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Inequality Aversion and the Optimal Composition of Government Expenditure*

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Abstract

This paper examines the choice of government expenditure on public goods and transfer payments, in the form of a pension, in an overlapping generations model. Government expenditure is tax-financed on a pay-as-you-go basis. A utilitarian judge chooses expenditures to maximize a social welfare function. The nonlinear solution is found to involve the ratio of a welfare-weighted average income, which depends on the inequality aversion of the judge, to arithmetic mean income. An approximation for this ratio is found which produces explicit solutions for the optimal composition. The result is used to obtain an indication of 'implicit' inequality aversion for a range of countries.

JEL code: D72, H41, H53, H11

Keywords: Overlapping Generations,Composition of Government Expenditure, Utilitarian Social Welfare Function

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1 Introduction

A number of recent studies emphasise the importance of unobserved cultural factors in shaping tax and expenditure policies. Alesina and Glaeser (2004), on the basis of cross-national survey data, conjecture that different preferences for a more equal income distribution in Europe and the U.S. may play an important role in accounting for the considerable differences in their expenditure policies. Corneo and Grüner (2002), using a large international survey, present empirical evidence of systematic differences across countries in the preference for redistributive policies. To understand the way in which different views regarding inequality may influence the composition of expenditure, this paper carries out a normative exploration of the expenditure mix which would be chosen by a utilitarian judge having a degree of inequality aversion.

The framework of analysis is that of the choice of expenditure on non-rival and non-excludable public goods and transfer payments, in the form of an unconditional pension, in a growing economy with overlapping generations. Expenditure is tax-financed on a pay-as-you-go basis. The utilitarian judge is regarded as maximising a social welfare function expressed in terms of the utilities of heterogenous individuals, subject to the pay-as-you-go financing constraint. By taking a growth context and focusing on the expenditure composition for a given tax rate,[1] the model also allows for a range of factors other than the judge's inequality aversion which may contribute to the division of expenditures, such as consumers' preference, tax rate, interest rate, and the growth rates of income and population. The transfer payment in the model is referred as a pension, since it is received in the second period of life. However, it may be thought of more broadly as a standard type of income transfer since it involves a decision regarding income shifting between periods within the life cycle as well as intra- and inter-generational

[1] Since the emphasis of the present study is on the composition of expenditure, rather than its total, the income tax rate is assumed to be exogenously fixed. In practice, tax and expenditure policies are debated separately.

redistribution.

The optimal expenditure ratio, defined as the ratio of transfer payments (here the pension) to public goods expenditure, is shown to depend on a welfare-weighted mean income in relation to arithmetic mean income, along with individuals' preferences for public goods, the discount rate, population and income growth rates, and the tax rate. However, a closed-form solution for the expenditure ratio is not available, as the welfare-weighted mean income in general depends on the choice of pension expenditure.[2] A useful approximation is therefore considered for the welfare-weighted mean income, using an explicit form for the welfare function. This produces an explicit approximate solution for the optimal expenditure ratio, and simulations show that the approximation performs well. Comparative statics based on the approximate solution are shown to be as expected; in particular, a higher inequality aversion gives rise, ceteris paribus, to the transfer payment forming a larger proportion of total government expenditure.

It is of course unreasonable to suggest that, in practice, expenditure decisions arise from the optimising plans of a single omniscient and omnipotent judge. However, it is of interest to consider the hypothetical question of what value judgements could be considered to be implicit in the observed expenditure policies across countries. The approximate closed-form solution for the optimal expenditure ratio allows an implied value of inequality aversion to be obtained for a range of countries, given data for other determinants of the expenditure ratio for each country. The results show that different degrees of inequality aversion are evident among the countries examined. The average implied inequality aversion for Scandinavian countries is consistently higher than that of Commonwealth countries, while the U.S. has a lower degree of inequality aversion than average Scandinavian countries and Commonwealth countries. These results are consis-

[2] It is known from the optimal tax literature that explicit solutions cannot be obtained, even in the simple case where a proportional tax finances only an unconditional transfer in a static framework.

tent with Lambert, Millimet and Slottje (2003), where they measure the degree of inequality aversion for a large sample of countries from income inequality indexes. Germany has the highest implied inequality aversion among all countries. Significant inequality aversion in Germany was also found in Lambert, *et al.* (2003) and Schwarze and Härpfer (2007), where the latter used regional life-satisfaction data to estimate inequality aversion in Germany. These results suggest that variations in expenditure patterns indeed reflect different social preferences regarding inequality which may result from cultural differences across countries. This study then provides a structural way to tackle the conjecture of Alesina and Glaeser (2004).

In view of the complexity of decision making in practice and the frequent need for compromises, it is typically far from straightforward to 'reconstruct' the values which may be implied by actual policies. The evaluation of implicit values can thus help to judge the consistency of actual policies with certain stated objectives of policy makers and commentators. This study therefore relates to a large literature which attempts to recover implicit value judgements in government tax and expenditure policies. For example, early attempts to impute a value of inequality aversion implicit in government tax decisions include Christiansen and Jansen (1978) and Stern (1977).[3] More recent studies include Madden (1992, 1995) and Cragg (1991) in the context of indirect taxes, and Oliver and Spadaro (2004) and Spadaro (2007) in the context of direct taxation.[4] Schwarze and Härpfer (2007) and Lambert *et al.* (2003), as mentioned above, provide empirical examinations of attitudes toward inequality. By using a structural model to infer inequality aversion for a range of countries, the present study obtains results which are consistent with these earlier studies.

