INDIRECT TAXATION AND PROGRESSIVITY:
REVENUE AND WELFARE CHANGES

by

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Indirect Taxation and Progressivity: Revenue and Welfare Changes

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Abstract
This paper compares the disproportional effects of indirect taxation using two alternative measures, tax-progressivity and welfare-progressivity. In the context of an indirect tax imposed on a single good, tax-progressivity requires the taxed good to be luxury. In contrast, welfare-progressivity requires the equivalent variation as fraction of total expenditure to rise with total expenditure. Sufficient conditions for welfare-progressivity are derived for both the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS). When the parameters of the direct utility functions are held constant, imposing homogeneous preferences, the condition required for welfare-progressivity is the same as that required for tax-progressivity, namely that the taxed good is a luxury. Parameter constancy also implies a particular pattern for the variation in budget shares with total expenditure, which is unique for each demand system. When parameters are allowed to vary with total expenditure, according to a general budget share relationship, which enables preference heterogeneity amongst households, welfare-progressivity is independent of tax-progressivity for both models, giving rise to possible conflicts in tax and welfare disproportionality. The empirical application of these conditions to New Zealand data shows that many such cases of conflict can arise. Furthermore, conflicting results are also obtained when examining the effects of the overall indirect tax structure. The majority of conflicts arise where tax-regressivity exists at the same time as welfare-progressivity.

JEL Classification H23; H22: H31
Keywords Tax progressivity; equivalent variations; budget shares
1 Introduction

An indirect tax is described as locally tax-progressive if a household’s average tax rate (the ratio of tax paid to total expenditure) rises with total household expenditure. By this definition, any tax imposed on a luxury good is tax-progressive. As total expenditure rises, the budget share attributed to the luxury good rises and consequently so must the average tax rate. Variations in the degree of tax-progressivity over a range of total expenditure levels depend only on the variation in the budget share attributed to the taxed good. Hence, tax-progressivity does not necessarily reflect the degree of welfare loss, as the latter depends, among other things, on the compensated price elasticity of demand for the good. Thus welfare-progressivity, which arises when the welfare loss, expressed as a fraction of total expenditure, rises with total expenditure, is not solely dependent on the variation in the budget share attributed to the taxed good. It is therefore not immediately obvious that tax-progressivity implies welfare-progressivity.

The aim of this paper is to consider the conditions under which a tax imposed on a luxury (or necessity) good is consistent with an increasing (or decreasing) ratio of the welfare loss to total expenditure. The welfare measure examined is the Hicksian equivalent variation. Of course, a good may change from being a luxury over a certain range of total expenditure to being a necessity over another range. For this reason the analysis is concerned only with the local measure of progressivity, which is unique to a chosen level of total expenditure.

Section 2 examines the case where the parameters of the direct utility function of a demand system are assumed to remain fixed as total expenditure varies. This assumption restricts households to have the same basic preferences, in spite of differing levels of total expenditure. Welfare-progressivity is examined for two demand systems; the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS),
where expressions for the welfare changes are readily obtained. Fixing the parameters of
the direct utility functions implies a particular form of variation in the budget shares of
each good as total expenditure rises. This implied variation, which is different for the
LES than for the AIDS, is shown in section 3. A more general form of variation in budget
shares, which has been found to be useful in empirical work, is therefore suggested which
combines elements implicit in both of these demand systems. The imposition of the more
general form leads the parameters of the direct utility function to vary with total
expenditure, allowing for heterogeneous preferences among households and therefore
more realistic analysis. Using the extended LES, section 4 uses data obtained from New
Zealand’s *Household Economic Surveys (HES)* to provide empirical examples of cases
where the directions of tax and welfare disproportionality conflict for the case of a tax
imposed on a single good and New Zealand’s indirect tax structure as a whole.
Conclusions are provided in section 5. First, for completeness the following subsection
presents the results relating to tax revenue for a single good.

### 1.1 Tax Revenue

Let $m$ denote a household’s total expenditure and $w_i = p_i x_i / \sum_{i=1}^{n} p_i x_i$ the budget share
devoted to the $i$th good, with $p_i$ and $x_i$ the price and quantity demanded of the good
respectively. If $t_i$ is the tax-inclusive *ad valorem* tax rate on the $i$th good, total indirect
tax revenue is $T = m \sum_{i=1}^{n} t_i w_i$. The Musgrave-Thin (1948) measure of local liability
progression is the elasticity of tax paid with respect to total expenditure (the ratio of the
marginal to the average tax rate), $\eta$, given by:

$$\eta = 1 + \sum_{i=1}^{n} \left( \frac{t_i w_i}{\sum_i t_i w_i} \right) \hat{w}$$

---

3 The tax-inclusive *ad valorem* tax rate is the ratio of tax paid to the tax-inclusive price of the good.
where \( \dot{w}_i = \frac{dw_i}{dm} \) is the elasticity of the budget share with respect to total expenditure.

