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Bargaining Order in a Multi-Person Bargaining Game*

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Abstract

This paper studies a complete-information bargaining game with one buyer and multiple sellers of different "sizes" or bargaining strengths. The bargaining order is determined by the buyer. If the buyer can commit to a bargaining order, there is a unique subgame perfect equilibrium outcome where the buyer bargains in order of *increasing* size – from the smallest to the largest. If the buyer cannot commit to a bargaining order and the sellers are sufficiently different, there is also a unique subgame perfect equilibrium outcome again with the order of increasing size. *Journal of Economic Literature* Classification Number: C78.

Keywords: multi-person bargaining, bargaining order

1 Introduction

Consider a scenario in which a real estate developer must acquire land from multiple sellers. The sellers' lots are of different sizes with a larger lot giving a higher flow of payoffs to its owner. Such situations are quite common. For example, in Chongqing, China, the construction of a retail mall required 280 separate negotiations. The project was suspended for three years because one out of the 280 residents refused to sell his property to the developer.[1] Columbia University's expansion plan in West Manhattanville is another prominent example. The 6.3-billion, 17-acre project acquired land from 67 property owners. The whole negotiation lasted for a long period from 2002 to 2010, and the negotiation on the last three properties alone took more than three years.[2] What should the buyer (developer) do when she needs to purchase land from multiple sellers who own different sized lots? In particular, which seller should she bargain with first, the one with a large lot or a small lot? This paper examines the corresponding non-cooperative bargaining game. The results suggest that the buyer should bargain with

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[1] The negotiation started in 2004, and eventually the owner sold the house in 2007. See French (2007) or "Nail House in Chongqing Demolished", *China Daily,* April 3, 2007.

[2] See Williams (2008), Chung (2009) and Sieff (2010).

the seller of the smallest lot first, especially when the sizes of the lots are quite different. This paper does not try to explain the delay in the examples above. Delay happened even when there is only one seller left in the first example, perhaps due to incomplete information.[3]

While the model studied here is couched in the language of a single developer negotiating with multiple sellers, it is applicable to a variety of other bargaining scenarios. For example, consider an airline that must bargain with two separate unions, pilots and flight attendants, in order to end a strike. Both unions are necessary for the airline to operate but their outside options differ.[4] Which union should the firm negotiate with first? A similar question can be asked about the negotiation between a manufacturer and a group of upstream suppliers producing parts at different costs.[5] The key characteristics common to these scenarios are: the one-to-many aspect of the negotiation; the fact that an agreement with *all* sellers is necessary to reap any economic gains; and, finally, the "size" differences among the sellers.

In this paper, bargaining strength is measured by the size of the outside/inside option[6] available to a seller when bargaining with the buyer. A seller with a large lot is stronger than a seller with smaller lot in the sense that, in equilibrium, the price received by the large seller is higher than that received by the small seller. There are other notions of bargaining strength, of course. For instance, one may measure bargaining strength by how patient a seller is and different sellers may have different discount rates. Alternatively, it may have to do with the likelihood of making first offers (see, for instance, Li, 2010).

It is useful to begin with a simple example. Consider a scenario with one developer and two farmers. All parties share a discount factor of $\delta = 2/3$. Farmer 1 owns a large lot of land that produces 1/8 units of harvest each period; and farmer 2 owns a small lot of land that produces 1/18 units of harvest each period. A lot does not produce any harvest once it is sold to the developer. The developer must purchase both lots to build a mall that produces 1/3 units of profit each period. It is easy to see that the present value of all harvests is $v_1 = 3/8$ for farmer 1, $v_2 = 1/6$ for farmer 2, and the present value of all profits of the mall is 1.

Negotiations are sequential and in any period, the developer negotiates with only one farmer. The developer first offers a price, which the farmer may accept or reject. If the offer is accepted, the developer proceeds to negotiate with the other farmer in the next period (in a standard two-player alternating offer bargaining game). If the offer is rejected, the farmer makes a counter-offer in the next period, which the developer may accept or reject. If the developer accepts this offer, she proceeds to negotiate with the other farmer. If the developer rejects the offer, she picks a farmer, which could be the same as in the previous period, and negotiates with him in the same fashion, and so on. We would like to know which farmer the developer should bargain with first.

Because the developer can pick any remaining farmer to negotiate with, there is no restriction on the choice of bargaining orders. However, there is a unique subgame perfect equilibrium outcome, where the developer bargains with farmer 2 until he agrees and then

[3] See, for instance, Admati and Perry (1987) for how incomplete information can lead to delay.

[4] An outside option is the payoff that a player receives if he leaves the negotiation. This paper focuses on inside options, but our qualitative results would not be affected if sellers had outside options instead.

[5] Bargaining between a manufacturer and its upstream suppliers is discussed, for example, in Blanchard and Kermer (1997) and Bedrey (2009).

[6] An inside option is the payoff received by a seller while negotiations are ongoing (see, for example, Muthoo, 1999).

with farmer 1, and farmer 2 sells his land in period 1 at a price of $p_2 = 1/5$, and farmer 1 sells his land in period 2 at a price of $p_1 = 5/8$.[7] The prices are explained below. The payment to the first farmer is a sunk cost to the buyer, so after farmer 2 sells his land, the surplus is $1 - v_1$, which is the difference between the value of the mall and the value of farmer 1's land. Farmer 1 gets the same $\delta/(1+\delta)$ of the surplus as in Rubinstein bargaining game,[8] which implies that his selling price is $v_1 + \frac{\delta}{1+\delta}(1 - v_1) = 5/8$. Excluded the price for farmer 1, the remaining value of mall is $\delta(1-p_1)$ evaluated in the first period. As a result, the surplus for farmer 2 and the buyer is $\delta(1-p_1) - v_2$, which is also the difference between the remaining value of the mall and the value of farmer 2's land. Similarly, farmer 2 and the buyer split this surplus as in Rubinstein bargaining game, therefore the price for farmer 2 is $v_2 + \frac{\delta}{1+\delta}[\delta(1-p_1) - v_2] = 1/5$.

