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Sustainable Preferences and Damage Abatement: Value Judgments and Implications for Consumption Streams

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Abstract

This paper examines the implications of adopting alternative value judgements when evaluating future consumption streams in the context of damage abatement. The paper focusses on a form of 'sustainable preferences' designed to avoid either a dictatorship by present or by future generations which can arise when using a 'standard' social welfare function. Numerical examples are reported, based on a simple growth model, under alternative damage abatement parameters and welfare functions. The results illustrate how sustainable preferences effectively reduce the damages on future consumption by shifting consumption from the present to the future. This implies an intergenerational trade-off. An explicit policy of damage abatement under a standard social welfare function implies a similar intergenerational trade-off. However, the results suggest that damage abatement does not penalise current generations as much under sustainable preferences as it does under standard value judgements.

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1 Introduction

This paper examines the implications of adopting alternative value judgements when evaluating consumption streams over a very long period. In particular the use of a social welfare function with a positive and constant pure time preference rate is compared with value judgements reflecting ‘sustainable preferences’, following Chichilnisky (1997), Heal (1998) and Li and Lofgren (2000). This involves the use of a social welfare function for which neither the present nor the future is favoured over the other (Chichilnisky, 1997, p. 468). It contrasts with a ‘standard approach’ in which either the present or the future dominates, depending on whether the time preference rate of the judge is positive or zero.

The alternative forms of social welfare function are used to examine, within the framework of a simple growth model, the resulting optimal time path of consumption. The context involves a policy designed to reduce some form of damage, arising in the long term, by devoting current and future resources to abatement.[1] The abatement policy imposes costs on current and future generations in order to reduce damages that are expected to increase over a long time horizon. Generic abatement and damage functions are used, so no attempt is made to model the damage-generating process from any particular phenomenon such as climate change or population ageing. Rather emphasis is on trying to understand the nature of and implications of adopting alternative value judgements. This is important in view of the extensive and heated debates in a range of contexts involving long term plans. The arguments suggest that the standard form of social welfare function used to evaluate consumption streams reflects value judgements with which not everyone would agree.

Section 2 briefly discusses the general notion of sustainability adopted in the growth model applied later in this paper. Section 3 briefly presents the ‘standard model’ and discusses the familiar ethical dilemma arising from discounting. Koopmans (1960) pointed out that time preference is required so that the infinite future does not completely dominate the present. Here the dilemma becomes clear: discounting solves this problem but means that the distant future is disregarded in favour of the present. Section 4 describes the value judgements described by Chichilnisky as involving

[1] The related issues of uncertainty and irreversibility surrounding the long term effects of phenomena such as climate change and population ageing are ignored here, thereby avoiding questions about option values; for discussion of these see Arrow and Fisher (1974).

'sustainable preferences'[2], along with the variant of Li and Lofgren (2000), whose slightly different approach results in a similar optimal path towards an identical stationary solution as the time horizon becomes infinitely large. Section 5 describes the growth model, along with the specification of damages and abatement functions, used to examine alternative preferences. Section 6 presents numerical results of applying alternative value judgements for different values of key parameters, and Section 7 concludes.

2 The Concept of Sustainability

If sustainability is to be made operational it must be defined and it must be measured. As Solow (1992, p. 163) said about sustainability, 'talk without measurement is cheap'. The Brundtland Commission (United Nations, 1987) defined sustainable development as development that, 'meets the needs of the present without compromising the ability of future generations to meet their own needs'.

Some critics have suggested that the Brundland definition is too vague to be of use as a practical guide to planning; see Stavins et al. (2003). For example, a society living forever at a minimum subsistence level of consumption would satisfy the Brundtland requirement, but it would obviously be wasteful in terms of foregone opportunities to use resources to improve well-being. However, others argue that the notion of sustainable development is inevitably vague, but not necessarily meaningless (Solow, 2005). One definition, suggested by Solow (1992), that is both imprecise but meaningful defines sustainable development as an obligation to leave behind a generalised capacity to create well-being. This implies an obligation to give future generations the capacity to be as well off as the present by preserving the existing capacity for material development. That is, future generations are not owed any particular thing - rather they are owed a capacity to enjoy a level of well-being at least equal to that of the present. As Aghion and Howitt (1998) put it, 'sustainability doesn't require that any *particular* species of owl or any *particular* species of fish or any *particular* tract of forest be preserved'. The implication is that all forms of capital, reproducible capital and natural capital for example, are substitutable to some extent in generating well-being.

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[2] She cited Solow's term, 'intertemporally equitable preferences' as an alternative description.

This is the general interpretation of sustainability implicitly adopted in this paper. The growth model developed and applied below has one generalised consumption good and one generalised form of capital. Hence damages to one form of capital which results in a loss of consumption can be compensated by building up other forms of capital which can replace the lost consumption. However, this general interpretation is not sufficient to define a unique sustainable path of consumption over time and therefore among generations. For example, with technical progress many paths of consumption would be sustainable in the sense that future generations are at least as well off as current generations; but some paths would see future generations better off than they would be under other paths. There is therefore a need to go further than Solow's definition of sustainability in order to define a unique consumption path. This requires an explicit valuation of future well-being that imposes, as Chichilnisky (1997) puts it, neither a 'dictatorship of the present' nor a 'dictatorship of the future'. Before considering such value judgements in detail, the following section discusses the standard form of welfare function that is extensively used in cost-benefit studies.

3 A Standard Welfare Function and Time Preference

Consider a time stream of consumption per capita, C_t, over the period $t = 1,...,T$, where T represents a long time horizon. For simplicity, assume zero population growth and homogeneous consumption needs of the population.[3] A 'standard' approach is to examine the implications of adopting an additive Paretian social welfare function, representing the value judgements of an independent judge, of the form:

$$V = \sum_{t=1}^{T} (1+\rho)^{1-t} W(C_t) \qquad (1)$$

Here $W(C)$ is a weighting function representing the weight attached by the judge to consumption and ρ is the constant pure rate of time preference.[4] Alternative value judgements can be specified by the selection of different forms of W (in particular with different degrees of concavity) and values of ρ. In the vast majority of studies, the

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[3] If consumption needs differ with respect to age then C requires further clarification. In particular, it matters whether the social welfare function is expressed in terms of average consumption per equivalent person, or in terms of the ratio of average consumption to the average equivalent size; see Creedy and Guest (forthcoming).

