On error bounds for multivariate distributions

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Abstract

In the present paper we extend some error bounds developed for approximations to univariate distributions to a multivariate framework.

1 Introduction

1A. Dhaene & De Pril (1994) presented a framework for approximations to univariate aggregate claims distributions, incorporating the approximations of Kornya, De Pril, and Hipp, and within this framework they developed general results for error bounds. Some of the results of Dhaene & De Pril (1994) were reformulated within the framework of De Pril transforms and further discussed by Dhaene & Sundt (1998). Dhaene & Sundt (1997) discussed some error bounds without introducing the De Pril transform.

Sundt (1998*b*) extended the definition of the De Pril transform to multivariate functions and discussed its properties within that framework.

In the present paper we shall extend some of the approximations and error bounds of Dhaene & De Pril (1994) and Dhaene & Sundt (1997, 1998) to the multivariate case, utilising the multivariate De Pril transform where appropriate.

In Section 2 we recapitulate some notation, definitions, and results from Sundt (1998*b*) in connection with the multivariate De Pril transform. Section 3 gives a general discussion of approximations to aggregate claims distributions. Sections 4 and 5 are devoted to multivariate extensions of error bounds; in Section 4 we consider the bounds of Dhaene & Sundt (1997) whereas the topic of Section 5 is the bounds of Dhaene & De Pril (1994) and Dhaene & Sundt (1998).

1B. In the present paper we shall represent probability distributions by their probability functions. Therefore, for convenience, we shall refer to a probability function as a distribution.

We make the convention that $\sum_{i\in S}=0$ and $\prod_{i\in S}=1$ when the set S is empty.

2 The multivariate De Pril transform

2A. Let m be a positive integer. We denote a column vector by a bold-face letter and its elements by the corresponding italic with the number of the element indicated by a subscript; furthermore, we let the italic with subscript $\cdot$ denote the sum of the elements, e.g. $\mathbf{x}=(x_1,\ldots,x_m)'$ and $x_{\cdot}=\sum_{j=1}^m x_j$. For two $m\times 1$ vectors $\mathbf{x}$ and $\mathbf{y}$, by $\mathbf{y}\leq\mathbf{x}$ we shall mean that $y_j\leq x_j$ for $j=1,\ldots,m$, and by $\mathbf{y}<\mathbf{x}$ that $y_j\leq x_j$ for $j=1,\ldots,m$ with strict inequality for at least one j. By $\mathbf{0}$ we shall mean the vector of which all elements are equal to 0.

Let $\mathbb{N}_1$ denote the set of all non-negative integers, and let

$$\mathbb{N}_m=\left\{\mathbf{x}=(x_1,\ldots,x_m)':x_j\in\mathbb{N}_1;\ j=1,\ldots,m\right\}$$

$$\mathbb{N}_{m+}=\{\mathbf{x}\in\mathbb{N}_m:\mathbf{x}>\mathbf{0}\}.$$

Furthermore we let $\mathcal{P}_m$ and $\mathcal{F}_m$ denote the classes of respectively distributions and functions on $\mathbb{N}_m$, $\mathcal{P}_{m0}$ and $\mathcal{F}_{m0}$ the classes of respectively distributions and functions on $\mathbb{N}_m$ with a positive mass at $\mathbf{0}$, and $\mathcal{P}_{m+}$ and $\mathcal{F}_{m+}$ the classes of respectively distributions and functions on $\mathbb{N}_{m+}$.

