; if the Outflow Rate exceeds the Inflow Rate, the Unemployment Rate will fall, and; if the Outflow Rate equals the Inflow Rate, the Unemployment Rate will remain constant.

Note, in passing, that the above implies that the Unemployment rate will evolve over time according to the rule⁵

$$UR_t = UR_{t-1} + (INR_{t-1} - OTR_{t-1})$$

and that given (1), we may write:

$$UR_t = INR_{t-1} + (1 - \phi_t)UR_{t-1} \quad (3)$$

3.3 The Inflow Rate (INR)

Throughout the paper we will adopt the 'Keynesian assumption' that the Inflow Rate can be regarded as exogenous – we envisage that it reflects variations in the demand for labour which in turn will be driven by fluctuations in the rate of growth of GDP (and changes in technology). For now we will simply write:

$$INR_t = f(\text{time}) \quad (4)$$

3.4 A first look at some dynamics

Some intuitive feeling for how these equations interact with each other may be obtained if we envisage a world in which we have a classical business cycle in which the Unemployment Rate moves up and down with a recurrent wave-like motion. Over the course of a recession the Unemployment Rate will reach its 'trough level' (where it is momentarily, at least, constant), it then rises (which must be because the Inflow Rate exceeds the Outflow Rate, and the rise in the Unemployment Rate will persist only so long as this is true) by a large or small amount until it reaches its peak (where, again, the Unemployment Rate is momentarily, at least, constant). When the Unemployment Rate

has troughed or ‘bottomed out’ the Inflow and Outflow Rates must be equal. As the recession begins the Inflow Rate will rise relative to the Outflow Rate (of course the recession begins because the Inflow Rate rises relative to the Outflow Rate) and indeed the Inflow Rate will keep rising until (at least) the peak is reached. For the Unemployment Rate to peak, the Outflow Rate must rise over the course of the recession, and it has to keep rising until it has caught up with the Inflow Rate. Once it has caught up, the Unemployment Rate will stop rising and it (the Unemployment Rate) will have peaked (i.e. the recession will be at an end). Another way to put all this is to say that over the course of a recession we move from a situation where the Inflow Rate and the Outflow Rate are both equal and momentarily constant, through a number of successive periods where the Inflow Rate exceeds the Outflow Rate and then to a situation where the Inflow Rate and the Outflow Rate are again both equal – albeit at a higher level than they were before the recession began – and (momentarily, at least) constant. Clearly we can tell the story with appropriate changes to direction of movement and relative sizes of the Inflow Rate and the Outflow Rate to explain the behaviour in a recovery or boom period.

3.5 Towards a formal model

In providing a formal exposition of the model we will look at three cases: Case 1. Where the outflow probability (ϕ) is exogenous and constant while the Inflow Rate follows a rectangular time path jumping from a boom to a recession and back again. Case 2. Where the outflow probability (ϕ) is endogenous and varies with the Unemployment Rate while the Inflow Rate again follows a rectangular time path, and Case 3. Where the outflow probability (ϕ) is endogenous and varies with the Unemployment Rate while the Inflow Rate follows a sine wave. (This case is examined in an Appendix to the paper.)

In each case we will ask questions of the model including: What will be the time paths of the Unemployment Rate and the Outflow Rate as they react to the evolution of the Inflow Rate (which we are taking as exogenous)? What can we say about the level of the Unemployment Rate when it is at its peak and trough (ie its amplitude)? What will be the amount of time between peaks and/or troughs in the Unemployment Rate (ie its frequency or period)? What will be the delay (if any) between the Inflow Rate and the Unemployment Rate? If the Outflow Rate tracks the Inflow Rate with a lag, what determines the length of the lag? (Remember that it is persistent or systematic differences between the Inflow Rate and the Outflow Rate which drive the Unemployment Rate up or down – hence the importance of investigating the length of the delay between the Inflow Rate and the Outflow Rate.) What will be the long-run effect of a change in the Inflow Rate on the Unemployment Rate and the Outflow Rate? Is the behaviour of the model consistent with the ‘law of flows’, which holds that the Inflow Rate and the Outflow Rate are cointegrated with a cointegrating vector of $(1, -1)$?

We begin with the case where the Outflow Rate expressed simply as a constant proportion of the Unemployment Rate. Although this is counter to one of our stylised facts, it allows us to better see the analytics of a solution for the variables of interest and also to see that the results of numerical simulation are in accord with the analytical solution. We then look at a more realistic model where the Outflow Rate is a non-linear and inverse function of the Unemployment Rate. That case we will explore by numerical simulation alone.

4. Case 1: The behaviour of the model when ϕ is a constant and exogenous

Here, we will treat the Inflow Rate as exogenous, the outflow probability (ϕ) as a constant and both the Outflow Rate and the Unemployment Rate as endogenous.

Since ϕ is constant, and we are really only interested in the time paths of the Inflow Rate, the Outflow Rate and the Unemployment Rate, we can work with three equations in the model (recall that stocks such as the Unemployment Rate are measured at the start of a period whereas flows such as the Inflow Rate and the Outflow Rate are measured over the period).⁶ The equations which make up the model will be:

$$INR_t = f(\text{time}) \quad (4)$$

$$UR_t = INR_{t-1} + (1 - \phi)UR_{t-1} \quad (3')$$

$$OUTR_t = \phi \times UR_t \quad (1')$$

which is a recursive model.

4.1 What is the relationship between UR and INR?

Given (3'), the Unemployment Rate will track the movements in the Inflow Rate with a lag. Also, since $OUTR_t = \phi \times UR_t$, the Outflow Rate will also track the movements in the Inflow Rate with a lag.

