ISSN 0819-2642
ISBN 0 7340 2616 1

THE UNIVERSITY OF MELBOURNE

DEPARTMENT OF ECONOMICS

RESEARCH PAPER NUMBER 959

FEBRUARY 2006

TESTING FOR RATE-DEPENDENCE AND ASYMMETRY IN INFLATION UNCERTAINTY: EVIDENCE FROM THE G7 ECONOMIES

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Testing for Rate-Dependence and Asymmetry in Inflation Uncertainty: Evidence from the G7 Economies*

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Abstract

The Friedman-Ball hypothesis implies a link between the inflation rate and inflation uncertainty. In this paper we employ a new test for the joint null hypothesis of no dependence effects and no asymmetry in the G7 inflation volatility. The results show that higher inflation rates operate additively via the conditional variance of inflation to induce greater inflation uncertainty in the U.S., U.K. and Canada. In addition, positive inflationary shocks are found to generate greater inflation uncertainty than negative shocks of a similar magnitude in the U.K. and Canada.

Keywords: Friedman-Ball hypothesis, Asymmetry, Davies' Problem

J.E.L. Codes: E390

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* This research was funded by the Australian Research Council under the Discovery-Projects scheme grant number DP0664286
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1. Introduction

There exists an extensive literature arguing that inflation volatility may be positively correlated with the average rate of inflation.[1] Should such a relation exist, high inflation is likely to be associated with reduced welfare and possibly even lower output growth, a view popularized by Friedman (1977). Ball (1992) provides a formal model of Friedman's hypothesized causal link between inflation and inflation uncertainty (the Friedman-Ball rate dependence hypothesis, henceforth).

Much of the empirical work that tests the Friedman-Ball hypothesis employs generalized autoregressive conditional heteroscedasticity (GARCH) models.[2] These models offer a direct test for the statistical significance of the time variation of inflation's conditional variance (Engle 1982); the causal relationship between inflation and inflation uncertainty, rate dependence (Grier and Perry, 1998); and differences in the response of inflation uncertainty to the sign and size of inflationary shocks, an asymmetry effect (Daal et al., 2005). Cox, Ingersoll and Ross (1985) provide two models for inflation dynamics both of which imply rate dependence; a positive correlation between the level of the rate of inflation and the volatility of inflation.

In this paper, we present a new specification for rate dependence and asymmetry in inflation uncertainty and employ a diagnostic test developed by Henry et al. (2004) to detect these empirical features. Rate dependence is explicitly parameterized in our specification with the lagged rate of inflation entering as an explanatory variable in the conditional variance equation. With the exception of Kontonicas (2004), the parametric specification of rate dependence is rarely considered in the inflation literature where rate dependence is commonly tested in a Granger causality framework where the relationship between the conditional variance of inflation and inflation rates is specified as a vector autoregression.[3] Asymmetry in inflation uncertainty arises when positive innovations to inflation have a larger impact on inflation uncertainty than negative innovations of equal absolute magnitude. Our specification and diagnostic testing procedure explicitly allows for both asymmetry and rate dependence.

An application of the test to the G7 inflation rates reveals that monthly inflation in the U.S., U.K. and Canada displays significant rate dependence, while asymmetric inflation uncertainty can only be detected in the U.K and Canadian inflation rates. The estimated inflation model, which allows for additive rate dependence and asymmetry, confirms these results.

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[1] Okun (1971) was one of the first papers to find evidence of a positive correlation between inflation variability and the average rate of inflation. Subsequent papers have yielded mixed results; see Davis and Kakago (2000).

[2] Nas and Perry (1998), Fountas (2001), Hwang (2001), Apergis (2004) and Kontonicas (2004) employ GARCH models in studying the relationship between inflation and its uncertainty.

[3] See, amongst others, Grier and Perry (1998) and Daal et al. (2005).

2. A test for level dependence and asymmetric inflation uncertainty

Consider the following model for inflation dynamics

$$\begin{gathered} y_t = \omega + \varepsilon_t \\ \varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t) \\ h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b\hat{y}_{t-1}^{\delta} + \alpha_2 \eta_{t-1}^2 \end{gathered} \tag{2}$$

where $\beta + \alpha_1 < 1$, β , $\alpha_i, b > 0$ for i = 0, 1 and 2, Ω_{t-1} is the information set at time *t-1* and $\eta_{t-1} = \max(0, \varepsilon_{t-1})$ Equation (2) captures the possibility of an asymmetric impact of positive inflationary shocks on inflation using the Glosten et al. (1993) threshold GARCH model where the magnitude of positive innovations on the conditional variance is $\alpha_1 + \alpha_2$ as compared with α_1 for negative innovations. The rate dependence is parameterized as $b\hat{y}_{t-1}^{\delta}$. The degree of dependence of inflation uncertainty on inflation rates is governed by both parameters b and δ.