Section 3 shows that no other bargaining order can arise in an equilibrium. To demonstrate the main idea, consider a deviation in which the developer chooses a "wrong" bargaining order: bargaining with farmer 1 in the first period. The unique equilibrium outcome of the resulting subgame is the following. Both the offer and counter-offer are rejected in the first two periods; the developer chooses to bargain with farmer 2 in the third period and the farmer 2 sells his land in the third period at a price of $p_2 = 1/5$; farmer 1 then sells his land in the fourth period at a price of $p_1 = 5/8$.

Notice that while the selling prices in the two orders are the *same*, there is a two-period delay if the bargaining starts with farmer 1. The prices must be the same because the bargaining game is also a proper subgame of the game where the developer chooses farmer 1 in the first period. Since each has a unique subgame perfect equilibrium outcome, the prices must agree. However, why does one order lead to a delay? To see why, consider what would happen if, when the bargaining starts with farmer 1, the developer offers farmer 1 the present value of the price he would get in period 4 plus the appropriate compensation for the foregone harvest in three periods; that is, $p_1' = 1/8 + \delta/8 + \delta^2/8 + \delta^2 p_1 = 0.45$. Farmer 1 would agree to this, and in the resulting negotiation with farmer 2, the price would be $p_2' = 1/2$. However, the present value of the two payments $p_1' + \delta p_2' = 0.78$ is greater than δ, the value of the mall to the developer, which is realized in period 2. While this shows that one particular deviation will not break the delayed equilibrium, the formal proof in Section 3 shows that no such deviation can prevent delay.

Even though the bargaining game is of complete information, the "wrong" bargaining order may cause delay in agreement. Several papers have described different reasons for delay in complete information bargaining. The reason discussed above is similar to Cai (2000). On the other hand, the reasons in Haller and Holden (1990) and Fernandez and Glazer (1991) are different from those identified here. Their models are standard two-person alternating bargaining games between a labor union and a firm, but the labor union can choose between production and strike when an offer is rejected. In an equilibrium with delay, the firm would rather wait several periods to avoid the "bad" equilibrium in which the union strikes once disagreement occurs. In our paper, production (building the mall) is not allowed while the bargaining is going on. Harvests are different from the production and the farmers receive them for sure during the bargaining.

Our model builds on Cai (2000) by introducing endogenous bargaining order and asymmetric sellers. His model is the extreme case of our game when all the farmers

[7] The numbers in the example are calculated according to the equilibrium derived later in the paper and are given only to illustrate the main findings. Technical details and calculations are postponed to Example 2.

[8] See Rubinstein (1982).

receive no harvest. The bargaining order is fixed and rotates among the sellers in his paper. He finds multiple stationary subgame perfect equilibrium outcomes, and delay can happen in some of them. In contrast, the sellers are asymmetric and the bargaining order is endogenous in our game, this results in a *unique* subgame perfect equilibrium outcome.

Few papers discuss bargaining orders as part of equilibrium. However, there are several papers with similar setups that allow endogenous bargaining orders but in a restricted way. Perry and Reny (1993) allow each player to decide when to make an offer, which implicitly allows for different bargaining orders. Stole and Zwiebel (1996), Noe and Wang (2004) and Bedrey (2009) study bargaining orders in finite bargaining games. Chatterjee and Kim (2005) focus on the orders in which the buyer cannot switch to another seller before an agreement. The literature on agenda formation[9] also discusses orders, but the orders have a different meaning: sequences of different issues or tasks. Contrary to our paper, this literature suggests the most important issue should be discussed first.[10]

Li (2010) also allows endogenous bargaining order, but his paper is very different from ours. In his paper, a seller's bargaining strength is measured his likelihood to make the first offer in each bargaining. He finds many equilibria and any selling order can be sustained. In our paper, a seller's bargaining strength is measured by the size of his outside/inside option, and the bargaining game has a unique equilibrium outcome.

The seller holdout problem[11] also has complementary sellers as in our setup. In this literature, it is argued that incomplete information often leads to holdout. However, holdout is not important in our setup because our bargaining is sequential and of complete information.[12]

The rest of the paper is organized as follows. Section 2 introduces the bargaining game where the buyer can commit to a bargaining order, and analyzes the equilibrium in the case of two sellers. Section 3 introduces the bargaining game in which the buyer cannot commit to a bargaining order, and examines the two-seller case of such a game. Section 4 extends the previous two sections to N-seller case. Section 5 discusses alternative assumptions and applications. Finally, Section 6 concludes the paper.

2 Bargaining with Commitment

Our model is a non-cooperative, infinite-horizon and complete-information bargaining game with endogenous bargaining order and asymmetric sellers. The game has $N+1$ players including one buyer, B, and a set of sellers, $\{1, 2, \cdots, N\}$. Each seller (he) has one lot of land, and the buyer (she) must purchase every lot in order to build a mall. In other words, the lots are perfect complementary for the buyer.

