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Public-Private Mix of Health Expenditure: A Political Economy Approach and A Quantitative Exercise*

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Abstract

This paper constructs a simple overlapping generations model to examine how the choice of public and private health expenditure is affected by preferences and economic factors under majority voting. In the model, agents with heterogeneous income decide how much to consume, save, and invest in private health care, and vote for the income tax to be used to finance public health. Agents' survival probabilities are endogenously determined by a CES composite of public and private health expenditure. For the two special cases that public and private health are complements or perfect substitutes, we show that the voting equilibrium is unique and locally stable. For the general case, we calibrate the model to Canadian data to conduct a quantitative analysis. Our results suggest that the public-private mix of health expenditure is quite sensitive to the degree of substitutability between private and public health and the relative effectiveness of public and private health. Using a sample of advanced democratic countries, we further infer these two parameters and construct the shares of public health in total health expenditure for each country, and find that the predicted values match the data quite well.

JEL code: D7, H51, I1

Keywords: Public-private mix, Health expenditure, Majority voting, Overlapping generations model

1 Introduction

Achieving a good health status in the overall population is one of the most important goals in every society, not only because it enhances people's life expectancy by reducing both the mortality and morbidity rates, but also because it can improve workers' labor productivity and hence allow a society to consume more output. There are various factors that can improve health status and life expectancy. Among those most important factors are public health care and private health care. The former includes public health policies or programs that provide public hospitals, immunization, disease control and diagnostic health screening, invest in new medical facilities, and promote healthy environment through, e.g., reducing air and water pollutions. The latter refers to private decisions on eating healthy food, taking preventive medicines and vitamins, and having preventive and diagnostic health screenings.

Although the total spending on public and private health care has been rising, and consequently health status has been improving in most countries, there are considerable variations in the mixture of public and private health spending over time and across countries. For instance, the share of public health in total health expenditure has increased by more than 10 percent since the 1970s in the U.S., Austria, Greece, and Japan, while it has decreased by more than 10 percent in the Czech Republic, Norway and the UK. Across OECD countries, the share of public health ranges from less than 50 percent (e.g., the U.S.) to more than 80 percent (e.g., Denmark, Norway, Sweden, Japan, the UK, the Czech Republic) in the 2000s.

This paper aims to shed light on the interaction between public and private financing of health care in a society. Understanding this relationship can help policy makers to allocate resources to health care more efficiently so as to achieve better health outcomes. This is particularly relevant for discussions among policy makers as well as economists regarding the role of private and public sectors in providing health care and their implications for efficiency and equity of the society.

In particular, we study how the public-private mix of health expenditure is chosen by people in a democratic society collectively, when people can choose between public and private health care in improving their life expectancies. We construct an overlapping generations model to explore how public and private health spending are determined by utility-maximizing agents with heterogenous income through majority voting, and how their decisions are shaped by the degree of substitutability between public and private health, income distribution and other economic factors, as well as preferences. Furthermore, the model is calibrated to conduct a quantitative exercise to investigate how well the model can explain the observed differences in the mixture of health expenditure across a group of advanced democratic countries.

In the model, agents live for three periods: childhood, young adulthood and old adulthood. In young adulthood, agents receive exogenous heterogeneous income, and they decide how much to consume, to save for old adulthood, and to invest in private health care, and they vote for the income tax to be used to finance public health. Agents' survival probabilities to old adulthood are endogenously determined by a CES composite of the public and private health expenditure. For the two special cases that public and private health are complements or perfect substitutes, we show that the voting equilibrium is unique and locally stable, and derive the closed form solution for the steady state majority choice of tax rate. For the general case, we are able to show the existence of a voting equilibrium and derive the equations that implicitly determine the equilibrium majority choice of tax rate. Instructed by the equilibrium equations, we then calibrate the model and conduct a quantitative analysis.

The baseline values for parameters are calibrated to match moments of the Canadian data, including several important moments that capture the life expectancy, relative size and composition of health expenditure in Canada. The comparative static results suggest that the size of public health spending relative to national income and the share of public health in total health expenditure are quite sensitive to the degree of substitutability between private and public health and the share parameters in the CES function that indicate the relative importance of public and private health. We further infer these two parameters for each country using country-specific data, and construct the model predicted shares of public health in total health expenditure for each country in the sample. The results show that the predicted mixture of health expenditure matches the data quite well for the majority of countries, with an overall correlation of 0.44 between predicted shares of public health and corresponding data values. We then discuss several factors the model abstracts from that may have important implications for the public-private mix of health expenditure, such as demographic structure, pricing of health care services, and the composition of funds within public and private financing of health care.

The contributions of this paper are two-fold. The first contribution is a theoretical one. It is one of the few studies that aims to explain the coexistence of private and public health care. Epple and Romano (1996) and Gouveia (1997) are among the first to address this issue. Following the strand of literature on the socialization of commodities, they focus on the public provision of a private good –health care– through majority voting in a static micro-theoretic context. In their models, private and public health care are treated as perfect substitutes, and they directly enter the utility function as an ordinary consumption good. Our model also uses a voting mechanism to study the public and private mix of health expenditure, however we carry out the analysis in a dynamic macro-theoretic context that emphases the role of health care in improving life expectancy and allows for general substitutability between public and private health care. Lahiri and Richardson (2008) develop a similar political economy model as ours in which individuals vote on the division of tax revenues between public health and a lump sum transfer payment. Their focus, however, is on how public and private health spending impact on growth and wealth inequality in the long run. As the existence and uniqueness of a voting equilibrium cannot be analytically established, their analysis is primarily based on numerical simulations. Bethencourt and Galasso (2008) present another political economy model that incorporates both public and private health spending, but they address the political complementarities between public health and social security.

This paper also relates to a large literature that incorporates health and endogenous mortality into a standard growth model to examine their implications for growth, poverty, inequality of income or wealth, and so on. Most of this literature considers one type of health expenditure, either private health in the forms of physical resources, time and human capital (Blackburn and Cipriani, 2002; Chakraborty and Das, 2005; and Castelló-Climent and Doménech, 2008) or public health (Chakraborty, 2004; Áısa and Pueyo, 2006; and Osang and Sarkar, 2008). A few studies consider roles of both public and private health expenditure, such as Bhattacharya and Qiao (2007), and Gupta and Vermeulen (2010). Again, the focus is on the developmental implications of health spending rather than the mixture of health spending. In particular, when public health is considered, the public policy involved (such as the tax rate) is exogenously given rather than being a collective choice. Besides, most of the studies with endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004), do not have disparities in health status across agents, while our model generates heterogeneous life-expectancy as private health spending varies with income, which is heterogenous.

The second contribution is a quantitative one. The aforementioned literature on health is largely purely theoretical. Our paper provides the first quantitative study on the public-private mix of health expenditure, and the quantitative results are reasonably good. The model implies a distribution of private health expenditure that is much more skewed than the distribution of income, which is qualitatively consistent with the data. The computed income elasticities suggest that public health and total health are likely to be normal goods but private health is likely to be a luxury good. These results are in line with the stylized facts that countries or individuals tend to spend more on health when they are richer. The estimated elasticities of substitution between public and private health for some countries (such as the U.S. and the UK) receive some empirical support from country-specific studies. Finally, there is a relatively close match between the predicted shares of public health in total health spending and the corresponding data values for the group of 22 advanced democratic countries. These results suggest that our model provides a promising framework to study the determination of public and private health spending. The quantitative results also provide important insights for empirical work on health. The model has identified several important factors for the size and composition of health spending, such as the elasticity of substitution between public and private health and the relative importance of public and private health. These factors are largely ignored in the empirical literature on health, possibly due to their unobservability in the data. Our quantitative exercise provides a way to deduce the values for these important factors, which may be used for relevant empirical work.

Lahiri and Richardson (2008) also carry out some calibrations and numerical analysis. However, their analysis is not guided by a specific quantitative question and some key parameters, such as the elasticity of substitution between public and private health, are not calibrated to match the data. There is also an emerging quantitative literature on health related issues, such as De Nardi, French, and Jones (2010), Hsu and Lee (2011), and Jung and Tran (2010). None of them, however, focuses on the public-private mix of health expenditure.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 characterizes the voting equilibrium for the two special cases. Section 4 derives some analytical results for the general case and conducts a quantitative exercise. Section 5 concludes.

2 Model

We consider an overlapping generations economy with infinite number of periods. The economy is populated with a large number of agents who potentially live for three periods: childhood, young adulthood and old adulthood. In the first period, agents do not make any decisions. In the second period, agents receive exogenous income and make decisions on their consumption in young adulthood, saving for old adulthood, private health expenditure, and they vote for taxes that finance public health expenditure. Each young adult gives birth to one offspring in young adulthood, and thus the population size is constant. In their third period of life, agents simply consume what they have and exit the economy. All economic and political decisions are made in young adulthood.

Young adults in the same generation at time t are differentiated by their income, $y_{i,t}$, according to an exogenous probability distribution function $F_t(\cdot)$, where i refers to the ith young adult. The mean income at time t is thus given by $\bar{y}_t \equiv \int y dF_t(y)$.

Survival in childhood and young adulthood is certain, but survival in old adulthood endogenously depends upon an agent's health status in childhood. That is, we assume that young adult i's survival probability to old adulthood, $p_{i,t} \in (0,1)$, is a function of health capital acquired in childhood, $\hat{H}_{i,t-1}$:

$$p_{i,t} = p\left(\hat{H}_{i,t-1}\right), \tag{1}$$

where $\partial p_{i,t}/\partial \hat{H}_{i,t-1} > 0$, $\partial^2 p_{i,t}/\partial \hat{H}^2_{i,t-1} < 0$, $p_{i,t}(0) = 0$ and $\lim_{\hat{H}_{i,t-1}\to\infty} p_{i,t} = 1/\kappa < 1$. The health capital is defined as a CES composite of both public and private health expenditure:

$$\hat{H}_{i,t-1} = (\phi_H H^{\rho}_{t-1} + \phi_h h^{\rho}_{i,t-1})^{1/\rho}, \tag{2}$$

where H_{t-1} denotes per capita public health expenditure in period $t-1$, $h_{i,t-1}$ is the private health expenditure of agent i's parent in $t-1$. The share parameters $\phi_H \in (0,1)$ and $\phi_h \equiv 1 - \phi_H$ indicate the importance of public and private health expenditure, respectively, and $\rho \in [0,1]$ measures the elasticity of substitution between public and private health, which is a constant given by $\varepsilon \equiv 1/(1-\rho) \in [1,\infty)$.[1] The health technology specified in (2) is a reduced-form representation of the structure of public and private health care in a society. It allows for general substitutability between these two in forming the health capital of the society.

[1] The restriction that $\rho \in [0,1]$ is needed to establish the existence of a voting equilbirium in later sections. This restriction implies that public and private health care are substitutes, with elasticity of substitution within $[1,\infty)$. The empirical work of McAvinchey and Yannopoulos (1994) finds that private and public health care are substitues.

The assumption above implies that an agent's mortality later in life is determined by her health status in childhood, which depends on her parents' choices of health expenditure. Empirical evidence on the importance of health status in childhood for mortality later in life is well documented in Van Den Berg, Lindeboom and Portrait (2006) and Ferrie and Rolf (2011). Similar assumptions are also made in many other studies that link private health and longevity, such as Blackburn and Cipriani (2002), Castelló-Climent and Doménech (2008), Goulao and Perez-Barahona (2011) and Heijdra and Reijnders (2009). This assumption provides us with some analytical tractability in characterising the voting equilibrium. An alternative assumption in this literature is to ignore parents' influence on children's health status and assume that an agent's longevity is determined by her own health expenditure in adulthood, see Hall and Jones (2007) and Chakraborty and Das (2005).

The lifetime utility of agent i at time t is defined over her consumption in young adulthood, $c_{i,t} \in R_+$, consumption in old adulthood, $d_{i,t+1} \in R_+$, and health capital, $\hat{H}_{i,t} \in R_+$:[2]

$$U_{it} = \ln(c_{i,t}) + \beta p_{i,t} \ln(d_{i,t+1}) + \alpha \ln\left(\hat{H}_{i,t}\right), \tag{3}$$

where $\beta \in (0,1)$ is the subjective discount factor on expected utility from consumption in old adulthood, and $\alpha \in (0,1)$ is the utility weight attached to the health capital that would determine her offspring's survival probability.

