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INDIRECT TAXATION AND PROGRESSIVITY: REVENUE AND WELFARE CHANGES

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Indirect Taxation and Progressivity: Revenue and Welfare Changes[1]

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Abstract

This paper compares the disproportional effects of indirect taxation using two alternative measures, tax-progressivity and welfare-progressivity. In the context of an indirect tax imposed on a single good, tax-progressivity requires the taxed good to be luxury. In contrast, welfare-progressivity requires the equivalent variation as fraction of total expenditure to rise with total expenditure. Sufficient conditions for welfare-progressivity are derived for both the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS). When the parameters of the direct utility functions are held constant, imposing homogeneous preferences, the condition required for welfare-progressivity is the same as that required for tax-progressivity, namely that the taxed good is a luxury. Parameter constancy also implies a particular pattern for the variation in budget shares with total expenditure, which is unique for each demand system. When parameters are allowed to vary with total expenditure, according to a general budget share relationship, which enables preference heterogeneity amongst households, welfare-progressivity is independent of tax-progressivity for both models, giving rise to possible conflicts in tax and welfare disproportionality. The empirical application of these conditions to New Zealand data shows that many such cases of conflict can arise. Furthermore, conflicting results are also obtained when examining the effects of the overall indirect tax structure. The majority of conflicts arise where tax-regressivity exists at the same time as welfare-progressivity.

JEL Classification H23; H22: H31

Keywords Tax progressivity; equivalent variations; budget shares

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1 The views, opinions, findings, and conclusions or recommendations expressed in this Working Paper are strictly those of the authors. They do not necessarily reflect the views of the Reserve Bank of New Zealand.

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Indirect Taxation and Progressivity: Revenue and Welfare Changes

1 Introduction

An indirect tax is described as locally tax-progressive if a household's average tax rate (the ratio of tax paid to total expenditure) rises with total household expenditure. By this definition, any tax imposed on a luxury good is tax-progressive. As total expenditure rises, the budget share attributed to the luxury good rises and consequently so must the average tax rate. Variations in the degree of tax-progressivity over a range of total expenditure levels depend only on the variation in the budget share attributed to the taxed good. Hence, tax-progressivity does not necessarily reflect the degree of welfare loss, as the latter depends, among other things, on the compensated price elasticity of demand for the good. Thus welfare-progressivity, which arises when the welfare loss, expressed as a fraction of total expenditure, rises with total expenditure, is not solely dependent on the variation in the budget share attributed to the taxed good. It is therefore not immediately obvious that tax-progressivity implies welfare-progressivity.

The aim of this paper is to consider the conditions under which a tax imposed on a luxury (or necessity) good is consistent with an increasing (or decreasing) ratio of the welfare loss to total expenditure. The welfare measure examined is the Hicksian equivalent variation. Of course, a good may change from being a luxury over a certain range of total expenditure to being a necessity over another range. For this reason the analysis is concerned only with the local measure of progressivity, which is unique to a chosen level of total expenditure.

Section 2 examines the case where the parameters of the direct utility function of a demand system are assumed to remain fixed as total expenditure varies. This assumption restricts households to have the same basic preferences, in spite of differing levels of total expenditure. Welfare-progressivity is examined for two demand systems; the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS), where expressions for the welfare changes are readily obtained. Fixing the parameters of the direct utility functions implies a particular form of variation in the budget shares of each good as total expenditure rises. This implied variation, which is different for the LES than for the AIDS, is shown in section 3. A more general form of variation in budget shares, which has been found to be useful in empirical work, is therefore suggested which combines elements implicit in both of these demand systems. The imposition of the more general form leads the parameters of the direct utility function to vary with total expenditure, allowing for heterogeneous preferences among households and therefore more realistic analysis. Using the extended LES, section 4 uses data obtained from New Zealand's *Household Economic Surveys (HES)* to provide empirical examples of cases where the directions of tax and welfare disproportionality conflict for the case of a tax imposed on a single good and New Zealand's indirect tax structure as a whole. Conclusions are provided in section 5. First, for completeness the following subsection presents the results relating to tax revenue for a single good.

1.1 Tax Revenue

Let m denote a household's total expenditure and $w_i = p_i x_i / \sum_{i=1}^{n} p_i x_i$ the budget share devoted to the ith good, with p_i and x_i the price and quantity demanded of the good respectively. If t_i is the tax-inclusive *ad valorem* tax rate on the ith good, total indirect tax revenue is $T = m \sum_{i=1}^{n} t_i w_i$.[3] The Musgrave-Thin (1948) measure of local liability progression is the elasticity of tax paid with respect to total expenditure (the ratio of the marginal to the average tax rate), η, given by:

$$\eta = 1 + \sum_{i=1}^{n} \left(\frac{t_i w_i}{\sum_i t_i w_i} \right) \dot{w} \tag{1}$$

[3] The tax-inclusive *ad valorem* tax rate is the ratio of tax paid to the tax-inclusive price of the good.

where $\dot{w}_i = \frac{dw_i}{dm}\frac{m}{w_i}$ is the elasticity of the budget share with respect to total expenditure. Using the fact that the total expenditure elasticity is given by $e_i = 1 + \dot{w}_i$, a progressive system, for which the elasticity η must be greater than unity, generally requires luxuries to be taxed more heavily than necessities. If taxation is imposed on only one good, say good k, then $\eta = e_k$ and the Musgrave-Thin local progressivity measure is given simply by the total expenditure elasticity.

2 Welfare Changes with Fixed Parameters

This section compares tax- and welfare-progressivity for the case where the parameters of the utility function are fixed, thereby imposing homogeneous preferences on all households. The analysis proceeds by imposing a tax on a single commodity. The equivalent variation resulting from the tax, EV, is derived for each demand system and then the expression for EV/m is differentiated with respect to total expenditure, m, while holding the parameters of the direct utility function constant. The conditions required to achieve welfare-progressivity (or regressivity) are found by constraining the derivative to be strictly positive (or negative).

