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**Abstract**

This paper studies endogenous market formation in a financial trading model where strategic traders face information asymmetries and aggregate shocks. First, we show that negative participation externalities can arise for a large class of assets. In a decentralized process of market formation, the negative externalities limit competition between intermediaries. The model predicts that free entry into intermediation causes market fragmentation, but it is Pareto-superior to a single market. The model also predicts that the more intense the information asymmetry, the more a security tends to trade in fragmented markets.

**Keywords**: Asymmetric information, Aggregate shock, Imperfect competition, Market fragmentation, Network externality puzzle.

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# 1 Introduction

For many securities, trading occurs at multiple venues, rather than in a single large market. Recently, there is a growing interest in the study of fragmented markets.[1] This empirical work suggests that market fragmentation is a common and robust phenomenon. Yet we know little about the causes of fragmentation and its implications for welfare. In the market microstructure literature,[2] a popular view is that security trading should concentrate on a single venue due to positive liquidity externalities but intermediation costs (e.g., transaction costs, search costs etc) prevent any market from becoming too large. From this perspective, the level of market fragmentation that we observe today looks puzzling if we believe that intermediation costs have been substantially falling over the past few decades. Borrowing a phrase from Madhavan (2000), this is best summarized as *the network externality puzzle*: why is trading for the same security split across multiple trading venues? We revisit this issue in a financial trading model where strategic traders face information asymmetries and an aggregate shock. We show that securities may have finite optimal market size even in the absence of intermediation costs. We also show that the more intense the information asymmetry, the more a security tends to trade in fragmented markets.

Understanding what determines optimal market size is important for both theoretical and applied reasons. From a theoretical point of view, economists tend to think that a market with more traders is "thicker," and treat a thick market as an ideal benchmark. For example, it is customary to consider the limit as the number of traders goes to infinity to obtain asset pricing implications.[3] Considering such a limit may be justified normatively, if the thicker market is welfare-superior to the thinner market, or positively if someone has an incentive to create a thick market. In this paper, we argue that some types of securities can have very small optimal market size and never attract a large number of traders. For

[1] Brown, Mulherin, and Weidenmier (2008) and Cantillon and Yin (2010, 2011) study competition between exchanges. Ready (2009) and O'Hara and Ye (2011) study fragmentation of trades across alternative trading platforms.

[2] See Madhavan (2000) and Biais, Glosten, and Spatt (2005) for surveys.

[3] For example, see Corollary 1 in Madhavan (1992).

such a security, the thick market limit may not be a relevant benchmark. From an applied viewpoint, the securities industry is seeing the proliferation of new financial instruments and trading platforms, which seem to be causing more trading fragmentation.[4]

What makes optimal market size finite in our model is the trade-off between (i) the sharing of idiosyncratic risks and (ii) information aggregation by prices. The risk-sharing gains increase in the number of traders in one market. Given this, it may seem that the optimal market size is infinite and that it is efficient to have ALL traders in a single market. In our model, this would be true only if there were no information asymmetry about the security payoff. When the payoff is subject to information asymmetry, the ex ante gains from trade start decreasing as the number of traders increases, and the optimal market size is finite. This is because when prices reveal information about the payoff, it is impossible to trade risks that are resolved by prices –"the Hirshleifer effect" (Hirshleifer 1971).

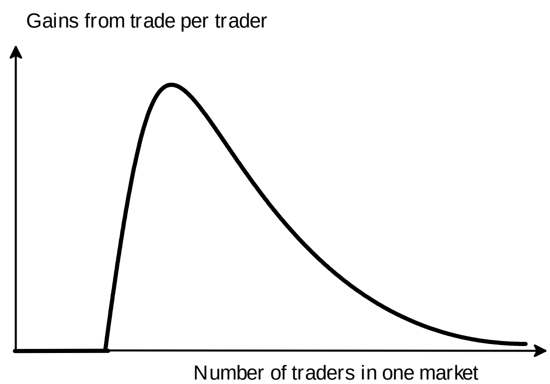



Figure 1. Hump-shaped gains from trade.

The hump-shape illustrated in **Figure 1** has implications both for efficiency of market structures and for competition between intermediaries. Suppose that each trader in a given market faces the same amount of operational costs. Having an additional trader in the market increases social welfare as long as his marginal contribution to the social surplus exceeds his marginal social cost. When the gains from trade per trader are monotonically increasing in the number of traders, the marginal social cost of one trader is a constant operational cost.

[4] O'Hara and Ye (2011) document the large variety of trading venues in US equity markets.

In this case, creating more than one market is inefficient, because it decreases the gains from trade without saving operational costs. When the gains from trade are decreasing in the number of traders, there is an additional social cost: each trader hurts the other traders' gains from trade. This makes it inefficient to have all traders in one market.[5]

Section 2 outlines the model. Section 3 derives a trade equilibrium in a one-market world and derives expression for the ex ante gains from trade. Section 4 studies a trade equilibrium and optimal market size for different types of assets. Section 5 studies a decentralized process of market formation. Each intermediary runs one market and competes in a Bertrand manner by setting entry fees. Traders decide in which market to participate. The entry fees are not necessarily bid down to marginal costs, because each intermediary knows that raising its fee will not drive all traders to his competitors. However, free entry into intermediation increases the number of intermediaries until each market attains the peak of the hump-shaped gains from trade. The result implies that free entry into intermediation causes more market fragmentation, but it is welfare-improving relative to a single market. Section 6 concludes. All proofs are in the Appendix.

# 2 Model Environment

This paper uses a noisy rational expectation equilibrium (REE) setup. There are $n+1$ traders indexed by $i \in \{1, ..., n+1\}$ in a given market. Traders have identical preferences and trade a risky asset with an unknown payoff $v$. Before trading, each trader receives (i) a risky-asset endowment $e_i$, and (ii) a private signal $s_i$ about $v$. Both are privately known. The sum of endowments $\sum_{i=1}^{n+1} e_i$ is the total amount of the risky asset in the market. One interpretation of the setup is inter-dealer trading, where dealers trade one another for their inventory management.

[5] A constant marginal cost is not necessary for market fragmentation to be efficient. As long as the gains from trade net of operation costs are not monotonically increasing, a single market is Pareto-dominated by fragmented markets.

The asset payoff $v$ has two components, $v_0$ and $v_1$:

$$v = \sqrt{1-t}v_0 + \sqrt{t}v_1, \quad t \in [0,1], \tag{1}$$

where $v_0$ and $v_1$ are independently normally distributed with mean zero and variance $\tau_v^{-1}$. Each trader observes a signal $s_i = v_1 + \varepsilon_i$, where $\varepsilon_i$ is unobserved noise normally distributed with mean zero and variance $\tau_\varepsilon^{-1}$. A constant $t$ in (1) measures the degree of information asymmetry. When $t$ is zero, the signal $s_i$ provides no information about the payoff $v = v_0$. When $t$ is positive, there is something to learn from signals. When $t$ is one, the entire payoff $v = v_1$ is subject to information asymmetry.

The endowment $e_i$ also has two components, $x_0$ and $x_i$:

$$e_i = \sqrt{1-u}x_0 + \sqrt{u}x_i, \quad u \in [0,1], \tag{2}$$

where $x_0$ and $\{x_i\}_{i=1}^{n+1}$ are independently normally distributed with mean zero and variance $\tau_x^{-1}$. The first component, $x_0$, is an aggregate shock to the initial position, while the second component, $x_i$, is an idiosyncratic shock. Trader $i$ knows the realized $e_i$ but does not observe $x_0$ and $x_i$ separately. A constant $u$ in (2) determines the relative importance of two shocks to the endowment. If $u$ is zero, there is no diversifiable risk and no trade can happen. When $u$ is positive, there is a risk-sharing opportunity. When $u$ is one, endowments $e_i = x_i$ are distributed i.i.d., and there are maximum potential gains from risk sharing.

Two parameters $(t, u) \in [0,1] \times [0,1]$ determine types of the risky asset.[6] The parameterization (1) and (2) ensures that ex ante variances of $v$ and $e_i$ do not depend on $(t, u)$. However, the asset type makes a difference when the asset is traded. An asset with a larger $t$ is subject to the higher degree of information asymmetry. An asset with a larger $u$ is subject to the lower degree of aggregate shock to the endowment. This is illustrated in **Figure 2**.

[6] By generalizing a security's payoff structure and an endowment structure, our framework nests Diamond and Verrecchia (1981), Pagano (1989), Madhavan (1992), and Ganguli and Yang (2009).

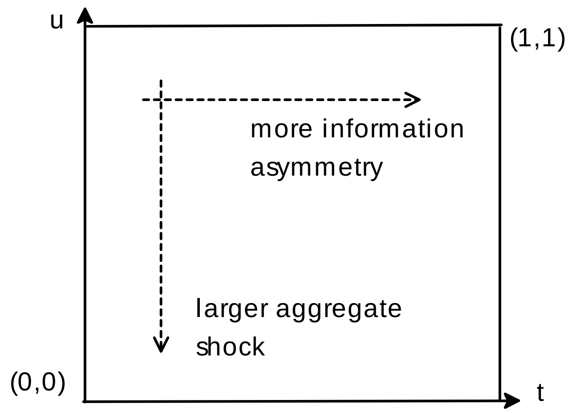



Figure 2. Type of assets.

Assets with $t = 1$ have been extensively studied in many “insider trading” models. Although this case is of particular interest, we study a general case with residual uncertainty $v_0$ in the payoff. It turns out that some equilibrium properties for the assets with $t = 1$ are not shared by the assets with $t$ arbitrarily close to one. Also, assuming $u = 1$ excludes the situation in which traders' initial positions are affected by a common factor (e.g., business cycle). We will show that some results for the assets with $u = 1$ are not robust to an arbitrarily small aggregate shock. However, our main result is that optimal market size is finite for all assets except those with $t = 0$.

To summarize, the $3+2(n+1)$ random variables $\left(v_0, v_1, x_0, \{x_i\}_{i=1}^{n+1}, \{\varepsilon_i\}_{i=1}^{n+1}\right)$ are normally and independently distributed with zero means, and variances

$$Var[v_0] = Var[v_1] = \tau_v^{-1},\ Var[x_0] = Var[x_i] = \tau_x^{-1},\ Var[\varepsilon_i] = \tau_\varepsilon^{-1}.$$

Each trader has exponential utility with a risk-aversion coefficient $\rho > 0$

$$U(\pi_i) = -\exp(-\rho\pi_i),$$

where $\pi_i$ is trader $i$'s profit. Profits are the sum of the payoff from the new position $q_i + e_i$ and the payment or the receipt for trading $q_i$, and hence $\pi_i = v(q_i + e_i) - pq_i$. After observing the private information $H_i = (e_i, s_i)$, each trader chooses her order $q_i(p; H_i)$.

Following Kyle (1989), we characterize a Nash equilibrium in demand functions and also compare it to a price-taking equilibrium. Because an order can be explicitly conditioned on a market-clearing price $p$, traders' strategies internalize information conveyed by the price. This solution concept captures the idea that rational traders make inferences from prices, which are consistent with the trading outcome. The market-clearing price satisfies

$$\sum_{i=1}^{n+1} q_i (p; H_i) = 0,$$

subject to a market-clearing rule described in the Appendix. To make explicit the dependence of the market-clearing price and an allocation on the strategies of traders, write $p = p(q)$ and $q_i = q_i(q)$, where $q = (q_1, ..., q_{n+1})$ is a vector of strategies. A rational expectations equilibrium with imperfect competition is defined as a $q$ that satisfies

$$E[U((v - p(q)) q_i(q) + vx_i)] \geq E[U((v - p(q')) q_i(q') + vx_i)] \tag{3}$$

for any $q'$ differing from $q$ only in the $i$-th component, for all $i \in \{1, ..., n+1\}$. We call this equilibrium *a trade equilibrium* in this paper. For a sake of comparison, *a price-taking equilibrium* is defined by replacing $p(q')$ with $p(q)$ in (3).

The absence of noise traders facilitates welfare analysis based on the ex ante gains from trade (henceforth GFT). Let $E_i[\cdot]$ denote trader $i$'s conditional expectation $E[\cdot|H_i, p]$ based on his private information and the market-clearing price.

**Definition 1 (gains from trade)**

*Define the interim profit* $\Pi_i$ *by* $E_i[\exp(-\rho\pi_i)] = \exp(-\rho\Pi_i)$.

*Define the interim no-trade profit* $\Pi_i^{nt}$ *by* $E_i[\exp(-\rho v e_i)] = \exp(-\rho\Pi_i^{nt})$.

*Define the interim GFT by* $G_i \equiv \Pi_i - \Pi_i^{nt}$.

*Define the ex ante GFT,* $G$, *by* $E[\exp(-\rho G_i)] = \exp(-\rho G)$.

Each trader's reservation value is determined by the value of no-trade profit $ve_i$ *after* receiving private information. Accordingly, the ex ante GFT, $G$, is defined by the ex ante certainty equivalent value of the interim GFT, $G_i$. We denote the interim profit, the interim GFT, and the ex ante GFT in a price-taking equilibrium by $\Pi_i^{pt}$, $G_i^{pt}$, and $G^{pt}$. Finally, we define the optimal market size by the number of traders that maximizes the ex ante GFT.