[4] This is often referred to as a 'utility discount rate', from the different context of individual lifetime optimisation. Here, W is not a utility function.

implications of allowing W to take the isoelastic form $W\left(C_t\right) = C_t^{1-\beta} / \left(1-\beta\right)$ are considered. Hence β is the constant 'elasticity of marginal valuation', which can be interpreted in terms of a constant relative aversion to variability over time. Maximisation of V subject to a wealth constraint gives rise to the familiar Euler equation for optimal consumption growth at t:

$$g_t = \frac{1}{\beta}\left(r_t - \rho\right) \tag{2}$$

where g_t is the growth rate of consumption and r_t represents the rate of interest (and the marginal product of capital, net of depreciation, in a closed economy model). Rearranging this gives the 'Ramsey equation', $r_t = \rho + \beta g_t$, so that along the optimal path, the judge equates the marginal product of capital (the return from saving), r_t, with the marginal cost of saving, represented by $\rho + \beta g_t$ which is often called the consumption discount rate. There is no necessary relationship between ρ and β on ethical grounds, but in a small open economy the two are related by the condition that $\rho = r_t - \beta g_t$ where r_t, β and g_t are all given. This condition ensures that consumption growth cannot deviate from output growth permanently as that would imply either permanently accumulating or decumulating foreign assets.

A number of authors have argued that a pure time preference rate of zero should be imposed. This clearly involves an attempt, using various rhetorical devices, to impose their own value judgements. Famous examples include Ramsey (1928, p.543) and Pigou (1932, p. 25).[5] However, Ramsey (1928) realised that without discounting, infinite utility streams would be non-convergent and therefore could not be ordered. His solution was to measure utility over time as a cumulative sum of the distance from a 'bliss' level of utility, but the main problem with this approach is the arbitrariness of the level of bliss.

One way of achieving a partial ordering of infinite utility streams without discounting is the overtaking criterion; see von Weizacker (1965). This says that utility stream A is preferred to utility stream B if, after some finite time period, T, the cumulative utility of stream A is remains greater than stream B for all time $t > T$. However this is only a partial ordering of utility streams because one stream may oscillate above and below another stream indefinitely – it may never permanently

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[5] See also Padilla (2002) and Caplin and Leahy (2000).

overtake. Also, rather than replacing the need for discounting, the overtaking criterion comes close to Koopmans' axiomatic defence of discounting, because it implies that, for example, utility stream A:{0,1,0,0,….} is preferred to stream B:{0,0,1,0,0,…}. Stream B is stream A lagged one period. Hence stream A overtakes stream B in period 1 but is identical thereafter. Thus the preference for the overtaking stream reflects a time preference, as noted by Heal (1998).

The failure to rank all utility streams also applies to the Rawlsian criterion, which ranks the maxi-min utility stream above all others but fails to rank the others among each other. Similarly, an objective function that ranks the satisfaction of basic needs above all other outcomes fails to rank other outcomes.

The standard welfare function discussed above imposes a constant time preference rate. However, a number of authors have suggested using a rate which declines over time, that is, a hyperbolic time preference function (Laibson, 1996). Indeed, a feature of sustainable preferences discussed in the following section is that they imply a form of hyperbolic preferences. Both constant and hyperbolic time preference functions imply discount factors, $(1+\rho_t)^{1-t}$, that decline at a decreasing rate; that is, the second derivative with respect to time is positive.

However, within this framework it is also possible to consider value judgements such that the time preference function is logistic. This implies that the judge's concern, refected in the discount factor, at time 0 for the well-being of individuals living in time $t > 0$ declines relatively slowly as t is increased, but then begins to decline at an increasing rate. Hence the judge cares almost as much about generations in the near future as the present generation, but this concern at some point begins to diminish more rapidly. This decline could not accelerate forever as the discount factor cannot be negative, so it would have to tail off after some point, with a point of inflexion. A logistic time preference rate function describes such intertemporal preferences.[6] Their implications for the optimal consumption path are briefly reported below.

[6] Appendix Figures A1 and A2 compare the time preference functions, ρ_t, and discount factors, $(1+\rho_t)^{1-t}$, respectively, for the case of logistic preferences and the case of sustainable preferences discussed in the next section.

4 Sustainable Preferences

This section describes alternative social welfare function specifications which embody sustainable preferences and thus place a positive value on very long run outcomes. Subsections 4.1 and 4.2 examine in turn the value judgements specified by Chichilnisky (1997) and Li and Lofgren (2000).

4.1 A Chichilnisky Social Welfare Function

Chichilnisky (1997) proposed an approach which assigns declining weights over time and then some extra weight to the last period. The judge's evaluation function thus consists of a weighted average of two terms: the sum of discounted values where the pure rate of time preference declines over time and the (undiscounted) value in the final period, T, which the judge chooses to be long way into the future. Hence the social welfare function is of the form[7]:

$$V = \theta \sum_{t=1}^{T} \left(1+\rho_t\right)^{1-t} W\left(C_t\right) + \left(1-\theta\right) W\left(C_T\right) \qquad \rho_t{}' < 0; 0 < \theta < 1 \qquad (3)$$

In explaining the choice of a declining, or hyperbolic discount rate in the first term in (3), represented by $\rho_t{}' < 0$, Chichilnisky (p.468) refers to experimental evidence that the relative weight that people give to two subsequent periods in the future is inversely related to the distance of the two periods from today. This evidence suggest that it is worth examining the implications of a judge adopting such value judgements.[8]

Chichilnisky (1997) proved that a hyperbolic discount rate in (3) is a necessary condition for an optimal path to exist in the limit as T approaches infinity. A constant discount rate would not yield a solution. Adding the second term in (3), $\left(1-\theta\right)W\left(C_T\right)$, gives explicit recognition to consumption in the very long run, at time T. Taking a weighted average of the two terms implies a trade-off between the present and the future, yet neither need dominate completely.

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[7] Chichilnisky's objective function differs from (3) in that in her model the weighting function, which she calls utility, *U*, rather than *W* is derived from both consumption, *c*, and a flow of services from the stock of natural capital, *s*. This implies an optimal combination of *c* and *s* at any time *t*. However, dropping *s* from the utility function, as we do here, doesn't affect the notion of sustainable preferences, the key ingredients of which are a declining discount rate applied to *U*() and the second additive term in (3).

[8] It does not of course support the argument that evaluations *should* take this form (there is no legitimate route from 'is' to 'ought').

These preferences are subject to the standard criticism of time inconsistency that applies to hyperbolic preferences since these are reflected in the first term in (3). But this criticism is weak when the objective is a socially optimal consumption path. From a social choice perspective, Heal (1998) argued that over time, new generations arrive and older ones drop out of the choice process, so there is no reason why the preferences of generations who have dropped out should be imposed on new generations in the name of time consistency.