Using the fact that the total expenditure elasticity is given by \( e_i = 1 + \dot{w}_i \), a progressive system, for which the elasticity \( \eta \) must be greater than unity, generally requires luxuries to be taxed more heavily than necessities. If taxation is imposed on only one good, say good \( k \), then \( \eta = e_k \) and the Musgrave-Thin local progressivity measure is given simply by the total expenditure elasticity.

2 Welfare Changes with Fixed Parameters

This section compares tax- and welfare-progressivity for the case where the parameters of the utility function are fixed, thereby imposing homogeneous preferences on all households. The analysis proceeds by imposing a tax on a single commodity. The equivalent variation resulting from the tax, \( EV \), is derived for each demand system and then the expression for \( EV / m \) is differentiated with respect to total expenditure, \( m \), while holding the parameters of the direct utility function constant. The conditions required to achieve welfare-progressivity (or regressivity) are found by constraining the derivative to be strictly positive (or negative).

2.1 The Linear Expenditure System

The Linear Expenditure System (LES) has direct utility functions of the form:

\[
U = \prod_{i=1}^{n} (x_i - \gamma_i)^{\beta_i} \tag{2}
\]

where \( \gamma_i \) is committed consumption with \( x_i > \gamma_i, 0 \leq \beta_i \leq 1, \sum_{i=1}^{n} \beta_i = 1 \). Define \( C = \sum_{i=1}^{n} p_i \gamma_i \) as total committed expenditure, and let \( B = \prod_{i=1}^{n} \left( \frac{p_i}{\beta_i} \right)^{\beta_i} \). It can be shown that when prices change from \( p^0 \) to \( p^1 \), the equivalent variation is:

\[4\] For details of the LES see, for example, Brown and Deaton (1973), Powell (1974) and Creedy (1998a, b). Muellbauer (1974) used the fixed parameter LES to examine the distributional effects of inflation in the UK.
If \( \hat{p}_i \) denotes the proportionate change in the price of the \( i \)th good, then

\[
\frac{C^1}{C^0} = 1 + \sum_{i=1}^{n} s_i \hat{p}_i \quad \text{where} \quad s_i = \left( \frac{p_i^0 \gamma_i}{\sum_{i=1}^{n} p_i^0 \gamma_i} \right). 
\]

The term \( \frac{B^1}{B^0} \) simplifies to

\[
\frac{B^1}{B^0} = \prod_{i=1}^{n} \left( \frac{p_i^1}{p_i^0} \right)^{\beta_i} = \prod_{i=1}^{n} (1 + \hat{p}_i)^{\beta_i}.
\]

Consider imposing a tax on a single good, \( k \), giving rise to a proportionate change in the price of the good, \( \hat{p}_k \), with \( \hat{p}_i = 0 \) for all other goods. The term \( \frac{C^1}{C^0} \) simplifies to \( \frac{C^1}{C^0} = 1 + s_k \hat{p}_k \) and \( \frac{B^1}{B^0} \) becomes \( \frac{B^1}{B^0} = (1 + \hat{p}_k)^{\beta_k} \). Let \( c_k = p_k^0 \gamma_k \) denote the initial level of committed expenditure on good \( k \). Hence the equivalent variation as a proportion of total expenditure is:

\[
EV = m - C^0 \left[ 1 + \frac{B^0}{B^1} \left( \frac{m \gamma_k}{C^0} - \frac{C^1}{C^0} \right) \right]
\]

Differentiating equation (4) with respect to total expenditure, while holding the parameters of the direct utility function, \( \beta_k \) and \( \gamma_k \) constant, using the property that \( \beta_i = e_i w_i \), and simplifying gives the familiar condition that welfare-progressivity requires \( e_k > 1 \), the same condition required for local tax-progressivity.

### 2.2 The Almost Ideal Demand System

The Almost Ideal Demand System (AIDS) is based on the expenditure function:

\[
\log E(p, U) = a(p) + b(p)U
\]

where the two price indices \( a(p) \) and \( b(p) \) are defined as:

\[
a(p) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{l=1}^{n} \gamma_{il} \log p_i \log p_l
\]

\[
b(p) = \beta_0 \prod_{i=1}^{n} p_i^{\beta_i}
\]

---

5 For further details of the AIDS see, Deaton and Muellbauer (1980) and Creedy (1998b).

6 The parameter \( \beta \) must be distinguished from its use in defining the LES above.
Denoting income by \( m \), \( m = \sum_{i=1}^{n} p_i x_i = E(p, U) \), the direct utility function takes the form,

\[
U = \left( \log m - a \right) / b .
\]  

(8)

It can be shown that when prices change from \( p^0 \) to \( p^1 \), the equivalent variation is:

\[
EV = m - \exp\left( a\left(p^0\right) + b\left(p^0\right) U^1 \right)
\]  

(9)

Defining the price index \( P = \exp a(p) \), this becomes:

\[
EV = m - P^0 \exp\left( b\left(p^0\right) \log\left( \frac{m}{P^1} \right) \right)
\]  