[3] See also Mera (1969), Moreh (1981) and Brent (1984). Ravallion (1988) examined inequality aversion implicit in regional disbursement policies.

[4] By examining the shape of the implicit welfare function, Spadaro (2007) found that it was concave, thereby supporting the idea that tax policy could be regarded as the outcome of an optimising process. Implicit values in the choice of adult equivalent scales were examined by van de Ven and Creedy (2005).

The paper is arranged as follows. Section 2 describes the economic environment. Section 3 examines the optimal choice of expenditure of the utilitarian judge. Section 4 describes the approximation which produces the closed-form solution for the expenditure ratio and discusses the compartive statics. Implicit value judgements for a range of countries are investigated in Section 5. Brief conclusions are in section 6.

2 The Economic Environment

This section describes the overlapping generations model in which a pure public good and a transfer payment, in the form of a pension, are tax-financed on a pay-as-you-go basis.

Each individual is assumed to live for two periods, a working and a retirement period, so that the economy is populated by two overlapping cohorts in any given period. Individuals have identical preferences but are heterogenous with respect to incomes. A young individual i, born at time t, works in the first period and receives an exogenously fixed income, $y_{i,t}$. Income is taxed at rate τ, which is the same for all individuals and is assumed to be exogenously determined. In period t, a young individual, i, allocates disposable income between current consumption, $c_{1i,t}$, and savings, $s_{i,t}$. In the second period of life, the individual finances consumption of private goods, $c_{2i,t+1}$, using the unconditional and untaxed pension from the government, b_{t+1}, and the return on savings, $(1+r)s_{i,t}$, where r is the constant interest rate at which individuals can borrow or lend. The price of the private consumption goods is normalised to unity, so that c also denotes private consumption expenditure.

The quantity of pure public good in period t is denoted as $Q_{G,t}$, which can be consumed by all individuals in period t. Assume that the production of public goods involves a constant unit cost, p. Then government expenditure on public goods in t is simply $G_t \equiv pQ_{G,t}$.

Suppose all individuals have Cobb-Douglas preferences, expressed for person i as:

$$U_{i,t} = c_{1i,t} c_{2i,t+1}^{\beta} Q_t^{\gamma} Q_{t+1}^{\gamma\beta}, \tag{1}$$

where $0 < \beta = 1/(1+\rho) < 1$ is the discount factor, ρ is the time preference rate, and γ is the utility weight attached to consumption of public goods, reflecting the individuals' preference for public goods relative to private goods.

In view of the inter-generational transfers in addition to the income shifting and intra-generational redistribution, it is desirable to allow for population and income growth. Suppose the average income of young individuals grows at a constant rate of ω over time, and there is constant growth, at the rate n, in the population, where N_t denotes the number of individuals born in period t. It is assumed that income growth involves an equal proportional change at all income levels and that population growth involves an equal proportional change in population frequencies at each income level, so that ω and n are independent.

3 The Optimal Choice of a Utilitarian Judge

This section investigates the optimal choice of expenditure by a utilitarian judge who aims to maximise a social welfare function. It begins by describing each individual's consumption and saving choice conditional on government expenditures, which yields indirect utility as a function of expenditure terms. It then defines the social welfare function in terms of individual utilities and characterises the optimal expenditure ratio, defined as the ratio of total expenditure on pensions to that on public goods, that maximises the social welfare function.

From Section 2, the lifetime budget constraint of individual i is given by:

$$c_{1i,t} + \frac{c_{2i,t+1}}{(1+r)} = (1-\tau) y_{i,t} + \frac{b_{t+1}}{(1+r)} \equiv M_{i,t}. \tag{2}$$

This form allows for the fact that tax-financed public goods are non-excludable so that individuals are not charged at the point of consumption. The consumption

plans, conditional on the values of public expenditure and the pension, are given, using the standard properties of Cobb-Douglas utility functions, as

$$c_{1i,t} = M_{i,t}/(1+\beta), \quad c_{2i,t+1} = \beta(1+r)M_{i,t}/(1+\beta).$$

So the indirect utility function, $V_{i,t}$, is:

$$V_{i,t} = \left(\frac{M_{i,t}}{1+\beta}\right)\left(\frac{\beta(1+r)M_{i,t}}{(1+\beta)}\right)^{\beta} Q_t^{\gamma} Q_{t+1}^{\gamma\beta}. \tag{3}$$

The utilitarian judge, recognizing that individuals' consumption and saving decisions are conditional on its expenditure choices, chooses a sequence of expenditures on public goods and pensions, $\{b_{t+1}, G_{t+1}\}_{t=0}^{\infty}$, ($b_0$ and G_0 are given), to maximise a social welfare function, subject to the government budget constraint in every period.

