BUYER-SUPPLIER INTERACTION, ASSET SPECIFICITY, AND PRODUCT CHOICE

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Abstract

The goal of this paper is to explore how the demand for specific investments may affect the product variety in a bilateral duopolistic industry. In the literature on the hold-up problem, it is generally assumed that the degree of specificity of investments is either exogenously determined or chosen by the suppliers. We develop a model where the degree of specificity of investments is endogenously determined through the product choices of both buyers and suppliers. In an environment where input prices are determined by bilateral negotiations, the existence of alternative buyers causes suppliers to choose less-than-fully-specialized input types. We show that their location and investment choices crucially depend on the degree of product differentiation in the downstream market. This implies that the buyers may choose to increase their own competition by producing more similar products in order to increase the suppliers’ investment incentives. We also analyze the impact of capacity constraints on the structure of the upstream market and show that if the suppliers do not face capacity constraints, only one supplier serves the downstream market in equilibrium.

JEL Classification: L11, L23, R12

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1 Introduction

It has long been recognized that the incompleteness of contracts may result in reduced incentives to invest in the context of bilateral relationships (Klein et al., 1978; Williamson, 1985; Hart, 1995). The extent of underinvestment depends on whether there exist alternative uses for the investments made. More specific investments have higher values for the buyers for which they are intended, but are less valuable in other uses. A large literature in economics discusses how firms may rely on alternative governance structures, such as vertical integration or long-term supply arrangements, to encourage specific investments.

The goal of this paper is to explore how the demand for specific investments may affect the product variety in a bilateral duopolistic industry. We consider a model of differentiated products where specific investments arise because there is an ideal input type corresponding to each possible variety of a final good.1 Suppliers of inputs make two kinds of decisions: they choose what type of input to produce and how much effort to put into it. The degree of specificity of their input to a given buyer in the downstream market depends on how suitable the input is for the purposes of that buyer as well as how suitable it is for the purposes of alternative buyers. This implies that the degree of product differentiation in the downstream market plays a significant role in determining the alternative uses suppliers have for their inputs. If there are downstream firms which demand similar inputs, suppliers have better alternative uses for their inputs and, hence, exert a higher effort in developing them.

In this context, the analysis focuses on two questions. First, what is the impact of downstream buyers’ choice of product differentiation on the location and investment choices of upstream suppliers? Second, how does the downstream buyers’ concern about the investment incentives of the upstream suppliers alter their product choices? Both questions are analyzed

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1It is emphasized in the product management literature that the distinctiveness of two final goods may significantly depend on the distinctiveness of the inputs used in their production (Robertson and Ulrich, 1998). For example, the distinctiveness of two cars may substantially depend on the type of engine components used in their production. Krishnan and Ulrich (2001) explain the close links that exist between firms’ choices of inputs and their decisions regarding which product variety to produce.
in a model where input prices are determined by bilateral negotiations, which is a common occurrence in markets for intermediate goods.

Capacity constraints play a key role in determining whether suppliers have alternative uses for their inputs. We consider two types of industries, one where suppliers are capacity constrained and one where they are not. The existence of a capacity-constrained supplier confers a positive externality on the other suppliers in an input industry. If a supplier has limited capacity and cannot completely meet the input demand of its buyer, rival suppliers may be left with an outside option that increases their bargaining power. If the suppliers do not have capacity constraints, then a buyer’s demand is more likely to be satisfied by its preferred supplier. This implies that rival suppliers may have no alternative uses for their inputs in equilibrium. However, the buyers can always turn to an alternative supplier if they cannot reach an agreement with their own supplier.

The results show that with capacity constraints, the existence of alternative buyers causes suppliers to choose less-than-fully-specialized input types. Their investment incentives crucially depend on the degree of product differentiation in the downstream market. This implies that the buyers may choose to increase their own competition by producing more similar products in order to increase the suppliers’ investment incentives. Hence, in contrast with the maximum differentiation result of d’Aspremont et al. (1979), firms may not always choose to be maximally differentiated in an effort to decrease the competition between themselves.

The results also reveal that the existence of capacity constraints has an important impact on the structure of the upstream market. Specifically, if the suppliers do not face capacity constraints, only one supplier serves the downstream market in equilibrium. The buyers are willing to use an input type that is different from their ideal variety in order to benefit from the higher effort that the supplier puts into the developing the input.

The Harvard Business School case on Crown Cork and Seal Company, Inc. (CCS) illustrates how the hold-up problem can result in strategic location choice (Gordon, Reed and
Hamermesh, 1977).\(^2\) In the 1960s and 1970s, CCS made metal cans for the soft-drink industry. In the metal can industry, suppliers generally located their plants next to customers in order to minimize transportation costs. This exposed them to hold-up. CCS was "unusual in that it set up no plants to service a single customer" (Gordon et al., 1977, p. 11). Instead, it made sure there were a number of customers for which it could provide products near its plants. Hence, although CCS specialized in customer service, it deliberately located between potential buyers in order to avoid the hold-up problem. It tailored the specifications of the cans to the customers’ requirements, but stood ready to modify them if necessary.

The results of this paper also suggest one explanation for the use of common inputs, i.e., platform sharing, which has become a common practice in the automobile industry (Robertson and Ulrich, 1998). According to our model, firms may choose to produce similar products and, thus, use similar inputs in order to improve their suppliers’ investment incentives.

This paper contributes to both the literature on the hold-up problem and the literature on product choice by bringing the issues raised in them together in a single model and showing the interdependence between them. In the literature on the hold-up problem, the endogenous choice of input specificity has been explored by Choi and Yi (2000), Church and Gandal (2000), McLaren (2000), and Grossman and Helpman (2002).\(^3\) One of the core elements of these papers is that an integrated supplier chooses to produce a more specific input than an unintegrated supplier. Hence, vertical integration confers a negative externality on the other firms in the industry.\(^4\) This paper differs by demonstrating the impact of specific investments on another aspect of firm behavior, that of product choice. Previous papers assume that the buyers’ product choices are exogenously determined and analyze the suppliers’ choice of specificity. We analyze how the degree of specificity and the suppliers’ investment incentives

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\(^2\) I am grateful to Michael Raith for pointing out this case study to me.


\(^4\) Riordan and Williamson (1985) also analyze the impact of asset specificity on firm boundaries. However, they do not consider how the decisions of other firms in the industry may affect the behavior of a specific firm.
depend on the strategic product choices made by firms on both sides of the market.

This paper is also related to Inderst and Wey (2003a), who focus on horizontal mergers in bilaterally oligopolistic industries. They show that firms may merge either to enhance their bargaining power or to affect their suppliers’ incentives to make non-contractible investments while we show that firms may strategically choose their locations in order to improve either their bargaining power or their suppliers’ investment incentives.

There exists an extensive literature on product differentiation and economic geography. Current theories of horizontal product differentiation have been very much influenced by the “Main Street” model of Hotelling (1929). Ever since d’Aspremont et al. (1979) challenged the “Principle of Minimum Differentiation” argued by Hotelling (1929), there have been numerous attempts to establish the validity of this principle.5 In the economic geography literature, several papers focus on the so-called “Marshallian externalities” and increasing returns to explain the formation of economic clusters.6

In this literature, the current paper is most closely related to Belleflamme and Toulemonde (2003). They illustrate that downstream and upstream firms may choose to agglomerate in an effort to decrease their costs because the upstream firms experience economies of scale. This paper differs by focusing on the role played by non-contractible investments in product choice. We analyze how capacity constraints may affect the firms’ product choices.

The paper proceeds as follows. We present the details of the model in the next section. Section 3 discusses the bargaining outcome. Sections 4 and 5 present the results on the product and investment choices of the upstream suppliers and downstream buyers respectively. Section 6 concludes by suggesting avenues for future research. All the proofs are in the Appendix.

5See, for example, de Palma, et al. (1985), Friedman and Thisse (1993), and Rhee, et al. (1992).
6In his pioneering work, Marshall (1890) offered several reasons, such as mass-production and the availability of specialized inputs, in order to explain the phenomenon of localized industries. See the discussion in Fujita and Thisse (1996) regarding the literature based on his ideas.
2 Model

2.1 Consumers and production technology

We employ the setting of d’Aspremont et al. (1979), who modify the standard Hotelling (1929) model of spatial competition by assuming quadratic consumer transportation costs. The market region is described by the unit interval \([0, 1]\). Consumers are uniformly distributed along the line market and the location of a consumer is denoted by \(x \in [0, 1]\). Two downstream buyers, \(B_1\) and \(B_2\), produce differentiated products. They choose to locate at distances \(b_1\) and \(b_2\) from the two ends of the unit line. Without loss of generality, we assume that \(B_1\) is located to the left of \(B_2\): \(0 \leq b_1 \leq 1 - b_2 \leq 1\). We can interpret the location choices of the firms as product design selections within the space of horizontal characteristics.