(10)

Consider imposing a tax on only good, \( k \), as before. The term \( b\left(p^0\right)/b\left(p^1\right) \) simplifies to \( b\left(p^0\right)/b\left(p^1\right) = \left(1 + \dot{p}_k\right)^{\beta_k} \), and the equivalent variation as a fraction of total expenditure is:

\[
\frac{EV}{m} = 1 - \exp\left( \left(1 + \dot{p}_k\right)^{-\beta_k} \log\left( \frac{m}{P^1} \right) - \log\left( \frac{m}{P^0} \right) \right)
\]  

(11)

Applying the approximation, \(-x = 1 - \exp(x)\), gives:

\[
\frac{EV}{m} = \log\left( \frac{m}{P^0} \right) - \left(1 + \dot{p}_k\right)^{-\beta_k} \log\left( \frac{m}{P^1} \right)
\]  

(12)

Differentiating equation (12) with respect to total expenditure, holding the parameter of the direct utility function, \( \beta_k \), constant, using the property that \( \beta_k = w_k (e_k - 1) \), and simplifying again gives the condition that \( e_k > 1 \) is required for a tax placed on the good to be locally welfare-progressive.
3 Welfare Changes with Variable Parameters

3.1 Budget Shares

The above analysis has held the parameters of the direct utility functions constant while considering the variation in the equivalent variation with total expenditure. This must imply a certain relationship between the budget share attributed to each good and the level of total expenditure, so that \( w_i \) must be a function of \( m \). For example, in the case of the LES, it has been stated that \( \beta_i = e_i w_i \), and using \( e_i = 1 + \hat{w}_i \), gives:

\[
\frac{w_i + m \frac{d w_i}{d m}}{m} = \beta_i
\]

(13)

\[
\frac{d (mw_i)}{d m} = \beta_i
\]

The solution to this differential equation is thus:

\[
mw_i = m \beta_i + d_i
\]

(14)

where \( d_i \) is a constant of integration. The budget share is therefore related to total expenditure as:

\[
w_i = \beta_i + \frac{d_i}{m}
\]

(15)

In the case of the AIDS, it has been stated above that \( \beta_i = w_i (e_i - 1) \), which gives:

\[
\beta_i = w_i \hat{w}_i = m \left( \frac{dw_i}{dm} \right)
\]

so that \( \left( \frac{dw_i}{dm} \right) = \beta_i / m \). Integration therefore leads to:

\[
w_i = h_i + \beta_i \log m
\]

(16)

where \( h_i \) is a constant of integration.\(^7\)

Empirical studies of budget data have found a more general form to be useful, which combines both the elements of the LES and AIDS models, whereby for good \( i \):

\[
w_i = \delta_{i1} + \delta_{i2} \log m + \frac{\delta_{i3}}{m}
\]

(17)

\(^7\) This form was explicitly discussed by Deaton and Muellbauer (1980), who also referred briefly to the more general form discussed below. They also pointed out that ordinary least squares regression estimates, for each commodity group at a time, would automatically satisfy the required adding up condition.
In practice, the parameters, \( \delta_{ii}, \delta_{2i} \) and \( \delta_{3i} \) can be estimated by regressing budget shares for good \( i \), obtained from sample surveys, on households’ total expenditure levels.\(^8\) Using this specification, the total expenditure elasticity for the good, \( e_i = 1 + \dot{w}_i \), is:

\[
e_i = \left( \frac{\delta_{ii} + \delta_{2i} (1 + \log m)}{\delta_{ii} + \delta_{2i} \log m + \delta_{3i}} \right) \tag{18}
\]

This more general budget share relationship leads the parameters of the LES and the AIDS to be functions of total expenditure, \( m \), which in turn allows for heterogeneous preferences among households. The consequence of this allowance for welfare-progressivity is explored in the following subsections.

### 3.2 The Extended Linear Expenditure System

In the LES, the parameter, \( \beta_i \), can be expressed as a function of \( m \) by substituting the above expression for \( e_i \) into \( \beta_i = e_i w_i \). Using this result to replace \( \beta_i \) in the expression for \( EV/m \) in equation (4), differentiating and then simplifying, leads the required condition for welfare-progressivity to become:

\[
e_k > 1 + \delta_{2k} \hat{p}_k \left( 1 + \frac{e_k}{\xi} \right) \tag{19}
\]

where \( \xi \) is the Frisch parameter, defined as the elasticity of marginal utility of total expenditure with respect to total expenditure; see Frisch (1959). This is assumed to be constant. For details of the derivation, see the Appendix.

Identifying good \( k \) as a luxury good is no longer sufficient to establish welfare-progressivity. Similarly, taxing a necessity is not sufficient to ensure welfare-regressivity. It is not surprising that the condition depends crucially on the parameter, \( \delta_{2k} \), as this is the additional parameter introduced into the LES budget share relationship, compared with the fixed parameter case. Setting this parameter to zero leads to the condition

\(^8\) This approach may not satisfy regularity conditions, in that some predicted budget shares at very low values of total expenditure can be negative, but this does not present problems in practice.
derived for the fixed parameter case, namely that $e_k > 1$. The condition shown in equation (19) is explored below in further detail for necessities and luxuries in turn.