Agent i draws income $y_{i,t}$ from the exogenous distribution, pays income taxes at uniform rate τ_t, spends her disposable income on consumption in young adulthood, private savings, and private health spending. To deal with the mortality risk, we follow the strand of literature (e.g., Chakraborty (2004)) that assumes a perfectly competitive annuities market for private savings. This implies that the gross rate of return on private savings is given by $(1 + r_{t+1})/\bar{p}_t$, where $\bar{p}_t$ is the average survival probability, and $1 + r_{t+1}$ is the exogenous gross interest rate. In old adulthood, agent i simply consumes her private saving and any interest income earned.

[2] We implicitly assume that the utility of the dead is an infinitely negative number.

The budget constraints of agent i in young and old adulthood, respectively, are given by:

$$\begin{aligned} c_{i,t} + s_{i,t} + h_{i,t} &= (1-\tau_t) y_{i,t}, &(4)\\ d_{i,t+1} &= \frac{(1+r_{t+1})}{\bar{p}_t} s_{i,t}. &(5) \end{aligned}$$

Given a tax rate τ_t, agent i's utility maximization problem is to choose $s_{i,t}$ and $h_{i,t}$ to maximize (3) subject to (4) and (5).

Public health expenditure, H_t, is financed by income taxes collected from young adults in period t. Government budgets are balanced in every period:

$$H_t = \tau_t \bar{y}_t. \tag{6}$$

The tax rates prevailing in each period are endogenously determined by a majority voting mechanism. That is, in period t, each young adult votes on her preferred tax rate, which would be the tax rate that maximizes her indirect utility. The collective choice of the tax rate, τ_t, is determined by the majority rule.

It is not easy to characterize the voting equilibrium analytically for the general form of health capital in (2). Therefore we first consider two special cases: $\rho = 0$ and $\rho = 1$. For each case, we are able to show the existence, uniqueness, and stability of the equilibrium tax rate under majority voting, as well as some analytical results regarding how the equilibrium tax rate (ratio of public health expenditure to mean income) and the public-private mix of health expenditure depend on important economic variables. We then provide a quantitative exercise for the general case.

3 Special Cases

3.1 Public and Private Health are Complements

We first consider the special case that $\rho = 0$ in (2), then the health capital takes the Cobb-Douglas form:

$$\hat{H}_{i,t-1} = H_{i,t-1}^{\phi_H} h_{it-1}^{\phi_h}. \tag{7}$$

Under this form, public and private health care are substitutable, but have a low elasticity of substitution 1, or in other words, they are more complementary or supplementary in forming the society's health capital. In this sense, we refer to this case as public and private health being complements in Cobb-Douglas form. In practice, public and private health care are complementary in many ways. For instance, public health expenditure that reduces air or water pollution can complement private health investment via nutritious food or regular exercise in reducing individuals' risk of getting communicable diseases.

Solving the utility maximization problem formulated earlier yields agent i's optimal private saving and private health spending:

$$s_{it} = \frac{\beta p_{i,t}}{(1 + \beta p_{i,t})} \left((1 - \tau_t) y_{i,t} - h_{i,t} \right), \tag{8}$$

$$h_{i,t} = \frac{\alpha \phi_h}{1 + \beta p_{i,t} + \alpha \phi_h} (1 - \tau_t) y_{i,t}. \tag{9}$$

Eq. (8) implies that an increase in the probability of survival increases private savings as young adults who expect to live longer are effectively more patient and more willing to save for old adulthood consumption. It is also clear from (9) that private health expenditure is a normal good. What is relatively new here is that an increase in an agent's survival probability reduces her private health investment in her offspring. This occurs because the agent's own adulthood consumption competes with the private health investment.

To derive the equilibrium tax rate preferred by a majority of voters, we first find the indirect utility of agent i by substituting Eq. (8) and (9) into the utility function (3):

$$V_{i,t} = (1 + \beta p_{i,t} + \alpha\phi_h) \ln (1 - \tau_t) + \alpha\phi_H \ln (\tau_t) + \Gamma_c, \tag{10}$$

where Γ_c is an expression of parameters and variables that are either exogenously given or predetermined, and $p_{i,t}$ is given by (1) and (2).[3] Eq. (10) implies that the voters' choices in period t are over a single dimension (τ_t). It can be shown that all voters' preferences are single-peaked in τ_t, since $\partial^2 V_{it}/\partial\tau_t^2 = -(1 + \beta p_{i,t} + \alpha\phi_h) / (1 - \tau_t)^2 - \alpha\phi_H/\tau_t^2 < 0$, and hence the median voter (young adult with median income) is the decisive voter.[4] The majority voting equilibrium tax rate, $\tau_{m,t}$, is then determined by the following condition:

$$\frac{\partial V_{m,t}}{\partial \tau_{m,t}} = -\frac{(1 + \beta p_{m,t} + \alpha\phi_h)}{(1 - \tau_{m,t})} + \frac{\alpha\phi_H}{\tau_{m,t}} = 0,$$

where $V_{m,t}$ is the indirect utility of the median voter at time t, such that

$$\tau_{m,t} = \frac{\alpha\phi_H}{1 + \alpha + \beta p_{m,t}}. \tag{11}$$

Making use of Eq. (9) and (11), we can derive the private health spending of the median voter in period t as

$$h_{m,t} = (\phi_h/\phi_H)\, \tau_{m,t}\; y_{m,t},$$

such that the ratio of median private health to public health expenditure ($H_t = \tau_{m,t}\, \bar{y}_t$) and the health capital, respectively, are simply given by:

$$\frac{h_{m,t}}{H_t} = \frac{\phi_h}{\phi_H} \frac{y_{m,t}}{\bar{y}_t}, \tag{12}$$

[3] $\Gamma_c = (1 + \beta p_{i,t} + \alpha\phi_h) \ln \left(\frac{y_{i,t}}{1+\beta p_{i,t}+\alpha\phi_h} \right) + \beta p_{i,t} \ln \left(\frac{(1+r_{t+1})\beta p_{i,t}}{\bar{p}_t} \right) + \alpha \ln \left(\bar{y}^{\phi_H} (\alpha\phi_h)^{\phi_h} \right).$

[4] For the theory of majority voting, see Muller (2003).

and

$$\hat{H}_{m,t-1} = \Phi_{m,t-1}\tau_{m,t-1}, \tag{13}$$

where $\Phi_{m,t-1} = [(\phi_h/\phi_H)(y_{m,t-1}/\bar{y}_{t-1})]^{\phi_h}\bar{y}_{t-1}$. Note that in this special case the ratio of median private health to public health expenditure is independent of the equilibrium tax rate. In fact, it is the product of two ratios: the share of private relative to public health expenditure in the composite health service, and the ratio of median income to mean income, which is an important measure of income inequality. It is obvious from (12) that the median voter prefers to have higher private health relative to public health when income inequality is lower ($y_{m,t}/\bar{y}_t$ is higher), and when the share of private relative to public health expenditure is higher.

Note that Eq. (11) characterizes the majority choice of tax rate in a recursive manner, that is, $\tau_{m,t}$ depends on the majority choice of tax rate in the previous period, $\tau_{m,t-1}$. An increase in $\tau_{m,t-1}$ reduces $\tau_{m,t}$ because an increase in $\tau_{m,t-1}$ increases the survival probability, $p_{m,t} = p(\hat{H}_{m,t-1})$, via its positive effect on the health capital, $\hat{H}_{m,t-1}$ in (13). That is, the first derivative of $\tau_{m,t}$ with respect to $\tau_{m,t-1}$ is negative. It can also be shown that the second derivative of $\tau_{m,t}$ with respect to $\tau_{m,t-1}$ is positive. Hence Eq. (11) implies a unique steady state of the majority choice of tax rate, if we consider an economy with a time-invariant distribution of income, $F_t(\cdot) = F(\cdot)$ for all t.

To further characterize the steady state equilibrium, we consider the following parametric form for the survival probability $p_{i,t}$:[5]

$$p_{i,t} = p\left(\hat{H}_{i,t-1}\right) = \frac{\hat{H}_{i,t-1}}{1+\kappa\hat{H}_{i,t-1}}, \tag{14}$$

where $\kappa > 1$. This form implies that the survival probability is strictly increasing and strictly

[5] A similar functional form for the probability of survival is assumed in Chakraborty (2004).

concave in health capital. Then Eq. (11) can be written as:

$$\tau_{m,t} = \frac{\alpha\phi_H\left(1+\kappa\Phi_m\tau_{m,t-1}\right)}{\left(1+\alpha\right)\left(1+\kappa\Phi_m\tau_{m,t-1}\right)+\beta\Phi_m\tau_{m,t-1}}, \tag{15}$$

where $\Phi_m = [(\phi_h/\phi_H)(y_m/\bar{y})]^{\phi_h}\bar{y}$. The unique steady state value of the majority voting equilibrium tax rate, denoted as τ_m, is then given by:

$$\tau_m = \frac{-\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]+\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\left[\left(1+\alpha\right)\kappa+\beta\right]\alpha\phi_H\Phi_m}}{2\left[\left(1+\alpha\right)\kappa+\beta\right]\Phi_m}. \tag{16}$$

Proposition 1 summarizes the results above and establishes some important properties of τ_m. A detailed proof is given in the Appendix.

Proposition 1 *When public and private health expenditure are complements in Cobb-Douglas form, there is a unique steady state voting equilibrium. Given the parametric form for survival probability in (14), the unique steady state tax rate, τ_m, is given by (16) and it has the following properties: (i) $\tau_m \in (0,1)$; (ii) τ_m is locally stable and converged with damped oscillations; (iii) τ_m falls with a rise in $y_m/\bar{y}$ (keeping $\bar{y}$ constant); (iv) τ_m falls with a rise in $\bar{y}$ (keeping $y_m/\bar{y}$ constant).*

The result in (iii) implies that when income inequality is higher, a majority of voters tend to vote for higher tax rates. This finding is consistent with a large literature that models majority voting over the tax rate; see Meltzer and Richard (1981) and Krusell and Rios-Rull (1999) for examples. In those models, tax revenues are used to finance some type(s) of redistributive expenditure such as transfer payments and pensions. In our model, public health, financed by tax revenues, also has a redistributive role because the coverage is universal while the contributions to public health are associated with individuals' income. The result in (iv) states that the majority choice of tax rate decreases with a society's average income, suggesting that the society can meet its finance needs with a lower tax rate as it becomes richer.

3.2 Public and Private Health are Perfect Substitutes

Public and private health can also be highly substitutable. Due to the diversity of tastes and needs, individuals may prefer private health care over public health care, or vice versa, even though they provide similar health services. For instance, individuals may prefer private hospital over public hospital as private hospital provides faster access or nicer environment than public hospital does. So we consider another special case that public and private health are perfect substitutes, that is, we let $\rho = 1$ in (2) and hence the elasticity of substitution $\varepsilon = \infty$. The health capital then takes the linear form:

$$\hat{H}_{i,t-1} = \phi_H H_{t-1} + \phi_h h_{i,t-1}, \tag{17}$$

where the share parameters ϕ_H and ϕ_h also indicate the effectiveness of public and private health, respectively.

Solving the individual's maximization problem yields agent i's optimal private saving and private health spending:

$$s_{it} = \frac{\beta p_{i,t}}{1+\beta p_{i,t}} \left((1-\tau_t) y_{i,t} - h_{i,t}\right), \tag{18}$$

$$h_{i,t} = \max\left\{0, \frac{\alpha y_{i,t}}{1+\beta p_{i,t}+\alpha}\left(1-\tau_t\left(1+\frac{(1+\beta p_{i,t})}{\alpha}\frac{\phi_H}{\phi_h}\frac{\bar{y}_t}{y_{i,t}}\right)\right)\right\}. \tag{19}$$

As in the complements case, an increase in the probability of survival, $p_{i,t}$, increases private savings, s_{it}, and reduces private health spending, $h_{i,t}$, when $h_{i,t}$ is positive. Private health spending, $h_{i,t}$, is positive as long as the following condition holds:

$$\frac{y_{i,t}}{\bar{y}_t}\frac{\alpha}{1+\beta p_{i,t}} > \frac{\phi_H}{\phi_h}\frac{\tau_t}{1-\tau_t}. \tag{20}$$

This condition is more likely to hold when agent i's income is higher, or when her probability of survival, $p_{i,t}$, is lower. The intuition for the latter result is that a lower survival probability leads to lower private savings for old adulthood and hence more income in young adulthood can be allocated to private health investment.