2.1 The Linear Expenditure System

The Linear Expenditure System (LES) has direct utility functions of the form:[4]

$$U = \prod_{i=1}^{n} \left(x_i - \gamma_i \right)^{\beta_i} \tag{2}$$

where γ_i is committed consumption with $x_i > \gamma_i, 0 \le \beta_i \le 1, \sum_{i=1}^{n} \beta_i = 1$. Define $C = \sum_{i=1}^{n} p_i \gamma_i$ as total committed expenditure, and let $B = \prod_{i=1}^{n} \left(\frac{p_i}{\beta_i} \right)^{\beta_i}$. It can be shown that when prices change from p^0 to p^1, the equivalent variation is:

[4] For details of the LES see, for example, Brown and Deaton (1973), Powell (1974) and Creedy (1998a, b). Muellbauer (1974) used the fixed parameter LES to examine the distributional effects of inflation in the UK.

$$EV = m - C^0 \left[1 + \frac{B^0}{B^1} \left(\frac{m}{C^0} - \frac{C^1}{C^0} \right) \right] \tag{3}$$

If $\dot{p}_i$ denotes the proportionate change in the price of the ith good, then $C^1 / C^0 = 1 + \sum_{i=1}^{n} s_i \dot{p}_i$ where $s_i = \left(p_i^0 \gamma_i \right) / \sum_{i=1}^{n} p_i^0 \gamma_i$. The term B^1 / B^0 simplifies to $B^1 / B^0 = \prod_{i=1}^{n} \left(p_i^1 / p_i^0 \right)^{\beta_i} = \prod_{i=1}^{n} \left(1 + \dot{p}_i \right)^{\beta_i}$.

Consider imposing a tax on a single good, k, giving rise to a proportionate change in the price of the good, $\dot{p}_k$, with $\dot{p}_i = 0$ for all other goods. The term C^1 / C^0 simplifies to $C^1 / C^0 = 1 + s_k \dot{p}_k$ and B^1 / B^0 becomes $B^1 / B^0 = \left(1 + \dot{p}_k \right)^{\beta_k}$. Let $c_k = p_k^0 \gamma_k$ denote the initial level of committed expenditure on good k. Hence the equivalent variation as a proportion of total expenditure is:

$$\frac{EV}{m} = 1 - \frac{C^0}{m} - \left(1 + \dot{p}_k \right)^{-\beta_k} \left(1 - \frac{C^0}{m} - \frac{c_k}{m} \dot{p}_k \right) \tag{4}$$

Differentiating equation (4) with respect to total expenditure, while holding the parameters of the direct utility function, β_k and γ_k constant, using the property that $\beta_i = e_i w_i$, and simplifying gives the familiar condition that welfare-progressivity requires $e_k > 1$, the same condition required for local tax-progressivity.

2.2 The Almost Ideal Demand System

The Almost Ideal Demand System (AIDS) is based on the expenditure function:[5]

$$\log E\left(p, U \right) = a\left(p \right) + b\left(p \right) U \tag{5}$$

where the two price indices $a\left(p \right)$ and $b\left(p \right)$ are defined as:[6]

$$a\left(p \right) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{l=1}^{n} \gamma^*_{il} \log p_i \log p_l \tag{6}$$

$$b\left(p \right) = \beta_0 \prod_{i=1}^{n} p_i^{\beta_i} \tag{7}$$

[5] For further details of the AIDS see, Deaton and Muellbauer (1980) and Creedy (1998b).
[6] The parameter β must be distinguished from its use in defining the LES above.

Denoting income by m, $m = \sum_{i=1}^{n} p_i x_i = E(p,U)$, the direct utility function takes the form,

$$U = (\log m - a)/b \,. \tag{8}$$

It can be shown that when prices change from p^0 to p^1, the equivalent variation is:

$$EV = m - \exp\left(a\left(p^0\right) + b\left(p^0\right)U^1\right) \tag{9}$$

Defining the price index $P = \exp a(p)$, this becomes:

$$EV = m - P^0 \exp\left(\frac{b\left(p^0\right)}{b\left(p^1\right)} \log\left(\frac{m}{P^1}\right)\right) \tag{10}$$

Consider imposing a tax on only good, k, as before. The term $b\left(p^0\right)/b\left(p^1\right)$ simplifies to $b\left(p^0\right)/b\left(p^1\right) = (1+\dot{p}_k)^{\beta_k}$, and the equivalent variation as a fraction of total expenditure is:

$$\frac{EV}{m} = 1 - \exp\left(\left(1+\dot{p}_k\right)^{-\beta_k} \log\left(\frac{m}{P^1}\right) - \log\left(\frac{m}{P^0}\right)\right) \tag{11}$$

Applying the approximation, $-x = 1 - \exp(x)$, gives:

$$\frac{EV}{m} = \log\left(\frac{m}{P^0}\right) - \left(1+\dot{p}_k\right)^{-\beta_k} \log\left(\frac{m}{P^1}\right) \tag{12}$$

Differentiating equation (12) with respect to total expenditure, holding the parameter of the direct utility function, β_k, constant, using the property that $\beta_k = w_k\left(e_k - 1\right)$, and simplifying again gives the condition that $e_k > 1$ is required for a tax placed on the good to be locally welfare-progressive.

3 Welfare Changes with Variable Parameters

3.1 Budget Shares

The above analysis has held the parameters of the direct utility functions constant while considering the variation in the equivalent variation with total expenditure. This must imply a certain relationship between the budget share attributed to each good and the level of total expenditure, so that w_i must be a function of m. For example, in the case of the LES, it has been stated that $\beta_i = e_i w_i$, and using $e_i = 1 + \dot{w}_i$, gives:

$$\begin{aligned} w_i + m\frac{dw_i}{dm} &= \beta_i \\ \frac{d\left(mw_i\right)}{dm} &= \beta_i \end{aligned} \tag{13}$$

The solution to this differential equation is thus:

$$mw_i = m\beta_i + d_i \tag{14}$$

where d_i is a constant of integration. The budget share is therefore related to total expenditure as:

$$w_i = \beta_i + \frac{d_i}{m} \tag{15}$$

In the case of the AIDS, it has been stated above that $\beta_i = w_i\left(e_i - 1\right)$, which gives $\beta_i = w_i \dot{w}_i = m\left(dw_i / dm\right)$, so that $\left(dw_i / dm\right) = \beta_i / m$. Integration therefore leads to:

$$w_i = h_i + \beta_i \log m \tag{16}$$

where h_i is a constant of integration.[7]

Empirical studies of budget data have found a more general form to be useful, which combines both the elements of the LES and AIDS models, whereby for good i:

$$w_i = \delta_{1i} + \delta_{2i} \log m + \frac{\delta_{3i}}{m} \tag{17}$$

[7] This form was explicitly discussed by Deaton and Muellbauer (1980), who also referred briefly to the more general form discussed below. They also pointed out that ordinary least squares regression estimates, for each commodity group at a time, would automatically satisfy the required adding up condition.