Now, given (3') we may write for UR_{t-1} ,

$$UR_{t-1} = INR_{t-2} + (1 - \phi)UR_{t-2}$$

Substitution of the above into (3') gives

$$UR_t = INR_{t-1} + (1 - \phi)INR_{t-2} + (1 - \phi)^2 UR_{t-2} \quad (5)$$

Likewise, given (3') we can write for UR_{t-2}

$$UR_{t-2} = INR_{t-3} + (1-\phi)UR_{t-3}$$

Substitution of the above into (5) gives

$$UR_t = INR_{t-1} + (1-\phi)INR_{t-2} + (1-\phi)^2 INR_{t-3} + (1-\phi)^3 UR_{t-3}$$

If we continue in this fashion, continuously backwards substituting for the Unemployment Rate further and further back in time, we will obtain

$$UR_t = \sum_{i=0}^n (1-\phi)^i INR_{t-1-i} + (1-\phi)^n UR_{t-n} \quad (6)$$

Given that $0 < \phi < 1$, it must be the case that at some point as $n \rightarrow \infty$, $(1-\phi)^n$ becomes negligible leaving the expression:

$$UR_t = \sum_{i=0}^n (1-\phi)^i INR_{t-1-i} \quad (7)$$

Which is to say that with a constant transition probability (ϕ), the Unemployment Rate at any moment in time is a geometric distributed lag function³ of past Inflow Rates. The relationship is such that a once and for all increase (decrease) in the Inflow Rate will result in a monotonic increase (decrease) in the Unemployment Rate, rather in the fashion of a Koyck-type stock adjustment function.

4.2 What can we say about the sum of the weights which relate UR to lagged values of INR?

The sum (S_n) of the weights in (7) will equal

$$S_n = 1 + (1-\phi) + (1-\phi)^2 + \dots + (1-\phi)^{n-1} \quad (8)$$

If we multiply both sides of the above by $(1-\phi)$, we get

$$(1-\phi)S_n = (1-\phi) + (1-\phi)^2 + \dots + (1-\phi)^n \quad (9)$$

Subtracting (9) from (8) and rearranging, we have

$$S_n = \frac{1}{\phi} - \frac{(1-\phi)^n}{\phi} \quad (10)$$

Given that $0 < \phi < 1$, it must be the case that as $n \rightarrow \infty$, $S_n \rightarrow 1/\phi$ and so the long run response of the Unemployment Rate to a once and for all increase or decrease in the Inflow Rate is to not only move in the same direction but (sooner or later) to change by an amount equal to $1/\phi$ times the change in the Inflow Rate.

4.3 What is the relationship between OUTR and INR?

Since $OUTR_t = \phi \times UR_t$, then, given (3'), we may write:

$$OUTR_t = \phi INR_{t-1} + (1-\phi)OUTR_{t-1} \quad (11)$$

Also, given (7), we may write:

$$OUTR_t = \sum_{i=0}^n \phi (1-\phi)^i INR_{t-1-i} \quad (12)$$

Which is to say that with a constant transition probability (ϕ), the Outflow Rate at any moment in time is a geometric distributed lag function of past Inflow Rates. The relationship is such that a once and for all increase (decrease) in the Inflow Rate will result in a monotonic increase (decrease) in the Outflow Rate as it moves into equality with the new level of the Inflow Rate.

4.4 What can we say about the sum of the weights which relate OUTR to the lagged values of INR?

The sum (S_n) of the weights in (12) will equal

$$S_n = \phi + \phi(1-\phi) + \phi(1-\phi)^2 + \phi(1-\phi)^3 + \dots + \phi(1-\phi)^{n-1} \quad (13)$$

and so it also is a geometric progression.

If we multiply both sides of the above by $(1-\phi)$, we get

$$(1-\phi)S_n = \phi(1-\phi) + \phi(1-\phi)^2 + \phi(1-\phi)^3 + \dots + \phi(1-\phi)^n \quad (14)$$

Subtracting (14) from (13) and rearranging, we have

$$S_n = 1 - (1-\phi)^n \quad (15)$$

Given that $0 < \phi < 1$, it must be the case that as $n \rightarrow \infty$, $S_n \rightarrow 1$ and so the long run response of the Outflow Rate to a once and for all increase or decrease in the Inflow Rate is to move in the same direction as the change in the Inflow Rate and (sooner or later) by the same amount as the change in the Inflow Rate. This means that we have here the explanation for the law of flows – the finding that the Inflow Rate and the Outflow Rate are cointegrated with a cointegrating vector of $(1, -1)$. Note also that the sums of weights and related findings apply independent of the time path of the Inflow Rate which we have not had to specify.

4.5 A mathematical approach to the determination of the lag of OUTR behind INR

It is important to study the determinants of the lag of *OUTR* behind *INR* because it is persistent or systematic differences between the Inflow Rate and the Outflow Rate which drive the Unemployment Rate up or down. One way to approach this matter is to pose the traditional question which has been asked of Koyck-type or geometric distributed lag models, namely: What determines the mean⁷ or median lag of the Outflow Rate behind the Inflow Rate? Given that in the preceding section we developed a statement for the value

of the sum of the weights at any date as well as in the 'long-run', it is easiest to derive the expression for the median lag.