Substituting the expression for private savings in (18) into the utility function, we obtain the indirect utility for agent i:

$$V_{i,t} = (1 + \beta p_{i,t}) \ln ((1 - \tau_t) y_{i,t} - h_{i,t}) + \alpha \ln (\phi_H H_t + \phi_h h_{i,t}) + \Gamma_{PS},$$

where $\Gamma_{PS} = -(1 + \beta p_{i,t}) \ln (1 + \beta p_{i,t}) + \beta p_{i,t} \ln (\beta (1 + r_{t+1}) p_{i,t} / \bar{p}_t)$, which is independent of the current tax rate, τ_t. It can be shown that $\partial^2 V_{i,t} / \partial \tau_t^2 < 0$ (regardless of whether $h_{i,t}$ is zero or positive) such that voters' preferences are single-peaked in τ_t. Then the equilibrium tax rate under majority voting, $\tau_{m,t}$, is the tax rate that maximizes the median voter's indirect utility.

If the following condition holds (obtained by rearranging (20)),

$$\tau_{m,t} \leq \frac{\alpha}{\alpha + (1 + \beta p_{m,t}) \frac{\phi_H}{\phi_h} \frac{\bar{y}}{y_m}} \equiv \widehat{\tau}_{m,t}, \tag{21}$$

$h_{m,t}$ is given by the second expression in the bracket in (19), so we obtain the following expression for $\partial V_{m,t} / \partial \tau_{m,t}$:

$$\frac{\partial V_{m,t}}{\partial \tau_{m,t}} = (1 + \beta p_{m,t} + \alpha) \frac{y_{m,t} \left(\frac{\phi_H}{\phi_h} \frac{\bar{y}_t}{y_{m,t}} - 1 \right)}{(1 - \tau_{m,t}) y_{m,t} + \frac{\phi_H}{\phi_h} H_t}.$$

Assume that the ratio of median to mean income is constant over time, i.e., $y_{m,t} / \bar{y}_t = y_m / \bar{y}$, and define $\digamma \equiv (\phi_H / \phi_h)(\bar{y} / y_m)$. If $\digamma = 1$, $\partial V_{m,t} / \partial \tau_{m,t} = 0$ for any $\tau_{m,t} \in [0, \widehat{\tau}_{m,t}]$ such that the equilibrium tax rate is not unique. Hence, we focus on the case that $\digamma \neq 1$.

First consider $\digamma < 1$, then $\partial V_{m,t} / \partial \tau_{m,t} < 0$ for all t, so the median voter would prefer a zero tax rate in every period, i.e. $\tau_{m,t} = 0$ for all t. Consequently, public health spending is zero in all periods and all young adults choose their own maximum levels of private health. The intuition for this result is as follows. When $\digamma < 1$, the degree of income inequality is relatively low (low $\bar{y}/y_m$) or public health is relatively less effective (low ϕ_H/ϕ_h), or both. Hence a majority of voters tend to vote for a low level of public health, which serves as a redistribution device in the model, and replace public health with private health as they are perfect substitutes in improving longevity.

Next we consider $\digamma > 1$, then $\partial V_{m,t}/\partial \tau_{m,t} > 0$ for all t, so the median voter prefers a tax rate that is as high as possible such that $\tau_{m,t} = \widehat{\tau}_{m,t}$ for all t. Consequently the private health spending of the median voter is zero in every period, i.e. $h_{m,t} = 0$ for all t, while public health spending is given by $H_t = \widehat{\tau}_{m,t}\bar{y}_t$. Eq. (20) implies that in a given period young adults with income and survival probability lower than those of the median voter also have zero private health spending, while young adults with income and survival probability higher than those of the median voter have positive private health spending.

Note that the equation $\tau_{m,t} = \widehat{\tau}_{m,t}$ defines a recursive relationship in equilibrium tax rates over time:

$$\tau_{m,t} = \frac{\alpha}{\alpha + [1 + \beta p(\phi_H \tau_{m,t-1}\bar{y}_{t-1})]\,\digamma}. \tag{22}$$

As in the complement case, $\tau_{m,t}$ depends on $\tau_{m,t-1}$ negatively at a diminishing rate such that there is a unique steady state tax rate. With the functional form specified in (14) for the survival probability, the unique steady state tax rate is given by:

$$\tau_m = \frac{-\left[\alpha(1-\kappa\phi_H\bar{y}) + \digamma\right] + \sqrt{\left[\alpha(1-\kappa\phi_H\bar{y}) + \digamma\right]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+\digamma) + \beta\digamma]}}{2\phi_H\bar{y}[\kappa(\alpha+\digamma) + \beta\digamma]}. \tag{23}$$

Proposition 2 summarizes the results above and establishes some equilibrium properties. A detailed derivation of the main results is given in the Appendix.

Proposition 2 *When public and private health expenditure are perfect substitutes, there is a unique steady state voting equilibrium when* $\digamma \equiv (\phi_H/\phi_h)(\bar{y}/y_m) \neq 1$*. For* $\digamma < 1$*, the steady state tax rate is given by* $\tau_m = 0$ *such that public health spending is zero. For* $\digamma > 1$*, the private health spending of the median voter is zero and, given the parametric form in (14) for survival probabilities, the unique steady state tax rate* τ_m *is given by (23) and it has* *the following properties: (i) $\tau_m \in (0,1)$; (ii) τ_m is locally stable and converged with damped oscillations; (iii) τ_m rises with a rise in $y_m/\bar{y}$ (keeping $\bar{y}$ constant); (iv) τ_m falls with a rise in $\bar{y}$ (keeping $y_m/\bar{y}$ constant).*

The comparative static property stated in (iii) is different to that found in the complement case and in most of the existing literature. It states that the majority choice of tax rate increases with a reduction in the degree of income inequality. This may sound puzzling. However, note that when there is a reduction in income inequality (a fall in $\bar{y}/y_m$), for $F > 1$ to hold, ϕ_H/ϕ_h needs to be relatively large, implying that public health is much more effective relative to private health. Therefore a majority of individuals tend to vote for a higher tax rate to substitute public health for private health .

4 General Case: A Quantitative Exercise

Now consider the general form of health capital in (2) with $\rho \in [0,1]$. Agent i's utility maximization problem yields the following equations:

$$s_{it} = \frac{\beta p_{i,t}}{(1+\beta p_{i,t})}\left((1-\tau_t)y_{i,t} - h_{i,t}\right),$$

$$\frac{1}{(1-\tau_t)y_{i,t} - s_{i,t} - h_{i,t}} = \alpha \frac{\phi_h h_{i,t}^{\rho-1}}{\phi_H H_t^\rho + \phi_h h_{i,t}^\rho}.$$

Combining these two equations gives

$$\frac{(1+\beta p_{i,t})}{(1-\tau_t)y_{i,t} - h_{i,t}} = \frac{\alpha \phi_h h_{i,t}^{\rho-1}}{\phi_H H_t^\rho + \phi_h h_{i,t}^\rho}. \tag{24}$$

It is not easy to solve for $h_{i,t}$ explicitly from (24), however, it can be shown that $\partial h_{i,t}/\partial y_{i,t} > 0$, so private health is a normal good. Utilizing (24), agent i's indirect utility can be written as

$$V_{it} = \left(1 + \beta p_{i,t} + \frac{\alpha}{\rho}\right)\ln\left((1-\tau_t)y_{i,t} - h_{i,t}\right) - \frac{\alpha(1-\rho)}{\rho}\ln(h_{i,t}) + \Gamma,$$

where Γ is an expression that is independent of $h_{i,t}$. We can show that voters' preferences are single-peaked as well, though this is not as straightforward as in the special cases (see the Appendix for a detailed proof). Therefore, the majority choice of tax rate, $\tau_{m,t}$, satisfies $\partial V_{m,t}/\partial \tau_{m,t} = 0$, which simplifies to

$$\frac{H_t}{h_{m,t}} = \left(\frac{\phi_H}{\phi_h} \frac{\bar{y}_t}{y_{m,t}} \right)^{\frac{1}{1-\rho}}. \tag{25}$$

Recall that the expression $(\phi_H/\phi_h)(\bar{y}_t/y_{m,t})$ is in fact the $\digamma$ defined earlier. From Eq. (24), $h_{m,t}$ is implicitly determined by

$$\frac{1-\tau_{m,t}}{\tau_{m,t}} \frac{y_{m,t}}{\bar{y}_t} \frac{H_t}{h_{m,t}} = 1 + \frac{1+\beta p_{m,t}}{\alpha} \left[1 + \frac{\phi_H}{\phi_h} \left(\frac{H_t}{h_{m,t}} \right)^{\rho} \right], \tag{26}$$

where the survival probability, $p_{m,t}$, using the parametric form of $p(\cdot)$ in (14), is given by

$$p_{m,t} = p\left(\hat{H}_{m,t-1}\right) = \frac{1}{\kappa + \frac{1}{\left[\phi_H + \phi_h \left(\frac{h_{m,t-1}}{H_{t-1}}\right)^{\rho}\right]^{1/\rho} H_{t-1}}}, \tag{27}$$

and recall that $H_t = \tau_{m,t}\bar{y}$.

Eq. (25)-(27) characterize the equilibrium tax rate. With y_m, $\bar{y}$ and the parameters given, the steady state value τ_m is implicitly determined by substituting H/h_m using (25) into (26) and (27), then substituting p_m using (27) into (26).

For this general case, we are able to obtain some analytical results. First, we can show that τ_m decreases with average income $\bar{y}$, keeping income inequality $y_m/\bar{y}$ unchanged (see the Appendix for the proof). This is consistent with what we found in the two special cases above. Second, the ratio of median private health expenditure to per capita public health expenditure (h_m/H), as characterized in Eq. (25), decreases with an increase in income inequality (lower $y_m/\bar{y}$) and with an increase in ϕ_H, which indicates the importance of public health relative to private health, and decreases (increases) with an increase in the elasticity of substitution between private and public health $1/(1-\rho)$ if $\digamma$ is greater than one (less than one). However, it is difficult to derive further analytical results. Instead, we calibrate the parameters and conduct a quantitative analysis.

4.1 Baseline Calibration

Parameters of the model include: the discount factor (β), the interest rate (r), the utility weight attached to the health capital (α), the parameter in the parametric form of survival probability (κ), the share parameters in the CES function of health capital (ϕ_H, $\phi_h \equiv 1-\phi_H$), and the parameter measuring the degree of substitution between public and private health (ρ). We calibrate these parameters to match certain characteristics of the Canadian data (2000-2009). Canada is chosen for its well-established universal public health care system as well as data availability.

First, we set a period in the model to be 30 years, that is, childhood is from 0 to 30 years old, young adulthood is from 31 to 60 years old, and old adulthood is from 61 to 90 years old. Then β and r can be set to match the average annual real interest rate in Canada, which is around 2.51% according to *World Development Indicator-2011*. That is, $r = (1 + 0.0251)^{30} - 1$, and $\beta = 1/(1+r)$. To calibrate κ, note that the parametric form given in (14) implies that the maximum survival probability is given by $1/\kappa$, which would be achieved when the health capital approaches infinity. So we set κ to match a maximum survival probability of 0.9, i.e., $\kappa = 1/0.9$.[6]

The parameters α, ρ and ϕ_H are not directly deducible from the data. We calibrate them jointly using the steady state versions of the three equations that characterize the solution of the model, namely, Eq. (25)-(27). Note that the steady state value of public health expenditure H also needs to be calibrated, as it is not unit-free and cannot be determined from the data.[7] We calibrate α, ρ, ϕ_H, and H to match four moments of the Canadian data.

The first moment is the average median survival probability, p_m. In fact in the model $p_{m,t}$

[6] The sensitivity analysis below shows that the equilibrium tax rate and the public-private mix of health expenditure are not sensitive to κ.