Heal (1998) shows that as $T \to \infty$, the effect on the optimal path of the term $(1-\theta)W(C_T)$ drops out. In other words in the limit the optimal path converges to the stationary solution from maximising (3) without the second additive term. This limiting solution is what Chichilnisky and Heal call the "green golden rule" which is the analogue to the Phelps golden rule in a Ramsey model. The green golden rule applies where natural capital generates a flow of services that yield utility directly in addition to the utility derived from a general consumption good. In the Ramsey model utility is derived only from the general consumption good.

4.2 A Modified Li and Lofgren Social Welfare Function

This subsection describes the social welfare function reflecting sustainable preferences introduced by Li and Lofgren (2000), with minor differences.[9] It is assumed that society is composed of two representative individuals who have utility functions specified over *T* years, where *T* is large and spans multiple future life spans on the basis that the individuals care about their offspring.[10] One individual discounts the future at a constant rate and the other does not discount. Individual 1 who is the discounter has a utility function, V_1:

$$V_1 = \sum_{t=1}^{T} (1+\rho)^{1-t} U(C_{1,t}, A_t) \qquad (4)$$

In this case ρ represents the pure rate of time preference of the individual, and *U* is the utility at time *t* derived from consumption of $C_{1,t}$ and damage abatement, A_t (discussed further below). Damage abatement is a public good and therefore both representative individuals receive the same level of A_t.

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[9] The differences are that the model here is in discrete time, replaces the conservationist in Li and Lofgren with a more generic person who may be thought of as a conservationist in the context of climate change, and damage abatement, *A*, replaces the environmental capital stock.

[10] Following Barro (1974), all generations are effectively linked if parents care about their offspring, in which case they plan their consumption as though they will live forever.

Individual 2 has a utility function, V_2, of the form:

$$V_2 = \sum_{t=1}^{T} U\left(C_{2,t}, A_t\right) \qquad (5)$$

The judge's evaluation, or social welfare, function is assumed to be a weighted average of the utilities of the two individuals:

$$V = \alpha V_1 + \left(1-\alpha\right) V_2 \qquad 0 \le \alpha \le 1 \qquad (6)$$

This approach is therefore different from the approach in Section 2 where the judge applied a weighting function to per capita consumption that reflected the judge's preferences rather than the preferences of individuals in the society. That approach implied that no matter how individuals in society may actually discount the future in their private consumption decisions and whatever the degree of concavity of their utility functions (the elasticity of marginal utility), the welfare function embodies only value judgements of the judge. Here, in (4), the pure time preference rate reflects the properties of individual 1, rather than those of the judge. However, the judge's preferences are not irrelevant – they are embodied in the relative preferences for each individual's welfare as reflected in the parameter α in (6).[11]

It is shown in the Appendix that the social welfare function (6) implies a hyperbolic effective social rate of time preference. The Appendix also shows that the rate of decline of the effective social rate of time preference depends on the consumption shares of the two individuals which in turn depends on the judge's preference parameter, α. The effect on the path of optimal consumption of α is investigated by using the growth model presented in the following section.

5 A Closed Economy with Damage and Abatement Functions

This section presents a model in which there are damages in the form of future costs which can be to some extent avoided by a costly abatement policy. The model is used in the next section to examine the implications of adopting the modified Li and Lofgren evaluation function.

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[11] It would be possible to rewrite the model such that individuals 1 and 2 are representing types, so that α measures the population share of type 1 individuals. If this latter interpretation is taken, the welfare function is in fact equivalent to a 'classical utilitarian' evaluation function, that is, a simple sum of individuals' utilities. However, this would require aggregate consumption, in the specification of the following subsection, to be expressed as a weighted sum rather than a simple sum of the two consumption values.

5.1 Structure of the Model

Consider a closed economy which incurs in period t a cost, A_t, of abating a potential future cost. This may be thought of as a pollution abatement cost or a cost of policies designed to reduce problems associated with, say, population ageing. The precise context is not important here as the aim is simply to investigate the way in which evaluation using a form of sustainable preferences affects policy judgements.

The judge maximises (6) subject to the closed economy accounting constraint:

$$Y_t = C_t + I_t + A_t \tag{7}$$

where Y_t is output, C_t is aggregate consumption equal to $C_{1,t} + C_{2,t}$, and I_t is investment. Abatement is a policy variable and is assumed to be a constant proportion, ε, of GDP lagged one period. Hence $A_t = \varepsilon Y_{t-1}$. Investment is given by:

$$I_t = K_t - (1-\delta) K_{t-1} \tag{8}$$

where K_t is the capital stock and δ is a constant rate of depreciation. The final capital stock at time T is subject to the constraint that $K_T = K^*$. Output is a function of the capital stock and damages, D_t, net of abatement:[12]

$$Y_t = \frac{K_t^{\gamma}}{(1+D_t)} \tag{9}$$

The damages function is given by the logistic form:

$$D_t = \frac{\mu C_{t-1}^{\omega} - A_t z}{1+e^{-x(t-0.5T)}} \tag{10}$$

The parameter μ is a damage scaling factor and ω is the elasticity of damages with respect to (lagged) total consumption. The parameter, z, is an abatement effectiveness term, measuring how much a given damage abatement expenditure, A_t, actually reduces damages. Abatement expenditure reduces damages both directly, through the term $A_t z$, and indirectly by diverting resources from consumption, thereby reducing C_t.[13]

The damage function in (10) generates a pattern of damages which increases slowly for an initial period then accelerates for an intermediate period before slowing down to approach an asymptote in the long run. This is consistent with typical modelling approaches such as Stern (2007, p.665) which assumed that damages cease to

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[12] Population growth is assumed to be zero, so labour inputs do not need to be included explicitly here.
[13] Environmental damages are obviously positively related to consumption but this is probably not true of ‘damages’ from population ageing.

increase after the year 2200 implying that the problem is contained after this time. The length and acceleration of damages in (10) can be varied through the parameter *x*. However there is no attempt here to model the underlying process generating damages as a function of variables other than time, such as temperature change or demographic change. This process simply assumed to be captured in a stylised way by a logistic function.[14]

Damage abatement policies with respect to climate change and population ageing impose costs on current and future generations in order to reduce damages that are expected to increase over time. In the case of climate change, the policy is a mechanism to reduce greenhouse gases (such as a carbon tax or tradable pollution permits). In the case of population ageing the mechanisms include measures to smooth the fiscal costs of ageing and tax incentives to boost labour force participation. Again, these mechanisms are not modelled in detail here.