**Definition 2 (optimal market size)**

$$n_{pt}^* \equiv \arg\max_{n\geq 1} G^{pt} \text{ and } n^* \equiv \arg\max_{n\geq 1} G. \tag{4}$$

The optimal market size is defined with respect to traders' welfare without any reference to the costs of organizing a market. This reflects our emphasis on the role of asset types and trading rules, rather than intermediation costs, in determining market structures. A popular view is that (4) is infinite due to positive externalities but that costs of intermediation prevent any market from becoming too large. There is no doubt that those costs are important determinants of market structures, but in this paper we would like to point to another force inherent in security trading that can make (4) finite. In this sense, the way we define the optimal market size is meant only to make our point clear. The next section derives expression for $G^{pt}$ and $G$. Section 4 studies the optimal market size.

# 3 Trade Equilibrium and Ex Ante Gains From Trade

This section collects common properties of a symmetric linear equilibrium

$$q_i\left(p; H_i\right) = \beta_s s_i - \beta_e e_i - \beta_p p \tag{5}$$

with non-negative constants $\beta_s, \beta_e, \beta_p$. The characterization is standard in the literature, and details are gathered in the Appendix. The important feature of the equilibrium is information sharing through prices. Let $Var_i[\cdot]$ denote trader $i$'s conditional variance operator $Var[\cdot|H_i, p]$. Let $\tau \equiv \left(Var_i\left[v\right]\right)^{-1}$ and $\tau_1 \equiv \left(Var_i\left[v_1\right]\right)^{-1}$. Normality and symmetry make

$\tau$ and $\tau_1$ constants independent of the realizations of $p$, $H_i$, and $i$. The precision $\tau_1$ is bounded from below by the prior precision $\tau_v + \tau_\varepsilon$ and from above by the full information precision $\tau_v + (n+1)\tau_\varepsilon$. Depending on how much information about $v_1$ is revealed by the market-clearing price, there exists an endogenous number $\varphi \in [0,1]$ such that

$$\tau_1 = \tau_v + \tau_\varepsilon + n\varphi\tau_\varepsilon. \tag{6}$$

The endogenous variable $\varphi$ measures the fraction of the other $n$ traders' private signals revealed by prices. If $\varphi$ is zero, prices do not reveal any information about $v_1$. If $\varphi$ is one, prices reveal all the private signals in the market. Therefore, $n\varphi$ measures the total amount of information each trader learns from prices.

For assets with positive $t$, the equilibrium characterization boils down to the characterization of $k \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$, which solves the cubic equation[7]

$$\begin{aligned} F(k) \equiv\ & \left\{\frac{\rho^2}{\tau_\varepsilon \tau_x} u\{1+(1-u)n\}k^2 + 1\right\}\left\{\sqrt{t}k - \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)\right\} \\ & + n\left\{(1-u)\sqrt{t}k - (1-t)\frac{\tau_\varepsilon}{\tau_v}\right\} = 0. \end{aligned} \tag{7}$$

Note that $k \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$ measures a relative weight put on $e_i$ as opposed to $s_i$ in the order (5). Because $F(k) < 0$ for $k \leq 0$ and $F(k) > 0$ for sufficiently large $k$, (7) has at least one positive solution.[8] The solution depends on primitive parameters $\frac{\rho^2}{\tau_\varepsilon \tau_x}$, $\frac{\tau_\varepsilon}{\tau_v}$, the number of traders $n$, and the asset type $(t,u)$. We will do comparative statics with respect to $n$ and $(t,u)$ in the next section. Also, Section 5 will endogenize $n$.

Let $\bar{s} \equiv \frac{1}{n+1}\sum_{i=1}^{n+1} s_i$ and $\bar{e} \equiv \frac{1}{n+1}\sum_{i=1}^{n+1} e_i$. **Proposition 1** characterizes a trade equilibrium for assets with positive $t$, given a solution $k \in (0,\infty)$ to (7).

[7] See proof of **Proposition 1** for the derivation of (7).

[8] In the appendix, we characterize all solutions to (7) and show that when there are multiple trade equilibria they are Pareto-ranked.

**Proposition 1 (trade equilibrium)**

**(a)** *The amount of information sharing is*

$$\varphi = \left[1 + \frac{\rho^2}{\tau_\varepsilon \tau_x} k^2 u\{1 + (1-u)n\}\right]^{-1}. \tag{8}$$

*The second-order condition of the traders' problem is satisfied if and only if* $1 < n$ *and*

$$\frac{n+1}{n-1} < \frac{\rho^2}{\tau_\varepsilon \tau_x} u\{1 + (1-u)n\} k^2. \tag{9}$$

**(b)** *The optimal order is* $q_i(p; H_i) = \frac{\frac{n-1}{n} - 2\varphi}{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)} \left\{\sqrt{t}\frac{\tau_\varepsilon}{\rho} s_i - \sqrt{t} k e_i - \frac{\tau_1}{\rho(1+n\varphi)} p\right\}$,

*where* $\tau_1$ *is given by* (6).

**(c)** *The equilibrium price is* $p^* = \sqrt{t}\frac{\tau_\varepsilon(1+n\varphi)}{\tau_1}(\overline{s} - k\frac{\rho}{\tau_\varepsilon}\overline{e})$ *and the quantity traded is* $q_i(p^*; H_i) = \frac{\sqrt{t}\left(\frac{n-1}{n} - 2\varphi\right)}{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)} \left\{\frac{\tau_\varepsilon}{\rho}(s_i - \overline{s}) - k(e_i - \overline{e})\right\}$.

First, recall that $\frac{\beta_e}{\beta_s} = \frac{\rho}{\tau_\varepsilon} k$ and that $k$ measures a relative weight put on $e_i$ as opposed to $s_i$. Therefore, other things equal, a larger $k$ implies the lower level of information sharing $\varphi$ and increased trading volume measured by $E[|q_i(p^*; H_i)|]$. Second, the existence of the trade equilibrium depends on whether the second-order condition is satisfied. **Proposition 1(a)** shows that it cannot be satisfied for bilateral trading ($n = 1$). As $n$ increases, the left-hand side of (9) decreases to one, so the existence of the equilibrium is guaranteed for sufficiently large $n$ *if the right-hand side of (9) is NOT bounded* in $n$. We show in the Appendix that, for assets with $t = 1$ (and only for such assets), the right hand side of (9) can be bounded in $n$. For such assets, model parameters other than $n$ need to satisfy certain conditions for the trade equilibrium to exist even with arbitrarily large $n$. In this section, we assume that (9) is satisfied and focus on common properties of the trade equilibrium. **Lemma 1** compares the trade equilibrium with a price-taking equilibrium.

**Lemma 1 (price-taking equilibrium)**

**(a)** *The second-order condition is always satisfied.*

**(b)** $q_i$ *and* $q_i(p^*)$ *are obtained by replacing* $\frac{n-1}{n} - 2\varphi$ *with* $1 - \varphi$ *in Proposition 1.*

**(c)** $\varphi$ *and* $p^*$ *are the same as in Proposition 1.*

In the price-taking equilibrium, traders do not internalize the impact of their demand on a market-clearing price. As a result, the price-taking equilibrium always exists without any parameter restriction, and the three constants $\beta_s, \beta_e, \beta_p$ are larger compared with those in the trade equilibrium. This implies that imperfect competition reduces trading volume relative to that in the price-taking equilibrium, while it does not affect the amount of information sharing. In any equilibrium of the form (5), the inference from the price is affected by the ratio of constants $\beta_s, \beta_e, \beta_p$, but not by their level. Because the price impact affects the three constants proportionally, it does not distort the information revealed by prices.[9] Because beliefs are the same in both equilibria but there is less trading in the trade equilibrium, the GFT are smaller in the latter. **Lemma 2** confirms this intuition.

**Lemma 2 (interim gains from trade)**

**(a)** *In a price-taking equilibrium, the interim GFT are* $G_i^{pt} = \frac{\tau}{2\rho}(E_i[v] - p - \frac{\rho}{\tau}e_i)^2$.

**(b)** *In a trade equilibrium, the interim profit is* $\Pi_i = \widetilde{\lambda}\Pi_i^{nt} + (1-\widetilde{\lambda})\Pi_i^{pt}$, *where* $\widetilde{\lambda} \equiv \left(\frac{\varphi + \frac{1}{n}}{1-\varphi}\right)^2 \in [0,1]$. *The interim GFT are* $G_i = (1-\widetilde{\lambda})G_i^{pt}$.

The interim profit $\Pi_i$ is a weighted sum of $\Pi_i^{nt}$ and $\Pi_i^{pt}$, whose explicit solutions are given in the Appendix. Using **Lemma 2**, we can obtain analytical expressions for the ex ante GFT. Let $A(n) \equiv \frac{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)}{1+\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)} \in [0,1]$ and $B(n) \equiv \frac{u}{1+(1-u)n\varphi} \in [0,1]$.

[9] See page 623 in Madhavan (1992) for more discussion on this point.

**Proposition 2 (ex ante gains from trade)**

**(a)** *In a price-taking equilibrium,*

$$G^{pt} = \frac{1}{2\rho} \log \left( 1 + \frac{\rho^2}{\tau_v \tau_x} \frac{n(1-\varphi)}{1+n} A(n)B(n) \right). \tag{10}$$

**(b)** *In a trade equilibrium,*

$$G = \frac{1}{2\rho} \log \left( 1 + \frac{\rho^2}{\tau_v \tau_x} \frac{\frac{n-1}{n} - 2\varphi}{1-\varphi} A(n)B(n) \right) < G^{pt}. \tag{11}$$

**(c)** *If $\varphi$ does not increase in $n$, then both $\frac{n(1-\varphi)}{1+n}$ and $\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}$ increase in $n$.*

*If $n\varphi$ increases in $n$, then $A(n)B(n)$ decreases in $n$.*

We are interested in how the ex ante GFT, $G$, depend on market size $n$. In (11), $G$ depends on $n$ through two channels: $\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}$ and $A(n)B(n)$. **Proposition 2 (c)** indicates why the ex ante GFT can exhibit the hump-shape as in **Figure 1**. Recall that $\varphi$ measures informativeness of the price *per signal*, and $n\varphi$ measures *total information revealed by the price* from each trader's perspective. Although information revelation occurs as an equilibrium phenomenon in our model, publicly revealed information still reduces tradeable risk from the ex ante point of view. Therefore, *other things equal*, larger $n\varphi$ decreases welfare.[10] This is captured by $A(n)B(n)$. However, because risk-sharing gains increase in $n$ (captured by the first channel), other things are not equal. This trade-off creates the hump-shape.[11] In the next section, we show that for almost all assets the ex ante GFT is decreasing in $n$ for sufficiently large $n$.

[10] See Schlee (2001) for a detailed analysis of the Hirshleifer effect in an exchange economy.

[11] Several papers studied the similar trade-off in fixed market structures. Pithyachariyakul (1986) compares the Walrasian system and the monopolistic market-making system and shows the trade-off between the two systems. Naik, Neuberger, and Viswanathan (1999) show that trade disclosure regulation can reduce welfare in a dealership environment. Marin and Rahi (2000) analyze the trade-off in a security design problem. However, none of these works studies traders' participation decision or competition between intermediaries.

## 4 Optimal Market Size

The next proposition is the main result of this paper.

**Proposition 3 (negative externalities in a large market)**

**(a)** *For all assets with $t \in (0,1]$, $G$ is decreasing in $n$ for sufficiently large $n$, and the optimal market size $n^*$ is finite.*

**(b)** *For assets with $(t,u) \in (0,1) \times (0,1]$, $\lim_{n\to\infty} \varphi = 0$ and $\lim_{n\to\infty} G^{pt} = \lim_{n\to\infty} G > 0$.*

**(c)** *For assets with $t = u = 1$, given $1 < \frac{\rho^2}{\tau_\varepsilon \tau_x}$,*

$\lim_{n\to\infty} G^{pt} = \lim_{n\to\infty} G = 0$ *and* $0 < \lim_{n\to\infty} nG < \lim_{n\to\infty} nG^{pt} < \infty$.

**Proposition 3(a)** shows robustness of negative externalities in a thick market. In the context of our model, they are a norm rather than an exception. Results **(b)** and **(c)** show that for the assets with $t \in (0,1)$, the GFT decrease for large $n$ but have positive limits, while the asset with $t = u = 1$ has the ex ante GFT *disappearing* at the rate $\frac{1}{n}$ and hence exhibits the stronger negative externality. For the rest of this section, we characterize assets with $t = 0$ and assets with $t = u = 1$. Although both were studied well in the literature, their contrasting implications for market structures have never been pointed out. We fill this gap and also provide further discussion about the model's implication for market fragmentation. Readers who are not interested in technical details can skip the rest of this section.

**Lemma 3 (symmetric information asset: $t = 0$)**

**(a)** $E_i[v] = 0$, $\tau = \tau_v$.

**(b)** *In a price-taking equilibrium,* $q_i(p; H_i) = -(e_i + \frac{\tau_v}{\rho} p)$, $p^* = -\frac{\rho}{\tau_v}\bar{e}$,

$q_i(p^*; H_i) = -(e_i - \bar{e})$, *and* $G^{pt} = \frac{1}{2\rho} \log\left(1 + \frac{\rho^2}{\tau_v \tau_x} \frac{n}{1+n} u\right)$

**(c)** *In a trade equilibrium,* $q_i(p; H_i) = -\frac{n-1}{n}(e_i + \frac{\tau_v}{\rho} p)$, $p^* = -\frac{\rho}{\tau_v}\bar{e}$,

$q_i(p^*; H_i) = -\frac{n-1}{n}(e_i - \bar{e})$, *and* $G = \frac{1}{2\rho} \log\left(1 + \frac{\rho^2}{\tau_v \tau_x} \frac{n-1}{n} u\right)$.

