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TESTING FOR A LEVEL EFFECT IN SHORT-TERM INTEREST RATES

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Testing for a Level Effect in Short-Term Interest Rates*

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Abstract

There is an extensive theoretical and empirical literature discussing the link between short-term interest rate volatility and interest rate levels. We present an LM based test for the presence of a level effect which is robust to the presence of unidentified nuisance parameter under the null of no level effect. We provide extensive Monte-Carlo evidence on the performance of this test under various DGPs. When applied to data on the 3-month US Treasury Bills rate, the test reports significant evidence of a level effect.

Keywords: Level Effects; LM Tests; Davies Problem
J.E.L. Reference Numbers: C12; G12; E44

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1 Introduction

Single factor models of the term structure of interest rates imply that all rates move in the same direction over any short time interval. This theoretical approach to modelling the term structure of interest rates assumes a plausible process for the short term interest rate that governs the dynamics of the entire term structure. In the empirical literature on short term interest rates there is strong evidence that the volatility of short-term interest rates is positively correlated with the level of the short term rate of interest. This tendency for interest rates to be more volatile as short term rates rise is commonly referred to as the level effect. The parameterisation of recent theoretical and empirical models of the term structure of interest rate reflect the importance of the level effect in constructing an adequate conditional characterisation of the dynamics of the short term rate of interest. Dothan (1978), Brennan and Schwartz (1977, 1980) add Cox, Ingersoll and Ross (1985), *inter alia*, all specify the volatility of short-term interest rates as a function of short-rate levels. Empirical models of the short rate estimated by Chan, Karolyi, Longstaff and Sanders (1992), Brenner, Harjes and Kroner (1996), Dewachter (1996), and Koedijk et al. (1997), *inter alia* also reported a strongly significant dependence in the short rate volatility on short-rate levels.

Despite the apparent importance of the level effect, as yet no method has been developed to specifically detect the dependence of short rate volatility on short-rate levels. The main difficulty in testing for the null of no levels effect arises from the potential presence of an unidentified nuisance parameter under the null hypothesis. As a result, the current practice solely relies on post-modelling misspecification tests to evaluate the adequacy of the model. Brenner *et al.* (1996), for example, use the conditional moment test of Woodridge (1990) to test for model misspecification associated with a level effect. Koedijk *et al.* (1997) employ a Lagrange multiplier misspecification test from Wooldridge (1994) and Bollerslev and Wooldridge (1992) that is robust to deviations from normality in the data. In testing for the presence of a level effect these authors all assume a specific value for the unidentified exponent parameter associated with the lagged interest rate level. However, given that the true value of the exponent parameter is not known *a priori*, this approach may lead to invalid inference when the true value of the exponent parameter differs from the assumed value.

This paper presents a test for the presence of a level effect that is robust to the presence of an unidentified nuisance parameter under the null hypothesis. The test is based on the Lagrange multiplier principle and follows the approach for testing for the null of linearity against the alternative of a smooth transition autoregressive (STAR) model discussed in Luukonnen *et al.* (1988). Under the null hypothesis of no level effect, the problems associated with an unidentified nuisance parameter are overcome by taking a Taylor series approximation around the nuisance parameter. The test, therefore, has the advantage of not requiring the practitioner to assume any known value for the actual value of the exponent parameter of the interest rate level. In addition, the level effect test statistic is shown to be approximately distributed as a Chi-square variate with one or two degrees of freedom under the null hypothesis of no level effect.

A series of Monte Carlo experiments suggest that the test is largely free from size distortions and has desirable power for the sample sizes typically used in studies of short-term interest rate dynamics. The test is also relatively robust to variance asymmetry, the degree of persistence in the conditional variance, the presence of mean reversion in the short rate and to multiplicative levels effects.

The organization of this paper is as follows. Section 2 discusses the formulation of the levels effect in short-term interest rates and the develops the level effect diagnostic test. Section 3 describes the design and conduct of the Monte Carlo experiments to study the small and large sample properties of the level effect test. This section also discusses sensitivity analyses for neglected asymmetry in volatility, different degrees of persistence in the level effect and the conditional variance, and mean reversion in the short-rate process. In the fourth section the level effect test is applied to the U.S. three-month Treasury bill data sampled at a weekly frequency and the results are used to aid the development of empirical models of short-term interest rate dynamics. Section 5 summarizes and concludes.

2 Short Rate Models and the Level Effect Test

2.1 Short Rate Models with Level Effects

Chan, Karolyi, Longstaff and Sanders (1992) (CKLS, hereafter) propose the general non-linear process for short-term interest rates, $\{r_t, t \geq 0\}$, written as

$$dr = (\mu + \lambda r)\, dt + \phi r^{\delta} dW. \tag{1}$$

Here r represents the level of the short-term interest rate, W is a Brownian motion and μ, λ and δ are parameters. The drift component of short-term interest rates is captured by $\mu + \lambda r$ while the variance of unexpected changes in interest rates equals $\phi^2 r^{2\delta}$. The parameter ϕ is a scale factor and δ controls the degree to which the interest rate level influences the volatility of short-term interest rates. By placing restrictions on δ, the Chan *et al.* (1992) model nests many of the existing interest rate models. For example, when $\delta = 0$ then (1) reduces to the Vasicek (1977) model, while $\delta = 1/2$ yields the Cox, Ingersoll and Ross (1985) model, see Chan *et al.* (1992) *inter alia* for further details.

Brenner, Harjes and Kroner (1996) (BHK, hereafter) argue that by allowing ϕ^2 to be a time varying function of the information set, Ω, it gives rise to a superior conditional characterisation of short term interest rate changes. CKLS and BHK, *inter alia,* consider the Euler-Maruyama discrete time approximation to (1) written as

$$\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_t. \tag{2}$$

Here Ω_{t-1} represents the information set available at time $t-1$ and $E\left(\varepsilon_t|\Omega_{t-1}\right) = 0$. Letting h_t represent the conditional variance of the short-term interest rate then $E\left(\varepsilon_t^2|\Omega_{t-1}\right) \equiv h_t = \phi^2 r_{t-1}^{2\delta}$. The sole source of conditional heteroscedasticity in (2) is through the level of the interest rate and thus excludes the information arrival process.

One common approach to capturing the effect of news is the GARCH(1,1) model

$$h_t = \alpha_0 + \beta h_{t-1} + \alpha_1 \varepsilon_{t-1}^2. \tag{3}$$

The innovation ε_t represents a change in the information set from time $t-1$ to t and can be treated as a collective measure of news. In (3) only the magnitude of the innovation is important in determining h_t. BHK extend (2) to allow for volatility clustering caused by information arrival using

$$\begin{aligned} \Delta r_t &= \mu + \lambda r_{t-1} + \varepsilon_t. \\ E\left(\varepsilon_t|\Omega_{t-1}\right) &= 0,\ E\left(\varepsilon_t^2|\Omega_{t-1}\right) \equiv h_t = \phi_t^2 r_{t-1}^{2\delta} \\ \phi_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \phi_{t-1}^2 \end{aligned} \tag{4}$$

Equation (4) defines the multiplicative level effect model given that the conditional volatility of the short-rate chage is multiplicatively dependent on the short rate levels. In high information periods, when the magnitude of ε_t is largest then the sensitivity of volatility to the level of short term interest rates is highest. Under the restriction $\alpha_1 = \beta = 0,$ (4) collapses to (2) and volatility depends on levels alone. Furthermore when $\delta = 0$ then there is no levels effect. Testing the null of no level effect (i.e. $\delta = 0$) does not pose any problem in the multiplicative level effect model.

An alternative approach to modelling volatility clustering and levels effects is the additive level effect model

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}. \tag{5}$$

Under the null hypothesis $\alpha_1 = \beta = 0$, volatility depends on interest rate levels alone. If $b = 0$ then there is no levels effect, however under this null the parameter δ is unidentified and so tests of the null hypothesis $H_0 : b = 0$ will have a non-standard distribution, see Davies (1987) for further details. To accommodate the Davies problem, other authors test the null $H_0 : b = 0$ assuming δ is known. For instance Longstaff and Schwartz (1992) and BHK, *inter alia*, assume $\delta = 1.0$ while Bekaert, Hodrick and Marshall (1997) assume $\delta = 0.5$.

2.2 The Level Effect Test

To develop a test for the null of no level effect we consider a short-rate model that is free from mean reversion:

$$\Delta r_t = \varepsilon_t$$

$$\varepsilon_t | \Omega_{t-1} \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}. \tag{6}$$

where $\beta + \alpha_1 < 1$, and $\beta, \alpha_i, b > 0$ for $i = 0$ and 1. There is little empirical evidence of mean reversion in short term interests rates, see Rose (1988), Shea (1992), Brenner *et al.* (1996), and Rodrigues and Rubia(2004), *inter alia*. We relax the assumption of no mean reversion in our Monte-Carlo experiments below. The null hypothesis is that there is no level effect. This implies that the short rate follows a GARCH(1,1) process. Our alternative hypothesis is of a GARCH(1,1) with a level effect. These hypotheses may be formulated as follows

$$H_0 \ : \ b = 0$$
$$H_1 \ : \ b \neq 0.$$

If $b = 0$ then there is no levels effect, however under this null the parameter δ is unidentified and so tests of the null hypothesis $H_0 : b = 0$ will have a non-standard distribution, see Davies (1987) for further details. To address this problem we linearize (6). Sequentially substituting for h_{t-1} in (6) and taking a first order Taylor series expansion about δ^* yields

$$h_t = \sum_{i=1}^{t-1} \beta^{i-1}\alpha_o + \sum_{i=1}^{t-1} \beta^{i-1}\alpha_1\varepsilon_{t-i}^2 + \beta^{t-1}h_1 + \sum_{i=1}^{t-1} \beta^{i-1} b r_{t-i}^{\delta^*}(1 - \delta^* \ln r_{t-i}) + \sum_{i=1}^{t-1} \beta^{i-1}\gamma r_{t-i}^{\delta^*} \ln r_{t-i}. \quad (7)$$

The null hypothesis of no level effect is reformulated as $H_0 : b = \gamma = 0$ where $\gamma = b\delta$. Assuming that the residual ε_t is conditionally normally distributed, a Lagrange Multiplier test statistic under the null hypothesis is