2B. Sundt (1998*b*) defined the *De Pril transform* φ_f of a function $f\in\mathcal{F}_{m0}$ by the recursion

$$\varphi_f(\mathbf{x})=\frac{1}{f(\mathbf{0})}\left(x_{\cdot}f(\mathbf{x})-\sum_{\mathbf{0}<\mathbf{y}<\mathbf{x}}\varphi_f(\mathbf{y})f(\mathbf{x}-\mathbf{y})\right).\qquad(\mathbf{x}\in\mathbb{N}_m)$$

When φ_f is given, we can evaluate f recursively by

$$f(\mathbf{x})=\frac{1}{x_{\cdot}}\sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{x}}\varphi_f(\mathbf{y})f(\mathbf{x}-\mathbf{y}),\qquad(\mathbf{x}\in\mathbb{N}_m)\qquad(2.1)$$

2C. The convolution $f*g$ of two distributions f and g on $\mathbb{N}_m$ is defined by

$$(f*g)(\mathbf{x})=\sum_{\mathbf{0}\leq\mathbf{y}\leq\mathbf{x}}f(\mathbf{y})g(\mathbf{x}-\mathbf{y}).\qquad(\mathbf{x}\in\mathbb{N}_m)$$

Sundt (1998b) showed that if $f, g \in \mathcal{F}_{m0}$, then

$$\varphi_{f*g} = \varphi_f + \varphi_g. \tag{2.2}$$

2D. For $p \in \mathcal{F}_{10}$ and $h \in \mathcal{F}_{m+}$ we define the compound function $p \vee h \in \mathcal{F}_{m0}$ by

$$(p \vee h)(\mathbf{x}) = \sum_{n=0}^{x.} p(n)\, h^{n*}(\mathbf{x}). \qquad (\mathbf{x} \in \mathbb{N}_m)$$

Sundt (1998b) showed that

$$\varphi_f(\mathbf{x}) = x. \sum_{y=1}^{x.} \frac{\varphi_p(y)}{y} h^{y*}(\mathbf{x}). \qquad (\mathbf{x} \in \mathbb{N}_{m+}) \tag{2.3}$$

3 Approximations

3A. For $j = 1, \ldots, t$ let $f_j{=}p_j \vee h_j$ with $p_j \in \mathcal{P}_{10}$ and $h_j \in \mathcal{P}_{m+}$. We want to evaluate $f = *_{j=1}^{t} f_j$. From (2.2) and (2.3) we obtain

$$\varphi_f(\mathbf{x}) = \sum_{j=1}^{t} \varphi_{f_j}(\mathbf{x}) = x. \sum_{j=1}^{t} \sum_{y=1}^{x.} \frac{\varphi_{p_j}(y)}{y} h_j^{y*}(\mathbf{x}). \qquad (\mathbf{x} \in \mathbb{N}_{m+}) \tag{3.1}$$

Thus we can first find φ_f by (3.1) and then evaluate f recursively by (2.1). However, evaluation of φ_f by (3.1) can be rather time-consuming; when $x.$ is large we need to perform long summations involving high order convolutions of the h_j's. Therefore it is tempting to for each j approximate p_j by some function $q_j \in \mathcal{F}_{10}$ such that $\varphi_{q_j}(y) = 0$ for all y greater than some positive integer r. Let $g_j = q_j \vee h_j$ and $g = *_{j=1}^{r} g_j$. Then

$$\varphi_g(\mathbf{x}) = \sum_{j=1}^{t} \varphi_{g_j}(\mathbf{x}) = x. \sum_{j=1}^{t} \sum_{y=1}^{r} \frac{\varphi_{q_j}(y)}{y} h_j^{y*}(\mathbf{x}). \qquad (\mathbf{x} \in \mathbb{N}_{m+})$$

In the univariate case ($m = 1$) such approximations have been studied by Dhaene & De Pril (1994), Dhaene & Sundt (1998), and Sundt, Dhaene, & De Pril (1998). For the special case where all the p_i's are Bernoulli distributions there are several earlier papers; we mention in particular De Pril (1989).

3B. For further discussion on the following three classes of approximations we refer to Dhaene & De Pril (1994) and Dhaene & Sundt (1998).

A simple approximation is the De Pril approximation where we for each j replace $\varphi_{p_j}(y)$ with zero for all y greater than r and keep $p(0)$ unchanged.

In the Kornya approximation we apply the same approximation to φ_{p_j}, but now we determine $q_j(0)$ such that q_j sums to one like a probability distribution.