**(d)** $n^*_{pt} = n^* = \infty$.

When $t = 0$, the information about the payoff is symmetric and the equilibrium price reveals no information about the asset payoff.[12] As long as there is a diversifiable risk ($u > 0$), the ex ante GFT *per trader* increase in the number of traders with an upper bound. This positive externality exists because each trader creates more risk sharing opportunity for the other traders due to an idiosyncratic component of his endowment. In the limit as $n$ approaches infinity, everyone trades to hold the average position. Note that trading volume and the ex ante GFT are lower in a trade equilibrium than in a price-taking equilibrium, but the difference disappears as $n$ increases. Therefore, a thick market is not only Pareto-superior to a thin market, but also it provides a better approximation to a price-taking equilibrium.

Next, we turn to the asset with $t = u = 1$, which is particularly tractable because both $k$ and $\varphi$ are independent of $n$.[13] Let $c_s \equiv \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_v} \in (0, 1)$ for notational convenience.

**Lemma 4 (Asset with $t = u = 1$)**

**(a)** $k = 1$ *is the unique solution to (7) and* $\varphi = \left[1 + \frac{\rho^2}{\tau_\varepsilon \tau_x}\right]^{-1}$.

*A trade equilibrium exists if and only if* $1 < n$ and $\frac{n+1}{n-1} < \frac{\rho^2}{\tau_\varepsilon \tau_x}$.

**(b)** *In a trade equilibrium,* $q_i(p; H_i) = \left(\frac{n-1}{n} - 2\varphi\right) \left\{\frac{\tau_\varepsilon}{\rho} s_i - e_i - \frac{\tau_\varepsilon}{\rho} \frac{1+c_s n\varphi}{c_s(1+n\varphi)} p\right\}$,

$p^* = \frac{c_s(1+n\varphi)}{1+c_s n\varphi}(\overline{s} - \frac{\rho}{\tau_\varepsilon}\overline{e})$ *and* $q_i(p^*; H_i) = \left(\frac{n-1}{n} - 2\varphi\right) \left\{\frac{\tau_\varepsilon}{\rho}(s_i - \overline{s}) - (e_i - \overline{e})\right\}$.

**(c)** $G^{pt} = \frac{1}{2\rho} \log\left(1 + \frac{\rho^2}{\tau_v \tau_x} \frac{n(1-\varphi)}{1+n} \frac{1-c_s}{1+c_s n\varphi}\right)$ *is maximized at* $n^*_{pt} = \sqrt{\frac{1}{c_s\varphi}}$.

$G = \frac{1}{2\rho} \log\left(1 + \frac{\rho^2}{\tau_v \tau_x} \frac{\frac{n-1}{n} - 2\varphi}{1-\varphi} \frac{1-c_s}{1+c_s n\varphi}\right)$ *is maximized at*

$$n^* = \frac{1}{1-2\varphi} + \sqrt{\frac{1}{1-2\varphi}\left(\frac{1}{c_s\varphi} + \frac{1}{1-2\varphi}\right)} > n^*_{pt}.$$

First, for the existence of a trade equilibrium, some parameter restriction is required. For example, with three traders, $3 < \frac{\rho^2}{\tau_\varepsilon \tau_x}$ must be satisfied. Even with infinite number of traders, we need $1 < \frac{\rho^2}{\tau_\varepsilon \tau_x}$. When this condition is satisfied, a trade equilibrium exists for all

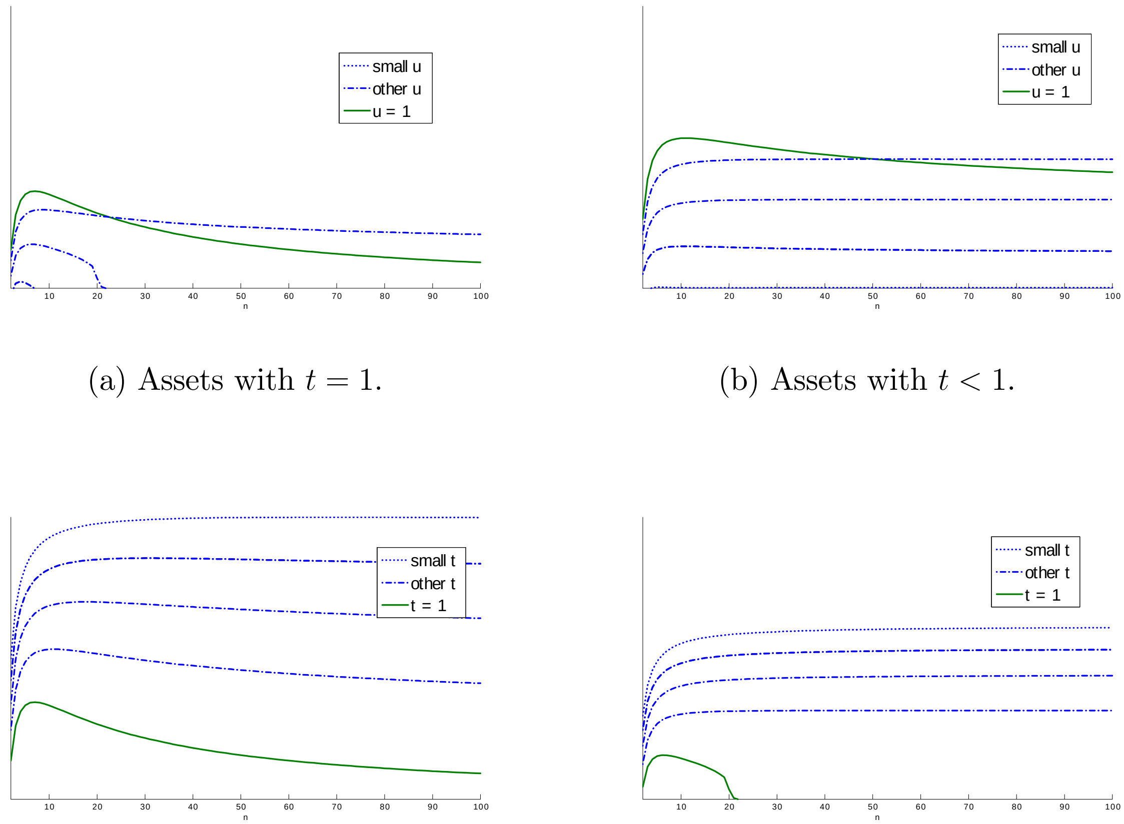
[12] Pagano (1989) studies the special case $u = 1$.

[13] See Diamond and Verrecchia (1981) and Madhavan (1992).

$n > \frac{1}{1-2\varphi}$, but otherwise it does not exist for any $n$. Second, recall that the optimal order and the quantity traded in a price-taking equilibrium are obtained by replacing $\left(\frac{n-1}{n} - 2\varphi\right)$ with $(1-\varphi)$ in the result **(b)**. Therefore, two equilibria are distinct no matter how large $n$ is. The result **(c)** provides explicit solutions for the optimal market size. It is finite for both equilibria, but it is larger in the trade equilibrium. Recall that the same amount of information is revealed in two equilibria. The trading volume is larger in the price-taking equilibrium because in the trade equilibrium traders withhold a larger fraction of their endowments to reduce their price impact. Therefore, more traders are required in order to achieve the same level of risk sharing as in the price-taking equilibrium. This makes the optimal market size in the trade equilibrium relatively larger. Finally, it is tempting to argue that a thick market is "informationally efficient" because $p^*$ converges to $v = v_1$ almost surely as $n$ increases to infinity. However, this theoretical prediction will not have much relevance, unless traders are forced or subsidized to participate in a thick market.

**Lemma 3** and **4** present contrasting results about the nature of externalities. Economides and Siow (1988) and Pagano (1989) study endogenous market participation in the absence of information asymmetry. In their models, the optimal market size is infinite as in **Lemma 3**, but exogenous transaction costs or search costs limit the market size. Therefore, their models predict the rise of a single market as transaction and search costs decrease. Our model shows that for assets with positive $t$, the optimal market size is finite because of endogenous negative externalities among traders. Therefore, even if transaction and search costs decrease, market fragmentation may survive for assets subject to information asymmetries.

To conclude this section, **Figure 3** shows $G$ for different types of assets.




(a) Assets with $t = 1$.

(b) Assets with $t < 1$.

(c) Assets with $u = 1$.

(d) Assets with $u < 1$.

Figure 3. Gains from trade for different assets. The ex ante gains from trade $G(n)$ is measured on the vertical axis as a function of the number of traders $n$. Assets with $t = 1$ have no residual risk $v_0$. Assets with $u = 1$ are not subject to aggregate shock $x_0$.

# 5 Endogenous Market Structure

To endogenize a market structure, we use a two-stage game played by traders and intermediaries at the ex ante stage. The idea is that market formation would take much more time compared to trading. In essence, we assume that traders and intermediaries can commit to a market structure determined before the realization of private information, in order to avoid the cost of redesigning it in every information state. The information asymmetry matters to the extent that its impact on trading is rationally anticipated at the ex ante stage.

Let $N = \{1, .., \overline{n}\}$ be the set of potential traders, and let $J = \{0, 1, .., j_{\max}\}$ denote the set of markets. A market structure for given $N$ is a partition of $N$ such that $N = \underset{j \in J}{\cup} N_j$ and $N_j \cap N_k = \emptyset$ for any $j \neq k$. We use $N_0$ for the set of traders who do not participate in any markets and $N_j$ with $j > 0$ for a set of traders in a market $j$. A lowercase letter $n_j$ denotes the number of traders in $N_j$. The GFT for each trader in $N_j$ are given by $G(n_j)$, while $C(n_j)$ is the total operational cost faced by an intermediary who runs a market $j$ with $n_j$ traders. The curvature of $C(\cdot)$ is one obvious determinant of the market structure. For example, other things equal, a rapidly increasing cost function would trivially make each market small. In order to focus on the implication of the endogenous negative externality represented by the shape of $G(\cdot)$, we assume $C(n) = cn$. This assures that market fragmentation is not due to ad hoc assumptions on the cost function.[14]

The market formation game proceeds in two steps. First, intermediaries simultaneously set entry fees. Second, traders simultaneously decide which market to participate in, or not to participate in any markets, taking the fees as given. Note that there is no room for horizontal differentiation for intermediaries because all traders are ex ante identical. This contrasts our analysis to that of Economides and Siow (1988), which is a model of horizontal differentiation (or spatial competition). In subsection 5.1, we analyze traders' participation decision for given fees. Subsections 5.2 and 5.3 study monopoly and competing intermediaries' decision making.

We assume that traders receive endowments and signals only if they participate in markets. If traders participate in any market, they will enjoy the monetary equivalent value of $G_i$. Therefore, traders decide whether or not to participate in any market based on the ex ante GFT $G$ and the fees set by intermediaries. Let the fees be denoted by $\left\{\phi_j\right\}_{j \in J}$, where $\phi_0 \equiv 0$. Given $\left\{\phi_j\right\}_{j \in J}$, trader $i$ chooses a participation pattern $r_i = \{r_{i,j}\}_{j \in J}$ to solve

$$\max_{r_i} \sum_{j \in J \backslash \{0\}} r_{i,j} \left\{ G\left( \sum_{i \in N} r_{i,j} \right) - \phi_j \right\} \tag{12}$$

[14]Qualitative results hold for more general cost functions, as long as $G(n) - \frac{C(n)}{n}$ has a single peak.

$$\text{s.t. } r_{i,j} \in \{0,1\} \text{ for all } j \in J \text{ and } \sum_{j \in J} r_{i,j} = 1. \tag{13}$$

*A participation equilibrium* for given $\{\phi_j\}_{j \in J}$ is $\{r_i^*\}_{i \in N}$ such that for all $i \in N$, $r_i^*$ solves (12) subject to (13) given $r_{-i}^*$. Note that (13) constrains each trader to participate in at most one market. We do not formally analyze an extension where traders can participate in multiple markets, but offer some discussion at the end of this section. We also focus on a pure strategy equilibrium. In the next subsection, we introduce an equilibrium selection criterion such that traders in each market do not regret their choice given a realized market structure. This rules out a mixed-strategy equilibrium.

The equilibrium strategies $\{r_i^*\}_{i \in N}$ determine a market size

$$n_j(\phi_j, \phi_{-j}) = \sum_{i \in N} r_{i,j}^* \tag{14}$$

for each $j$. From intermediary $j$'s perspective, (14) is a demand function. Given (14), intermediaries compete by setting fees $\{\phi_j\}_{j \in J}$. Intermediary $j$'s problem is

$$\max_{\phi_j} (\phi_j - c)n_j, \text{ s.t. } n_j = n_j(\phi_j, \phi_{-j}). \tag{15}$$

*A fee-setting equilibrium* is $\{\phi_j^*\}_{j \in J}$ such that for all $j \in J \backslash \{0\}$, $\phi_j^*$ solves (15) given $\phi_{-j}^*$.

## 5.1 Participation equilibrium

There are many participation equilibria, including unreasonable ones. For example, for any $\{\phi_j\}_{j \in J}$, there is an equilibrium $r_{i,0}^* = 1$ for all $i \in N$ (no trader participates in any market no matter what the fees are). This equilibrium exists due to a coordination failure, and has nothing to do with negative externalities among traders. Given our view that the market formation is a long-run process, it is reasonable to expect at least tacit cooperative behavior from traders. Accordingly, we employ the following equilibrium selection criterion.