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]'\right\}^{-1}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\right\} \quad (8)$$

where

$$\frac{\partial h_t}{\partial \varpi'} = \left[\sum_{i=1}^{t-1}\hat{\beta}^{i-1}, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\varepsilon_{t-i}^2, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\tilde{h}_{t-i}, \sum_{i=1}^{t-1}\hat{\beta}^{i-1} r_{t-i}^{\delta^*}(1 - \delta^* \ln r_{t-i}), \sum_{i=1}^{t-1}\hat{\beta}^{i-1} r_{t-i}^{\delta^*} \ln r_{t-i}\right], \quad (9)$$

$\tilde{h}_t$ is the conditional variance under the null of GARCH(1,1), ϖ' is the vector of parameters $(\alpha_0, \alpha_1, \beta, b, \gamma)$, and $\hat{\beta}$ is the estimated parameter β in the GARCH(1,1) model. The LM test statistic (8) is asymptotically equivalent to $T \cdot R^2$ from the outer product auxiliary regression of

$$\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]$$

on X_t where

$$X_t' = \frac{1}{\tilde{h}_t}\begin{bmatrix} \sum_{i=1}^{t-1}\hat{\beta}^{i-1} \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\varepsilon_{t-i}^2 \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\tilde{h}_{t-i} \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1} r_{t-i}^{\delta^*}(1 - \delta^* \ln r_{t-i}) \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1} r_{t-i}^{\delta^*} \ln r_{t-i} \end{bmatrix}. \quad (10)$$

Here T is the sample size and R^2 is the coefficient of determination from the regression (10). We refer to this test statistic as $LM(\delta^*)$ since it is computed using a set of values derived from theoretical short rate models for $\delta^* = \{0, 0.5, 1, 1.5\}$[1]. The test is approximately distributed as a Chi-square with two degrees of freedom for $\delta^* = \{0.5, 1, 1.5\}$ although we provide simulated critical values to allow for the approximation error [2].

Preliminary Monte Carlo experiments reveal that the empirical size of $LM(\delta^*)$ is significantly larger than the nominal size. This size distortion may result from a violation of the usual orthogonality conditions. The normalized residuals, $\tilde{v}_t \equiv \varepsilon_t / \sqrt{h_t}$, should be orthogonal to

$$\frac{1}{\tilde{h}_t}\left[\sum_{i=1}^{t-1}\hat{\beta}^{i-1}, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\varepsilon_{t-i}^2, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\tilde{h}_{t-i}\right], \tag{11}$$

yet in practice exact orthogonality may not always hold because of the highly nonlinear structure of the model. In the event that these orthogonality conditions fail to hold, the empirical size of the test statistic may be distorted (see Engle and Ng , 1993, pp.1759). To correct for the apparent upward bias in the empirical size of the test statistic, we employ the method introduced by Eitrheim and Teräsvirta (1996) and Engle and Ng (1993, pp. 1759). The procedure involves replacing $\tilde{v}_t$ with a variable that is guaranteed to be orthogonal to (11). This is done by:

1. Regressing

$$\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]$$

on (11). The residuals from this regression, $\{\tilde{e}_t\}_{t=1}^T$, will, by construction, be orthogonal to (11).

2. Then regress $\tilde{e}_t$ on X_t specified in equation (10) and compute the regression R^2. The test statistic which is labelled $LM_1(\delta^*)$ is set equal to $T \cdot R^2$ and is approximately distributed as a Chi-square with two degrees of freedom.

[1] The choice of $\delta^* = \{0.5, 1.0, 1.5\}$ arises from theoretical short rate models. When $\delta^* = 0.5$, the conditional variance is similar to the square root (SR) process that appears in Cox, Ingersoll and Ross (CIR) (1985) single-factor general equilibrium term structure model. For $\delta^* = 1.0$, the model conditional variance then resembles that of Dothan's (1978) model, the Geometric Brownian Motion process of Black and Scholes (1973), and Brennan and Schwartz (1980) model. While for $\delta^* = 1.5$, the conditional variance is equivalent to the model that CIR (1980) used to study variable rate (VR) securities. We also consider $\delta^* = 0$ because it simplifies $\frac{\partial h_t}{\partial \varpi'}$ in equation (9) to $\left[\sum_{i=1}^{t-1}\hat{\beta}^{i-1}, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\varepsilon_{t-i}^2, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\tilde{h}_{t-i}, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\ln r_{t-i}\right]$.

[2] In the case of $\delta^* = 0$, the fourth regressor $\sum_{i=1}^{t-1}\hat{\beta}^{i-1} r_{t-i}^{\delta^*}(1 - \delta^* \ln r_{t-i})$ in X_t' simplifies to $\sum_{i=1}^{t-1}\hat{\beta}^{i-1}$ which is exactly identical to the first regressor $\sum_{i=1}^{t-1}\hat{\beta}^{i-1}$, hence either one of the regressors may be dropped. In this case the test statistic would be approximately distributed as a Chi-square with one degree of freedom.

3 A Monte Carlo Experiment

3.1 The Simulated Size of the Test Statistic

To study the simulated size of the level effect test statistic, we generate data from the simple GARCH(1,1) process

$$\Delta r_t = \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \qquad \text{where } v_t \backsim i.i.d.N(0,1) \tag{12}$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$$

We examine the effect of increasing persistence in the conditional variance on the simulated size of $LM(\delta^*)$ and $LM_1(\delta^*)$ test statistics. Following Engle and Ng's (1993) Monte Carlo study, we employ three sets of parameter values:

1. model H (for high persistence), where $(\alpha_0, \beta, \alpha_1) = (0.01, 0.9, 0.09)$ and $\alpha_1 + \beta = 0.99$
2. model M (for medium persistence), where $(\alpha_0, \beta, \alpha_1) = (0.05, 0.9, 0.05)$ and $\alpha_1 + \beta = 0.95$
3. model L (for low persistence), where $(\alpha_0, \beta, \alpha_1) = (0.2, 0.75, 0.05)$ and $\alpha_1 + \beta = 0.80$

To mitigate the effect of start-up values in all the experiments, we discard the first 500 observations yielding samples of 500, 1000 and 3000 observations, drawn with 10,000 replications. Upon generating the data, we estimate a GARCH(1,1) specification by maximizing the log-likelihood function using the Broyden, Fletcher, Goldfarb and Shanno (BFGS)[3] algorithm. The level effect test is then calculated on the resulting standardised residuals using the test statistics $LM(\delta^*)$ and $LM_1(\delta^*)$ for $\delta^* = \{0, 0.5, 1.0, 1.5\}$. Because of the highly non-linear structure of the models, in a small fraction of these replications, the convergence criterion is not satisfied. In such cases, new replications are added to ensure that there are 10,000 converged replications. To conserve space we report the results for the $LM_1(\delta^*)$. We note that there is an upward bias in the empirical size of the uncorrected $LM(\delta^*)$ test regardless of the degree of persistence in the conditional variance for all samples. These results are available upon request from the authors.

- Table 1 here -

The results, presented in Table 1, suggest that for all data generating processes the corrected test, $LM_1(\delta^*)$, exhibits some size distortions for small samples

[3] The BFGS algorithm with numerical derivatives is discussed in Judd (1988, pp. 114).

of 500 and 1000 observations. However, for a sample of 3000 observations the empirical size of $LM_1(\delta^*)$ is close to the nominal size for the parameter value of $\delta^* = 0.5, 1.0$ and 1.5. There is evidence that the empirical size of $LM_1(\delta^*)$ marginally decreases with a less persistent conditional variance.

3.2 Sensitivity Analyses for the Simulated Size

To assess the robustness of the level effect test in the presence of a neglected asymmetric volatility, we perform the following sensitivity analyses. Following Engle and Ng (1993), two types of asymmetric GARCH processes are considered; they are the EGARCH model due to Nelson (1991) and the GJR specification due to Glosten *et al.* (1993). The specifications are as follows:

EGARCH Model

$$\varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1) \tag{13}$$

$$\log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right]$$

GJR Model

$$\varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1) \tag{14}$$

$$h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot \left[|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}\right]^2.$$

The parameter values used in these Monte-Carlo experiments are taken from Engle and Ng (1993).

- Table 2 here -

The results in Table 2 suggest that, for a sample of 1000 observations, the level effect test statistic is robust in the presence of an EGARCH asymmetry, however this is not true in the case of threshold asymmetry of the GJR type. Nevertheless, for the larger sample size there is no apparent distortion in the empirical size of $LM_1(\delta^*)$. This suggests that the empirical size of the test statistic is robust to EGARCH and GJR asymmetric data generating processes.

We also perform sensitivity analyses for the level effect test when the data generating process displays means reversion using:

$$\Delta r_t = -0.01 r_{t-1} + \varepsilon_t \ , \ \varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1) \tag{15}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}.$$

Here the speed of short-rate mean reversion is governed by the value 0.01 which is commonly cited in the empirical short-rate models of C*KLS* and

BHK, *inter alia*. The experiment was performed for a sample of 3000 observations.

- Table 3 here -

Table 3 reports the empirical size of the test statistic for the stationary data generating process. The results suggest a minor upward bias in the empirical size of $LM_1(\delta^*)$. The results, however, are indicative that the level effect test is robust to mean-reversion in the data generating process[4]. There is further evidence that the empirical size of the test statistic also declines with a less persistent conditional variance.

We also consider the effects of both GJR-type asymmetry and stationarity on the simulated size of the level effect test. The simulated size is unaffected by the presence of both asymmetry and stationarity. The results are not reported in the paper to conserve space, but they are available upon request from the authors.

3.3 The Simulated Power of the Test Statistic

The next Monte Carlo experiment studies the simulated power of $LM_1(\delta^*)$. The data are generated according to

$$\Delta r_t = \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t} \cdot \upsilon_t \quad \text{where} \quad \upsilon_t \sim i.i.d.N(0,1)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}. \tag{16}$$

We examine the effect of increasing persistence in the conditional variance on the simulated power of $LM_1(\delta^*)$ test statistic using the same parameters values as in the empirical size of $LM_1(\delta^*)$ test. In addition, the simulated power is illustrated for differing degrees of persistence in the level effect through changing the values of b and δ. The set of parameter values are $b = \{0.01, 0.5, 0.99\}$ and $\delta = \{0, 0.5, 1.0, 1.5\}$. The size-adjusted power results across different degrees of persistence in the conditional variance, for a sample

[4] One potential explanation for the observed upward bias in the empirical size is the failure to account for the first order derivative in the conditional variance function with respect to the mean equation parameter λ. This is pertinent when computing the vector $\frac{\partial h_t}{\partial \varpi'}$ defined in equation (9). The cause of such bias might be worthy of an investigation in future research to accommodate different degrees of mean-reversion in the short rate process. However, given the lack of evidence of mean reversion in actual data (Shea, 1992, Rose, 1998 and Mankiw and Miron,1986 *inter alia*) we leave this agenda for future research.

of 3000 observations, are reported in Table 4 using the simulated critical values which are reported in Table 5.

