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ABSTRACT

This paper investigates commitment credit risk and valuation in connection with their risk-adjusted balance used in computing the bank's capital requirement mandated by the Bank for International Settlements (the BIS). The value of the European commitment put is obtained as a power series solution to a two-factor model of the marked-to-market value of the credit line, the indebtedness value x, and its mean-reverting volatility, V. Once computed, the put is combined with the line fees to determine the commitment net value, and subsequently, the exercise-contingent bank exposure to commitment credit risk. The major pattern which emerges from the put estimates is that the stochastic volatility model generates lower commitment put values for any (x-V) correlation than does the corresponding B-S put formula, except for some at-the-money or slightly in-the-money put options with negative correlations. This is because the level of mean volatility is lower for stochastic volatility commitment put values than for the corresponding B-S ones. The numerical simulations are next used to ascertain how commitment credit risk is affecting the banks' capital requirement. According to the BIS accounting-based procedure, the risk-adjusted balance of short-term commitments is nil; this is not the case when the same risk-adjusted balance is computed by way of the option-based procedure. Beyond capital sufficiency, the approach also has the advantage of sizing up the impact of commitment credit risk on the bank's future profits.

Key words: stochastic volatility commitment put option, commitment net value, capital sufficiency and banks' exposure to commitment credit risk.

Journal of Economic Literature: classifications G13 and G21.

PRICING THE STOCHASTIC VOLATILITY PUT OPTION OF BANKS' CREDIT LINE COMMITMENTS

I. INTRODUCTION

Banks carry off-balance sheet substantial amounts of short- and long-term credit line (CL) commitments which, due to credit risk, are subject to the capital adequacy guideline mandated by the Bank for International Settlements (the BIS) . This raises two questions: 1) What is the market value of credit commitments? and 2) Do banks incur any liabilities when offering this type of credit, and if so, how is their exposure to commitment risk computed?

Thakor *et al.* (1981) have shown that there exists an isomorphic correspondence between commitment contracts and equity put options: when the rate defined in a CL commitment is lower than that on an equivalent spot loan, the borrower receives the line face value but is only indebted for its lower marked-to-market value (from now on, the latter will be referred to as the **indebtedness value**). The borrower's claim on the lending bank constitutes a valuable commitment put option. The aggregate value of still unused commitments is reported as an off-balance sheet entry to the bank's annual consolidated balance sheet and is subject to regular (monthly, quarterly and, by law, annual) audits. As Merton (1977) has argued for related loan guarantees, the time remaining to commitment maturity can be interpreted as the length of time until the next audit of these off-balance sheet contracts. In that case, the boundary condition of the commitment put is Max $(L - x_T, 0)$, where L denotes the CL par value and x its indebtedness value at the annual audit date, T. The value of the European commitment put thus captures the bank's notional liability for carrying off-balance sheet commitments at the annual audit date. In this research, we examine the most prevalent type of CL commitments, those with a floating-rate formula devised as "stochastic index cost of funds plus a fixed forward markup". And amongst those, we concentrate on the class of prime-rate commitments with an original term to maturity less than one year. These short term commitments finance working capital, trade and commerce.

In recent years, several researchers have derived alternative formulas for valuing bank credit line commitments. Bartter and Rendleman (1979), Thakor *et al.* (1981) and Ho and Saunders (1983) derived option-like expressions for fixed-rate CL commitments, Thakor (1982) and Chateau (1990) obtained valuation formulas for variable-rate credit commitments and

Hawkins (1982) priced revolving credit lines. All chose however to retain the assumption that the volatility of the indebtedness value diffusion is constant. In actuality, the indebtedness-value volatility may vary stochastically and may or may not be correlated with the indebtedness value itself. Fortunately, there have been advances in this area of research. Numerical solutions for stochastic volatility stock returns had been proposed by Johnson and Shanno (1987), Scott (1987) and Wiggins (1987); and Melino and Turnbull (1990) and Vetzal (1997) proposed numerical solutions for foreign currency options and bond options, respectively. Ball and Roma (1994) report that the power series methodology of Dothan (1987) and Hull and White (1987 and 1988) provides mathematically tractable and easy to implement solutions, even when the two underlying diffusions exhibit nonzero correlation. Ball and Roma simulations also suggest that the power series procedure is easier to implement than the exact but cumbersome Fourier inversion method proposed by Heston (1993) or Stein and Stein (1991), who incidentally also offered approximations to their inversion solutions. Another exact solution (a general equilibrium rather than arbitrage solution) was provided by Longstaff and Schwartz (1992) for European bond options on discount bonds. Granted these observations, this research sets out: (i) to derive a power series solution of the European commitment put in a two-factor model; (ii) to uncover in simulation experiments value differences between the stochastic volatility commitment put and the corresponding Black and Scholes (1973) constant volatility put formula; (iii) to introduce the random volatility put option in two additional concepts, the commitment net value and the bank's exposure to commitment credit risk; and (iv) to examine, in the light of the simulated values, two of the policy implications of short-term unused commitments: their impact on the bank's risk-based capital requirement and on its future profits. All this is worked out for European random volatility commitment puts generated by the fixed markup of short-term credit lines with a floating prime-rate formula.

We begin the formal analysis by introducing diffusion processes for the indebtedness value and its variance rate. The presence of the volatility factor is based on the statistical evidence pointing out to the fact that the indebtedness-value volatility is varying stochastically with a mean-reverting tendency. Since the indebtedness-value volatility is not spanned by assets in the economy, its market price of risk must enter explicitly the bivariate partial differential equation (PDE) of commitment put pricing. The volatility risk premium is typically assumed to be zero and the terminal boundary condition jointly with the two-dimensional PDE determine the particular solution to the commitment put option. The solution proposed here is tractable and remains plausible even when the indebtedness value is correlated with its mean-reverting volatility: it consists in a power series approximation along the lines of Hull and White (1988). Once computed, the put value is combined with the bank's commitment fees in order to determine the commitment net value, the CNV, and subsequently, the bank's exposure to commitment credit risk. The novelty for both concepts is that an indicator function captures the commitment exercise and a takedown parameter accounts for the proportion of the exercised lines that is effectively mobilized.

Numerical simulations are next used to ascertain the above theoretical considerations. The purpose of the simulation is threefold. Firstly, to uncover value differences between the commitment put computed with the stochastic volatility formula and the constant volatility formula when: i) the indebtedness-value volatility is reverting at different speeds to its long-term mean level; and ii) the correlation between the indebtedness value and its variance rate varies in the interval [-1 to +1]. Secondly, to detect the existence of any patterns of systematic under- and/or over-valuation of the proposed put values with respect to the corresponding B-S put values. And thirdly, to determine the commitment net value and the bank's risk-adjusted exposure on the basis of put estimates, reasonable line fees, as well as the conditional proportion of credit lines exercised and effectively drawn down. The numerical values are finally used to examine two among the policy implications of commitment pricing. The first implication examines whether the risk-adjusted balance of short-term commitments used in the computation of the bank's capital requirement should be determined by the accounting-based procedure mandated by the BIS or by the alternative and option-based valuation proposed here. The other policy implication looks at the effect of the commitment potential liability on the bank's future profitability.

The rest of the paper is organized as follows. In Section 2, we value the European commitment put and define commitment net values and the bank's credit risk exposure. Simulation results are presented in Section 3 and used in Section 4 to articulate two policy implications of commitment pricing. The paper concludes in Section 5 with a short summary.