[7] The calibrated value for H is later used to determine a scale factor to scale down average incomes across countries.

and $h_{m,t}$ refer to the survival probability and the private health expenditure, respectively, of the median voter (the individual who has the median income) in period t. Recall that private health is a normal good and the survival probability is strictly increasing in private health expenditure, so $h_{m,t}$ and $p_{m,t}$ are the median private health expenditure and median survival probability in period t as well. Using the data from *World Health Statistics - 2011*, we find that the median age at which people would die, conditional on that they have survived over 60 years but under 90 years, is 83.526. So p_m is set to $(83.526 - 60)/(90 - 60) = 0.784$.[8]

The second moment is the ratio of average annual public health to national income, represented by $\tau_m \equiv H/\bar{y}$ in the model, which is 6.83% according to *OECD Health Dataset - 2011*. The third moment is the average annual share of public health expenditure in total health expenditure, $H/(H+\bar{h})$, where $\bar{h}$ denotes the average private health expenditure. This moment is equal to 70.14 percent for Canada according to *OECD Health Dataset - 2011*. The fourth moment is the ratio of median to mean private health expenditure, $h_m/\bar{h}$, which is 0.6962 according to *National Household Survey* from *Statistics Canada*.[9] Besides, the income inequality measure that is taken as exogenous, $y_m/\bar{y}$, is equal to 0.8659 according to *OECD.Stat Extracts - 2011*. Given the moments above, the ratio of per capita public health to median private health expenditure, H/h_m, appearing in Eq. (25) to (27), is obtained as $H/h_m = 1/\left[\left(h_m/\bar{h}\right)\left(\bar{h}/H\right)\right]$.

The calibration procedure is as follows. First, α can be determined independently of ρ and ϕ_H, by combining Eq. (25) and (26):

$$\alpha = \frac{(1+\beta p_m)\left(1+\frac{H}{h_m}\frac{y_m}{\bar{y}}\right)}{\frac{1-\tau_m}{\tau_m}\frac{y_m}{\bar{y}}\frac{H}{h_m}-1}. \tag{28}$$

For a given H, ρ and ϕ_H are solved jointly from Eq. (25) and (27). We assume that the

[8] World Health Organization provides a life-table for all countries. The life time is 100 years and the table reports data by age intervals x to $x+n$, where $x = 0, 1, 5, ..., 95$ and 100. The life-table considers a sample of 100000 people for each country and reports age-specific death rates calculated from data on death among individuals aged between x and $x+n$.

[9] National Household Survey is not publicly available and is purchased from Statistics Canada. Available at: http:// http://cansim2.statcan.gc.ca/

distribution of income is a log-normal distribution with parameters μ and σ, and calibrate μ and σ to match the scaled mean income and the degree of inequality in the Canadian data, that is, $\mu = \ln(\bar{y} \cdot (y_m/\bar{y}))$, where $\bar{y}$ is given by H/τ_m, and $\sigma = \sqrt{2\ln(\bar{y}/y_m)}$. Then we draw 20,000 income realizations from this distribution, and for each income draw, $y_{i,t}$, we solve the corresponding private health expenditure, $h_{i,t}$, from (24).[10] Then $\bar{h}$ is calculated as the mean of $h_{i,t}$'s, and a value for the ratio of private health to public health $\bar{h}/H$ is obtained. If this value is different from the value of $\bar{h}/H$ in the data, another H is chosen and the process described above is repeated, until the computed value of $\bar{h}/H$ matches its data counterpart.

The calibrated parameters for the Canadian economy are given by: $\alpha = 0.1386$, $\rho = 0.6162$ and $\phi_H = 0.58$, $H = 9.15$ (and hence the scaled average income of Canadians is given by $\bar{y} = H/\tau_m$). Recall that α is an agent's utility weight assigned to the health capital that would determine her offspring's survival probability, so it indicates the degree of altruism towards offspring. The calibrated value of α is about 40 percent of the median effective discount factor βp_m. This value is slightly lower than the values used in most of the quantitative studies that consider altruism toward children's education or future earnings.[11] The value of ρ implies an elasticity of substitution between public and private health of 2.6. The value of ϕ_H is a bit higher than 0.5, suggesting that public health plays a slightly larger role than private health in the society's health capital. Due to the lack of relevant empirical or quantitative studies, we cannot compare the calibrated values for ρ and ϕ_H with other studies. Nevertheless, these values seem realistic.

Under the baseline calibration, the numerical derivatives of public health (H), mean private health ($\bar{h}$), and total health ($H + \bar{h}$) with respect to average income ($\bar{y}$) can be calculated, and hence the income elasticities of total health, public health, and private health

[10] In solving for $h_{i,t}$, we assume that $h_{i,t-1} = h_{i,t}$ in the determination of $p_{i,t}$ such that $h_{i,t}$ is the only unknown in Eq. (24).

[11] Most of the literature that considers altruism assume parents care about their children's education or children's future income rather than children's health status in a life-cycle model. In their numerical exercises, most studies assume a relatively high degree of parental altruism (e.g., 0.65 in Osang and Sarkar (2008) and Raut (2003), and 0.5 in Kalemli-Ozcan (2002)), and a few studies assume a relatively low degree of altruism (e.g., 0.3 in Tang and Zhang (2007) and 0.1 in Pecchenino and Utendorf (1999)).

expenditure are computed as 1.0074, 0.9692 and 1.0901, respectively. The income elasticity of health expenditure has been an important subject addressed in the empirical literature. The values of the income elasticity of total health expenditure produced by the model is in line with most of the empirical finding that the income elasticity estimates are around one or greater than one (see, e.g., Newhouse (1977), Leu (1986), Parkin et al. (1987), Gerdtham et al. (1992), and Hitiris and Posnett (1992)). Although the value of income elasticity of public health expenditure obtained in our model (0.9692) is higher than the value estimated by Di Matteo and Di Matteo (1998) (0.77), our result is consistent with their conclusion that Canadian public health expenditure is not a luxury good. Our result that income elasticity of private health expenditure exceeds one also receives some empirical support, as studies on some private medical care, such as eyeglasses and plastic surgery, find income elasticities that are substantially greater than one (see e.g., Andersen and Benham (1970), Silver (1970), Scanlon (1980), Sunshine and Dicker (1987), and Parker and Wong (1997)).

In the model economy, private health expenditure and hence the probability of surviving to old adulthood are heterogeneous across individuals. In fact heterogeneity in steady state is one advantage of the model. Most existing studies that model endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004) and Hall and Jones (2007), do not have disparities in health status across agents. Based on $20,000$ income draws, Figure 1 plots the kernel density and cumulative distribution functions for income, private health expenditure, and survival probability in the steady state. A notable feature from the figure is that the distribution of private health expenditure is much more skewed than the distribution of income, with about 30 percent of the population having zero or close to zero private health expenditure. However, the distribution of survival probabilities appears quite symmetric, due to the contribution of public health. These qualitative features are broadly consistent with the data, though we do not have individual level data to conduct a quantitative comparison.

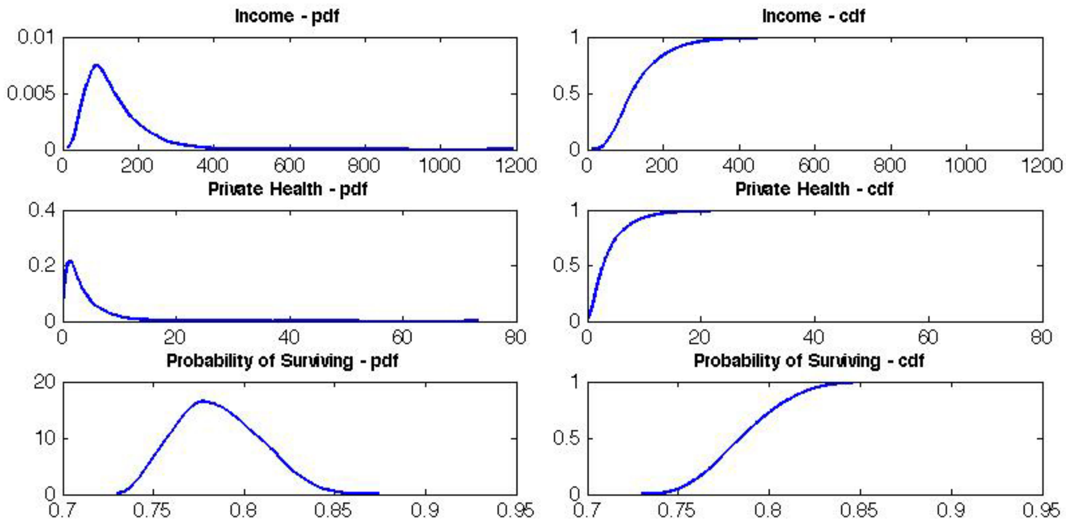

Figure 1: Distribution of Income, Private Health Spending and Probability of Surviving

4.2 Comparative Statics

Next we conduct a numerical exercise to investigate the comparative static properties of the model. The aim is to see how the majority choice of tax rate, which in the model measures the size of public health relative to national income, and the public-private mix of health expenditure respond to variations in the primitives of the model, in particular, how sensitive they are to each variation.

Specifically, we examine how the majority choice of tax rate, τ_m, the ratio of median private health to public health expenditure, h_m/H, and the share of public health in total health expenditure, $H/(H+\bar{h})$, respond to changes in parameters α, β, κ, ϕ_H , ρ, and the statistics that characterise the distribution of income, $y_m/\bar{y}$ and $\bar{y}$. Baseline values for these parameters are the ones described in the calibration above. We consider 5, 10 and 15 percent variations of each parameter around its baseline value, with all other parameters kept at their baseline values. For each variation, h_m/H is determined by (25), and τ_m is solved from (25) to (27). To get the mean private health expenditure $\bar{h}$, we follow the same procedure as in the baseline calibration, then the share of public health in total health spending is calculated as $H/(H+\bar{h})$, where H equals the computed τ_m times $\bar{y}$.

Table 1 summarizes the results from the numerical exercise. It is found that the ratio of per capita public health to average income, τ_m, decreases with β, $y_m/\bar{y}$, and $\bar{y}$, and increases with κ, α, ρ and ϕ_H. In terms of the magnitude of change, τ_m is most sensitive to variations in ϕ_H, also sensitive to variations in α, ρ, and $y_m/\bar{y}$, and less sensitive to β, κ, and $\bar{y}$. The median survival probability, p_m, decreases with β and κ, and increases with α, ρ, ϕ_H, $y_m/\bar{y}$ and $\bar{y}$. In terms of magnitude, p_m is relatively more sensitive to κ and $\bar{y}$, followed by α, ρ, ϕ_H and $y_m/\bar{y}$, and it is least sensitive to β. The ratio of median private health to per capital public health spending, h_m/H, does not vary with β, κ, α and $\bar{y}$, while decreases with ρ and ϕ_H and increases with $y_m/\bar{y}$. Again, it is most sensitive to ϕ_H, and also quite sensitive to variations in ρ and $y_m/\bar{y}$. The share of public health in total health expenditure, $H/(H+\bar{h})$, increases with β, α, ρ and ϕ_H, decreases with κ, and has no clear relationship to $y_m/\bar{y}$ and $\bar{y}$. It is quite sensitive to ϕ_H as well as ρ, while not sensitive at all to other parameters.

