ISSN 0819-2642
ISBN 0 7340 2600 5

THE UNIVERSITY OF MELBOURNE

DEPARTMENT OF ECONOMICS

RESEARCH PAPER NUMBER 944

JULY 2005

SNOBS AND QUALITY GAPS

by

Suren Basov

Department of Economics
The University of Melbourne
Melbourne Victoria 3010
Australia.

Snobs and quality gaps

Suren Basov

Department of Economics, The University of Melbourne, Melbourne, Victoria 3010, Australia (e-mail: s.basov@econ.uimelb.edu.au)

Summary. In this paper I revisit the Mussa and Rosen (1978) model. However, unlike Mussa and Rosen, I assume that there is a positive mass of the consumers of the highest possible type. I call them snobs. I prove that snobs consumers are served efficiently and the product line decreases in the mass of the serious consumers. Moreover, if the mass of the serious consumers is more than some critical level then they are the only consumers who are served at equilibrium.

Keywords and Phrases: screening, product line, efficiency.

JEL Classification Numbers: C0, D8.

1 Introduction

The topic of imperfect information first attracted researches in the middle of the seventies. Probably the first paper in the area is Mirrlees (1971). Some of the early papers (e. g. Adams and Yellen, 1976) used a finite number of types. For an excellent review of such models and the main result in that area, the so called constraint reduction theorem, one can consult the still unpublished but available on the web notes of Stole (2000). From Mussa and Rosen (1978) on the interest firmly shifted to the models with a continuum of types. Similar models, arising from different economic situations, were considered by Mirman and Sibley (1980), Baron and Myerson (1982), Sappington (1983), and Maskin and Riley (1984) among others. For a textbook treatment of the model with a continuum of types see, for example, Fudenberg and Tirole (1992).

Models with a discrete set and a continuum of types share some common features. In both types of models the top type is served efficiently and the lowest type gets the reservation utility.[1] One important difference, however, is that in discrete models it is possible that only one type, the highest one, is

[1]Provided that the participation constraint is not type dependent. See Jullien (2000) for a discussion.

served at equilibrium, while in the models with a continuum of types the set of types served at equilibrium always includes more than one type (in fact, continuum of types).

In this paper I consider a model in which the distribution of types is absolutely continuous on $(0, 1)$ but possesses an atom at one. I call that consumers of the highest possible type *snobs*. The upper bound of the type space reflects the price at which another luxury good of a fixed quality can be pirchased. For example, assume that a monopolist is a car producer that produces cars of different qualities. The General Motors, for instance, produces Cadillac, Buick, Oakland, Chevrolet, which differ in quality and target different segments of the market. Caddilac is designed for the snobs and a big part of utility they derive from driving the car comes fron showing off. However, if the price of a Cadillac goes too high, the snobs might choose another way to show off. For example, she may buy a yacht. Then the maximal price anyone will be willing to pay for a Caddilac will be limited by the price of a yacht. Therefore, though a priori the valuations of snobs of the car per se might differ, after including the value of showing off and optimizing over the ways to do so, the will tend to cluster at some value, which I normalize to be one. As a result, one will observe a probability mass at one.

Another way to justify the mass at one is to assume that there are two types of the consumers: privater and industrial. The marginal utility for the industrial consumers is simply equal to the marginal product, which in turn is determined by the most efficient current technology. I will normalize the marginal utility of the industrial consumers to be one. I will also assume that there is a continuum of private consumers whose tastes for quality are distributed on $(0, 1)$ according to a continuous density function. Existence of the industrial quality goods is a widely observed phenomenon. Google search produces 131,000 matches for the industrial quality and advertises a variety of goods such as xeroxes, water filters, saw blades, and many others. These goods are usually available at industrial quality or ordinary quality levels. The model developed in this paper explains the prevalence of industrial quality goods without assuming that there is a gap in tastes between private and institutional consumers. This feature is rather attractive, since some private consumers may have rather high demands for quality.

The first result is that, probably not surprisingly, the top types are still served efficiently and the lowest type earns no information rents. More interesting result is that the size of the optimal product line decreases in the mass of serious consumers. Moreover, if the mass of serious consumers is above some critical level, the serious consumers are the only ones served at equilibrium. For a wide class of distributions this threshold level depends only on the limit of the density of not serious consumers at the right end of the distribution and is unrelated to the finer details of the distribution. Moreover, the optimal quality is discontinuous at the right hand of the distribution, i. e. there is a quality gap. It is worth mentioning that the results will remain qualitatively the same if the quality probability mass is concentrated in the interior point of the type space, rather than at its right hand. Such a formulation will allow for some private consumers have a higher valuation for the quality than do the industrial consumers. I will continue to assume that the positive mass is on the right end for simplicity.