Each consumer purchases exactly one unit from the firm offering it at the lower effective price, namely the mill price plus the transportation cost. A consumer of type \(x\) purchasing from buyer \(B_i\) gets utility

\[ U(x, b_i, p_i) = S - p_i - t(x - b_i)^2, \quad i = 1, 2 \] (1)

where \(S\) represents the gross surplus from consumption, \(p_i\) stands for the price at which buyer \(i\) offers its good, and \(t\) is a measure of consumer loyalty. When \(t = 0\), the consumer simply chooses the product with the lowest price and is indifferent about which variety is consumed. Disutility costs vary quadratically with the distance between the product produced by the firm and that which is most preferred by the consumer. As shown in d’Aspremont et al. (1979), this assumption prevents discontinuities in the profit functions of the firms.

The profit functions of the downstream firms have the general form

\[ \pi_i(p_1, p_2) = (p_i - c_i) D_i(p_1, p_2) - F_i, \quad i = 1, 2 \] (2)

where \(c_i\) and \(F_i\) denote the marginal and fixed cost of production respectively. \(D_i(p_1, p_2)\) stands for the demand for buyer \(B_i\’s\) product. We assume that the magnitudes of \(S\) and \(F_i\) are such that the market is covered, i.e., all consumers purchase.
Each firm in the downstream industry, $B_i$, can decrease its fixed cost of production by procuring a specialized input from a supplier in an upstream industry. For example, the input can be a software developed to organize the databases of the firm in a better way. We assume that the inputs decrease the buyers’ fixed cost of production, rather than their marginal cost of production, for tractability reasons.\(^7\)

Each type of final good has an ideal input associated with it. The unit interval represents both the space of possible input types and the space of possible final goods. The upstream industry consists of two suppliers, $S_1$ and $S_2$, which choose to locate at distances $s_1$ and $s_2$ from the two ends of the unit line. By choosing where to locate, the suppliers decide for which variety of the final good to produce an ideal input. A buyer located at $b_i$ gets the maximum benefit from using the input produced by the supplier located at $b_i = s_i$. Buyer $B_i$ can also use an input produced by a supplier located at a different point along the unit interval, but the benefit from using a less-than-ideal input decreases with the degree of differentiation between the locations of the buyer and the supplier.

More precisely, let $F_i = K_i$ represent $B_i$’s fixed cost of production if it does not purchase any inputs. Consider buyer $B_i$ and supplier $S_i$ located at $b_i$ and $s_i$ from one end of the unit interval respectively. We assume that the benefit $B_i$ receives from using an input produced by $S_i$ is a decreasing and concave function of the distance between $B_i$ and $S_i$. That is, the marginal disutility from using an input variety that is different from the ideal one is an increasing function of the distance between $s_i$ and $b_i$, $|b_i - s_i|$. Specifically, each unit of input that $B_i$ procures from $S_i$ reduces its fixed cost of production by $e_i \left[1 - (b_i - s_i)^2\right]$, where $e_i$ is the amount of investment made by $S_i$. This implies that if $b_i = s_i$, the buyer receives a

\(^7\)If the suppliers invest to decrease the buyers’ marginal cost of production instead of their fixed cost of production, the locations of the buyers can affect the investment decisions of the suppliers in two different ways. A change in the locations of the buyers would cause a change in both the amount of profits made in the downstream market and the outside options of the firms. As in McLaren (2000), our current set-up allows us to differentiate between these two and concentrate on the latter one. Although it would be natural to allow the inputs to affect the marginal cost of production, it would result in a substantial amount of additional complication without improving our illustration of the main points of the paper.
fixed cost reduction of $e_i$. The buyers can mix and match the inputs produced by different suppliers. For example, if $B_i$ purchases one unit from supplier $S_i$ and one unit from supplier $S_j$, its fixed cost is reduced by $e_i \left[1 - (b_i - s_i)^2\right] + e_j \left[1 - (1 - s_j - b_i)^2\right]$. We assume the parameter values are such that the total cost reduction that can be obtained from the inputs is less than or equal to $K_i$.

The input prices and purchases are determined by bilateral negotiations in the input markets. The specific value that $F_i$ takes in (2) is determined as a result of this process. The details of the bargaining process are explained in Section 2.3.

In the input development stage, the suppliers choose both the input variety they would like to produce and their effort level. We assume that the suppliers’ cost of effort is $e_i^2$. The environment is one of incomplete contracting. Due to the uncertain nature of the innovation process, the parties cannot sign enforceable ex ante contracts for the delivery of a specific innovation (Aghion and Tirole, 1994). The precise nature of the required input is revealed only ex post and the parties cannot commit to refrain from renegotiation. For example, Helper and Levine (1992) state that in the automotive industry, contracts with suppliers are necessarily incomplete because of the complexity of the parts.

As is well known, the absence of ex ante contracts results in the hold-up problem in the investment stage. We assume that vertical integration is not a desirable way of dealing with the potential underinvestment problem because of internal incentive and coordination problems. Hence, we focus on the impact of the hold-up problem on the firms’ product choices.

2.2 Timeline

The firms play the following four-stage non-cooperative game.

\footnotesize
8 Such an argument is in line with the models developed in Hart and Moore (1999) and Segal (1999) in order to define contractual incompleteness precisely.

8 They state that "since engineering changes are common, the part actually produced by the supplier is often not the same as the part that was contracted for" (p. 567).
Stage 1: The buyers choose their locations along the unit line. After the location choices are made, each buyer contacts a supplier and provides the supplier with the characteristics of the input it requires. This information also allows the supplier to choose any location in the input space. Without loss of generality, assume that \( B_i \) contacts \( S_i \), \( i = 1, 2 \).

Stage 2: The suppliers simultaneously decide both where to locate along the unit line and how much to invest in the development of a prototype. We assume that the suppliers cannot observe each other’s location choices before making their investment decisions.

Stage 3: After the suppliers choose their locations and develop their prototypes, the buyers and suppliers meet in the market for inputs. They negotiate and contract over the quantities to be delivered and the payments to be paid. The details of the bargaining process are given in Section 2.3.

Stage 4: After the trade of inputs takes place, the buyers simultaneously choose the output prices at which they are willing to sell their products.

2.3 Ex-post bargaining

We consider a simple bilateral bargaining game that allows us to demonstrate the main points of the paper.\(^\text{10}\) The bargaining game consists of two stages. In the first stage, buyer \( B_i \) approaches the supplier it has contacted after choosing its location and they bargain over price. If the negotiations between \( B_i \) and \( S_i \) fail, they can try to find other partners in the second stage of bargaining and bargain with them. Specifically, \( S_i \) can try to approach \( B_j \) and \( B_i \) can try to approach \( S_j \), where \( i \neq j \). We assume that once the first-stage negotiations fail, \( B_i \) and \( S_i \) cannot meet to bargain again. If the second round of negotiations also fails,

\(^\text{10}\) See Grossman and Helpman (2002) and McLaren (2000) for two alternative simple price determination mechanisms. Grossman and Helpman (2002) consider a similar two-stage bargaining model, where they assume that in the first stage, the negotiations between the parties can fail with some exogenous probability. McLaren (2000) assumes that buyers submit bids for the inputs of the suppliers in a model of technological uncertainty. He assumes that inputs may turn out to be of high or low quality, and high-quality non-specific inputs create a higher value than low-quality specific inputs. See, on the other hand, de Fontenay and Gans (2005) for a more fully specified bargaining game, where each upstream-downstream pair negotiates sequentially over the quantity supplied and a price. All these approaches would yield qualitatively similar results.
there are no more opportunities for trade.

We assume the parties split the surplus from bargaining equally in both rounds of negotiations. During the negotiations, the parties’ outside options make a difference in their bargaining positions. In the first stage of the bargaining game, the parties negotiate knowing that if their negotiations fail, they may have the option of dealing with an alternative partner in the second stage. Therefore, the second stage of the bargaining game determines the values of the parties’ outside options in the first stage of the bargaining game.

The next section demonstrates that whether the suppliers are capacity-constrained plays a critical role in the ability of the firms to find alternative partners in the second stage of the bargaining game.

3 Bargaining Outcomes

We are interested in finding the subgame perfect Nash equilibrium of the game outlined in Section 2.1. Therefore, we work backward from the determination of the output prices.

Consider the final-stage subgame, where the buyers compete by choosing prices for given locations \( b_1 \) and \( 1 - b_2 \). They maximize (2). The firms’ demand functions can be determined by defining the indifferent consumer. We get

\[
D_1 (p_1, p_2) = \frac{(p_2 - p_1)}{2t (1 - b_1 - b_2)} + \frac{(1 - b_1 - b_2)}{2} + b_1
\]

and

\[
D_2 (p_1, p_2) = 1 - D_1 (p_1, p_2).
\]

After substituting for these demand functions in the profit functions of the firms given in (2), we can solve for the Nash equilibrium in prices. Assuming symmetry, we have \( c_1 = c_2 = c \) and \( K_1 = K_2 = K \). Solving the first-order conditions for profit maximization in prices simultaneously yields the result that buyer \( B_i \)'s equilibrium price is equal to

\[
p_i^* (b_i, b_j) = c + \frac{t (3 + b_i - b_j) (1 - b_i - b_j)}{3}.
\]
It is straightforward to check that the second-order condition is satisfied. The equilibrium price level is not affected by the suppliers’ location and investment choices because the inputs bought affect the fixed costs of production only.