In practice, the parameters, δ_{1i}, δ_{2i} and δ_{3i} can be estimated by regressing budget shares for good i, obtained from sample surveys, on households' total expenditure levels.[8] Using this specification, the total expenditure elasticity for the good, $e_i = 1 + \dot{w}_i$, is:

$$e_i = \frac{\left(\delta_{1i} + \delta_{2i}\left(1 + \log m\right)\right)}{\left(\delta_{1i} + \delta_{2i}\log m + \dfrac{\delta_{3i}}{m}\right)} \tag{18}$$

This more general budget share relationship leads the parameters of the LES and the AIDS to be functions of total expenditure, m, which in turn allows for heterogeneous preferences among households. The consequence of this allowance for welfare-progressivity is explored in the following subsections.

3.2 The Extended Linear Expenditure System

In the LES, the parameter, β_i, can be expressed as a function of m by substituting the above expression for e_i into $\beta_i = e_i w_i$. Using this result to replace β_i in the expression for EV/m in equation (4), differentiating and then simplifying, leads the required condition for welfare-progressivity to become:

$$e_k > 1 + \delta_{2k}\dot{p}_k\left(1 + \frac{e_k}{\xi}\right) \tag{19}$$

where ξ is the Frisch parameter, defined as the elasticity of marginal utility of total expenditure with respect to total expenditure; see Frisch (1959). This is assumed to be constant. For details of the derivation, see the Appendix.

Identifying good k as a luxury good is no longer sufficient to establish welfare-progressivity. Similarly, taxing a necessity is not sufficient to ensure welfare-regressivity. It is not surprising that the condition depends crucially on the parameter, δ_{2k}, as this is the additional parameter introduced into the LES budget share relationship, compared with the fixed parameter case. Setting this parameter to zero leads to the condition

[8] This approach may not satisfy regularity conditions, in that some predicted budget shares at very low values of total expenditure can be negative, but this does not present problems in practice.

derived for the fixed parameter case, namely that $e_k > 1$. The condition shown in equation (19) is explored below in further detail for necessities and luxuries in turn.

3.2.1 Necessities

When good k is a necessity, the necessary and sufficient conditions for welfare-progressivity and regressivity of the tax are:

Condition i) $\delta_{2k} > 0$

This is a sufficient condition for welfare-regressivity.

Condition ii) $\delta_{2k} < 0$

This is a necessary condition for welfare-progressivity. The sufficient condition is:

$$\delta_{2k} < \frac{\xi}{\dot{p}_k} \frac{(e_k - 1)}{(\xi + e_k)} < 0 \qquad (20)$$

For necessities, the constraint $|\xi| > 1$ ensures total committed expenditure is non-negative, and is sufficient to ensure committed expenditure on each good is also non-negative.

3.2.2 Luxuries

For a luxury good, the necessary and sufficient conditions for welfare-progressivity and regressivity of the tax are:

Condition i) $\delta_{2k} > 0$

This is a necessary condition for welfare-regressivity. The sufficient condition is:

$$\delta_{2k} > \frac{\xi}{\dot{p}_k} \frac{(e_k - 1)}{(\xi + e_k)} > 0 \qquad (21)$$

Condition ii) $\delta_{2k} < 0$

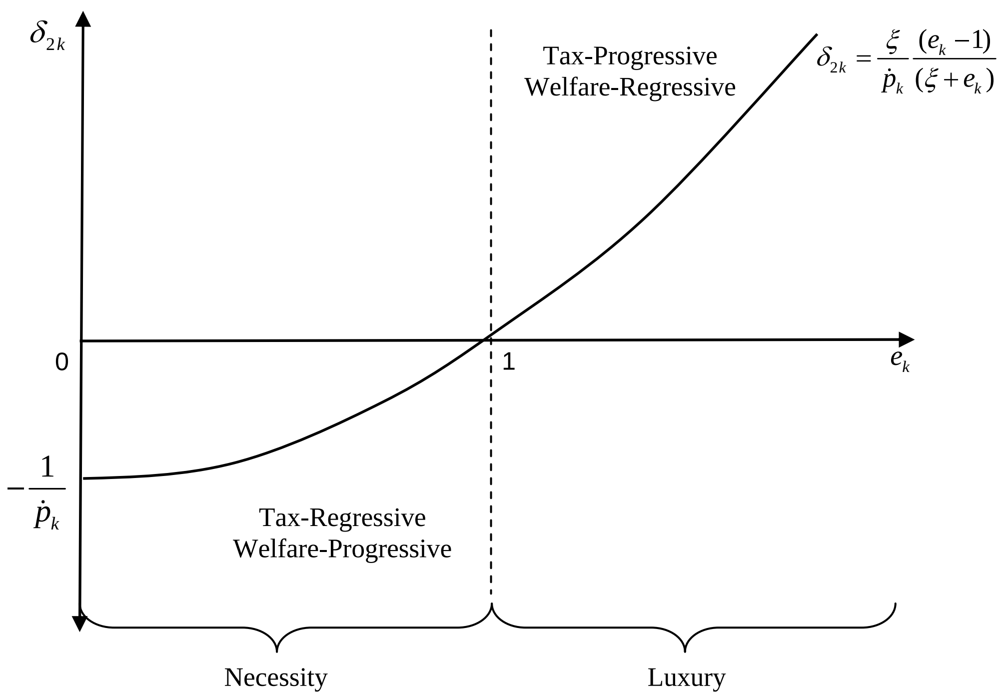
This is a sufficient condition for welfare-progressivity.