**Definition 3 (stable participation)**

*A participation equilibrium is not stable if the resulting market structure satisfies one of the two conditions: (i)* $\exists j,k \in J\backslash\{0\}$ *and* $n \in \{1,..,n_j\}$ *s.t.* $G(n_j) - \phi_j < G(n+n_k) - \phi_k$. *(ii)* $\exists j \in J\backslash\{0\}$ and $n \in \{1,..,n_0\}$ *s.t.* $0 < G(n+n_j) - \phi_j$.

**Definition 3** stipulates that any subsets of traders in each market is free to move if there is a better alternative. We focus on stable participation equilibria where no such move is possible. This stability requirement excludes a trivial equilibrium where no trader participates in any markets, as well as a mixed-strategy equilibrium where traders randomly choose a market. Let $0 \leq \underline{\phi} \equiv \lim_{n\to\infty} G(n) < \overline{\phi} \equiv G(n^*)$, where $n^* \equiv \arg\max_n G(n)$. By setting $\phi_j > \overline{\phi}$, intermediary $j$ does not attract any traders. On the other hand, setting $\phi_j < c$ would only cause a loss. Hence, we can focus on $\phi_j \in [c, \overline{\phi}]$. **Lemma 5** is a necessary condition for the existence of the stable participation equilibrium.

**Lemma 5 (participation equilibrium)**

*In a stable participation equilibrium given* $\{\phi_j\}_{j\in J}$,

$$G(n_j) - \phi_j \geq \max\{G(n_k+1) - \phi_k, 0\} \text{ for all } j \in J\backslash\{0\} \text{ and } k \in J\backslash\{0,j\}. \tag{16}$$

*Additionally, either one of the following two conditions holds:*

$$\text{(i) For all } j \in J\backslash\{0\},\ G(n_j) \geq G(n_j+1). \tag{17}$$

*(ii)* $n_0 = 0$ *and there exists one* $j \in J\backslash\{0\}$ *s.t.* $G(n_j) < G(n_j+1)$ *and*

$$G(n_k+1) - \phi_k \leq G(n_j) - \phi_j < \overline{\phi} - \phi_j \leq G(n_k) - \phi_k \text{ for all } k \in J\backslash\{0,j\}. \tag{18}$$

**Lemma 5** shows that there are two types of stable participation equilibria. The first type satisfies (17), i.e., all markets operate on the decreasing part of $G$. The second type satisfies (18), i.e., all but one market operates on the decreasing part of $G$. When there is more than one market, the second type equilibrium is asymmetric in that the smallest market offers the smallest net benefits to traders compared to all other markets. Traders in the smallest market do not want to move to other markets because if they do, the larger markets will be so "congested". Although the possibility of the asymmetric equilibrium is interesting, it is implausible in the context of our model for two reasons. First, intermediaries are symmetric and there is no obvious way to determine which market will be the small one. Second, this equilibrium requires that $G$ should be quickly decreasing. The numerical evaluation of $G$ shows that it is unlikely to be satisfied (see **Figure 3**).[15] Therefore, we focus on the first type of equilibrium characterized by (16) and (17). Note however that even in the second type of equilibrium, all but one market will be operating at the decreasing part of $G$.

For the rest of our analysis, we use an approximation to get around issues associated with the integer restriction.[16] The conditions (16) and (17) are approximated by

$$G(n_j) - \phi_j = G(n_k) - \phi_k \geq 0 \text{ for all } j, k \in J \backslash \{0\}, \tag{19}$$

$$n^* \leq \ n_j \text{ for all } j \in J \backslash \{0\}. \tag{20}$$

In a stable participation equilibrium characterized by (19) and (20), all markets operate at the decreasing part of $G$ and provide the same net benefit to traders. The indifference condition (19) implicitly defines demand functions for intermediaries.

[15] A possible exception is the assets with $t = 1$, $u < 1$, and $\frac{\rho^2}{\tau_\varepsilon \tau_x} < \frac{4}{u}$, studied in the Appendix.
[16] This approximation was also used by Economides and Siow (1988).

## 5.2 Monopoly

When there is a single intermediary ($j_{\max} = 1$), the nature of demand function faced by the intermediary depends on the relative size of the potential number of traders $\overline{n}$ and the optimal market size $n^*$. If $\overline{n} \leq n^*$, the intermediary faces a completely elastic demand function: setting $\phi \leq G(\overline{n})$ attracts all traders $\overline{n}$ while setting $\phi > G(\overline{n})$ attracts no trader. As a result, the market size is always $\overline{n}$ as long as $G(\overline{n}) > c$. On the other hand, if $n^* < \overline{n}$, the demand function has an inelastic part: setting $\phi \in (G(\overline{n}), \overline{\phi}]$ would attract only some traders $n(\phi) = G^{-1}(\phi) \in [n^*, \overline{n})$. In this part of the demand function, raising a fee reduces, but does not entirely eliminate, the number of participating traders. The next proposition characterizes the monopoly market size for a large $\overline{n}$.

**Proposition 4 (monopoly market)**

*Assume a large $\overline{n} > n^*$. The monopoly market size $n^m$ is always larger than $n^*$. If $c \in \left(\underline{\phi}, \overline{\phi}\right)$, some traders are excluded from the market and $n^m \in (n^*, n^c)$, where $n^c$ is the larger solution to $G(n) = c$.*

If negative externalities are strong enough that $\underline{\phi} \equiv \lim_{n \to \infty} G(n) < c$, then $\overline{n} - n^m$ traders are excluded from the market as long as $\overline{n} > n^m$. For example, the asset with $t = u = 1$ has $\underline{\phi} = 0$, and the exclusion of traders occurs for any $c \in (0, \overline{\phi})$. For other assets, the relative size of $c$ and $\underline{\phi}$ determines whether the exclusion occurs or not. Generally, assets with large $t$ and small $u$ have lower GFT, so the exclusion is more likely for such assets. Note that the monopoly market is larger than the optimal size, regardless of whether the exclusion occurs. At the optimal market size, there is no externality, because $G'(n^*) = 0$. But the total surplus is still increasing in the market size, because an additional trader would add $G(n^*) - c > 0$ to the total surplus. Therefore, the intermediary maximizes the profit by having more than $n^*$ traders. The next subsection considers whether competition between intermediaries can solve this problem.

### 5.3 Competing intermediaries

Suppose that there are $j_{\max} \geq 2$ competing intermediaries and a large number of potential traders $\overline{n} > n^* j_{\max}$. We characterize a fee-setting equilibrium assuming that intermediaries rationally anticipate that a stable participation equilibrium will be played once they set fees. The indifference condition (19) implies that each intermediary offers the same level of net benefit to traders in a participation equilibrium. Let it be denoted by $U$.[17]

$$G\left(n_j\right) - \phi_j = U \geq 0. \tag{21}$$

Because $G$ is invertible at $n_j$ by (20), (21) determines a demand function $n_j(\phi_j) = G^{-1}(U + \phi_j)$. By substituting (21) into (15), intermediary $j$ solves

$$\max_{\phi_j} \; \{G\left(n_j(\phi_j)\right) - c - U\} n_j(\phi_j). \tag{22}$$

Note that $G - c$ is the surplus per trader. Therefore, the equilibrium value of $U$ determines the share of surplus left for each trader. There are two possibilities. First, if $U = 0$, then some traders must be both excluded from any markets ($n_0 > 0$) and weakly better off by not participating. Second, if $U > 0$, then all traders must be participating in markets ($n_0 = 0$). **Proposition 5** summarizes competition between intermediaries.

**Proposition 5 (competing intermediaries)**

*Either one of the following two conditions holds in a fee-setting equilibrium:*

*(i)* $U = 0$*. For all* $j \in J \backslash \{0\}$*,* $(G(n_j) - c)\, n_j$ *is maximized at* $n_j = n^m \in \left(n^*, \frac{\overline{n}}{j_{\max}}\right)$ *and* $\phi_j^* = G\left(n^m\right)$*.*

*(ii)* $U = H\left(\frac{\overline{n}}{j_{\max}}\right) > 0$*, where* $H(n) \equiv G(n) + G'(n)n - c$*. For all* $j \in J \backslash \{0\}$*,* $(G(n_j) - c)\, n_j$ *is increasing at* $n_j = \frac{\overline{n}}{j_{\max}}$

[17] This is the market utility approach commonly used in the competitive search literature (Galenianos and Kircher 2012). Alternatively, we could analyze each intermediary's strategic influence on the value of $U$. A previous version of the paper considers this and obtains a qualitatively similar result.

*and* $\phi_j^* = c - G'\left(\frac{\overline{n}}{j_{\max}}\right)\frac{\overline{n}}{j_{\max}} \in \left(c, G\left(\frac{\overline{n}}{j_{\max}}\right)\right)$.

**Proposition 5** shows that competing intermediaries either (i) behave as if each of them is a monopoly, leaving some traders excluded from markets and extracting all the surplus from participating traders, or (ii) accommodate all traders, extracting some, but not all, surplus from traders. In both cases, intermediaries' market power comes from the negative externality among traders. **Figure 4** illustrates the latter case with two intermediaries.

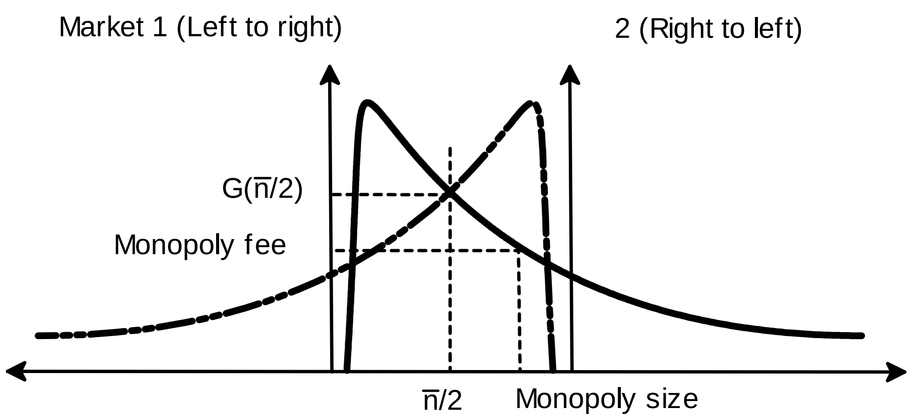



Figure 4. Two markets.

The length between the two vertical axes represents the number of potential traders $\overline{n}$. The size of market 1 is measured from the left axis to the right, while the size of market 2 is measured in the opposite direction. In the figure, the shared market size is smaller than the monopoly size (i.e., $n^* < \frac{\overline{n}}{2} < n^m$). The equilibrium fee is $\phi^* = c - G'\left(\frac{\overline{n}}{2}\right)\frac{\overline{n}}{2} < G\left(\frac{\overline{n}}{2}\right)$. At the shared market size, the total profit is increasing in the market size, which creates an incentive to attract more traders by setting the fee lower than $G\left(\frac{\overline{n}}{2}\right)$. However, such an incentive is not strong enough to "compete away" profits. Because the slope of $G$ at the shared market size is negative, i.e., the demand is inelastic, the equilibrium fee is higher than marginal costs, and thus intermediaries earn positive profits.[18]

[18]This result is related to Kreps and Scheinkman (1983), who show that Bertrand competition combined with capacity constraints yields Cournot outcomes. Intermediaries in our model are not subject to physical capacity constraints, but they use *a trading rule*, which creates negative externalities among traders. This works as an *endogenous* capacity constraint. Generally, negative externalities within the demand side undermine the incentive of the supply side to attract more demand, and weaken the power of price competition.

What is the market structure consistent with zero profits? Free entry into intermediation will make a shared market size $\frac{\overline{n}}{j_{\max}}$ smaller and closer to $n^*$. Because total surplus is increasing near $n = n^*$, the case (ii) in **Proposition 5** applies, and the equilibrium fee approaches $\phi_j^* = c - G'(n^*)\, n^* = c$. Therefore, entry continues until $j^* \equiv \frac{\overline{n}}{n^*}$ markets are established and all traders participate in one of the markets.[19] This achieves the maximum total surplus $\{G(n^*) - c\}\overline{n}$. Thus, free entry into intermediation achieves the socially optimal market structure. For assets with the small optimal market size $n^*$, $j^*$ is larger and free entry leads to a more fragmented market structure. This is typically the case for assets subject to the high level of information asymmetry.

## 5.4 Discussion

Some readers have questioned whether the information leakage across markets (e.g., some traders can be allowed to observe prices in the other markets, or to submit orders to multiple markets) would affect normative implications of the current analysis. A formal analysis is beyond the scope of this paper, but we suspect that the force toward market fragmentation would be robust to such extensions. First, suppose trading in different markets occurs at different times so that traders can observe prices in the other markets that opened earlier. Traders have an incentive to observe other markets' prices to reduce the perceived risk of the payoff. From the ex ante perspective, however, this reduces GFT, because such an observation increases information revelation without increasing risk sharing. Therefore, there will be demand for trading in the first market, where no such observation is possible. When the first market becomes large enough, the GFT start decreasing and the second market will open. The force for fragmentation still exists.

Next, consider submitting orders to multiple markets. This not only provides additional information but also provides additional risk sharing across markets. At the ex ante stage,

[19] If $j \geq j^*$ intermediaries enter, there is no stable participation equilibrium characterized by (19) and (20). This problem can be avoided by assuming that $j'$ markets are randomly chosen, where $j'$ is the maximum integer that satisfies $\overline{n} > n^* j'$. With this assumption, free entry will drive the expected profits down to zero, but the selected $j'$ intermediaries will make small profits.

these two forces create the same trade-off as in the benchmark model studied in this paper. Therefore, if everyone submits orders to all markets, multiple markets essentially become one integrated market and the ex ante gains from trade will be decreasing for a sufficiently large number of traders. Thus, some intermediaries will find it profitable to open a market that is separate from the integrated markets. In sum, as long as the traders' ex ante participation decision is relevant, there will be a force toward fragmented markets for assets with $t > 0$.