Brief intuitions are as follows. A higher discount factor, β, implies that individuals care more about their old-age consumption, so they prefer a lower tax rate in order to have more disposable income to save for old age. As a consequence, the survival probabilities decline. An increase in κ implies a lower survival probability (at any given level of health capital), so to improve survival probabilities, individuals tend to vote for a higher level of public health. A higher α means a higher weight is attached to the health capital in the utility function so that individuals prefer a higher tax rate to achieve a higher level of public health. As a result survival probabilities also increase. A higher ρ implies a greater substitutability between public and private health. With a ϕ_H greater than 0.5, individuals tend to substitute private health with public health and vote for a higher tax rate, or in other words, public health crowds out private health when ρ increases. Consequently, the ratio of median private health to public health expenditure falls and the share of public health in total health expenditure rises. A higher ϕ_H indicates that the importance of public health rises relative to private health in the formation of health capital and thus, individuals vote

Table 1: Variation in Tax and the Ratio for Alternative Parameters

	β	κ	α	ρ	ϕ_H	$\frac{y_m}{\bar{y}}$	$\bar{y}$
Majority Choice of Tax Rate (τ_m)							
-15%	0.0709	0.0660	0.0592	0.0640	0.0496	0.0726	0.0687
-10%	0.0700	0.0668	0.0622	0.0653	0.0563	0.0712	0.0685
-5%	0.0691	0.0676	0.0653	0.0667	0.0626	0.0697	0.0684
Baseline	0.0683	0.0683	0.0683	0.0683	0.0683	0.0683	0.0683
$+5\%$	0.0675	0.0690	0.0713	0.0701	0.0733	0.0669	0.0682
$+10\%$	0.0667	0.0696	0.0743	0.0721	0.0774	0.0655	0.0681
$+15\%$	0.0659	0.0703	0.0772	0.0743	0.0809	0.0641	0.0680
Median Survival Probability (p_m)							
-15%	0.7879	0.8974	0.7689	0.7830	0.7779	0.7805	0.7674
-10%	0.7866	0.8563	0.7745	0.7834	0.7793	0.7817	0.7735
-5%	0.7854	0.8187	0.7796	0.7838	0.7814	0.7830	0.7791
Baseline	0.7842	0.7842	0.7842	0.7842	0.7842	0.7842	0.7842
$+5\%$	0.7830	0.7524	0.7885	0.7847	0.7876	0.7854	0.7889
$+10\%$	0.7818	0.7230	0.7924	0.7853	0.7914	0.7867	0.7932
$+15\%$	0.7806	0.6958	0.7960	0.7859	0.7954	0.7879	0.7972
Ratio of Median Private Health to Per Capita Public Health (h_m/H)							
-15%	0.2964	0.2964	0.2964	0.3753	0.7393	0.1941	0.2964
-10%	0.2964	0.2964	0.2964	0.3507	0.5463	0.2252	0.2964
-5%	0.2964	0.2964	0.2964	0.3244	0.4031	0.2593	0.2964
Baseline	0.2964	0.2964	0.2964	0.2964	0.2964	0.2964	0.2964
$+5\%$	0.2964	0.2964	0.2964	0.2665	0.2166	0.3366	0.2964
$+10\%$	0.2964	0.2964	0.2964	0.2349	0.1570	0.3799	0.2964
$+15\%$	0.2964	0.2964	0.2964	0.2015	0.1125	0.4266	0.2964
Ratio of Public to Total Health Spending ($H/(H+\bar{h})$)							
-15%	0.7031	0.7038	0.7031	0.6674	0.5155	0.7029	0.7026
-10%	0.7032	0.7036	0.7032	0.6781	0.5827	0.7055	0.7028
-5%	0.7032	0.7034	0.7032	0.6900	0.6457	0.7012	0.7006
Baseline	0.7032	0.7032	0.7032	0.7032	0.7032	0.7032	0.7032
$+5\%$	0.7033	0.7031	0.7033	0.7181	0.7549	0.7005	0.7018
$+10\%$	0.7034	0.7030	0.7034	0.7346	0.8006	0.7003	0.7005
$+15\%$	0.7035	0.7029	0.7035	0.7529	0.8404	0.6986	0.7032

for a higher tax rate, which leads to a lower ratio of median private to public health and a higher share of public health.

The relationship between τ_m and $y_m/\bar{y}$ is standard: lower income inequality leads to a lower preferred tax rate. Hence, following an increase in $y_m/\bar{y}$, the ratio of median private health to public health expenditure increases. The negative effect of $\bar{y}$ on τ_m confirms the analytical result in the special and general cases. The strong positive effect of $\bar{y}$ on the survival probability is in line with the observation that wealthier countries have higher life expectancy than poorer countries, as well as with the empirical literature. For instance, Preston (1975) finds that average income contributes positively to life expectancy, and Kennelly et al. (2003) provide evidence that per capita income and public health expenditure are both positively associated with improved health outcomes. There are some other studies that find a strong positive effect of wealth on life expectancy (see, e.g., Deaton and Paxson (2001), and Attanasio and Emmerson (2003)).

4.3 A Quantitative Exercise: Cross-Country Analysis

As described in the Introduction, there are considerable differences in the public-private mix of health expenditure across OECD countries. Next, instructed by the model, we will conduct a few cross-country quantitative exercises to explore what might account for the differences in the composition of health expenditure for a sample of 22 OECD countries with the highest index of democracy.[12] These countries have relatively similar economic and political backgrounds.

4.3.1 Income Distribution and the Public-Private Mix of Health

First of all, we want to examine the role of income distribution, including average income as well as income inequality, in accounting for the observed differences in the mixture of

[12] *Polity IV* dataset provides an index of democracy for all countries. This index is between 0 and 10. Our sample includes OECD countries with the highest index of democracy (9 and 10). However, not all countries with the index of 9 and 10 are included in our sample; because of data limitations. Figure 3 shows all countries in our sample.

health expenditure. Income has traditionally been viewed as one of the most important determinants of total health expenditure. The seminal paper by Newhouse (1977) finds that per capita income can explain much of the cross-national variations in the per capita health expenditure of developed countries; Hitiris and Posnett (1992), using a substantially larger sample than used in previous studies, confirm a strong positive effect of income on per capita health expenditure in OECD countries. Ettner (1996) and Di Matteo and Di Matteo (1998) provide country-specific evidence, where the former finds a large positive effect of income on health status in the U.S., and the latter find that one of the key determinants of Canadian per capita provincial government health expenditure is real provincial per capita income. However, there are few empirical studies that investigate how income affects the mixture of public and private health expenditure. One of them is Di Matteo (2000), which studies the public-private mix of health expenditure in Canada and finds that an important determinant of the split is the share of individual income held by the top quintile of the income distribution–a measure of income inequality.In the quantitative exercise, we assume that countries only differ in their average income ($\bar{y}$) and ratio of median to mean income ($y_m/\bar{y}$), with all other factors the same as Canada. We then calculate the predicted shares of public health in total health expenditure ($H/(H+\bar{h})$) for each country and compare them with the corresponding data values. Specifically, we first solve τ_m from (25) - (27), using the baseline values for parameters α, β, κ, ϕ_H, ρ, and country-specific values for $\bar{y}$ and $y_m/\bar{y}$, where $\bar{y}$ is scaled down using the same scaling factor implied for Canada. The data for $y_m/\bar{y}$ and $\bar{y}$ (PPP-based per capita GDP Constant 2000, $\bar{y}_{PPP}$) for each country are obtained from *OECD.Stat Extracts - 2011* and *World Development Indicator-2011*, respectively. Then we calculate the share of public health for each country in the same way as we did in the baseline calibration, assuming income follows a log-normal distribution for all countries and calibrating the country-specific μ and σ to match each country's scaled $\bar{y}$ and $y_m/\bar{y}$.

Figure 2 plots the predicted ratios of public health to national income (τ_m) and shares of public health in total health expenditure versus their data counterparts for the 22 countries.

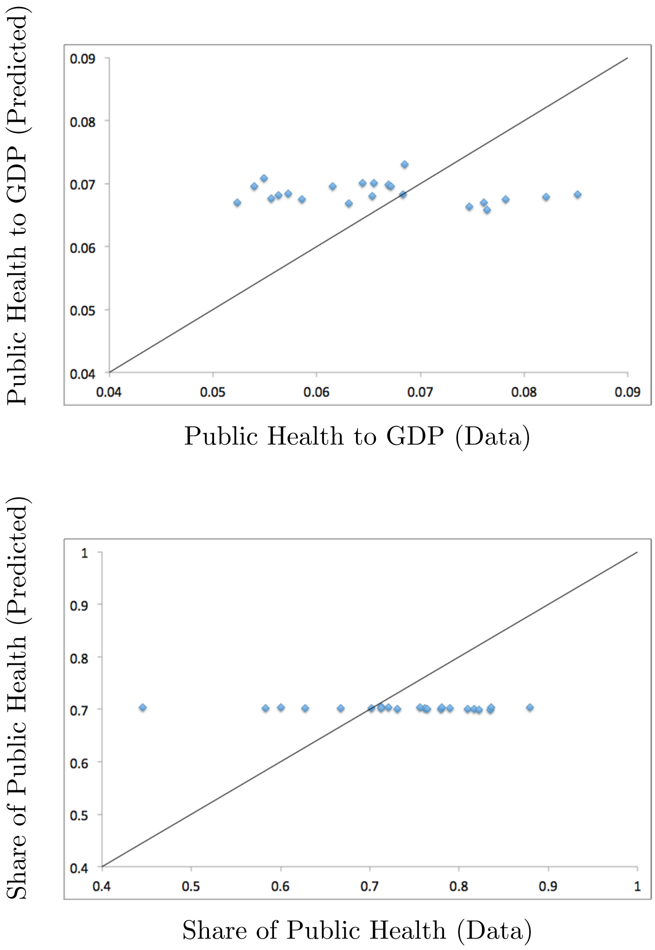

Figure 2: Prediction v.s. Data for a Sample of Democratic Countries

It is clear that the predicted shares of public health in total health do not exhibit much variation across countries, and nor are they close to their data counterparts. Hence, income distribution does not seem to play a role in accounting for the observed differences in the public-private mix of health expenditure. The predicted ratios of public health to national income are closer to their data counterparts and exhibit a bit more variation across countries, suggesting that income distribution might play a minor role in accounting for the size of public health expenditure relative to national income.

These results should be interpreted with caution, since we only consider differences in the mean and variance of income distributions and ignore variations in all other factors across countries. A possible reason for the insignificant role of income distribution is that it is not appropriate to assume that the parameters α, β, κ, ϕ_H, ρ take the same values for each country as Canada. In particular, the comparative static results highlight that the composition of health expenditure is very sensitive to the degree of substitutability between public and private health spending, measured by ρ, and the relative importance of public health, measured by ϕ_H. Hence, in the next quantitative exercise we calibrate these two parameters for each country in our sample and re-predict the public-private mix of health expenditure.

Another possible explanation is that in the model income is exogenously given such that there is no feedback effects between income and health expenditure: an individual with higher income spends more on health care and higher health spending leads to higher productivity and income.

4.3.2 Public-Private Mix of Health: Model v.s. Data

In the following quantitative exercise we aim to answer the quantitative question outlined in the Introduction: how well does the model explain the observed differences in the public-private mix of health expenditure across countries? That is, we use the model to predict the shares of public health in total health expenditure and compare the predicted values with their data counterparts.

The first quantitative exercise shows that when the parameters (other than $\bar{y}$ and $y_m/\bar{y}$) are assumed to take the same values as Canada for each country, the predicted mixture of health expenditure does not match the data. The comparative static results suggest that the mixture of health expenditure is quite sensitive to ρ and ϕ_H; while not sensitive at all to β, α and κ. Thus, in this exercise we calibrate ρ and ϕ_H for each country to match two counry-specific moments: the ratio of public health to national income $H/\bar{y}$, and the median survival probability p_m. Due to the lack of data, in particular the data for H/h_m, we are not able to calibrate β, α and κ for each country. Instead, we assume that they take the baseline values for each country. Given β, α and κ, for each country ρ and ϕ_H are solved jointly from Eq. (26) and (27), with H/h_m given by (25) and the country-specific $y_m/\bar{y}$ and scaled $\bar{y}$. Then the share of public health in total health spending for each country is solved in the same way as we did in the first quantitative exercise.

Table 2 reports for each country the calibrated values for ϕ_H, ρ and the implied elasticity of substitution ($\varepsilon = 1/(1-\rho)$), the predicted shares of public health in total health expenditure ($H/(H+\bar{h})$), and the ratios of the predicted shares to the corresponding data values.

According to Table 2, the calibrated values of ϕ_H are above 0.5 for all countries in the sample, suggesting that public health plays a relatively more important role than private health in forming the health capital of the society to promote life expectancy. The calibrated values of ρ vary substantially across countries, ranging from 0.011 to 0.827, implying that the elasticities of substitution between private and public health expenditure, ε, vary between 1.011 and 5.78 in the sample. Public and private health are considered to be more of complements if $\rho \in [0, 0.4]$ or equivalently $\varepsilon \in [1, 1.6]$. Such countries include: Australia, Finland, Greece, Italy, Japan, Netherlands, New Zealand, Portugal, Spain, Switzerland and UK. This result suggests that for these countries a Cobb-Douglas form can be a good approximation for the production function of health capital. Public and private health are more of substitutes, with $\rho \in [0.7, 1]$ or $\varepsilon > 3.33$, in the following countries: Austria, Czech Republic, Denmark, Germany, Ireland, Norway and U.S. For these countries a linear form may be a good approximation for the health technology. The other countries, including Canada, France, Hungary and Sweden, have ρ close to 0.5 or ε close to 2.