In the third stage of the game, the parties decide whether to trade after observing the location and investment choices made in the previous stages of the game. They know what the equilibrium prices will be in the last stage of the game. We are interested in solving for the subgame perfect equilibrium of the bargaining game specified in Section 2.3. We start in the next section by considering cases where the production technology of the buyers is such that they can make use of more units of the input than the suppliers can provide them with. This implies that the suppliers are capacity-constrained relative to the buyers’ needs and total demand in the market exceeds total supply.\textsuperscript{11}

3.1 Capacity Constraints

To keep the analysis simple, suppose the suppliers can produce at most one unit of input while the buyers can make use of up to two units of the input.\textsuperscript{12} Hence, after supplier $S_i$ chooses its location and invests $e_i$ to develop the input, it brings the input to the marketplace for trade. The marginal cost of producing a second unit is formidable high.

The existence of capacity constraints ensures that a supplier can always find another buyer to deal with after a failed negotiation. Since each buyer can use up to two units of the input and each supplier has only one unit to sell, $S_i$ can offer its unit for sale to $B_j$ if it cannot agree with $B_i$ in the first stage of the bargaining game, where $i,j = 1,2$ and $i \neq j$.

Proposition 1 presents the subgame perfect equilibrium of the bargaining game as a function

\textsuperscript{11}Hart and Tirole (1990) also consider the role capacity constraints play in the context of vertical interactions. In contrast with the current analysis, they analyze the impact of capacity constraints on the incentives to vertically integrate. They refer to the cases of capacity constraints and no capacity constraints as scarce supplies and scarce needs respectively. See also Inderst and Wey (2003b) who analyze the impact of buyer power in the context of both capacity constraints and no capacity constraints.

\textsuperscript{12}Our goal is to distinguish between cases when total demand in the input market exceeds total supply and cases when it does not. To achieve this, we fix the buyers’ demand and vary the amount the suppliers can supply. Assuming that the buyers can use more than two units of input would not change the results in any substantial way.
Proposition 1  Given $b_1, b_2, s_1, s_2, e_1$ and $e_2$, consider the inequality $C(i)$ defined as

$$e_i \left[ \left( 1 - (b_i - s_i)^2 \right) - \frac{1}{2} \left( 1 - (1 - b_j - s_j)^2 \right) \right] \geq 0,$$

where $i, j = 1, 2$ and $i \neq j$. The label $C(i)$ stands for the case of capacity constraints.

(i) If $C(i)$ holds for $i = 1, 2$, then $B_i$ and $S_i$ trade one unit in the first stage of the bargaining game. Their payoffs are

$$W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_i \left[ \left( 1 - (b_i - s_i)^2 \right) - \frac{1}{2} \left( 1 - (1 - b_j - s_j)^2 \right) \right]$$

(ii) If $C(i)$ holds but $C(j)$ does not, then $B_i$ buys one unit from $S_i$ in the first stage of the bargaining game and another unit from $S_j$ in the second stage of the bargaining game. $B_j$ does not buy any inputs. The payoffs are

$$W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_i \left[ \left( 1 - (b_i - s_i)^2 \right) + \frac{1}{2} \left( 1 - (1 - b_j - s_j)^2 \right) \right]$$

$$W_{B_j} = (p_j^* - c) D_j (p_1^*, p_2^*) - K$$

(iii) If $C(i)$ does not hold for $i = 1, 2$, $B_i$ buys one unit from $S_j$ (where $i \neq j$) in the second stage of the bargaining game. The payoffs are

$$W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_j \left( 1 - (1 - s_j - b_i)^2 \right)$$

$$W_{S_i} = \frac{1}{2} e_i \left( 1 - (1 - b_j - s_i)^2 \right).$$
Proposition 1 states the necessary and sufficient condition for trade to take place between Bi and Si. The inequality C (i) given in (6) implies that Bi and Si trade if the surplus created is nonnegative. The surplus created is the difference between the benefit of Si’s input to Bi and Si’s outside option. If Si does not trade with Bi, it can trade with Bj in the second stage of the bargaining game. Hence, trade takes place between Bi and Si if it results in a higher value than what Si can earn by trading with Bj. Whether or not Bj and Sj trade has no bearing on the decision of Bi and Si since Bj can purchase a second unit of input from Si even if it has purchased one unit of input from Sj in the first period.

To clarify the implication of the assumption that the suppliers are capacity-constrained relative to the buyers’ needs, consider the case of Si. Suppose the rival supplier, Sj, has a capacity constraint which implies that it supplies less than Bj can make use of. Hence, Si can always find an alternative buyer to negotiate with in the second stage of the bargaining game if the negotiation in the first stage fails. This improves Si’s bargaining position in the first stage if Si has a capacity constraint relative to Bi’s need. If Si is not capacity-constrained, it can sell units to Bj whether or not it trades with Bi and the potential trade with Bj does not have the impact of improving Si’s bargaining position in the first-stage negotiation with Bi.

Similar reasoning implies that since Bi can make use of more units of input than Si can supply, whether or not Bi trades with Sj in the second stage does not have a bearing on its bargaining position in the first stage. We can see this in case (ii) above, where C (i) holds but C (j) does not. Since C (j) does not hold, Bj and Sj do not trade in the first stage and Bi can (and does) trade with Sj in the second stage. This option increases Bi’s payoff by \( \frac{1}{2}e_j \left( 1 - (1 - s_j - b_i)^2 \right) \). However, a comparison of WBi in case (i) with WBi in case (ii) shows that it does not increase the amount Bi receives of the surplus created in the negotiation with Si.

Since the buyers do not have any outside options, the inequality C (i) does not depend on the location and investment choices of the alternative supplier in the market. Taking the
derivative of the left hand side of \( C(i) \) with respect to \( s_i \) shows that it is decreasing in \( s_i \) for \( s_i \geq b_i - (1 - b_i - b_j) \), which, as shown in the Section 4, always holds in equilibrium. Hence, trade is less likely to take place between parties that are located further away from each other.

Having a constrained capacity helps the suppliers. If \( C(i) \) holds, \( S_i \)’s payoff is given by its share of the surplus created when it sells its unit of input to \( B_i \) and the value of its outside option. If \( C(i) \) does not hold, \( S_i \)’s payoff is determined by its share of the surplus created when it sells its unit of input to \( B_j \). This is given in case (iii). In this case, the supplier does not receive the value of its outside option because trade takes place in the second stage of the bargaining game. As stated in Section 2.3, if the second-stage negotiations fail, there are no more opportunities for trade.

The buyers’ payoffs depend on how many units of input they can purchase. In case (i), each obtain one unit in the first stage of the bargaining game. In case (ii), \( C(j) \) does not hold and both suppliers sell their inputs to \( B_i \). \( B_j \) does not obtain any inputs. In case (iii), each buyer purchases one unit in the second stage of the bargaining game.

### 3.2 No Capacity Constraints

Consider now the case when the suppliers are not capacity-constrained relative to the buyers’ needs. We continue to assume that the buyers can make use of up to two units of the input. The suppliers first invest to develop a prototype. Once they spend \( e^2_i \) to develop the prototype, the cost of producing additional units of the input is zero.\(^\text{13}\)

The existence of unconstrained suppliers implies that the buyers can always find an alternative supplier if the negotiations in the first stage of the bargaining game fail. The subgame perfect equilibrium of the bargaining game is presented in Proposition 2.

\(^\text{13}\)The more general case where the suppliers have convex costs of production is not treated in this paper. Analyzing the cases of capacity constraints and no capacity constraints separately allows us to emphasize their different implications and avoid unnecessary complications. Assuming that the suppliers have convex costs of production would not change the results qualitatively.
Proposition 2 Given \( b_1, b_2, s_1, s_2, e_1, \) and \( e_2 \), consider the inequality \( NC(i) \) defined as

\[
e_i \left( 1 - (b_i - s_i)^2 \right) - \frac{1}{2} e_j \left( 1 - (1 - s_j - b_i)^2 \right) \geq 0, \quad (15)
\]

where \( i, j = 1, 2 \) and \( i \neq j \). The label \( NC(i) \) stands for the case of no capacity constraints.