In the Hipp approximation we determine q_j such that its moments up to order r match the corresponding moments of p_j (assuming that these moments exist and are finite), that is,

$$\sum_{x=0}^{\infty} x^i q_j(x) = \sum_{x=0}^{\infty} x^i p_j(x). \qquad (i = 0, 1, \ldots, r)$$

This matching of moments is discussed by Dhaene, Sundt, & De Pril (1996) and Sundt, Dhaene, & De Pril (1998).

3C. When approximating a function $f \in \mathcal{F}_m$ by another function $g \in \mathcal{F}_m$ we are of course interested to get some idea of the accuracy of the approximation. As a measure of accuracy we introduce

$$\epsilon(f, g) = \sum_{\mathbf{x} \in \mathbb{N}_m} |f(\mathbf{x}) - g(\mathbf{x})|.$$

In the univariate case this measure has been discussed by i.a. De Pril (1989), Dhaene & De Pril (1994), and Dhaene & Sundt (1997, 1998).

We shall divide our discussion of the error measure ϵ into two sections.

In Section 4 we consider bounds in which De Pril transforms do not appear. Here we generalise error bounds discussed by Dhaene & Sundt (1997) in the univariate case.

In Section 5 we consider bounds based on the De Pril transform. We first generalise a general bound for $\epsilon(f, g)$ deduced by Dhaene & De Pril (1994) in the univariate case. Then we extend some results presented by Dhaene & Sundt (1998) in the univariate case.

4 General error bounds

For $f \in \mathcal{F}_m$ we introduce

$$\nu(f) = \sum_{x=0}^{\infty} f(x).$$

When such quantities appear in our formulae, it will always be tacitly assumed that they exist and are finite. If $f \in \mathcal{P}_m$, then $\nu(f) = 1$.

Lemma 4.1. *If $f, g \in \mathcal{F}_m$ such that $\nu(|f|) < \infty$ and $\nu(|g|) < \infty$, then*

$$\nu(f * g) = \nu(f)\,\nu(g).$$

Proof. We have

$$\nu(f*g) = \sum_{\mathbf{x}\in\mathbb{N}_m}^{\infty} (f*g)(\mathbf{x}) = \sum_{\mathbf{x}\in\mathbb{N}_m} \sum_{\mathbf{0}\le\mathbf{y}\le\mathbf{x}} f(\mathbf{y})\, g(\mathbf{x}-\mathbf{y}) =$$

$$\sum_{\mathbf{y}\in\mathbb{N}_m} f(\mathbf{y}) \sum_{\mathbf{x}\ge\mathbf{y}} g(\mathbf{x}-\mathbf{y}) = \nu(f)\,\nu(g). \qquad \text{Q.E.D.}$$

In the univariate case Lemma 4.1 was proved as Lemma 4.1 in Dhaene & Sundt (1997) in exactly the same way, except that in the multivariate version we have to take more care with the summations. Other results that we will present in the following, are analogous extensions of univariate results, and if the proofs are equally trivial extensions of the univariate case, then we shall leave the proofs to the readers. This goes in particular for the following lemma, which extends Lemma 4.2 in Dhaene & Sundt (1997).

Lemma 4.2. *For $f, g, h \in \mathcal{F}_m$ we have*

$$\epsilon(f*h, g*h) \le \epsilon(f,g)\,\nu(|h|).$$

The following theorem is proved in the univariate case as Theorem 4.1 in Dhaene & Sundt (1997). By application of Lemmas 4.1 and 4.2 the proof trvially extends to the multivariate case.