Table 2: Calibrated Values and Computed Share of Public to Total Health Expenditure for Each Country

Country	ϕ_H	ρ	ε	$\frac{h_m}{H}$	Model $\frac{H}{H+h}$	Data $\frac{H}{H+h}$	$\frac{\text{Model}}{\text{Data}}$
Australia	0.602	0.024	1.025	0.581	0.6055	0.66725	0.9074
Austria	0.568	0.792	4.808	0.153	0.7888	0.76144	1.0359
Canada	0.58	0.6162	2.606	0.296	0.7027	0.7014	1.0018
Czech Republic	0.502	0.81	5.263	0.453	0.6112	0.8792	0.6951
Denmark	0.557	0.827	5.780	0.190	0.7931	0.835	0.9498
Finland	0.633	0.021	1.021	0.514	0.6357	0.7306	0.8702
France	0.705	0.629	2.695	0.067	0.8906	0.79	1.1273
Germany	0.602	0.763	4.219	0.103	0.8292	0.7793	1.0639
Greece	0.578	0.023	1.024	0.617	0.5811	0.6001	0.9682
Hungary	0.502	0.651	2.865	0.622	0.5484	0.7126	0.7694
Ireland	0.507	0.72	3.571	0.543	0.5754	0.7642	0.7529
Italy	0.706	0.02	1.020	0.337	0.7099	0.7558	0.9393
Japan	0.709	0.011	1.011	0.359	0.7106	0.817	0.8698
Netherlands	0.541	0.313	1.456	0.680	0.5625	0.628	0.8957
New Zealand	0.72	0.013	1.013	0.323	0.7236	0.7803	0.9273
Norway	0.554	0.809	5.236	0.173	0.7719	0.8361	0.9232
Portugal	0.73	0.02	1.020	0.275	0.7348	0.7212	1.0186
Spain	0.614	0.032	1.033	0.542	0.6179	0.7127	0.8671
Sweden	0.665	0.513	2.053	0.211	0.7968	0.8221	0.9693
Switzerland	0.633	0.284	1.397	0.408	0.6802	0.5831	1.1660
UK	0.697	0.022	1.022	0.346	0.7005	0.81	0.8648
U.S.	0.504	0.822	5.618	0.295	0.6522	0.446	1.4623

Recall that ρ and ϕ_H capture the interaction of public and private health care in forming the health capital of the society. Our quantitative exercise provides a way to estimate these parameters for each country. The variations in the estimates suggest that the interaction of public and private health care differs substantially across countries in our sample. As there is little empirical work that looks at the interaction between public and private health, we are not able to assess whether our estimates are in line with the existing institutional arrangements of the health care systems in each country. However, there are some country-specific empirical studies which provide some empirical support for our estimates. For instance, Propper (2000) examines the choice between public and private health care using the British Household Panel Survey between 1991 and 2000, and finds that private health services appear to be complementary to public health services. Cutler and Gruber (1996a and 1996b) and Gruber and Simon (2008) study the impact on private health insurance of an increase in the coverage of Medicaid and Medicare in the U.S., and find that the increase in public health coverage has crowded out private health insurance substantially, suggesting that public and private health are substitutes.

The last column of Table 2 reports the ratio of predicted shares of public health in total health expenditure to their data counterparts for each country. Further, Figure 3 plots the predicted versus actual shares of public health for each country, where the solid line in the figure corresponds to the 45-degree line. As shown in Table 2 and Figure 3, our model predicts the best for 10 out of 21 countries, including Australia, Austria, Denmark, Germany, Greece, Italy, New Zealand, Norway, Portugal, and Sweden (Canada is excluded as it is the benchmark country with a perfect match). For these countries the ratios of predicted shares to actual values are within $[0.9, 1.1]$. The model also predicts reasonably well for countries such as Finland, France, Japan, Netherlands, Spain, Switzerland and UK, with the ratios within $[0.8, 1.2]$, and it is not bad for Hungary and Ireland. The worst prediction is for the other two countries, the U.S. and the Czech Republic, with the share of public health over-predicted for the former and under-predicted for the latter. The correlation between the predicted and actual shares of public health is 0.44 for the whole sample, and is 0.63 if we exclude the U.S. and the Czech Republic from the sample.

We consider the U.S. and the Czech Republic to be outliers in the quantitative analysis. Among the countries in the sample, the U.S. has the highest total health expenditure relative to GDP (15 percent of GDP, on average, in 2000's) and the lowest share of public health in total health expenditure (45 percent), while the Czech Republic has the highest share of

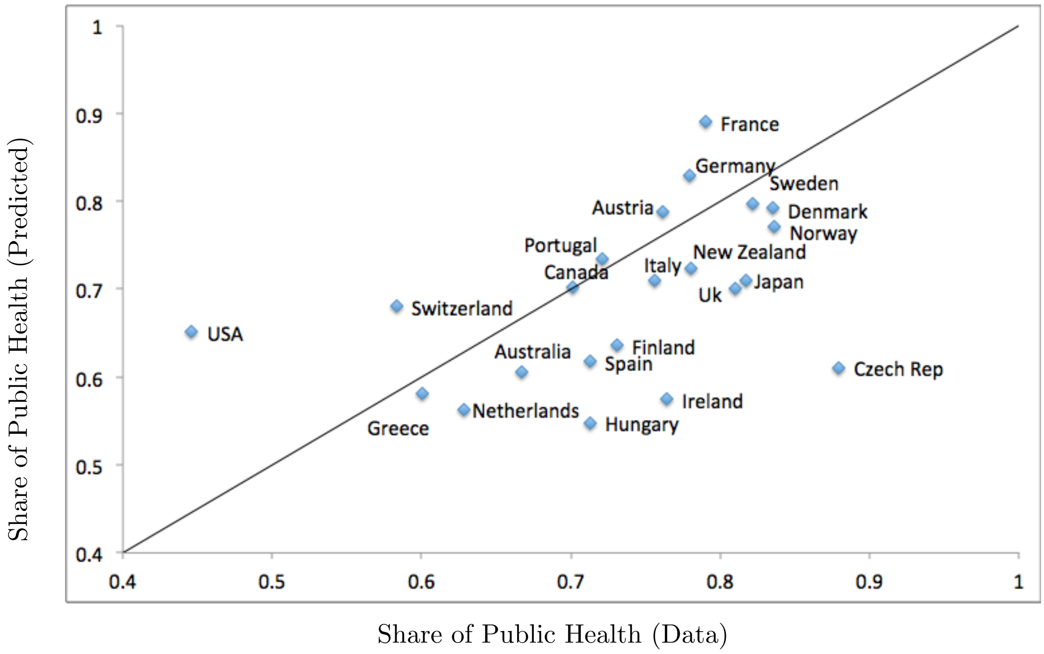

Figure 3: Share of Public Health in a Sample of Democratic Countries

public health in total health expenditure (88 percent). Our quantitative results over-predict the role of public health for the U.S. and under-predict for the Czech Republic. This may be due to some specific features of the health care systems in these two countries which are not captured very well by the model. Public health is treated as having universal coverage in the model, and the U.S. is the only country in the sample that does not have a universal public health care system. The two main public health programs in the U.S., Medicare and Medicaid, only provide coverage to particular groups of people: Medicare is for people who are above 65 years of age or permanently disabled, and Medicaid is for low-income families. For the Czech Republic, due to regulations, health care is highly centralized and there is only little role for the private sector in providing health care.

The gap between the predicted public-private mixture of health expenditure and the data may be due to many other factors that the model abstracts from. Countries differ in many factors that may potentially shape the structure of their health care systems, including institutional and demographical differences, as well as differences in people's preferences for various types of health services and in the pricing of health care services. For example, different countries utilize different policies, regulations, and market mechanisms (such as outsourcing of public health care and public-private partnerships) for health care, and hence public and private sectors play diverse roles in funding and providing health care across countries. As a consequence, the interaction of public and private health in the overall health care system is a complex issue. Indeed, a taxonomy of health care systems across countries is a difficult task, as discussed in Joumard, André and Nicq (2010).

Due to the complexity of the issue, an extensive discussion is beyond the scope of this paper. Nevertheless, next we discuss several factors that may be important for the public-private mix of health expenditure but are not considered in our model, including the demographic structure of the population, the structure of public financing and private financing of health care, as well as the pricing of health care services.

The demographic structure of the population may have important implications for the composition of health care spending. Health expenditure is typically highly concentrated, in particular, old people account for a much larger fraction of total health spending relative to their share in the population. So an ageing population implies a higher demand for health care services, especially for public long-term health care. In fact, for countries in our sample, most of which have an ageing population, there is a positive correlation (0.22) between the shares of the population above 65 years of age and the shares of public health in total health expenditure.

The focus in this study is on the composition of financing of health care: public versus private financing. We abstract from the different types of public and private financing of health care, as observed in the data. According to *OECD Health Dataset - 2011*, government revenues and social insurance are the two main sources of finance for public health care and out-of-pocket and private health insurance are the two main sources of private funding of health care. Figure 4 compares the compositions of public and private financing of health care across countries in the sample. It is clear that the composition of public financing differs substantially across countries. For some countries (e.g., Denmark, Australia, and

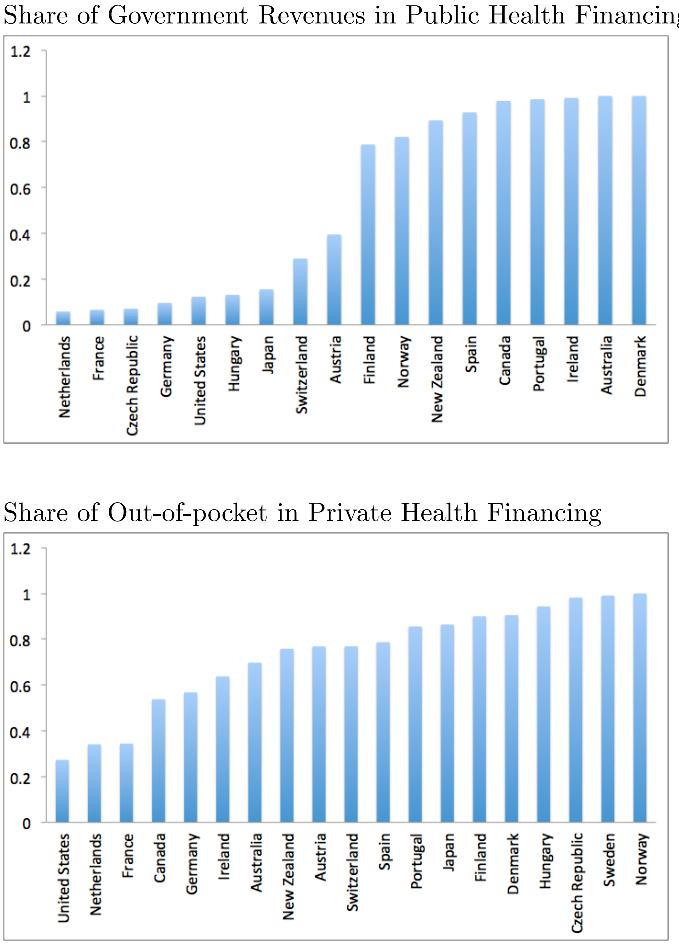

Figure 4: Composition of Private and Public Health Financing for a Sample of Democratic Countries

Ireland) government revenue accounts for more than 95 percent of public financing, while for some other countries (e.g., Netherlands, France and Germany) public health care relies mostly on social insurance based funding. For the financing of private health care, the out-of-pocket contributions range from 30 percent to almost 100 percent. Understanding these differences within each type of financing of health care is important for us to understand the observed differences in the public-private mixture of the overall financing of health care across countries.

Last but not least, the pricing of health care services also has important implications for the mixture of health expenditure. An increase in the prices of private health care services would lead to a higher share of private health expenditure if the demand for private health care is relatively inelastic. Hence differences in the relative prices of health care services across countries, which are well observed in the data (Gerdtham and et al (1992) and Gerdtham and Jonsson (1991)), contribute to the observed differences in the mixture of health expenditure across countries.

5 Conclusions

Despite the large variations in the public-private mix of health expenditure across countries, factors that critically affect the composition of health expenditure have rarely been examined analytically and empirically in the existing literature. In this study, we examined, in the context of a simple overlapping generations model, how the public-private mix of health spending is determined through majority voting and how this decision is affected by various preference and economic factors. Further, we calibrated the model to conduct a quantitative exercise. The quantitative results are in line with the data, in particular, the predicted mixture of health expenditure matches the data reasonably well for a group of advanced democratic countries, suggesting that the model provides a promising framework to study the choice of public and private spending on health care.