II. VALUATION OF CREDIT COMMITMENTS WITH A STOCHASTIC VOLATILITY

II.a. Problem statement

Consider a bank that writes at date 0 (for instance, at the date of its annual report) an off-balance sheet commitment contract for a credit line (CL) with the following features: i) the commitment period, [0, T], is one year; ii) the CL face (= constant accounting) value, L, is standardized at $100; iii) loan duration, $[T, T_1]$, is one year from date T if the credit line is drawn down; and iv) the commitment floating prime-rate formula is devised as: "index cost of funds plus a fixed forward markup" [1]. Illustrated numerically, the $100 one-year CL has a time-0 rate formula $[c + m]_0$, where the variable forward rate, say 6.0% p.a., is made up of a 4.5%-p.a. stochastic cost of funds (c, the rate on certificates of deposits (CDs) is generally used as exogenous index) and a fixed markup m of 1.5% p.a. This fixed markup signals to the market the creditworthiness of prime-rate borrowers at the time of commitment writing; as it only hedges credit risk[2], the corporate borrower either bears the funding risk, c, or takes an offsetting position in some interest-rate futures contract. As compensation for commitment writing, the bank collects split fees: namely, an upfront commitment fee of 1/4 of 1% p.a., f_0^u, or here 25 cents per $100 of line face value, and an identical but exercise-contingent usage fee, f_T^e [3]. The commitment contract itself results from a mechanism for optimal risk sharing between

[1] For non-prime commitments, the forward markup is adjusted for add-ons or discounts: consult Morgan (1993) for the magnitude of such spreads over the prime rate.

[2] Markup risk should not be confused with the risk of default by counterparties to off-balance sheet transactions such as swaps (see, e.g., Das [1995], Duffee [1996], Jarrow and Turnbull [1995] or Hull and White [1995]). In the latter case, this settlement risk is very similar to the one faced by the bank after the commitment has been exercised and the credit line drawn down: it holds a vulnerable counterparty call as the borrower may default on loan principal and interests.

[3] In a well-known American variant, the upfront fee is charged in conjunction with a fee for either the amount actually borrowed and/or the unused portion of the commitment. In this case, the borrower who opts for a spot loan is charged the latter administrative cost in addition to the fee on the commitment unused balance. According to Shockley (1995) for the years 1989 and 1990, the mean upfront fee on corporate credit commitments was 27.4 basis points while the mean annual fee on commitment unused balances was 25.2 basis points. With some simple

DECISION CHART OF A BANK CREDIT COMMITMENT WITH A FIXED FORWARD MARKUP.

a) initial situation at t=0: contractual terms and the bank annual audit dates.

one year commitment period

\$100 one-year CL with a 1.5% p.a. fixed markup (m_0)

```
|-------------------------------+-----------------------------|
t=0                             T                             T1
```

t=0: initial audit; upfront commitment fee: f_0^u = 25¢

T: next audit

T_1

b) situation at the valuation date s: T - s = 6 months, time left to the next audit.

6 months

1 year on-balance sheet loan + defaultable repayment call

```
|---------------X--------------+-----------------------------|
t=0             s              T                             T1
```

s ↓ valuation date

T: I, exercise indicator and d, takedown parameter; f_T^e = 25¢: exercise-contingent usage fee

T_1

the bank and the borrower as in Thakor and Udell [1987], among others[4]: the screening device resolves the asymmetries of information between these parties and the presence of adverse selection gives rise to split fees. The analysis thus focuses on valuing the components "fees + commitment put", at date $s \in [0,T]$, with $T - s = \tau$ being Merton's (1977) length of time until the next audit. Once credit is funded under a commitment, the resultant loan is reported as an on-balance-sheet corporate loan; the bank then holds a "vulnerable" repayment call on the firm's assets since the latter may default on loan principal and interests. The important features of a

adjustments, the model can accommodate the American variant.

[4] Self-selection as a screening device with optimal fee mix is also examined in Avery and Berger (1991), James (1981), and Bhattacharya and Thakor (1993) and references therein. The model formal examination is beyond the scope of this research.

fixed markup commitment are stylized in the decision chart above; for the sake of continuity, we shall refer to this numerical illustration in the rest of the paper.

II.b. Indebtedness value and its stochastic volatility

Thakor *et al.* (1981) were the first to define the marked-to-market value of a credit line, an economic value often referred to as the **indebtedness value,** x. With regard to the above problem statement, the indebtedness value at date T is computed as

$$x_T = L\exp\{(m_0 - m_T)(T_1 - T)\} \qquad \text{with } x_0 = L, \tag{1}$$

where L is the line par value, $(T_1 - T)$ is loan duration once the commitment has been exercised and $(m_0 - m_T)$ is the difference between m_0, the fixed forward markup set at date 0 when the commitment was written, and $m_T = (\ell_T - c_T)$, the date-T stochastic spot markup defined as the difference between the prime rate in the spot credit market, ℓ_T, and the funding rate in the CD market, c_T. At date T, the commitment holder decides to draw on the line only if *ceteris paribus*[5] $m_0 < m_T$, namely when the initial markup is less than the (stochastic) spot markup computed from primary credit and funding rates. For instance, when our illustrative 1.5% forward markup is combined with, say, a 2.5% spot markup, the markup differential in eq. (1) is negative at -1%; it follows that $x_T < L$, namely the indebtedness value is less than the line par value. This inequality gives rise to the commitment put option. To the extent the markup differential at date 0 is null[6], the initial indebtedness value is equal to the line par value, x(0)

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[5] Any banking decision taken at the margin considers the upfront fee as a fixed, and thus sunk, cost. To maintain here the neutrality of the trade-off between spot loan and credit under a commitment, we assume that the usage fee due at the exercise date, f_T^e, matches the administrative cost, c_T^a, that the borrower will pay for a spot loan. Otherwise, the markup differential $(m_0 - m_T)$ becomes $(m_0 + f_T^e - m_T - c_T^a)$ with eq. (1) adjusted accordingly. Recently, Houston and Venkataraman (1994 and 1996) have also examined the borrower's trade-off between loan commitments and the firm's other short and long term debts.

[6] This assumes that the bank sets at t=0 its forward markup (m_f) equal to the mean spot markup of the prime-rate class, namely $m_f = m_0 = E(m_{i0})$, where subscript i refers to the i-th representative bank. According to this approach, the forward markup is an unbiased estimate of

= L. The diffusion process for x in eq. (1) is given by[7]

$$dx(t) = x[\mu_x dt + \sigma_x(t)dz_x(t)], \tag{2}$$

where μ_x and σ_x^2 are the instantaneous drift and instantaneous variance of the indebtedness-value distribution and $dz_x(t)$ the differential of the Wiener process $z_x(t)$. To what extent is the indebtedness-value volatility, $\sigma_x(t)$, changing over time ? The statistical evidence reported in Chateau (1995, p. 74) is already hinting at the presence of some stochastic volatility in the indebtedness value diffusion:

> "...the indebtedness-value changes show a slightly negative mean tendency ($\mu_{\Delta x}$ = -0.0002). Yet, the size of the monthly average change is swamped by the size of the volatility of indebtedness-value variations captured either by the standard deviation of the process ($\sigma_{\Delta x}$ = 0.0080) or by the drift parameter of the absolute-change diffusion process ($\mu_{|\Delta|}$ = 0.00524)."

Insert Figure 1 about here

To assess the indebtedness-value volatility, 29 annual estimates (σ_{tx}: t = 1,.., 29) of the volatility

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the spot mean markup, and hence the near-market for spot credit markups is at least weakly efficient.