In the pay-as-you-go financing structure, income tax revenue in each period must be sufficient to finance the transfer payments to those currently retired along with the public good. Hence, at time t the government budget constraint is given by:

$$G_t + N_{t-1}b_t = \tau N_t \bar{y}_t. \tag{4}$$

The social welfare function is defined as the following discounted sum of all generations' utilities:

$$SW = \sum_{t=-1}^{\infty} \delta^t W_t, \tag{5}$$

where δ is the government's discount factor, which could be different from private individuals' discount factor, β. The social welfare associated with generation t, W_t, is defined as a function of utilities of all individuals born in period t:

$$W_t = W(V_{1,t}, ..., V_{i,t}, ...V_{N_t,t}), \tag{6}$$

where $V_{i,t}$ is defined in (3), and the function W is time-invariant, individualistic and paretean. To ensure the problem is well defined, it is also assumed that W

is additively separable, and strictly increasing and (weakly) concave in each of its argument.[5]

A simplification arises from the fact that the imposition of pay-as-you-go financing as well as the two-period overlapping generations nature of the model ensures that maximisation of the social welfare function in (5) is equivalent to the maximisation of W_t in every period t.

Then the first-order conditions for choice of b_{t+1}, for $t = 0, 1, \ldots$, are given by:

$$\sum_{i=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \left(\frac{\partial V_{i,t}}{\partial b_{t+1}} + \frac{\partial V_{i,t}}{\partial G_{t+1}} \frac{dG_{t+1}}{db_{t+1}} \right) = 0. \tag{7}$$

Define $v_{i,t} = (\partial W_t / \partial V_{i,t}) / (\partial V_{i,t} / \partial b_{t+1})$ as the welfare weight attached to an increase in i's income. Then:

$$\sum_{i=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial G_{t+1}} = \sum_{i=1}^{N_t} v_{i,t} \left(\frac{\partial V_{i,t} / \partial G_{t+1}}{\partial V_{i,t} / \partial b_{t+1}} \right). \tag{8}$$

Hence, writing $\tilde{v}_{i,t} = v_{i,t} / \sum_{i=1}^{N_t} v_{i,t}$, where $\sum_{i=1}^{N_t} \tilde{v}_{i,t} = 1$, and noting that from the government budget constraint in (4), $dG_{t+1}/db_{t+1} = -N_t$, substitution in (7) for each t gives:

$$\sum_{i=1}^{N_t} \tilde{v}_{i,t} \left(\frac{\partial V_{i,t} / \partial G_{t+1}}{\partial V_{i,t} / \partial b_{t+1}} \right) = \frac{1}{N_t}, \tag{9}$$

This result does not depend on the precise form of V or W, but further progress can be made using (3) above, which gives:

$$\frac{\partial V_{i,t}}{\partial G_{t+1}} = \frac{\beta \gamma V_{i,t}}{G_{t+1}}, \tag{10}$$

$$\frac{\partial V_{i,t}}{\partial b_{t+1}} = \frac{(1+\beta) V_{i,t}}{(1+r)(1-\tau) y_{i,t} + b_{t+1}}. \tag{11}$$

Substituting into (9) and writing $\tilde{y}_t = \sum_i^{N_t} \tilde{v}_{i,t} y_{i,t}$ gives:

$$\frac{\beta \gamma \{(1+r)(1-\tau)\tilde{y}_t + b_{t+1}\}}{(1+\beta) G_{t+1}} = \frac{1}{N_t}. \tag{12}$$

[5] The social welfare function defined this way is consistent with requirements for time-consistent policy. See Calvo and Obstfeld (1988) and Ambler (2000) for further discussion on this issue in overlapping generations models. A simple form of W_t adopted by many studies, such as Ghiglino (2000), defines W_t as the weighted sum of lifetime utilities of individuals in cohort t.

The term $\tilde{y}_t$ is a welfare-weighted average of individual incomes with weights, $\tilde{v}_{i,t}$. Substituting the government budget constraint into (12) gives the optimal per capita expenditure on the pension, b_{t+1}. Therefore the ratio of total expenditure on pension to total expenditure, $B_{t+1} \equiv N_t b_{t+1}/\tau N_{t+1}\bar{y}_{t+1}$, is given by:

$$B_{t+1} = \frac{1}{(1+\beta+\beta\gamma)}\left\{(1+\beta) - \beta\gamma\frac{(1+r)(1-\tau)}{\tau\,(1+n)\,(1+\omega)}\frac{\tilde{y}_t}{\bar{y}_t}\right\}. \tag{13}$$

The share of total expenditure on public goods, $G_{t+1}/\tau N_{t+1}\bar{y}_{t+1}$, is thus simply obtained as $1 - B_{t+1}$. Therefore the ratio of total expenditure on pensions to that on public goods, R_{t+1}, is:

$$R_{t+1} = \frac{\left((1+\beta)/\beta\gamma\right)(1+n)(1+\omega) - (1+r)(\frac{1-\tau}{\tau})\,(\tilde{y}_t/\bar{y}_t)}{(1+n)(1+\omega) + (1+r)(\frac{1-\tau}{\tau})\,(\tilde{y}_t/\bar{y}_t)}. \tag{14}$$

This result shows how the optimal expenditure ratio, R_{t+1}, depends, *inter alia*, on the ratio of the welfare-weighted mean income to the arithmetic mean income at time t as well as parameters regarding population growth, income growth, the tax rate and preferences.[6] This optimal choice is characterised by a balanced growth path, along which all aggregate endogenous variables grow at the same rate as aggregate income, and per capita variables grow at the same rate as average income.