When good k is a luxury, the constraint $|\xi| > 1$ is no longer sufficient to ensure committed expenditure on good k is non-negative. Instead, the required constraint, using equation (A3) in the Appendix is $|\xi| > e_k$.

These conditions are illustrated in Figure 1. The welfare-disproportionality of the tax is determined by the solid upward sloping line, which forms part of a hyperbola. Any point lying above this line indicates that the tax imposed on good k is welfare-regressive, while any point lying below the line indicates welfare-progressivity. This line is contrasted against the dotted line positioned at $e_k = 1$ which determines the tax-disproportionality of the indirect tax. Because of the differences between the two lines, there are two areas in which tax and welfare disproportionality can conflict, as shown in the figure. Raising the tax rate imposed on the good shifts the curve upward, while lowering the degree of convexity. The curve continues to pass through the point (1,0).

The conditions in this sub-section have been expressed in terms of the total expenditure elasticity of the good, e_k, so that a direct comparison with the simple condition for tax-progressivity could be made. However, e_k depends on the parameters, $\delta_{1k}, \delta_{2k}, \delta_{3k}$, of the budget share relationship, so it may be argued that a more complete expression would establish the precise ranges of total expenditure over which tax-progressivity and welfare-progressivity simultaneously arise. This is unfortunately rather cumbersome analytically, although, as shown in section 4, ranges can readily be obtained numerically using the above conditions.

Figure 1 - Tax and Welfare Disproportionality in the Extended LES



3.3 The Extended Almost Ideal Demand System

In the extended model, the budget shares vary with total expenditure according to the general form given above. The implied variation in β_i is therefore:

$$\begin{aligned}\beta_i &= w_i\left(e_i - 1\right) \\ \beta_i &= \delta_{2i} - \frac{\delta_{3i}}{m}\end{aligned} \tag{22}$$

Substituting for β_k into the expression for EV/m, shown in equation (12), differentiating and simplifying, as shown in further detail in the Appendix, gives the condition that welfare-progressivity requires:

$$e_k > 1 - \frac{\delta_{3k}}{mw_k}\log\left(\frac{m}{P^1}\right) \tag{23}$$

Hence, as with the LES, identifying good k as a luxury may no longer be sufficient to establish the welfare-progressivity of the tax. The conditions are given below for necessities and luxuries in turn.[9]

3.3.1 Necessities

The conditions characterising welfare-progressivity and regressivity are:

Condition i) $\delta_{3k} > 0$

This is a necessary condition for welfare-progressivity. The sufficient condition is:

$$\delta_{3k} > \frac{mw_k\left(1-e_k\right)}{\log\left(\frac{m}{P^1}\right)} > 0 \tag{24}$$

Condition ii) $\delta_{3k} < 0$

This is a sufficient condition for welfare-regressivity.

[9] These conditions are based on the assumption that $m > P^1$.

3.3.2 Luxuries

When good k is a luxury, the necessary and sufficient conditions for the welfare-progressivity and regressivity of the tax are:

Condition i) $\delta_{3k} > 0$

This is a sufficient condition for welfare-progressivity.

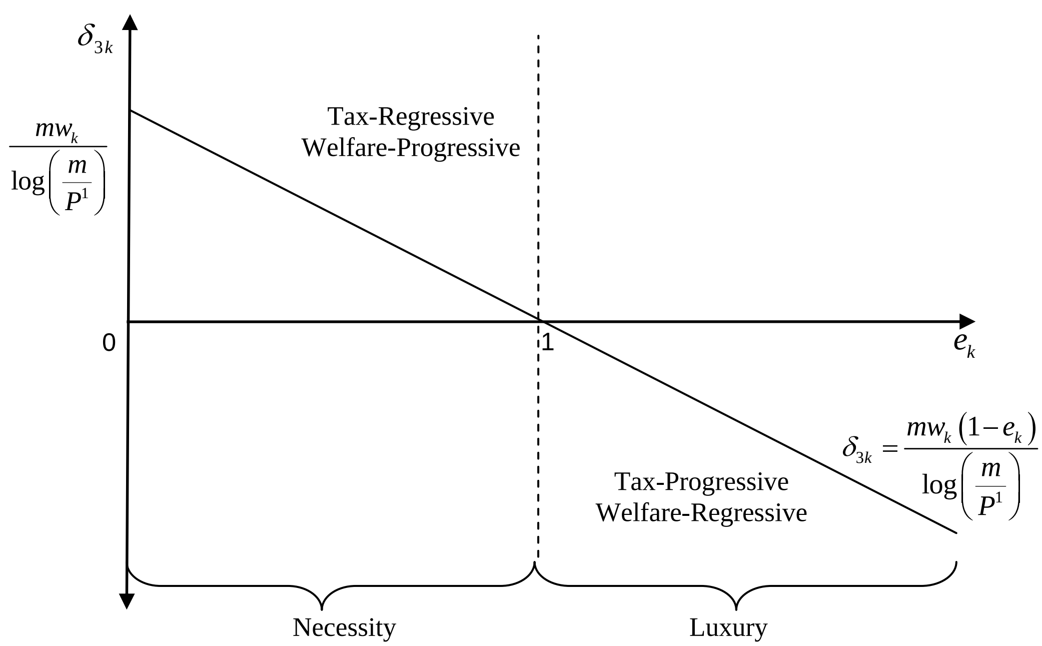
Condition ii) $\delta_{3k} < 0$

This is a necessary condition for welfare-regressivity. The sufficient condition is:

$$\delta_{3k} < \frac{mw_k\left(1-e_k\right)}{\log\left(\frac{m}{P^1}\right)} < 0 \qquad (25)$$

These conditions depend crucially on the parameter, δ_{3k}, as this is the additional term introduced by the more general budget share relationship. The conditions for tax and welfare disproportionality for the extended AIDS are summarised in Figure 2.

Figure 2 - Tax and Welfare Disproportionality in the Extended AIDS



4 Empirical Examples

The previous section showed that when allowing for heterogeneous preferences using either the extended LES or AIDS, a tax imposed on a luxury good, which is tax-progressive, may also be welfare-regressive. Using the extended LES, this section provides empirical examples of such cases where tax and welfare disproportionality conflict, using New Zealand data. Subsection 4.1 considers the case of imposing a tax on a single good, while subsection 4.2 compares the tax and welfare disproportionality of the current indirect tax system as a whole.