All the players share the same discount factor $\delta \in (0, 1)$.[13] Seller i's land is of value v_i, which is the present value of a constant flow of harvests. Therefore, seller i's harvest for one period is of value $v_i(1-\delta)$, and is received at the end of each period while the land is still in his possession. In the literature of bargaining, the harvest is referred as to seller i's inside option. It is assumed that $v_1 > v_2 > \cdots > v_N > 0$. If every unit area

[9] See, for example, Fershtman (1990), Winter (1997), Reinhard and Matthias (2001) and Flamini (2007).

[10] See, for example, Winter (1997) and Flamini (2007).

[11] See, for example, Mailath and Postelwaite (1990), Menezes and Pitchford (2004) and Chowdhury and Sengupta (2008).

[12] If the buyer can make simultaneous contingent offers, holdout may arise. See Section 5.6.

[13] If the sellers are different only in their discount factors, it can be verified that the buyer would be indifferent among different orders because they all give her the same payoff.

of land is equally productive, the assumption implies that seller i's land is of larger size than seller $(i+1)$'s. The mall produces a constant profit each period, and the present value of the profits is normalized to 1.

2.1 Timing

The buyer first announces a sequence $((i_1, n_1), (i_2, n_2), ...)$ of pairs, where i_j is a seller and n_j is a positive integer or ∞. This sequence means that the buyer will bargain with seller i_1 for n_1 periods (if $n_1 < \infty$) or until that seller sells his land (if $n_1 = \infty$), then switch to seller i_2 for n_2 periods (if $i_1 \neq i_2$) for n_2 periods or until seller i_2 sells his land. We are going to focus on the bargaining orders before the first agreement, and it is represented by an infinite sequence of sellers, $i_1, i_2, ...$ where $i_t \in \{1, 2, \cdots, N\}$ for all t. Starting with the first seller in the sequence, the buyer bargains with one seller over the price in each round of bargaining. Each round has one or two periods. In the first period, the buyer suggests a price to the seller, the seller then decides to either accept it or reject it. If the seller accepts, this round ends with only one period. Otherwise, the seller suggests another price in the second period, which the buyer must either accept or reject. If an agreement is reached, the buyer pays the seller the agreed price right away and the seller leaves the game permanently before the harvests of the period are realized. If the seller's suggestion is rejected, the bargaining moves to the next round in which the buyer bargains with the second seller in the sequence in the same fashion. At the end of each period, every remaining seller receives a harvest from his land.

Note that the bargaining order cannot be revised afterwards, which is relaxed in the next section. Hereafter, the above game is referred as to the "N-seller game with commitment", or the "N-seller game" where no ambiguity results. The bargaining order in the N-seller game specifies only the order of sellers before the first agreement. After the first agreement, the game has only $N-1$ sellers and the buyer chooses an order for the resulting $(N-1)$-seller game, and so on.

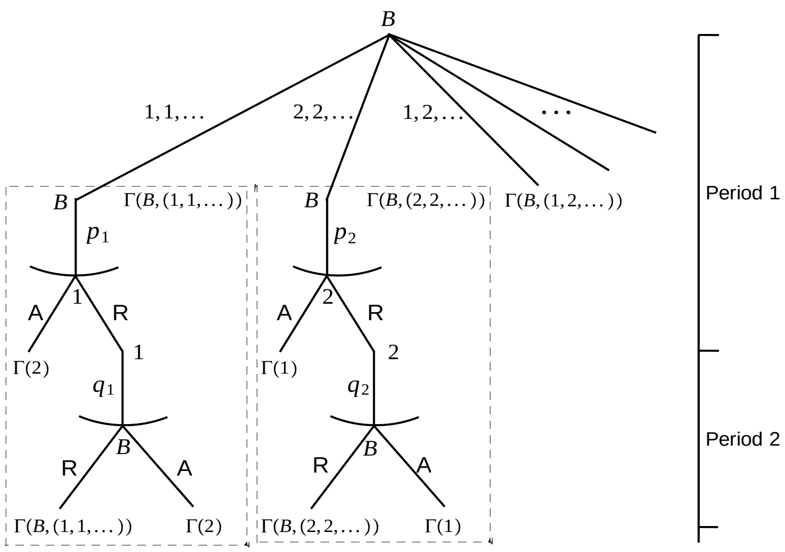

Figure 1. Bargaining with Commitment

Let $\Gamma(j,(i_1,i_2,...))$ denote the two-seller game with the *fixed* bargaining order $i_1, i_2, ...$, where player $j \in \{i_1, B\}$ makes the first offer. Bargaining order $i_1, i_2, ...$ means that the buyer bargains with seller i_t in period $2t+1$ if no agreement has been achieved. For example, bargaining order $1, 1, ...$ means the buyer bargains with seller 1 until he agrees. Let $\Gamma(i)$ represent the one-seller game between seller i and the buyer who makes the first offer. Figure 1 demonstrates the game tree for the two-seller game.

Two features of the model are very important. First, the buyer bargains with only one seller at a time. This bargaining is very popular in reality, especially when it is costly to communicate with all the sellers at the same time. In the example by Coase (1960), a railway company has to bargain with the farmers along a railway track. It is difficult to make simultaneous offers to multiple farmers when they are located far away from each other. Moreover, this assumption is commonly used in the recent bargaining studies such as Cai (2000, 2003) and Noe and Wang (2004). Second, the payments are made immediately after the corresponding agreements. In other words, the contracts between the buyer and sellers are cash-offer contracts. These contracts are widely used by real estate developers;[14] moreover, they are also studied in recent bargaining literature[15]. However, there are also situations that do not satisfy these assumptions, and the consequences of relaxing the assumptions are discussed in Section 5.