# 6 Conclusion

We showed that negative participation externalities can arise for a large class of assets subject to information asymmetries and aggregate shocks. This result implies that negative externalities in financial markets may be more relevant than usually believed. We also showed that negative externalities, regardless of their source, have an important implication for the endogenous formation of market structure. In related works, Spiegel and Subrahmanyam (1992) show that when traders are limited to using market orders, negative externalities can arise due to price volatility. Foucault and Menkveld (2008) argue that order fragmentation enhances liquidity by reducing limit order congestion. Identifying and quantifying different sources of negative externalities are important tasks for better understanding of trading behavior and the design of market structures.

Empirical studies of trading fragmentation suggest that some traders have different needs from others, which causes sorting of traders into different markets (Ready 2009; Cantillon and Yin 2010). The current model leaves no room for such horizontal differentiation, because traders are identical when markets are formed. As Economides and Siow (1988) showed, however, incorporating horizontal differentiation is likely to amplify the force for market fragmentation.

# APPENDIX

The appendix has three parts. The first part describes the market-clearing rule used in section 2. The second part presents a detailed characterization of a trade equilibrium for different assets. First, we study equilibrium multiplicity and how equilibria are Pareto-ranked. Second, we study the rate at which prices aggregate information. These results are used to prove the main results of this paper. The third section contains all proofs.

## 1. Market-clearing rule

See page 321 in Kyle (1989) for more details. A limit order $q_i(p; H_i)$ is allowed to be any convex-valued, upper-hemicontinuous correspondence that maps prices $p$ into non-empty subsets of the closed infinite interval $[-\infty, \infty]$. An intermediary calculates the set of market-clearing prices and quantity allocation. An allocation with infinite trade is assumed to be market-clearing if and only if there is at least one positive and one negative infinite quantity at that price. If a market-clearing price exists, the intermediary chooses the price with minimum absolute value and the market-clearing quantity allocation that minimizes the sum of squared quantities traded. If there is positive excess demand at all prices, $p = \infty$ is announced and all buyers receive negative infinite utility. If there is negative excess demand at all prices, $p = -\infty$ is announced and all sellers receive negative infinite utility. This guarantees that infinite prices and quantities do not occur in equilibrium.

## 2. Equilibrium multiplicity and speed of information aggregation

Define a class of asset $\{(t,u)|X(t,u) = m\}$ parameterized by $m \in [0, \infty]$, where

$$X(t,u) \equiv \frac{1-t}{1-u}\frac{c_s}{1-tc_s}. \tag{23}$$

This parameterization includes $\{(t,u)|X(t,u) = 0\} \equiv \{(t,u)|t = 1, u \in (0,1)\}$ (assets without $v_0$) and $\{(t,u)|X(t,u) = \infty\} \equiv \{(t,u)|u = 1, t \in (0,1)\}$ (assets without $x_0$).

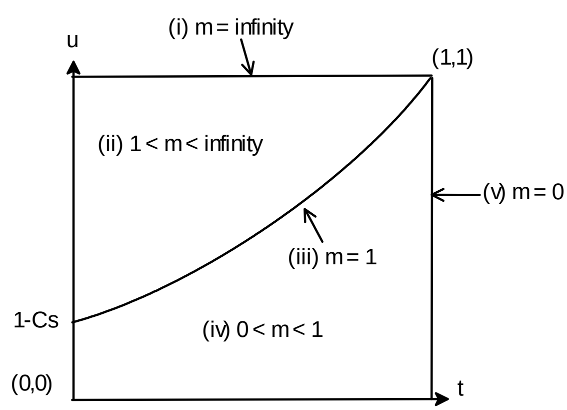



Figure A1. Asset classification.

**Proposition A1 (Multiplicity)**

**(a)** *Assets with* $m \geq \frac{1}{9}$ *do not have multiple equilibria.*
*There is a unique trade equilibrium for sufficiently large* $n$.

**(b)** *For assets with* $m < \frac{1}{9}$, (7) *has multiple solutions if and only if (i)* $n > \frac{8}{(1-u)(1-9m)}$ *and (ii)* $\frac{\rho^2}{\tau_\varepsilon \tau_x} \in (\alpha_n^-, \alpha_n^+)$, *where* $\alpha_n^\pm$ *are defined in the proof.*
*If multiple solutions satisfy* (9)*, an equilibrium with a larger* $k$ *is Pareto-superior.*

**(c)** *For assets with* $m \in (0, \frac{1}{9})$*, a trade equilibrium exists for sufficiently large* $n$.

**(d)** *For assets with* $m = 0$*, if* $\frac{\rho^2}{\tau_\varepsilon \tau_x} < \frac{4}{u}$*, all trade equilibria disappear as* $n$ *approaches some finite value. Otherwise, there are multiple equilibria for sufficiently large* $n$.

A necessary condition for multiplicity is that an asset must have low value of $m = X(t, u)$, i.e., high $t$ and low $u$ such that the asset is subject to *both* information asymmetry and an aggregate shock. For assets with $m < \frac{1}{9}$, two more conditions are required for equilibrium multiplicity. First, $n$ must be larger than the threshold value $\frac{8}{(1-u)(1-9m)}$, which is increasing in $u$ (and hence in $t$) for a fixed $m$. Second, $\frac{\rho^2}{\tau_\varepsilon \tau_x}$ must be in the region $(\alpha_n^-, \alpha_n^+)$, which depends on $n$. This region is well defined in the limit as $n$ goes to infinity. Therefore, if $\frac{\rho^2}{\tau_\varepsilon \tau_x}$ lies in the limiting region, equilibrium multiplicity survives in the limit. **Proposition A1(b)** also shows that multiple equilibria are Pareto-ranked by the size of the solution to (7). Given our interest in the optimal market size, it is natural to select the equilibrium that is not Pareto-dominated. Assets with $m \in (0, \frac{1}{9})$ always have one solution to (7) that constitutes a trade equilibrium for sufficiently large $n$. Therefore, we can consider the limit as $n$ increases for these assets given our equilibrium selection.

For assets with $m = 0$, it is possible that a trade equilibrium exists for small $n$, but it disappears as $n$ increases. This happens when $\frac{\rho^2}{\tau_\varepsilon \tau_x} < \lim_{n\to\infty} \alpha_n^- = \frac{4}{u}$ so that there is a unique solution to (7) for sufficiently large $n$. When the equilibrium disappears at a finite $n$, the price impact increases to infinity, i.e., the market becomes infinitely illiquid, and both trading volume and the GFT approach zero. Therefore, there is a particularly strong negative externality for assets with $m = 0$ and $\frac{\rho^2}{\tau_\varepsilon \tau_x} < \frac{4}{u}$. **Figure 3** (a) and (d) contain these assets. For all other assets, an equilibrium exists for sufficiently large $n$.

**Proposition A2** characterizes the speed of information aggregation for different assets. It is convenient to classify assets into five groups: (i) $m = \infty$, (ii) $m \in (1, \infty)$, (iii) $m = 1$, (iv) $m \in (0, 1)$, and (v) $m = 0$. These are shown in **Figure A1**.

**Proposition A2 (information aggregation)**

**(a)** *$n\varphi$ is increasing in $n$ for all groups (i) to (v).* $\lim_{n\to\infty} \varphi = 0$ *for groups (i) to (iv).*
**(b)** *For (i), $\varphi$ decreases in $n$ at the rate $n^{-\frac{2}{3}}$.*
**(c)** *For (ii), the unique solution $k$ increases in $n$ with an upper bound $\frac{m(1-tc_s)}{\sqrt{t}(1-c_s)}$. $\varphi$ decreases in $n$ at the rate $(k^2 n)^{-1}$.*
**(d)** *For (iii), there is a unique solution $k = \frac{1-tc_s}{\sqrt{t}(1-c_s)}$. $\varphi$ decreases in $n$ at the rate $n^{-1}$.*
**(e)** *For (iv), the largest solution $k$ decreases in $n$ with a lower bound $\frac{m(1-tc_s)}{\sqrt{t}(1-c_s)}$. $\varphi$ decreases in $n$ at the rate $(k^2 n)^{-1}$.*
**(f)** *For (v), the largest solution $k$ decreases in $n$. If $\frac{\rho^2}{\tau_\varepsilon \tau_x} \geq \frac{4}{u}$, $k$ converges to a positive value and $\varphi$ decreases in $n$ at the rate $(k^2 n)^{-1}$. If $\frac{\rho^2}{\tau_\varepsilon \tau_x} < \frac{4}{u}$, $k$ converges to zero at the rate $n^{-1}$ and $\varphi$ increases so that (9) is violated for all $n$ greater than a finite threshold value.*

Information aggregation is fastest for the group (v) with $\frac{\rho^2}{\tau_\varepsilon \tau_x} < \frac{4}{u}$, because $\varphi$ increases in $n$. However, this is the case where any trade equilibrium disappears as $n$ increases. Therefore, the total informativeness $n\varphi$ cannot grow without bound. Recall that the asset with $t = u = 1$ has constant $\varphi$ and, given $1 < \frac{\rho^2}{\tau_\varepsilon \tau_x}$, a unique trade equilibrium exists and does not disappear as $n$ increases (**Lemma 4**). This is the second-fastest information aggregation, and $n\varphi$ is increasing in $n$ linearly. As a result of the fast information aggregation, the GFT for this asset disappear in the limit. For all other assets, $\varphi$ decreases to zero, while $n\varphi$ increases. The group (i) has unbounded $n\varphi$. For the remaining groups [(ii)-(iv) and (v) with $\frac{\rho^2}{\tau_\varepsilon \tau_x} \geq \frac{4}{u}$], $n\varphi$ is increasing in $n$ with a finite limit. Thus, an arbitrarily small aggregate shock sets a finite limit for the amount of information revealed by prices.

The groups (ii)–(iv) have the same limit speed of information aggregation as $n$ goes to infinity, but the speed differs for finite $n$. The difference depends on whether $k$ is increasing or decreasing in $n$. Recall that a larger $k$ means a larger relative weight put on an endowment as opposed to a signal. Hence, if $k$ is increasing in $n$, a price aggregates information more slowly compared with the case where $k$ is decreasing. This difference matters only for finite $n$. However, all of these assets have finite optimal market sizes, so the difference in the speed of information aggregation may be empirically relevant.

## 3. Proofs

**Proof of Proposition A1**
First, define

$$X_n \equiv \frac{\frac{1}{n} + \frac{(1-t)c_s}{1-tc_s}}{\frac{1}{n} + 1 - u}.$$

For any $n \geq 1$, $X_n \lesseqgtr 1$ if and only if $X(t,u) \lesseqgtr 1$, where $X(t,u) = \frac{1-t}{1-u}\frac{c_s}{1-tc_s}$ was defined in (23). Also, $\lim_{n\to\infty} X_n = X(t,u)$ and $X_n$ is increasing (decreasing, constant) in $n$ for assets with $X(t,u) > 1$ $(< 1, = 1)$. We use the following fact:

$$\begin{aligned} a_3x^3 + a_2x^2 + a_1x + a_0 &= 0 (a_3 \neq 0) \text{ has three distinct real solutions if and only if} \\ 0 &< \Delta \equiv -4a_1^3 a_3 + (a_1 a_2)^2 - 4a_0 a_2^3 + 18 a_0 a_1 a_2 a_3 - 27(a_0 a_3)^2. \end{aligned}$$

Let $N_u \equiv 1 + (1-u)n$. Apply the fact for (7) to obtain

$$\begin{aligned} \Delta &= \frac{\rho^2}{\tau_\varepsilon \tau_x} u N_u^2 \times \left[ t \left\{ \frac{\rho^2}{\tau_\varepsilon \tau_x} u \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^2 \right\} N_u^2 \right. \\ &\quad -2\frac{\rho^2}{\tau_\varepsilon \tau_x} u \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right) \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right) \left\{ 2\frac{\rho^2}{\tau_\varepsilon \tau_x} u \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^2 - 9t \right\} N_u \\ &\quad \left. -27\frac{\rho^2}{\tau_\varepsilon \tau_x} tu \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right)^2 \right]. \end{aligned}$$

Let $\alpha \equiv \frac{\rho^2}{\tau_\varepsilon \tau_x}$ and write terms in a square bracket as a quadratic function of $\alpha$:

$$\begin{aligned} \widetilde{\Delta}(\alpha) &\equiv -4t^2 N_u^2 - 4\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^3 \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right) u^2 \alpha^2 \\ &\quad + \left\{ \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^2 N_u^2 + 18\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right) N_u - 27\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right)^2 \right\} tu\alpha. \end{aligned}$$

$\widetilde{\Delta}(\alpha) = 0$ has two real solutions if and only if

$$\begin{aligned} 0 < &\left[ \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^2 N_u^2 + 18\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right) N_u \right. \\ &\left. -27\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right)^2 \right]^2 - 64\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^3 \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)\right) N_u^3. \end{aligned} \tag{24}$$

Otherwise, $\widetilde{\Delta}(\alpha) \leq 0$ for all $\alpha$.