The measure of equivalent variation in the extended LES is computed using data collected from households who participated in the 1995, 1996, 1997, 1998 and 2001 *Household Economic Surveys* (HES).[10] The weekly total expenditure data were adjusted to 2001 prices using the consumer price index (CPI). There were very few changes in indirect tax rates over this period. The surveys were then pooled to form one large database, providing approximately 13,500 households. Each household was placed into one of 18 demographic groups, shown in Table A1, which were further sub-divided into smoking and non-smoking households. A positive weekly expenditure on tobacco was sufficient for a household to be designated as a smoking household. The division into smoking and non-smoking households was found to improve substantially the fit of the estimated budget share relationships. Using equation (17), the budget shares collected from the HES were regressed on the household's total expenditure levels for 22 commodity groups in turn, which are listed in Table A2. Thus, a total of 792 (22x18x2) regressions were performed. The following results were obtained using a value of $\xi = -2$.[11]

4.1 A Single Tax

A tax (and proportional price) increase of 10 percent was imposed on each commodity group in turn, and the conditions derived in section 3 were examined for a range of total

[10] Surveys have only been conducted tri-annually since 1998. For further details of this approach, see Creedy and Sleeman (2005).

[11] Tulpule and Powell (1978) used a value of ξ=-1.82 when calculating elasticities at average income for Australia, based on the work of Williams (1978), and this value was adopted by Dixon *et al.* (1982) in calibrating a general equilibrium model. The slightly higher absolute value was used here to avoid some negative committed expenditures.

expenditure levels, which were allowed to vary in $1 increments from $200 to $1500 per week. The directions of tax and welfare-progressivity were found to conflict in a total of 250 cases. Table 1 provides the number of conflicts which arose for each commodity group. The majority of conflicts occur where the tax revenue indicated local regressivity, but the equivalent variation indicated progressivity. In these cases, the revenue collected from the tax as a fraction of total expenditure was falling, while at the same time, the welfare loss from the tax as a fraction of total expenditure was rising.

Table 1 – Conflicting Movements in Tax and Welfare Disproportionality

Commodity Group	Overall	Tax-Regressive Welfare-Progressive	Tax-Progressive Welfare-Regressive
Rent	27	26	1
Other Expenditure	24	21	3
Vehicle Supplies etc	24	23	1
Household Equipment	23	19	4
Petrol	22	19	3
Services	21	9	12
Medical, Cosmetic etc	20	16	4
Alcohol	12	9	3
Public Transport in NZ	12	9	3
Food Outside Home	12	12	0
Household Services	11	6	5
Pay to Local Authorities	10	0	10
Furnishings	6	5	1
Food	5	4	1
Cigarettes and Tobacco	4	4	0
Children's Clothing	4	3	1
House Maintenance	4	1	3
Overseas Travel	3	0	3
Adult Clothing	3	1	2
Vehicle Purchase	2	1	1
Domestic Fuel and Power	1	0	1
Recreational Vehicles	0	0	0
Total	250	188	62

Tables 2 and 3 provide further details of the demographic groups and the ranges of total expenditure over which conflicts occurred. As it is not possible to show all cases, only those where the range of total expenditure exceeded $2 are provided in the two tables. It is clear from these results that the possibility of conflicting indications of disproportionality when using tax and welfare measures is far from negligible. Further, there appears no particular pattern in terms of the expenditure ranges or the demographic groups for which these conflicts can arise. Thus, regardless of the type of good on which a tax is imposed, the disproportionality of the tax should not be decided on without first evaluating the welfare losses of the proposed reform for a selection of demographic groups and total expenditure levels.

Table 2 - Cases of Tax-Regressivity and Welfare-Progressivity

		Expenditure Range, m					Expenditure Range, m		
	Rent					**Other Expenditure**			
S	Single Adult & 1 Child	376	-	381	S	Adult Couple and 3 Children	985	-	988
	Single Adult & 2 Children	369	-	373		3 Adults & No Children	831	-	834
	Single Adult & 3 Children	413	-	420		3 Adults and 2+ Children	1052	-	1055
	4+ Adults & No Children	1084	-	1089	NS	Adult Couple and 2 Children	947	-	951
	4+ Adults & 2+ Children	697	-	700		Adult Couple and 3 Children	1238	-	1242
NS	Single Adult & 1 Child	285	-	288		3 Adults & No Children	1094	-	1098
	Single Adult & 2 Children	332	-	336		3 Adults and 2+ Children	1175	-	1180
	Single Adult & 3 Children	247	-	250		4+ Adults & No Children	1011	-	1015
	Single Adult & 4+ Children	441	-	448					
						Food			
	Services				NS	Adult Couple $ 4+ Children	256	-	259
S	3 Adults & No Children	981	-	986		4+ Adults & 2+ Children	293	-	296
NS	65+ Single	907	-	913					
	Adult Couple and 2 Children	931	-	937		**Food Outside Home**			
	3 Adults & 1 Child	1116	-	1124	NS	3 Adults & No Children	1441	-	1444
	3 Adults and 2+ Children	1425	-	1435					

Table 3 - Cases of Tax-Progressivity and Welfare-Regressivity

		Expenditure Range, m					Expenditure Range, m		
	Food					**Services**			
NS	Single Adult & 4+ Children	698	-	704	S	4+ Adults & No Children	706	-	709
						4+ Adults & 2+ Children	965	-	969
	Rent								
S	Single Adult & 4+ Children	939	-	947					

4.2 The Indirect Tax System

Having analysed the case of a single tax, the question arises as to whether a conflict between the direction of tax and welfare disproportionality could arise when considering a large number of indirect taxes combined, such as the effective current structure of indirect taxes in New Zealand. The rates are summarised in Table A2.