The expression in (14) cannot of course be regarded as providing a closed-form solution for the expenditure ratio, as the welfare-weighted mean income in general depends on the value of b. However, a useful approximation is considered in the following subsection, using an explicit form for the welfare function.

4 Approximating the Optimal Expenditure Ratio

This section shows how an approximation for the optimal expenditure ratio can be obtained by approximating the welfare-weighted average income, $\tilde{y}_t$, using a

[6] The expenditure ratio does not depend on the constant cost of producing the public good, p. This property arises from the Cobb-Douglas form of utility functions.

widely adopted social welfare function which takes the iso-elastic form:

$$W_t = \frac{1}{1-\varepsilon}\sum_{i=1}^{N_t} V_{i,t}^{1-\varepsilon}, \tag{15}$$

where $\varepsilon > 0$ denotes the relative inequality aversion of the judge.[7] From (15), $\partial W_t/\partial V_{i,t} = V_{i,t}^{-\varepsilon}$, for all $\varepsilon > 0$, and obtaining $\partial V_{i,t}/\partial b_{t+1}$ from (3), the welfare weights, $v_{i,t} = (\partial W_t/\partial V_{i,t}) \,/\, (\partial V_{i,t}/\partial b_{t+1})$, are:[8]

$$v_{i,t} = \frac{(1+\beta)\left(\frac{\beta^\beta (1+r)^\beta Q_t^\gamma Q_{t+1}^{\beta\gamma}}{(1+\beta)^{(\beta+1)}}\right)^{1-\varepsilon}}{(1+r)}\left((1-\tau)y_{i,t} + \frac{b_{t+1}}{(1+r)}\right)^{\beta-\varepsilon(1+\beta)}. \tag{16}$$

This form is clearly intractable, but it is reasonable to suppose that b_{t+1} is small relative to $y_{i,t}$.(put a footnote here) In this case an approximation, $\tilde{y}_{A,t}$, for the welfare-weighted mean is obtained as the ratio of two fractional moments:

$$\tilde{y}_{A,t} = \frac{\frac{1}{N_t}\sum y_{i,t}^{1+\beta-\varepsilon(1+\beta)}}{\frac{1}{N_t}\sum y_{i,t}^{\beta-\varepsilon(1+\beta)}}. \tag{17}$$

Suppose further that $y_{i,t}$ is lognormally distributed as $\Lambda(y_{i,t}|\,\mu_t, \sigma_t^2)$, with mean and variance of logarithms of μ_t and σ_t^2 respectively. Using the properties of the lognormal moment generating function, it can be found that:[9]

$$\frac{\tilde{y}_{A,t}}{\bar{y}_t} = \exp\left[\{2\beta - 2\varepsilon\,(1+\beta)\}\,\frac{\sigma_t^2}{2}\right]. \tag{18}$$

Higher ε therefore reduces $\tilde{y}_{A,t}$ relative to $\bar{y}_t$, by giving a lower weight to higher incomes. Appendix A reports simulation analyses which show that this does indeed provide a good approximation when used with equation (14).

It is now possible to discuss comparative static properties of the model. Examination of (14), using (18) to approximate $\tilde{y}_t/\bar{y}_t$, shows that a higher tax rate is

[7] For $\varepsilon = 1$ this takes the form $W_t = \sum_{i=1}^{N_t} \log V_{i,t}$.

[8] The results clearly depend on the precise cardinalisation of individuals' utility functions.

[9] The vth moment about the origin is given by $\mu_v = \exp\left(v\mu + v^2\sigma^2/2\right)$, and thus $\bar{y} = \mu_1 = \exp\left(\mu + \sigma^2/2\right)$; see Aitchison and Brown (1957, p. 8).

associated with a greater proportion of expenditure being devoted to transfer payments, whereby R_{t+1}, is increasing in τ at a decreasing rate. An increase in the tax rate gives the government more income to spend on both types of expenditure, but the increase in pension expenditure is relatively higher than the increase in public goods expenditure.[10] Population and income growth have similar effects, as they appear in (14) only together in the form $(1+n)(1+\omega)$: they are perfect substitutes for each other. Increases in ω and n increase the share of expenditure on pension and decreases the share of expenditure on public goods, and also their ratio. With income growth or population growth, tax revenues of the government are increased such that the government is able to spend more on both types of expenditure. However, the increase in the expenditure on pensions is higher than the expenditure on public goods.

An increase in the interest rate has a negative effect on the share of expenditure on pensions and on the ratio of pensions to public goods, but increases total expenditure on public goods. A higher interest rate leads to more private savings by individuals at a young age. This results in a higher public goods expenditure and a lower ratio of pension to public goods expenditure.