Note that $1 + (1-t)\frac{\tau_\varepsilon}{\tau_v} = \frac{1}{1-c_s} - t\frac{c_s}{1-c_s} = \frac{1-tc_s}{1-c_s}$ and $1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n) = \frac{1-tc_s}{1-c_s} + \frac{(1-t)c_s}{1-c_s}n$. Hence, $\frac{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n)}{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}}\frac{1}{N_u} = \frac{1+\frac{(1-t)c_s}{1-tc_s}n}{1+(1-u)n} = X_n$. Divide (24) by $\left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)^4 N_u^4$ to obtain:

$$\begin{aligned} H(X_n) &\equiv \left(1 + 18X_n - 27X_n^2\right)^2 - 64X_n \\ &= \left(1 + 18X_n - 27X_n^2 + 8\sqrt{X_n}\right)\left(1 + 18X_n - 27X_n^2 - 8\sqrt{X_n}\right) \\ &= \left(1 + 18X_n - 27X_n^2 + 8\sqrt{X_n}\right)\left(\frac{1}{3} - \sqrt{X_n}\right)\left\{\left(3\sqrt{X_n} + 1\right)(9X_n - 5) + 8\right\}. \end{aligned} \tag{25}$$

A necessary condition for (7) to have three solutions is $H(X_n) > 0$. Also, because $\alpha > 0$, $\widetilde{\Delta}(\alpha) = 0$ must have a positive solution. This requires $1 + 18X_n - 27X_n^2 > 0 \Leftrightarrow$

$X_n \in \left[0, \frac{1}{3}\left(1+\sqrt{\frac{4}{3}}\right)\right)$. In this range, $H(X_n) > 0$ if and only if $X_n < \frac{1}{9}$. For assets with $\frac{1}{9} \leq X(t,u)$, $X_n > X(t,u) \geq \frac{1}{9}$ for all $n$. Therefore (7) does not have multiple solutions. For assets with $X(t,u) = m < \frac{1}{9}$, $X_n$ approaches $m$ from above as $n$ increases, and $X_n < \frac{1}{9} \Leftrightarrow 1 + \frac{(1-t)c_s}{1-tc_s} n < \frac{1}{9}(1+(1-u)n) \Leftrightarrow \left\{\frac{1-u}{9} - \frac{(1-t)c_s}{1-tc_s}\right\} n > \frac{8}{9} \Leftrightarrow (1-u)(1-9m)n > 8 \Leftrightarrow n > \frac{8}{(1-u)(1-9m)}$. When this is satisfied, $\widetilde{\Delta}(\alpha) = 0$ has two positive solutions

$$\alpha_n^{\pm} = \frac{(1-m(1-u))(c_s - m(1-u))}{8c_s u X_n}\left\{1 + 18X_n - 27X_n^2 \pm \sqrt{H(X_n)}\right\}. \tag{26}$$

If $\alpha \in (\alpha_n^-, \alpha_n^+)$, then $\Delta > 0$ and (7) has three solutions. $\alpha \in \{\alpha_n^-, \alpha_n^+\}$ is a knife-edge case where (7) has two real solutions. Note that for $m \in (0, \frac{1}{9})$, $\lim_{n\to\infty} \alpha_n^{\pm}$ is well-defined, with $X_n$ replaced with its limit $m$ in (26). Therefore multiple solutions can persist in the limit if $\alpha \in \left(\lim_{n\to\infty} \alpha_n^-, \lim_{n\to\infty} \alpha_n^+\right)$. Suppose multiple solutions to (7) all satisfy the second-order condition (9). From (8), the larger solution $k$ implies lower $\varphi$. Because lower $\varphi$ raises $G^{pt}$ and $G^p$ for a fixed $n$, the equilibrium with the higher value of $k$ is Pareto-superior to the one with the lower value of $k$. **Proposition A2** shows that for assets with $m \in (0, \frac{1}{9})$, any solution to (7) is bounded below by a positive number as $n$ increases. Therefore, for sufficiently large $n$, (9) is satisfied.

For assets with $m = 0$, $X_n = \frac{\frac{1}{n}}{\frac{1}{n} + 1 - u} = N_u^{-1}$ and $X_n < \frac{1}{9} \Leftrightarrow N_u > 9 \Leftrightarrow n > \frac{8}{1-u}$. Also,

$$\alpha_n^{\pm} = \frac{1}{8u}\left\{N_u + 18 - 27N_u^{-1} \pm \sqrt{(N_u + 18 - 27N_u^{-1})^2 - 64N_u}\right\}.$$

Note that $\lim_{n\to\infty} \alpha_n^{\pm} = (\frac{4}{u}, \infty)$. Therefore, if $\alpha \leq \frac{4}{u}$, there is a unique solution to (7) for sufficiently large $n$. **Proposition A2** shows that this unique solution is decreasing in $n$ at the rate faster than $n^{-\frac{1}{2}}$ with the limit zero. Therefore, (9) is violated for sufficiently large $n$. If $\alpha > \frac{4}{u}$, there are multiple solutions to (7) for sufficiently large $n$, and the largest solution is bounded away from zero.

**Proof of Proposition A2**

Recall the five asset groups indexed by $\{(t,u)|X(t,u) = m\}$: (i) $m = \infty \Leftrightarrow u = 1$ and $t \in (0,1)$, (ii) $m \in (1, \infty)$, (iii) $m = 1$, (iv) $m \in (0,1)$, and (v) $m = 0 \Leftrightarrow t = 1$ and $u \in (0,1)$.

First, we show that $k^*$, the solution to (7), is increasing in $n$ for (i) and (ii), independent of $n$ for (iii), and decreasing in $n$ for (iv) and (v), where the largest solution is selected if there are multiple solutions. Because (7) is linear in $n$, it can be written as

$$F(k) = \frac{\partial F}{\partial n} n + \left(\alpha u k^2 + 1\right)\left\{\sqrt{t}k - \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)\right\}, \tag{27}$$

where $\frac{\partial F}{\partial n} =$

$$\begin{cases} -\left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right) \text{ if } u=1, \\ (1-u)\left\{(\alpha u k^2+1)\left(\sqrt{t}k-\left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)\right)+\left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)-\frac{1-t}{1-u}\frac{\tau_\varepsilon}{\tau_v}\right\} \text{ otherwise.} \end{cases}$$

Consider assets with $m > 1 \Leftrightarrow \left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)(1-u) < (1-t)\frac{\tau_\varepsilon}{\tau_v}$. A brief inspection of (7) reveals that $k^*$ satisfies $\sqrt{t}k^* \in \left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}, \frac{(1-t)\frac{\tau_\varepsilon}{\tau_v}}{1-u}\right)$ for $m \in (1,\infty)$ and $\sqrt{t}k^* > 1+(1-t)\frac{\tau_\varepsilon}{\tau_v}$ for $m=\infty$. Let $\frac{\partial F}{\partial n}\Big|_{k^*}$ denote $\frac{\partial F}{\partial n}$ evaluated at $k^*$. From (27), $\frac{\partial F}{\partial n}\Big|_{k^*} < 0$ because $F(k^*)=0$ and the second term is positive. Similarly, consider assets with $m < 1 \Leftrightarrow \left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)(1-u) > (1-t)\frac{\tau_\varepsilon}{\tau_v}$. For these assets, $\sqrt{t}k^* \in \left(\frac{(1-t)\frac{\tau_\varepsilon}{\tau_v}}{1-u}, 1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)$. From (27), $\frac{\partial F}{\partial n}\Big|_{k^*} > 0$. Finally, consider assets with $m=1 \Leftrightarrow \left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)(1-u) = (1-t)\frac{\tau_\varepsilon}{\tau_v}$. For these assets, $\sqrt{t}k^* = \frac{(1-t)\frac{\tau_\varepsilon}{\tau_v}}{1-u} = 1+(1-t)\frac{\tau_\varepsilon}{\tau_v}$. Thus, $k^*$ is independent of $n$.

For assets with $m > 1$, the solution $k^*$ is unique (**Proposition A1**) and therefore $F'(k^*) > 0$. By the implicit function theorem, $k^*$ is increasing in $n$. For assets with $m < 1$, if the solution $k^*$ is unique, it satisfies $F'(k^*) > 0$. If there are three solutions, the largest solution is chosen and it satisfies $F'(k^*) > 0$. In the knife-edge case where there are two solutions, the larger solution satisfies either $F'(k^*) > 0$ or $F'(k^*) = 0$. In the latter case, the solution disappears with the slight increase in $n$ and the selected solution discontinuously drops to the lower value. Hence, the selected $k^*$ is decreasing in $n$ for assets with $m < 1$. From the discussion above, the relevant bound for $k^*$ is $\frac{1}{\sqrt{t}}\frac{(1-t)\frac{\tau_\varepsilon}{\tau_v}}{1-u} = \frac{m}{\sqrt{t}}\left\{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right\} = \frac{m}{\sqrt{t}}\frac{1-tc_s}{1-c_s}$.

Next, we show that $n\varphi$ is increasing in $n$. From (8),

$$\begin{aligned} \frac{1}{n\varphi} &= \frac{1}{n}\left(\alpha u k^2 + N_u\right) \\ &= \frac{1}{n}\left(\alpha u k^2 + 1\right) + 1 - u. \end{aligned}$$

For groups (iii), (iv), (v), $k^*$ is not increasing in $n$ and hence $n\varphi$ is increasing in $n$.

For groups (i) and (ii), $F(k)=0$ implies

$$\frac{1}{n}\left(1+\alpha u N_u k^2\right) = \frac{(1-t)\frac{\tau_\varepsilon}{\tau_v} - (1-u)\sqrt{t}k}{\sqrt{t}k - \left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)}.$$

Because $\frac{1}{n}\left(1+\alpha u N_u k^2\right) = \frac{1}{n}\left(\alpha u k^2 + N_u - (1-u)n(1-\alpha u k^2)\right)$,

$$\begin{aligned} \frac{1}{n}\left(\alpha u k^2 + N_u\right) &= \frac{1}{n}\left(1+\alpha u N_u k^2\right) + (1-u)(1-\alpha u k^2) \\ &= \frac{(1-t)\frac{\tau_\varepsilon}{\tau_v} - (1-u)\sqrt{t}k}{\sqrt{t}k - \left(1+(1-t)\frac{\tau_\varepsilon}{\tau_v}\right)} + (1-u)(1-\alpha u k^2). \end{aligned}$$

This is decreasing in $n$ because $k^*$ is increasing in $n$ for groups (i) and (ii).

Finally we characterize behavior of $\varphi$. For groups (ii), (iii), (iv), the results are obvious from (8) and the results on $k^*$. For group (i), the unique $k^*$ solves

$$F(k; u = 1) = \left(\alpha k^2 + 1\right)\left\{\sqrt{t}k - \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)\right\} - \left(1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}\right)n = 0.$$

Therefore, $\sqrt{t}k^* > 1 + (1-t)\frac{\tau_\varepsilon}{\tau_v}$ and $k^*$ increases in $n$ without a bound at the rate $n^{\frac{1}{3}}$. From (8), $\varphi$ decreases in $n$ at the rate $n^{-\frac{2}{3}}$.

For group (v), there may be multiple $k^* \in (0,1)$ that solve

$$F(k; t = 1) = \left(\alpha u k^2 + 1\right)(k-1) + (1-u)\left\{\left(\alpha u k^2 + 1\right)(k-1) + 1\right\} n = 0.$$

Note that $(\alpha u k^2 + 1)(k-1) + 1 = k\left(\alpha u k^2 - \alpha u k + 1\right)$. Therefore, as $n$ increases, $k^*$ must be approaching one of the solutions to $k\left(\alpha u k^2 - \alpha u k + 1\right) = 0$. If $(\alpha u)^2 - 4\alpha u < 0 \Leftrightarrow \alpha < \frac{4}{u}$, then $\alpha u k^2 - \alpha u k + 1 > 0$ for all $k$ and $k^*$ must be approaching to zero at the rate $\frac{1}{n}$. If $\alpha \geq \frac{4}{u}$, by the selection of the largest solution, $k^*$ approaches to $\frac{1}{2}\left(1 + \sqrt{1 - \frac{4}{\alpha u}}\right) \in [\frac{1}{2}, 1)$.