### 3.2.1 Necessities

When good $k$ is a necessity, the necessary and sufficient conditions for welfare-progressivity and regressivity of the tax are:

**Condition i)** $\delta_{2k} > 0$

This is a sufficient condition for welfare-regressivity.

**Condition ii)** $\delta_{2k} < 0$

This is a necessary condition for welfare-progressivity. The sufficient condition is:

$$\delta_{2k} < \frac{\bar{e}_k (e_k - 1)}{\bar{p}_k (\bar{e}_k + e_k)} < 0$$

(20)

For necessities, the constraint $|\bar{e}_k| > 1$ ensures total committed expenditure is non-negative, and is sufficient to ensure committed expenditure on each good is also non-negative.

### 3.2.2 Luxuries

For a luxury good, the necessary and sufficient conditions for welfare-progressivity and regressivity of the tax are:

**Condition i)** $\delta_{2k} > 0$

This is a necessary condition for welfare-regressivity. The sufficient condition is:

$$\delta_{2k} > \frac{\bar{e}_k (e_k - 1)}{\bar{p}_k (\bar{e}_k + e_k)} > 0$$

(21)

**Condition ii)** $\delta_{2k} < 0$

This is a sufficient condition for welfare-progressivity.

When good $k$ is a luxury, the constraint $|\bar{e}_k| > 1$ is no longer sufficient to ensure committed expenditure on good $k$ is non-negative. Instead, the required constraint, using equation (A3) in the Appendix is $|\bar{e}_k| > e_k$.

These conditions are illustrated in Figure 1. The welfare-disproportionality of the tax is determined by the solid upward sloping line, which forms part of a hyperbola. Any
point lying above this line indicates that the tax imposed on good $k$ is welfare-regressive, while any point lying below the line indicates welfare-progressivity. This line is contrasted against the dotted line positioned at $e_k = 1$ which determines the tax-disproportionality of the indirect tax. Because of the differences between the two lines, there are two areas in which tax and welfare disproportionality can conflict, as shown in the figure. Raising the tax rate imposed on the good shifts the curve upward, while lowering the degree of convexity. The curve continues to pass through the point (1,0).

The conditions in this sub-section have been expressed in terms of the total expenditure elasticity of the good, $e_k$, so that a direct comparison with the simple condition for tax-progressivity could be made. However, $e_k$ depends on the parameters, $\delta_{1k}, \delta_{2k}, \delta_{3k}$, of the budget share relationship, so it may be argued that a more complete expression would establish the precise ranges of total expenditure over which tax-progressivity and welfare-progressivity simultaneously arise. This is unfortunately rather cumbersome analytically, although, as shown in section 4, ranges can readily be obtained numerically using the above conditions.

**Figure 1 - Tax and Welfare Disproportionality in the Extended LES**

\[
\delta_{2k} = \frac{\xi (e_k - 1)}{\hat{p}_k (\xi + e_k)}
\]
3.3 The Extended Almost Ideal Demand System

In the extended model, the budget shares vary with total expenditure according to the general form given above. The implied variation in $\beta_i$ is therefore:

$$\beta_i = w_i (e_i - 1)$$

$$\beta_i = \delta_{2i} - \delta_{3i} \frac{m}{m}$$

Substituting for $\beta_k$ into the expression for $EV/m$, shown in equation (12), differentiating and simplifying, as shown in further detail in the Appendix, gives the condition that welfare-progressivity requires:

$$e_k > 1 - \frac{\delta_{3k}}{m} \log \left( \frac{m}{P^i} \right)$$

Hence, as with the LES, identifying good $k$ as a luxury may no longer be sufficient to establish the welfare-progressivity of the tax. The conditions are given below for necessities and luxuries in turn.9

3.3.1 Necessities

The conditions characterising welfare-progressivity and regressivity are:

**Condition i)** $\delta_{3k} > 0$

This is a necessary condition for welfare-progressivity. The sufficient condition is:

$$\delta_{3k} > \frac{m w_i (1 - e_k)}{m} > 0$$

**Condition ii)** $\delta_{3k} < 0$

This is a sufficient condition for welfare-regressivity.

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9 These conditions are based on the assumption that $m > P^i$. 
3.3.2 Luxuries

When good \( k \) is a luxury, the necessary and sufficient conditions for the welfare-progressivity and regressivity of the tax are:

*Condition i*) \( \delta_{3k} > 0 \)

This is a sufficient condition for welfare-progressivity.