The utility function $U(C_t, A_t)$ is assumed to be additively separable in C_t and A_t on the assumption that people's valuation of another unit of abatement is independent of the level of consumption and *vice versa*. Each separable part in the utility function is assumed to take the isoelastic form. Hence $U\left(C_{1,t}, A_t\right) = \frac{C_{1,t}^{1-\beta}}{1-\beta} + \Phi \frac{A_t^{1-\beta}}{1-\beta}$ and $U\left(C_{2,t}, A_t\right) = \frac{C_{2,t}^{1-\beta}}{1-\beta} + \Phi \frac{A_t^{1-\beta}}{1-\beta}$ where Φ is a parameter reflecting relative preferences for abatement. In this context the parameter β reflects the elasticity of marginal utility of consumption of the individuals themselves (in contrast with the elasticity of marginal valuation of the independent judge in the standard approach discussed in Section 3). Since A_t is a common resource, both individuals consume the same amount at any time: A_t. The two individuals are assumed to have the same parameters in their utility functions and therefore the only distinguishing feature in their utility functions is the pure rate of time preference, which is zero for individual 2.

The optimal plan chosen by the independent judge, which maximises *V* subject to the constraints give above, is obtained by forming the following Lagrangian:

[14] Nordhaus (1994), for example, models damages as a function of temperature changes.

$$\begin{aligned} L &= \alpha\sum_{t=1}^{T}U_1\left(C_{1,t},A_t\right)\left(1+\rho\right)^{1-t}+\left(1-\alpha\right)\sum_{t=1}^{T}U_2\left(C_{2,t},A_t\right) \\ &+\sum_{t=1}^{T}\lambda_t\left(Y_t-\left(C_{1t}+C_{2,t}\right)-\left[K_t-\left(1-\delta\right)K_{t-1}\right]-A_t\right) \\ &+\phi\left(K_T-K^*\right) \end{aligned} \tag{11}$$

It is therefore necessary to derive the Euler equation describing the growth rate of aggregate consumption for each time period. In the Appendix it is shown that this is given by the following, where a dot above the variable indicates a first difference such that $\dot{C}_t = C_t - C_{t-1}$:

$$\frac{\dot{C}_t}{C_t}=\frac{1}{\beta}\left[Y'\left(K\right)-\delta-\rho\left(1-\frac{C_{2,t}}{C_t}\right)+\dot{\Omega}_t\right] \tag{12}$$

where:

$$\begin{aligned} \Omega_t &= \frac{\partial Y_t}{\partial D_t}\frac{\partial D_t}{\partial C_{1,t}}=\frac{\partial Y_t}{\partial D_t}\frac{\partial D_t}{\partial C_{2,t}} \\ &=\left(\frac{-K^{\gamma}}{\left(1+D_t\right)^2}\right)\left(\frac{\mu\omega C_{t-1}^{\omega-1}}{1+e^{-x(t-0.5T)}}\right) \end{aligned} \tag{13}$$

Examination of (12), and comparison with (2), shows that, although the social welfare function does not contain an explicit pure time preference rate of the judge, it actually implies an effective time preference rate in each period of $\rho\left(1-C_{2,t}/C_t\right)$. It therefore depends on the ratio of person 2’s consumption to total consumption in that period. Furthermore, it is shown that $\lim_{t\to\infty} C_{2,t}/C_t \to 1$, so that ultimately the individual who does not discount dominates completely. Hence the judge’s implied pure time preference rate is hyperbolic, with an asymptote of zero.

5.2 A Solution Procedure

The values of *C, K* and *Y* are solved as follows. An initial steady state is assumed with *C, K* and *Y* constant and $D_0 = 0$. The initial capital stock, K_0, and initial output, Y_0, are determined from the production function, (9), given an assumed initial value of (K_0/Y_0). Initial consumption, C_0, is given by $Y_0 - I_0$ where $I_0 = \delta K_0$. The steady state is then shocked by allowing D_t to follow the damages function given above. The new steady state is found by a shooting algorithm in which an arbitrary initial level of consumption is chosen and variables solved forward using the above Euler equation for consumption and the equations for investment, output and capital stock. Repeated initial values of consumption are chosen until the target level of capital stock, $K_T = K_0$, is achieved.

6 Implications of Alternative Value Judgements

An explicit policy of damage abatement $(\varepsilon > 0)$ is one way of protecting future consumption from the costs of damages. Another (not mutually exclusive) way is for the social judge to apply alternative value judgements, reflected in the parameter α in the intertemporal social welfare function that represents sustainable social preferences. Cases where $0 \leq \alpha < 1$ represent sustainable preferences as defined here, and the case where $\alpha = 1$ represents the standard approach in which the social rate of time preference is constant.[15] This section applies the growth model presented in the previous section to consider the implications for C_t of such alternative value judgements, compared with the implications of an explicit damage abatement policy. Both methods of protecting future consumption – damage abatement and sustainable preferences – shift consumption forward in time. However, in the latter case the cost of damages on consumption is compensated by creating a higher stock of capital.

6.1 Calibration of the Model

The benchmark parameter values are given in Table 1. The first two parameters in the table, μ and ω, determine the scale of damages and their responsiveness to consumption. The parameter, x, is the logistic function parameter determining the underlying response of damages over time. Next are the damage abatement expenditure, ε, and damage effectiveness, z, parameters. The parameters are chosen such that the damage function before and after abatement is consistent with the magnitude and pattern of climate change damages projected in Stern (2007).

The remaining parameters relate to production function and preferences. The values of these parameters are typical values used in such models. The solution is found by searching numerically for a new value of consumption at the time that the damages shock is revealed (t = 1) that ensures that the terminal condition is met.

[15] Furthermore, the utility functions in the Li and Lofgren approach are considered to be weighting functions of the judge in the standard model.

Table 1 Benchmark Parameter Values

Scaling factor for damages, μ	0.25
Elasticity of damages with respect to consumption, ω	0.20
Rate of increase in damages, x	0.08
Damage abatement expenditure as a ratio of GDP, ε	0.01
Abatement effectiveness, z	5.0
Time horizon, T	200
Capital elasticity of output, γ	0.25
Initial capital to output ratio, $(K/Y)_0$	3.0
Depreciation rate, δ	0.05
Elasticity of marginal valuation, β	2.0
Time preference rate, ρ	0.035

6.2 Numerical Results

Figure 1 shows four illustrative series of damages as a percentage of GDP. The series represent damages with and without abatement for each of two values, $\alpha = 1$ and $\alpha = 0.5$. The damages ratio depends on α because damages depend on the path of aggregate consumption which depends on α, and also because GDP depends on α through its effect on capital accumulation (lower α implies greater capital accumulation and therefore higher GDP). Gross damages (that is, without abatement) rise from zero to 20 percent of GDP after 100 years for $\alpha = 1$ and to 23 per cent for $\alpha = 0$. Damage abatement reduces net damages by about 5 per cent of GDP. These levels of gross damages are consistent with the projected damages from climate change in Stern (2006) of 5 to 20 percent, and also similar to the projected costs to GDP per capita of population ageing in OECD countries - a cost of between 10 and 15 per cent is commonly projected for OECD countries (see for example Martins et al., 2005).