2.2 Payoffs

An outcome is denoted as $(p_1, p_2, ..., p_N, t_1, t_2, ..., t_N)$, where seller i sells his land at price p_i in period t_i. If seller i never sells his land, t_i is infinity and p_i is zero. The present value of t periods of seller i's harvests is denoted as

$$H_{i,t} \equiv v_i (1-\delta) \sum_{s=1}^{t} \delta^{s-1}.$$

Given an outcome, seller i's payoff is

$$\pi_i = H_{i,t_i-1} + \delta^{t_i-1} p_i \tag{1}$$

where the first term is the present value of the harvests before the land is sold, and the second term is present value of the payment from the buyer. Since there is no harvest when the buyer owns the land, the buyer's payoff is

$$\pi_B = \delta^{\max(t_1,\cdots,t_N)-1} - \sum_{i=1}^{N} \delta^{t_i-1} p_i \tag{2}$$

where the first term is the present value of the mall and the second term is the present value of the payments to the sellers.

The assumption that the buyer cannot reap the harvests from the land represents the fact that the buyer cannot fully utilize the land as the sellers do before the mall is built. Take the land purchasing case in Chongqing for example. The sellers received utilities by living in their houses, but the buyer could not get all of those utilities even if she owns the houses.

[14] See "Nail House in Chongqing Demolished", *China Daily,* April 3, 2007.
[15] See Krishna and Serrano (1996) and Cai (2000, 2003).

2.3 Equilibrium

Throughout the paper, "equilibrium" refers to subgame perfect equilibrium. We only consider the two-seller case for the rest of this section and in Section 3, and all the analysis is extended to the N-seller case in Section 4.

Since the payment to the first seller is a sunk cost to the buyer, the subgame after the first purchase is the one-seller game between the buyer and the remaining seller. Therefore, let us first examine the one-seller game with land value v_i.

In the one-seller game, the buyer has to bargain with the only seller every period, so the game is simply a two-person alternating bargaining game with inside options only available to the seller. The unique equilibrium of such a game is characterized in Lemma 1, which is a straightforward adaptation of Proposition 6.1 in Muthoo (1999).

Lemma 1. *Let* $\left(p_i^1, q_i^1\right)$ *be the solution to*

$$
\begin{aligned}
p_i &= H_{i,1} + \delta q_i && (3)\\
1 - q_i &= \delta\left(1 - p_i\right). && (4)
\end{aligned}
$$

In the one-seller game between the buyer and seller i*, there is a unique equilibrium in which*

i) the seller offers price q_i^1 *and accepts price no less than* p_i^1*,*

ii) the buyer offers price p_i^1 *and accepts price no more than* q_i^1*.*

Similar to Proposition 6.1 in Muthoo (1999), the seller and the buyer should be indifferent between accepting and rejecting the other player's offer in the unique equilibrium. (3) means that the buyer offers such that the seller is indifferent between accepting and rejecting, and (4) means that the seller offers such that the buyer is indifferent between accepting and rejecting.

The superscript denotes the number of sellers in the game where the variables are considered, but it is omitted where no ambiguity results. (3) and (4) implies that the equilibrium price is

$$
p_i^1 = v_i + \frac{\delta}{1+\delta}\left(1 - v_i\right), \tag{5}
$$

which is always higher than seller's value of the land v_i. Moreover, the equilibrium payoffs are $v_i + \frac{\delta}{1+\delta}\left(1 - v_i\right)$ for the seller and $\frac{1}{1+\delta}\left(1 - v_i\right)$ for the buyer, so they split the surplus $1 - v_i$ as in the Rubinstein bargaining game.

Lemma 1 characterizes the unique equilibrium after the first purchase in the two-seller game, so we focus only on the strategies *before* the first purchase for the rest of this section.

Lemma 2. *Let* (p_2^2, q_2^2) *be the solution of the following equations*

$$
\begin{aligned}
p_2 &= H_{2,1} + \delta q_2 && (6)\\
\delta\left(1 - p_1^1\right) - q_2 &= \delta\left(\delta\left(1 - p_1^1\right) - p_2\right) && (7)
\end{aligned}
$$

If

$$
\delta v_1 + (1+\delta)\, v_2 \le \delta, \tag{8}
$$

the following strategies constitute an equilibrium in the game $\Gamma\left(B, (2, 2, ...)\right)$*:*

i) seller 2 suggests price q_2^2 *and accepts price no less than* p_2^2*,*

ii) the buyer suggests price p_2^2 *to seller 2 and accepts price no more than* q_2^2 *from seller 2.*

Proof: According to (6), seller 2 is indifferent between accepting and rejecting. In particular, if seller 2 accepts p_2 in the current period, his payoff is the left hand side of (6). If seller 2 rejects p_2, he receives a harvest at the end of current period and accepts q_2 in the next period, then his payoff is the right hand side of (6).

Similarly, the buyer is indifferent between accepting and rejecting according to (7). In particular, if the buyer accepts q_2, she pays q_2 to seller 2 in the current period, pays p_1^1 to seller 1 and receives the value of the mall in the next period, so her payoff is the left hand side of (7). If the buyer rejects q_2, she pays p_2 to seller 2 in the next period, pays p_1^1 to seller 1 and receives the value of the mall two periods later, then her payoff is the right hand side of (7).