(i) If \( NC(i) \) holds for \( i = 1, 2 \), then \( B_i \) and \( S_i \) trades two units in the first stage of the bargaining game. \( B_i \) and \( S_i \)'s payoffs are

\[
W_{B_i} = (p_i^* - c) D_i (p_i^*, p_j^*) - K + e_i \left( 1 - (b_i - s_i)^2 \right) + \frac{1}{2} e_j \left( 1 - (1 - s_j - b_i)^2 \right) \quad (16)
\]

\[
W_{S_i} = e_i \left( 1 - (b_i - s_i)^2 \right) - \frac{1}{2} e_j \left( 1 - (1 - s_j - b_i)^2 \right). \quad (17)
\]

(ii) If \( NC(i) \) holds but \( NC(j) \) does not, then \( B_i \) buys two units from \( S_i \) in the first stage and \( B_j \) buys two units from \( S_i \) in the second stage of the bargaining game. The payoffs are

\[
W_{B_i} = (p_i^* - c) D_i (p_i^*, p_j^*) - K + e_i \left( 1 - (b_i - s_i)^2 \right) + \frac{1}{2} e_j \left( 1 - (1 - s_j - b_i)^2 \right) \quad (18)
\]

\[
W_{B_j} = (p_j^* - c) D_j (p_i^*, p_j^*) - K + e_i \left( 1 - (1 - s_i - b_j)^2 \right) \quad (19)
\]

\[
W_{S_i} = e_i \left( 2 - (b_i - s_i)^2 - (1 - b_j - s_i)^2 \right) - \frac{1}{2} e_j \left( 1 - (1 - s_j - b_i)^2 \right) \quad (20)
\]

\[
W_{S_j} = 0. \quad (21)
\]

(iii) If \( NC(i) \) does not hold for \( i = 1, 2 \), \( B_i \) buys two units from \( S_j \), where \( i \neq j \), in the second stage of the bargaining game. \( S_i \) and \( B_i \)'s payoffs are

\[
W_{B_i} = (p_i^* - c) D_i (p_i^*, p_j^*) - K + e_j \left( 1 - (1 - s_j - b_i)^2 \right) \quad (22)
\]

\[
W_{S_i} = e_i \left( 1 - (1 - b_j - s_i)^2 \right). \quad (23)
\]

The inequality \( NC(i) \) given in (15) implies that in the case of no capacity constraints, the suppliers do not have any outside options. The gains from trade are determined by the difference between the benefit of \( S_i \)'s input to \( B_i \) and \( B_i \)'s outside option. When the suppliers are not capacity-constrained, the buyers always have an outside option and the value of their outside option depends on the alternative supplier’s location and investment choices. Hence,
unlike \( C(i) \), \( NC(i) \) also depends on \( s_j \) and \( e_j \). An increase in \( e_j \) or a decrease in \( s_j \) (which causes \( S_j \) to locate closer to \( B_i \)) makes \( B_i \)'s outside option more attractive. Hence, it is more likely to hold as \( e_i \) increases or as \( S_i \) moves closer to \( B_i \), and is less likely to hold as \( e_j \) increases or as \( S_j \) moves closer to \( B_i \).

Proposition 2 indicates that depending on whether \( NC(i) \) holds for \( i = 1, 2 \), either the buyers deal with different suppliers or they both deal with the same supplier in equilibrium. In case (i), the buyers procure inputs from different suppliers in the first stage of the bargaining game. Their payoffs are given by their share of the surplus created and the value of their outside option. In case (ii), both buyers procure inputs from the same supplier in different stages of the bargaining game. The second supplier makes zero. Finally, in case (iii), the buyers procure inputs from different suppliers in the second stage of the bargaining game. Hence, they receive their share of the surplus created but not the value of their outside option.

4 Product and Investment Choices in the Upstream Market

In the second stage of the game outlined in Section 2.1, the suppliers simultaneously choose the input variety they would like to produce and their investment levels. In this section we analyze their choices first in the case of capacity constraints and then in the case of no capacity constraints.

4.1 Capacity Constraints

Supplier \( S_i \) maximizes its earnings net of investment costs taking \( b_i \), \( b_j \), \( s_j \) and \( e_j \) as given. Its earnings in the input market for any given set of \( b_i \), \( b_j \), \( s_i \), \( s_j \), \( e_i \) and \( e_j \) are presented in Proposition 1. Since the suppliers choose their locations after the buyers, their decisions reflect the extent to which their production is customized to a specific buyer. Their optimal location and investment choices are presented in Proposition 3.
Proposition 3 For given values of \( b_i \) and \( b_j \), \( S_i \)'s equilibrium choices of location and investment are

\[
s_i^c (b_i, b_j) = b_i + \frac{(1 - b_i - b_j)}{3}
\]

(24)

and

\[
e_i^c (b_i, b_j) = \frac{9 - 2(1 - b_i - b_j)^2}{24}.
\]

(25)

Evaluating \( C(i) \) given in (6) at these values reveals that in equilibrium each supplier sells its unit of input in the first stage of the bargaining game. It is more advantageous for the suppliers to trade in the first stage of the bargaining process when they can use the possibility to trade with the alternative buyer in the second stage to strengthen their bargaining position.

The location choice given in Proposition 3 implies that the suppliers always locate between the two buyers. Fully specialized inputs maximize the surplus within the intended interaction while minimizing the value of the outside option. The suppliers pick their locations in the input space strategically in order to improve their bargaining positions. The expression for \( s_i^c (b_i, b_j) \) indicates that while choosing their locations, the suppliers try to balance the value created within the relationship and the value of their outside option. How closely they locate to the buyer with which they intend to trade depends on the degree of product differentiation in the downstream market, which is represented by \( 1 - b_i - b_j \).

Corollary 1 The suppliers choose to produce less specialized inputs as the degree of product differentiation in the downstream market increases.

To illustrate, consider for a moment the scenario where we fix the distance between the supplier and one of the buyers, and increase the degree of product differentiation in the downstream market by moving the other buyer further away. This causes the input of the supplier to become more specific even though the distance between its location and the location of its intended trading partner does not change. The supplier’s input still has the same benefit for the intended buyer, but the payoff the supplier expects to get is lower because
the value of its outside option is reduced. This causes the supplier to re-adjust its location to
compensate for the increase in the specificity of its input. Hence, when the buyers choose to
produce more differentiated products, the suppliers choose to produce less specialized inputs
because they do not want to produce an input type that is too unsuitable for the purposes
of the alternative buyer in the market.

We next consider how the investment choices of the suppliers depend on the product
choices of the buyers. Let $\gamma = (1 - b_i - b_j)$. The partial derivative of $e_i^c(b_i, b_j)$ with respect
to $\gamma$ is

$$\frac{\partial e_i^c(b_i, b_j)}{\partial \gamma} = \frac{(1 - b_i - b_j)}{6} < 0$$

(26)

for $1 - b_j - b_i > 0$. Thus, except for cases when the buyers are located at the same point,
i.e., when $b_i = 1 - b_j$, $S_i$’s investment choice increases as the buyers approach each other.

**Corollary 2** The suppliers’ investment choices increase as the degree of product differentia-
tion in the downstream market decreases.

As the degree of product differentiation decreases, both the distance between $S_i$ and $B_i$,
and the distance between $S_i$ and $B_j$ decreases. This is because after observing $b_i$ and $b_j$,
the suppliers always choose to locate between the two buyers. The decrease in the distance
between $S_i$ and $B_i$ causes the parties to get a higher benefit from the inputs traded. The
decrease in the distance between $S_i$ and $B_j$ causes the outside option of $S_i$ to improve. Hence,
both changes enhance the suppliers’ investment incentives.

Corollary 2 implies that the quality of the inputs to which the buyers have access depends
critically on how differentiated their products are. The firms negotiate the price of the input
after the investment costs are sunk, at which time the input has limited alternative uses.
This leaves the suppliers in a weak bargaining position. A decrease in the degree of product
differentiation between the two final goods results in an increase in the value of the suppliers’
outside options. The suppliers’ chances of recovering the sunk costs of their investment are
better if the products of the buyers are more similar to each other. As the threat of hold-up decreases, the suppliers’ incentives to invest increase.

The finding of Vukina and Leegommonchai (2006) provides an illustration of this result. They analyze the link between asset specificity and underinvestment in the broiler industry and show that underinvestment in this industry is negatively related to the number of processors competing for grower services in a given area. We analyze in Section 5 how the possibility of underinvestment may affect the location decisions of the buyers.

4.2 No Capacity Constraints

If the suppliers are not capacity-constrained, the equilibrium location and investment choices are given in Proposition 4.

**Proposition 4** For given values of $b_i$ and $b_j$, in the unique equilibrium of the game, $S_i$ sets

$$s_i^{nc}(b_i, b_j) = b_i + \frac{(1 - b_i - b_j)}{2}$$

and

$$e_i^{nc}(b_i, b_j) = 1 - \frac{(1 - b_i - b_j)^2}{4}.$$  \hspace{1cm} (27) (28)

$S_j$ chooses $e_j^{nc}(b_i, b_j) = 0$.

Hence, in contrast with the case of capacity constraints, there is only one supplier choosing a positive amount of investment in equilibrium. Both buyers purchase inputs from the same supplier, which chooses to produce an input type that is exactly in the middle between the two buyers’ ideal input types. In the proof of Proposition 4 we show that the other supplier does not find it profitable to enter the market by investing a positive amount. Even in cases when it could enter and produce a fully specialized input type, the buyers prefer to use a common input type that is not fully specialized for their purposes in order to benefit from the increased investment incentives of the single supplier. When both buyers buy inputs from the
same supplier, that supplier has increased incentives to invest because of the high demand it faces.