An increase in income inequality, as measured by σ^2, reduces $\tilde{y}_A/\bar{y}$, from (18), and thus increases the proportion of expenditure devoted to the pension. Hence more basic inequality implies a more redistributive expenditure policy. This result is consistent with models examining majority voting over the tax rate, with an unconditional transfer payment, in which a uniform result is that more basic inequality leads to the choice of a more redistributive tax and transfer structure; for example, see Meltzer and Richard (1981). Similarly, an increase in inequality aversion, ε, also reduces $\tilde{y}_A/\bar{y}$ and therefore rises the share of expenditure on redistribution.

[10] However, in a static model with labour supply, transfers and a public good, Creedy and Moslehi (2009) find a concave relationship between the share of transfers and the tax rate. This arises from adverse incentive effects of the tax and transfer system.

The comparative static results with respect to the weight attached to public goods, γ, suggest that an increase in the preference for public goods unambiguously increases the share of expenditure on public goods, but decreases share of expenditure on pensions and the ratio of pension to public goods expenditure. Analytic results relating to β were found to be equivocal, but the sensitivity analysis reported below demonstrates that B and R fall as β increases.

The comparative static properties shown the direction of changes, but do not show the sensitivity the optimal expenditure composition with respect to changes in parameters. The baseline value for all parameters, τ, r, β, ω, n, σ^2 and ε, were set to the average of the sample reported in Table 4. The baseline value for the weight attached to public goods is 1. The parameter variations considered are 10 and 20 per cent changes around the baseline values. Table 1 reports changes in the ratio of total expenditure on the pension to total expenditure, along with R, around their baseline values. In each case only one parameter at a time is varied, with all other parameters kept at their baseline values.

Table 1: Variation in Share of Expenditure on Pensions, B and The Ratio R for Alternative Parameters

		τ	r	β	ω	n	σ^2	ε	γ
	-20%	0.433	0.502	0.526	0.480	0.491	0.461	0.416	0.569
	-10%	0.468	0.499	0.509	0.489	0.493	0.479	0.459	0.531
B	Baseline	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
	$+10\%$	0.519	0.493	0.485	0.502	0.498	0.511	0.526	0.463
	$+20\%$	0.538	0.490	0.476	0.509	0.500	0.525	0.551	0.432
	-20%	0.764	1.007	1.108	0.924	0.965	0.856	0.711	1.319
	-10%	0.879	0.995	1.038	0.957	0.974	0.920	0.850	1.133
R	Baseline	0.984	0.984	0.984	0.984	0.984	0.984	0.984	0.984
	$+10\%$	1.079	0.972	0.941	1.009	0.993	1.045	1.111	0.862
	$+20\%$	1.165	0.960	0.908	1.037	1.002	1.105	1.229	0.760

The results show that the share of expenditure on pensions, public goods and their ratio are quite sensitive to variations in the tax rate, the weight attached to public goods, inequality aversion, the variance of the distribution of income and

the discount factor. They are less sensitive to changes in the interest rate and the growth rates of population and income.

5 An application of the model: Implicit inequality aversion

This section considers the following hypothetical question. Supposing that actual expenditure policies were determined by a utilitarian judge of the type considered above, what degree of inequality aversion is implied? This type of question has previously been considered in the context of static models of taxation. Actual policy making is of course far too complicated to be captured by such a simple model, and in practice value judgements are extremely difficult to articulate precisely and thus are never expressed explicitly by policy makers. But information about implicit value judgements can provide a useful input into rational policy analysis.

The present framework allows an implied value, $\hat{\varepsilon}$, of inequality aversion to be obtained. Using the approximation for $\tilde{y}_A/\bar{y}$ in (18) above, $\hat{\varepsilon}$ is, after some rearrangement:

$$\hat{\varepsilon} = \frac{1}{(1+\beta)}\left(\beta + \frac{1}{2}\frac{\log(\tilde{y}_A/\bar{y})}{(-\sigma_t^2/2)}\right). \tag{19}$$

For the lognormal distribution, median income is $y_m = e^{\mu}$, so the ratio of median to mean income is given by $\log(y_m/\bar{y}_t) = -\sigma_t^2/2$. Hence (19) can be seen to involve a ratio of two terms consisting of logarithms of income ratios, one containing the welfare weighted mean and the other involving the median income.

Starting from equation (13), it is possible to express $\tilde{y}/\bar{y}$ for the relevant period in terms of the other variables and the expenditure share B. However, a problem is raised by the difficulty of obtaining detailed comparable data on the composition of government expenditure for a wide range of countries. A major difficulty in the present context is that inadequate information about expenditure components is available. Ideally, separate details regarding pension expenditure are required,

but only aggregate expenditure on transfer payments can be obtained: the use of the aggregates effectively requires an assumption that the composition of transfers is similar across countries.[11] Hence the results presented here should be treated with much caution. The data used here are described in detail in Appendix B.