The median lag is the value at time T for which the fraction of the total adjustment which has been completed at time T is $\frac{1}{2}$ of the total. So we are looking for an expression which satisfies the condition that

$$\frac{\text{T-period response}}{\text{Long-run response}} = \frac{1}{2}$$

We have already established that the long-run response is unity (see the text under equation (15)) and equation (15) itself tells us the sum of the responses up to any period, eg the T^{th} period. Given all this, the above may be written as

$$\frac{1 - (1 - \phi)^T}{1} = \frac{1}{2}$$

which may be solved for the value of T which satisfies the equality,

$$T = \frac{\ln(1/2)}{\ln(1 - \phi)} \quad (16)$$

This expression tells us that T and ϕ are inversely related, and so we may say that the larger is ϕ , the smaller will be the delay and the smaller is ϕ , the longer will be the delay. Now, the longer the delay the larger (cet par) the gap between the Inflow Rate and the Outflow Rate at any moment in time and so the larger the change in the Unemployment Rate in each period. In other words the value by which the Unemployment Rate rises over the course of a recession will be higher, the lower is ϕ .⁸

4.6 A graphical approach to the determination of the lag of OUTF behind INR

We have already noted that it is persistent or systematic differences between the Inflow Rate and the Outflow Rate which drives the Unemployment Rate up or down. The larger

and the more prolonged the difference, the larger will be the change in the Unemployment Rate. Another way to approach this matter is to ask: What determines the length of the delay between when the Inflow Rate reaches its peak (or trough) and when the Outflow Rate reaches its peak or trough? In Section 4.6 we approached this matter by deriving analytically an expression for the median lag. We saw there (equation (16)) that the delay is inversely related to ϕ . While that result must also apply here there is also a neat graphical explanation which can be given to show that the delay and ϕ are inversely related.

Depicted in the diagram below is a stylized version of the levels of the Inflow Rate and the Outflow Rate around the point where the Unemployment Rate peaks (ie the business cycle trough) we see that the Outflow Rate reaches its peak (at D) after the Inflow Rate reaches its peak (at B) and that at its peak the Outflow Rate has the same value as the Inflow Rate. Given this it is appropriate to approach the determination of the size of the delay of D behind B by taking a linear approximation. At its peak the Inflow Rate has the value indicated by B. When the Inflow Rate is at its peak, the Outflow Rate has the value indicated by A. The length of the delay is given to us by the amount of time it takes the Outflow Rate to rise by the distance $(B - A)$ which is equal to the distance $(D - C)$. Using the linear approximation then we see that the amount of time it takes for the Outflow Rate to rise by the required amount (the time taken being the distance $(C - A)$) depends upon the slope of the line which joins A and D.

[FIGURE 1 NEAR HERE]

To establish the determinants of this slope, we may reason as follows: Given that $OUTR = \phi \times UR$, it must be true that $dOUTR/dt = \phi \times (dUR/dt)$. We also know that $(dUR/dt) = (INR - OUTR)$ and so we may write $(dOUTR/dt) = \phi \times (INR - OUTR)$ which

implies that $((INR - OUTR)/(dOUTR/dt)) = (1/\phi)$. So the time it would take the Outflow Rate to rise by a distance equal to the difference between the value of the Inflow Rate when it is at its peak and the value which the Outflow Rate is at when the Inflow Rate is at its peak, is equal to $(INR - OUTR)/(dOUTR/dt)$ and this ratio is equal to $(1/\phi)$.

All of which is to say that in this case also, the larger is ϕ , the smaller will be the length of the delay between when the Inflow Rate reaches its peak (or trough) and when the Outflow Rate reaches its peak or trough and the smaller is ϕ , the longer will be this delay. Now, the longer the delay the larger (cet. par.) will be the gap between the Inflow Rate and the Outflow Rate at any moment in time and so the larger the change in the Unemployment Rate in each period. In other words the value by which the Unemployment Rate rises over the course of a recession will be higher, the lower is ϕ . This result is in accord with the findings of our investigation of the relationship between the median lag and ϕ in the previous sub-section.

4.7 What determines the peak and trough (and thus the mean) value(s) of UR?

The Unemployment Rate will be at its maximum or minimum over the cycle when (conditional on the Inflow rate) the rate of change in the Unemployment Rate is zero and this will be when the Inflow Rate (which can be either 'high' or 'low') is equal to the Outflow Rate, in other words, when, in long-run equilibrium:

$$INR = OUTR = \phi \times UR$$

and so the value of the Unemployment Rate at its peak and trough will be

$$UR = \frac{INR}{\phi} \tag{17}$$

Given the values of the Inflow Rate when it peaks and troughs, the lower is ϕ , the higher will be both the peak and trough value of the Unemployment Rate, in other words, the lower is ϕ , the higher the average Unemployment Rate will be. Essentially this is because for a given inflow, the lower is the probability of exiting unemployment per member of the pool, the larger the pool of Unemployment will have to be in order to generate enough outflow to match inflow.

Notice that in deriving our results we have not had to say anything about the behavior over time of the Inflow Rate, only that it is exogenous. We will now look at two sets of simulation results. We begin with the case where ϕ is constant and the Inflow Rate behaves in a 'rectangular fashion'. We then look at the more realistic case where ϕ varies inversely with the Unemployment Rate.

5. Simulation results of the model with ϕ constant and where *INR* follows a rectangular time path jumping from booms to recessions and back again

Here, we will continue to treat the Inflow Rate as exogenous, ϕ as a constant and both the Outflow Rate and the Unemployment Rate as endogenous. The aim is to mimic our analytical results with numerical simulation and in particular, to see the effects of different values of ϕ upon the evolution of the Outflow Rate and the Unemployment Rate.

Now, imagine that the Inflow Rate is exogenous and behaves in a rectangular fashion such that it spends some time in a state of boom then jumps to a state of recession and spends some time in that state before switching back to the boom state for a time etc etc.

Our equation for the Inflow Rate will be:

$$INR_t = a + b \times D_t \quad (4')$$

where D is a dummy which has a value of 0 when the economy is in a boom and a value of 1 when the economy is in a recessions

The time path for the Inflow Rate used in the simulation experiments is set out in Figure 2 below. In our simulation experiments a is set equal to 0.021 and b is set equal to 0.008. The reason for choosing these values is that observing the recession of the early 80's in Australia we find a mean value for the Inflow Rate of 0.025, a minimum of 0.021 and a high of 0.029. We have arbitrarily set the period from peak to peak or trough to trough at 48 months (4 years).