[7] To obtain eq. (2), combine eq. (1) with the spot markup process defined in the text, of which the diffusion is given by:

$$dm_t = \mu_m dt + \sigma_m dz_m(t),$$

where constant μ_m and σ_m are the markup instantaneous drift and instantaneous dispersion, respectively, and $dz_m(t)$ is the differential of the Wiener process $z_m(t)$. Applying Itô's lemma to eq. (1) yields eq. (2) in the text with $\mu_x \equiv [-\mu_m(T_1-T) + \frac{1}{2}\sigma_m^2(T_1-T)^2]$, $\sigma_x \equiv [-\sigma_m(T_1-T)]$ and $dz_x \equiv dz_m$.

were computed from the monthly indebtedness-value observations over the period 1966-1995[8]; this is presented in Figure 1. Visual inspection of the figure reveals that far from being constant, the indebtedness-value volatility is varying stochastically over time in the range [0.0021 to 0.0147] and tends to revert to a longer-term mean value of 0.0061 (0.0061 corresponds to an annualized volatility of 2.1% p.a. since $\sigma_x = s_x\sqrt{12}$ for the monthly data used). Granted this observation, we shall assume that the variance rate of x, $V \equiv \sigma_x^2$, follows the diffusion process

$$dV(t) = \alpha dt + \xi\sqrt{V(t)}dz_V(t), \tag{3}$$

where α and ξ are independent of x, the volatility of volatility ξ is constant and $dz_V(t)$ is the differential of the Wiener process $z_V(t)$. Unanticipated changes in the indebtedness value and its variance rate are correlated, i.e., $dz_x(t)dz_V(s) = \rho dt$ for $t=s$ and 0 otherwise. To the extent that the drift rate of V is effectively mean reverting, we can posit

$$\alpha = a + bV, \tag{4}$$

where a and b are positive and negative constants, respectively. In (4) b governs the rate of reversion to the long-term mean level (-a/b) and appropriate value choices for a and b allow us to control the speed of reversion (quicker or slower) to the long-run mean level. Two additional expressions related to V are of relevance in this context. The first one, the expected volatility at time $s \in [0,T]$, conditional on the time-0 volatility, V(0), is given by

$$E\{V(s,0)\} = (-a/b) + [V(0) - (-a/b)]e^{bs} \tag{5}$$

and the other, the average expected variance rate over [0, T], $\bar{V}$, is given by

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[8] We follow the computation procedure advocated by Thakor et al. (1982) and Chateau (1995). The examination of indebtedness-value actual and simulated distributions is beyond the scope of this research. Consult Melino and Turnbull (1990) and Vetzal (1997) on this topic.

$$\bar{V} = 1/T \int_0^T E\{V(s,0)\}ds. \tag{6}$$

According to eq. (5), the expected volatility is reverting to the longer-term mean level as any deviations, shown in the second term on the RHS, are dying away exponentially. Eq. (5) is next introduced in eq. (6) to define the average expected variance rate. It results from the above information that relative changes in the indebtedness value, dx/x, will be drawn from a normal distribution with mean $\mu_x dt$ and variance (V + dV)dt. The diffusions proposed here are different from those retained by Thakor (1982) and Chateau (1990) where the indebtedness-value process with a constant volatility was both lognormal and stationary. We now turn to valuing the commitment put option when the variance rate of the underlying indebtedness value is stochastic.

II.c. Commitment put value

We intend to value the European commitment put option on an indebtedness value with exercise value L and maturity date T. In a risk-neutral world, the indebtedness value, x, and its variance rate, V, are assumed to obey the following stochastic processes:

$$dx = x[rdt + \sqrt{V}dz_x] \tag{7}$$

and

$$dV = \alpha dt + \xi\sqrt{V}dz_V. \tag{8}$$

According to Garman (1976) or Dothan (1987), if the (quasi-[9]) traded asset, x, and its non-traded variance rate, V, obey the processes given in eqs. (7) and (8), the commitment put value, P, must satisfy a bivariate fundamental PDE that includes the market price of volatility risk, λ_V, since V is not spanned by assets in the economy. We thus have

[9] Although the indebtedness value is not likely to trade directly, the difficulty is overcome (i) by appealing to Merton's (1973) intertemporal CAPM as advocated by Thakor *et al.* (1981) or (ii) by observing that the spot markup constitutes a **quasi-price** as it results from actual (equilibrium) prices in continuous primary lending and funding markets, as in Chateau (1995).

$$P_t + \tfrac{1}{2}Vx^2 P_{xx} + \rho V\xi x\, P_{xV} + \tfrac{1}{2}\xi^2 V\, P_{VV} - rP + rx\, P_x + [\alpha - \lambda_V \xi\sqrt{V}]\, P_V = 0, \qquad (9)$$

where subscripts to P denote the time and partial argument derivatives of the commitment put option and r is the (constant) instantaneous risk-free rate of interest. From here on and without loss of generality[10], we make the not unreasonable assumption that the volatility has zero systematic risk, i.e., $\lambda_V = 0$: the previous equation then becomes

$$P_t + \tfrac{1}{2}Vx^2 P_{xx} + \rho V\xi x\, P_{xV} + \tfrac{1}{2}\xi^2 V\, P_{VV} - rP + rx\, P_x + \alpha\, P_V = 0 \qquad (10)$$

and the solution to eq. (10) has to satisfy the terminal boundary condition of the commitment put option

$$P(x, V, T) = \max [\, 0, L - x \,]. \qquad (11)$$

In order to solve this value problem, we now introduce an analytical artifice inspired by the early exercise premium of American put options: the value of the random volatility commitment put, $P(\nabla)$, is equal to the value of the Black-Scholes constant volatility put, P_{BS}, plus a volatility correction B: namely,

$$P(\nabla) = P_{BS} + B. \qquad (12)$$

Substituting then eq. (12) into eq. (10) and using the one-factor PDE of the standard Black-Scholes commitment put, we obtain the equation that the correction must satisfy:

$$B_t + \tfrac{1}{2}Vx^2 B_{xx} + \rho V\xi x\, B_{xV} + \tfrac{1}{2}\xi^2 V\, B_{VV} - rB + \rho V\xi\, P_{xV} + \tfrac{1}{2}\xi^2 V\, P_{VV}$$
$$+ rx\, B_x + \alpha\, B_V + \alpha\, P_V = 0. \qquad (13)$$

---

[10] If λ_V is assumed to be different from zero, we can adjust the risk-neutralized drift of V in eq. (4) for either a or b; namely $a^* = a - \lambda_V \xi V^{1/2}$ or, alternatively, $b^* = b - \lambda_V \xi V^{1/2}$. The developments from eq. (9) onwards then remain valid except that a or b is replaced by a^* or b^*.