As a result, neither seller 2 nor the buyer would deviate in the subgame $\Gamma(B, (2, 2, ...))$, so the lemma is proved. ■

Equations (6) and (7) imply that the equilibrium price for seller 2 is

$$p_2^2 = v_2 + \frac{\delta}{1+\delta}\left[\delta(1-p_1^1) - v_2\right]. \tag{9}$$

Substituting (5) and (9) into (1) and (2) gives the equilibrium payoffs for the buyer and seller 2

$$\pi_B^* = \frac{1}{1+\delta}\left[\delta\frac{1}{1+\delta}(1-v_1) - v_2\right] \tag{10}$$

$$\pi_2^* = v_2 + \frac{\delta}{1+\delta}\left[\delta\frac{1}{1+\delta}(1-v_1) - v_2\right] \tag{11}$$

Since Lemma 1 shows that seller 1 sells at price p_1^1 in period 2, his equilibrium payoff is

$$\pi_1^* = v_1 + \delta\frac{\delta}{1+\delta}(1-v_1) \tag{12}$$

After the first purchase, the surplus for seller 1 is $\frac{\delta}{1+\delta}(1-v_1)$ evaluated in period 2 as in Lemma 1. Therefore, with the surplus for seller 1 excluded, the mall worth $\frac{1}{1+\delta}(1-v_1)$ in period 2 or $\delta\frac{1}{1+\delta}(1-v_1)$ in period 1. As a result, the agreement with seller 2 produces a surplus of $\delta\frac{1}{1+\delta}(1-v_1) - v_2$. It is easy to see from (10) and (11) that the buyer and seller 2 also split the surplus $\frac{\delta}{1+\delta}(1-v_1) - v_2$ as in the Rubinstein bargaining game, where (8) guarantees that this surplus is not negative. Therefore, (8) also implies that seller 2 and the buyer are not worse off by participating the bargaining. Intuitively, (8) requires that the land values cannot be too large; otherwise the mall is not profitable for the buyer and the early seller.

Proposition 1. *In the two-seller game with commitment, if the mall is profitable*[16] *as*

[16] Because the sellers agree sequentially, the mall may not be profitabe (for the buyer and first seller) even if it is efficient to build it ($v_1 + v_2 < 1$). We would not have this type of inefficiency if the buyers can make simultaneous or contingent offers (see Section 4).

in (8), there is a unique equilibrium outcome and a unique equilibrium bargaining order where the buyer bargains with the smaller seller until an agreement is reached.

Proof: A sketch of the proof is presented below, and the full proof is in Appendix A. The proof consists of five claims. First, in an equilibrium with agreement in the first period, any equilibrium payoff for the buyer can be reached. Second, consider the equilibria of the subgames with either bargaining order $1, 1, ...$ (the buyer bargains with seller 1 until he agrees) or $2, 2, ...$ (the buyer bargains with seller 2 until he agrees). In these equilibria, the supremum of the buyer's equilibrium payoffs, $\bar{\pi}_B$, can be approached. Third, if the buyer commits to order $2, 2, ...$, the corresponding subgame has a unique equilibrium with an agreement in the first period; when the buyer commits to order $1, 1, ...$, the resulting subgame has either a unique equilibrium with agreement in the first period or equilibria with no agreement at all. Fourth, if the buyer chooses order $1, 1, ...$, her payoff is less than $\bar{\pi}_B$. Fifth, only if the buyer chooses order $2, 2, ...$, the buyer's equilibrium payoff reaches $\bar{\pi}_B$. ■

When δ approaches 1, seller i's harvest in each period $v_i(1-\delta)$ and the mall's profit in each period $1-\delta$ converge to 0. It is easy to see that Proposition 1 holds even for δ close to 1.

It is surprising that the equilibrium outcome[17] is unique because multiple equilibrium outcomes are prevalent in similar bargaining games.[18] For example, if the buyer follows a bargaining order that alternates between the sellers, there are multiple equilibrium outcomes by the same analysis in Theorem 1 of Cai (2000). In our model, uniqueness results because the bargaining order is endogenous. Since the buyer does not choose the alternating order in the equilibria, the multiple equilibrium outcomes do not arise.

It is important to understand the intuition behind this choice. Consider the extreme case in which δ is close to 1. Then, according to (5) and (10), the buyer and the seller split the surplus evenly in each bargaining. Suppose the selling order is small-large, where seller 2 sells in the first period and seller 1 sells in the second. Since δ is close to 1, the total surplus of the mall is approximately $1-v_1-v_2$. The surplus for the second period is $1-v_1$, half of which goes to the last seller according to (5). What is left is the surplus for the first period, $(1-v_1)/2-v_2$, half of which is the buyer's payoff according to (10). Suppose the selling order is large-small, where seller 1 sells in the first period and seller 2 sells in the second. The surplus for the first period is $(1-v_2)/2-v_1$, which is lower than in the small-large order. Since the buyer receives a half of the surplus for the first period, her payoff is lower than in the small-large order. This intuition also works for any δ, which is the content of Claim 4.