**Proof of Proposition 1**

Given (5), from the market-clearing condition, $0 = \sum_{j\neq i} q_j + q_i = \beta_s \sum_{j\neq i} s_j - \beta_e \sum_{j\neq i} e_j - n\beta_p p + q_i$. From this, $\frac{n\beta_p p - q_i}{n\beta_s} = v_1 + \overline{\varepsilon}_{-i} - \frac{\beta_e}{\beta_s}\overline{e}_{-i}$, where $\overline{\varepsilon}_{-i}$ and $\overline{e}_{-i}$ are the average of noise in signals and endowments held by all traders except $i$. Because $\overline{e}_{-i} = \sqrt{1-u}x_0 + \sqrt{u}\overline{x}_{-i}$ and $e_i = \sqrt{1-u}x_0 + \sqrt{u}x_i$, by Bayes' rule, $E_i[v_1] = \frac{\tau_\varepsilon}{\tau_1}s_i + \frac{(Var[v_1|h_i])^{-1}}{\tau_1}h_i$, where $h_i = \frac{n\beta_p p - q_i}{n\beta_s} + (1-u)\frac{\beta_e}{\beta_s}e_i$, $\tau_1 = \tau_v + \tau_\varepsilon + (Var[v_1|h_i])^{-1}$, and $(Var[v_1|h_i])^{-1} = n\left[\tau_\varepsilon^{-1} + u\left\{1 + (1-u)n\right\}\left(\frac{\beta_e}{\beta_s}\right)^2 \tau_x^{-1}\right]^{-1} = n\tau_\varepsilon\varphi$. Here, $\varphi = \left[1 + u\left\{1 + (1-u)n\right\}\left(\frac{\beta_e}{\beta_s}\right)^2 \frac{\tau_\varepsilon}{\tau_x}\right]^{-1}$. Therefore, $E_i[v] = \sqrt{t}E_i[v_1]$ and

$$E_i[v_1] = \frac{\tau_\varepsilon(1-\varphi)}{\tau_1}s_i + \frac{\tau_\varepsilon\varphi}{\tau_1}\left\{1 + (1-u)n\right\}\frac{\beta_e}{\beta_s}e_i + \frac{\tau_\varepsilon\varphi(n+1)}{\tau_1}\frac{\beta_p}{\beta_s}p. \tag{28}$$

Given the conjecture (5) and the market-clearing condition $\sum_{j\neq i} q_j + q_i = 0$, $\sum_{j\neq i} q_j = \beta_s \sum_{j\neq i} s_j - \beta_e \sum_{j\neq i} e_j - n\beta_p p$. Solving for the market-clearing price, we obtain

$$p = p_i + \lambda q_i, \tag{29}$$

where $p_i \equiv \frac{\beta_s}{\beta_p}\overline{s}_{-i} - \frac{\beta_e}{\beta_p}\overline{e}_{-i}$ and $\lambda \equiv \frac{1}{n\beta_p}$. Trader $i$ maximizes $E_i[-\exp(-\rho\pi_i)]$. Because of the normality of $v$ conditional on each trader's information, the objective becomes

$$E_i[v](q_i + e_i) - \frac{\rho}{2}Var_i[v](q_i + e_i)^2 - pq_i \tag{30}$$

subject to (29). The first-order condition and the second-order condition are

$$E_i[v] - \frac{\rho}{\tau}(q_i + e_i) = p_i + 2\lambda q_i = p + \lambda q_i, \tag{31}$$

$$2\lambda + \frac{\rho}{\tau} > 0, \tag{32}$$

where $Var_i[v] = \tau^{-1}$ is used. From (31), we obtain

$$q_i^* = \frac{E_i[v] - p - \frac{\rho}{\tau}e_i}{\lambda + \frac{\rho}{\tau}}. \tag{33}$$

Using $E_i[v] = \sqrt{t}(k_s s_i + k_e e_i + k_p p)$, where $k_s, k_e, k_p$ are given in (28), we obtain

$$q_i^* = \frac{\sqrt{t}k_s s_i - \left(\frac{\rho}{\tau} - \sqrt{t}k_e\right)e_i - \left(1 - \sqrt{t}k_p\right)p}{\lambda + \frac{\rho}{\tau}}. \tag{34}$$

This is the best response of trader $i$ when the other traders use (5). By equating coefficients of (5) and (34), we have three equations

$$\beta_s = \frac{\tau_\varepsilon}{\lambda\tau + \rho}(1-\varphi)\frac{\tau}{\tau_1}\sqrt{t}, \tag{35}$$

$$\beta_e = \frac{\rho}{\lambda\tau + \rho}\left(1 - \varphi\frac{\tau_\varepsilon}{\rho}\{1 + (1-u)n\}\frac{\beta_e}{\beta_s}\frac{\tau}{\tau_1}\sqrt{t}\right), \tag{36}$$

$$\beta_p = \frac{\tau}{\lambda\tau + \rho}\left(1 - \varphi\frac{(n+1)\tau_\varepsilon}{\tau_1}\frac{\beta_p}{\beta_s}\sqrt{t}\right). \tag{37}$$

Solving for $\frac{\beta_p}{\beta_s}$ shows that $\frac{\beta_p}{\beta_s} = \frac{\tau_1}{\sqrt{t}\tau_\varepsilon(1+n\varphi)}$. Using (37) and $\lambda = \frac{1}{n\beta_p} = \frac{\tau_\varepsilon(1+n\varphi)\sqrt{t}}{n\beta_s\tau_1}$, obtain $\beta_s = \left(\frac{n-1}{n} - 2\varphi\right)\sqrt{t}\frac{\tau_\varepsilon}{\rho}\frac{\tau}{\tau_1}$. Similarly, solving for $\frac{\beta_e}{\beta_s}$ shows that $\sqrt{t}\frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s} = \frac{\tau_1}{\tau}\frac{1}{1+(1-u)n\varphi}$. Define $k \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s} = \frac{\tau_1}{\sqrt{t}\tau}\frac{1}{1+(1-u)n\varphi}$. Using $\frac{\tau}{\tau_1} = \frac{1}{\tau_1}\left\{(1-t)\frac{1}{\tau_v} + t\frac{1}{\tau_1}\right\}^{-1} = \frac{\tau_v}{\tau_v + (1-t)\tau_\varepsilon(1+n\varphi)} = \frac{1}{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)}$, the expressions for $\beta_s$, $\beta_e$, $\beta_p$ are obtained.

To characterize $k$, plug $\varphi = \left[1 + \{1 + (1-u)n\}k^2\frac{\rho^2}{\tau_\varepsilon\tau_x}\right]^{-1}$ into $k = \frac{\tau_1}{\sqrt{t}\tau}\frac{1}{1+(1-u)n\varphi}$. After simplification, (7) is obtained. Plug $\lambda = \frac{1}{n\beta_p}$ into (32) to show that the second-order condition is satisfied if and only if $n > 1$ and $\varphi < \frac{1}{2}\frac{n-1}{n}$, the latter of which is equivalent to $\frac{n+1}{n-1} < u\{1 + (1-u)n\}\frac{\rho^2}{\tau_\varepsilon\tau_x}k^2$. To obtain the equilibrium price and the quantity traded, use $p^* = \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e}$ and $q_i(p^*) = \beta_s(s_i - \overline{s}) - \beta_e(e_i - \overline{e})$.

**Proof of Lemma 1**

A price-taking equilibrium is characterized by setting $\lambda = 0$ in (31) and (32). Hence the second-order condition is always satisfied. The three equations (35), (36), (37) show that ratios $\frac{\beta_p}{\beta_s}$ and $\frac{\beta_e}{\beta_s}$ are not affected by the value of $\lambda$. Therefore, characterization of $k$, $\varphi$, and $p^*$ are the same. Using $\frac{\beta_p}{\beta_s} = \frac{\tau_1}{\sqrt{t}\tau_\varepsilon(1+n\varphi)}$ and (37) with $\lambda = 0$, obtain $\beta_s = (1-\varphi)\sqrt{t}\frac{\tau_\varepsilon}{\rho}\frac{\tau}{\tau_1}$.

**Proof of Lemma 2**

By plugging the optimal order (33) into the interim profit (30) and simplifying, obtain $\Pi_i = \left(1-\left(\frac{\lambda\tau}{\rho+\lambda\tau}\right)^2\right)\left\{\frac{\tau}{2\rho}\left(E_i[v]-p\right)^2+pe_i\right\}+\left(\frac{\lambda\tau}{\rho+\lambda\tau}\right)^2\left(E_i[v]e_i-\frac{\rho}{2\tau}e_i^2\right)$. Clearly, the interim no-trade profit is $\Pi_i^{nt} = E_i[v]e_i - \frac{\rho}{2\tau}e_i^2$. By setting, $\lambda = 0$, $\Pi_i^{pt} = pe_i + \frac{\tau}{2\rho}(E_i[v]-p)^2$. It is straightforward to show $G_i^{pt} \equiv \Pi_i^{pt} - \Pi_i^{nt} = \frac{\tau}{2\rho}(E_i[v]-p-\frac{\rho}{\tau}e_i)^2$ and $G_i = \Pi_i - \Pi_i^{nt} = \left(1-\left(\frac{\lambda\tau}{\rho+\lambda\tau}\right)^2\right)\Pi_i^{pt}-\Pi_i^{nt}$. Finally, $\frac{\lambda\tau}{\rho+\lambda\tau} = \frac{1}{\rho\beta_p\frac{n}{\tau}+1}$ and $\rho\beta_p = \frac{\tau_1}{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)}\frac{\frac{n-1}{n}-2\varphi}{1+n\varphi} = \tau\frac{\frac{n-1}{n}-2\varphi}{1+n\varphi}$. Hence, $\rho\beta_p\frac{n}{\tau} = \frac{n-1-2n\varphi}{1+n\varphi}$ and $\rho\beta_p\frac{n}{\tau}+1 = \frac{n-n\varphi}{1+n\varphi} = \frac{n(1-\varphi)}{1+n\varphi}$. Therefore, $\frac{\lambda\tau}{\rho+\lambda\tau} = \frac{\varphi+\frac{1}{n}}{1-\varphi}$. It is easy to verify that $\frac{\varphi+\frac{1}{n}}{1-\varphi} < 1$ if the second-order condition (9) is satisfied, and that $\frac{\varphi+\frac{1}{n}}{1-\varphi} \nearrow 1$ when parameters change such that (9) approaches equality.

**Proof of Proposition 2**

Write $E_i[v] = \gamma_s s_i + \gamma_e e_i + \gamma_p p$, where $\gamma_s = \sqrt{t}k_s$, $\gamma_e = \sqrt{t}k_e$, $\gamma_p = \sqrt{t}k_p$ for $k_s, k_e, k_p$ given in (28). Then $G_i^{pt} = \frac{\tau}{2\rho}(E_i[v]-p-\frac{\rho}{\tau}e_i)^2 = \frac{\tau}{2\rho}\left\{\gamma_s s_i - (\frac{\rho}{\tau}-\gamma_e)e_i - (1-\gamma_p)p\right\}^2 = [s_i, e_i, p]^\top C[s_i, e_i, p]$, where $C \equiv \frac{\tau}{2\rho}[\gamma_s, -(\frac{\rho}{\tau}-\gamma_e), -(1-\gamma_p)]^\top[\gamma_s, -(\frac{\rho}{\tau}-\gamma_e), -(1-\gamma_p)]$ is a 3-by-3 matrix. Let $\Sigma \equiv Var[[s_i, e_i, p]]$. This is a symmetric 3-by-3 matrix $\begin{bmatrix} V_s & 0 & V_{sp} \\ & V_e & V_{ep} \\ & & V_p \end{bmatrix}$, where $V_s \equiv Var[s_i]$, $V_e \equiv Var[e_i]$, $V_p \equiv Var[p]$, $V_{sp} \equiv cov[s_i, p]$ and $V_{ep} = cov[e_i, p]$.

Apply the following formula: Given the $n$-dimensional random vector $z$ that is normally distributed with mean zero and variance-covariance matrix $\Sigma$,

$$E[-\exp(-\rho(z^\top Cz))] = -\left[\det\left(I_n + 2\rho\Sigma C\right)\right]^{-\frac{1}{2}},$$

where $I_n$ is the $n$-dimensional identify matrix. Therefore, $G^{pt} = \frac{1}{2\rho}\log\left[\det\left(I_n+2\rho\Sigma C\right)\right]$ and $G = \frac{1}{2\rho}\log\left[\det\left(I_n + 2\rho(1-\widetilde{\lambda})\Sigma C\right)\right]$.

First, $\Sigma C = \frac{\tau}{2\rho}\begin{bmatrix} \gamma_s C_1 & -(\frac{\rho}{\tau}-\gamma_e)C_1 & -(1-\gamma_p)C_1 \\ -\gamma_s C_2 & (\frac{\rho}{\tau}-\gamma_e)C_2 & (1-\gamma_p)C_2 \\ \gamma_s C_3 & -(\frac{\rho}{\tau}-\gamma_e)C_3 & -(1-\gamma_p)C_3 \end{bmatrix}$, where $C_1 = \gamma_s V_s - (1-\gamma_p)V_{sp}$, $C_2 = (\frac{\rho}{\tau}-\gamma_e)V_e + (1-\gamma_p)V_{ep}$ and $C_3 = \gamma_s V_{sp} - (\frac{\rho}{\tau}-\gamma_e)V_{ep} - (1-\gamma_p)V_p$. Using $V_p = \frac{\gamma_s}{\gamma_p}\frac{\varphi(1+n)}{1-\varphi}V_{sp} - \frac{\gamma_e}{\gamma_p}\frac{1+n}{1+(1-u)n}V_{ep}$, $V_{sp} = \frac{\gamma_s}{\gamma_p}\frac{\varphi(1+cn)}{1-\varphi}V_s$, $V_{ep} = -\frac{\gamma_e}{\gamma_p}V_e$, $V_s = \frac{1}{c\tau_v}$ and $V_e = \frac{1}{\tau_x}$, algebra shows that $C_1 = \frac{\gamma_s}{\tau_\varepsilon}\frac{n}{1+n}$, $C_2 = \frac{\rho}{\tau_x\tau}\left\{1-\frac{1+n\varphi}{1+n}\frac{1+(1-u)n}{1+(1-u)n\varphi}\right\}$ and $C_3 = 0$. Hence, $\det\left(I_n+2\rho\Sigma C\right) = 1+\tau\left\{\gamma_s C_1 + (\frac{\rho}{\tau}-\gamma_e)C_2\right\}$ and $\det\left(I_n + 2\rho(1-\widetilde{\lambda})\Sigma C\right) = 1 + (1-\widetilde{\lambda})\tau\left\{\gamma_s C_1 + (\frac{\rho}{\tau}-\gamma_e)C_2\right\}$.