Finally, B is expanded in a Taylor series in ξ:

$$B = f_0 + f_1\xi + f_2\xi, \tag{14}$$

where each $f_i = f(x, V, t)$. Along the lines of Hull and White (1988, eqs. (16) to (18)), we obtain:

$$f_0 = P(\bar{V}) - P_{B\text{-}S}, \tag{14a}$$
$$f_1 = (\rho/h^2\delta)\{\alpha(1 - e^{\delta} + \delta e^{\delta}) + a(1 + \delta - e^{\delta})\} xP(\bar{V})_{x\bar{V}},$$
$$f_2 = \phi_1(T\text{-}t)^{1}xP(\bar{V})_{x\bar{V}} + \phi_2(T\text{-}t)^{2}P(\bar{V})_{\bar{V}\bar{V}} + \phi_3(T\text{-}t)^{2}xP(\bar{V})_{x\bar{V}\bar{V}} + \phi_4(T\text{-}t)^{3}P(\bar{V})_{\bar{V}\bar{V}\bar{V}},$$

where

$$\phi_1 = (\rho^2/h^4)\{\ \alpha[e^{\delta}(\tfrac{1}{2}\delta^2 - \delta + 1) - 1] + a(e^{\delta}[e^{\delta}(2 - \delta) - (2 + \delta)]\ \}, \tag{14b}$$
$$\phi_2 = 2\phi_1 + 1/(2h^4)[\ \alpha(e^{2\delta} - 2\delta e^{\delta} - 1) - \tfrac{1}{2}a(e^{2\delta} - 4e^{\delta} + 2\delta + 3)\ ],$$
$$\phi_3 = (\rho^2/2h^6)\ [\ \alpha(e^{\delta} - \delta e^{\delta} - 1) - a(1 + \delta - e^{\delta})\ ]^2,$$
$$\phi_4 = 2\phi_3,$$
$$\delta = h(T\text{-}t),$$

and where

$$P(\bar{V})_{x\bar{V}} = -N'(d_1)d_2/(2\bar{V}), \tag{14c}$$
$$P(\bar{V})_{\bar{V}\bar{V}} = [x\surd(T\text{-}t)N'(d_1)(d_1d_2 - 1)]/(4\bar{V}^{3/2}),$$
$$P(\bar{V})_{x\bar{V}\bar{V}} = [N'(d_1)(d_1 - d_1d_2^2 + 2d_2\ )]/(2\bar{V})^2,$$
$$P(\bar{V})_{\bar{V}\bar{V}\bar{V}} = \{x\surd(T\text{-}t)N'(d_1)[(d_1d_2 - 3)(d_1d_2 - 1) - (d_1^2 + d_2^2)]\}/(8\bar{V}^{5/2}),$$

are partial derivatives with respect to the subscripts x and $\bar{V}$. $N'(d_1)$ is the derivative of the cumulative distribution at d_1, and d_1 and d_2 are given by

$$d_1 = [\ln(x/L) + (r + \bar{V}/2)(T\text{-}t)]/[\bar{V}(T\text{-}t)]^{1/2}, \text{ and} \tag{14d}$$
$$d_2 = d_1 - [\bar{V}(T\text{-}t)]^{1/2}.$$

II.d. Commitment net value and credit risk exposure

Once computed, the put value can be combined with the commitment fees collected by the bank. According to problem statement II.a., the upfront fee, f_0^u, guarantees credit availability to the borrower. Her subsequent choice between spot loan and credit under a commitment depends on the trade-off between stochastic spot markup and fixed forward markup. This state-contingent decision is captured by an *exercise* indicator, $I = 1\{x_T < L\}$, that is equal to one if exercise occurs, and zero otherwise. Analytically, the following expression captures the commitment net value when computed at the valuation date s

$$CNV_1 = f_0^u \exp(r(s - t_0)) + I.f_T^e \exp(-r(T - s)) - P(x, V, s) \quad \text{when there is exercise} \quad (15a)$$

$$CNV_2 = f_0^u \exp(r(s - t_0)) \quad \text{in the absence of exercise} \quad (15b)$$

where both fees, f_0^u and f_T^e, are compounded and discounted respectively at the risk-free rate,r; the second fee is received only if the commitment is effectively exercised at T. This exercise mechanism deserves further scrutiny within the regulatory time frame of capital sufficiency.

Recall that the credit unit chosen in problem statement II.a. was standardized at $100. The advantage of this choice is twofold: 1) to circumvent the line partial takedown at the exercise date T[11] and 2) to obviate the deadweight loss due to compensating deposit balances (Hawkins, [1982]). A unit of standardized credit ($100) reduces commitment hedging to its pricing (markup) component as the quantity dimension is neutralized as in an equity put. Suppose, for argument sake, that the bank has written ten identical prime-rate commitments of $100 each for a grand total of $1,000. Suppose, moreover, that 60% of the commitments are exercised and, for those exercised, the average take down is 80% of each line maximum amount of $100. So, from the bank viewpoint, $480 out of the $1,000 offered are drawn down, that is a proportion, p, of 48%. We propose to account for this exercise-cum-takedown feature by introducing the

[11] Morgan (1993) indicates that between 1988 and 1990, the fraction of the loan limit actually borrowed by prime-rate borrowers is about 55%; unfortunately, he is not reporting the number of commitments left unexercised.

simplification[12]: fully drawn standardized credit units are used to cover the takedown proportion of the dollar total of aggregate commitments at the audit date, the complementary fraction being the dollar aggregate of all unexercised and thus undrawn commitments. Then, p, the conditional average proportion, is

$$p = E[\, d.I \mid I = 1] = E[\, d \mid I = 1] = E[\, d \mid x_T < L],$$

where E denotes the mathematical expectation and the exercise indicator, I, is combined with the takedown parameter, d. When there is full takedown of the \$100 credit unit, $d = 1$; in the absence of exercise and thus takedown, the complementary proportion is $(1 - p) = E[1 - d \mid x_T < L]$. For the sake of simplicity, we have selected a fixed proportion, p, and have reallocated partial takedown, $0 < d < 1$, to the two other proportions[13]. Granted the empirical evidence reported in Morgan [1993], we retain a proportion of $p = 0.5$: 50% of the dollar total of all commitments is drawn down and the other 50% is left unexercised. Combining these proportions with the CNVs from eqs. (15a) and (15b) allows us to determine the risk exposure that the bank faces when offering \$100 of credit under a commitment: namely

$$\text{Exposure} = p\,[f_0^{\$} \exp(r(s-t_0)) + I.f_T^{\$} \exp(-r(T-s)) - P(x, V, s)] + (1 - p)\,[f_0^{\$} \exp(r(s-t_0))]. \qquad (16)$$

European commitment put values, CNVs and the bank's exposure computed from eqs. (1) to (16)

[12] In actuality, the problem is more complex than that because: 1) the commitments have different initial maximum amounts; 2) some lines are completely drawn down, others are partially drawn down and in stages, and some are left unexercised all together; 3) draw downs are taking place on different dates. Our simplification constitutes but a proximate solution to the problem left unresolved by Thakor *et al.* (1981). Greenbaum and Venezia (1985) treat partial exercise from the borrower's viewpoint, but outside the commitment put framework.

[13] The product of the exercise indicator, I, and the takedown parameter, d, is reminiscent of the hazard rate model of Artzner and Delbaen (1995). As I and d are exogenously given here, the CL commitment is considered to be small with respect to the bank's aggregate volume of commitments.

will be construed as benchmarks which may change when some of the commitment-specific assumptions are relaxed. Other commitment options, i.e., with a formula such as [prime or Libor $\pm$ x basis points], can easily be valued from the above "generic" valuation programme, which is estimated in the next section.