In our model explicated above, the sellers' heterogeneity allows the selling orders to affect the surplus after the first purchase. In contrast, it might not be the case in which the sellers' heterogeneity does not allow this, and there could be multiple equilibrium outcomes. For example, Li (2010) considers the heterogeneity in sellers' probabilities to make the first offer, so no matter who sells first the surplus after the first purchase is the

[17] There are multiple equilibria in our bargaining game, but all of them have the same outcome. As in the proof of Claim 3, there is no agreement in $\Gamma(B, (1, 1, ...))$ if $\delta v_2 + (1+\delta) v_1 > \delta$. Therefore, given that any offer is rejected, it is seller 1's equilibrium strategy to offer any price no less than v_1. As a result, the subgame $\Gamma(B, (1, 1, ...))$ has many equilibria, so does the whole game.

[18] However, several papers also demonstrate unique subgame perfect equilibrium in multi-person bargaining games. See Jun (1987), Chae and Yang (1988) and Krishna and Serrano (1996).

same, and he finds multiple equilibrium outcomes with different bargaining orders.

Example 1. Consider a two-seller game with $\delta = 2/3$, $v_1 = 3/8$ and $v_2 = 1/6$. Proposition 1 implies that the buyer bargains with seller 2 until an agreement is reached. In the resulting subgame, the equilibrium strategies before the first purchase are

	accepts	offers
Seller 2	$\geq 1/5$	$13/60$
Buyer	$\leq 13/60$ from seller 2	$1/5$ to seller 2

where the prices are calculated according to Lemma 2. After the first purchase, the equilibrium strategies are given in Lemma 1. As a result, seller 2 sells at price $1/5$ in period 1 and seller 1 sells at price $5/8$ in period 2. According to (1) to (2), the payoffs are $13/24$ for seller 1, $1/5$ for seller 2 and $1/20$ for the buyer.

3 Bargaining without Commitment

The assumption that the buyer can commit to a bargaining order is relaxed in this section. In particular, the buyer does not fix a bargaining order in the beginning of the game. Instead, the buyer picks only one remaining seller at the beginning of each round, and bargains in the same pattern as in the case with commitment. If no agreement is reached in a round, the bargaining moves to the next round where the buyer picks a remaining seller and bargains with him in the same fashion. Note that the buyer can choose any remaining seller to bargain with, so there is no restriction on the bargaining order. From now on, the above game is referred as to the "N-seller game without commitment", or the "N-seller game" where no ambiguity results.

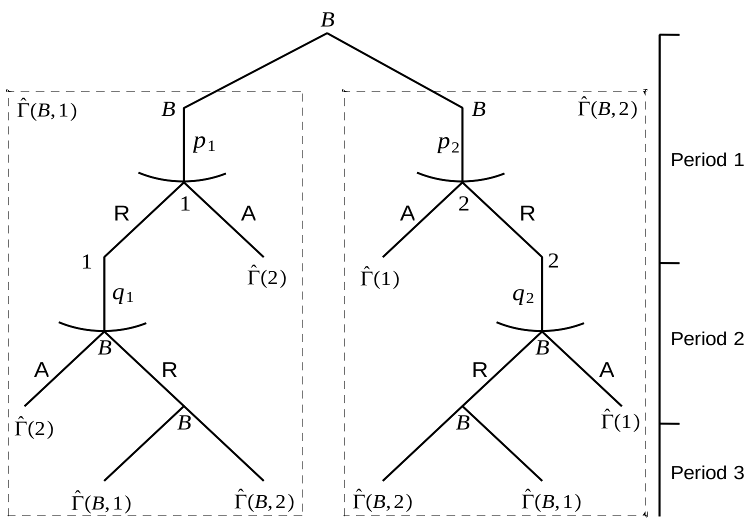

Figure 2. Bargaining without Commitment

We use $\hat{\Gamma}(j, j')$ to denote the subgame in which no seller has sold his land and player j suggests a price to player j' in the first period. $\hat{\Gamma}(i)$ represents the one-seller game between seller i and the buyer who makes the first offer. Figure 2 demonstrates the game tree for the two-seller game.

Given an equilibrium, a bargaining order in the N-seller game without commitment is an infinite sequence of sellers, where the buyer bargains with the tth seller if no agreement has been reached after the buyer bargains with the first $t-1$ sellers in the sequence. After the first agreement, the buyer follows a bargaining order in the resulting $(N-1)$-seller game, and so on.

In the rest of this section, the two-seller game is discussed in two cases. In the first case where the sellers have sufficiently different sizes, we have a unique equilibrium outcome. This case is discussed in the following three steps. We first discuss the subgame in which the buyer bargains with seller 2 first; then study the subgame where the buyer bargains with seller 1 first; finally compares the two scenarios and summarizes the case. In the second case where the sellers have similar sizes, we have multiple equilibrium outcomes and examples are provided.

3.1 Case 1: Sufficiently Different Sizes

Lemma 3 characterizes all the equilibria, and Lemma 4 shows the corresponding outcome is unique. Since the payment to the first seller is a sunk cost to the buyer, the subgame after the first purchase is a one-seller game between the buyer and the remaining seller. Since we already know the unique equilibrium in the one-seller game, the following lemma focuses on the strategies *before* the first purchase.