The quantitative exercise also revealed the importance of the degree of substitutability between public and private health and the relative effectiveness of public health vs. private health in explaining the composition of health expenditure, and provided a way to infer these factors from the data. Knowing about these factors can help policy makers to design or reform health care policies to achieve the goals of efficiency and equity in health care financing. For instance, if the elasticity of substitution between public and private health is high, i.e., the two types of health expenditure are more substitutable, an increase in one type of health expenditure is more likely to crowd out the other type of health expenditure. So any proposed policy change or reform with respect to the financing of health care should take this crowding out effect into consideration. In this respect, our study has important policy implications. To the best of our knowledge, this has not been explored in the existing literature.

As one of the first few attempts to formally examine the public-private mix of health care, this study utilizes a simple framework which incorporates voting in a dynamic macro-theoretic model. The model considers several important factors for an individual's decision regarding public and private health spending, such as income, the role of health care in improving health status, the substitutability between public and private health, as well as the relative effectiveness of public and private health. However, the model abstracts from a few dimensions that are potentially important, such as the prices of public relative to private health services, the age-dependent demand for health care services, and so on. These considerations are left for future research.

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Appendix: Proof

Proof of Proposition 1. By substituting $p_{m,t} = p(\hat{H}_{m,t-1}) = p\left(\Phi_m \tau_{m,t-1}\right)$ into (11), we obtain $\tau_{m,t} = \alpha\phi_H/[1+\alpha+\beta p\left(\Phi_m\tau_{m,t-1}\right)]$. Recall that $\Phi_m = [(\phi_h/\phi_H)(y_m/\bar{y})]^{\phi_h}\bar{y}$, $\partial p_{m,t}/\partial\hat{H}_{m,t-1} > 0$ and $\partial^2 p_{m,t}/\partial\hat{H}^2_{m,t-1} < 0$. Hence, by differentiating $\tau_{m,t} = \alpha\phi_H/[1+\alpha+\beta p\left(\Phi_m\tau_{m,t-1}\right)]$ with respect to $\tau_{m,t-1}$, we obtain

$$\frac{\partial\tau_{m,t}}{\partial\tau_{m,t-1}} = -\alpha\phi_H \frac{\beta\Phi_m \frac{\partial p_{m,t}}{\partial\hat{H}_{m,t-1}}}{\left(1+\alpha+\beta p\left(\Phi_m\tau_{m,t-1}\right)\right)^2} < 0,$$

and

$$\begin{aligned}\frac{\partial^2\tau_{m,t}}{\partial\tau^2_{m,t-1}} &= -\alpha\phi_H\beta\Phi_m \frac{\frac{\partial^2 p_{m,t}}{\partial\hat{H}^2_{m,t-1}}\left(1+\alpha+\beta p\left(\Phi_m\tau_{m,t-1}\right)\right) - 2\beta\Phi_m\left(\frac{\partial p_{m,t}}{\partial\hat{H}_{m,t-1}}\right)^2}{\left(1+\alpha+\beta p\left(\Phi_m\tau_{m,t-1}\right)\right)^3} \\ &> 0.\end{aligned}$$

Therefore, the the steady state tax rate, τ_m, is unique.

Next, we show that the steady state tax rate is given by (16). Since $\tau_{m,t} = \tau_{m,t-1} = \tau_m$ at the steady state, by using (15), we obtain the following quadratic function:

$$\tau_m^2\Phi_m[(1+\alpha)\kappa+\beta] + \tau_m\left[1+\alpha(1-\kappa\phi_H\Phi_m)\right] - \alpha\phi_H = 0$$

which implies

$$\tau_m = \frac{-\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right] \pm \sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2 + 4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m}}{2\left[(1+\alpha)\kappa+\beta\right]\Phi_m}.$$

It is shown below that the positive steady state tax rate is given by:

$$\tau_m = \frac{-\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right] + \sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2 + 4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m}}{2\left[(1+\alpha)\kappa+\beta\right]\Phi_m}.$$

τ_m has the following properties:

(i) $\tau_m \in (0,1)$: The denominator of τ_m is positive and so it suffices to show that the numerator of τ_m is also positive. Since $4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m > 0$,

$$\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m} > \left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right].$$

Therefore, we have $\tau_m > 0$. To show $\tau_m < 1$, it suffices to show that the numerator of τ_m is less than the denominator of τ_m. Since

$$\begin{aligned}
& \{\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m}\}^2 - \\
& \{2\left[(1+\alpha)\kappa+\beta\right]\Phi_m+\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]\}^2 \\
= \; & -4\left[(1+\alpha)\kappa+\beta\right]\Phi_m\left[\alpha(1-\phi_H)(1+\kappa\Phi_m)+1+(\kappa+\beta)\Phi_m\right] \\
< \; & 0,
\end{aligned}$$

the numerator of τ_m is less than the denominator of τ_m:

$$\begin{aligned}
& -\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]+\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\left[(1+\alpha)\kappa+\beta\right]\Phi_m\alpha\phi_H} \\
< \; & 2\left[(1+\alpha)\kappa+\beta\right]\Phi_m.
\end{aligned}$$

Therefore, $\tau_m < 1$.

(ii) τ_m is locally stable and converged with damped oscillations: by substituting $\tau_{m,t-1} = \tau_m$, $p_m = \tau_m\Phi_m/(1+\kappa\tau_m\Phi_m)$, and $\partial p_{m,t}/\partial\hat{H}_{m,t-1} = 1/(1+\kappa\tau_m\Phi_m)^2$ into $\partial\tau_{m,t}/\partial\tau_{m,t-1}$ obtained above, we have:

$$\begin{aligned}
& \left|\frac{\partial\tau_{m,t}}{\partial\tau_{m,t-1}}\right|_{\tau_m} = \frac{\alpha\phi_H\beta\Phi_m}{\left[(1+\alpha)+\left(\kappa\left(1+\alpha\right)+\beta\right)\Phi_m\tau_m\right]^2} \\
= \; & \frac{4\alpha\phi_H\beta\Phi_m}{\left[(1+\alpha)+\alpha\phi_H\kappa\Phi_m+\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\alpha\phi_H\Phi_m\left[(1+\alpha)\kappa+\beta\right]}\right]^2}.
\end{aligned}$$

It is obvious that $\left|\partial\tau_{m,t}/\partial\tau_{m,t-1}\right|_{\tau_m} < 1$, as $(1+\alpha)+\alpha\phi_H\kappa\Phi_m$ in the denominator is positive.

(iii) τ_m falls with a rise in $y_m/\bar{y}$ (keeping $\bar{y}$ constant): We first differentiate τ_m in (16) with respect to y_m:

$$\frac{\partial\tau_m}{\partial y_m} = \frac{\partial\Phi_m}{\partial y_m}\frac{\left((1+\alpha)\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m}-\widetilde{N}\right)}{2\left[(1+\alpha)\kappa+\beta\right]\Phi_m^2\sqrt{\left[1+\alpha-\alpha\phi_H\kappa\Phi_m\right]^2+4\left[(1+\alpha)\kappa+\beta\right]\alpha\phi_H\Phi_m}}$$

where

$$\widetilde{N} \equiv \alpha\phi_H\Phi_m[(1+\alpha)\,\kappa + 2\beta] + (1+\alpha)^2 > 0.$$

Since $\partial\Phi_m/\partial y_m > 0$, and it can be verified that

$$(1+\alpha)^2\,\{\sqrt{[1+\alpha-\alpha\phi_H\kappa\Phi_m]^2 + 4\,[(1+\alpha)\,\kappa+\beta]\,\alpha\phi_H\Phi_m}\}^2 < (\widetilde{N})^2, \tag{A.1}$$

we have $\partial\tau_m/\partial y_m < 0$ which implies $\partial\tau_m/\partial(y_m/\bar{y}) < 0$ when private and public health are complementary.

(iv) τ_m falls with a rise in $\bar{y}$ (keeping $y_m/\bar{y}$ constant): We first differentiate τ_m in (16) with respect to $\bar{y}$:

$$\frac{\partial\tau_m}{\partial\bar{y}} = \frac{\partial\Phi_m}{\partial\bar{y}}\frac{\left((1+\alpha)\sqrt{[1+\alpha-\alpha\phi_H\kappa\Phi_m]^2 + 4\,[(1+\alpha)\,\kappa+\beta]\,\alpha\phi_H\Phi_m} - \widetilde{N}\right)}{2\,[(1+\alpha)\,\kappa+\beta]\,\Phi_m^2\sqrt{[1+\alpha-\alpha\phi_H\kappa\Phi_m]^2 + 4\,[(1+\alpha)\,\kappa+\beta]\,\alpha\phi_H\Phi_m}}.$$

Since $\partial\Phi_m/\partial\bar{y} > 0$ and given the fact in (A.1), we also have $\partial\tau_m/\partial\bar{y} < 0$ when private and public health are complementary. ■

Proof of Proposition 2. The proof for the case when $\digamma = 1$ and $\digamma < 1$ is trivial. The proof for the case when $\digamma > 1$ is as follows.

First, by differentiating (22) with respect to $\tau_{m,t-1}$, it can be shown that

$$\frac{\partial\tau_{m,t}}{\partial\tau_{m,t-1}} = -\frac{\alpha\beta\digamma\phi_H\bar{y}_{t-1}\frac{\partial p_{m,t}}{\partial\hat{H}_{m,t-1}}}{(\alpha+\digamma+\beta\digamma p_{m,t})^2} < 0,$$

and

$$\frac{\partial^2\tau_{m,t}}{\partial\tau_{m,t-1}^2} = -\frac{\alpha\beta\digamma(\phi_H\bar{y}_{t-1})^2\left(\frac{\partial^2 p_{m,t}}{\partial\hat{H}_{m,t-1}^2}(\alpha+\digamma+\beta\digamma p_{m,t}) - 2\beta\digamma\left(\frac{\partial p_{m,t}}{\partial\hat{H}_{m,t-1}}\right)^2\right)}{(\alpha+\digamma+\beta\digamma p_{m,t})^3} > 0.$$

Therefore, the steady state tax rate, $\tau_m \equiv \widehat{\tau}_m$, is unique.

Next, we show that the steady state tax rate is given by (23). Since $\tau_{m,t} = \tau_{m,t-1} = \tau_m$ at the steady state, by substituting $p_m = \tau_m\phi_H\bar{y}/(1+\kappa\tau_m\phi_H\bar{y})$, using the parametric form in (14), into $\tau_m = \alpha/(\alpha+(1+\beta p_m)\digamma) \equiv \widehat{\tau}_m$ given by (22) at the steady state, we obtain the following quadratic function:

$$\tau_m^2 \phi_H \bar{y}[\kappa(\alpha + F) + \beta F] + \tau_m \left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right] - \alpha = 0$$

which implies

$$\tau_m = \frac{-\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right] \pm \sqrt{\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right]^2 + 4\alpha\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]}}{2\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]}.$$

It is shown below that the positive steady state tax rate is given by:

$$\tau_m = \frac{-\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right] + \sqrt{\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right]^2 + 4\alpha\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]}}{2\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]}.$$

τ_m has the following properties:

(i) $\tau_m \in (0, 1)$: Since the denominator of τ_m is positive, it suffices to show that the numerator of τ_m is also positive. Since $4\alpha\phi_H \bar{y}[\kappa(\alpha + F) + \beta F] > 0$, we have

$$\sqrt{\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right]^2 + 4\alpha\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]} > \left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right],$$

and thus, $\tau_m > 0$. To show $\tau_m < 1$, it suffices to show that the denominator of τ_m is larger than the numerator of τ_m. It is easy to verify that

$$\begin{aligned} & \{2\phi_H \bar{y}[\kappa(\alpha + F) + \beta F] + [\alpha(1 - \kappa\phi_H \bar{y}) + F]\}^2 \\ > \; & \{\sqrt{\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right]^2 + 4\alpha\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]}\}^2, \end{aligned}$$

so the denominator of τ_m is larger than the numerator of τ_m:

$$\begin{aligned} & 2\phi_H \bar{y}[\kappa(\alpha + F) + \beta F] \\ > \; & -\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right] + \sqrt{\left[\alpha(1 - \kappa\phi_H \bar{y}) + F\right]^2 + 4\alpha\phi_H \bar{y}[\kappa(\alpha + F) + \beta F]}, \end{aligned}$$

and thus, $\tau_m < 1$.