III. SIMULATION RESULTS

III.a. Simulation

Tables 1 and 2 contain the values of: (i) the European commitment put computed with eq. (12); (ii) commitment net values, the CNVs shown in eqs. (15a) and (15b); and (iii) the bank's exposure to commitment credit risk stylized in eq. (16). The simulation experiments are performed for a whole range of indebtedness values and (x-V) correlation values: these parameters are not empirical estimates, but are not atypical of the Canadian credit experience since 1966. The indebtedness value is set at x = \$100, \$99.5, \$99, \$98.5 and \$98, respectively: for a line par value of \$100, these slightly in-the-money indebtedness values simulate small increases in the spot markup of the class of prime-rate borrowers over the year-long commitment period. Further, the value of ρ, the correlation between the unanticipated fluctuations in the indebtedness value and those in its variance rate, ranges over the value domain [-1.0 to +1.0]. We choose to control the speed of reversion of the variance drift: when a = 0.004 and b = -2, the variance rate is slowly reverting to the long-run average level 0.002 as in Table 1, but when a = 0.02 is coupled with b = -10, it reverts more rapidly to the same mean level as in Table 2. The other common parameters of the simulations are: the initial variance rate, V(0) = 0.002, is corresponding to σ_x = 0.0447 (a very plausible 4.47% p.a.), T - s = τ = 0.5 year, the midpoint between two annual report dates, the volatility of volatility is ξ = 0.075, and the short-term risk-free rate, r = 0.04, is consistent with the 4.5% CD rate introduced in problem statement II.a.

Before reporting on the simulations, we first clarify the meaning of computed values. Consider the plausible scenario represented by entries 11 to 15 in column (3) of Table 1, in which the indebtedness value x is slightly in-the-money at \$99 and the (x-V) correlation is mildly negative at ρ= -0.2. The estimate P(∇) = 0.797 in entry 11 of column (3) means that the commitment put has an equilibrium value of ≐ 0.8% of the line par value if: (i) our prime-rate commitment with a 1.5%-p.a. fixed forward markup is priced when the stochastic spot markup is ≐ 2.5% p.a.; and (ii) the time remaining to commitment expiry is 6 months. Unlike the B-S estimate, $P_{B\text{-}S} = 0.814$, the presence of a stochastic variance rate in $P(\bar{V})$ decreases the put value from 0.814% to 0.797% of the line par value; that is, according to entry 12 of column (3), by 2% in terms of the B-S put value. The put estimate $P(\bar{V}) = 0.797$ is next used in entry 13 of column (3) to compute the CNV_1, (25¢)exp[(.04)(0.5)] + 25¢exp[-(0.04)(0.5)] - \$0.797 = -\$0.297 (both fees being compounded and discounted respectively as in eq. (15a)). This negative CNV_1 corresponds to a net notional discount of 29.7¢ per \$100 of line par value. However, if the commitment is left unexercised as reported in entry 14 of column (3), the CNV_2 of (25¢)exp[(.04)(0.5)] constitutes a premium of ≐ 25.5¢. Finally, with a conditional proportion of exercise-cum-takedown of 50%, the bank's risk-weighted exposure in entry 15 of column (3) turns out to be a net liability of 2.1¢ per \$100 of credit offered.

Insert Tables 1 and 2 about here

III.b. Put values, commitment net values and the bank's credit-risk exposure

Regarding commitment put values, at least three revealing tendencies are emerging from Tables 1 and 2. The first tendency is that for any (x-V) correlation, commitment put values computed with the stochastic volatility model, $P(\bar{V})$, are lower than those computed with the B-S constant volatility put formula, $P_{B\text{-}S}$, except for a subset of the tables that we now define. The regions in which $P(\bar{V})$ is greater (smaller) than $P_{B\text{-}S}$ can be defined by finding the (x-V) correlation value for which $P(\bar{V}) = P_{B\text{-}S}$ in each of the indebtedness-value scenarios considered. In Table 1 for instance, this equality occurs for the following pairs of x and ρ values: (100; -0.14), (99.5; -0.24), (99; -0.43), and (98.5; -1.05), respectively. This boundary is dividing Table 1 in two regions and in its upper-left corner, $P(\bar{V}) > P_{B\text{-}S}$; everywhere else, the reverse is true. The same pattern is repeated in Table 2, but the region in which $P(\bar{V}) > P_{B\text{-}S}$ is somewhat larger when the reversion to the variance mean level is faster. The most prevalent situation, $P(\bar{V}) < P_{B\text{-}S}$, seems to be explained by the fact that the level of mean volatility is lower in the random volatility model than in the B-S constant volatility put formula. A second pattern that emerges from the tables concerns the relative biases between the two put values, $P(\bar{V})$ and $P_{B\text{-}S}$. Visual inspection of Table 1 reveals that the bias range varies from [18.6% to -36.4%] for the scenario x = \$100 to [-3.6% to -2.6%] for the scenario x = \$98. In other terms, when the indebtedness value is moving progressively in-the-money, the bias intervals are narrowing from a range comprising a mixture of over- and under-pricing of $P(\bar{V})$ with regard to $P_{B\text{-}S}$, to a narrower range exhibiting mild $P(\bar{V})$ underpricing exclusively. The pattern is repeated in Table 2 where bias ranges are generally tighter in the case of a faster reversion to the variance mean level. The third pattern concerns x moving deeper in-the-money: as one moves from an even indebtedness value (when x = \$100 = L) to deeper in-the-money indebtedness values, commitment put values are increasing continuously, as expected. Yet, European commitment put estimates present a downward bias for x values below 99 because the intrinsic value (namely, the difference between the line par value L and x itself) is larger than both put estimates, $P(\bar{V})$ and $P_{B\text{-}S}$. European commitment put values are also lower than the corresponding American values that capture the borrower's early exercise option; this is to be expected since the latter is not constrained by the regulatory time frame (the audit dates) of the capital-adequacy problem examined here. In a sense, both $P(\bar{V})$ and $P_{B\text{-}S}$ constitute conservative estimates when used later on in computing the commitment risk-adjusted balance that enters the date-T calculation of the bank's capital requirement.

In a chain reaction, commitment put values are affecting the CNVs and the bank's risk exposure. The commitment net values are reported in Tables 1 and 2 as CNV_1 and CNV_2, respectively. For in-the-money indebtedness values, commitment put values comprise both time and intrinsic values and the resultant CNVs comprise both the "time and intrinsic" components of any off-balance sheet exposure. In this regard, scenario x = \$99 corresponding to entries 11 to 15 in Table 1 is again representative: when the line is exercised and fully drawn, the CNV_1 in entry 13 of column (3) constitutes a net notional discount of 29.7¢ per \$100 of credit provided. Or, to put it differently, a 1-billion tranche of short term commitments carries with it an off-balance sheet liability of 2.97 million. If the commitment is left unexercised on the other hand, as for entry 14 in column (3), the bank collects a 25.5¢ premium per \$100; it is indeed the same CNV_2 in all cases. Finally, a 50% probability of commitment exercise with full credit take-down is assumed for entry 15 in column (3); for this very plausible scenario, the bank's exposure is approximately 2.1¢ per $100 of credit offered: more concretely, if the bank were to carry off-balance sheet a $1-billion tranche of short-term unused commitments, it ought to simultaneously report a notional liability of $0.21 million. Irrespective of the (x-V) correlation, the credit risk exposure mainly constitutes a liability, as soon as x is below $99.5. Under rational pricing, the bank should strive to achieve at least a break-even exposure: to wipe out any negative exposure, the bank could then decide to increase future fees using any combinations of upfront and/or rear-end fees.

IV. CREDIT RISK

The above simulations are now used to articulate two of the policy implications of commitment pricing: the impact of off-balance sheet credit commitments on the bank's risk-based capital requirement and their simultaneous impact on its future profitability. These points are now examined in turn.

IV.a. Capital sufficiency

The *Basle capital rules* are linked solely to credit risk[14]: they required that standard risk-adjusted balances be determined for each off-balance sheet and on-balance sheet instruments and their aggregate value be weighted against a definition of regulatory capital. To calculate risk-adjusted values, off-balance sheet contractual amounts are initially converted by way of credit conversion factors to "credit equivalent amounts"; which in turn are weighted by appropriate "principal risk factors" to determine risk-adjusted balances. Since the end of 1992, a minimum total capital requirement of 8% applies to such balances[15]. To illustrate our subject,

[14] An additional guideline regarding market risk is presently under consideration: see Bank for International Settlements (1995): "Proposal to Issue a Supplement to the Basle Capital Accord to Cover Market Risk." *Basle Committee on Banks' Supervision*, April.