*Condition ii*) \( \delta_{3k} < 0 \)

This is a necessary condition for welfare-regressivity. The sufficient condition is:

\[
\delta_{3k} < \frac{mw_k(1-e_k)}{\log \left( \frac{m}{P^*} \right)} < 0
\]  

(25)

These conditions depend crucially on the parameter, \( \delta_{3k} \), as this is the additional term introduced by the more general budget share relationship. The conditions for tax and welfare disproportionality for the extended AIDS are summarised in Figure 2.

**Figure 2 - Tax and Welfare Disproportionality in the Extended AIDS**
4 Empirical Examples

The previous section showed that when allowing for heterogeneous preferences using either the extended LES or AIDS, a tax imposed on a luxury good, which is tax-progressive, may also be welfare-regressive. Using the extended LES, this section provides empirical examples of such cases where tax and welfare disproportionality conflict, using New Zealand data. Subsection 4.1 considers the case of imposing a tax on a single good, while subsection 4.2 compares the tax and welfare disproportionality of the current indirect tax system as a whole.

The measure of equivalent variation in the extended LES is computed using data collected from households who participated in the 1995, 1996, 1997, 1998 and 2001 Household Economic Surveys (HES). The weekly total expenditure data were adjusted to 2001 prices using the consumer price index (CPI). There were very few changes in indirect tax rates over this period. The surveys were then pooled to form one large database, providing approximately 13,500 households. Each household was placed into one of 18 demographic groups, shown in Table A1, which were further sub-divided into smoking and non-smoking households. A positive weekly expenditure on tobacco was sufficient for a household to be designated as a smoking household. The division into smoking and non-smoking households was found to improve substantially the fit of the estimated budget share relationships. Using equation (17), the budget shares collected from the HES were regressed on the household’s total expenditure levels for 22 commodity groups in turn, which are listed in Table A2. Thus, a total of 792 (22x18x2) regressions were performed. The following results were obtained using a value of $\xi = -2$. \(^{11}\)

4.1 A Single Tax

A tax (and proportional price) increase of 10 percent was imposed on each commodity group in turn, and the conditions derived in section 3 were examined for a range of total

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\(^{10}\) Surveys have only been conducted tri-annually since 1998. For further details of this approach, see Creedy and Sleeman (2005).

\(^{11}\) Tulpule and Powell (1978) used a value of $\xi=-1.82$ when calculating elasticities at average income for Australia, based on the work of Williams (1978), and this value was adopted by Dixon et al. (1982) in calibrating a general equilibrium model. The slightly higher absolute value was used here to avoid some negative committed expenditures.
expenditure levels, which were allowed to vary in $1 increments from $200 to $1500 per week. The directions of tax and welfare-progressivity were found to conflict in a total of 250 cases. Table 1 provides the number of conflicts which arose for each commodity group. The majority of conflicts occur where the tax revenue indicated local regressivity, but the equivalent variation indicated progressivity. In these cases, the revenue collected from the tax as a fraction of total expenditure was falling, while at the same time, the welfare loss from the tax as a fraction of total expenditure was rising.

Table 1 – Conflicting Movements in Tax and Welfare Disproportionality

<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>Overall</th>
<th>Tax-Regressive Welfare-Progressive</th>
<th>Tax-Progressive Welfare-Regressive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>27</td>
<td>26</td>
<td>1</td>
</tr>
<tr>
<td>Other Expenditure</td>
<td>24</td>
<td>21</td>
<td>3</td>
</tr>
<tr>
<td>Vehicle Supplies etc</td>
<td>24</td>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>Household Equipment</td>
<td>23</td>
<td>19</td>
<td>4</td>
</tr>
<tr>
<td>Petrol</td>
<td>22</td>
<td>19</td>
<td>3</td>
</tr>
<tr>
<td>Services</td>
<td>21</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Medical, Cosmetic etc</td>
<td>20</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>Alcohol</td>
<td>12</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Public Transport in NZ</td>
<td>12</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Food Outside Home</td>
<td>12</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Household Services</td>
<td>11</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Pay to Local Authorities</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Furnishings</td>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Food</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Cigarettes and Tobacco</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Children's Clothing</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>House Maintenance</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Overseas Travel</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Adult Clothing</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Vehicle Purchase</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Domestic Fuel and Power</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Recreational Vehicles</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>250</strong></td>
<td><strong>188</strong></td>
<td><strong>62</strong></td>
</tr>
</tbody>
</table>

Tables 2 and 3 provide further details of the demographic groups and the ranges of total expenditure over which conflicts occurred. As it is not possible to show all cases, only those where the range of total expenditure exceeded $2 are provided in the two tables. It is clear from these results that the possibility of conflicting indications of
disproportionality when using tax and welfare measures is far from negligible. Further, there appears no particular pattern in terms of the expenditure ranges or the demographic groups for which these conflicts can arise. Thus, regardless of the type of good on which a tax is imposed, the disproportionality of the tax should not be decided on without first evaluating the welfare losses of the proposed reform for a selection of demographic groups and total expenditure levels.