It was found that a variety of cases exist where tax revenues from the current indirect tax structure indicate tax-regressivity, while the welfare losses from the taxes indicate progressivity. Figures 3 to 5 provide examples of this occurrence for three demographic groups. In each case, the current structure of indirect taxes, taken together, is tax-regressive over all ranges of total expenditure, but welfare-progressive over higher ranges. Reliance on tax revenue alone thus gives an incomplete indication of progressivity.

Figure 3 - Tax and Welfare Disproportionality of the Indirect Tax System: 65+ Single Adults, Non-Smoking Households

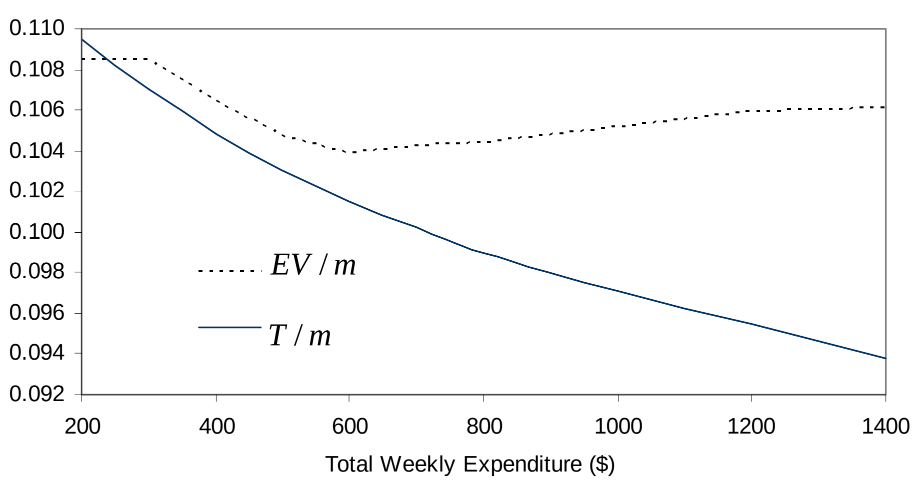


Figure 4 - Tax and Welfare Disproportionality of the Indirect Tax System: Single Adult, No Children, Non-Smoking Households

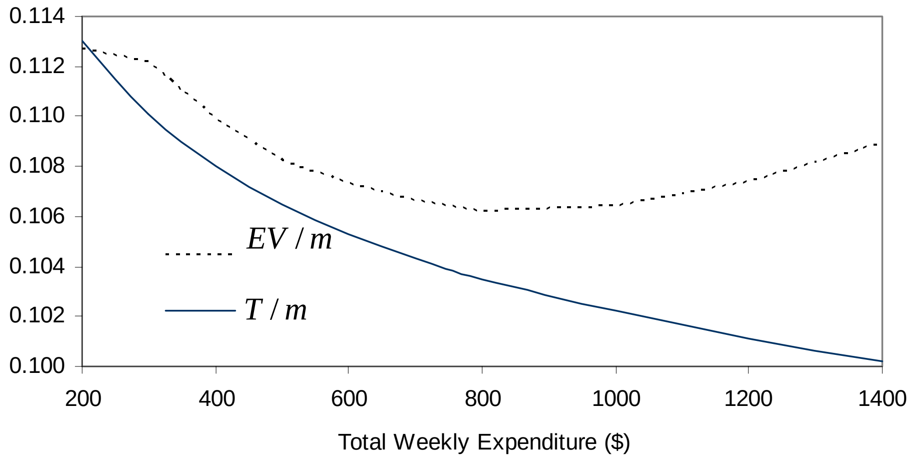
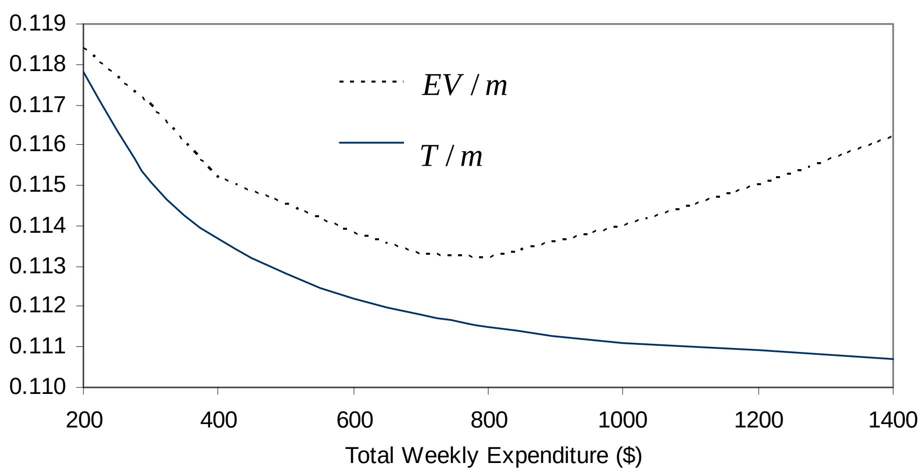


Figure 5 - Tax and Welfare Disproportionality of the Indirect Tax System: Adult Couple, One Child, Non-Smoking Households



5 Conclusions

This paper has considered the disproportional effects of indirect taxation using two alternative local measures, tax-progressivity and welfare-progressivity. In the context of an indirect tax imposed on a single good, welfare-progressivity requires the welfare loss from the tax as a fraction of total expenditure to rise with total expenditure. When the parameters of the direct utility function are held constant over all levels of total expenditure, it was shown that for both the AIDS and the LES, the condition for welfare-progressivity is the same as that required for tax-progressivity, namely that the taxed good is a luxury. However, this constancy (or preference homogeneity) implies a particular pattern for the variation in budget shares with total expenditure, which is different for each demand system.

When parameters are allowed to vary, using a general budget share relationship applied to both models, enabling heterogeneous preferences amongst households, it was found that tax- and welfare-progressivity can conflict. The empirical application of these conditions to New Zealand data showed that many such cases can arise. Furthermore, conflicting results were obtained when examining the disproportionality of the effective indirect tax structure in NZ (allowing for GST and Excise taxes). The majority of conflicts were found to arise where tax-regressivity existed at the same time as welfare-progressivity. The results show the importance of allowing for heterogeneous preferences in welfare analysis and further suggest that care should be taken when judging the effects of indirect taxes on the basis of tax revenue alone.