Lemma 3. *Let $\left(p_2^2, q_2^2\right)$ be the solution of (6) and (7), and $\left(p_1^2, q_{B1}^2\right)$ be the solution of the following equations*

$$\begin{aligned} p_1 &= H_{1,3} + \delta^3 p_1^1 && (13) \\ \delta\left(1 - p_2^1\right) - q_{B1} &= \delta\left(\delta\left(1 - p_1^1\right) - p_2^2\right) && (14) \end{aligned}$$

If (8) and

$$v_1 - v_2 > \frac{\delta}{1+\delta} - \frac{1+2\delta}{1+\delta} v_2, \tag{15}$$

for any $\left(q_1^2, p_{B1}^2\right)$ such that $q_1^2 > q_{B1}^2$ and $p_{B1}^2 < p_1^2$, the strategies below constitute an equilibrium in the game $\hat{\Gamma}(B, 2)$,

i) seller 2 suggests price q_2^2 and accepts price no less than p_2^2,

ii) seller 1 suggests price q_1^2 and accepts price no less than p_1^2,

iii) the buyer bargains with seller 2 until an agreement is reached; suggests price p_2^2 to seller 2 and p_{B1}^2 to seller 1; accepts price no more than q_2^2 from seller 2 and no more than q_{B1}^2 from seller 1.

Proof: Notice that the outcome is the same as in Lemma 2, so the equilibrium payoffs are π_B^*, π_2^* and π_1^* given in (10) to (12). The left hand side of (14) is the buyer's payoff if she accepts price q_{B1}^2 from seller 1, and the right hand side is $\delta\pi_B^*$, so the buyer is indifferent between accepting q_{B1}^2 and rejecting it. As a result, the buyer accepts price no higher than q_{B1}^2 from seller 1. However, (15) implies $q_{B1}^2 < v_1$, so seller 1 cannot afford any price that the buyer would accept, therefore seller 1 suggests q_1^2 which is below the buyer's threshold of acceptance, q_{B1}^2, and does not deviate.

The left hand side of (13) is seller 1's payoff if he accepts p_1^2. If seller 1 rejects p_1^2, he gets π_1^* with two periods of delay, which is the right hand side of (13). Hence, the equation means seller 1 is indifferent between accepting p_1^2 and rejecting it. As a result, seller 1 accepts price no less than p_1^2. However, (15) implies that accepting any price above p_1^2 is not profitable for the buyer, therefore the buyer offers p_{B1}^2, which is below seller 1's threshold of acceptance, and does not deviate.

As in (14), the buyer is indifferent between accepting q_2^2 from seller 2 and rejecting it. So the buyer would not change her threshold of acceptance, q_2^2, and seller 2 would not change his offer q_2^2.

As in (13), seller 2 is indifferent between accepting p_2^2 from seller 2 and rejecting it. So the buyer would not deviate from her threshold of acceptance, p_2^2, and seller 2 would not deviate from his offer p_2^2. ■

Proposition 2. *In the two-seller game without commitment, if the mall is profitable as in (8) and the sellers' sizes are significantly different as in (15), there is a unique equilibrium outcome and a unique bargaining order where the buyer bargains with the smaller seller until an agreement is reached.*

The proof of this lemma is in Appendix B. It is an extension of the uniqueness proof for the Rubinstein bargaining game.[19] In our model, seller 2 always sells first because the buyer gets a negative payoff otherwise. Given the price accepted by seller 2, everything is known according to the unique equilibrium of Rubinstein bargaining game. As a result, the price for seller 2 is the only parameter to be determined for the set of possible outcomes and therefore for the set of possible payoffs as well. Once the set of possible payoffs is characterized by the single parameter, the rest of the proof is parallel to the proof for Rubinstein bargaining game.

Condition (15) requires that, fix the size of the smaller lot, the difference between the sizes should be sufficiently large. Similar to the discussion of (8) after Lemma 2, the buyer and seller 1 split a surplus of $\delta \frac{1}{1+\delta}(1-v_2)-v_1$ if seller 1 sells his land first. However, (15) implies that the surplus is negative, so seller 1 does not sell first otherwise the buyer would have a negative payoff. Therefore, both the offer and counter-offer would be rejected when the buyer bargains with seller 1 first, and $\hat{\Gamma}(B,1)$ would have a delay of two periods in the equilibria. (15) guarantees the unique equilibrium outcome. If this condition is violated, there are multiple equilibrium outcomes with different bargaining orders as will be shown in Case 2.

Let us explain the implications of Lemma 3 and Proposition 2. Lemma 3 characterizes all the equilibria, which share the same outcome $\left(p_1^1, p_2^2, 2, 1\right)$ and the same bargaining order $2, 2, \ldots$. Since the equilibrium outcome is unique according to Proposition 2, all the strategies are uniquely determined by backward induction except q_1^2 and p_{B1}^2. As a result, Lemma 3 also describes all the equilibria of $\hat{\Gamma}(B,2)$.

The game $\hat{\Gamma}(B,1)$ is a proper subgame of $\hat{\Gamma}(B,2)$, so the equilibria and the equilibrium outcome are inherited from Lemma 3. In particular, if there is no agreement in period 1 or 2, then the buyer chooses seller 2 to bargain with and seller 2 sells in period 3 and

[19] See Shaked and Sutton (1984).

seller 1 sells in period 4. As a result, there is a delay of two periods and the payoffs are

$$\begin{aligned}\pi_1^{*\prime} &= H_{1,2}+\delta^2\pi_1^* \\ \pi_2^{*\prime} &= H_{2,2}+\delta^2\pi_2^* \\ \pi_B^{*\prime} &= \delta^2\pi_B^*\end{aligned}$$

This means that delay can happen if a "wrong" order was chosen, which is also true in the game with commitment as discussed after Proposition 1. However, it can be avoid if the buyer chooses the bargaining order in which the buyer bargains with the smallest remaining seller until an agreement is reached.