Information about σ^2 can be obtained from the reported values of the Gini coefficient, again using properties of the lognormal distribution (as described in Appendix B). Data for the Gini measure are not available for each year for each country, and so the average 2000-06 was used. The growth rate of income and population as well as the real interest rate were obtained from the average annual growth rates of GDP per capita and population, and the average annual real deposit interest rate between 2000 and 2006 for each country. The value of β is obtained, for each country, by assuming that time preference rate is equal to the real interest rate. It is necessary to adjust the various rates were so that they correspond to the overlapping generations framework used here.

Information can be obtained from various data sources about all these variables, except for the value of γ, the relative weight attached to public goods in the utility function. Independent evidence on preferences for public goods is obviously notoriously difficult to obtain in view of the well-known problems of preference revelation with non-rival and non-excludable goods. Hence it is possible only to obtain, for a given country at any time, a value of $\tilde{y}/\bar{y}$ conditional on an assumed value of γ, also consider the sensitivity of results to variations in γ. In considering sensible values for γ, it was decided to use a 'benchmark case' of $\gamma = 1$; that is, consumers have the same preference for public goods as for private goods. Then variations of 10 per cent and 20 per cent around this value were considered. The results reported here are for the extreme case where individuals' preferences are assumed to be the same in all countries.

[11] Also, not all non-transfer expenditure is on pure public goods. Furthermore, transfer expenditure is in practice endogenous, since governments only set levels and qualifying conditions for the receipt of benefits, not total expenditure.

Table 2: Implied Inequality Aversion for Alternative Values of γ

$\gamma = 0.8$		$\gamma = 0.9$		$\gamma = 1$		$\gamma = 1.1$		$\gamma = 1.2$	
Country	ε	Country	ε	Country	ε	Country	ε	Country	ε
Ireland	0.30	Ireland	0.49	Ireland	0.68	Ireland	0.87	Ireland	1.09
Greece	0.35	Greece	0.57	Greece	0.79	Greece	0.99	Greece	1.20
Hungary	0.44	Hungary	0.79	Hungary	1.13	Hungary	1.45	Hungary	1.75
Czech Re	0.49	Czech Re	0.90	Czech Re	1.30	Czech Re	1.68	US	1.98
Iceland	0.61	Iceland	1.08	Iceland	1.50	US	1.85	Czech Re	2.07
Belgium	0.99	Belgium	1.30	Belgium	1.60	Poland	1.86	Poland	2.09
Netherlands	1.01	Poland	1.36	Poland	1.61	Belgium	1.90	Italy	2.16
Poland	1.08	UK	1.42	Italy	1.67	Iceland	1.90	Belgium	2.20
UK	1.11	Netherlands	1.42	US	1.71	Italy	1.91	NZ	2.21
Finland	1.16	Italy	1.44	UK	1.72	NZ	1.98	Iceland	2.27
NZ	1.19	NZ	1.47	NZ	1.73	UK	2.01	UK	2.29
Italy	1.20	US	1.56	Netherlands	1.81	Netherlands	2.19	Netherlands	2.55
US	1.41	Finland	1.60	Spain	2.01	Spain	2.31	Canada	2.61
Spain	1.43	Spain	1.72	Finland	2.05	Canada	2.38	Spain	2.64
Norway	1.51	Canada	1.93	Canada	2.16	Finland	2.50	Finland	2.97
France	1.54	Norway	2.02	Australia	2.42	Australia	2.74	Australia	3.07
Austria	1.68	France	2.02	Austria	2.49	Austria	2.89	Austria	3.29
Canada	1.69	Austria	2.09	Norway	2.50	Norway	2.96	Norway	3.41
Australia	1.77	Australia	2.10	France	2.50	France	2.99	France	3.50
Denmark	2.30	Denmark	2.86	Switzerland	3.33	Switzerland	3.66	Switzerland	4.01
Switzerland	2.69	Switzerland	3.01	Denmark	3.41	Denmark	3.94	Denmark	4.48
Germany	3.24	Germany	3.73	Germany	4.21	Germany	4.71	Germany	5.24
Scandinavian Countries									
Average	1.66		2.16		2.65		3.14		3.62
Coeff Var	0.35		0.30		0.26		0.24		0.21
Commonwealth Countries									
Average	1.44		1.73		2.01		2.28		2.54
Coeff Var	0.24		0.20		0.17		0.16		0.15
All Countries									
Average	1.33		1.68		2.01		2.35		2.69
Coeff Var	0.56		0.47		0.42		0.39		0.37

Table 2 shows the implied values of inequality aversion, ε, for each country in the sample. For each value of γ the countries are ranked in increasing order, that is from low implied inequality aversion to high implied aversion. Furthermore, the countries are divided into various groups. The Scandinavian countries includes Denmark, Finland, Norway. The group of Commonwealth countries includes the U.K., Australia, Canada and New Zealand.

From the generally large values of implied aversion reported in Table 2, it seems highly likely that the inability to measure a more restricted concept of transfer payments has imparted a significant upward bias. It is therefore perhaps more useful to concentrate on the relative orders of magnitude for different countries. There is much consistency in the rankings as γ is varied. The average implied inequality aversion for Scandinavian countries is consistently higher than that of Commonwealth countries, and the former are more homogeneous, as shown by the lower coefficient of variation. The relative positions of the two groups of countries accord with expectations. Countries within each group become more homogeneous with regard to the implicit inequality aversion as γ is increased. This can be seen by the reduction in the coefficient of variation within each group.