In testing for the rate dependence of inflation uncertainty, the test statistic under the null of no rate dependence (*b=0*) does not follow a standard distribution because of the unidentified nuisance parameter δ (see Davies, 1987). Henry et al. (2004) show how a first order Taylor series approximation around δ can circumvent the nuisance parameter problem. Using the Lagrange Multiplier (LM) principle, they show that the joint test statistic for the null of no rate dependence and asymmetry is asymptotically equivalent to the $T \cdot R^2$ from the auxiliary regression of $\left(\varepsilon_t^2 / \tilde{h}_t - 1\right)$ on a vector of regressors comprising the GARCH terms $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1}$, $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \varepsilon_{t-i}^2$ and $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \tilde{h}_{t-i}$, the linearised terms $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \hat{y}_{t-i}^{\delta *} (1 - \delta^* \ln \hat{y}_{t-i})$ and $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \hat{y}_{t-i}^{\delta *} \ln y_{t-i}$, and the asymmetry term $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \eta_{t-i}^2$.[4] Here T is the sample size and R^2 is the coefficient of determination from the auxiliary regression. δ^*={0.5, 1.0, 1.5, 2.0} is an approximation of the true parameter δ and $\tilde{h}_t$ is the GARCH(1,1) specification under the null. The test statistic is asymptotically distributed as a Chi-square variate with three degrees of freedom. The test for the null of no rate dependence can be performed in a similar fashion by omitting the asymmetric term in equation (1) with the resulting LM test statistic distributed as a Chi-square variate with two degrees of

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[4] The summations arise from the sequential backward substitution of h_{t-1}. In practice, $\hat{\varepsilon}_t$ is obtained from the residuals of the regression of y_t on a constant.

freedom (see Henry and Suardi, 2004). Following Eitrheim and Teräsvirta (1996) to adjust for possible distortion in the test statistic's nominal size, Henry et al. (2004) propose running a regression of $\left(\varepsilon_t^2 / \tilde{h}_t - 1\right)$ on the GARCH terms $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1}$, $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \varepsilon_{t-i}^2$ and $1/\tilde{h}_t \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \tilde{h}_{t-i}$ and using the resulting residuals $\{\hat{e}_t\}_{t=1}^{T}$ to run the auxiliary regression.[5]

3. Results

Monthly inflation rates are calculated by taking log differences of the consumer price index (CPI) obtained from the International Financial Statistics (IFS) CD-ROM. The US CPI starts from 1950:01, Italy (IT) and Germany (GM) start from 1951:01, Canada (CA) and Japan (JP) start from 1957:01, and the UK and France (FR) start from 1960:01. All data ends in 2004:06 thus yielding a sample with more than 500 observations in each country.

Results of the diagnostics tests are presented in table 1. Panel A of table 1 shows summary statistics of the G7 inflation rates. Italy has the highest average monthly inflation rate of 0.53%, while Germany has the lowest. Japan's monthly inflation rate shows the greatest variability followed by the U.K. and Italy. All inflation rates are found to be stationary at 5% significance level when tested with the Augmented Dickey Fuller (ADF) unit root tests.[6]

Panel B reports evidence of time variation in the conditional variance of the respective G7 inflation rates. The ARCH test for the null of no serial correlation in the squared residuals up to the 12th order lag is rejected at all levels of significance. The test for rate dependence in inflation uncertainty shows that only U.S., U.K. and Canadian inflation volatility appear to be significantly correlated with the inflation rate. Likewise, the results for the joint test indicate that the null of no rate dependence and no asymmetry in inflation uncertainty is rejected for the U.K., U.S. and Canada for all δ^* values and at the 10% significance level.

An AR(p)MA(1,12)-GARCH(1,1) model with level dependence and asymmetry is estimated for each country:

$$y_t = \theta_0 + \sum_{i=1}^{p} \theta_i y_{t-i} + \varepsilon_t + \rho_1 \varepsilon_{t-1} + \rho_{12} \varepsilon_{t-12}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b \hat{y}_{t-1}^{\delta} + \alpha_2 \eta_{t-1}^2 \qquad (3)$$

[5] The size bias corrected empirical critical values for both the joint test and level dependence test are shown to approximate a Chi-square distribution with three and two degrees of freedom for all levels of significance respectively.

[6] Results for unit root tests are not reported here but are available upon request from the authors.

where the variables are defined as in equation (2). Following Daal et al. (2005), we include the MA(1,12) process which serves to provide a parsimonious ordering of the AR process and to account for possible seasonality in the data. We employ the Akaike Information Criteria (AIC) to determine the optimal lag length for the AR process. To conserve space, we only report the coefficient estimates of the conditional variance specifications in Table 2.[7]

The parameters α_1 and β_1 are significant for all countries with the lowest persistence in the conditional variance reflected in Germany's inflation uncertainty. There is little evidence of asymmetric volatility in these data. The estimates of α_2 are only significant for the U.K. and Canada. The magnitude of $\hat{\alpha}_2$ for Canada suggests that such asymmetry is unlikely to be economically important.