Table 2 - Cases of Tax-Regressivity and Welfare-Progressivity

<table>
<thead>
<tr>
<th>Rent</th>
<th>Expenditure Range, m</th>
<th>Other Expenditure</th>
<th>Expenditure Range, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Single Adult &amp; 1 Child 376 - 381</td>
<td>S Adult Couple and 3 Children 985 - 988</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Single Adult &amp; 2 Children 369 - 373</td>
<td>3 Adults &amp; No Children 831 - 834</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Single Adult &amp; 3 Children 413 - 420</td>
<td>3 Adults and 2+ Children 1052 - 1055</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>4+ Adults &amp; No Children 1084 - 1089</td>
<td>NS Adult Couple and 2 Children 947 - 951</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>4+ Adults &amp; 2+ Children 697 - 700</td>
<td>Adult Couple and 3 Children 1238 - 1242</td>
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</tr>
<tr>
<td>NS</td>
<td>Single Adult &amp; 1 Child 285 - 288</td>
<td>3 Adults &amp; No Children 1094 - 1098</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Single Adult &amp; 2 Children 332 - 336</td>
<td>3 Adults and 2+ Children 1175 - 1180</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Single Adult &amp; 3 Children 247 - 250</td>
<td>4+ Adults &amp; No Children 1011 - 1015</td>
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<tr>
<td>S</td>
<td>Single Adult &amp; 4+ Children 441 - 448</td>
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<table>
<thead>
<tr>
<th>Services</th>
<th>Expenditure Range, m</th>
<th>Food</th>
<th>Expenditure Range, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 3 Adults &amp; No Children 981 - 986</td>
<td>NS Adult Couple $ 4+ Children 256 - 259</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NS 65+ Single 907 - 913</td>
<td>4+ Adults &amp; 2+ Children 293 - 296</td>
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<td></td>
</tr>
<tr>
<td>Adult Couple and 2 Children 931 - 937</td>
<td>Food Outside Home</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Adults &amp; 1 Child 1116 - 1124</td>
<td>NS 3 Adults &amp; No Children 1441 - 1444</td>
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<td></td>
</tr>
<tr>
<td>3 Adults and 2+ Children 1425 - 1435</td>
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<td></td>
</tr>
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Table 3 - Cases of Tax-Progressivity and Welfare-Regressivity

<table>
<thead>
<tr>
<th>Food</th>
<th>Expenditure Range, m</th>
<th>Services</th>
<th>Expenditure Range, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>Single Adult &amp; 4+ Children 698 - 704</td>
<td>S 4+ Adults &amp; No Children 706 - 709</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4+ Adults &amp; 2+ Children 965 - 969</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Single Adult &amp; 4+ Children 939 - 947</td>
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</table>
4.2 The Indirect Tax System

Having analysed the case of a single tax, the question arises as to whether a conflict between the direction of tax and welfare disproportionality could arise when considering a large number of indirect taxes combined, such as the effective current structure of indirect taxes in New Zealand. The rates are summarised in Table A2.

It was found that a variety of cases exist where tax revenues from the current indirect tax structure indicate tax-regressivity, while the welfare losses from the taxes indicate progressivity. Figures 3 to 5 provide examples of this occurrence for three demographic groups. In each case, the current structure of indirect taxes, taken together, is tax-regressive over all ranges of total expenditure, but welfare-progressive over higher ranges. Reliance on tax revenue alone thus gives an incomplete indication of progressivity.

Figure 3 - Tax and Welfare Disproportionality of the Indirect Tax System:
65+ Single Adults, Non-Smoking Households
Figure 4 - Tax and Welfare Disproportionality of the Indirect Tax System:
Single Adult, No Children, Non-Smoking Households

Figure 5 - Tax and Welfare Disproportionality of the Indirect Tax System:
Adult Couple, One Child, Non-Smoking Households
5 Conclusions

This paper has considered the disproportional effects of indirect taxation using two alternative local measures, tax-progressivity and welfare-progressivity. In the context of an indirect tax imposed on a single good, welfare-progressivity requires the welfare loss from the tax as a fraction of total expenditure to rise with total expenditure. When the parameters of the direct utility function are held constant over all levels of total expenditure, it was shown that for both the AIDS and the LES, the condition for welfare-progressivity is the same as that required for tax-progressivity, namely that the taxed good is a luxury. However, this constancy (or preference homogeneity) implies a particular pattern for the variation in budget shares with total expenditure, which is different for each demand system.

When parameters are allowed to vary, using a general budget share relationship applied to both models, enabling heterogeneous preferences amongst households, it was found that tax- and welfare-progressivity can conflict. The empirical application of these conditions to New Zealand data showed that many such cases can arise. Furthermore, conflicting results were obtained when examining the disproportionality of the effective indirect tax structure in NZ (allowing for GST and Excise taxes). The majority of conflicts were found to arise where tax-regressivity existed at the same time as welfare-progressivity. The results show the importance of allowing for heterogeneous preferences in welfare analysis and further suggest that care should be taken when judging the effects of indirect taxes on the basis of tax revenue alone.
Appendix: Further Details of the LES and the AIDS

This appendix provides some further details regarding the derivation of the conditions for welfare-progressivity.