- Table 4 here -

Apart from the case of $b = 0.01$ and $\delta = 0.5$, across different combinations of b and δ the test rejects the null hypothesis of no levels effects in at least 95% of simulations for each data generating process. The level effect test also displays significant size adjusted power across all δ^* values considered.

– Table 5 here –

Empirical values, reported in Table 5, are obtained from simulations used to examine the empirical size of the corrected level effect test statistic. Given that these values are relatively close to the relevant $\chi^2(2)$ variate, the $\chi^2(2)$ may be a useful approximation to the true distribution of the test, especially for large samples exceeding 1000 observations.

3.4 Sensitivity Analyses for the Simulated Power

The simulated power of the level effect test in the presence of a neglected asymmetry is computed using the following conditional variance specification in the DGP (16):

EGARCH Model

$$\varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1) \tag{17}$$

$$\log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right] + br_{t-1}^{\delta}$$

GJR Model

$$\varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1) \tag{18}$$

$$h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot \left[|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}\right]^2 + br_{t-1}^{\delta}.$$

Panels A-C of table 6 report the power of $LM_1(\delta^*)$ for a sample size of 3000 in the presence of an unparameterised EGARCH specification. The power of the test is computed using the three sets of critical values reported in Table 5, corresponding to the different degrees of persistence in the GARCH process. The results indicate that the tests display significant power across various data generating processes However, when $b = 0.01$, that is when the level effect is weakest, the power of the test is substantially reduced. The results also suggest that the empirical power of $LM_1(\delta^*)$ varies across the values of b and δ considered. Holding the value of b constant the powers of the test declines marginally as the value of δ increases. In contrast, with δ constant the test displays increasing power as the value of b increases.

- Table 6 here -

Table 7 reports the empirical power of $LM_1(\delta^*)$ for a sample size of 3000 in the presence of a neglected asymmetry in volatility of the GJR type. With one exception, the results are similar to those presented in Table 6. The exception occurs when $b = 0.01$ and the test displays increasing power as the magnitude of δ increases. For instance, consider $b = 0.01$ and $\delta = 0.5$, in which case the power of the test at the 5% significance level for $\delta^* = 0.5$ is 21.68%. By increasing δ to 1.0 and holding all other parameters constant, the power of the test at the same level of significance increases to 99.94%.

- Table 7 here -

We also perform a sensitivity analysis on the empirical power of $LM_1(\delta^*)$ when the DGP displays mean reversion. The data r_t is generated as follows:

$$\Delta r_t = -0.01 r_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1) \tag{19}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}.$$

Table 8 presents the results for the empirical power of $LM_1(\delta^*)$ for a sample size of 3000 observations. The results largely follow those reported in Table 4. The only noticeable difference occurs for a weakly persistent level effect that is when $b = 0.01$ and $\delta = 0.5$, where there appears to be an improvement in the power of $LM_1(\delta^*)$.

- Table 8 here -

In the presence of GJR-type asymmetry and mean reversion, the power of $LM_1(\delta^*)$ improves for the case of a weakly persistent level effect (i.e. for $b = 0.01$ and $\delta = 0.5$). The remainder of the results are largely consistent with those reported in Tables 8. These results are again not reported to save space but they are available from the authors upon request.

Finally, we check for the robustness of the test's empirical power to DGPs that display (i) a multiplicative level effect, or (ii) a DGP that follows the CKLS model. The data under each type of specification is generated as follows:

$$CKLS : \Delta r_t = \varepsilon_t \quad , \quad \varepsilon_t = v_t \cdot r_{t-1}^{\delta/2} \quad \text{where } v_t \backsim i.i.d.N(0,1), \tag{20}$$

$$Multiplicative\ Levels\ Effect : \Delta r_t = \varepsilon_t \ \ , \ \varepsilon_t = \sqrt{h_t} \cdot v_t \cdot r_{t-1}^{\delta/2} \quad \text{where } v_t \backsim i.i.d.N(0,1), \tag{21}$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}.$$

The results reported in Table 9 show that $LM_1(\delta^*)$ has good power even when the conditional variance of the DGP does not follow a GARCH process but contains a levels effect of the type documented in the CKLS model. Allowing for mean reversion in the short rate level marginally improves the power of the test. Tables 10 presents results that suggest that the test has power against a multiplicative levels effect, with or without mean reversion in the short rate level.

- Table 9 here -

- Table 10 here -

4 Empirical Application

4.1 Data Description and Diagnostic Tests Results

U.S. three-month Treasury bill yields, sampled at a weekly frequency over the period January 5, 1965 to November 4, 2003 yielding 2027 observations, were obtained from the FRED II database maintained by the Federal Reserve Bank of St. Louis.[5]

Figure 1 plots the level (r_t) and change (Δr_t) in the data. Visual inspection of Figure 1 suggests that the short rate (i) is most volatile between the period of 1979 and 1982 which coincides with the period of change in Federal Reserve monetary policy, (ii) that the volatility of Δr_t increases with the level of the short rate and (iii) that Δr_t displays volatility clustering.

- Figure 1 here -

Table 11 presents summary statistics for the data series. There is strong evidence of a unit root in the levels r_t. However, the change in short rate appears stationary. Δr_t display strong evidence of excess kurtosis. The Bera Jarque (1982) test for the normality of Δr_t is significant. Engle's (1982) LM test for ARCH, performed using the squared residuals from a fifth order autoregression provides strong evidence of conditional heteroskedasticity in Δr_t. The $LM_1(\delta^*)$ test for a level effect suggests that there is strong evidence that the volatility of Δr_t is dependent on the lagged short rate level. Engle

[5] http://research.stlouisfed.org/fred2/

and Ng (1993) sign and size bias tests suggest that there is no statistical evidence supporting asymmetric volatility in the short rate change.

- Table 11 here -

The results from the diagnostic tests suggest that an appropriate empirical model of the U.S. short-term interest rate should capture the characteristics of volatility clustering and a levels effect. We start with the estimation of the CKLS short rate specification. In the CKLS model the conditional heteroscedasticity depends on the lagged short rate level alone. This model, however, fails to specify the news arrival process that may be necessary to generate the volatility clustering usually associated with short-term interest rates. Nonetheless, the CKLS model provides a useful benchmark for the purpose of comparing with short rate models that explicity account for both the level effects and the news arrival process. The CKLS discrete-time econometric specification of the short rate is

$$r_t - r_{t-1} = \mu + \lambda r_{t-1} + \varphi CRASH + \varepsilon_t$$
$$E(\varepsilon_t) = 0, \quad E(\varepsilon_t^2) = b r_{t-1}^{\delta}. \tag{22}$$

The model is estimated using the Generalised Method of Moments (GMM) technique of Hansen (1982).

The parameter estimates of CKLS model are reported in column one of Table 12. The coefficients μ and λ are not significantly different from zero at conventional levels of significance suggesting that the short rate does not display the property of mean reversion. This result, however, has to be treated with caution since under the null of no mean reversion (i.e. $\lambda = 0$), r_t has a stochastic trend which implies that the estimated t-ratio for $\hat{\lambda}$ is biased. When using the Dickey-Fuller critical values[6], the estimate $\hat{\lambda}$ is not significantly different from zero. The coefficient of the $CRASH$ dummy, φ, is significant at the 5% significance level implying that the expected change in the short rate is higher during the stock market crash in 1989. The conditional variance also appears to depend on the lagged short rate level. The coefficient δ is significant at 1% significance level. Under the null that the model (22) is true, Hansen's (1982) J-test for the goodness-of-fit that is Chi-square distributed with one degree of freedom fails to reject the null at all conventional levels of significance. Some care should be taken in evaluating the result of the Hansen J-test because the test is sensitive to the number and choice of instruments used to estimate the model[7]. Nonetheless, there

[6] The Dickey-Fuller critical values with intercept are 3.440, 2.866 and 2.569 at 1%, 5% and 10% significance levels.

[7] In the GMM estimation, our instruments are the $CRASH$ dummy and r_{t-1}.

is evidence that the model is misspecified based on the presence of fifth order serial correlation in both the residuals and the squared residuals.

- Table 12 about here -

We then estimate short rate models that accommodate both the news arrival process and the short rate levels effect. In addition to capturing a level effect, the conditional volatility is allowed to respond asymmetrically to the sign and size of the short rate innovation in the AsyGARCHL model, specified as:

$$r_t - r_{t-1} = \mu + \lambda r_{t-1} + \varphi CRASH + \varepsilon_t \tag{23}$$
$$E(\varepsilon_t^2|\Omega_{t-1}) = h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1} + \alpha_2\eta_{t-1}^2 + b\,(r_{t-1}/10)^{\delta}$$

where $\eta_{t-1} = \min(0, \varepsilon_{t-1})$. For completeness, we also estimate the multiplicative level effects short rate model that is consistent with the asymmetric time varying parameter model of BHK. The specification is as follows

$$r_t - r_{t-1} = \mu + \lambda r_{t-1} + \varphi CRASH + \varepsilon_t$$
$$E\left(\varepsilon_t^2|\Omega_{t-1}\right) \equiv h_t = \phi_t^2(r_{t-1}/10)^{\delta} \tag{24}$$
$$\phi_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\phi_{t-1}^2 + \alpha_2\eta_{t-1}^2.$$

Both models stated in equations (23) and (24) are estimated using quasi maximum likelihood (QML) methods. Bollerslev and Wooldridge (1992) argue that the asymptotic standard errors resulting from QML are robust to departures from the normality assumption. We report these robust standard errors in the tables below.

Parameter estimates for the short rate models are reported in column two to six of Table 11. The asymmetric additive levels model (AsyGARCHL) specified in equation (23) nests the Asymmetric GARCH model (AsyGARCH), the GARCH model with level effects (GARCHL) and the GARCH model. The asymmetric multiplicative levels model in equation (24) nests the GARCH model with multiplicative levels effects. There is no evidence of mean reversion in short term interest rates in any of the models considered, with $\hat{\mu}$ and $\hat{\lambda}$ being insignificantly different from zero[8] and $\hat{\lambda}$ having the wrong sign. The coefficient of the $CRASH$ dummy, $\hat{\varphi}$, is significant at 5% significance level only in the models where the level effect is specified. Interestingly, in the variance equation, the sum of the GARCH parameters, $\hat{\alpha}_1 + \hat{\beta}_1$ is close to one only in cases where the level effect is not specified in the model. In

[8] Again, here, one has to be cautious in interpreting the t-test results since under the null of no mean reversion, r_t is non-stationary and the usual t-test does not apply.

other words, modelling the dependence of the conditional variance of short rate change on the short rate level helps to dampen the persistence of the shock. This result is again consistent with that of Brenner *et al.* (1996). The asymmetry coefficient $\hat{\alpha}_2$, which relates to the sign and size bias of the innovation is statistically insignificantly different to zero in all the models except for the AsyGARCHL model, where the estimate is significant but the magnitude is virtually zero[9].