The parameters b and δ, which capture rate dependence in inflation, are significant for the U.K., U.S. and Canada at the 1% significance level. Given the problems of unidentified nuisance parameters, Davies' (1987) bound approach is employed to determine the significance of *b*, details of which are provided in the appendix. Of the three countries that display rate dependence in inflation, the impact of inflation rates on inflation uncertainty may be greatest for the U.K. given the estimated values of b and δ.

Our results for inflation rate dependence differ to some extent from those of Daal et al. (2005) who found evidence to support the Friedman-Ball hypothesis for all countries except Germany. The contrasting results are most likely due to the difference in the econometric specification and tests employed in this study.

4. Concluding Remarks

This paper examines the Friedman-Ball hypothesis, which suggests a causal link between the inflation rate and inflation uncertainty, allowing for an asymmetric response in inflation uncertainty to positive and negative shocks. We employ a recently developed test for the null hypothesis of no rate dependence and/or asymmetry in the variance of the G7 inflation rates and estimate a specification that allows for these features in the data. We find that rate dependence is present in U.S., U.K. and Canadian inflation rates, while asymmetric inflation uncertainty is prevalent only in the U.K. and Canadian inflation rates.

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[7] Results for the mean specification estimates are available from the authors upon request. The MA(1,12) coefficients are significant at 1% level implying the presence of seasonal effects in the data for all countries. The autoregressive coefficients are also largely significant at 5% level of significance.

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Table 1: Summary Statistics and Diagnostic Tests of Inflation

Panel A: Summary Statistics

Countries	US	FR	UK	IT	JP	CA	GM
Mean	0.32	0.41	0.51	0.53	0.31	0.35	0.24
Standard deviation	0.32	0.44	0.64	0.56	0.73	0.39	0.34
Minimum	-0.46	-0.86	-1.63	-0.86	-1.56	-0.86	-1.66
Maximum	1.79	3.28	4.22	3.10	4.10	2.59	1.71
Panel B: Diagnostic Tests							
ARCH(12)	102.010	97.668	118.439	116.545	74.888	115.900	65.327
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Level Dependence Test							
$\delta^* = 0.5$	5.899	1.0561	15.828	1.8387	2.6895	9.3573	0.7248
	[0.0524]	[0.6178]	[0.0004]	[0.3987]	[0.2606]	[0.0092]	[0.6959]
$\delta^* = 1.0$	7.9310	1.3073	13.080	2.8263	0.8662	6.6838	3.9464
	[0.0189]	[0.5200]	[0.0014]	[0.2433]	[0.6484]	[0.0353]	[0.1390]
$\delta^* = 1.5$	5.126	1.9937	20.194	0.8254	1.0943	7.3869	1.0880
	[0.0771]	[0.3690]	[0.0000]	[0.6618]	[0.5785]	[0.0248]	[0.5804]
$\delta^* = 2.0$	5.4151	4.1177	19.7575	0.3589	1.4970	9.0926	0.9728
	[0.0513]	[0.1276]	[0.0000]	[0.8357]	[0.4730]	[0.0106]	[0.6014]
Joint Test for Level Dependence and Asymmetry							
$\delta^* = 0.5$	6.333	1.159	15.996	2.561	3.305	11.989	3.7576
	[0.0964]	[0.5599]	[0.0018]	[0.4643]	[0.3469]	[0.0074]	[0.2888]
$\delta^* = 1.0$	11.986	1.851	12.763	8.488	4.728	9.565	2.6252
	[0.0074]	[0.6038]	[0.0051]	[0.0369]	[0.1927]	[0.0226]	[0.4531]
$\delta^* = 1.5$	10.197	1.872	20.551	8.276	5.509	8.991	1.5271
	[0.0169]	[0.5992]	[0.0001]	[0.0406]	[0.1192]	[0.0294]	[0.6760]
$\delta^* = 2.0$	7.8603	4.1118	20.4619	3.3123	5.5082	10.7546	0.3553
	[0.0489]	[0.2496]	[0.0001]	[0.3459]	[0.1208]	[0.0131]	[0.9493]

Notes: Figures reported in [.] are p-values. The level effect test statistic tests under the null of no levels effect is distributed as Chi-square with 2 degrees of freedom for all δ^* values. The joint test statistic under the null of no levels effect and no asymmetry is distributed as Chi-square with 3 degrees of freedom.