A notable change in ranking by implied inequality aversion, as γ is increased, is for the US. This country moves from around the middle to around the bottom quartile, which is perhaps more consistent with a priori expectations. Germany has the highest implied inequality aversion for all values of γ, which may perhaps be regarded as consistent with its 'social-market model'. Significant inequality aversion in Germany was also found by Schwarze and Härpfer (2007). Furthermore, Lambert *et al.* (2003) suggested that countries which have lower population growth tend to have higher inequality aversion, and indeed Germany has a much lower population growth rate than the other countries examined. Similarly, Scandinavian countries have relatively low population growth rates. Exceptions to this result are for the former communist countries of Hungary, Poland and the Czech

Republic, which have low population growth rates combined with relatively low implicit inequality aversion. The low implied inequality aversion for Ireland is consistent with the findings of Madden (1992, 1995).[12]

6 Conclusions

This paper examined the optimal composition of government expenditure, as chosen by an independent utilitarian judge, in a two-period overlapping generations model. The main focus was on public goods and a transfer payment, in the form of a pension, which are financed by income tax on a pay-as-you-go basis. The choice of expenditure by the judge who maximises a social welfare function is complicated by the fact that the pension involves a combination of income-shifting between phases of the life cycle with both inter-generational and intra-generational transfers. The latter arises because the basic pension is unrelated to income whereas the tax is proportional to income. The former arises from the pay-as-you-go feature of financing whereby each generation can benefit from productivity and thus income growth accruing to the following generation. This modelling framework, despite of its simplicity, therefore offers useful insights into the nature of the policy choices involved.

The analysis is a normative exercise in welfare economics rather than a positive attempt to explain differences in the composition of government expenditure in different countries. There is no suggestion that policies are actually based on the 'simple' maximisation of a clearly stated social welfare function. The same point of view is of course taken in all models which examine optimal policies, however defined. Nevertheless it is of interest to consider what values are implied by actual policies, using a 'what if' approach. Thus, if outcomes are considered as arising from maximisation of a welfare function, what extent of inequality aversion do they reveal?

[12] Madden's results were, however, confined to implications of indirect tax structures.

Expressions for the choice of expenditure levels and their ratio were found to depend on the ratio of a welfare-weighted average income to arithmetic mean income in each generation. This welfare-weighted arithmetic mean income level was shown to depend on the degree of relative inequality aversion of the judge. Using an approximation for this welfare-weighted mean income, it was thus possible to produce, for a range of countries, implicit values of the inequality aversion parameter, conditional on assumed preferences of individuals for public goods relative to private goods. The implicit values of inequality aversion, revealed by the policies of different countries, were generally found to accord with expectations and also consistent with previous studies. However, their absolute values are likely to be biased upwards by the difficulty of measuring transfer payments corresponding to the more limited concept specified in the overlapping generations model constructed.

Appendix A: Testing the Approximation

It is important to test the value of the approximation derived in subsection 4. Hence values of the expenditure components using the approximation $\tilde{y}_A$ were compared with those obtained using a simulated population of size 15000 drawn at random from a lognormal distribution with $\mu = 9.0$ and $\sigma^2 = 0.32$.[13] Using the simulated distribution, a range of values of b_{t+1} were investigated. For each b_{t+1} the government budget constraint was used to obtain G_{t+1} and the resulting values were used to calculate each individual's level of utility. These were then used to obtain social welfare, using the iso-elastic function with a specified inequality aversion parameter, ε. Finally, given a number of W_t measures for different b_{t+1}, the maximum was determined, giving the optimal composition. Table 3 gives the results for a range of degrees of inequality aversion, with other parameters set at the average of the sample reported in Table 4 of Appendix B. These results show that the approximation gives values of expenditure levels and ratios which are reasonably close to those obtained using a large simulated population.

Table 3: Composition of Government Expenditure: Alternative Solutions

	Approximation			Simulation		
ε	b_{t+1}	B_{t+1}	R	b_{t+1}	B_{t+1}	R
			$\gamma = 0.9$			
1.81	2813.4	0.51	1.03	2660	0.48	0.93
2.01	2934.52	0.53	1.13	2730	0.49	0.97
2.21	3041.62	0.55	1.22	2790	0.50	1.01
			$\gamma = 1$			
1.81	2537.23	0.46	0.85	2420	0.44	0.78
2.01	2739.88	0.50	0.98	2530	0.46	0.84
2.21	2908.26	0.53	1.11	2630	0.48	0.91
			$\gamma = 1.10$			
1.81	2418.73	0.44	0.77	2280	0.41	0.70
2.01	2557.45	0.46	0.86	2350	0.42	0.74
2.21	2680.14	0.48	0.94	2420	0.43	0.77

[13] This implies $y_m/\bar{y} = 0.85$ which is the average of the sample.