Figure 1 Damages as Percentage of GDP

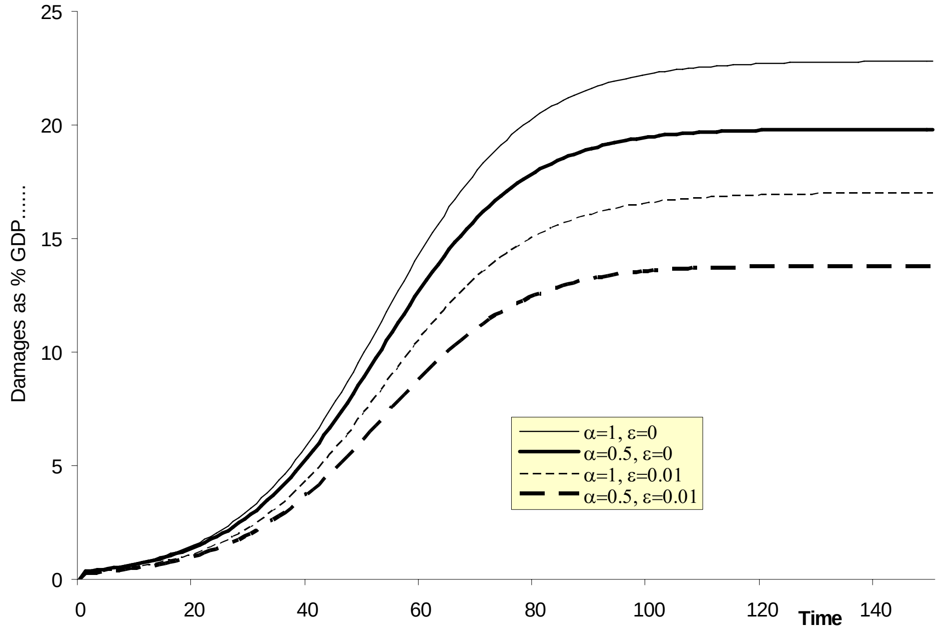


Figure 2 Consumption with Alternative Welfare Functions: Damages with Zero Abatement