Finally, $\tau\left\{\gamma_s C_1 + (\frac{\rho}{\tau}-\gamma_e)C_2\right\} = (1-\varphi)\frac{n}{1+n}\frac{\rho^2}{\tau_x\tau}\left\{1-\frac{(1-u)(1+n\varphi)}{1+(1-u)n\varphi}\right\}$ $= (1-\varphi)\frac{n}{1+n}\frac{\rho^2}{\tau_v\tau_x}\frac{1+(1-t)\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)}{1+\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)}\frac{u}{1+(1-u)n\varphi}$. Also, $(1-\widetilde{\lambda})\frac{n}{1+n} = \left(1-\left(\frac{\varphi+\frac{1}{n}}{1-\varphi}\right)^2\right)\frac{n}{1+n} = \frac{1-2\varphi-2\varphi\frac{1}{n}}{(1-\varphi)^2}\frac{n}{1+n} = \frac{1-2\varphi(1+\frac{1}{n})}{(1-\varphi)^2}\frac{n}{1+n} = \frac{\frac{n}{1+n}-2\varphi}{(1-\varphi)^2}$. This completes the proof of (a) and (b).

To show (c), note that both $\frac{n(1-\varphi)}{1+n}$ and $\frac{\frac{n}{1+n}-2\varphi}{1-\varphi}$ are increasing in $n$ and decreasing in $\varphi$.

Hence, if $\varphi$ does not depend on $n$ or decreases in $n$, $\frac{n(1-\varphi)}{1+n}$ and $\frac{\frac{n}{1+n}-2\varphi}{1-\varphi}$ are increasing in $n$. $A(n)B(n)$ is clearly decreasing in $n\varphi$.

**Proof of Proposition 3**

(a) It suffices to show that $\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}A(n)B(n)$ is decreasing for sufficiently large $n$. From **Proposition A2**, for assets with $t=1$, $u<1$, and $\frac{\rho^2}{\tau_\varepsilon\tau_x}<\frac{4}{u}$, $\varphi$ increases in $n$ and there is a finite value of $\widetilde{n}$ such that $\frac{\widetilde{n}-1}{\widetilde{n}}=2\varphi$ (and the second-order condition is violated for all $n\geq\widetilde{n}$). Since $A(n)B(n)$ is decreasing in $n\varphi$, $\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}A(n)B(n)$ must be decreasing for $n$ close to $\widetilde{n}$. For all other types of assets, **Proposition A2** shows that $\varphi$ is decreasing in $n$, $n\varphi$ is increasing in $n$. Therefore, by **Proposition 2(c)**, $\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}$ is increasing in $n$ while $A(n)B(n)$ is decreasing in $n$.

We show that $\left(\frac{n-1}{n}-2\varphi\right)n^2\frac{\partial}{\partial n}\left\{\log\left(\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}A(n)B(n)\right)\right\}<0$ for sufficiently large $n$. First, $\left(\frac{n-1}{n}-2\varphi\right)n^2\frac{\partial}{\partial n}\left\{\log\left(\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}A(n)B(n)\right)\right\}=1-n\left\{(n+1)(1-\varphi)\varphi'+cK(n-1-2n\varphi(n\varphi)')\right\}$, where $K\equiv\frac{t}{\{1-tc+(1-t)cn\varphi\}(1+cn\varphi)}+\frac{1-u}{c\{1+(1-u)n\varphi\}}$ and $\varphi'\equiv\frac{\partial\varphi}{\partial n}$. Because $\varphi'=\frac{1}{n}((n\varphi)'-\varphi)$,

$$\begin{aligned}&(n+1)(1-\varphi)\varphi'+cK(n-1-2n\varphi(n\varphi)')\\=\;&cK(n-1)-\frac{n+1}{n}(1-\varphi)\varphi+\left\{\frac{n+1}{n}(1-\varphi)-2cKn\varphi\right\}(n\varphi)'.\end{aligned}\tag{38}$$

It suffices to show that (38) is positive and not decreasing in $n$ for sufficiently large $n$. For assets with $u=1$ and $t<1$, **Proposition A2** shows that $n\varphi$ increases at the rate $n^{\frac{1}{3}}$. Hence, $K=\frac{t}{\{1-tc+(1-t)cn\varphi\}(1+cn\varphi)}$ decreases at the rate $n^{-\frac{2}{3}}$ and $K(n-1)$ increases at the rate $n^{\frac{1}{3}}$. Because $Kn\varphi$ decreases at the rate $n^{-\frac{1}{3}}$, (38) is positive for sufficiently large $n$. For all other assets, **Proposition A2** shows that $n\varphi$ increases to a finite limit. Hence, $K$ and $Kn\varphi$ are bounded and $(n\varphi)'$ approaches zero, while $K(n-1)$ is not bounded. Therefore, (38) is positive and not decreasing in $n$ for sufficiently large $n$.

(b) See proof of **Proposition A2** for $\lim\limits_{n\to\infty}\varphi=0$. For assets with $u<1$, $n\varphi$, and hence $A(n)B(n)$ has finite limits. For assets with $u=1$, $B(n)=1$ and $\lim\limits_{n\to\infty}n\varphi=\infty$ and hence $\lim\limits_{n\to\infty}A(n)=1-t$.

(c) See **Lemma 4** and note that $n\log\left(1+\frac{\rho^2}{\tau_v\tau_x}\frac{n(1-\varphi)}{1+n}\frac{1-c}{1+cn\varphi}\right)$ and $n\log\left(1+\frac{\rho^2}{\tau_v\tau_x}\frac{\frac{n-1}{n}-2\varphi}{1-\varphi}\frac{1-c}{1+cn\varphi}\right)$ have finite limits.

**Proof of Lemma 3**

The proof follows the same step with that of **Proposition 1**, **2** and **Lemma 1**, **2**, except that traders' beliefs are fixed by their prior beliefs, $E_i[v]=E[v]=0$ and $V_i[v]=V[v]=\frac{1}{\tau_v}$ and there is no need to characterize $\tau_1$ and $\varphi$. Because $G^{pt}$ is a monotonic transformation of $\frac{n}{n+1}$, it is monotonically increasing in $n$. The same goes for $G$ which is a monotonic transformation of $\frac{n-1}{n}$.

**Proof of Lemma 4**

First, set $t = u = 1$ in (7) to obtain $F(k; t = u = 1) = \left(\frac{\rho^2}{\tau_\varepsilon \tau_x} k^2 + 1\right)(k-1) = 0$, for which $k = 1$ is the unique solution. Use $k = 1$ in (8) and (9) to obtain $\varphi = \left[1 + \frac{\rho^2}{\tau_\varepsilon \tau_x}\right]^{-1}$ and verify the second-order condition. The optimal order, the equilibrium price, the quantity traded, and the ex ante GFT are obtained by plugging $t = u = k = 1$ in those in **Proposition 1**. Because $\varphi$ does not depend on $n$, $G^{pt}$ is maximized when $\frac{n}{1+n}\frac{1}{1+c_s n\varphi}$ is maximized, while $G$ is maximized when $\frac{\frac{n-1}{n}-2\varphi}{1+c_s n\varphi}$ is maximized. The first derivative of $\frac{n}{1+n}\frac{1}{1+c_s n\varphi}$ is $\frac{1-c_s\varphi n^2}{(1+n)^2(1+c_s n\varphi)^2}$. Hence, $G^{pt}$ is increasing for $n < \sqrt{\frac{1}{c_s\varphi}}$ and decreasing for $n > \sqrt{\frac{1}{c_s\varphi}}$. The first derivative of $\frac{\frac{n-1}{n}-2\varphi}{1+c_s n\varphi}$ is $\frac{1+2c_s\varphi n-(1-2\varphi)c_s\varphi n^2}{(1+n)^2(1+c_s n\varphi)^2}$. Two solutions for $1 + 2c_s\varphi n - (1-2\varphi)c_s\varphi n^2 = 0$ is $n^\pm = \frac{c_s\varphi \pm \sqrt{(c_s\varphi)^2 + (1-2\varphi)c_s\varphi}}{(1-2\varphi)c_s\varphi}$. Because the smaller solution is negative, $G$ is increasing for $n < \frac{c_s\varphi + \sqrt{(c_s\varphi)^2+(1-2\varphi)c_s\varphi}}{(1-2\varphi)c_s\varphi} = \frac{1}{1-2\varphi} + \sqrt{\frac{1}{1-2\varphi}\left(\frac{1}{c_s\varphi} + \frac{1}{1-2\varphi}\right)}$ and decreasing for $n > \frac{1}{1-2\varphi} + \sqrt{\frac{1}{1-2\varphi}\left(\frac{1}{c_s\varphi} + \frac{1}{1-2\varphi}\right)}$. Because $\frac{1}{1-2\varphi} > 1$, $\frac{1}{1-2\varphi} + \sqrt{\frac{1}{1-2\varphi}\left(\frac{1}{c_s\varphi} + \frac{1}{1-2\varphi}\right)} > \sqrt{\frac{1}{c_s\varphi}}$.

**Proof of Lemma 5**

First, (16) is the optimality condition for traders in $N_j$. Suppose neither (i) nor (ii) holds. Then one of the following two must be true: (a) $G(n_j) < G(n_j + 1)$ for two markets $j = 1, 2$; or (b) $G(n_j) < G(n_j + 1)$ for only one market and either $n_0 < 0$ or (18) is violated. For (a), if $G(n_1) - \phi_1 \leq G(n_2) - \phi_2$, any trader in $N_1$ has an incentive to move to market 2, and vice versa. Contradiction. For (b) If $n_0 > 0$, a trader in $N_0$ wants to participate in $N_j$. Note that $G(n_j) < \overline{\phi}$ and $n_j < n^* < n_k$ for all $k \neq j$. If there is $k \neq j$ such that $G(n_k + 1) - \phi_k > G(n_j) - \phi_j$, any trader in $N_j$ wants to move to $N_k$. If there is $k \neq j$ such that $\overline{\phi} - \phi_j > G(n_k) - \phi_k$, $n^* - n_j$ traders in $N_k$ want to move to $N_j$. Contradiction.

**Proof of Proposition 4**

Let $n(\phi)$ be the market size determined in a stable participation equilibrium for given $\phi \in [c, \overline{\phi}]$. From **Lemma 5**, either (i) the market operates at the decreasing part of $G$ so that $n^* \leq n(\phi) < \overline{n}$ and $G(n(\phi)) = \phi$, or (ii) the market operates at the increasing part of $G$ so that $n(\phi) = \overline{n} < n^*$ and $\phi \leq G(\overline{n})$. Therefore, the intermediary faces the demand

$$n(\phi) = \begin{cases} 0 & if \quad \phi > \overline{\phi} \\ 0 & if \quad \phi \in (G(\overline{n}), \overline{\phi}] \text{ and } \overline{n} \leq n^* \\ G^{-1}(\phi) & if \quad \phi \in (G(\overline{n}), \overline{\phi}] \text{ and } n^* < \overline{n} \\ \overline{n} & if \quad \phi \leq G(\overline{n}) \end{cases}.$$

First define $H(n) \equiv \frac{\partial\{(G(n)-c)n\}}{\partial n} = G(n) + G'(n)n - c$. This is the derivative of the total surplus created by a single market. At the optimal market size, $G'(n^*) = 0$ and $H(n^*) = G(n^*) - c > 0$. Because the total surplus is still increasing in the market size, the intermediary maximizes the profit by having more than $n^*$ traders.

If $c \in (\underline{\phi}, \overline{\phi})$, the GFT curve intersects with the marginal cost curve twice, so $n^c > n^*$ exists. By setting $\phi = \overline{\phi}$, the profit is $(\overline{\phi} - c)n^* > 0$. By setting $\phi = c$, the profit is 0.

Because the demand changes continuously with $\phi$ in $[c, \overline{\phi}]$, there is an optimal fee in $[c, \overline{\phi}]$ that maximizes the profit. Because the demand function is invertible, the intermediary's problem can be written as $\max_{n \in [n^*, n^c]} (G(n) - c)n$. The first-order condition is $H(n) = G'(n)n + G(n) - c = 0$. Note that $H(n^*) = \overline{\phi} - c > 0$ and $H(n^c) = G'(n^c)n^c < 0$. Therefore, $n^m \in (n^*, n^c)$ maximizes the total surplus. If $H(n) > 0$ for all $n \geq n^*$, by setting $\phi = G(\overline{n}) > c$, the profit is $(G(\overline{n}) - c)\overline{n}$. This is unbounded in $\overline{n}$ because $G(\overline{n}) - c > H(\overline{n}) > 0$. Therefore, the monopoly market size is $\overline{n}$ for sufficiently large $\overline{n}$.

**Proof of Proposition 5**

The first-order condition to (22) is $\{H(n_j) - U\} \frac{\partial n_j(\phi_j)}{\partial \phi_j} = 0$, where $\frac{\partial n_j(\phi_j)}{\partial \phi_j} = \frac{1}{G'(n_j)} < 0$ by the implicit function theorem. Therefore, the optimality of $\phi_j^*$ requires $H(n_j) = U$ in equilibrium and $\phi_j^* = G(n_j) - U = G(n_j) - H(n_j) = c - G'(n_j) n_j$. First, for $U = H(n_j) = 0$ to hold in equilibrium, the total surplus must be maximized at $n_j$ and some traders must be excluded from markets. Therefore, $n_j = n^m \leq \frac{\overline{n}}{j_{\max}}$. Second, for $U = H(n_j) > 0$ to hold in equilibrium, the total surplus must be increasing at $n_j$ and all traders must participate in markets. Therefore, $n_j = \frac{\overline{n}}{j_{\max}}$. Hence, in a symmetric equilibrium

$$\phi_j^* = \begin{cases} G(n^m) & \text{if } n^m \leq \frac{\overline{n}}{j_{\max}} \text{ solves } H(n) = 0, \\ c - G'\left(\frac{\overline{n}}{j_{\max}}\right) \frac{\overline{n}}{j_{\max}} & \text{otherwise.} \end{cases}$$

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