Table 2: Estimates of the Conditional Variance Equation with Level Dependence and Asymmetry

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b\hat{y}_{t-1}^{\delta} + \alpha_2 \eta_{t-1}^2$$

	US	FR	UK	IT	JP	CA	GM
α_0	3.42E-07*	1.63E-07	1.85E-07*	2.53E-08*	1.47E-06	4.71E-0.6*	2.01E-07*
	(1.01E-07)	(1.25E-07)	(4.01E-08)	(1.11E-0.8)	(3.51E-05)	(2.15E-0.6)	(1.10E-08)
β	0.819*	0.956*	0.954*	0.876*	0.774*	0.499*	0.356*
	(0.069)	(0.025)	(0.001)	(0.029)	(0.0007)	(0.131)	(0.172)
α_1	0.065*	0.035*	0.017*	0.107*	0.021*	0.151*	0.210*
	(0.023)	(0.012)	(0.002)	(0.027)	(0.0009)	(0.061)	(0.101)
b	0.011*	1.21E-05	0.034*	0.006	0.001	0.014*	2.31E-06
	(0.006)	(0.03)	(0.015)	(0.005)	(0.0008)	(0.006)	(0.078)
δ	1.077*	0.586	1.403*	1.621	1.557	1.545*	0.376
	(0.341)	(2.370)	(0.235)	(1.217)	(1.150)	(0.376)	(1.283)
α_2	0.003	0.004	0.011*	1.79E-03	1.18E-04	0.001*	2.31E-05
	(0.047)	(0.044)	(0.005)	(1.21E-03)	(1.97E-03)	(0.0003)	(1.74E-04)
$H_0 : b = 0$	P(LR>37.371) =6.250E-08	P(LR>5.250) =0.154	P(LR>56.711) =2.961E-12	P(LR>5.971) =0.113	P(LR>3.714) = 0.294	P(LR>39.784) =1.184E-08	P(LR>1.563) =0.667

Notes: Standard errors are reported in (.).* indicates that the coefficient is significant at 5% significance level. The conditional variance specification is defined in equation (2).

Appendix to Davies' (1987) Upper-Bound Test

Let δ be a vector of dimension v from some parameter space Ω that is identified under the alternative hypothesis. The likelihood ratio statistic as a function of δ is

$$LR(\delta) = 2[\ln L_1(\delta_1) - \ln L_0(\delta_0)], \tag{A1}$$

where $L_1(\delta_1)$ denotes the likelihood value of the objective function evaluated at δ_1 which is the estimated δ value under the alternative hypothesis, and $L_0(\delta_0)$ is the maximum likelihood value derived under the null hypothesis (when δ is not identified). Further assume that δ^* is the argmax of $L_1(\delta)$ such that the likelihood function under the alternative hypothesis evaluated at δ^* is denoted by $L_1(\delta^*)$ then

$$\sup_{\delta \in \Omega} LR(\delta) = 2[\ln L_1(\delta^*) - \ln L_0(\delta^*)]. \tag{A2}$$

Let Q be the empirically observed value of $2[\ln L_1(\delta^*) - \ln L_0(\delta^*)]$. Davies (1987) shows that the significance of Q has an upper bound given by

$$P\left[\sup_{\delta \in \Omega} LR(\delta) > Q\right] \le \Pr\left[\chi_v^2 > Q\right] + G \cdot Q^{(v-1)/2} \cdot \exp^{-(Q/2)} \cdot \frac{2^{-v/2}}{\Gamma(v/2)} \tag{A3}$$

where $\Gamma(\cdot)$ denotes the gamma function, G is defined as

$$G = \int_{\delta_U}^{\delta_L} \left| \frac{\partial LR(\delta)^{1/2}}{\partial \delta} \right| d\delta$$
$$= \left|\mathrm{LR}(\delta_1)^{1/2} - \mathrm{LR}(\delta_\mathrm{L})^{1/2}\right| + \left|\mathrm{LR}(\delta_2)^{1/2} - \mathrm{LR}(\delta_1)^{1/2}\right| + \ldots + \left|\mathrm{LR}(\delta_\mathrm{U})^{1/2} - \mathrm{LR}(\delta_\mathrm{n})^{1/2}\right|, \tag{A4}$$

and $\delta_L, \delta_1, ..., \delta_n, \delta_U$ are the turning points of LR(δ). By assuming that there is a single peak in the likelihood ratio function, Davies shows that G simplifies to $2Q^{1/2}$ which in turn simplifies (A3) to

$$P\left[\sup_{\delta \in \Omega} LR(\delta) > Q\right] \le \Pr\left[\chi_v^2 > Q\right] + Q^{v/2} \cdot \exp^{-(Q/2)} \cdot \frac{2^{-v/2}}{\Gamma(v/2)}. \tag{A5}$$

Note that v =1 in our case since δ is the only unidentified parameter.