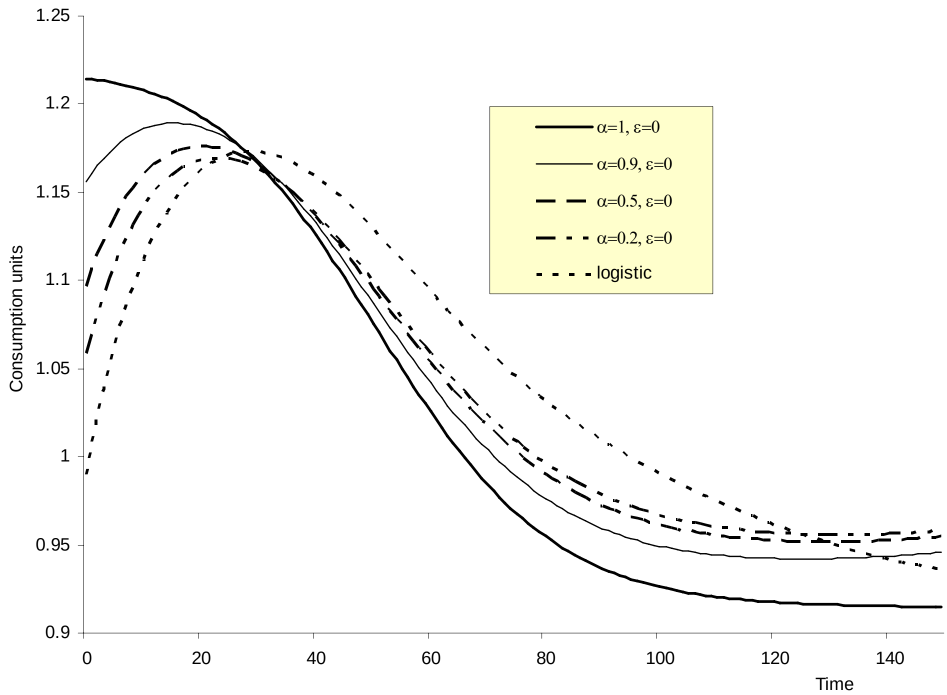


Figure 3 Consumption with Damage Abatement Policy: Constant Discount Rate

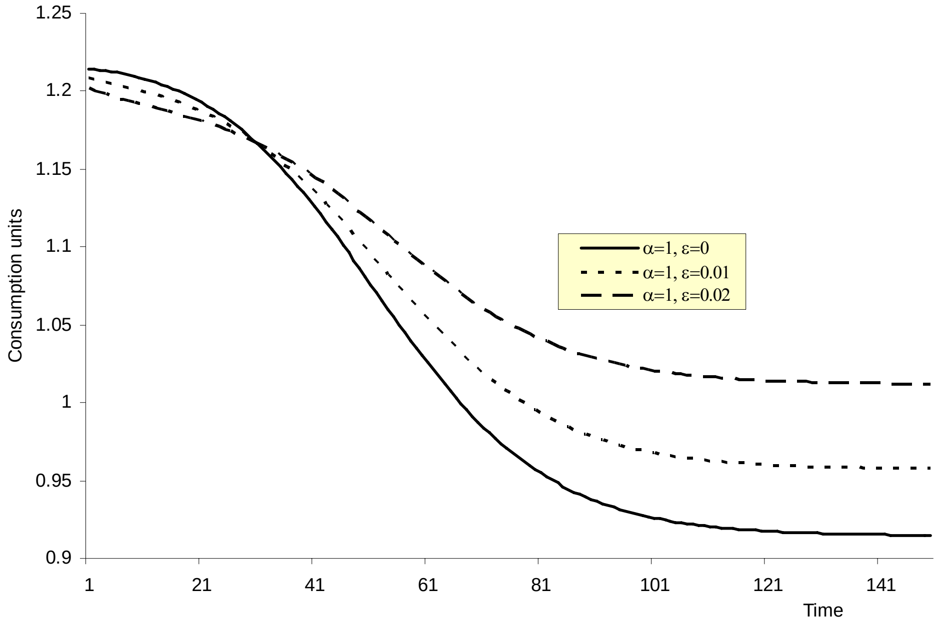


Figure 4 Consumption with Damage Abatement: Sustainable Preferences

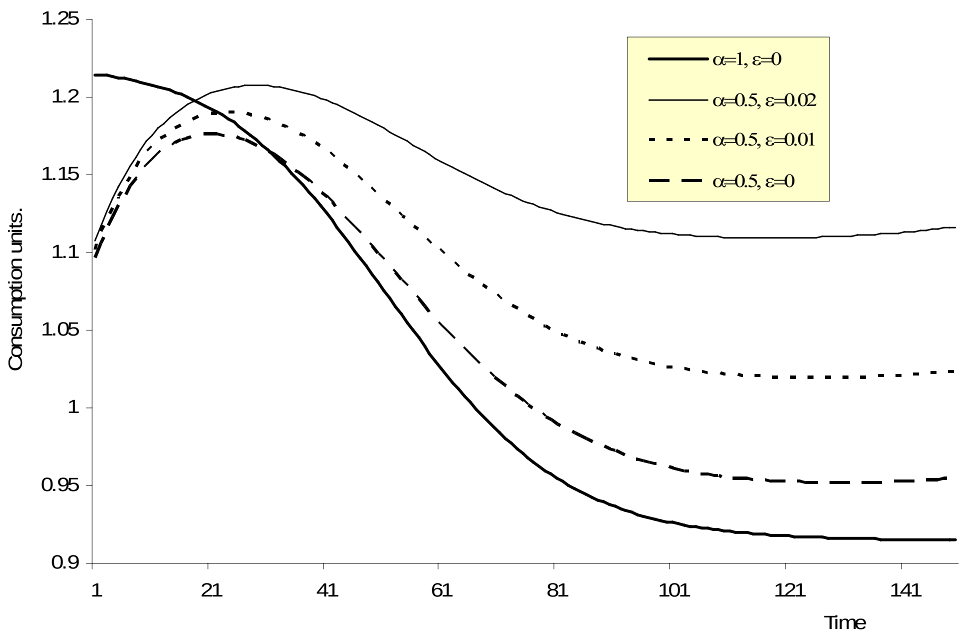


Figure 2 plots consumption per capita under alternative models of preferences: the standard model in which $\alpha = 1$, the sustainable preferences model (for $\alpha = 0.2$, $\alpha = 0.5$, and $\alpha = 0.9$), and logistic preferences for the functional form illustrated in Figure A1. There is no damage abatement in Figure 2 – abatement is introduced in Figure 3. In all cases in Figure 2 consumption is ultimately lower as a result of the damages shock.[16] Compared with conventional preferences ($\alpha = 1$), consumption under sustainable preferences ($\alpha < 1$) is initially lower, but ultimately higher. This is because the social rate of time preference in the sustainable preference model , $\rho\left(1 - C_{2,t}/C_t\right)$ is less than ρ, which is in turn due to the effect on social welfare of the utility of the individual who has a zero rate of pure time preference. A lower social rate of time preference implies a lower desire to substitute present consumption for future consumption. The result is a flatter consumption path. Consumption remains higher in the long run because a higher capital stock is created by sacrificing consumption in the early years.

The higher consumption under sustainable preferences implies higher damages, since damages are a function of consumption (10), but the resulting negative feedback effect on consumption is outweighed by the positive effect on consumption of the higher capital stock created by sacrificing consumption earlier on. Sustainable preferences therefore protect consumption in the long run compared with the outcome under standard value judgements.

Comparing the three cases of sustainable preferences in Figure 2 ($\alpha = 0.2$, $\alpha = 0.5$, and $\alpha = 0.9$), the lower the value of α the greater the fall in current consumption and the higher the level of consumption in the long run. This is because a lower value of α implies a higher relative weight given to person who does not discount the future. Yet even a small regard for that person's welfare (for example, α=0.9) implies an appreciable reduction in consumption. The time stream of consumption under sustainable preferences is also different in that it does not decline monotonically. It follows a hump shape which is more accentuated the lower is the value of α. This can be traced to the behaviour of three terms in the Euler equation, $Y'\left(K\right)$, $\frac{C_{2,t}}{C_t}$ and $\dot{\Omega}_t$, all of which depend on α.

[16] 'Lower consumption' means lower than the level before the damages shock. If allowance were made for labour productivity growth, consumption may still be higher in absolute terms notwithstanding damages.

Figure 2 includes a ‘logistic’ path of consumption, derived from a logistic time preference function discussed at the end of Section 3 and illustrated in Appendix Figure A1. This generates an even larger initial reduction in consumption than in the sustainable preferences model, because the concern for the well-being of close descendants is highest for this preference specification. The implications of the logistic preference function are not discussed further.

Figure 3 introduces damage abatement at two levels: $\varepsilon = 0.01$ and $\varepsilon = 0.02$, and shows that damage abatement protects consumption in the long run in the same way as sustainable preferences, as discussed in Figure 2 – that is, by shifting consumption from present generations to future generations. However, this is achieved by damage reduction directly through abatement, rather than through capital creation. The sacrifice in initial consumption is lower, however, to achieve a given increase in future consumption. This is partly because abatement expenditure has a dual effect on damages. There is a direct reduction, through the term $A_t z$ in the damage function (10), and an indirect reduction by diverting resources away from consumption which lowers damages through the term μC_{t-1}^{ω} in (10).

Figure 4 shows the combined effect of damage abatement and sustainable preferences on consumption. Given sustainable preferences (denoted in this case by $\alpha = 0.5$), no further loss of current consumption is incurred by introducing damage abatement, yet the gain to future generations is greater. This is evident from a comparison of the three series in Figure 4 for $\alpha = 0.5$. Among these three cases, greater damage abatement does not reduce current consumption but it significantly increases future consumption. Certainly current consumption is lower in all three cases than under conventional preferences (denoted by the $\alpha = 1$ series), but there is no further loss of current consumption by introducing damage abatement. This is different from the case of the standard welfare function, shown in Figure 3, where a higher damage abatement implies a greater loss of consumption.

The numerical examples shown in Figure 4 therefore suggest that the terms of the intergenerational trade-off implied by damage abatement policy are different under sustainable preferences compared with the standard welfare function. The suggestion is that the cost to current generations is relatively smaller under sustainable preferences.

The terms of the intergenerational trade-off under both social welfare functions depend on the parameters in Table 1, in particular, the abatement effectiveness parameter, z. The base case value of 5 was chosen in order to generate a reduction of damages of 25 percent, from about 20 percent of GDP to about 15 percent of GDP, after 100 years. As a sensitivity check, the abatement effectiveness parameter is halved to 2.5. In this case, under conventional preferences, damage abatement results in a cost to current generations that is larger and lasts longer than it does for $z = 5$. However, for sustainable preferences (and assuming $\alpha = 0.5$) damage abatement leaves current generations no worse off, which is the same result reported for the $z = 5$ case. This supports the suggestion that damage abatement does not penalise current generations as much under sustainable preferences as under the standard welfare function.