Theorem 4.1. *For $f_j, g_j \in \mathcal{F}_m$ for $j = 1, \ldots, t$, we have*

$$\epsilon\left(\mathop{*}_{j=1}^{t} f_j, \mathop{*}_{j=1}^{t} g_j\right) \le \sum_{j=1}^{t} \epsilon(f_j, g_j) \left(\prod_{i=j+1}^{t} \nu(|f_i|)\right) \prod_{i=1}^{j-1} \nu(|g_i|). \tag{4.1}$$

In the special case when $f_j \in \mathcal{P}_m$ for $j = 1, \ldots, t$, (4.1) reduces to

$$\epsilon\left(\mathop{*}_{j=1}^{t} f_j, \mathop{*}_{j=1}^{t} g_j\right) \le \sum_{j=1}^{t} \epsilon(f_j, g_j) \prod_{i=1}^{j-1} \nu(|g_i|), \tag{4.2}$$

and if in addition $g_j \in \mathcal{P}_m$ for $j = 1, \ldots, t$, then

$$\epsilon\left(\mathop{*}_{j=1}^{t} f_j, \mathop{*}_{j=1}^{t} g_j\right) \le \sum_{j=1}^{t} \epsilon(f_j, g_j).$$

The following two theorems are trivial multivariate extension of the univariate Theorems 5.1 and 5.4 in Dhaene & Sundt (1997).

Theorem 4.2. *For $p, q \in \mathcal{F}_1$ and $h \in \mathcal{F}_{m+}$ with $\nu(|h|) \le 1$, we have*

$$\epsilon(p \vee h, q \vee h) \le \epsilon(p, q). \tag{4.3}$$

Theorem 4.3. *For $p \in \mathcal{F}$ and $h, k \in \mathcal{F}_{m+}$ with $\nu(|h|) \le 1$ and $\nu(|k|) \le 1$, we have*

$$\epsilon(p \vee h, p \vee k) \le \nu(|p|)\,\epsilon(h, k).$$

5 Error bounds based on the De Pril transform

5A. For $f, g \in \mathcal{F}_{m+}$, let

$$\delta_0(f,g) = \sum_{\mathbf{x}\in\mathbb{N}_{m+}} \frac{\left|\varphi_f(\mathbf{x}) - \varphi_g(\mathbf{x})\right|}{x.}; \qquad \delta(f,g) = \delta_0(f,g) + \left|\ln \frac{f(\mathbf{0})}{g(\mathbf{0})}\right|.$$

In the univariate case these quantities were introduced in Dhaene & Sundt (1998).

In the univariate case the following theorem was proved by Dhaene & De Pril (1994) and reformulated in the context of De Pril transforms by Dhaene & Sundt (1998). The present proof follows to a great extent the proof of Dhaene & De Pril (1994).

Theorem 5.1. *Let $f \in \mathcal{P}_{m0}$ and $g \in \mathcal{F}_{m0}$. Then*

$$\epsilon(f,g) \leq e^{\delta(f,g)} - 1. \tag{5.1}$$

Proof. When $\delta(f,g) = \infty$, the theorem trvivially holds. Let us therefore turn to the case $\delta(f,g) < \infty$.

We define $a \in \mathcal{F}_{m0}$ by

$$\varphi_a = \varphi_g - \varphi_f; \qquad a(\mathbf{0}) = \frac{g(\mathbf{0})}{f(\mathbf{0})}. \tag{5.2}$$

Then $\varphi_g = \varphi_f + \varphi_a$, and consequently $g = f * a$. For $\mathbf{x} \in \mathbb{N}_m$ we obtain

$$f(\mathbf{x}) - g(\mathbf{x}) = (1 - a(\mathbf{0}))\, f(\mathbf{x}) - \sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{x}} a(\mathbf{y})\, f(\mathbf{x}-\mathbf{y}), \tag{5.3}$$

which gives

$$\begin{aligned}
&\epsilon(f,g) = \sum_{\mathbf{x}\in\mathbb{N}_m} |f(\mathbf{x}) - g(\mathbf{x})| \leq \\
&|1 - a(\mathbf{0})| \sum_{\mathbf{x}\in\mathbb{N}_m} f(\mathbf{x}) + \sum_{\mathbf{x}\in\mathbb{N}_{m+}} \sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{x}} |a(\mathbf{y})|\, f(\mathbf{x}-\mathbf{y}) = \\
&|1 - a(\mathbf{0})| + \sum_{\mathbf{y}\in\mathbb{N}_{m+}} |a(\mathbf{y})| \sum_{\mathbf{x}\geq\mathbf{y}} f(\mathbf{x}-\mathbf{y}) = |1 - a(\mathbf{0})| + \sum_{\mathbf{y}\in\mathbb{N}_{m+}} |a(\mathbf{y})|,
\end{aligned}$$