[15] The BIS capital guideline is formalized as follows:

$$\frac{\qquad}{\Sigma_i\ (c_i\ u_i)\ v_i + \Sigma_j\ y_j\ v_j} \geq .08$$

data regarding commitments and loans are presented in Table 3 for a large international bank, the Royal Bank of Canada as at October 31, 1996.

Insert Table 3 about here

Consider now how commitments to extend credit with an original term to maturity up to 1 year are treated under the BIS guideline. According to line (3) of Table 3, their credit equivalent amount is nil since the credit-conversion factor applied to the commitment contractual amount ($62.1 billion on line (1)) is 0%; and their risk-adjusted balance is accordingly also nil as the risk factor for such commitments is also 0%. Thus, still unused short-term commitments do not affect the risk-adjusted capital requirement at any point in time or on a continuous basis. The same is not true for longer-term commitments and on-balance sheet loans, however. On line (5), the risk-adjusted balance of over-one-year commitments is $13.5 billion and that of other (mainly corporate) loans is $68.4 billion, and, in both cases, the principal risk weight is 100% according to line (4). On line (1) also, the contractual amount of short-term commitments ($62.1 billion) is larger than that of longer-term commitments ($28.9 billion) and sizeable at any rate with regard to on-balance sheet corporate loans, $74.2 billion shown on line (3). More concretely, the balance-sheet amount of outstanding loans is $74.2 billion while the total amount of off-balance sheet unused commitments is $91 billion, with $62.1 billion or 68.2% of them being riskless according to the BIS accounting-based valuation of credit risk.

At this juncture, we are in a position to offer a market-based alternative to the accounting-based approach mandated by the BIS: simply combine the numerical values obtained in Tables 1 and 2 with the data presented in Table 3. As off-balance sheet commitments constitute put options, they should be treated, in terms of capital sufficiency, in the same way as the bank's

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where e refers to the bank's total regulatory capital, $0 \leq c_i \leq 1$ and u_i to the credit conversion factor and contractual amount, respectively, of the i-th off-balance sheet instrument, $0 \leq v_{i \text{ or } j} \leq 1$ to the principal risk weight (i: for off-balance sheet instruments; j: for on-balance sheet instruments) and y_j to the contractual amount of the j-th on-balance sheet instrument. The bank's risk-adjusted balance is $\Sigma_i (c_i u_i) v_i + \Sigma_j y_j v_j$, and 0.08 denotes the minimum ratio of total regulatory capital to risk-adjusted balance in force since the end of 1992.

other off-balance sheet derivative instruments, i.e., the over-the-counter foreign exchange and interest rate contracts[16]. Presently, the latter contractual amounts are converted to credit-equivalent amounts by adding (i) the **current** exposure, i.e. the difference between the present marked-to-market value and the contractual nominal value, and (ii) an amount for potential **future** exposure on the basis of their residual term to maturity. This credit-equivalent amount is next weighted by a principal risk factor (ranging from 20% up to 50%) to arrive at the instrument risk-adjusted balance.

If a similar approach is extended to credit commitments, we obtain: intrinsic value (L - x) of the commitment put (i.e., today's credit-risk exposure) + time component of commitment put, if any (corresponding to the potential future credit-risk exposure) = European commitment put value (namely the cost of contractual credit risk). This contractual risk (or credit-equivalent amount) is next weighted by the proportion of credit units exercised to yield the risk-adjusted balance for short-term unused commitments. The suggested approach is illustrated numerically in Table 4 below.

Insert Table 4 about here

Consider the contractual amount (L = \$62.1 billion) of short-term unused commitments reported earlier in Table 3 in conjunction with the scenario x = \$99 of Table 1: this indebtedness value is fairly representative as it captures a mild decline in the creditworthiness of prime-rate borrowers with 6 months, $\tau = 0.5$[17], remaining to the next audit date. According to entry 11

[16] Foreign currency and interest rate futures, being exchange-traded and subject to margin requirements, are deemed to carry no additional credit risk. Notice that in the BIS procedure, the credit risk of off-balance sheet commitments is linked to that of on-balance sheet loans rather than to the credit risk of the other derivative instruments that remain off-balance sheet, as is the case here.

[17] In actuality, the aggregate value of short-term commitments is recorded at the date of the bank's annual report: the midpoint between two annual reports corresponds to a valuation date six months before commitment expiry. As Merton (1977) has argued for related loan guarantees, the time remaining to commitment maturity can be interpreted as the length of time until the next

in column (3) of Table 1, the credit-equivalent amount for \$62.1 billion of short-term unused commitments is:

\$62.1 billion x 0.00797 (= the commitment put value per \$ **billion**) = \$494.94 million.

This adjustment for contractual risk is shown on line (3) of Table 4, and with an exercise-cum-takedown proportion of 50%, the risk-adjusted balance of short-term commitments shown on line (5) is:

\$494.94 million x (0.5) = \$247.47 million.

While not shown in Table 4, the same procedure also applies to the commitments with a term longer than one year. The approach advocated above can be formalized in the following

PROPOSITION: (1) Compute the indebtedness value of short-term unused credit commitments, x, and subtract it from the line par value, L; (2) add this current commitment exposure (L - x) to the potential future exposure captured by the time component, if any, of the European commitment put value; and (3) weight this credit-equivalent amount by the experience-based proportion (say 50%) of all commitments taken down to arrive at the option-based risk-adjusted balance of this off-balance sheet instrument.

IV.b. Impact on banks' profitability

Capital sufficiency concentrates on commitment credit risk, uniquely. To the extent we wish to consider commitment risk and offsetting benefits (fees), we ought to examine the impact of the commitment net liability on the bank's profits. This notional liability is captured under the heading Exposure in Tables 1 and 2. Again for \$100 of credit offered under a commitment, we have:

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audit of these off-balance sheet contracts.

$$\begin{aligned} \text{Exposure} &= p\,[f_0^u \exp(r(s - t_0)) + L.f_T^c \exp(-r(T - s)) - P(x, V, s)] + (1 - p)\,[f_0^u \exp(r(s - t_0))], \\ &= 0.5 \times CNV_1 \quad + \quad 0.5 \times CNV_2, \text{ and} \\ &= 0.5\,(-0.297) \quad + \quad 0.5\,(0.255) \quad = -0.021 \text{ per \$100 of line par value,} \end{aligned}$$

where the first line reproduces eq. (16) of the valuation programme and the second explains how the exposure is calculated in Tables 1 and 2. The last line shows the result for entry 15 of column (3) in Table 1: the presence off-balance sheet of a $100 commitment creates a net notional liability of 2.1¢ per $100 of credit offered. If this scenario is again applied to the contractual amount (L = $62.1 billion) of short-term commitments reported in Table 3, we obtain:

$$\text{Exposure} = 0.5\,[\,62.1\,(-0.00297)] + 0.5\,[\,62.1\,(0.00255)] = -0.01304,$$

where the first component captures the impact when the commitments are exercised and the second when they remain unexercised. In this particular case, the bank's exposure to $62.1 billion of off-balance short-term unused commitments corresponds to a notional loss of the order of $13.04 million (also shown on line (6) in Table 4). This option-based illustration is thus uncovering the following systematic pattern: ever slight declines in the creditworthiness of prime-rate borrowers adversely affect the bank's future profits through the commitment off-balance sheet notional loss. Don't negative markup differentials and in-the-money indebtedness values carry intrinsic risk, and so, give rise to a commitment exposure that lowers the bank's future profitability ?