Tests of the significance of the level effect coefficient, $\hat{b}$, are difficult to perform since δ is unidentified under the null of no level effect (i.e. $b = 0$), inference based on the t-test is invalid. Informally, the magnitude of $\hat{b}$ suggests that h_t is positively correlated with r_{t-1}. We address the problems of an unidentified parameter when formally testing for the significance of $\hat{b}$ by employing the Davies' (1987) bound method. This approach is applicable when a vector δ of dimension v from some parameter space Ω is only identified under the alternative hypothesis. Define the likelihood ratio statistic as a function of δ:

$$LR(\delta) = 2[\ln L_1(\delta_1) - \ln L_0(\delta_0)], \tag{25}$$

where $L_1(\delta_1)$ denotes the likelihood value of the objective function evaluated at δ_1 which is the estimated value of δ under the alternative hypothesis, and $L_0(\delta_0)$ is the maximum likelihood value derived under the null hypothesis (when δ is not identified). Suppose $\tilde{\delta}$ is the argmax of $L_1(\delta)$ and denote the likelihood function under the alternative hypothesis evaluated at $\tilde{\delta}$ by $L_1(\tilde{\delta})$. Then

$$\sup_{\delta\in\Omega} LR(\delta) = 2[\ln L_1(\tilde{\delta}) - \ln L_0(\tilde{\delta})]. \tag{26}$$

Let the empirically observed value of $2[\ln L_1(\tilde{\delta}) - \ln L_0(\tilde{\delta})]$ be denoted by Q. Davies shows that the significance of Q has an upper bound given by:

$$\Pr\left[\sup_{\delta\in\Omega} LR(\delta) > Q\right] \leq \Pr\left[\chi^2_v > Q\right] + V \cdot Q^{(v-1)/2} \cdot \exp^{-(v/2)} \cdot \frac{2^{-v/2}}{\Gamma(v/2)} \tag{27}$$

where $\Gamma(.)$ denotes the gamma function and G is defined as

$$\begin{aligned} G &= \int_{\delta_U}^{\delta_L} |\frac{\partial LR(\delta)^{1/2}}{\partial \delta}| d\delta \qquad (28) \\ &= |LR(\delta_1)^{1/2} - LR(\delta_L)^{1/2}| + |LR(\delta_2)^{1/2} - LR(\delta_1)^{1/2}| \\ &\quad +... + |LR(\delta_U)^{1/2} - LR(\delta_n)^{1/2}|, \end{aligned}$$

[9]The likelihood ratio test statistics for the AsyGARCHL and GARCHL in the additive and multiplicative models are both zero which suggests that there is no asymmetry in the conditional variance.

where $\delta_L, \delta_1, ..., \delta_n, \delta_U$ are the turning points of $LR(\delta)$. Davies obtained a quick rule by assuming that there is a single peak in the likelihood ratio. In such a case, G simplifies to $2Q^{1/2}$ which in turn simplifies inequality (27) to

$$\Pr\left[\sup_{\delta\in\Omega} LR(\delta) > Q\right] \leq \Pr\left[\chi_v^2 > Q\right] + Q^{v/2} \cdot \exp^{-(v/2)} \cdot \frac{2^{1-v/2}}{\Gamma(v/2)}. \qquad (29)$$

We adopt this quick rule in our testing procedure by estimating the model under the alternative hypothesis to obtain $L_1(\tilde{\delta})$, Q and G. Note that in equation (29) v is the number of identified parameters under the alternative hypothesis and $\Gamma(.)$ denotes the gamma function. Here $v = 1$ (since δ is identified under the alternative hypothesis) so that the upper bound significance value for the likelihood ratio test is 2.55×10^{-14} (for the AsyGARCHL model vs. AsyGARCH model) and 1.29×10^{-28} (for the GARCHL model vs. GARCH model) (see Table 12). These results suggest that the null of no level effect is rejected at all conventional significance levels and that the short rate level indeed influences the conditional variance of the short rate change. In the multiplicative levels models, the significance of $\hat{\delta}$ further supports the presence of level effects in the short rate models. Taken together, these results therefore concur with the diagnostic test results which suggest a correct model of the short rate change should specify level effects.

- Table 13 here -

To test that the model is not misspecified we employ the Ljung-Box test for the standardised residuals and the standardised squared residuals. There is clear evidence of fifth order serial correlation in the standardised residuals, although no fifth order serial correlation is present in the squared standardised residuals. In addition, we employ the conditional moment tests of Newey (1985) to detect possible sources of misspecification in the variance specification of the model. The conditional moment tests are fit using orthogonality conditions implied by the correct specification about the distribution of the standardised residuals, $z_t = \varepsilon_t/\sqrt{h_t}$ and the squared standardised residuals, z_t^2. A correct model would suggest that $E(z_t|\Omega_{t-1}) = 0$ and $E(z_t^2|\Omega_{t-1}) = 0$. In other words, z_t and z_t^2 are uncorrelated with any variable known at time $t-1$. Our assumption of normality in applying the quasi maximum likelihood method is tested in the first four orthogonality conditions. The diagnostic tests results presented in Table 13 satisfy the assumptions that the mean of the short rate standardised residuals is zero for all the empirical short rate models. The standardised residuals variances are also close to one. In addition, the short rate standardised residuals are leptokurtic. When considered individually, the squared standardised residuals of the short rate are also free of serial correlation. We then proceed to test the joint conditional moments of the model. Following Greene (2000), let θ represent the k-dimensional vector of parameters with corresponding estimates $\hat{\theta}$, and $\hat{z}_t = \hat{\varepsilon}_t / \sqrt{\hat{h}_t}$ represent the vector of standardised residuals, then the estimated vector of restrictions $p\left(\hat{\theta}\right)$ may be written as

$$p\left(\hat{\theta}\right) = \begin{bmatrix} r_1\left(\hat{\theta}\right) \\ \cdot \\ \cdot \\ r_J\left(\hat{\theta}\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T} m_1\left(\hat{z}_t\right) \\ \cdot \\ \cdot \\ \frac{1}{T}\sum_{t=1}^{T} m_J\left(\hat{z}_t\right) \end{bmatrix}. \tag{30}$$

Note that $p\left(\hat{\theta}\right)$ is a J-dimensional vector, where J represents the number of restrictions to be tested. Let $\hat{M}$ denote the $T \times J$ matrix

$$\hat{M} = \begin{bmatrix} m_1\left(\hat{z}_1\right) & \cdot & m_J\left(\hat{z}_1\right) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ m_1\left(\hat{z}_T\right) & \cdot & m_J\left(\hat{z}_T\right) \end{bmatrix}. \tag{31}$$

The log likelihood function is $l\left(\theta\right) = \sum_{t=1}^{T} l_t\left(\theta\right)$. The partial derivative of $l_t\left(\theta\right)$ with respect to θ_i, evaluated at the estimated parameter values is

$$d_{t,i} = \frac{\partial l_t\left(\theta\right)}{\partial\left(\theta_i\right)}|_{\theta=\hat{\theta}}. \tag{32}$$

Let D be the matrix of first derivatives of the realizations of the log-likelihood function with respect to the parameters, evaluated at the estimated parameters

$$D = \begin{bmatrix} d_{1,1}\left(\hat{\theta}\right) & \cdot & d_{1,k}\left(\hat{\theta}\right) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ d_{T,1}\left(\hat{\theta}\right) & \cdot & d_{T,k}\left(\hat{\theta}\right) \end{bmatrix}. \tag{33}$$

The null hypothesis is $H_0 : p\left(\theta\right) = 0$. Let $\hat{V}$ represent the variance-covariance matrix of $p\left(\hat{\theta}\right)$. The test statistic, which is distributed as $\chi^2\left(J\right)$ is

$$W = p\left(\hat{\theta}\right)' \hat{V}^{-1} p\left(\hat{\theta}\right). \tag{34}$$

The variance-covariance matrix of $p\left(\hat{\theta}\right)$ may be calculated as

$$\hat{V} = \frac{1}{T^2}\left[\hat{M}'\hat{M} - \hat{M}'\hat{D}\left(\hat{D}'\hat{D}\right)^{-1}\hat{D}'\hat{M}\right]. \tag{35}$$

The results of these moment tests are displayed in Table 13. There is no evidence of up to fourth or ARCH in the standardised residuals. In contrast, for all short rate models, there is evidence of serial correlation in the standardised residuals of the short rate when tested individually or jointly up to the fourth order. A joint test for the overall significance in the orthogonality conditions excluding the last four moments (m13 to m16) which relate to serial correlation, yields a p-value of 1.00 for all models. However, when these last four moments are included, the marginal significance level drops to 0.0000. There is strong evidence that the mean equation is misspecified in these short rate models. B*HK* report similar findings of serial correlation in z_t.

It is also interesting to note that a simple test of the hypothesis that h_t depends on r_{t-1} is significant only in the case when the level effects are not modelled in the conditional variance of the short rate. The test for the appropriateness of asymmetric variance models consistently fails to reject the null of no asymmetric volatility. The conditional moment test for asymmetry is computed using $I(\varepsilon_{t-1} < 0) \cdot \varepsilon_{t-1}$ where $I(\cdot)$ is an indicator dummy that takes the value one when $\varepsilon_{t-1} < 0$ and zero otherwise. Finally, in all the models considered there is no evidence of a structural break in the variance process caused by the change in monetary policy in the 1979-1981 period or for the stock market crash of 1987.

To address the problem of serial correlation in the standardised residuals, we re-specify the mean equation for the various short rate models discussed above and estimate:

$$\Delta r_t = \mu + \lambda r_{t-1} + \sum_{i=1}^{4} \rho_i \Delta r_{t-i} + \varphi CRASH + \varepsilon_t \tag{36}$$

whilst the variance equation specification is retained as before in the case of multiplicative and additive level effects. A lag order of four is chosen based on the Akaike (1974) and Schwarz (1979) Information Criteria. The results regarding the statistical significance of the parameter estimates and the model specifications are qualitatively similar to those reported in Tables 12 and 13. The only difference is that with (36) there is no evidence of fifth order serial correlation in both the standardised and the squared standardised residuals. These results are reported in Table 14.