As in the case of capacity constraints, the supplier’s investment choice increases as the buyers locate close to each other.

\[
\frac{\partial e_i^c(b_i, b_j)}{\partial \gamma} = -\frac{(1 - b_i - b_j)}{2} < 0
\]  

given \(1 - b_j - b_i > 0\). However, unlike the case of capacity constraints, the investment choice is affected by the location choices of the buyers simply because the supplier sells inputs to both of the buyers. The trade partners get a higher benefit from the inputs traded if they are located closer together. Hence, the supplier’s payoff increases as the degree of product differentiation in the downstream market decreases not because its outside option is improved, but because it can benefit more from the units it sells to the buyers.

5 Product Choices in the Downstream Market

In the first stage of the game, the buyers choose their locations to maximize their payoffs. Our goal in this section is to analyze how vertical interactions affect the product choices of buyers of inputs. Again, we first consider the case of capacity constraints and then compare the results with those under no capacity constraints.

5.1 Capacity Constraints

Substituting for the suppliers’ equilibrium location and investment choices from Proposition 3 in \(C (i)\) reveals that this inequality always holds for all \(b_i, b_j \in [0, 1]\). Hence, buyer \(B_i\) maximizes (7) taking \(b_j\) as given. After substituting for \(s_i^c(b_i, b_j)\) and \(e_i^c(b_i, b_j)\) in (7), we get

\[
W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_i^c(b_i, b_j) \left(1 - \frac{1}{2} e_i^c(b_i, b_j) \right) - \frac{1}{4} e_i^c(b_i, b_j) \left(1 - \frac{1}{2} e_i^c(b_i, b_j) \right) .
\]  

(30)
This expression illustrates the several effects that shape the location choices of the buyers. The first two terms, \((p^*_i - c)D_i(p^*_1, p^*_2) - K\), represent the profit level of \(B_i\) if it does not purchase any inputs. It is easy to verify that this is decreasing in \(b_i\). In order to relax the price competition between themselves, the firms would prefer to locate as far from each other as possible. We refer to this as the competition effect.

The third term, \(\frac{1}{2}e^c_i(b_i, b_j) \left(1 - \left((b_i - s_i^c(b_i, b_j))^2\right)\right)\), represents the benefit \(B_i\) gets from using \(S_i\)'s input. This benefit has two components. First, it depends on the level of investment made by \(S_i\). Corollary 2 implies that \(e^c_i(b_i, b_j)\) is increasing in \(b_i\) because an increase in \(b_i\) results in a decrease in the degree of product differentiation in the downstream market. Hence, in order to increase the investment incentives of their suppliers, the buyers prefer to locate closer to each other. This is the investment effect. Suppliers can increase their bargaining power by locating closer to the alternative buyer. However, doing so means they decrease the benefit they get from the buyer they trade with. If this buyer locates closer to the alternative buyer, the supplier benefits without bearing any costs.

The benefit \(B_i\) gets from using \(S_i\)'s input also depends on how suitable \(S_i\)'s input is for \(B_i\)'s purposes, which is represented by \(\left(1 - (b_i - s_i^c(b_i, b_j))^2\right)\). It is easy to verify that this term is increasing in \(b_j\). This specificity effect reflects the fact that the buyers always like to locate close to the suppliers they trade with. They know that for given values of \(b_i\) and \(b_j\), the suppliers always choose locations between them. The specificity effect indicates that for a given value of \(b_j\), as \(b_i\) increases, the distance between \(B_i\) and \(S_i\) decreases and the buyer benefits from using an input which is more suitable for its purposes.

The last term in (30) implies that buyer \(B_i\)'s payoff is decreasing in the outside option of \(S_i\). In order to have a strong bargaining position, \(B_i\) would like the outside option of \(S_i\) to be as low as possible. The value of \(S_i\)'s outside option decreases as the buyers move away from each other, i.e., as \(b_i\) decreases. This is because as \(b_i\) decreases, \(s_i\) also decreases and \(S_i\)'s input becomes less suitable for the needs of the alternative buyer in the market. This bargaining effect causes the buyers to prefer to locate as far from each other as possible.
We now analyze the location choices of the buyers. The location choice of \( B_i \) depends on which of the four effects dominate. Writing (30) more explicitly, we define \( B_i \)’s maximization problem as

\[
\max_{b_i} W_{B_i} = \frac{81 - 4 (1 - b_i - b_j)^4}{864} + \frac{t (3 + b_i - b_j)^2 (1 - b_i - b_j)}{18} - K. \tag{31}
\]

Given the symmetry between the firms, we focus on symmetric equilibrium. Note that when \( t = 0 \), the optimal product choices are \( b^c_i = b^c_j = b^c = \frac{1}{2} \). That is, when the consumers are indifferent about which variety they consume, the firms prefer to locate in the middle. In the proof of Proposition 5, we show that as \( t \) increases, \( b^{nc} \) monotonically decreases. Hence, we can state the following result.

**Proposition 5** Suppose the suppliers face capacity constraints. For sufficiently low values of \( t \), the investment and specificity effects dominate the competition and bargaining effects, and the buyers choose to be less than maximally differentiated.

A low \( t \) value means that the buyers can more easily attract the consumers with the most extreme tastes. This implies that even with maximum differentiation, the buyers face intense competition because the consumers care less about consuming a variety that is different from their ideal one. Proposition 5 states that as \( t \) decreases and the buyers cannot effectively decrease the rivalry between them by locating further away from each other, the investment and specificity effects start to play relatively more important roles in their location decisions. Hence, they may choose less than maximum differentiation in order to increase the investment incentives of the suppliers and in order to be able to use more suitable inputs.\(^{14}\)

\(^{14}\)To isolate the investment and specificity effects, one can consider a limited version of the current model where the suppliers cannot locate anywhere they want along the unit line and they have to locate with one of the buyers. This may be because they need location-specific technological information that the buyers can give to them in order to locate at a certain point. Assuming \( s_i = b_i \), one can show that the buyers would like to approach each other purely in order to improve the outside options of their suppliers.
5.2 No Capacity Constraints

We now consider the buyers’ product choices in the first stage of the game assuming the suppliers do not have capacity constraints. The results in Section 4 indicate that for all \( b_i, b_j \in [0, 1] \), there is only one supplier making a positive amount of investment in equilibrium. Hence, buyers \( B_i \) and \( B_j \) maximize (18) and (19) respectively.

Substituting for the equilibrium location and investment choices reveals that these two payoff functions are symmetric. Buyer \( B_i \) solves

\[
\max_{b_i} W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + e_i^{nc} (b_i, b_j) \left( 1 - (b_i - s_i^{nc} (b_i, b_j))^2 \right) \quad (32)
\]

\[
= \frac{(3 - b_i - b_j)^2 (1 + b_i + b_j)^2}{16} + \frac{t (3 + b_i - b_j)^2 (1 - b_i - b_j)}{18} - K. \quad (33)
\]

The product choice of \( B_i \) is affected by the competition, investment, and specificity effects as specified above. The competition effect is negative while the investment and specificity effects are positive. However, the reason for the investment effect is different in this case from the reason for the investment effect in the case of capacity constraints. Unlike in the case of capacity constraints, a single supplier provides inputs to both buyers. It invests more when the buyers are located closer together because the value created from the use of the inputs is higher.

As in the case of capacity constraints, it is straightforward to show that when \( t = 0 \), the optimal location choices are \( b_i^{nc} = b_j^{nc} = b^{nc} = \frac{1}{4} \). Hence, we again have the result that for sufficiently low values of \( t \), the investment and specificity effects dominate the competition effect.

**Proposition 6** Suppose the suppliers face no capacity constraints. For sufficiently low values of \( t \), the investment and specificity effects dominate the competition effect, and the buyers choose to be less than maximally differentiated.

The proof of Proposition 6 is similar to the proof of Proposition 5 and is omitted. It shows
that as the measure of consumer loyalty, $t$, decreases, the buyers have increased incentives to agglomerate in the middle.

These results indicate that once we extend the classic Hotelling model with quadratic costs to take into account the dynamics of the interactions between upstream suppliers and downstream buyers, buyers dealing with small-scale suppliers which face capacity constraints may choose intermediate locations in the product space in order to improve the outside options of their suppliers. On the other hand, buyers dealing with suppliers that are not capacity-constrained relative to the buyers’ needs may choose intermediate locations in the product space in order to be close to the only supplier in the market which is located in the middle.

6 Conclusion

Under incomplete contracts, the investment decisions of parties depend critically on the specificity of the investments. This paper has investigated the link between investment decisions and product choices of firms in the context of bilateral duopoly. We have considered an environment where buyers care about the suitability of the inputs they use for their own purposes and input prices are determined by bilateral negotiations. While making their investment decisions, suppliers decide both what type of input to produce and how much effort to put into it. The degree of specificity is endogenously determined in the model through the product choices made by all the firms in the market.