[FIGURE 2 NEAR HERE]

Since ϕ is constant and we are really only interested in the time paths of the Inflow Rate, the Outflow Rate and the Unemployment Rate we can work with three equations in the model (remember that stocks such as the Unemployment Rate are measured at the start of a period whereas flows such as the Inflow Rate and the Outflow Rate are measured over the period). The equations in the model will be:

$$INR_t = a + b \times D_t \quad (4')$$

$$UR_t = INR_{t-1} + (1 - \phi)UR_{t-1} \quad (3')$$

$$OUTR_t = \phi \times UR_t \quad (1')$$

We have simulated three versions of the model using EViews V4.0. The components of the models are identical except for the value of ϕ . In Table 1 below we set out the value of ϕ used in each simulation and various labour market outcomes associated

with that model. In each case (allowing for the discrete time nature of the numerical simulation) the predictions of the analytical model examined above hold.

[TABLE 1 NEAR HERE]

Notice that the wavelength or period from peak to peak or trough to trough in the Outflow Rate and in the Unemployment Rate are not affected by ϕ (it depends solely upon the behavior of the Inflow Rate). Notice also that while the amplitude (range) of the Outflow Rate is exactly equal to that of Inflow Rate no matter what the value of ϕ (the law of flows is at work again), both the mean and the amplitude of the Unemployment Rate is dependent on ϕ . The higher is ϕ , the lower is both the mean and the amplitude of the Unemployment Rate. All time paths are periodic: the Inflow Rate, the Unemployment Rate and the Outflow Rate have the same wavelength (time from peak to peak or trough to trough) in all cases. We also see that the Unemployment Rate has a greater amplitude than both the Inflow Rate and the Outflow Rate, and the amplitude of the Unemployment Rate is inversely related to the size of ϕ . Not surprisingly, given what has just been said, the lag of the Outflow Rate behind the Inflow Rate is inversely related to the size of ϕ . It is also clear that, regardless of the value of ϕ , the long-run effect of a once and for all change in the Inflow Rate is to change the Outflow Rate by an equal amount.

To illustrate what is happening in the simulations the time path of the three variables for the case where ϕ is 0.25 is given in Figure 3 below. This is typical of all of the simulations (they differ only in the length of the delay of the *OUTR* path behind the *INR* path).

[FIGURE 3 NEAR HERE]

6. Case 2: A model where ϕ is endogenous and varies with UR and where INR follows a rectangular time path jumping from booms to recessions and back again.

In this section we will treat the Inflow Rate as exogenous and the Outflow Rate, the Unemployment Rate and ϕ as endogenous. We know from empirical work (eg Dixon, Freebairn & Lim (2002)) that ϕ is endogenous and varies inversely with the Unemployment Rate. It would also appear that it is reasonable as a first approximation to regard the elasticity of ϕ with respect to the Unemployment Rate as constant over time (ibid). Accordingly we will assume that

$$\phi_t = \alpha UR_t^\beta \quad (18)$$

where β is negative and appears to be in the order of -0.5 . Combining the above with equation (1) gives an expression for the Outflow Rate:

$$OUTR_t = \alpha \times UR_t^{(1+\beta)} \quad (19)$$

Again, our system will consist of three equations, which in this case will be:

$$INR_t = a + b \times D_t \quad (4')$$

$$UR_t = INR_{t-1} + UR_{t-1} - OUTR_{t-1} = INR_{t-1} + UR_{t-1} (1 - \alpha \times UR_{t-1}^\beta) \quad (3'')$$

$$OUTR_t = \alpha \times UR_t^{(1+\beta)} \quad (19)$$

We will, with one exception, in this case eschew analytical solutions and move directly to a numerical solution. The exception concerns the determinants of the size of the Unemployment Rate at Business Cycle peaks and Troughs.

6.1 What determines the peak and trough (and thus the mean) value(s) of UR ?

We know that the Unemployment Rate will be constant at its cyclical peaks and troughs and that this will occur when the Inflow Rate is equal to the Outflow Rate. This means that in this model the Unemployment Rate will be at its maximum or minimum when

$$INR = OTR = \alpha \times UR^{1+\beta} \quad (20)$$

and so the value of the Unemployment Rate at its peak and trough will be

$$UR = \left(\frac{INR}{\alpha} \right)^{\frac{1}{1+\beta}} \quad (21)$$

Given the value of β and the time path of the Inflow Rate, the lower is α , the higher will be both peak and trough the Unemployment Rate, in other words, the lower is α , the higher the average unemployment rate will be.

We turn now to look at the role of β . Obviously if $\beta = 0$ we get the same answer as for Case 1 examined above. What can we say will happen when β does not equal zero? ⁹ Since β lies between zero and minus one, $(1/(1+\beta))$ must be positive and greater than 1 and will become larger the nearer is β to -1. The value of $[(INR/\alpha)^{(1/(1+\beta))}]$ will be smaller, the larger is the size of the exponent $((1/(1+\beta)))$.¹¹ All of which is to say, the nearer is β to -1, the smaller (ceteris paribus) will be both peak and trough the Unemployment Rate, in other words, nearer is β to -1, the lower (cet. par.) the average Unemployment Rate will be. (This makes intuitive sense as the nearer is β to -1, the larger is outflow being stemmed or reduced as the Unemployment Rate rises, this will contribute to a larger pool of Unemployment at any moment of time given the Inflow Rate.)

7. Simulation results of the model with $\phi_t = \alpha \times UR_t^\beta$ and where *INR* follows a rectangular time path jumping from booms to recessions and back again

Earlier I have explained the reasoning behind the choice of the numerical values which generate the evolution of the Inflow Rate over time. Here it is necessary to discuss the calibration of the equation for the Outflow Rate (equation (19) above).