The paper is organized in a following well. In Section 2 I introduce the model and discuss its general properties. In Section 3 I solve an example. Section 4 concludes.

2 The model

Consider a continuum of consumers each of whom is interested in buying at most one unit of an indivisible good. Different units of the goods may, however, differ in quality, x. The marginal rate of substitution between quality and money, α, does not depend on quality but differs across the consumers, i. e. the utility has a form

$$u(\alpha, x, t) = \alpha x - t, \tag{1}$$

where t is the amount paid to the monopolist. I assume that α is private information of the consumer. However, it is common knowledge that α is distributed on $[0, 1]$ according to a measure

$$\mu = (1-\gamma)\lambda + \gamma\delta_1, \tag{2}$$

where $\gamma \in (0, 1)$ is the mass of the serious consumers, measure λ is absolutely continuous with respect to the Lebesgue measure with the Radon-Nykodim derivative $f(\cdot)$, while δ_1 is the Dirac's measure concentrated at one. We assume that f is twice differentiable and strictly positive and weakly increasing on $[0, 1)$ and

$$\lim_{\alpha \to 1} f(\alpha) \equiv f(1) < \infty. \tag{3}$$

Let $F(\cdot)$ be the cumulative distribution function, corresponding to density $f(\cdot)$. Define the virtual type

$$v(\alpha) = \alpha - \frac{1 - F(\alpha)}{f(\alpha)}, \tag{4}$$

and assume that it is strictly increasing in α. This assumptions allows us to concentrate on the so-called relaxed problem, i. e. the problem in which we drop the constraint that the allocation $x(\cdot)$ is increasing (see, Mussa and Rosen, 1978).

The utility of the outside option is same across the consumers and is normalized to be zero. The cost of production is convex in quality and linear across consumers and is given by a twice differentiable, strictly increasing, convex function $c(x)$. I assume that

$$c(0) = c'(0) = 0. \tag{5}$$

The above consideration can be summarized by the following model. The monopolist select a measurable function $t : R \to R$ to solve

$$\max_{t(\cdot)} \int_0^1 (t(x(\alpha)) - c(x(\alpha)))d\mu(\alpha), \tag{6}$$

subject to

$$\begin{aligned} x(\alpha) &\in \arg\max(u(\alpha, x) - t(x)) && (7) \\ \max(u(\alpha, x) - t(x)) &\geq 0. && (8) \end{aligned}$$

The first of this constraints is known as the incentive compatibility constraint and guarantees that each consumer selects optimally, while the second is the individual rationality or participation constraint that states that each consumer should get utility, which is at least as large as that of the outside option.

Introducing the consumer surplus by

$$s(\alpha) = \max_{x \geq 0}(\alpha x - t(x)) \tag{9}$$

one can write the monopolist's objective as:

$$\max \int_0^1 (\alpha x - c(x) - s) d\mu(\alpha). \tag{10}$$

Using (2) this expression can be transformed to:

$$(1-\gamma) \int_0^1 (\alpha x - c(x) - s) f(\alpha) d\alpha + \gamma(x(1) - c(x(1)) - s(1)). \tag{11}$$

Note that equation (9) and the envelope theorem[2] imply:

$$s'(\alpha) = x(\alpha) \tag{12}$$

for all $\alpha \in (0,1)$. It is also possible to show that $s(\cdot)$ is absolutely continuous. Therefore,

$$s(1) = s(0) + \int_0^1 x(\alpha) d\alpha. \tag{13}$$

Transforming the first integral in (11) using integration by parts (see, Mussa and Rosen, 1978) and taking into account (13) the monopolist's objective

[2]See Milgrom and Segal (2002) for the most general formulation of the envelope theorem.

takes the form:

$$(1-\gamma)\int_0^1 [(v(\alpha)-c(x))f(\alpha)-\frac{\gamma}{1-\gamma}]d\alpha+\gamma(x(1)-c(x(1))-s(0). \quad (14)$$

The optimality conditions now imply,

$$s(0)=0. \quad (15)$$

i. e. the lowest type gets her reservation utility,

$$c'(x(1))=1, \quad (16)$$

the "no distortion at the top" property and

$$c'(x)=\max(v_\gamma(\alpha),0) \quad (17)$$

for $\alpha\in[0,1)$. Here

$$v_\gamma(\alpha)=v(\alpha)-\frac{\gamma}{(1-\gamma)f(\alpha)}. \quad (18)$$

Note, that under our assumption on the distribution of types $v_\gamma(\alpha)$ increases in α for all γ. Therefore, since the cost in convex in x, the allocation defined by (17) is increasing and therefore, implementable. Also note that