(ii) τ_m is locally stable, and converged with damped oscillations: by substituting $\tau_{m,t-1} =$

τ_m, $\bar{y}_{t-1} = \bar{y}$, and $p_m = \tau_m \phi_H \bar{y}/(1 + \kappa\tau_m\phi_H\bar{y})$ into $\partial\tau_{m,t}/\partial\tau_{m,t-1}$ obtained above, we have:

$$\begin{aligned}
&\left|\frac{\partial\tau_{m,t}}{\partial\tau_{m,t-1}}\right|_{\tau_m} \\
= &\frac{4\alpha\phi_H\bar{y}\beta F}{\left\{[\alpha(1+\kappa\phi_H\bar{y})+F] + \sqrt{[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]}\right\}^2},
\end{aligned}$$

and thus, it suffices to show that the denominator of $\left|\partial\tau_{m,t}/\partial\tau_{m,t-1}\right|_{\tau_m}$ is larger than the numerator of $\left|\partial\tau_{m,t}/\partial\tau_{m,t-1}\right|_{\tau_m}$:

$$\begin{aligned}
&\left\{[\alpha(1+\kappa\phi_H\bar{y})+F] + \sqrt{[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]}\right\}^2 \\
&-4\alpha\phi_H\bar{y}\beta F \\
= \ &[\alpha(1+\kappa\phi_H\bar{y})+F]\left\{[\alpha(1+\kappa\phi_H\bar{y})+F] +\right. \\
&\left. 2\sqrt{[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]}\right\} + \\
&[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}\kappa(\alpha+F) \\
> \ &0.
\end{aligned}$$

Thus, τ_m is locally stable, and converged with damped oscillations.

(iii) τ_m rises with a rise in $y_m/\bar{y}$ (keeping $\bar{y}$ constant): We first differentiate τ_m in (23) with respect to y_m:

$$\frac{\partial\tau_m}{\partial y_m} = -\frac{\partial F}{\partial y_m}\frac{\left(\begin{array}{c} N_{y_m/\bar{y}} - [\kappa(\alpha+F)+\beta F](\alpha+F) - \\ \alpha\beta\sqrt{[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]} \end{array}\right)}{2\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]^2\sqrt{[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]}},$$

where

$$N_{y_m/\bar{y}} \equiv \left(\begin{array}{c} \alpha[\kappa(\alpha+F)+\beta F]\phi_H\bar{y}(3\kappa+2\beta) + (\kappa+\beta)\,[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + \\ \alpha\kappa\phi_H\bar{y}(\kappa+\beta)\sqrt{[\alpha(1-\kappa\phi_H\bar{y})+F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha+F)+\beta F]} \end{array}\right).$$

Since $\partial F/\partial y_m < 0$, and it can be verified that

$$\begin{aligned} &(N_{y_m/\bar{y}})^2 \\ > \;& \{[\kappa(\alpha + F) + \beta F](\alpha + F) + \alpha\beta\sqrt{[\alpha(1 - \kappa\phi_H\bar{y}) + F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha + F) + \beta F]}\}^2, \end{aligned}$$

if ϕ_H is large enough such that $\phi_H > 1/(\kappa\bar{y})$. Therefore, $\partial\tau_m/\partial y_m > 0$, which implies $\partial\tau_m/\partial(y_m/\bar{y}) > 0$ when private and public health are perfect substitutes.

(iv) τ_m falls with a rise in $\bar{y}$ (keeping $y_m/\bar{y}$ constant): We first differentiate τ_m in (23) with respect to $\bar{y}$:

$$\frac{\partial\tau_m}{\partial\bar{y}} = \frac{[\kappa(\alpha + F) + \beta F]\{(\alpha + F)\sqrt{[\alpha(1 - \kappa\phi_H\bar{y}) + F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha + F) + \beta F]} - N_{\bar{y}}\}}{2\phi_H\bar{y}^2[\kappa(\alpha + F) + \beta F]^2\sqrt{[\alpha(1 - \kappa\phi_H\bar{y}) + F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha + F) + \beta F]}}$$

where

$$N_{\bar{y}} \equiv (\alpha(1 - \kappa\phi_H\bar{y}) + F)(\alpha\kappa\phi_H\bar{y} + \alpha(1 - \kappa\phi_H\bar{y}) + F) + 2\alpha\phi_H\bar{y}(\kappa(\alpha + F) + \beta F) > 0.$$

Since it can be verified that

$$(\alpha + F)^2\{[\alpha(1 - \kappa\phi_H\bar{y}) + F]^2 + 4\alpha\phi_H\bar{y}[\kappa(\alpha + F) + \beta F]\} < (N_{\bar{y}})^2,$$

$\partial\tau_m/\partial\bar{y} < 0$. ■

Proof of Single-peaked Preferences for the General Case. From Eq. (24), we find that

$$\frac{\partial h_{i,t}}{\partial\tau_t} = -y_{i,t}\frac{\Theta\rho\left(\frac{\tau_t\bar{y}_t}{h_{i,t}}\right)^{\rho-1}\frac{\bar{y}}{y_{i,t}} + 1}{\frac{(1+\beta p_{i,t}+\alpha)}{\alpha}\rho + (1-\rho)(1-\tau_t)\frac{y_{i,t}}{h_{i,t}}} < 0,$$

and

$$\frac{\partial^2 h_{i,t}}{\partial\tau_t^2} = \frac{\begin{array}{c}\left\{y_{i,t}(1-\rho)\Theta\bar{y}^\rho\rho\left(\frac{h_{i,t}-\tau_t\frac{\partial h_{i,t}}{\partial\tau_t}}{h_{i,t}^2}\right)\left(\frac{\tau_t}{h_{i,t}}\right)^{\rho-1}\right\} \\ \left\{\frac{1}{y_{i,t}}\left(\frac{\tau_t}{h_{i,t}}\right)^{-1}\frac{(1+\beta p_{i,t}+\alpha)}{\alpha} + \Theta\left(\frac{\tau_t}{h_{i,t}}\right)^{\rho-1}\frac{\bar{y}^\rho}{y_{i,t}} + 1\right\}\end{array}}{\left(\frac{(1+\beta p_{i,t}+\alpha)}{\alpha} + (1-\rho)\Theta\bar{y}^\rho\left(\frac{\tau_t}{h_{i,t}}\right)^\rho\right)^2} > 0,$$

where $\Theta = ((1 + \beta p_{i,t})/\alpha)(\phi/(1-\phi))$. By differentiating the indirect utility function with respect to the tax rate, we obtain the following first-order condition:

$$\frac{\partial V_{i,t}}{\partial \tau_t} = \left(1 + \beta p_{i,t} + \frac{\alpha}{\rho}\right) \frac{-\left(y_{i,t} + \frac{\partial h_{i,t}}{\partial \tau_t}\right)}{(1-\tau_t)y_{i,t} - h_{i,t}} - \alpha \frac{(1-\rho)}{\rho} \frac{\frac{\partial h_{i,t}}{\partial \tau_t}}{h_{i,t}} = 0,$$

which implies

$$H_t = \left(\frac{\phi}{1-\phi} \frac{\bar{y}_t}{y_{i,t}}\right)^{\frac{1}{1-\rho}} h_{i,t}, \tag{A.2}$$

after substituting the expression for $\partial h_{i,t}/\partial \tau_t$ obtained above into $\partial V_{i,t}/\partial \tau_t = 0$. Note that (A.2) implicitly determines a unique tax rate, τ_t, that is preferred by agent i, since $H_t = \tau_t \bar{y}_t$ is strictly increasing in τ_t, while $h_{i,t}$ is strictly decreasing in τ_t as shown above.

Then $\partial^2 V_{it}/\partial \tau_t^2$ is given by

$$\begin{aligned}
\frac{\partial^2 V_{it}}{\partial \tau_t^2} &= \left\{\left(1 + \beta p_{i,t} + \frac{\alpha}{\rho}\right) \frac{-\frac{\partial^2 h_{i,t}}{\partial \tau_t^2}\left((1-\tau_t)y_{i,t} - h_{i,t}\right) - \left(y_{i,t} + \frac{\partial h_{i,t}}{\partial \tau_t}\right)^2}{\left((1-\tau_t)y_{i,t} - h_{i,t}\right)^2}\right\} \\
&\quad - \left\{\alpha \frac{(1-\rho)}{\rho} \frac{\frac{\partial^2 h_{i,t}}{\partial \tau_t^2} h_{i,t} - \left(\frac{\partial h_{i,t}}{\partial \tau_t}\right)^2}{(h_{i,t})^2}\right\} \\
&= \left\{\frac{\alpha y_{i,t}}{\rho \Psi^3 \left((1-\tau_t)y_{i,t} - h_{i,t}\right)^2 h_{i,t}^3 \tau_t}\right\} \Pi.
\end{aligned}$$

Note that

$$\left\{\frac{\alpha y_{i,t}}{\rho \Psi^3 \left((1-\tau_t)y_{i,t} - h_{i,t}\right)^2 h_{i,t}^3 \tau_t}\right\} > 0,$$

since

$$\Psi = \frac{(1 + \beta p_{i,t} + \alpha)\rho}{\alpha} + \frac{(1-\rho)(1-\tau_t)\, y_{i,t}}{h_{i,t}} > 0.$$

And it can be shown that

$$\begin{aligned}
\Pi &= -\Omega y_{i,t}(\Psi - \Omega)^2 \Psi h_{i,t}^3 \tau_t - (1-\rho)(\Lambda)[\Omega h_{i,t} + \\
&\quad (1-\rho)\left((1-\tau_t)y_{i,t} - h_{i,t}\right)]\left((1-\tau_t)y_{i,t} - h_{i,t}\right) + \\
&\quad (1-\rho)\,\Omega^2 y_{i,t} \Psi \left((1-\tau_t)y_{i,t} - h_{i,t}\right)^2 h_{i,t}\tau_t < 0,
\end{aligned}$$

where $\Omega = ((1+\beta p_{i,t})\rho)/\alpha + 1$, and

$$\Lambda = (\Omega - 1)(\Psi h_{i,t})^2 + \Omega y_{i,t}\tau_t \Psi h_{i,t}(\Omega - 2) + (\Omega y_{i,t})^2 (1-\tau_t)\tau_t.$$

So $\partial^2 V_{it}/\partial \tau_t^2 < 0$, and hence the voters' preferences are single-peaked. ■

Proof of $\partial \tau_m / \partial \bar{y} < 0$ for the General Case. From (26), define the function at the steady state as $\boldsymbol{\Upsilon}$:

$$\boldsymbol{\Upsilon} = \frac{1-\tau_m}{\tau_m}\frac{y_m}{\bar{y}}\frac{H}{h_m} - 1 - \frac{1+\beta p_m}{\alpha}\left[1 + \frac{\phi_H}{\phi_h}\left(\frac{H}{h_m}\right)^{\rho}\right] = 0,$$

where

$$p_m = p\left(\hat{H}_m\right) = \frac{1}{\kappa + \frac{1}{\left[\phi_H + \phi_h\left(\frac{h_m}{H}\right)^{\frac{\rho}{1-\rho}}\right]\tau_m \bar{y}}}.$$

Thus,

$$\frac{\partial p_m}{\partial \tau_m} = \frac{C_1 \frac{1}{\bar{y}}\left(\frac{1}{\tau_m}\right)^2}{\left(\kappa + \frac{C_1}{H}\right)^2}, \text{ and } \frac{\partial p_m}{\partial \bar{y}} = \frac{C_1 \frac{1}{\tau_m}\left(\frac{1}{\bar{y}}\right)^2}{\left(\kappa + \frac{C_1}{H}\right)^2},$$

where $C_1 = 1/\left(\phi_H + \phi_h (h_m/H)^{\frac{\rho}{1-\rho}}\right)$. By implicit function theorem, we thus obtain

$$\frac{\partial \tau_m}{\partial \bar{y}} = -\frac{\frac{\partial \boldsymbol{\Upsilon}}{\partial \bar{y}}}{\frac{\partial \boldsymbol{\Upsilon}}{\partial \tau_m}} = -\frac{-\frac{\beta}{\alpha}\left[1 + \frac{\phi_H}{\phi_h}\left(\frac{H}{h_m}\right)^{\rho}\right]\frac{\partial p_m}{\partial \bar{y}}}{-\frac{1}{\tau_m^2}\frac{y_m}{\bar{y}}\frac{H}{h_m} - \frac{\beta}{\alpha}\left[1 + \frac{\phi_H}{\phi_h}\left(\frac{H}{h_m}\right)^{\rho}\right]\frac{\partial p_m}{\partial \tau_m}} < 0,$$

keeping $y_m/\bar{y}$ unchanged, and the fact that H/h_m is independent of the tax rate implied by (25). ■