The Linear Expenditure System

Committed expenditure for good $i$ can be written in the form:

$$c_i = p_i \gamma_i = \frac{mw_i(1 + \eta_i)}{1 - \beta_i} \quad (A1)$$

where $\eta_i$ is the own-price elasticity of demand for good $i$. The own-price elasticities are:

$$\eta_i = e_i \left( \frac{1}{\xi} - w_i \left( 1 + \frac{e_i}{\xi} \right) \right) \quad (A2)$$

where $\xi$ denotes the Frisch parameter.\(^{12}\) Using equation (A2), committed expenditure on good $i$ becomes:\(^{13}\)

$$c_i = mw_i \left( 1 + \frac{e_i}{\xi} \right) \quad (A3)$$

Total committed expenditure takes the form:

$$C = \sum_{i=1}^{n} p_i \gamma_i = m \sum_{i=1}^{n} \left( w_i + \frac{1}{\xi} w_i e_i \right) \quad (A4)$$

As $\xi$ is not commodity dependent, the ‘adding-up’ conditions, $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} e_i w_i = 1$ hold to give:

$$C = m \left( 1 + \frac{1}{\xi} \right) \quad (A5)$$

Differentiating equation (4) with respect to total expenditure, while holding the parameters of the direct utility function, $\beta_k$ and $\gamma_k$, constant, gives:

---

\(^{12}\) It is possible to allow the Frisch parameter to vary with total expenditure, as in Creedy (1998a).

\(^{13}\) Using $w_i = (p_i x_i) / m$, the equation can alternatively be written as, $\frac{C}{x_i} = \left( 1 + \frac{e_i}{\xi} \right)$ which is the ratio of committed to actual consumption.
\[
\frac{d}{dm}\left(\frac{EV}{m}\right) = \frac{1}{m^2} \left[ C_0 \bigg\{ \left(1 + \dot{p}_k\right)^{\beta_k} \left( C_0 + c_k \dot{p}_k \right) \bigg\} \right] \quad (A6)
\]

The tax imposed on good \( k \) is welfare-progressive when this change is strictly positive, which occurs when:

\[
(1 + \dot{p}_k)^{\beta_k} > 1 + \frac{c_k}{C_0} \dot{p}_k \quad (A7)
\]

Using the above results, and assuming the Frisch parameter, \( \xi \), to be constant, gives the condition for welfare-progressivity as:

\[
(1 + \dot{p}_k)^{w_k \epsilon_k} > 1 + \frac{w_k (\xi + e_k)}{(\xi + 1)} \dot{p}_k \quad (A8)
\]

Taking logs of both sides, applying the approximation \( \log(1+x) = x \), and simplifying further gives the familiar condition that welfare-progressivity requires \( e_k > 1 \), the same condition required for local tax-progressivity.

For the variable parameter case, using the general budget share relationship, equation (4) becomes:

\[
\frac{EV}{m} = \frac{1}{\xi} \left[ (1 + \dot{p}_k)^{\left[ \delta_k + \delta_{2k}(\log m + 1) \right]} \left[ 1 + \dot{p}_k \left[ \xi \left( \delta_k + \delta_{2k} \log m + \delta_{1k} \left( \log m + 1 \right) \right) \right] - 1 \right] \right] \quad (A9)
\]

Differentiating this equation with respect to \( m \) and applying the approximation \( \log(1+x) = x \), gives:

\[
\frac{d}{dm}\left(\frac{EV}{m}\right) = \frac{\dot{p}_k}{m} \left(1 + \dot{p}_k\right)^{-\beta_k} \left[ \left( \frac{\delta_k}{m} - \frac{\delta_{2k}}{m} \right) - \delta_{2k} w_k \dot{p}_k \left(1 + \frac{e_k}{\xi} \right) \right] \quad (A10)
\]

Welfare-progressivity is implied when the term in square brackets is strictly positive. Substituting for \( \delta_{2k} - \left( \delta_{3k} / m \right) \) with \( m (d w_k / dm) \) and then dividing by \( w_k \) leads the first term in the bracket to become \( (e_k - 1) \). Thus, welfare-progressivity occurs when:

\[
e_k > 1 + \delta_{2k} \dot{p}_k \left(1 + \frac{e_k}{\xi} \right) \quad (A11)
\]
The Almost Ideal Demand System

From Shephard’s lemma, the budget share for good $i$ takes the form:

$$ w_i = \frac{\partial \log E(p, U)}{\partial \log p} \tag{A12} $$

Hence, using the expenditure function in equation (5), the budget share, $w_i$, in the AIDS is defined by:

$$ w_i = p_i \frac{\partial a(p)}{\partial p_i} + U p_i \frac{\partial b(p)}{\partial p_i} \tag{A13} $$