Appendix: Further Details of the LES and the AIDS

This appendix provides some further details regarding the derivation of the conditions for welfare-progressivity.

The Linear Expenditure System

Committed expenditure for good i can be written in the form:

$$c_i = p_i \gamma_i = \frac{m w_i (1+\eta_{ii})}{1-\beta_i} \tag{A1}$$

where η_{ii} is the own-price elasticity of demand for good i. The own-price elasticities are:

$$\eta_{ii} = e_i \left(\frac{1}{\xi} - w_i \left(1 + \frac{e_i}{\xi} \right) \right) \tag{A2}$$

where ξ denotes the Frisch parameter.[12] Using equation (A2), committed expenditure on good i becomes:[13]

$$c_i = m w_i \left(1 + \frac{e_i}{\xi} \right). \tag{A3}$$

Total committed expenditure takes the form:

$$C = \sum_{i=1}^{n} p_i \gamma_i = m \sum_{i=1}^{n} \left(w_i + \frac{1}{\xi} w_i e_i \right). \tag{A4}$$

As ξ is not commodity dependent, the 'adding-up' conditions, $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} e_i w_i = 1$ hold to give:

$$C = m \left(1 + \frac{1}{\xi} \right) \tag{A5}$$

Differentiating equation (4) with respect to total expenditure, while holding the parameters of the direct utility function, β_k and γ_k, constant, gives:

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[12] It is possible to allow the Frisch parameter to vary with total expenditure, as in Creedy (1998a).

[13] Using $w_i = (p_i x_i)/m$, the equation can alternatively be written as, $\frac{\gamma_i}{x_i} = \left(1 + \frac{e_i}{\xi}\right)$ which is the ratio of committed to actual consumption.

$$\frac{d}{dm}\left(\frac{EV}{m}\right)=\frac{1}{m^2}\left(C^0-\left(1+\dot{p}_k\right)^{-\beta_k}\left(C^0+c_k\,\dot{p}_k\right)\right) \tag{A6}$$

The tax imposed on good k is welfare-progressive when this change is strictly positive, which occurs when:

$$\left(1+\dot{p}_k\right)^{\beta_k}>1+\frac{c_k}{C^0}\,\dot{p}_k \tag{A7}$$

Using the above results, and assuming the Frisch parameter, ξ, to be constant, gives the condition for welfare-progressivity as:

$$\left(1+\dot{p}_k\right)^{w_k e_k}>1+\frac{w_k\left(\xi+e_k\right)}{\left(\xi+1\right)}\,\dot{p}_k \tag{A8}$$

Taking logs of both sides, applying the approximation $\log\left(1+x\right)=x$, and simplifying further gives the familiar condition that welfare-progressivity requires $e_k>1$, the same condition required for local tax-progressivity.

For the variable parameter case, using the general budget share relationship, equation (4) becomes:

$$\frac{EV}{m}=\frac{1}{\xi}\left[\left(1+\dot{p}_k\right)^{-\left[\delta_{1k}+\delta_{2k}\left(\log m+1\right)\right]}\left(1+\dot{p}_k\left[\xi\left(\delta_{1k}+\delta_{2k}\log m+\frac{\delta_{3k}}{m}\right)+\left(\delta_{1k}+\delta_{2k}\left(\log m+1\right)\right)\right]\right)-1\right] \tag{A9}$$

Differentiating this equation with respect to m and applying the approximation $\log\left(1+x\right)=x$, gives:

$$\frac{d}{dm}\left(\frac{EV}{m}\right)=\frac{\dot{p}_k}{m}\left(1+\dot{p}_k\right)^{-\beta_k}\left[\left(\delta_{2k}-\frac{\delta_{3k}}{m}\right)-\delta_{2k}w_k\,\dot{p}_k\left(1+\frac{e_k}{\xi}\right)\right] \tag{A10}$$

Welfare-progressivity is implied when the term in square brackets is strictly positive. Substituting for $\delta_{2k}-\left(\delta_{3k}/m\right)$ with $m\left(dw_k/dm\right)$ and then dividing by w_k leads the first term in the bracket to become $\left(e_k-1\right)$. Thus, welfare-progressivity occurs when:

$$e_k>1+\delta_{2k}\dot{p}_k\left(1+\frac{e_k}{\xi}\right) \tag{A11}$$

The Almost Ideal Demand System

From Shephard's lemma, the budget share for good i takes the form:

$$w_i = \frac{\partial \log E\left(p,U\right)}{\partial \log p} \tag{A12}$$

Hence, using the expenditure function in equation (5), the budget share, w_i, in the AIDS is defined by:

$$w_i = p_i \frac{\partial a\left(p\right)}{\partial p_i} + Up_i \frac{\partial b\left(p\right)}{\partial p_i} \tag{A13}$$

Using equations (6) and (7), it follows that:

$$w_i = \alpha_i + \sum_k \gamma_{ik} \log p_k + \beta_i \log\left(\frac{m}{P}\right) \tag{A14}$$

The total expenditure elasticity, e_i, for good i is thus:

$$\begin{aligned} e_i &= \frac{dw_i}{dm}\frac{m}{w_i} \\ e_i &= \frac{\beta_i + w_i}{w_i} \end{aligned} \tag{A15}$$

which enables the parameter β_i to be expressed as:

$$\beta_i = w_i\left(e_i - 1\right) \tag{A16}$$

For the fixed-parameter case, welfare-progressivity is examined by differentiating equation (12) with respect to total expenditure, holding the parameter of the direct utility function, β_k, constant, to give:

$$\frac{d}{dm}\left(\frac{EV}{m}\right) = \frac{1}{m}\left(1 - \left(1 + \dot{p}_k\right)^{-\beta_k}\right) \tag{A17}$$

Hence welfare-progressivity is implied when $1 > \left(1 + \dot{p}_k\right)^{-\beta_k}$. Taking logs of both sides and applying the approximation, $\log\left(1 + x\right) = x$, gives the condition:

$$\beta_k \dot{p}_k > 0 \tag{A18}$$

Using $\beta_k = w_k\left(e_k - 1\right)$ and substituting in equation (A18) again gives the condition that $e_k > 1$.