Log linear regression analysis of time series data for Gross Flows and the Unemployment Rate for Persons in Australia, suggests that a reasonable model with $\alpha = 0.11$ and $\beta = -0.45$ is appropriate. Another way to calibrate the model would be to establish values of α and β which satisfy the law of flows, in other words, values of α and β which satisfy equation (20) above. Now some stylized facts would be that cyclical peak values for *INR* and *UR* are of the order of 0.029 and 0.095 respectively. Using equation (20) we can establish some pairs of values for α and β which, when combined with our suggested values for *INR* and *UR*, satisfy the equality in equation (20). Such pairs would be: $\beta = -0.33, \alpha = 0.14$; $\beta = -0.45, \alpha = 0.11$; ¹⁰ and, $\beta = -0.50, \alpha = 0.09$.

We have simulated three models containing equations (19), (3'') and (4') above. The models are identical except for the values of α and β . Table 2 reports the values of α and β used in each simulation and various labour market outcomes associated with each model. We have included a fourth case with $\beta = -0.45, \alpha = 0.14$ so that, by comparing the results of this run with those where $\beta = -0.33, \alpha = 0.14$, we can see the effects of a change in β , holding other things (the equation for *INR* and the value of α) equal.

[TABLE 2 NEAR HERE]

Notice that the wavelength or period is not affected by α or β (it depends solely upon the behavior of the Inflow Rate) but that the amplitude and mean of the

Unemployment Rate are dependent on α and β . The higher is α , given β and the Inflow Rate, the lower is both the mean and the amplitude of the Unemployment Rate. The nearer is β to -1, given α and the Inflow Rate, the lower is both the mean and the amplitude of the Unemployment Rate. Related to this is the finding that the higher is α , given β and the Inflow Rate, the shorter is the lag of the Outflow Rate behind the Inflow Rate and the nearer is β to -1, given α and the Inflow Rate, the shorter is the lag of the Outflow Rate behind the Inflow Rate. It is also clear that, regardless of the values of α and β , the long-run effect of a once and for all change in the Inflow Rate is to change the Outflow Rate by an equal amount.

Again, all time paths are periodic with the Inflow Rate, the Unemployment Rate and the Outflow Rate have the same wavelength in all cases. To illustrate what is happening the time path of the three variables for the case where α is 0.11 and β is -0.45 is given in Figure 4 below. The time paths displayed there are typical of all of the simulations.

[FIGURE 4 NEAR HERE]

8. Conclusions

The paper has sought to 'explain' certain stylised facts in relation to worker flows into and out of Unemployment. In particular it has sought to show why the empirical evidence shows that the long-run effect of a change in the Inflow Rate upon the Outflow Rate is to change the Outflow Rate by exactly the same amount as the Inflow Rate has changed. In addition it has sought to identify the 'proximate' determinants of the amplitude and the frequency of fluctuations in the Unemployment Rate over the course of the business cycle.

It was found that the transition probability (and/or the parameters that together determine the transition probability) is a determinant of the amplitude, and thus the mean value of, the Unemployment Rate but not the frequency of fluctuations in the Unemployment Rate. However, it is found that the Inflow Rate is a determinant of both the amplitude and the frequency of fluctuations in the Unemployment Rate. We were also able to explain why it is that researchers find that the Inflow Rate and the Outflow Rate are cointegrated with a cointegrating vector of (1, -1) as it has been shown that, regardless of the value of the transition probability or the time path for the Inflow Rate, the long-run effect of a once and for all change in the Inflow Rate is always to change the Outflow Rate by an equal amount.

Whereas most of the research in this area has focused on the determinants of outflow from unemployment (I am thinking in particular of models of matching and search) on the basis of this study, we must conclude that the most important task is to focus on modeling the determinants of the Inflow Rate. It is virtually self evident that this will, of necessity, involve a restatement of Keynes's theory of employment in a flows context.

Bibliography

- Balakrishman, R and Michelacci, C. 2001. Unemployment Dynamics Across OECD Countries, *European Economic Review*, vol. 45, 135-65
- Burda, M. and Wyplosz, C. 1994. Gross Worker and Job Flows in Europe, *European Economic Review*, vol. 38, pp 1287-315
- Burgess, S. 1994. Matching Models and Labour Market Flows, *European Economic Review*, vol. 38, 809-16
- Dixon, R. 2001. Australian Labour Force Data: How Representative is the 'Population Represented by the Matched Sample'? *Economic and Labour Relations Review*, vol. 12, 303-30
- Dixon, R., Freebairn, J. and Lim, G. 2002. Why are recessions as deep as they are? The behaviour over time of the outflow from unemployment: A new perspective, mimeo, Department of Economics at the University of Melbourne.
- Griliches, Z. 1967. Distributed Lags: A Survey, *Econometrica*, vol. 35, 16-49
- Jones, S. and Riddell, W. 1998. Gross Flows of Labour in Canada and the United States, *Canadian Public Policy*, vol. 24 (February Supplement), S103-20

Appendix. Case 3: A Model where ϕ is endogenous and varies with UR and where INR follows sine wave

We will again treat the Inflow Rate as exogenous and the Outflow Rate, the Unemployment Rate and ϕ as endogenous. Our expression for the Outflow Rate is again,

$$OR_t = \alpha \times UR_t^{(1+\beta)} \quad (19)$$

A slightly more realistic example would have the Inflow Rate moving in a wave-like motion rather than jumping from recession state to boom state. The obvious model to use for the Inflow Rate is that of a sine-wave. In this case the model remains identical to that described above except that our equation for the Inflow Rate is:

$$INR_t = a + b \times \sin(c \times time) \quad (4'')$$

where a is the mean value of the Inflow Rate over time, b determines the amplitude as it is the size of the maximum (and minimum) displacement above or below a (so the peak and trough values of the Inflow Rate will be $(a + b)$ and $(a - b)$ respectively; and c – the angular velocity – determines the ‘wavelength’ or ‘period’, i.e. the time from peak to peak (the period being $(2\pi/c)$). The function is periodic and continuous. The equation for the Inflow Rate used in the simulation experiments and its time path are set out in the chart below.