-Table 14 here-

On the basis of a series of Likelihood Ratio Tests we are able to reject the hypothesis that the volatility of the US-T Bill rate responds asymetrically to positive and negative shocks. Similarly, we are able to reject the hypothesis that the levels effect alone is responsible for the conditional heteroscedasticity observed in Δr_t. Our preferred specifications for Δr_t contain a level effect. We are unable to distinguish between the multiplicative or additive levels effect models as these models are not nested. It is unclear what effect an unidentified parameter under the null would have on the performance of a non-nested test designed to distinguish across these models. We leave this matter on the agenda for future research.

5 Summary and Conclusion

This paper develops a test for the presence of a level effect in short term interest rates. Under the null hypothesis of no level effect the test, which is based on the Lagrange multiplier principle, is not operational because of the potential presence of an unidentified parameter. This problem is overcome by performing a Taylor's series approximation around the nuisance parameter. The resulting test statistic is approximately distributed Chi-square with one or two degrees of freedom under the null hypothesis. Monte-Carlo evidence suggests that, in a large sample, the test statistic is free from significant size distortions. The power of the test appears impressive and is largely robust to the degree of persistence in the GARCH process as well as the strength of the levels effect. Moreover, the empirical size and power of the test appear to be robust to unparameterised asymmetry in volatility when the levels effect is highly persistent[10]. The test also has good power in detecting the presence of a level effect when the DGP contains either a multiplicative levels effect or follows the CKLS model.One caveat of the test is its low power when subjected to an EGARCH asymmetry and a weak levels effect.

A series of Monte Carlo experiments show that the performance of the level effect test is largely unaffected by the presence of mean-reversion in the data generating process. The empirical size of the level effect test also little if any distortion if the data generating process displays both asymmetry and mean reversion. This is important since there is scant evidence of mean reversion in the literature modelling short-term interest rate dynamicss.

[10] In view of relatively poor power of $LM_1(\delta^*)$ when there is neglected asymmetry and weakly levels effect, Henry, Olekalns and Suardi (2004) develop a joint test for both levels effect and asymmetry.

There are limitations to the level effect test developed in this paper. The test results are reasonably reliable for a large sample of 3000 observations or more. The test is also developed under the assumption of normality, yet many financial series violate the normality condition and are leptokurtic in distribution. In such cases, the performance of the level effect test may be subjected to distortions. Hence future work will consider robustifying the test under deviations from normality.

Finally, both the level effect test and the Engle and Ng (1993) tests for asymmetry in volatility were applied to a sample of three-month U.S. Treasury bill yields. While the Engle and Ng (1993) tests fail to detect the presence of asymmetric volatility in the short rate, the $LM_1(\delta^*)$ test provides evidence of a level effect.

Asymmetric GARCH models with additive and multiplicative level effects are estimated and the results concur with those of the diagnostics. Inference about the presence of a levels effect, based on Davies' (1987) bound test which overcomes the problem of an unidentified parameter under the null, confirms the evidence of a level effect. There is little evidence of asymmetry. There is no statistical evidence supporting mean-reversion in the U.S. short rate. Similarly there is no evidence of structural instability due to the change in Federal Reserve operating procedures over the period October 1979-October 1982.

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Table 1[11]: Simulated Size of $LM_1(\delta^*)$

$$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Persistence Sample Size		*H* 500	 1000	 3000	*M* 500	 1000	 3000	*L* 500	 1000	 3000
$\delta^* = 0.0$	1%	0.00	0.88	0.30	3.05	1.94	0.36	1.45	0.55	0.22
	5%	0.00	4.03	1.88	6.97	6.51	1.60	15.79	2.38	1.17
	10%	0.20	7.65	3.72	12.68	10.71	3.52	28.65	5.28	2.58
$\delta^* = 0.5$	1%	4.18	4.16	1.08	56.73	6.61	1.02	62.32	2.77	0.84
	5%	31.84	13.48	5.08	82.68	19.25	5.06	87.63	11.44	3.85
	10%	52.10	22.09	10.05	91.45	28.93	9.65	94.22	20.14	7.76
$\delta^* = 1.0$	1%	2.44	3.99	1.06	73.04	6.28	1.00	77.12	1.73	0.63
	5%	49.86	13.21	5.06	89.92	18.76	5.04	91.90	8.42	3.58
	10%	64.31	21.98	10.01	92.01	28.53	9.68	94.07	16.86	7.18
$\delta^* = 1.5$	1%	2.64	3.64	1.04	77.16	5.75	1.02	84.01	5.76	0.58
	5%	63.83	12.78	5.01	88.54	17.68	5.02	91.05	17.70	3.29
	10%	73.29	21.15	10.03	90.90	27.55	9.37	93.30	27.60	6.84

[11] Notes to Table 1: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 2: Simulated Size of $LM_1(\delta^*)$ with Asymmetric DGP

$$\Delta r_t = \varepsilon_t \ , \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where v_t \backsim i.i.d.N(0,1)$$

$$\text{EGARCH: } \log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right]$$

$$\text{GJR: } h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot \left[\left|\varepsilon_{t-1}\right| - 0.23 \cdot \varepsilon_{t-1}\right]^2$$

Asymmetry		EGARCH		GJR	
Sample Size		1000	3000	1000	3000
$\delta^* = 0.0$	1%	0.64	0.33	0.22	0.25
	5%	2.38	1.51	1.46	1.27
	10%	4.60	3.19	3.11	3.07
$\delta^* = 0.5$	1%	1.36	1.45	18.40	1.10
	5%	5.70	5.35	30.77	5.33
	10%	10.54	10.09	38.94	10.10
$\delta^* = 1.0$	1%	1.81	1.31	6.42	1.16
	5%	6.05	5.37	15.64	5.01
	10%	10.90	9.85	24.15	10.14
$\delta^* = 1.5$	1%	1.92	1.86	2.77	1.01
	5%	5.30	5.08	9.36	4.60
	10%	9.01	9.00	16.54	9.38

Table 3[12]: Simulated Size of $LM_1(\delta^*)$ with mean reverting DGP
$\Delta r_t = -0.01 r_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \quad$ where $v_t \backsim i.i.d.N(0,1)$
$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$

Persistence		H	M	L
$\delta^* = 0.0$	1%	0.62	0.41	0.28
	5%	3.25	2.32	1.67
	10%	6.08	4.58	3.47
$\delta^* = 0.5$	1%	1.21	1.18	0.96
	5%	5.57	5.42	5.15
	10%	11.66	10.63	10.16
$\delta^* = 1.0$	1%	2.05	0.98	0.81
	5%	5.84	4.99	4.32
	10%	11.24	9.91	9.75
$\delta^* = 1.5$	1%	1.21	0.93	0.70
	5%	5.67	5.05	4.29
	10%	11.27	10.24	9.87

[12]Notes to Table 3: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 4[13]: Size-Adjusted Power of $LM_1(\delta^*)$ for Sample 3000

$$\Delta r_t = \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \qquad \text{where } v_t \backsim i.i.d.N(0,1)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}$$

Level Effect		$b = 0.01$			$b = 0.5$			$b = 0.99$		
Persistence		H	M	L	H	M	L	H	M	L
Panel	A					$\delta = 0.5$				
$\delta^* = 0.0$	1%	76.89	80.64	36.61	99.97	99.92	99.71	99.99	99.97	99.81
	5%	82.73	86.78	55.31	99.99	99.98	99.87	100	99.99	99.90
	10%	87.44	90.41	65.76	100	99.99	99.94	100	99.99	99.97
$\delta^* = 0.5$	1%	55.35	66.76	19.79	99.85	99.94	99.83	99.98	99.85	99.96
	5%	77.02	82.68	37.95	99.90	99.98	99.93	100	99.91	99.98
	10%	85.43	89.29	48.95	99.92	99.99	99.96	100	99.96	99.98
$\delta^* = 1.0$	1%	46.56	64.68	22.91	99.88	99.93	99.91	99.95	99.91	99.96
	5%	68.04	81.52	42.60	99.95	99.99	99.94	99.99	99.99	99.98
	10%	77.70	88.00	54.51	99.99	100	99.98	100	100	100
$\delta^* = 1.5$	1%	43.21	56.35	25.46	99.88	99.89	99.89	99.97	99.91	99.95
	5%	64.93	77.24	46.56	99.95	99.96	99.94	99.99	99.98	99.96
	10%	75.70	85.36	58.64	99.98	99.98	99.97	100	99.99	99.96
Panel	B					$\delta = 1.0$				
$\delta^* = 0.0$	1%	98.71	98.16	96.12	99.99	99.90	99.73	99.97	99.73	99.64
	5%	99.31	99.10	98.32	100	99.98	99.90	99.98	99.80	99.85
	10%	99.56	99.43	98.84	100	99.99	99.95	99.98	99.85	99.90
$\delta^* = 0.5$	1%	100	100	99.93	100	100	100	100	100	100
	5%	100	100	99.98	100	100	100	100	100	100
	10%	100	100	99.98	100	100	100	100	100	100
$\delta^* = 1.0$	1%	100	100	99.96	100	100	100	100	100	100
	5%	100	100	99.99	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	100	100	99.96	100	100	100	100	100	100
	5%	100	100	99.99	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
Panel	C					$\delta = 1.5$				
$\delta^* = 0.0$	1%	99.39	99.35	94.53	99.83	99.71	99.74	99.78	99.74	99.22
	5%	99.64	99.61	97.78	99.93	99.83	99.87	99.86	99.83	99.58
	10%	99.75	99.77	98.84	99.94	99.99	99.90	99.93	99.90	99.68
$\delta^* = 0.5$	1%	100	100	99.97	99.99	100	100	100	100	99.97
	5%	100	100	99.99	100	100	100	100	100	99.99
	10%	100	100	100	100	100	100	100	100	99.99
$\delta^* = 1.0$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100

[13]Notes to Table 4: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1) = (0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 5[14]: Empirical Critical Values from Sample Size 3000