that is,

$$\epsilon(f,g) \leq |1 - a(\mathbf{0})| + \sum_{\mathbf{y}\in\mathbb{N}_{m+}} |a(\mathbf{y})|. \tag{5.4}$$

We see that

$$\left|1-a\left(\mathbf{0}\right)\right|=\left|1-e^{\ln\frac{g(\mathbf{0})}{f(\mathbf{0})}}\right|\leq e^{\left|\ln\frac{g(\mathbf{0})}{f(\mathbf{0})}\right|}-1=e^{\delta(f,g)-\delta_0(f,g)}-1,$$

and insertion in (5.4) gives

$$\epsilon\left(f,g\right)\leq e^{\delta(f,g)-\delta_0(f,g)}-1+\sum_{\mathbf{y}\in\mathbb{N}_{m+}}\left|a\left(\mathbf{y}\right)\right|. \tag{5.5}$$

By (2.1) we get

$$a\left(\mathbf{x}\right)=\frac{1}{x.}\sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{x}}\varphi_a\left(\mathbf{y}\right)a\left(\mathbf{x}-\mathbf{y}\right),$$

from which we obtain

$$\left|a\left(\mathbf{x}\right)\right|\leq\frac{1}{x.}\sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{x}}\left|\varphi_a\left(\mathbf{y}\right)\right|\left|a\left(\mathbf{x}-\mathbf{y}\right)\right|.$$

Let $b=p\vee h$, where $h\in\mathcal{P}_{m+}$ is given by

$$h\left(\mathbf{x}\right)=\frac{\left|\varphi_a\left(\mathbf{x}\right)\right|}{x.\delta_0\left(f,g\right)}\qquad\left(\mathbf{x}\in\mathbb{N}_{m+}\right)$$

and p is the Poisson distribution with parameter $\delta_0\left(f,g\right)$, that is,

$$p\left(n\right)=\frac{\left(\delta_0\left(f,g\right)\right)^n}{n!}e^{-\delta_0(f,g)}.\qquad\left(n\in\mathbb{N}_1\right)$$

Then, by formula (3.8) in Sundt (1998*a*), b satisfies the recursion

$$b\left(\mathbf{x}\right)=\frac{1}{x.}\sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{x}}\left|\varphi_a\left(\mathbf{y}\right)\right|b\left(\mathbf{x}-\mathbf{y}\right).$$

We shall prove by induction that

$$\left|a\left(\mathbf{x}\right)\right|\leq b\left(\mathbf{x}\right)e^{\delta(f,g)}.\qquad\left(\mathbf{x}\in\mathbb{N}_m\right) \tag{5.6}$$

We have

$$\left|a\left(\mathbf{0}\right)\right|\leq\left|1-a\left(\mathbf{0}\right)\right|+1\leq e^{\delta(f,g)-\delta_0(f,g)}=b\left(\mathbf{0}\right)e^{\delta(f,g)},$$

that is, the hypothesis holds for $\mathbf{x}=\mathbf{0}$. Now let us assume that it holds for all $\mathbf{x}$ such that $\mathbf{0}\leq\mathbf{x}<\mathbf{v}$. Then

$$\left|a\left(\mathbf{v}\right)\right|\leq\frac{1}{v.}\sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{v}}\left|\varphi_a\left(\mathbf{y}\right)\right|\left|a\left(\mathbf{v}-\mathbf{y}\right)\right|\leq$$

$$\frac{1}{v.}\sum_{\mathbf{0}<\mathbf{y}\leq\mathbf{v}}\left|\varphi_a\left(\mathbf{y}\right)\right|b\left(\mathbf{v}-\mathbf{y}\right)e^{\delta(f,g)}=b\left(\mathbf{v}\right)e^{\delta(f,g)},$$

that is, the hypothesis also holds for $\mathbf{x} = \mathbf{v}$, and hence it holds for all $\mathbf{x} \in \mathbb{N}_m$.