[FIGURE 5 NEAR HERE]

The reason for choosing these parameter values is that observing the recession of the early 80's we find a mean value for the Inflow Rate of 0.025, a minimum of 0.021 and a high of 0.029. The value of c (0.1309) has been set such that the period from peak to peak or trough to trough is 48 months (4 years).

Again, our system will consist of three equations, which in this case will be:

$$INR_t = a + b \times \sin(c \times time) \quad (4'')$$

$$UR_t = INR_{t-1} + UR_{t-1} - OUTR_{t-1} = INR_{t-1} + UR_{t-1} (1 - \alpha \times UR_{t-1}^\beta) \quad (3''')$$

$$OUTR_t = \alpha \times UR_t^{(1+\beta)} \quad (19)$$

We have simulated 3 models containing equations (19), (4'') and (3''') above. The models are identical except for the values of α and β .

[TABLE 3 NEAR HERE]

Notice that the wavelength or period is not affected by α or β (it depends solely upon the behavior of the Inflow Rate) but, as we saw with Case 2 reported in the main text of the paper, that the amplitude and mean of the Unemployment Rate is dependent on α and β . The higher is α , given β and the Inflow Rate, the lower is both the mean and the amplitude of the Unemployment Rate. The nearer is β to -1, given α and the Inflow Rate, the lower is both the mean and the amplitude of the Unemployment Rate. Related to this is the finding that the higher is α , given β and the Inflow Rate, the shorter is the lag of the Outflow Rate behind the Inflow Rate and the nearer is β to -1, given α and the Inflow Rate, the shorter is the lag of the Outflow Rate behind the Inflow Rate.

All time paths are sine waves; the Inflow Rate, the Unemployment Rate and the Outflow Rate have the same wavelength in all cases. To illustrate what is happening the time path of the three variables for the case where α is 0.11 and β is -0.45 is given in Figure 6 below. This is typical of all of the simulations of this version of the model.

[FIGURE 6 NEAR HERE]

NOTES

1. That the Inflow Rate, the Outflow Rate and the Unemployment Rate move up and down together is the case for Australian flows data (Dixon, Freebairn & Lim, 2002), and also for flows data from France, Germany, Spain and the USA (Burda & Wyplosz, 1994, p 1289f; Balakrishman & Michelacci, 2001, p 145), Canada (Jones & Riddell, 1998, p S106f) and the UK (Burgess, 1994, p 811; Balakrishman & Michelacci, 2001, p 145).
2. The term 'counter-cyclical' is often used to describe this when the cycle in Gross Domestic Product is being used as the reference point. Our reference point is the cycle in the Unemployment Rate.
3. An intuitive explanation for this finding is that while the probability that an individual unemployed person will find employment in any period falls in recessions and rises in recoveries, the total number unemployed will rise in recessions and fall in recoveries. This second effect is dominant with the result that the number of unemployed persons finding jobs rises in recessions and falls in recoveries.
4. The reason for expressing Inflow and Outflow relative to the size of the Labour Force is that movements in the Unemployment Rate up or down are driven by the size of the Inflow into Unemployment expressed as a proportion of the Labour Force relative to the size of the Outflow from Unemployment expressed as a proportion of the Labour Force. See Dixon (2002) for a formal proof and a statement of the assumptions under which this holds.
5. While Inflow and Outflow rates are measured during a period, the Unemployment Rate will be measured at the beginning of the period.
6. For this reason, while some may write: $OUTR_t = \phi \times UR_{t-1}$, we prefer to work with $OUTR_t = \phi \times UR_t$.
7. Griliches (1967, p 19) has shown that the mean lag in a model like this is $(1-\phi)/\phi$. Which is to say that the larger is ϕ , the smaller will be the delay and the smaller is ϕ , the longer will be the delay.
8. This result is consistent with that arrived at above when we saw that the long run response of the Unemployment Rate to a change in the Inflow rate is to not only move in the same direction but (sooner or later) by an amount which is determined by the inverse of ϕ .
9. In practice β lies between zero and minus one so we will restrict our discussion to that range.
10. That the pairs of values consistent with our 'equilibrium condition' include the pair given by regression analysis of actual data is at one with the 'law of flows' and suggests that our set of stylised facts are appropriate.
11. Provided that (IR/α) is less than 1.