In the variable parameter case, substituting the expression for β_k into equation (12) gives:

$$\frac{EV}{m} = \log\left(\frac{m}{P^0}\right) - \left(1+\dot{p}_k\right)^{-\left[\delta_{2k} - \frac{\delta_{3k}}{m}\right]} \log\left(\frac{m}{P^1}\right) \qquad \text{(A19)}$$

Differentiating with respect to total expenditure, m:

$$\frac{d}{dm}\left(\frac{EV}{m}\right) = \frac{1}{m}\left(1+\left(1+\dot{p}_k\right)^{-\beta_k} \log\left(1+\dot{p}_k\right)\frac{\delta_{3k}}{m}\log\left(\frac{m}{P^1}\right) - \left(1+\dot{p}_k\right)^{-\beta_k}\right) \qquad \text{(A20)}$$

Constraining the derivative to be strictly positive, welfare-progressivity is implied when:

$$1 > \left(1+\dot{p}_k\right)^{-\beta_k}\left(1-\log\left(1+\dot{p}_k\right)\frac{\delta_{3k}}{m}\log\left(\frac{m}{P^1}\right)\right) \qquad \text{(A21)}$$

Taking logarithms of this expression:

$$0 > -\beta_k \log\left(1+\dot{p}_k\right) + \log\left(1-\log\left(1+\dot{p}_k\right)\frac{\delta_{3k}}{m}\log\left(\frac{m}{P^1}\right)\right) \qquad \text{(A22)}$$

Applying the approximation $\log\left(1+x\right) = x$ gives:

$$0 > -\beta_k \dot{p}_k - \dot{p}_k \frac{\delta_{3k}}{m}\log\left(\frac{m}{P^1}\right) \qquad \text{(A23)}$$

Substituting for β_k, welfare-progressivity is found to occur when:

$$e_k > 1 - \frac{\delta_{3k}}{m w_k}\log\left(\frac{m}{P^1}\right) \qquad \text{(A24)}$$

Table A1 - Household Groups

No.	Household Group	Number of Households		Mean Total Expenditure ($)	
		Smoking	Non-Smoking	Smoking	Non-Smoking
1	65+ Single	16	1282	267	274
2	65+ Couple	224	1191	498	540
3	Single Adult & No Children	384	1098	406	437
4	Single Adult & 1 Child	148	239	400	403
5	Single Adult & 2 Children	148	181	428	438
6	Single Adult & 3 Children	59	75	468	475
7	Single Adult & 4+ Children	33	39	501	539
8	Adult Couple & No Children	966	2036	690	766
9	Adult Couple & 1 Child	381	643	668	763
10	Adult Couple & 2 Children	435	916	707	896
11	Adult Couple & 3 Children	207	458	805	844
12	Adult Couple & 4+ Children	98	195	673	822
13	3 Adults & No Children	319	456	975	992
14	3 Adults & 1 Child	122	157	898	1038
15	3 Adults & 2+ Children	117	134	826	920
16	4+ Adults & No Children	179	192	1311	1282
17	4+ Adults & 1 Child	65	60	1110	1129
18	4+ Adults & 2+ Children	47	47	1070	925
	Total	4093	9399		

Table A2 - Commodity Groups and Effective *Ad Valorem* Tax Rates

No.	Commodity Group	Tax Rate (%)	No.	Commodity Group	Tax Rate (%)
1	Overseas Travel	0	12	Household Services	12.5
2	Rent	0	13	Adult's Clothing	12.5
			14	Children's Clothing	12.5
3	Recreational Vehicles	6.3	15	Public Transport in NZ	12.5
4	Vehicle Purchases	7.1	16	Vehicle Supplies, Parts etc	12.5
			17	Medical, Cosmetic etc	12.5
5	Food	12.5	18	Services	12.5
6	Food Outside Home	12.5	19	Other Expenditure	12.5
7	Pay to Local Authorities	12.5			
8	House Maintenance	12.5	20	Alcohol	46.8
9	Domestic Fuel and Power	12.5	21	Petrol	71.8
10	Household Equipment	12.5	22	Tobacco	239.8
11	Furnishings	12.5			

References

Brown, J.A.C. and Deaton, A.S. (1973) Models of consumer behaviour. In Surveys of Applied Economics, Vol. I, pp. 177-268. London: Macmillan.

Creedy, J. (1998a) Measuring the welfare effects of price changes: a convenient parametric approach. Australian Economic Papers, 37, pp. 137-151.

Creedy, J. (1998b) Measuring Welfare Changes and Tax Burdens. Cheltenham: Edward Elgar.

Creedy, J. and Sleeman, C. (2005) Excise Taxation in New Zealand.

Deaton, A.S. and Muellbauer, J. (1980) Economics and Consumer Behaviour. Cambridge: Cambridge University Press.

Dixon, P., Parmenter, B.R., Sutton, J. and Vincent, D. P. (1982) ORANI: A Multisectoral Model of the Australian Economy. Amsterdam: North-Holland.

Frisch, R. (1959) A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model With Many Sectors. Econometrica, 27, pp. 177-196.

McLeod, R., Chatterjee, S., Jones, S., Patterson, D. and Sieper, T. (2001) Final Report, 2001 Tax Review. New Zealand Treasury.

Muellbauer, J. (1974) Prices and inequality: the United Kingdom experience. Economic Journal, 84, pp. 32-55.

Musgrave, R.A. and Thin, T. (1948) Income tax progression. Journal of Political Economy, 56, pp. 498-514.

Tulpule, A. and Powell, A. A. (1978) *Estimates of Household Demand Elasticities for the Orani Model.* IMPACT Project Preliminary Working Paper, OP-22.

Williams, R. A. (1978) The Use of Disaggregated Cross-Section Data in Explaining Shifts in Australian Consumer Demand Patterns Over Time. *Impact Project Research Paper*, SP-13.

Powell, A.A. (1974) Empirical Analytics of Demand Systems. Lexington, Massachusetts: Lexington Books.

Young, L. (2002) Ad valorem indirect tax rates in New Zealand. New Zealand Treasury Internal Paper (#435217).