Using equations (6) and (7), it follows that:

$$ w_i = \alpha_i + \sum_k \gamma_{ik} \log p_k + \beta_i \log \left( \frac{m}{P} \right) \tag{A14} $$

The total expenditure elasticity, $e_i$, for good $i$ is thus:

$$ e_i = \frac{dw_i}{dm} \frac{m}{w_i} = \frac{\beta_i + w_i}{w_i} \tag{A15} $$

which enables the parameter $\beta_i$ to be expressed as:

$$ \beta_i = w_i (e_i - 1) \tag{A16} $$

For the fixed-parameter case, welfare-progressivity is examined by differentiating equation (12) with respect to total expenditure, holding the parameter of the direct utility function, $\beta_k$, constant, to give:

$$ \frac{d}{dm} \left( \frac{EV}{m} \right) = \frac{1}{m} \left( 1 - (1 + \hat{p}_k)^{-\beta_k} \right) \tag{A17} $$

Hence welfare-progressivity is implied when $1 > (1 + \hat{p}_k)^{-\beta_k}$. Taking logs of both sides and applying the approximation, $\log (1 + x) = x$, gives the condition:

$$ \beta_k \hat{p}_k > 0 \tag{A18} $$
Using $\beta_k = w_k (e_k - 1)$ and substituting in equation (A18) again gives the condition that $e_k > 1$.

In the variable parameter case, substituting the expression for $\beta_k$ into equation (12) gives:

$$\frac{EV}{m} = \log \left( \frac{m}{P^i} \right) - \left( 1 + \hat{p}_k \right)^{\delta_{kk}} \log \left( \frac{m}{P^i} \right)$$

(A19)

Differentiating with respect to total expenditure, $m$:

$$\frac{d}{dm} \left( \frac{EV}{m} \right) = \frac{1}{m} \left( 1 + \hat{p}_k \right)^{-\beta_k} \log \left( 1 + \hat{p}_k \right) \frac{\delta_{kk}}{m} \log \left( \frac{m}{P^i} \right) - \left( 1 + \hat{p}_k \right)^{-\beta_k}$$

(A20)

Constraining the derivative to be strictly positive, welfare-progressivity is implied when:

$$1 > \left( 1 + \hat{p}_k \right)^{-\beta_k} \left( 1 - \log \left( 1 + \hat{p}_k \right) \frac{\delta_{kk}}{m} \log \left( \frac{m}{P^i} \right) \right)$$

(A21)

Taking logarithms of this expression:

$$0 > -\beta_k \log \left( 1 + \hat{p}_k \right) + \log \left( 1 - \log \left( 1 + \hat{p}_k \right) \frac{\delta_{kk}}{m} \log \left( \frac{m}{P^i} \right) \right)$$

(A22)

Applying the approximation $\log (1 + x) = x$ gives:

$$0 > -\beta_k \hat{p}_k - \hat{p}_k \frac{\delta_{kk}}{m} \log \left( \frac{m}{P^i} \right)$$

(A23)

Substituting for $\beta_k$, welfare-progressivity is found to occur when:

$$e_k > 1 - \frac{\delta_{kk}}{m w_k} \log \left( \frac{m}{P^i} \right)$$

(A24)
Table A1 - Household Groups

<table>
<thead>
<tr>
<th>No.</th>
<th>Household Group</th>
<th>Number of Households</th>
<th>Mean Total Expenditure ($)</th>
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<tr>
<td></td>
<td></td>
<td>Smoking</td>
<td>Non-Smoking</td>
</tr>
<tr>
<td>1</td>
<td>65+ Single</td>
<td>16</td>
<td>1282</td>
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<tr>
<td>2</td>
<td>65+ Couple</td>
<td>224</td>
<td>1191</td>
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<tr>
<td>3</td>
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<td>384</td>
<td>1098</td>
</tr>
<tr>
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<td>Single Adult &amp; 1 Child</td>
<td>148</td>
<td>239</td>
</tr>
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<td>5</td>
<td>Single Adult &amp; 2 Children</td>
<td>148</td>
<td>181</td>
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<td>6</td>
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<td>59</td>
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<td>Single Adult &amp; 4+ Children</td>
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<td>39</td>
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<td>8</td>
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<td>966</td>
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<td>381</td>
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<td>Adult Couple &amp; 2 Children</td>
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<td>916</td>
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<tr>
<td></td>
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Table A2 - Commodity Groups and Effective Ad Valorem Tax Rates

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<th>Tax Rate (%)</th>
<th>No.</th>
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<th>Tax Rate (%)</th>
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<td>Rent</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>14</td>
<td>Children’s Clothing</td>
<td>12.5</td>
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<tr>
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<td>Public Transport in NZ</td>
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<td>17</td>
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<tr>
<td>5</td>
<td>Food</td>
<td>12.5</td>
<td>18</td>
<td>Services</td>
<td>12.5</td>
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<td>Pay to Local Authorities</td>
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References