FIGURES

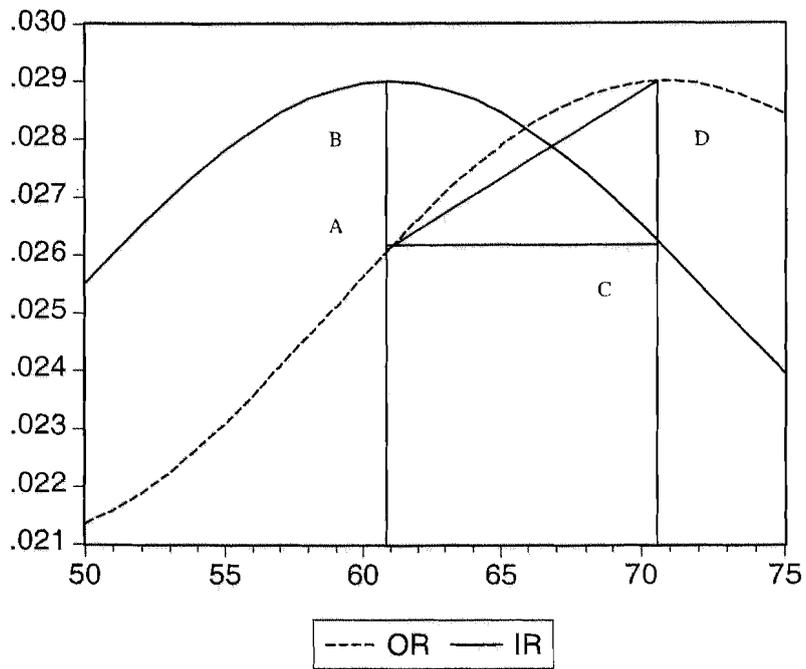


Fig. 1. *Stylised example of Inflow and Outflow Rates near their peaks*

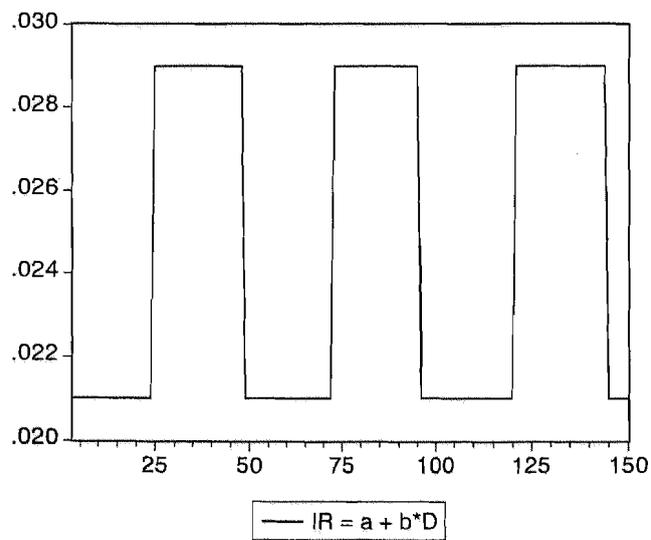


Fig. 2. *The time path of the inflow rate in simulations for case 1 and 2*

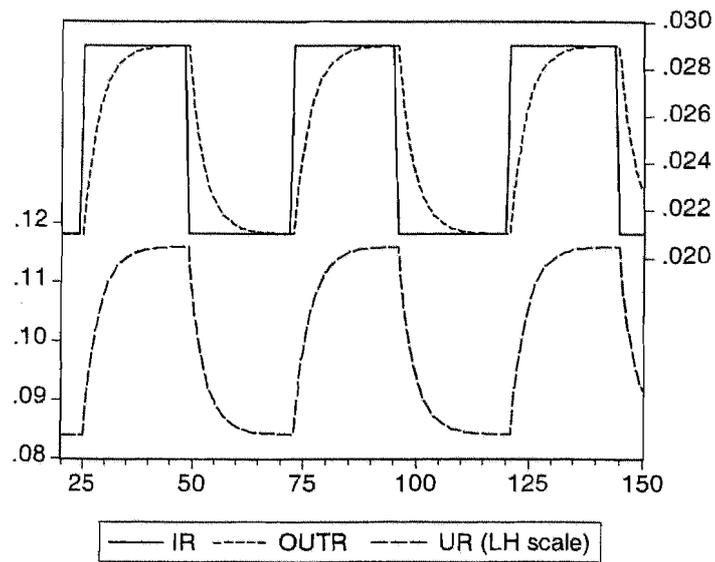


Fig. 3. Time paths of *INR*, *OUTR* and *UR* when $\phi = 0.25$

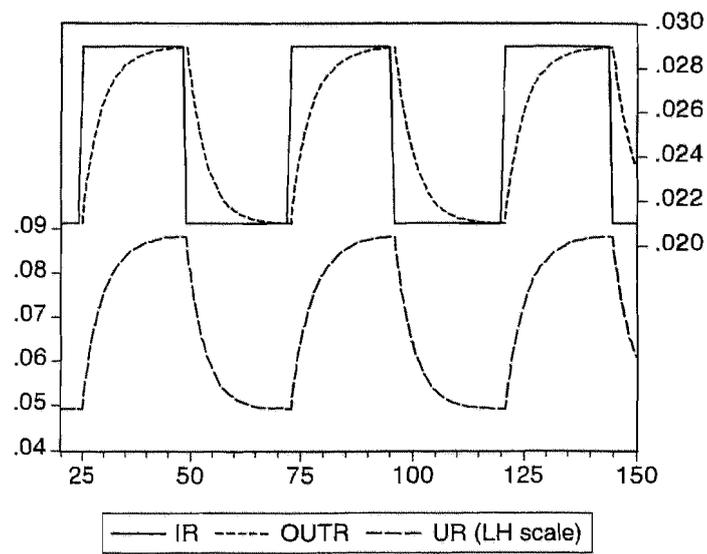


Fig. 4. Time paths of *INR*, *OUTR* and *UR* when $\alpha = 0.11$ and $\beta = -0.45$