We have shown that if the suppliers are capacity-constrained, the degree of product differentiation in the downstream market affects both the location and investment decisions of the suppliers. As the degree of product differentiation increases, the suppliers choose to produce input types that are less suitable for the purposes of the buyers with which they intend to trade. They trade off the value of the input for its intended buyer and the value of the input in its alternative uses. In order to balance these two forces, they refrain from
producing fully specialized inputs.

The suppliers’ investment levels are decreasing in the degree of product differentiation in the downstream market. This implies that the buyers may strategically commit to competition in order to alleviate lock-in.$^{15}$ Producing more similar products improves the bargaining positions and, hence, the investment incentives of their suppliers. Thus, in contrast with the maximum differentiation result of d’Aspremont et al. (1979), they may choose to be less-than-maximally differentiated.

If suppliers are not capacity-constrained, there is only one supplier making a positive amount of investment in the market. The supplier chooses to locate between the two buyers. The buyers may choose intermediate locations to increase the investment incentives of the supplier and to have access to more suitable inputs.

We conclude by mentioning some avenues along which the current analysis can be extended. First, it would be interesting to analyze the capacity choices of the suppliers. In a model that endogenizes the capacity choices of the suppliers, one could explore the effect of bargaining power on capacity choice. Second, we have assumed that the firms divide the gains from trade according to an exogenously determined sharing rule. In a model that endogenizes the sharing rule, one could analyze whether there are cases when either the buyers or the suppliers find it profitable to commit to having low bargaining power.

$^{15}$ Farrell and Gallini (1988) and Shepard (1987) also analyze the strategic use of commitment to competition in order to alleviate lock-in in different settings.
References


Appendix

This Appendix contains the proofs of Propositions 1-5.

1 Proof of Proposition 1

To solve for the subgame perfect equilibrium of the bargaining game, we start with the second stage and work backward. At the end of the first stage, there are 3 types of subgames.

1. Both pairs have reached an agreement in the first stage.
2. Only one of the pairs has reached an agreement in the first stage.
3. Neither pair has reached an agreement in the first stage.

We consider what happens in the second stage of the bargaining game in each type of subgame. In a subgame of type 1, no more trade takes place in the second stage because the suppliers do not have any more units to sell. In a subgame of type 2, only one of the pairs has reached an agreement. Suppose it is $B_i - S_i$. Since $S_i$ does not have any other units to sell in the second stage, $B_j$ cannot obtain any units from it. However, $B_i$ can get another unit from $S_j$.

For $B_i$ and $S_j$, the gains from trade are

$$e_j \left( 1 - (1 - b_i - s_j)^2 \right). \tag{A.1}$$

This is the benefit $B_i$ would get from using $S_j$’s unit of input. $B_i$ and $S_j$ receive the sum of their share of this amount and their outside option. $B_i$’s outside option is determined by the amount it would make in the downstream market without using $S_j$’s input. $S_j$’s outside option is 0 since it cannot find another trading partner after the second stage of the bargaining game.

In a subgame of type 3, both of the suppliers still have one unit to sell in the second stage. Consider the negotiation between $B_i$ and $S_j$ for $i, j = 1, 2$ and $i \neq j$. The gains from trade are as in (A.1). They receive the sum of their share of this amount and their outside option as described above.
We can now analyze the first stage of the bargaining game, where $B_i - S_i$ and $B_j - S_j$ bargain simultaneously. We would like to determine mutual best responses.

Consider the negotiation between $B_i$ and $S_i$. We consider two cases depending on whether or not $B_j$ and $S_j$ trade. Suppose $B_j$ and $S_j$ do not trade. What are $B_i$ and $S_i$’s best responses?

If they do not trade, we have a subgame of type 2 in the second stage, where $B_i$ buys a second unit from $S_j$. $B_i$ and $S_i$’s joint profits in the first stage are

$$W_{B_i} + W_{S_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right) + e_i \left( 1 - (b_i - s_i)^2 \right). \quad (A.2)$$

If they do not trade, we have a subgame of type 3. Their joint profits are

$$W_{B_i} + W_{S_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right) + e_i \left( 1 - (1 - b_j - s_i)^2 \right). \quad (A.3)$$

Subtracting (A.3) from (A.2) gives the gains from trade. Provided that $B_j$ and $S_j$ do not trade, it is a best response for $B_i$ and $S_i$ to trade in the first stage of the bargaining game if

$$e_i \left[ \left( 1 - (b_i - s_i)^2 \right) - \frac{1}{2} \left( 1 - (b_j - s_i)^2 \right) \right] \geq 0. \quad (A.4)$$

Suppose $B_j$ and $S_j$ trade. What are $B_i$ and $S_i$’s best responses? If they trade, we have a subgame of type 1 in the second stage of the bargaining game. $B_i$ and $S_i$’s joint profits are

$$W_{B_i} + W_{S_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + e_i \left( 1 - (b_i - s_i)^2 \right). \quad (A.5)$$

If they do not trade, we have a subgame of type 2 in the second stage, where $S_i$ negotiates with $B_j$ in the second stage and receives $(1/2) e_i \left( 1 - (1 - b_j - s_i)^2 \right)$. $B_i$ does not have any other input supplier it can turn to, so it does not buy any inputs. $B_i$ and $S_i$’s joint profits are

$$W_{B_i} + W_{S_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_i \left( 1 - (1 - b_j - s_i)^2 \right). \quad (A.6)$$

Subtracting (A.6) from (A.5), we get the gains from trade. Provided that $B_j$ and $S_j$ trade, it is a best response for $B_i$ and $S_i$ to trade in the first stage of the bargaining game if (A.4) holds.
We label the inequality in (A.4) as \( C(i) \). Whether or not \( B_j \) and \( S_j \) trade, \( B_i \) and \( S_i \) trade in the first stage of the bargaining game if \( C(i) \) holds. Hence, if both \( C(i) \) and \( C(j) \) hold, then both \( B_i - S_i \) and \( B_j - S_j \) will trade in the first stage of the bargaining game. \( B_i \) can expect payoff so if
\[
W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_i \left[ (1 - (b_i - s_i)^2) - \frac{1}{2} \left( 1 - (1 - b_j - s_i)^2 \right) \right] - \frac{1}{2} \left( 1 - (1 - b_j - s_i)^2 \right)
\]
which is the sum of its share of the gains from trade and its outside option, \((p_i^* - c) D_i (p_1^*, p_2^*) - K\). \( S_i \) can expect payoffs of
\[
W_{S_i} = \frac{1}{2} e_i \left[ (1 - (b_i - s_i)^2) - \frac{1}{2} \left( 1 - (1 - b_j - s_i)^2 \right) \right] + \frac{1}{2} e_i \left( 1 - (1 - b_j - s_i)^2 \right)
\]
\[
= \frac{1}{2} e_i \left[ (1 - (b_i - s_i)^2) + \frac{1}{2} \left( 1 - (1 - b_j - s_i)^2 \right) \right].
\]
We know that if \( B_i \) and \( S_i \) do not trade, we have a subgame of type 1 where \( S_i \) sells its unit of input to \( B_j \) in the second stage of the bargaining game. Hence, its outside option in the first stage is \((1/2) e_i (1 - (1 - b_j - s_i)^2)\). Its payoff is its share of the gains from trade defined in (A.4) and the value of its outside option.

If only \( C(i) \) holds, then \( B_j \) and \( S_j \) do not trade in the first stage of the bargaining game, but \( B_i \) and \( S_i \) do. We have a subgame of type 2 in the second stage of the bargaining game, where \( B_i \) can purchase a second unit from \( S_j \). Hence, \( B_i \)'s payoff is
\[
W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_i \left[ (1 - (b_i - s_i)^2) - \frac{1}{2} \left( 1 - (1 - b_j - s_i)^2 \right) \right] + \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right),
\]
where the last term represents the benefit of the input it purchases from \( S_j \). \( B_j \) has no inputs, so its payoff is
\[
W_{B_j} = (p_j^* - c) D_j (p_1^*, p_2^*) - K.
\]
Since \( S_i \) sells its unit of input to \( B_i \) in the first stage, it makes
\[
W_{S_i} = \frac{1}{2} e_i \left[ (1 - (b_i - s_i)^2) + \frac{1}{2} \left( 1 - (1 - b_j - s_i)^2 \right) \right]
\]
\[
(A.7)
\]
\[
(A.8)
\]
\[
(A.9)
\]
\[
(A.10)
\]
\[
(A.11)
\]
while \( S_j \) makes
\[
W_{S_j} = \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right)
\] (A.12)
since it sells its unit of input to \( B_i \) in the second stage.

If neither \( C(i) \) nor \( C(j) \) holds, then none of the pairs trade in the first stage of the bargaining game and a subgame of type 3 is reached in the second stage. \( B_i \) and \( S_j \)'s payoffs are
\[
W_{B_i} = (p_i^* - c) D_i (p_1^*, p_2^*) - K + \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right)
\] (A.13)
\[
W_{S_j} = \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right).
\] (A.14)

2 Proof of Proposition 2

To solve for the equilibrium of the bargaining game, we start with the second stage and work backward. At the end of the first stage, there are 6 types of subgames.