However, it is worth emphasising that the consumption paths resulting from all of these computations are the outcomes of optimal adjustments to consumption by an independent judge over a very long period of time. This framework is both a strength and a weakness. The strength is that the parsimony of the model allows the effects of ethical judgements and fundamental economic forces to be explored in a transparent way. It is worth investigating in a simple model how the optimal path responds to alternative assumptions about damages, damage abatement and ethical judgements about intergenerational equity. The weakness is that the model abstracts from many observed behavioural factors and exogenous forces, and it ignores distortions in markets that create deviations between the outcomes of a planned economy and a decentralised economy. These weaknesses restrict but do not disable the model as a tool for investigating the effects on consumption streams of alternative value in the context of damage abatement.

7 Conclusions

This paper investigated, using numerical examples based on a simple growth model, the effect on optimal consumption streams of sustainable preferences in the context of damage abatement. Sustainable preferences address the dilemma that arises in applying the standard social welfare function, with positive time preference, in dealing with damages arising from phenomena having very long run consequences such as climate change, nuclear waste disposal and population ageing. The perceived problem with discounting is that it discriminates against future generations. But the problem with not discounting is that it discriminates against present generations.

Sustainable preferences balance the interests of present and future generations by implying a declining, or hyperbolic, discount rate with respect to time.

The results indicate that sustainable preferences protect long run consumption in the face of long run damages, by shifting consumption from the present to the future. Even a small deviation from the standard welfare function (a value of α only slightly below 1) produces a ‘humped’ optimal profile of consumption over time, compared with a continuously decreasing profile with the standard social welfare function). A similar shift in consumption arises from a policy of damage abatement. However, the terms of the intergenerational trade-off implied by a policy of damage abatement are different under sustainable preferences compared with the standard approach. The examples suggest that damage abatement does not penalise current generations as much, if at all, under sustainable preferences as it does under the standard approach.

The basic position adopted here is that the appropriate role of economists is not to impose their own value judgements but to investigate the implications of adopting alternative value judgements and to clarify precisely what is involved in specifying social welfare functions. Within the context of the ‘standard’ approach to evaluating consumption streams, the range of value judgements is restricted (to variations in the elasticity of marginal valuation and the time preference rate) and dominance either by the present or a distant future generation is implied. The social welfare function implied by sustainable preferences therefore appears to offer a useful additional alternative when considering sensitivity analyses.

Appendix A. Derivation of Euler Equation for Growth Model

This Appendix derives the Euler equation for the growth model described in Section 5, with the modified Li and Lofgren (2000) specification of the social welfare function. From the Lagrangian in equation (11) above, the following first order conditions are derived for t=1,..,T-1:

$$\frac{\partial L}{\partial C_{1,t}} = \alpha \frac{\partial U_1}{\partial C_{1,t}} (1+\rho)^{1-t} - \lambda_t \left(1 - \frac{\partial Y_t}{\partial D_t} \frac{\partial D_t}{\partial C_{1,t}}\right) = 0 \tag{14}$$

$$\frac{\partial L}{\partial C_{2,t}} = (1-\alpha) \frac{\partial U_2}{\partial C_{2,t}} - \lambda_t \left(1 - \frac{\partial Y_t}{\partial D_t} \frac{\partial D_t}{\partial C_{2,t}}\right) = 0 \tag{15}$$

where $\frac{\partial D_t}{\partial C_{1,t}} = \frac{\partial D_t}{\partial C_{2,t}}$, and:

$$\frac{\partial L}{\partial K_t} = \lambda_t \left(Y'(K_t) - 1\right) + \lambda_{t+1}(1-\delta) = 0 \tag{16}$$

and for $t=T$:

$$\frac{\partial L}{\partial K_T} = \lambda_T \left(Y'(K_T) - 1\right) + \phi = 0 \tag{17}$$

The Euler equation can be obtained in the following three stages. First, from (14), and using the functional forms given in Section 5:

$$\begin{aligned} \frac{\partial L/\partial C_{1.t}}{\partial L/\partial C_{1,t+1}} &= \frac{\alpha \left(C_{1,t}^{-\beta}\right)(1+\rho)^{1-t}}{\alpha \left(C_{1,t+1}^{-\beta}\right)(1+\rho)^{-t}} = \frac{\lambda_t}{\lambda_{t+1}} \left(\frac{1-\Omega_t}{1-\Omega_{t+1}}\right) \\ &= \left(\frac{C_{1,t+1}}{C_{1,t}}\right)^{\beta} (1+\rho) = \frac{\lambda_t}{\lambda_{t+1}} \left(\frac{1-\Omega_t}{1-\Omega_{t+1}}\right) \end{aligned} \tag{18}$$

where $\Omega_t = \frac{\partial Y_t}{\partial D_t} \frac{\partial D_t}{\partial C_{1,t}} = \frac{\partial Y_t}{\partial D_t} \frac{\partial D_t}{\partial C_{2,t}}$ as defined in (13) above.

From (16) it can be seen that:

$$\frac{\lambda_t}{\lambda_{t+1}} = \frac{1-\delta}{1-Y'(K)} \tag{19}$$

and substituting into (18) and rearranging gives:

$$\frac{C_{1,t+1}}{C_{1,t}} = \left[\left(\frac{1-\delta}{\left(1-Y'\left(K_t\right)\right)\left(1+\rho\right)}\right)\left(\frac{1-\Omega_t}{1-\Omega_{t+1}}\right)\right]^{1/\beta} \tag{20}$$

Taking natural logarithms, using a dot to indicate a first difference and using the approximation for small values that $\log\left(1+x\right) \approx x$, gives:

$$\frac{\dot{C}_{1,t}}{C_{1,t}} = \frac{1}{\beta}\left[Y'\left(K_t\right) - \delta - \rho + \dot{\Omega}_t\right] \tag{21}$$

This result gives the Euler equation for person 1's consumption path. Except for the last term which depends on the damage and production functions, this takes the familiar form as given in equation (2) above, letting r_t in (2) equal $Y'\left(K_t\right) - \delta$. However, it is necessary to derive the Euler equation for total consumption, so the next stage involves obtaining a relationship between the growth rates of consumption for the two individuals. Given that $\lambda_t\left(1 - \frac{\partial Y_t}{\partial D_t}\frac{\partial D_t}{\partial C_{k,t}}\right)$ is the same for both k, then from the first-order condition:

$$\alpha\left(\frac{dU_1}{dC_{1,t}}\right)\left(1+\rho\right)^{1-t} = \left(1-\alpha\right)\left(\frac{dU_1}{dC_{2,t}}\right) \tag{22}$$