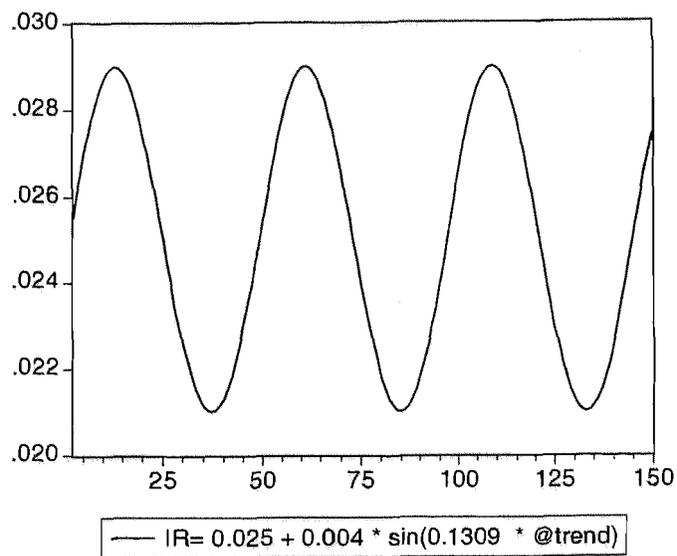


Fig. 5. Time path of IR in simulations for case 3

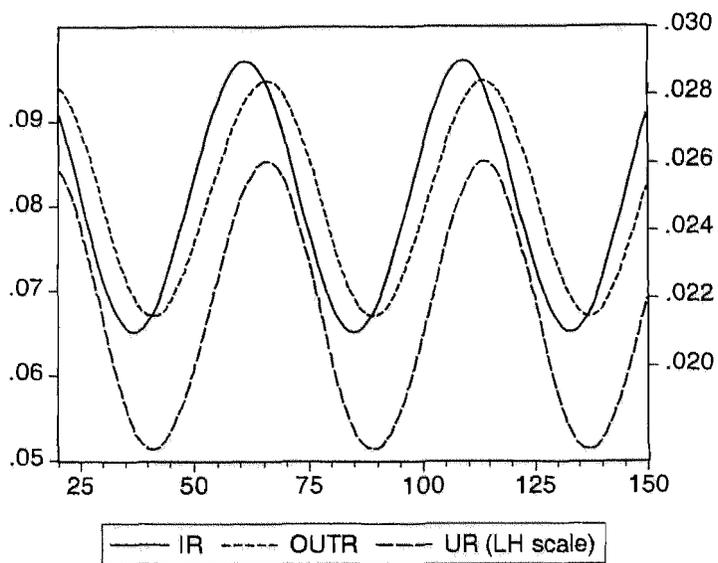


Fig. 6. Time paths of INR, OTR and UR when $\alpha = 0.11$ and $\beta = -0.45$

TABLES

Table 1. *Simulation output for the model with ϕ exogenous*

Value of ϕ	<i>INR</i> min. & max. and range over the cycle	<i>OUTR</i> min. & max. and range over the cycle	<i>UR</i> min. & max., mean and range over the cycle	Time from peak to peak in <i>INR</i> , <i>OUTR</i> & <i>UR</i> (months)	Time needed for <i>OUTR</i> to reach $\frac{1}{2}$ its long-run value (months)*
0.25	0.021, 0.029 range = 0.008	0.021, 0.029 range = 0.008	0.084, 0.116 mean = 1.000 range = 0.032	48, 48, 48	2.5
0.50	0.021, 0.029 range = 0.008	0.021, 0.029 range = 0.008	0.042, 0.058 mean = 0.500 range = 0.016	48, 48, 48	1
0.75	0.021, 0.029 range = 0.008	0.021, 0.029 range = 0.008	0.028, 0.039 mean = 0.335 range = 0.011	48, 48, 48	0.5

* The simulations are using discrete time and for that reason the length of time here will not be the same as the continuous time solution. Where the figure fell between two discrete time periods the mean number of periods has been reported. The analytic solution for $\phi = 0.25$ is 2.4 while for $\phi = 0.75$ it is 0.50 months.

Table 2. Simulation output for the model with ϕ endogenous

Value of α and β	<i>INR</i> min. & max. over the cycle	<i>OUTR</i> min. & max. over the cycle	<i>UR</i> min. & max. mean and amplitude over the cycle	Time from peak to peak in <i>INR</i> , <i>OUTR</i> & <i>UR</i> (months)	Time needed for <i>OUTR</i> to reach $\frac{1}{2}$ its long-run value (months)*
$\alpha = 0.14$ $\beta = -0.33$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.059, 0.095 mean = 0.077 amp = 0.018	48, 48, 48	2.5
$\alpha = 0.14$ $\beta = -0.45$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.032, 0.057 mean = 0.045 amp = 0.013	48, 48, 48	1.5
$\alpha = 0.11$ $\beta = -0.45$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.049, 0.088 mean = 0.069 amp = 0.020	48, 48, 48	3.0
$\alpha = 0.09$ $\beta = -0.50$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.055, 0.102 mean = 0.079 amp = 0.022	48, 48, 48	4.0

* The simulations are using discrete time and for that reason the number of periods here will not be the same as the continuous time solution. Where the figure fell between two discrete time periods the mean number of periods has been reported.

Table 3. Simulation output for the model with ϕ endogenous and with INR following a sine wave

Value of α and β	INR min. & max. over the cycle	OUTR min. & max. over the cycle	UR min. & max. mean and amplitude over the cycle	Time from peak to peak in INR, OUTR & UR (months)	Lag of OUTR behind INR at peaks (months)*
$\alpha = 0.14$ $\beta = -0.33$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.060, 0.094 mean = 0.077 amp = 0.017	48, 48, 48	4.5
$\alpha = 0.14$ $\beta = -0.45$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.032, 0.056 mean = 0.044 amp = 0.012	48, 48, 48	3.0
$\alpha = 0.11$ $\beta = -0.45$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.051, 0.086 mean = 0.069 amp = 0.017	48, 48, 48	4.5
$\alpha = 0.09$ $\beta = -0.50$	0.021, 0.029 mean = 0.025 amp = 0.004	0.021, 0.029 mean = 0.025 amp = 0.004	0.057, 0.099 mean = 0.078 amp = 0.021	48, 48, 48	5.5

* The simulations are using discrete time and for that reason the number of periods here will not be the same as the continuous time solution. Where the figure fell between two discrete time periods the mean number of periods has been reported.

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