1. Both pairs have traded 2 units in the first stage.
2. Both pairs have traded 1 unit in the first stage.
3. Both pairs have traded 0 units in the first stage.
4. One of the pairs has traded 2 units and the other one has traded 1 unit in the first stage.
5. One of the pairs has traded 2 units and the other pair has traded 0 units in the first stage.
6. One of the pairs has traded 1 unit and the other pair has traded 0 units in the first stage.

We consider what happens in the second stage of the bargaining game in each type of subgame.

In a subgame of type 1, no more trade takes place in the second stage because each buyer can use at most 2 units of input. In all the other types of subgames, either one or both of the pairs have traded less than 2 units in the first stage of the bargaining game. Since the
suppliers are not capacity-constrained, this implies that the buyers that have less than 2 units can obtain inputs in the second stage. Their ability to do so does not depend on whether or not trade took place between the other pair in the first stage.

Consider the negotiation between $B_i$ and $S_j$ in the second stage. For each unit that they negotiate over, the gains from trade are

$$e_j \left( 1 - (1 - b_i - s_j)^2 \right).$$

Hence, $B_i$ buys enough units from $S_j$ so that it has 2 units in total.

In the first stage of the bargaining game, $B_i - S_i$ and $B_j - S_j$ bargain simultaneously. We would like to determine mutual best responses. Consider the negotiation between $B_i$ and $S_i$. We consider three cases depending on whether $B_j$ and $S_j$ trade 0, 1 or 2 units. Suppose $B_j$ and $S_j$ do not trade. What are $B_i$ and $S_i$’s best responses? $S_i$ can negotiate with and sell 2 units to $B_j$ in the second stage whether or not it trades with $B_i$ in the first stage. It earns $2 \left( \frac{1}{2} \right) e_i \left( 1 - (1 - b_j - s_i)^2 \right) = e_i \left( 1 - (1 - b_j - s_i)^2 \right)$ from doing so. If $B_i$ and $S_i$ do not trade either, $B_i$ can negotiate with $S_j$. The gross benefit it gets from trading with $S_j$ is $e_j \left( 1 - (1 - s_j - b_i)^2 \right)$. Hence, $B_i$ and $S_i$’s joint profits in case of not trading are

$$W_{B_i} + W_{S_i} = (p_i^s - c) D_i (p_1^s, p_2^s) - K + e_j \left( 1 - (1 - b_i - s_j)^2 \right) + e_i \left( 1 - (1 - b_j - s_i)^2 \right).$$

(A.16)

If they trade 1 unit, $B_i$ can obtain one more unit from $S_j$ in the second stage. $B_i$ and $S_i$’s joint profits are

$$W_{B_i} + W_{S_i} = (p_i^s - c) D_i (p_1^s, p_2^s) - K + \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right) + e_i \left( 1 - (1 - b_j - s_i)^2 \right) + e_i \left( 1 - (1 - b_j - s_i)^2 \right).$$

(A.17)

Finally, if they trade 2 units, their joint profits are

$$W_{B_i} + W_{S_i} = (p_i^s - c) D_i (p_1^s, p_2^s) - K + 2e_i \left( 1 - (b_i - s_i)^2 \right) + e_i \left( 1 - (1 - b_j - s_i)^2 \right).$$

(A.18)
Comparing the joint profits, we can see that $B_i - S_i$ trade 2 units if

$$e_i \left(1 - (b_i - s_i)^2\right) - \frac{1}{2} e_j \left(1 - (1 - b_i - s_j)^2\right) \geq 0$$

(A.19)

and trade 0 units if (A.19) does not hold. (A.19) implies that since $B_i$ always has the option of trading with $S_j$, trade takes place between $B_i$ and $S_i$ if it creates a larger surplus than what $B_i$ can get by trading with $S_j$.

A similar analysis reveals that the best response of $B_i$ and $S_i$ in the cases when $B_j$ and $S_j$ trade 1 or 2 units is determined by (A.19) also. Hence, we find that for every possible strategy of $B_j$ and $S_j$, $B_i$ and $S_i$ trade 2 units if (A.19) holds and 0 units if it does not.

We label the inequality in (A.19) as $NC(i)$. If both $NC(i)$ and $NC(j)$ hold, then both pairs will trade 2 units in the first stage of the bargaining game. The payoffs of $B_i$ and $S_i$ are

$$W_{B_i} = (p_i^* - c) D_i(p_i^*, p_j^*) - K + e_i \left(1 - (b_i - s_i)^2\right) - \frac{1}{2} e_j \left(1 - (1 - b_i - s_j)^2\right)$$

(A.20)

$$W_{S_i} = e_i \left(1 - (b_i - s_i)^2\right) - \frac{1}{2} e_j \left(1 - (1 - b_i - s_j)^2\right).$$

(A.21)

If only $NC(i)$ holds, then $B_j$ and $S_j$ do not trade in the first stage of the bargaining game. The payoffs of the firms are

$$W_{B_i} = (p_i^* - c) D_i(p_i^*, p_j^*) - K + e_i \left(1 - (b_i - s_i)^2\right) + \frac{1}{2} e_j \left(1 - (1 - b_i - s_j)^2\right)$$

(A.22)

$$W_{B_j} = (p_j^* - c) D_j(p_i^*, p_j^*) - K + e_j \left(1 - (1 - b_j - s_i)^2\right)$$

(A.23)

$$W_{S_i} = e_i \left(2 - (b_i - s_i)^2 - (1 - b_j - s_i)^2\right) - \frac{1}{2} e_j \left(1 - (1 - b_i - s_j)^2\right)$$

(A.24)

$$W_{S_j} = 0.$$ 

(A.25)

If neither $NC(i)$ nor $NC(j)$ holds, then none of the pairs trades in the first stage of the
bargaining game. \(B_i\) and \(S_j\)'s payoffs are

\[
W_{B_i} = (p^*_i - c) D_i (p^*_1, p^*_2) - K + e_j \left( 1 - (1 - b_i - s_j)^2 \right) \tag{A.26}
\]

\[
W_{S_i} = e_i \left( 1 - (1 - b_j - s_i)^2 \right). \tag{A.27}
\]

3 Proof of Proposition 3

Supplier \(S_i\) has two options. It can either sell its input to buyer \(B_i\) in the first stage or to buyer \(B_j\) in the second stage. It chooses the option that yields the higher payoff.

If it serves \(B_i\), then its payoff function is given by (A.8). Taking the partial derivatives with respect to \(s_i\) and \(e_i\) and solving the first-order conditions give

\[
s_i = b_i + \frac{(1 - b_i - b_j)}{3} \tag{A.28}
\]

and

\[
e_i = \frac{9 - 2 (1 - b_i - b_j)^2}{24}. \tag{A.29}
\]

It is straightforward to check that the second-order condition is satisfied. Substituting these expressions in \(C(i)\) we can see that it holds. If \(S_i\) chooses these location and investment levels, its payoff is

\[
W_{S_i} = \frac{\left[ 9 - 2 (1 - b_i - b_j)^2 \right]^2}{576}. \tag{A.30}
\]

If \(S_i\) chooses \(s_i\) and \(e_i\) such that \(C(i)\) is violated, it serves \(B_j\) and its payoff function is given by

\[
W_{S_i} = \frac{1}{2} e_i \left( 1 - (1 - b_j - s_i)^2 \right) - e_i^2. \tag{A.31}
\]

Taking the partial derivatives with respect to \(s_i\) and \(e_i\) and solving the first-order conditions give

\[
s_i = 1 - b_j \tag{A.32}
\]

and

\[
e_i = \frac{1}{4}. \tag{A.33}
\]
Substituting these expressions in $C (i)$ we can see that it does not hold if $\frac{1}{2} - (1 - b_j - b_i)^2 < 0$. If $C (i)$ holds, then $S_i$ can choose $s_i$ and $e_i$ such that $C (i)$ does not hold. Doing so would yield a lower payoff than choosing (A.32) and (A.33). Hence, if $S_i$’s payoff when it chooses (A.32) and (A.33) is lower than (A.30), then the optimal choices are (A.28) and (A.29).

Substituting for the location and investment choices given by (A.32) and (A.33) in (A.31) gives $W_{S_i} = \frac{1}{16}$. This payoff is lower than the one in (A.30) if $\left[9 - 2 (1 - b_i - b_j)^2\right]^2 > 36$. This inequality always holds since $b_i, b_j \in [0, 1]$.

4 Proof of Proposition 4

We proceed in two steps.

(i) We first show that the location and investment choices given in Proposition 4 constitute an equilibrium. We do this by arguing that neither firm has an incentive to deviate.

In equilibrium, supplier $S_i$ earns the maximum possible level of profit by being the sole supplier of inputs to both of the buyers. Hence, it cannot do any better by deviating. $S_j$, on the other hand, makes 0. We need to check whether it can earn a positive profit by setting $s_j$ and $e_j$ such that it trades with buyer $B_j$ only, with buyer $B_i$ only, or with both of the buyers.