Persistence		H	M	L
$\delta^* = 0.0$	1%	3.956	4.766	5.233
	5%	2.935	3.301	3.218
	10%	2.194	2.389	2.289
$\delta^* = 0.5$	1%	10.558	9.925	8.835
	5%	6.733	6.289	5.485
	10%	5.033	4.537	4.095
$\delta^* = 1.0$	1%	10.053	9.651	8.362
	5%	6.474	6.093	5.262
	10%	4.864	4.518	3.982
$\delta^* = 1.5$	1%	10.061	9.821	8.133
	5%	6.478	6.057	5.150
	10%	4.844	4.461	3.854
$\chi^2(2)$	1%		9.210	
	5%		5.991	
	10%		4.605	

[14] Notes to Table 5: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 6[15]: Size- Adjusted Power of $LM_1(\delta^*)$ with EGARCH DGP for Sample Size 3000

$$\Delta r_t = \varepsilon_t \ , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where\ v_t \backsim i.i.d.N(0,1)$$

$$\log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right] + br_{t-1}^{\delta}$$

Level Effect		$b = 0.01$			$b = 0.5$			$b = 0.99$		
Persistence		H	M	L	H	M	L	H	M	L
Panel	A					$\delta = 0.5$				
$\delta^* = 0.0$	1%	6.70	3.39	2.61	99.65	99.50	99.42	98.90	99.81	99.77
	5%	9.70	7.92	8.20	99.80	99.79	99.78	99.91	99.89	99.90
	10%	15.14	13.36	14.23	99.89	99.85	99.86	99.92	99.92	99.92
$\delta^* = 0.5$	1%	0.81	0.97	1.43	97.98	98.24	98.67	99.30	99.44	99.56
	5%	3.65	4.56	6.57	99.21	99.29	99.44	99.80	99.81	99.88
	10%	8.50	10.75	13.50	99.51	99.60	99.65	99.90	99.91	99.91
$\delta^* = 1.0$	1%	1.09	1.18	1.86	98.19	98.31	98.77	99.28	99.36	99.49
	5%	4.43	5.39	7.63	99.24	99.31	99.49	99.76	99.80	99.82
	10%	9.27	10.77	13.88	99.54	99.59	99.68	99.87	99.87	99.92
$\delta^* = 1.5$	1%	1.56	1.67	2.63	94.84	95.13	96.86	98.86	98.92	99.32
	5%	4.66	5.51	7.92	98.11	98.45	98.81	99.69	99.70	99.77
	10%	8.94	10.62	14.37	98.87	99.01	99.24	99.80	99.83	99.87
$\delta^* = 0.0$	1%	15.98	12.10	10.53	98.88	98.53	98.30	99.25	99.47	99.60
	5%	22.62	19.84	20.38	99.24	99.16	99.18	99.04	99.41	99.56
	10%	29.27	27.25	28.15	99.41	99.38	99.41	98.89	99.42	99.58
Panel	B					$\delta = 1.0$				
$\delta^* = 0.0$	1%	15.98	12.10	10.53	98.88	98.53	98.30	99.25	99.47	99.60
	5%	22.62	19.84	20.38	99.24	99.16	99.18	99.04	99.41	99.56
	10%	29.27	27.25	28.15	99.41	99.38	99.41	98.89	99.42	99.58
$\delta^* = 0.5$	1%	3.99	4.76	6.30	95.32	95.76	98.69	97.46	97.82	98.32
	5%	11.20	12.79	16.15	97.94	98.11	98.89	98.98	99.07	99.22
	10%	18.49	21.63	25.06	98.69	98.54	99.06	99.31	99.47	99.56
$\delta^* = 1.0$	1%	4.16	4.66	6.52	92.88	93.37	94.87	96.46	96.71	97.57
	5%	10.86	12.14	15.54	96.71	97.04	97.71	98.49	98.64	98.90
	10%	17.83	20.01	23.72	98.03	98.24	98.51	99.13	99.23	99.43
$\delta^* = 1.5$	1%	3.69	3.84	6.07	89.16	94.99	96.95	95.47	95.72	97.05
	5%	9.50	10.71	14.51	89.64	95.59	97.31	98.09	98.29	98.79
	10%	15.80	17.64	21.56	92.48	96.65	97.90	98.91	98.98	99.24
Panel	C					$\delta = 1.0$				
$\delta^* = 0.0$	1%	28.96	24.51	22.54	96.38	95.69	95.25	96.35	95.64	95.43
	5%	36.64	33.39	34.11	97.17	97.14	97.05	97.40	97.00	97.08
	10%	44.80	42.56	43.56	97.92	97.71	97.80	98.03	97.89	97.98
$\delta^* = 0.5$	1%	12.72	13.58	15.00	91.50	91.97	92.93	90.13	90.80	91.92
	5%	19.94	21.62	25.35	94.54	94.90	95.56	93.86	94.31	95.05
	10%	27.94	31.09	34.55	95.94	96.33	96.81	95.42	95.74	96.09
$\delta^* = 1.0$	1%	13.19	13.52	16.15	84.10	84.76	87.03	87.89	88.36	90.00
	5%	20.63	22.13	26.75	90.21	90.89	92.34	92.27	92.82	93.82
	10%	28.51	30.98	35.15	92.89	92.34	94.17	94.22	94.75	95.44
$\delta^* = 1.5$	1%	12.38	12.69	15.67	75.13	75.90	80.60	82.34	82.82	85.89
	5%	20.29	21.74	26.26	84.44	85.30	87.42	88.77	89.74	91.42
	10%	28.05	30.83	35.62	88.16	89.07	90.48	92.04	92.67	93.87

[15] Notes to Table 6: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 7[16]: Size- Adjusted Power of $LM_1(\delta^*)$ for Sample 3000 with GJR Asymmetry:

$$\Delta r_t = \varepsilon_t \ , \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where\ v_t \backsim i.i.d.N(0,1)$$

$$h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2 + br_{t-1}^{\delta}$$

Level Effect		$b = 0.01$			$b = 0.5$			$b = 0.99$		
Persistence		H	M	L	H	M	L	H	M	L
Panel	A					$\delta = 0.5$				
$\delta^* = 0.0$	1%	29.85	23.92	21.16	99.91	99.83	99.79	99.75	99.66	99.62
	5%	39.45	35.69	36.47	99.95	99.92	99.94	99.82	99.79	99.80
	10%	48.20	45.82	47.14	99.99	99.97	99.98	99.89	99.88	99.88
$\delta^* = 0.5$	1%	8.17	9.54	12.61	99.99	99.91	99.94	99.89	99.91	99.93
	5%	21.68	24.23	36.47	99.98	99.98	99.98	99.94	99.95	99.99
	10%	32.59	36.76	47.14	99.98	99.98	99.99	99.99	99.99	99.99
$\delta^* = 1.0$	1%	12.29	13.39	17.3	99.92	99.92	99.96	99.94	99.96	99.97
	5%	25.49	27.64	33.61	99.98	99.98	99.98	99.97	99.98	100
	10%	36.61	39.72	44.60	99.98	99.98	99.99	100	100	100
$\delta^* = 1.5$	1%	8.65	9.14	14.49	99.85	98.87	98.91	99.87	99.89	99.91
	5%	22.93	25.72	32.56	99.93	98.94	98.97	99.93	99.94	99.97
	10%	35.07	38.91	45.09	99.98	98.98	98.98	99.97	99.98	99.98
Panel	B					$\delta = 1.0$				
$\delta^* = 0.0$	1%	97.98	96.96	96.11	99.30	99.13	99.08	99.63	99.51	99.46
	5%	98.91	98.60	98.69	99.47	99.42	99.42	99.74	99.70	99.71
	10%	99.47	99.32	99.42	99.62	99.58	99.62	99.77	99.75	99.76
$\delta^* = 0.5$	1%	99.88	99.91	99.91	100	100	100	100	100	100
	5%	99.94	99.94	99.94	100	100	100	100	100	100
	10%	99.95	99.95	99.95	100	100	100	100	100	100
$\delta^* = 1.0$	1%	99.94	99.94	99.95	100	100	100	100	100	100
	5%	99.98	99.99	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	99.94	99.96	99.99	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
Panel	C					$\delta = 1.5$				
$\delta^* = 0.0$	1%	93.51	91.49	90.11	99.71	99.66	99.62	99.81	99.79	99.78
	5%	95.65	95.04	95.16	99.79	99.76	99.77	99.91	99.87	99.88
	10%	97.18	100	100	99.84	99.83	99.84	99.94	99.94	99.94
$\delta^* = 0.5$	1%	99.92	99.92	99.95	99.98	99.98	99.98	99.96	99.96	99.96
	5%	99.96	99.96	99.96	99.98	99.98	99.98	99.97	99.97	99.98
	10%	99.96	99.98	99.98	99.99	99.99	99.99	99.99	99.99	99.99
$\delta^* = 1.0$	1%	100	100	100	99.98	99.98	99.98	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100

[16]Notes to Table 7: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 8: Size-Adjusted Power of $LM_1(\delta^*)$ for mean-reverting DGP Sample Size 3000:

$$\Delta r_t = -0.01 r_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}$$

Level Effect		$b = 0.01$			$b = 0.5$			$b = 0.99$		
Persistence		H	M	L	H	M	L	H	M	L
Panel	A				$\delta = 0.5$					
$\delta^* = 0.0$	1%	40.38	63.01	61.52	100	100	100	100	100	100
	5%	51.59	75.88	73.21	100	100	100	100	100	100
	10%	61.21	83.37	80.57	100	100	100	100	100	100
$\delta^* = 0.5$	1%	12.48	35.21	37.15	100	100	100	100	100	100
	5%	31.01	61.97	64.52	100	100	100	100	100	100
	10%	45.36	76.00	76.02	100	100	100	100	100	100
$\delta^* = 1.0$	1%	34.45	45.92	40.35	100	100	100	100	100	100
	5%	61.13	69.75	65.23	100	100	100	100	100	100
	10%	74.50	80.40	75.94	100	100	100	100	100	100
$\delta^* = 11.5$	1%	21.74	52.68	42.78	100	100	100	100	100	100
	5%	45.04	77.00	65.77	100	100	100	100	100	100
	10%	60.11	86.42	75.89	100	100	100	100	100	100
Panel	B				$\delta = 1.0$					
$\delta^* = 0.0$	1%	100	100	97.63	100	100	100	100	100	100
	5%	100	100	99.42	100	100	100	100	100	100
	10%	100	100	99.69	100	100	100	100	100	100
$\delta^* = 0.5$	1%	99.94	100	98.62	100	100	100	100	100	100
	5%	100	100	99.32	100	100	100	100	100	100
	10%	100	100	99.73	100	100	100	100	100	100
$\delta^* = 1.0$	1%	100	100	98.85	100	100	100	100	100	100
	5%	100	100	99.77	100	100	100	100	100	100
	10%	100	100	99.92	100	100	100	100	100	100
$\delta^* = 1.5$	1%	100	100	99.02	100	100	100	100	100	100
	5%	100	100	99.85	100	100	100	100	100	100
	10%	100	100	99.96	100	100	100	100	100	100
Panel	C				$\delta = 1.5$					
$\delta^* = 0.0$	1%	100	100	100	99.78	100	100	99.89	100	99.94
	5%	100	100	100	99.96	100	100	99.96	100	99.96
	10%	100	100	100	99.98	100	100	99.98	100	99.96
$\delta^* = 0.5$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.0$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100