Appendix B: Data Sources and Construction of Variables

The OECD Social Expenditure Database covers 30 OECD countries for the period 1980-2005. The share of social expenditure in percentage of total expenditure is obtained from this dataset. The social expenditure includes Old age, Survivors, Incapacity-related benefits, Health, Family, Active labour market programmes, Unemployment, Housing, and Other social policy areas.[14]

The source for the data for growth in income and population and real interest rate is *World Development Indicators* (2008).[15] Suppose the annual average growth of GDP per capita is x percent, and the length of a time period in this overlapping generation model is 20 years. The required adjusted growth rate is obtained as $(1+x)^{20}-1$. The same approach was used for population growth and the interest rate for each country. Also, by assuming that the time preference rate is equal to the real interest rate, $\rho = r$, then the discount factor for each countries, β, is obtained from $1/(1+r)^{20}$.

The tax rate obtained from *World Development Indicators* (2008). It includes tax revenue as well as social contributions. The ratio of tax revenue to GDP includes compulsory transfers to the central government for public purposes. The ratio of social contributions to GDP includes social security contributions by employees, employers, and self-employed individuals, and other contributions whose source cannot be determined. They also include actual or imputed contributions to social insurance schemes operated by governments.

Data on Gini inequality measures were obtained from *World Development Indicators* (2008) and *World Income Inequality Database* (2007).[16] Suppose income

[14] Aggregated data are described in the interpretative guide (see in particular sections 3.3, Annex 1 and Annex 4) which is available at http://stats.oecd.org/OECDStatDownloadFiles/_OECDSOCX2007InterpretativeGuide_En.pdf

[15] Data for interest rates in U.S., U.K. and Austria were obtained respectively from websites of Federal Reserve Bank of USA, Bank of England and Central Bank of Austria.

[16] Unfortunately, the income concept varies across countries. In some countries gross income

follows the lognormal distribution with mean and variance of logarithms of income being μ and σ^2. This variance can be obtained from the Gini index using $\sigma = \sqrt{2}\Phi^{-1}(\frac{Gini+1}{2})$, where Φ^{-1} is the inverse function of the standard normal cumulative distribution; see Aitchison and Brown (1957, p. 13). Since for most of the countries the Gini index is available only for selected years, the average of available years between 2000 and 2006 was used to calculate the ratio of median to mean income. Consequently, averages of growth of GDP per capita, population growth, the real interest rate, government expenditures and revenues were computed over the same period.

is considered and in others it is net income.

Table 4: Summary Information for Each Country and Group of Countries

Country	B	τ	r	β	ω	n	$Gini$	σ^2
Australia	0.50	0.25	0.11	0.90	0.46	0.30	30.56	0.31
Austria	0.53	0.36	0.27	0.79	0.39	0.10	29.15	0.28
Belgium	0.52	0.41	0.15	0.87	0.39	0.10	32.97	0.36
Canada	0.44	0.19	-0.12	1.14	0.47	0.21	32.56	0.35
Czech Republic	0.45	0.30	-0.14	1.16	1.37	0.01	25.33	0.21
Denmark	0.50	0.33	0.17	0.86	0.38	0.07	23.80	0.18
Finland	0.51	0.35	-0.03	1.03	0.83	0.06	26.88	0.24
France	0.54	0.40	0.18	0.85	0.33	0.14	27.50	0.25
Germany	0.57	0.28	0.36	0.74	0.32	0.01	28.31	0.26
Greece	0.44	0.34	-0.02	1.02	1.17	0.07	34.27	0.39
Hungary	0.44	0.34	0.41	0.71	1.27	-0.04	28.43	0.27
Iceland	0.38	0.29	0.88	0.53	0.74	0.33	25.00	0.20
Ireland	0.46	0.31	-0.55	2.24	1.23	0.45	34.28	0.39
Italy	0.51	0.35	-0.14	1.17	0.19	0.11	36.03	0.44
Netherlands	0.45	0.37	0.14	0.88	0.38	0.09	26.33	0.23
New Zealand	0.48	0.31	1.02	0.50	0.49	0.27	33.70	0.38
Norway	0.51	0.37	0.61	0.62	0.45	0.15	25.79	0.22
Poland	0.50	0.29	0.79	0.56	1.31	-0.03	34.49	0.40
Spain	0.53	0.26	-0.07	1.08	0.52	0.34	34.66	0.40
Switzerland	0.54	0.18	0.05	0.95	0.31	0.15	33.68	0.38
United Kingdom	0.51	0.36	0.39	0.72	0.56	0.10	33.00	0.35
United States	0.44	0.18	0.05	0.95	0.36	0.21	40.81	0.57
Scandinavian Countries								
Average	0.51	0.35	0.25	0.84	0.55	0.09	25.49	0.21
Coeff Variation	0.02	0.06	1.33	0.25	0.44	0.51	0.06	0.13
Commonwealth Countries								
Average	0.48	0.28	0.35	0.82	0.49	0.22	32.46	0.35
Coeff Variation	0.06	0.26	1.42	0.34	0.10	0.39	0.04	0.08
All Countries								
Average	0.49	0.31	0.20	0.92	0.63	0.15	30.80	0.32
Coeff Variation	0.09	0.21	1.83	0.39	0.60	0.89	0.14	0.30

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