V. CONCLUSIONS

This paper investigates commitment credit risk and valuation in connection with their risk-adjusted balance used in computing the bank's capital requirement mandated by the BIS. The pricing of the European commitment put is characterized by: i) the existence of a fixed forward markup and annual audit dates, and ii) a marked-to-market value of the credit line, or indebtedness value, that is varying stochastically. Once the correlated diffusions of the indebtedness value and its mean-reverting variance rate are integrated in a valuation programme, the European commitment put is priced as a power series approximation. Commitment net values and the bank's risk-adjusted exposure are next determined by combining the aforementioned put value, reasonable line fees, and a conditional exercise-cum-takedown proportion that captures the lines that are effectively mobilized. In simulation experiments, random volatility commitment put values exhibit biases with regard to the B-S constant-volatility formula. The magnitude and sign of the biases are governed by: i) the correlation between the indebtedness value and its volatility; ii) the indebtedness value moving slowly in-the-money; and iii) the speed of mean reversion in the variance rate drift.

The overall picture which emerges from the simulations is that the random volatility model generates in general lower commitment put values for any (x-V) correlation than the corresponding B-S put formula; the reverse is however true in a small upper-corner region of Tables 1 and 2 for at-the-money or slightly in-the-money put values with very negative (x-V) correlations. This characteristic pattern is due to the fact that the level of mean volatility is mostly lower in the stochastic volatility model proposed than in the corresponding B-S put formula. Numerical simulations are next used to ascertain how commitment credit risk is affecting the banks' capital requirement. According to the accounting-based procedure mandated by the BIS, the risk-adjusted balance of short-term commitments is nil; this is not the case however when the same risk-adjusted balance is computed by way of the option-based procedure proposed here. Beyond capital sufficiency, the procedure is also discovering the following systematic pattern: declines, ever slight, in the creditworthiness of prime-rate borrowers affect the bank's future profitability via the commitment off-balance sheet notional liability.

REFERENCES

Artzner, P., and F. Delbaen, 1995, Default risk insurance and incomplete markets, Mathematical Finance, 5, 187-195.

Avery, R.B. and A.N. Berger, 1991, Loan commitments and bank risk exposure, Journal of Banking and Finance, 15(1), 173--192.

Ball C.A. and A. Roma, 1994, Stochastic volatility option pricing, Journal of Financial and Quantitative Analysis, 29(4), 589-607.

Bartter, B.J. and R.J. Rendleman, 1979, Fee-based pricing of fixed rate bank loan commitments, Financial Management, Spring, 13--20.

Bhattacharya, S and A.V. Thakor, 1993, Contemporary banking theory, Journal of Financial Intermediation, 3, 2-50.

Black, F. and M.J. Scholes, 1973, The pricing of options and corporate liabilities, Journal of Political Economy, 81(3), 637--659.

Chateau, J.-P. D., 1990, Valuation of 'capped' variable rate loan commitments, Journal of Banking and Finance, 14(4), 717-728.

Chateau, J.-P. D., 1995, Valuation of floating-rate credit commitments: Theory and evidence, in Proceedings of the Administrative Sciences Association of Canada, D. Cyr editor, 16(1), 71-80.

Das, S.R., 1995, Credit risk derivatives, The Journal of Derivatives, Spring, 7-23.

Dothan, M., 1987, A random volatility correction for the Black-Scholes option-pricing formula, Advanced in Futures and Options Research, 2, 97-115.

Duffee, G.R., 1996, On measuring credit risks of derivative instruments, Journal of Banking and Finance, 20, 805-833.

Garman, M., 1976, A general theory of asset valuation under diffusion state processes, working paper, University of California, Berkeley.

Greenbaum, S.I. and I. Venezia, 1985, Partial exercise of loan commitments under adaptative pricing, The Journal of Financial Research, 8(4), 251-263.

Hawkins, G.D., 1982, An analysis of revolving credit agreements, Journal of Financial Economics, 10, 59-81.

Heston, S.L., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, The Review of Financial Studies, 6(2), 327-343.

Ho, T.S.Y. and A. Saunders, 1983, Fixed rate loan commitments, takedown risk and the dynamics of hedging with futures, Journal of Financial and Quantitative Analysis, 18, 499-516.

Houston, J.F. and S. Venkataraman, 1994, Information revelation, lock-in, and bank loan commitments, Journal of Financial Intermediation, 3, 355-378.

Houston, J.F. and S. Venkataraman, 1996, Liquidation under moral hazard: optimal debt maturity and loan commitments, Journal of Banking and Finance, 20, 115-133.

Hull, J. and A. White, 1987, The pricing of options on assets with stochastic volatilities, Journal of Finance, 42, 281-300.

Hull, J. and A. White, 1988, An analysis of the bias in option pricing caused by a stochastic volatility, Advanced in Futures and Options Research, 3, 29-61.

Hull, J. and A. White, 1995, The impact of default risk on the price of options and other derivative securities, Journal of Banking and Finance, 19, 299-322.

James, C., 1981, Self-selection and the pricing of bank services: an analysis of the market for loan commitments and the role of compensating balance requirements, Journal of Financial and Quantitative Analysis, 16(5), 725-746.

Longstaff, F.A., 1992, Interest rate volatility and the term structure: A two-factor general equilibrium model, Journal of Finance, 47, 1259-1282.

Melino, A. and S.M. Turnbull, 1990, Pricing foreign currency option with stochastic volatility, Journal of Econometrics, 45, 239-265.

Merton, R.C., 1973, An intertemporal capital asset pricing model, Econometrica, 41, 867-887.

Merton, R.C., 1977, An analytic derivation of the cost of the deposit insurance and loan guarantees, Journal of Banking and Finance, 1, 3-11.

Morgan, D. P., 1993, Financial contracts when costs and returns are private, Journal of Monetary Economics, 31(1), 129-46.

Scott, L.O., 1987, Option pricing when the variance changes randomly: Theory, estimation and an application, Journal of Financial and Quantitative Analysis, 22, 355-369.

Shockley, R.L., 1995, Bank loan commitments and corporate leverage, Journal of Financial

Intermediation, 4, 272-301.

Stein, E.M. and C.J. Stein, 1991, Stock price distributions with stochastic volatility, Review of Financial Studies, 4, 727-752.

Thakor, A., 1982, Toward a theory of bank loan commitments, Journal of Banking and Finance, 6(1), 55-84.

Thakor, A., H. Hong and S.I. Greenbaum, 1981, Bank loan commitments and interest rate volatility, Journal of Banking and Finance, 5(4), 497-510.

Thakor, A. and G.F. Udell, 1987, An economic rationale for the pricing structure of bank loan commitments, Journal of Banking and Finance, 11(2), 271-289.

Vetzal, K.R., 1997, Stochastic volatility, movements in short-term interest rates, and bond option values, Journal of Banking and Finance, 21, 169-196.

Wiggins, J.B., 1987, Option values under stochastic volatility: Theory and empirical estimates, Journal of Financial Economics, 19, 351-372.