Application of (5.6) gives

$$\sum_{\mathbf{x} \in \mathbb{N}_{m+}} |a(\mathbf{x})| \leq \sum_{\mathbf{x} \in \mathbb{N}_{m+}} b(\mathbf{x})\, e^{\delta(f,g)} = (1 - b(\mathbf{0}))\, e^{\delta(f,g)} =$$
$$e^{\delta(f,g)} - e^{\delta(f,g) - \delta_0(f,g)},$$

and by insertion in (5.5) we obtain (5.1).

This completes the proof of Theorem 5.1. Q.E.D.

The criticism raised at the end of Section 3 in Dhaene & Sundt (1998) against the univariate case of the error bound (5.1) is still valid in the multivariate case.

5B. For $f \in \mathcal{F}_m$ we define the cumulation operator Γ by

$$\Gamma f(\mathbf{x}) = \sum_{\mathbf{0} \leq \mathbf{y} \leq \mathbf{x}} f(\mathbf{y}) \,. \qquad (\mathbf{x} \in \mathbb{N}_m)$$

In the unviariate case the follwing theorem was proved as Corollary 1 in Dhaene & De Pril (1994). The proof easily extends to the multivariate case.

Theorem 5.2. *Let $f \in \mathcal{P}_{m0}$ and $g \in \mathcal{F}_{m0}$. Then*

$$|\Gamma f(\mathbf{x}) - \Gamma g(\mathbf{x})| \leq \left(e^{\delta(f,g)} - 1\right) \Gamma f(\mathbf{x}) \,. \qquad (\mathbf{x} \in \mathbb{N}_m) \tag{5.7}$$

As $\Gamma f(\mathbf{x}) \leq 1$, (5.7) immediately gives

$$|\Gamma f(\mathbf{x}) - \Gamma g(\mathbf{x})| \leq e^{\delta(f,g)} - 1.$$

Unfortunately the bound in (5.7) depends on $\Gamma f(\mathbf{x})$, that is, the quantity we want to approximate. Therefore Dhaene & De Pril (1994) proved the following corollary in the univariate case as their Corollary 2. The multivariate extension of their proof is trivial.

Corollary 5.1. *Let $f \in \mathcal{P}_{m0}$ and $g \in \mathcal{F}_{m0}$. If $\delta(f,g) < \ln 2$, then*

$$|\Gamma f(\mathbf{x}) - \Gamma g(\mathbf{x})| \leq \frac{e^{\delta(f,g)} - 1}{2 - e^{\delta(f,g)}} \Gamma g(\mathbf{x}) \,. \qquad (\mathbf{x} \in \mathbb{N}_m) \tag{5.8}$$

>From (5.8) we see that $\Gamma g(\mathbf{x}) \geq 0$ for all $\mathbf{x} \in \mathbb{N}_m$ if $\delta(f,g) < \ln 2$.

5C. As the bounds in (5.1), (5.7), and (5.8) are increasing in $\delta(f,g)$, we see that we can replace $\delta(f,g)$ with an upper bound.

In the univariate case the following theorem was contained in Theorem 5.2 in Dhaene & Sundt (1998). The proof easily extends to the multivariate case.