Table 9[17]: Size-Adjusted Power of $LM_1(\delta^*)$ for 300 observations CKLS DGP

$$\Delta r_t = \lambda r_{t-1} + \varepsilon_t \ \ , \ \ \varepsilon_t = v_t \cdot r_{t-1}^{\delta/2} \quad \text{where } v_t \backsim i.i.d.N(0,1)$$

Level Effect		$\delta = 0.5$	$\delta = 1.0$	$\delta = 1.5$
Panel	A		$\lambda = 0.0$	
$\delta^* = 0.0$	1%	99.20	98.40	99.31
	5%	99.78	99.27	99.57
	10%	99.88	99.49	99.69
$\delta^* = 0.5$	1%	99.85	100	100
	5%	99.94	100	100
	10%	99.97	100	100
$\delta^* = 1.0$	1%	99.91	100	100
	5%	99.98	100	100
	10%	100	100	100
$\delta^* = 1.5$	1%	99.94	100	100
	5%	99.99	100	100
	10%	100	100	100
Panel	B		$\lambda = -0.01$	
	1%	99.93	99.89	99.94
$\delta^* = 0.0$	5%	99.97	99.95	99.98
	10%	100	100	100
	1%	100	100	100
$\delta^* = 0.5$	5%	100	100	100
	10%	100	100	100
	1%	100	100	100
$\delta^* = 1.0$	5%	100	100	100
	10%	100	100	100
	1%	100	100	100
$\delta^* = 1.5$	5%	100	100	100
	10%	100	100	100

[17] Notes to Table 9: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 10[18]: Size-Adjusted Power of $LM_1(\delta^*)$ for 3000 observations with Multiplicative Level Effect DGP

$$\Delta r_t = \lambda r_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \cdot r_{t-1}^{\delta/2} \quad \text{where } v_t \backsim i.i.d.N(0,1)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Level Effect Persistence		$\delta = 0.5$ H	$\delta = 0.5$ M	$\delta = 0.5$ L	$\delta = 1.0$ H	$\delta = 1.0$ M	$\delta = 1.0$ L	$\delta = 1.5$ H	$\delta = 1.5$ M	$\delta = 1.5$ L
					$\lambda = 0.0$					
$\delta^* = 0.0$	1%	98.07	98.82	99.02	97.17	98.43	98.77	96.47	98.25	98.37
	5%	99.03	99.58	99.73	98.53	99.30	99.56	98.44	98.81	98.89
	10%	99.34	99.78	99.86	99.11	99.67	99.81	99.02	99.45	99.75
$\delta^* = 0.5$	1%	98.28	99.75	99.91	100	100	100	100	100	100
	5%	99.27	99.93	99.98	100	100	100	100	100	100
	10%	99.51	99.96	100	100	100	100	100	100	100
$\delta^* = 1.0$	1%	98.65	99.92	100	100	100	100	100	100	100
	5%	99.52	99.95	100	100	100	100	100	100	100
	10%	99.70	99.98	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	98.50	99.37	99.97	100	100	100	100	100	100
	5%	99.26	99.89	100	100	100	100	100	100	100
	10%	99.51	99.93	100	100	100	100	100	100	100
					$\lambda = -0.01$					
$\delta^* = 0.0$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 0.5$	1%	99.93	99.96	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.0$	1%	99.95	99.98	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	99.89	99.91	99.98	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100

[18] Notes to Table 10: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1)$ =$(0.01, 0.9, 0.09)$, M: $\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table 11[19]: Summary Statistics

Data Series	r_t	Δr_t
Mean	6.1022	-0.0014
Variance	7.3092	0.0665
Skewness	1.1051	-0.5434
Kurtosis	5.0226	20.9800
ADF(5)	-2.8718	-17.2931
PP(5)	-2.5476	-40.6889
KPSS(μ)	1.1550	0.1129
KPSS(τ)	0.7811	0.0224
Jarque-Berra $\sim \chi^2(2)$	758.13	27390.01
	[0.0000]	[0.0000]
ARCH(5)	62.8979	61.0037
	[0.0000]	[0.0000]
Ljung-Box statistic $Q(5)$	2.7986	0.5167
	[0.7310]	[0.9920]

Level Effect Test $LM_1(\delta^*)$ for Δr_t	
$\delta^* = 0.0$	14.6480
	[0.0001]
$\delta^* = 0.5$	17.0109
	[0.0002]
$\delta^* = 1.0$	17.9194
	[0.0001]
$\delta^* = 1.5$	18.6506
	[0.0001]

Engle and Ng's Asymmetry Tests for Δr_t	
Negative Sign	-0.5644
	[0.5725]
Negative Size	-1.2734
	[0.2030]
Positive Size	-0.0356
	[0.9716]
Joint Test $\sim \chi^2(3)$	4.4458
	[0.2172]

[19] Notes to Table 11: ADF(5) and PP(5) include an intercept and trend in the regressions. Both tests have 1%, 5% and 10% critical values of -3.9642, -3.4128 and -3.1284 respectively. KPSS(μ) 1%, 5% and 10% critical values are 0.739, 0.463 and 0.347 respectively. KPSS(τ) 1%, 5% and 10% critical values are 0.216, 0.146 and 0.119 respectively. $LM_1(\delta^* = 0) \sim \chi^2(1)$. p-values are reported in [.].

Table 12[20]: Empirical Models of the U.S. Short Rate

$$\Delta r_t = \mu + \lambda r + \varphi CRASH + \varepsilon_t$$
$$E(\varepsilon_t^2|\Omega_{t-1}) = h_t$$
$$\text{CKLS Model: } h_t = br_{t-1}^{\delta}$$
$$\text{Additive Model : } h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1} + \alpha_2\eta_{t-1}^2 + b\left(\frac{r_{t-1}}{10}\right)^{\delta}$$
$$\text{Multiplicative Model : } h_t = \phi_t^2 r_{t-1}^{\delta};$$
$$\phi_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\phi_{t-1}^2 + \alpha_2\eta_{t-1}^2$$

		Additive Levels Models				Multiplicative Levels Models	
	CKLS	AsyGARCHL	AsyGARCH	GARCHL	GARCH	AsyGARCHL	GARCHL
μ	0.0215	−0.0057	−0.0019	−0.0057	−0.0028	−0.0028	−0.0019
	(0.0234)	(0.0041)	(0.0049)	(0.0110)	(0.0060)	(0.0044)	(0.0049)
λ	−0.0037	0.0018	0.0006	0.0018	0.0007	0.0009	0.0008
	(0.0045)	(0.0011)	(0.0013)	(0.0009)	(0.0014)	(0.0011)	(0.0013)
φ	0.0559*	0.1122*	0.1129	0.1122*	0.1134	0.1189*	0.1143*
	(0.0148)	(0.0507)	(0.0958)	(0.0364)	(0.0998)	(0.0489)	(0.0429)
α_0		0.0002	0.0003*	0.0002	1.82×10^{-11}*	0.0011*	0.0011*
		(0.0001)	(0.0001)	(0.0001	(2.10×10^{-22})	(0.0004)	(0.0005)
β_1		0.7987*	0.8471*	0.7987*	0.9105*	0.8625*	0.8686*
		(0.0499)	(0.0304)	(0.0494)	(0.0215)	(0.0336)	(0.0312)
α_1		0.1628*	0.1529*	0.1628*	0.0895*	0.1375*	0.1314*
		(0.0423)	(0.0304)	(0.0428)	(0.0215)	(0.0336)	(0.0312)
α_2		1.23×10^{-15}*	0.0134			0.0458	
		(6.47×10^{-19})	(0.0211)			(0.0295)	
b	6.52×10^{-5}*	0.0081*		0.0081*			
	(3.32×10^{-5})	(0.0034)		(0.0030)			
δ	3.4325*	3.4489*		3.4489*		0.1128*	0.0989*
	(0.2463)	(0.9978)		(1.0669)		(0.0412)	(0.0432)
				Model Diagnostics			
L	·	2734.2977	2701.1121	2734.2977	2667.8615	2726.8174	2722.8223
Q(5)	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*
$Q^2(5)$	0.0000*	0.6695	0.6973	0.6695	0.4264	0.7484	0.7509
Hansen J- Test		$\chi^2_{(1)}$ =2.037533 [0.1535]					
Davies Bound Test							
$H_0: b = 0$		AsyGARCHL and AsyGARCH:			P(LR>66.3712) = 2.55×10^{-14}		
		GARCHL and GARCH:			P(LR>132.8724) = 1.29×10^{-28}		

[20]Notes to Table 12: Standard errors are reported in (.). * indicates that the coefficient is significant at 5% significance level. The p-values of the Ljung-Box test statistic are reported