$$v_\gamma(\alpha) < v(\alpha) \tag{19}$$

for all α. This implies three things. First, if type α is served under both conditions $\gamma = 0$ and $\gamma > 0$ the downward distortion is stronger under the second regime. Second, the exclusion region increases in γ in the set theoretic sense. Third, the product line defined by:

$$[0, v_\gamma(1)) \cup \{x(1)\} \tag{20}$$

is decreasing in γ and is not connected for $\gamma > 0$. Note that if

$$v_\gamma(1) \leq 0 \tag{21}$$

for all values of α, the snobs are the only ones served in the equilibrium. This happens if

$$\frac{\gamma}{1-\gamma} \geq f(1). \tag{22}$$

Note that γ depends only on the value of $f(1)$ (it is increasing in $f(1)$) and not on the finer details of the distribution. Note also that since $f(\cdot)$ is assumed to be non-decreasing and integrate to one, $f(1) \geq 1$, therefore in order to exclude all non-serious consumers γ should be at least $1/2$.

3 A numerical example

Let us consider a specific case of the mode of the previous section. Assume that

$$f(\alpha) = 1, \tag{23}$$

the cost of production is quadratic

$$c(x) = \frac{1}{2}x^2 \tag{24}$$

and $\gamma = 1/3$. Then

$$v_\gamma(\alpha) = 2\alpha - \frac{3}{2}. \tag{25}$$

The exclusion region is $[0, 3/4]$ which is a superset of the $[0, 1/3]$, the exclusion region for $\gamma = 0$. The product line is

$$[0, \frac{1}{2}) \cup \{1\}, \tag{26}$$

where qualities between in the range $[0, 1/2)$ are purchased by the consumers whose types belong to $[3/4, 1)$, while the serious consumers purchase the good of the quality one. A good with quality $x \in [0, 1/2)$ can be purchased at a price

$$t(x) = \frac{x^2 + 3x}{4}, \tag{27}$$

while the good of quality one can be purchased at

$$t(1) = \frac{15}{16}. \tag{28}$$

Note that the serious consumers are served efficiently and are indifferent between selecting their contract or purchasing good of quality $1/2$ at $t(1/2) = 7/16$. However, since the good of quality exactly $1/2$ is not offered by the monopolist, they strictly prefer their contract to any other deal offered by the market.

4 Conclusions

In this paper I take the first step to bridge the gap between screening models with discrete and continuous types. For this purpose, I revisit the

Mussa and Rosen (1978) model. However, unlike Mussa and Rosen, I assume that there is a positive mass of the consumers of the highest possible type. I call them serious consumers.

The main results of the paper are the following. The serious consumers are served efficiently and the product line decreases in the mass of the serious consumers. Moreover, if the mass of the serious consumers is more than some critical level then they are the only consumers who are served at equilibrium. For a wide class of distributions this threshold level depends only on the limit of the density of not serious consumers at the right end of the distribution and is unrelated to the finer details of the distribution. Moreover, the optimal quality is discontinuous at the right hand of the distribution, i. e. the optimal product line is not connected. The efficient quality always belongs to the optimal product line.

REFERENCES

W. J. Adams, and J. L. Yellen, 1976, Commodity bundling and the burden of monopoly, Quarterly Journal of Economics, 90,. 475-498

D. Baron, and R. Myerson, 1982, Regulating a monopolist with unknown cost, Econometrica, 50, 911-930.

D. Fudenberg, and J. Tirole, 1992, Game theory, (Cambridge, MA: MIT Press).

B. Jullien, 2000, Participation constraints in adverse selection models, Journal of Economic Theory, 93, 1-47.

T. Lewis, and D. Sappington, 1989, Countervailing incentives in agency theory, Journal of Economic Theory 49, 294-313.

A. Mas-Colell, M. D. Whinston, and J. R. Green, 1995, Microeconomic theory, (Oxford: Oxford University Press).

E. Maskin, and J. Riley, 1984, Monopoly with incomplete information, Rand Journal of Economics 15 171-196.

P. Milgrom, and I. Segal, 2002, Envelope theorems for arbitrary choice sets, Econometrica, 70, 583-601.

J. Mirrlees, 1971, An exploration in the theory of optimum income taxation, Review of Economic Studies, 38, 175-208.

L. J. Mirman, and D. Sibley, 1980, Optimal nonlinear prices for multiproduct monopolies, Bell Journal of Economics, 11, 659-675.

M. Mussa, and S. Rosen, 1978, Monopoly and product quality, Journal of Economic Theory 18, 301-317.

D. Sappington, 1983, Optimal regulation of the monopolist with unknown technological capabilities, Bell Journal of Economics, 15, 453-463.

L. A. Stole, 2000, Lectures on contracts and organizations,

http://gsblas.uchicago.edu/papers/lectures.pdf