Therefore, substituting for marginal utilities and rearranging gives:

$$C_{2,t} = \left[\left(\frac{1-\alpha}{\alpha}\right)\left(1+\rho\right)^{t-1}\right]^{1/\beta} C_{1,t} \tag{23}$$

For periods t and $t+1$, this gives:

$$\frac{C_{2,t+1}}{C_{2,t}} = \left(1+\rho\right)^{1/\beta}\left(\frac{C_{1,t+1}}{C_{1,t}}\right) \tag{24}$$

Again, taking logarithms and applying approximations:

$$\frac{\dot{C}_{2,t}}{C_{2,t}} = \frac{\rho}{\beta} + \frac{\dot{C}_{1,t}}{C_{1,t}} \tag{25}$$

Thus the optimal growth rate of person 2's consumption is equal to that of person 1, plus the ratio of 1's pure time preference rate to the common elasticity of marginal utility of consumption. Clearly, a higher value of the latter implies a higher aversion to variability of consumption over time, so the difference in the optimal growth rate for person 2 is correspondingly lower.

The third and final stage involves combining the above results to obtain the aggregate Euler equation. By definition:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{C}_{1,t}}{C_{1,t}}\frac{C_{1,t}}{C_t} + \frac{\dot{C}_{2,t}}{C_{2,t}}\frac{C_{2,t}}{C_t} \tag{26}$$

Substituting for $\frac{\dot{C}_{2,t}}{C_{2,t}}$ from (25) into (26) yields

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{C}_{1,t}}{C_{1,t}}\frac{C_{1,t}}{C_t} + \left(\frac{\rho}{\beta} + \frac{\dot{C}_{1,t}}{C_{1,t}}\right)\frac{C_{2,t}}{C_t} = \frac{\dot{C}_{1,t}}{C_{1,t}} + \frac{\rho}{\beta}\frac{C_{2,t}}{C_t} \tag{27}$$

Finally, substituting for $\frac{\dot{C}_{1,t}}{C_{1,t}}$ from (21) into (27) yields

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\beta}\left[Y'(K) - \delta - \rho\left(1 - \frac{C_{2,t}}{C_t}\right) + \dot{\Omega}_t\right] \tag{28}$$

This is the result given in (12) above, showing that $\rho\left(1 - \frac{C_{2,t}}{C_t}\right)$ can be interpreted as the implicit pure rate of time preference of the judge. From (25), $C_{2,t}$ grows faster than $C_{1,t}$, therefore

$$\lim_{t\to\infty}\frac{C_{2,t}}{C_t} \to 1 \tag{29}$$

which, substituting into (28), implies that the social rate of time preference is hyperbolic and asymptotes to zero over time (Li and Lofgren, 2000, p. 238).

Figure A1 Hyperbolic and Logistic Time Preference Rate Functions

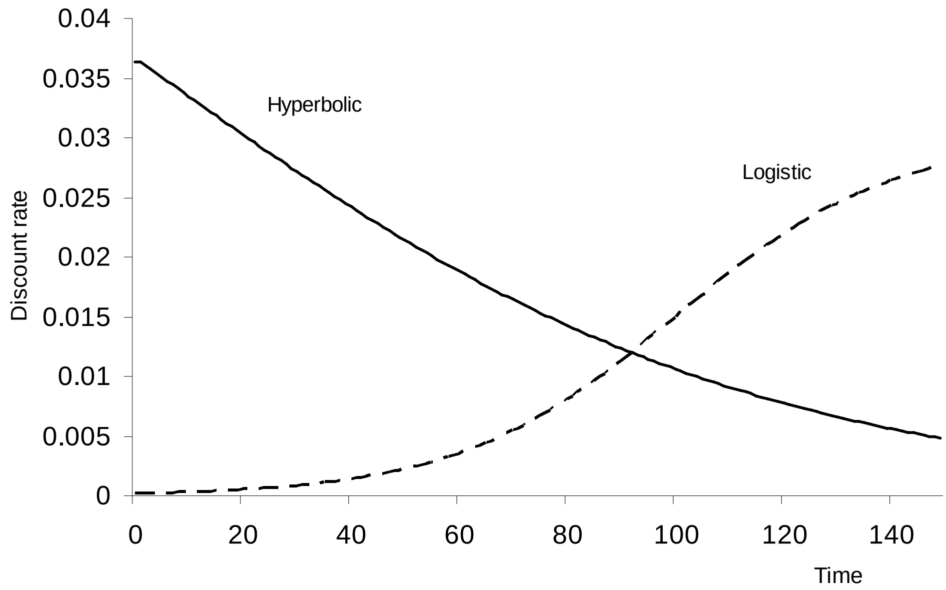
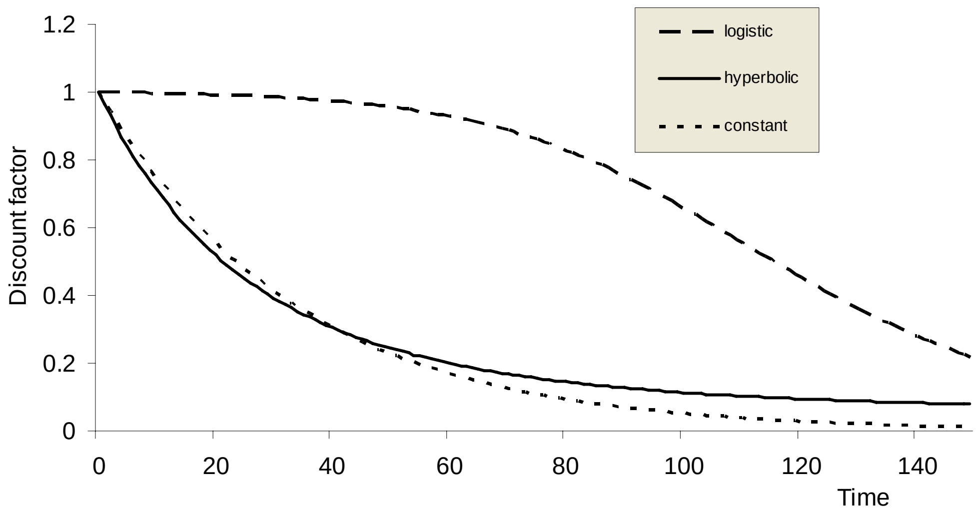


Figure A2 Alternative Discount Factors



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