TABLE 1

Theoretical values of: i) the European commitment put with a stochastic volatility, $P(\bar{V})$, and the B-S put value, P_{B-S}; ii) bias $B = [P(\bar{V}) - P_{B-S}]/P_{B-S}$ expressed in %; iii) commitment net value: CNV_1 when the commitment is exercised and CNV_2 when it is left unexercised; and iv) Exp. = exposure to commitment credit risk. Parameter definition: a and b = mean and reversion parameters of the variance drift; ξ = volatility of the variance rate; r = riskless rate of interest, in % p. a.; rho = indebtedness value-variance rate correlation; V(t) = variance rate, in % p.a.; τ = time to commitment expiry, in years; x = indebtedness value in $;

#		ρ	(1) -1	(2) -0.5	(3) -0.2	(4) 0.2	(5) 0.5	(6) 1.0
1	x = $100	$P(\bar{V})$	0.594	0.545	0.509	0.454	0.407	0.319
2	P_{B-S} = 0.501	B %	18.6	8.8	1.7	-9.3	-18.7	-36.4
3		CNV_1	-0.094	-0.045	-0.009	0.046	0.093	0.181
4		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
5		Exp.	0.081	0.105	0.123	0.151	0.174	0.218
6	x = $99.5	$P(\bar{V})$	0.711	0.669	0.639	0.595	0.558	0.488
7	P_{B-S} = 0.643	B %	10.5	4.0	-0.6	-7.5	-13.3	-24.1
8		CNV_1	-0.211	-0.169	-0.139	-0.095	-0.057	0.012
9		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
10		Exp.	0.022	0.043	0.058	0.08	0.099	0.133
11	x = $99	$P(\bar{V})$	0.85	0.818	**0.797**	0.766	0.741	0.695
12	P_{B-S} = 0.814	B %	4.4	0.6	**-2.0**	-5.8	-8.9	-14.5
13		CNV_1	-0.349	-0.318	**-0.297**	-0.266	-0.241	-0.195
14		CNV_2	0.255	0.255	**0.255**	0.255	0.255	0.255
15		Exp.	-0.047	-0.032	**-0.021**	-0.006	0.007	0.03
16	x = $98.5	$P(\bar{V})$	1.012	0.996	0.986	0.971	0.959	0.938
17	P_{B-S} = 1.014	B %	-0.2	-1.8	-2.8	-4.2	-5.4	-7.5
18		CNV_1	-0.512	-0.496	-0.486	-0.471	-0.459	-0.438
19		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
20		Exp.	-0.128	-0.121	-0.116	-0.108	-0.102	-0.91
21	x = $98	$P(\bar{V})$	1.202	1.206	1.209	1.211	1.212	1.214
22	P_{B-S} = 1.246	B %	-3.6	-3.2	-3.0	-2.8	-2.7	-2.6
23		CNV_1	-0.702	-0.706	-0.708	-0.711	-0.712	-0.714
24		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
25		Exp.	-0.233	-0.226	-0.227	-0.228	-0.229	-0.229

Common parameters: a = 0.004 and b = -2 characterise a slow reversion to the mean level; ξ = 0.075; V(0) = 0.002 (i.e., σ_x = 4.47% p.a.); r = 0.04; and T - t = τ = 0.5.

TABLE 2

Theoretical values of: i) the European commitment put with a stochastic volatility, $P(\bar{V})$, and the B-S put value, P_{B-S}; ii) bias B = $[P(\bar{V}) - P_{B-S}]/P_{B-S}$ expressed in %; iii) commitment net value: CNV_1 when the commitment is exercised and CNV_2 when it is left unexercised; and iv) Exp. = exposure to commitment credit risk. Parameter definition: a and b = mean and reversion parameters of the variance drift; ξ = volatility of the variance rate; r = short term rate, in % p. a.; rho = indebtedness value-variance rate correlation; V(t) = variance rate, in % p.a.; τ = time to commitment expiry, in years; x = indebtedness value in $;

#		ρ	(1) -1	(2) -0.5	(3) -0.2	(4) 0.2	(5) 0.5	(6) 1.0
1	x = $100	$P(\bar{V})$	0.55	0.526	0.509	0.485	0.446	0.431
2	P_{B-S} = 0.501	B %	9.9	5.0	1.7	-3.1	-6.9	-14.0
3		CNV_1	-0.05	-0.026	-0.09	0.015	0.034	0.069
4		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
5		Exp.	0.102	0.115	0.123	0.135	0.145	0.162
6	x = $99.5	$P(\bar{V})$	0.681	0.662	0.648	0.629	0.613	0.584
7	P_{B-S} = 0.643	B %	5.9	2.9	0.8	-2.2	-4.7	-9.2
8		CNV_1	-0.181	-0.161	-0.148	-0.129	-0.113	-0.084
9		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
10		Exp.	0.037	0.047	0.053	0.063	0.071	0.086
11	x = $99	$P(\bar{V})$	0.837	0.824	0.815	0.801	0.79	0.769
12	P_{B-S} = 0.814	B %	2.8	1.3	0.2	-1.5	-2.9	-5.4
13		CNV_1	-0.336	-0.324	-0.315	-0.301	-0.29	-0.269
14		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
15		Exp.	-0.041	-0.034	-0.03	-0.023	-0.018	-0.007
16	x = $98.5	$P(\bar{V})$	1.02	1.015	1.01	1.00	0.99	0.988
17	P_{B-S} = 1.014	B %	0.5	0.0	-0.3	-0.9	-1.5	-2.6
18		CNV_1	-0.52	-0.515	-0.511	-0.505	-0.499	-0.487
19		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
20		Exp.	-0.132	-0.13	-0.128	-0.125	-0.122	-0.116
21	x = $98	$P(\bar{V})$	1.21	1.238	1.24	1.241	1.24	1.38
22	P_{B-S} = 1.246	B %	-1.1	-0.7	-0.5	-0.5	-0.5	-0.6
23		CNV_1	-0.733	-0.738	-0.739	-0.741	-0.74	-0.738
24		CNV_2	0.255	0.255	0.255	0.255	0.255	0.255
25		Exp.	-0.239	-0.241	-0.242	-0.243	-0.243	-0.242

Common parameters: a = 0.02 and b = -10 characterize a fast reversion to the mean level; ξ = 0.075; V(0) = 0.002 (i.e., σ_x = 4.47% p.a.); r = 0.04; and T - t = τ = 0.5.

TABLE 3: **BIS accounting-based valuation of credit risk: from off-balance sheet commitments to on-balance sheet loans.**

	Off-balance sheet commitments		On-balance sheet loans
With an original term to maturity	≤ 1 yr	≥ 1 yr	
(1) Contractual amount. $ in billions	62.1	28.9	n.a.[1]
(2) Credit **conversion** factor. in %	0	50%	n.a.[1]
(3) Credit-equivalent amount. $ in billions	nil	14.4	74.2
(4) Principal **risk** factor. in %	0	100%	100%
(5) BIS risk-adjusted balance. $ in billions	nil	13.5	68.4

[1] n.a. = not applicable
Source: The Royal Bank of Canada annual report as at October 31. 1996.

TABLE 4: **Fair value or derivative-based risk valuation of off-balance sheet commitments with an original term to maturity less than one year.**

Concepts	Illustration
(1) Contractual amount. $ in billions	62.1
(2) Stochastic volatility commitment put value: entry 11 in column (3) of Table 1	0.00797 per billion
(3) Credit-equivalent amount. $ in millions	494.94
(4) Exercise-cum-takedown proportion. in percentage	50%
(5) **Option-based** risk-adjusted balance. $ in millions	247.47
(6) Bank's exposure to commitment credit risk. $ in millions	13.04

Figure 1: indebtedness value volatility, Canada 1966-1995