Table 13[21]: Newey (1985) Conditional Moment Misspecification Tests

	Orthogonality Conditions	Additive Levels Models AsyGARCHL	AsyGARCH	GARCHL	GARCH	Multiplicative Levels Models AsyGARCHL	GARCHL
		Sample Averages					
m1	$E[z_t]=0$	−0.0150 (−0.6773)	−0.0008 (−0.0359)	−0.0149 (−0.6773)	−0.0017 (−0.0706)	−0.0036 (−0.1607)	−0.0064 (−0.2815)
m2	$E[z_t^2]-1=0$	−0.0068 (−0.1314)	0.0207 (0.3950)	−0.0068 (−0.1314)	0.1717* (2.7329)	0.0168 (0.3221)	0.0519 (0.9647)
m3	$E[z_t^3]=0$	−0.0557 (−0.2489)	0.0687 (0.3075)	−0.0557 (−0.2489)	0.1059 (0.3466)	0.0372 (0.1645)	−0.0099 (−0.0422)
m4	$E[z_t^3]-4=0$	3.3334* (3.1021)	3.6261* (3.5723)	3.3334* (3.1022)	6.3619* (3.9032)	3.5532* (3.3589)	3.9821* (3.5865)
m5	$E[(z_t^2-1)(z_{t-1}^2-1)]=0$	0.0521 (0.3778)	0.0703 (0.6402)	0.0521 (0.3778)	0.3056 (1.8506)	0.0694 (0.5465)	0.1208 (0.8187)
m6	$E[(z_t^2-1)(z_{t-2}^2-1)]=0$	−0.0155 (−0.1922)	0.0531 (0.5677)	−0.0155 (−0.1922)	0.2968 (1.7558)	0.0062 (0.0748)	0.0458 (0.5011)
m7	$E[(z_t^2-1)(z_{t-3}^2-1)]=0$	−0.1368 (−1.0756)	−0.1316 (−1.2869)	−0.1368 (−1.0756)	0.0264 (0.2372)	−0.1190 (−1.7910)	−0.1155 (−1.6606)
m8	$E[(z_t^2-1)(z_{t-4}^2-1)]=0$	−0.0457 (−0.6421)	−0.0936 (−1.1784)	−0.0457 (−0.6421)	−0.0188 (−0.1623)	−0.0579 (−0.7538)	−0.0397 (−0.4764)
m9	$E[(z_t^2-1)\cdot r_{t-1}]=0$	0.0763 (0.2195)	0.6424* (1.9870)	0.0763 (0.2195)	1.3389* (3.0671)	0.2470 (0.6985)	0.5718 (1.5189)
m10	$E[(z_t^2-1)\cdot I(\varepsilon_{t-1}<0)\cdot\varepsilon_{t-1}]=0$	0.0007 (0.0886)	−0.0028 (−0.3236)	0.0007 (0.0886)	−0.0172 (−1.6517)	0.0036 (0.4578)	−0.0059 (−0.6505)
m11	$E[(z_t^2-1)\cdot FED]=0$	0.0227 (1.3350)	0.0362 (1.7375)	0.0227 (1.3349)	0.0353 (1.6405)	0.0095 (0.5700)	0.0189 (1.0428)
m12	$E[(z_t^2-1)\cdot CRASH]=0$	−0.0010 (−1.4145)	−0.0009 (−1.4145)	−0.0010 (−1.4145)	−0.0009 (−1.4145)	−0.0009 (−1.4146)	−0.0009 (−1.4145)
m13	$E[z_t\cdot z_{t-1}]=0$	0.0632* (2.7924)	0.0653* (2.7893)	0.0632* (2.7924)	0.0745* (2.6171)	0.0621* (2.6636)	0.0670* (2.7291)
m14	$E[z_t\cdot z_{t-2}]=0$	0.0322* (1.4711)	0.0320 (1.3763)	0.0322* (1.4711)	0.0333 (1.1681)	0.0316 (1.3921)	0.0354 (1.4844)
m15	$E[z_t\cdot z_{t-3}]=0$	0.0377 (1.8385)	0.0425* (2.0040)	0.0377 (1.8385)	0.0479 (1.8429)	0.0438* (2.0619)	0.0471* (2.1303)
m16	$E[z_t\cdot z_{t-4}]=0$	0.0791* (3.6711)	0.0852* (3.9394)	0.0791* (3.6711)	0.0909* (3.5561)	0.0843* (3.8409)	0.0894* (3.9056)
	Joint Moment Tests	Joint Test Statistic					
m5-m8		6.6726 [0.1542]	6.9905 [0.1363]	6.6069 [0.1581]	6.2664 [0.1801]	4.0108 [0.4045]	4.7056 [0.3188]
m13-m16		26.0186* [0.0000]	28.2182* [0.0000]	25.6695* [0.0000]	28.7725* [0.0000]	28.2567* [0.0000]	27.1043* [0.0000]
m1-m16		119.9267* [0.0000]	312.6975* [0.0000]	119.9404* [0.0000]	699.4688* [0.0000]	2722.3991* [0.0000]	404.8955* [0.0000]

[21] Notes to Table 13: The individual conditional moment test statistic for m1 to m16 is distributed as $\chi^2(1)$. The joint conditional moment tests for m5-m8 (or m13-m16) and m1-m16 are distributed as $\chi^2(4)$ and $\chi^2(16)$ respectively. * indicates that the null is rejected at 5% significance level. Figures in (.) and [.] are t-statistics and p-values respectively.

Table 14[22]: Models of the U.S. Short Rate Corrected for Serial Correlation

$$\Delta r_t = \mu + \lambda r_{t-1} + \Sigma_{i=1}^{4}\rho_i \Delta r_{t-i} + \varphi CRASH + \varepsilon_t$$

$$E(\varepsilon_t^2|\Omega_{t-1}) = h_t$$

$$\text{Additive Model}: h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1} + \alpha_2\eta_{t-1}^2 + b\left(\frac{\mathrm{r}_{t-1}}{10}\right)^{\delta}$$

$$\text{Multiplicative Model}: h_t = \phi_t^2 r_{t-1}^{\delta};$$

$$\phi_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\phi_{t-1}^2 + \alpha_2\eta_{t-1}^2$$

	Additive Levels	Multiplicative Levels
μ	-0.0034 (0.0038)	0.0005 (0.0045)
λ	0.0012 (0.0010)	0.0002 (0.0011)
ρ_1	0.0520* (0.0246)	0.0509 (0.0274)
ρ_2	0.0111 (0.0236)	0.0133 (0.0250)
ρ_3	0.0216 (0.0260)	0.0226 (0.0244)
ρ_4	0.1020* (0.0238)	0.1045* (0.0266)
φ	0.1038* (0.0211)	0.1053* (0.0263)
α_0	0.0002 (0.0002	0.0013* (0.0005)
β_1	0.7909* (0.0507)	0.8636* (0.0386)
α_1	0.1663* (0.0398)	0.1364* (0.0386)
b	0.0089* (0.0046)	
δ	3.5818* (1.0216)	0.1103* (0.0530)
Standardised Residual Diagnostics		
L	2734.7142	2724.6119
Q(5)	0.3146	0.1866
$Q^2(5)$	0.6879	0.7255
LR Test for Nested Models:		
Additive Models $H_0 : GARCHL$ vs. $H_1 : AsyGARCHL$		1.32×10^{-5}
Multiplicative Models $H_0 : GARCHL$ vs. $H_1 : AsyGARCHL$		3.34×10^{-3}
Davies Bound Test		
	GARCHL and GARCH:	P(LR>131.1692) = 3.00×10^{-28}
Joint Moment Tests		
m5-m8	6.6557 [0.1552]	4.2706 [0.3706]
m13-m16	9.1649 [0.0571]	9.9458 [0.0414]
m1-m16	98.3207* [0.0000]	251.3580* [0.0000]

[22]Note: See notes to Table 12 and 13. The LR test is distributed as a $\chi^2(1)$.

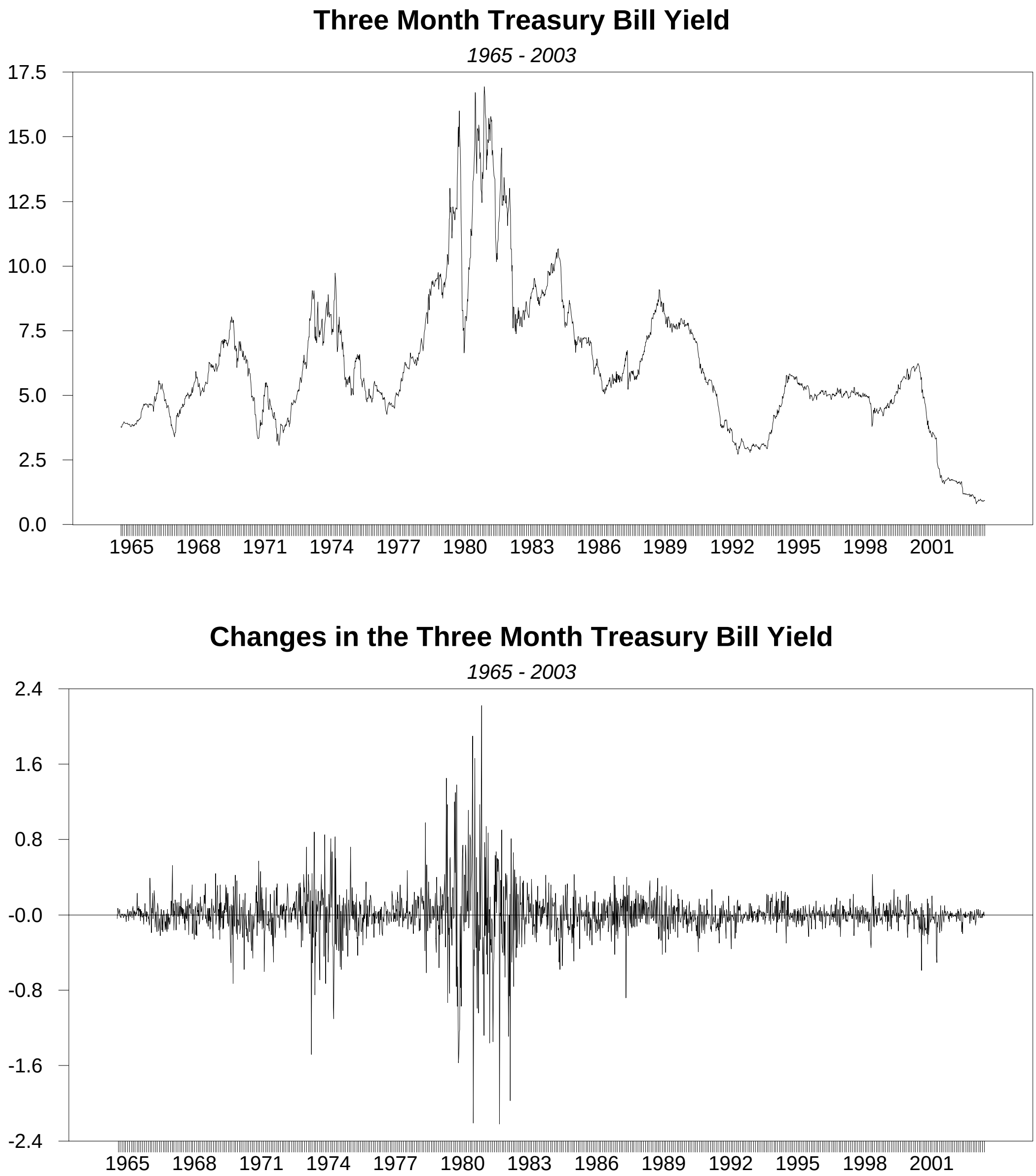

Figure 1: r_t and Δr_t