Theorem 5.3. *With $f_j \in \mathcal{P}_{m0}$ and $g_j \in \mathcal{F}_{m0}$ for $j = 1, \ldots, t$ we have*

$$\delta\left(\mathop{*}_{j=1}^{t} f_j, \mathop{*}_{j=1}^{t} g_j\right) \leq \sum_{j=1}^{t} \delta\left(f_j, g_j\right). \tag{5.9}$$

If $f = *_{j=1}^{t} f_j$ and $g = *_{j=1}^{t} g_j$, insertion of (5.9) in (5.1) gives

$$\epsilon(f, g) \leq e^{\sum_{j=1}^{t} \delta(f_j, g_j)} - 1.$$

On the other hand, insertion of (5.1) in (4.2) gives

$$\epsilon(f, g) \leq \sum_{j=1}^{t} \left(e^{\delta(f_j, g_j)} - 1\right) \prod_{i=1}^{j-1} \nu\left(|g_i|\right).$$

The comparison of these two bounds for $\epsilon(f, g)$ in subsection 5B in Dhaene & Sundt (1998) immediately carries over to the multivariate case.

The proof of the following theorem is a multivariate extension of the corresponding results in the univariate case in Theorem 6.2 in Dhaene & Sundt (1998).

Theorem 5.4. *Let $p \in \mathcal{P}_{10}$, $q \in \mathcal{F}_{10}$, and $h \in \mathcal{P}_{m+}$. Then*

$$\delta_0(p \vee h, q \vee h) \leq \delta_0(p, q) \tag{5.10}$$

$$\delta(p \vee h, q \vee h) \leq \delta(p, q). \tag{5.11}$$

Proof. By using (2.3) we obtain

$$\delta_0(p \vee h, q \vee h) = \sum_{\mathbf{x} \in \mathbb{N}_{m+}} \frac{\left|\varphi_{p \vee h}(\mathbf{x}) - \varphi_{q \vee h}(\mathbf{x})\right|}{x.} =$$

$$\sum_{\mathbf{x} \in \mathbb{N}_{m+}} \left|\sum_{y=1}^{x.} \frac{\varphi_p(y) - \varphi_q(y)}{y} h^{y*}(\mathbf{x})\right| \leq \sum_{\mathbf{x} \in \mathbb{N}_{m+}} \sum_{y=1}^{x.} \frac{\left|\varphi_p(y) - \varphi_q(y)\right|}{y} h^{y*}(\mathbf{x}) =$$

$$\sum_{y=1}^{x.} \frac{\left|\varphi_p(y) - \varphi_q(y)\right|}{y} \sum_{\mathbf{x} \in \mathbb{N}_{m+}} h^{y*}(\mathbf{x}) = \delta_0(p, q),$$

that is, (5.10) holds.

Furthermore,

$$\delta(p \vee h, q \vee h) = \delta_0(p \vee h, q \vee h) + \left|\ln \frac{(p \vee h)(\mathbf{0})}{(q \vee h)(\mathbf{0})}\right| \leq$$

$$\delta_0(p, q) + \left|\ln \frac{p(\mathbf{0})}{q(\mathbf{0})}\right| = \delta(p, q),$$

which proves (5.11).

This completes the proof of Theorem 5.4. Q.E.D.

We see that both by applying (5.11) in (5.1) and by applying (5.1) in (4.3) we obtain

$$\epsilon\,(p \vee h, q \vee h) \leq e^{\delta(f,g)} - 1.$$

In their Section 7 Dhaene & Sundt (1998) discuss improvements of the bounds in the case when approximating an infinitely divisible distribution $f \in \mathcal{P}_{10}$ with a function $g \in \mathcal{F}_{10}$. Extension to the multivariate situation is straight-forward and left to the readers.

5D. Let us now return to the situation of subsection 3A. By applying Theorems 5.3 and 5.4 successively we obtain

$$\delta\,(f, g) \leq \sum_{j=1}^{t} \delta\,(f_j, g_j) \leq \sum_{j=1}^{t} \delta\,(p_j, q_j)\,.$$

As we have now got a bound that depends on only the counting distributions and their approximations, the discussions on the univariate case in Section 5 of Dhaene & De Pril (1994) and Section 8 of Dhaene & Sundt (1998) are still valid in the multivariate case. In particular, for the multivariate extension of the approximations of De Pril, Kornya, and Hipp the error bounds given in Dhaene & De Pril (1994) are still valid.

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