If it trades with $B_j$, its payoff is

$$W_{S_j} = e_j \left(1 - (b_j - s_j)^2\right) - \frac{1}{2} \left(1 - (1 - b_j - s_i)^2\right) - e_j^2. \quad (A.34)$$

The unconstrained optimum is $s_j = b_j$ and $e_j = 1/2$. For $B_j$ to prefer to trade with $S_j$, $NC (j)$ must be holding. Given the equilibrium strategy of $S_i$, this condition is satisfied. Substituting for the relevant choices of $s_i, s_j, b_i$ and $b_j$, we find that $S_j$ makes

$$W_{S_j} = \frac{1}{4} - \frac{\left(4 - (1 - b_i - b_j)^2\right)^2}{32}, \quad (A.35)$$

which is negative for all $b_i, b_j \in [0, 1]$. Hence, this is not a profitable deviation.

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If it trades with $B_i$, it makes

$$W_{S_j} = e_j \left( 1 - (1 - b_i - s_j)^2 \right) - e_j^2. \quad (A.36)$$

The location and investment choices that maximize this payoff function are $s_j = 1 - b_i$ and $e_j = 1/2$. For $B_i$ to prefer to buy from $S_j$ instead of from $S_i$, it must be the case that $NC\ (i)$ is violated. That is,

$$e_i \left( 1 - (b_i - s_i)^2 \right) - \frac{1}{2} e_j \left( 1 - (1 - b_i - s_j)^2 \right) < 0. \quad (A.37)$$

Given the equilibrium strategy of $S_i$, this condition is not satisfied at the unconstrained optimum stated above. Supplier $S_j$ can choose $s_j$ and $e_j$ such that this condition is just satisfied. The constrained optimal choices are $s_j = 1 - b_i$ and $e_j = \left( 4 - (1 - b_i - b_j)^2 \right)^2 / 8$.

Evaluating $S_j$’s profit level at these choices, we get

$$W_{S_j} = \left( \frac{4 - (1 - b_i - b_j)^2}{8} \right) \left( 1 - \frac{4 - (1 - b_i - b_j)^2}{8} \right). \quad (A.38)$$

Again, since this is negative for all $b_i, b_j \in [0, 1]$, we do not have a profitable deviation.

Finally, $S_j$ can set $s_j$ and $e_j$ such that both buyers prefer to trade with it. This implies that its choices must violate $NC\ (i)$ and satisfy $NC\ (j)$. $S$’s payoff is given by

$$W_{S_j} = e_j \left( 2 - (b_j - s_j)^2 - (1 - b_i - s_j)^2 \right) - \frac{1}{2} e_i \left( 1 - (1 - b_j - s_i)^2 \right) - e_j^2.$$

Note that if we evaluate $NC\ (i)$ and $NC\ (j)$ at the unconstrained optimum of this payoff function, both of them would be satisfied. Hence, $B_i$ would prefer to trade with $S_i$ instead of $S_j$.

We proceed by considering values of $s_j$ and $e_i$ such that $NC\ (i)$ is just violated. Note that if we cannot find a profitable deviation that satisfies only one of the constraints, we cannot find a profitable deviation that satisfies both constraints. $S_j$ must set

$$e_j = \frac{\left( 4 - (1 - b_i - b_j)^2 \right)^2}{8 \left( 1 - (1 - b_i - \tilde{s}_j)^2 \right)}. \quad (A.39)$$
where $s_j$ represents the location choice that solves the constrained optimization problem. These choices of $s_j$ and $e_j$ constitute a profitable deviation if they result in a positive profit level. Substituting for the expression for $e_j$ in $S_j$’s payoff function we get

$$W_{S_j} = \frac{\left(4 - (1 - b_i - b_j)^2\right)^2}{8} \left[ \frac{(1 - (b_j - \tilde{s}_j)^2)}{(1 - (1 - b_i - \tilde{s}_j)^2) + \frac{3}{4} \left(4 - (1 - b_i - b_j)^2\right)^2} \right].$$

This expression is $\leq 0$ for all $\tilde{s}_j, b_i, b_j \in [0, 1]$.

Hence, we do not have any profitable deviations. The location and investment choices stated in Proposition 4 constitute an equilibrium.

(ii) We next show that the equilibrium stated in Proposition 4 is unique. To do this, we need to eliminate all other possible equilibria.

We start by establishing that in equilibrium, $NC(i)$ for $i = 1, 2$ never bind. To prove by contradiction, suppose not and suppose it is $NC(1)$ that binds in equilibrium. Then $S_2$ can increase its investment by $\varepsilon$ and cause a discontinuous upward jump in its payoff function. Since the cost of investment is continuous in the level of investment, there always exists a sufficiently small investment level that makes such a deviation profitable.

This result implies that we can focus on the interior solutions to the suppliers’ optimization problem as candidate equilibria. There are three types of candidate equilibria. We have already shown that one of them is an equilibrium. To eliminate the other two, it is sufficient to find a profitable deviation in each case.

Consider the candidate equilibrium where $S_i$ trades with $B_i$ for $i = 1, 2$ in the first stage of the bargaining game. The optimum choices are $s_i = b_i$ and $e_i = 1/2$. Note that these location and investment choices imply that $NC(i)$ is satisfied. Each supplier makes

$$W_{S_i} = \frac{1}{4} - \frac{(1 - (b_i - b_j)^2)}{4}.$$  

(A.41)

Now suppose $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such
that it serves both buyers. Does this result in a higher payoff level? It must be the case that NC (1) is violated and NC (2) holds. $S_2$ cannot set $s_2$ and $e_2$ at their unconstrained maximum levels since doing so does not violate NC (1). If it chooses $s_2$ and $e_2$ such that NC (1) is just violated, we get $e_2 = 1/\left(1 - (1 - b_1 - \tilde{s}_2)^2\right)$, where $\tilde{s}_2$ represents the constrained optimal choice for location. $S_2$ makes

$$W_{S_2} = \frac{1 - (b_2 - \tilde{s}_2)^2}{\left(1 - (1 - b_1 - \tilde{s}_2)^2\right)} + 1 - \frac{1 - (1 - b_1 - b_2)^2}{4} - \frac{1}{\left(1 - (1 - b_1 - \tilde{s}_2)^2\right)^2}. \quad (A.42)$$

Since it is sufficient to find only one profitable deviation, let us set $\tilde{s}_2 = 1/2$. The payoff function given in (A.42) always yields a higher value than the payoff function given in (A.41) for all $b_i, b_j \in [0, 1]$.

Now consider the case when $S_i$ trades with $B_j$ for $i, j = 1, 2$ and $i \neq j$ in the second stage of the bargaining game. The optimum location and investment choices are $s_i = 1 - b_j$ and $e_i = 1/2$. Each firm makes $W_{S_i} = 1/4$. Suppose $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such that it serves both buyers. It sets $s_2 = b_2 + (1 - b_1 - b_2)/2$ and $e_2 = 1 - (1 - b_1 - b_2)^2/4$. Note that NC (1) is violated and NC (2) is satisfied at these location and investment choices. Does $S_2$ earn a higher payoff level? Its payoff if it sells to both buyers is

$$W_{S_2} = \frac{\left(4 - (1 - b_i - b_j)^2\right)^2}{16} - \frac{1}{4}. \quad (A.43)$$

This is larger than $1/4$ for all $b_i, b_j \in [0, 1]$.

Hence, the equilibrium where $S_i$ sells to both buyers and $S_j$ sets $e_j = 0$ is the unique equilibrium.

5 Proof of Proposition 5

We can show that as $t$ increases from 0, $b^c$ changes in a monotonic way by using the implicit function theorem. Let $G(b_i, b_j; t)$ stand for the first derivative of $W_{B_i}$ with respect to $b_i$. It
is equal to
\[
\frac{(1 - b_i - b_j)^3 - 3t (3 + b_i - b_j)(1 + 3b_i + b_j)}{54}.
\] (A.44)

This expression is equal to 0 at an interior solution. We can solve for the symmetric equilibrium, \( b^c \), by setting \( b_i = b_j = b^c \). The implicit function theorem states
\[
\frac{\partial b^c}{\partial t} = -\frac{\partial G(b_i, b_j; t) / \partial t}{\partial G(b_i, b_j; t) / \partial b^c},
\] (A.45)

where the right hand side is evaluated at \( b^c \). Since the second-order condition is negative, the sign of \( \partial b^c / \partial t \) is equal to the sign of the numerator. In the numerator we have
\[
\frac{\partial G(b_i, b_j; t)}{\partial t} = -\frac{(3 + b_i - b_j)(1 + 3b_i + b_j)}{18} < 0.
\] (A.46)

Hence, \( b^c \) is a decreasing function of \( t \). As \( t \) increases from 0, \( b^c \) decreases from 1/2 to 0 in a continuous fashion.