It is easy to see that Proposition 2 holds when δ is close to 1.

Example 2. Consider Example 1 in the context of no commitment. Proposition 2 implies that the buyer always bargains with seller 2 before the first purchase. The strategies of an equilibrium are given below, and only the strategies before the first purchase are described.

	Accepts	Offers
Seller 1	≥ 0.45	3/4 (rejected)
Seller 2	$\geq 1/5$	13/60
Buyer	$\leq 3/10$ from seller 1	3/8 to seller 1 (rejected)
Buyer	$\leq 13/60$ from seller 2	1/5 to seller 2

The selling prices and payoffs are the same as in Example 1. Suppose the buyer bargains with seller 1 first, then seller 1's offer 3/4 and the buyer's offer 3/8 would be rejected in the first two periods, and the buyer would bargain with seller 2 in the next round.

3.2 Case 2: Similar Sizes

When the sellers have similar sizes, (15) could be violated. The outcome in Lemma 4 is still an equilibrium outcome when (15) is violated, but there are other equilibria with different outcomes and bargaining orders. Two other equilibria are described in the following two lemmas.

First, if we exchange sellers 1 and 2 in Lemma 3, it gives us another set of equilibria.

Lemma 5. *Let* $\left(p_1^{2\prime}, q_1^{2\prime}, p_2^{2\prime}, q_{B2}^{2\prime}\right)$ be the solution to

$$\begin{aligned}p_1 &= H_{1,1}+\delta q_1 \\ \delta\left(1-p_2^1\right)-q_1 &= \delta\left(\delta\left(1-p_2^1\right)-p_1\right) \\ p_2 &= H_{2,3}+\delta^3 p_2^1 \\ \delta\left(1-p_1^1\right)-q_{B2} &= \delta\left(\delta\left(1-p_2^1\right)-p_1\right).\end{aligned}$$

If (8), for any $\left(q_2^{2\prime}, p_{B2}^{2\prime}\right)$ *such that* $q_2^2 > q_{B2}^{2\prime}$ *and* $p_{B2}^2 < p_2^{2\prime}$*, the strategies below constitute an equilibrium in the two-seller game without commitment:*

i) seller 1 accepts price no less than $p_1^{2\prime}$ *and suggests price* $q_1^{2\prime}$*,*

ii) seller 2 accepts price no less than $p_2^{2\prime}$ *and suggests price* $q_2^{2\prime}$*,*

iii) the buyer bargains with seller 1 before the first purchase; accepts price no more than $q_1^{2\prime}$ *from seller 1 and no more than* $q_{B2}^{2\prime}$ *from seller 2; and suggests* $p_1^{2\prime}$ *to seller 1* *and* $p^{2\prime}_{B2}$ *to seller 2.*

The proof is the same as the proof for Lemma 3. In the equilibrium, seller 1 sells in period 1 and seller 2 sells in period 2, and the bargaining order is $1, 1, \ldots$.

Second, there is another set of equilibria in which no agreement is reached in the first two periods.

Lemma 6. *In the two-seller game without commitment, if (8) is satisfied, the following strategies constitute an equilibrium: the buyer chooses to bargain with seller 1 in the first period; everyone follows the strategies in Lemma 3 in* $\hat{\Gamma}(B, 1)$*; everyone follows the strategies in Lemma 5 in* $\hat{\Gamma}(B, 2)$*.*

Proof: Lemma 3 implies that the strategies in $\hat{\Gamma}(B, 2)$ in an equilibrium, and Lemma 5 implies that the strategies in $\hat{\Gamma}(B, 1)$ is also an equilibrium. By backward induction, the buyer chooses seller 1 to bargain with first because the buyer's payoff in $\hat{\Gamma}(B, 1)$ is higher than in $\hat{\Gamma}(B, 2)$. ■

As a result, there is no agreement in the first two periods and seller 2 and seller 1 sell in periods 3 and 4 respectively, and the corresponding bargaining order is $1, 2, 2, \ldots$

Using the equilibrium payoffs in Lemma 3, 5 and 6 as punishments, we can construct many other equilibria; moreover, there is a continuum of equilibria without delay.[20] However, it is difficult to find the full characterization of equilibria or equilibrium payoffs even for the two-seller game. The difficulty is briefly explained below. In order to find the full characterization, we need to find the minimum and maximum for each player's payoffs as in the three-person alternating bargaining game.[21] Both the selling prices and the length of delay could affect the bounds. For example, seller 2's minimum payoff could be reached by a low selling price with shorter delay or by a higher selling price but with longer delay. Moreover, the two factors interact with each other, which makes the problem even harder. Since the maximum length of delay is very likely to be increasing in δ, the difficulty remains even when δ approaches 1.

4 N-Seller Game

In this section, all the results in Section 2 and 3 are extended to the N-seller case by induction on the number of sellers. After the first purchase in the N-seller game, the resulting subgame is a $(N-1)$-seller game, so we only need to study the strategies before the first purchase. Recall that the analysis on the two-seller game also focuses on the strategies before the first purchase, so the analysis in this section is parallel to Section 2 and 3. However, in order to illustrate the conditions that are needed for our results in the N-seller case, two propositions are presented below while their proofs and the discussion of equilibrium prices and payoffs are included in the Appendices C and D.

[20] A similar analysis is used in the three-person alternating bargaining game. See, for instance, Herrero (1984) and Osborne and Rubinstein (1990). Since the equilibria with delay also demonstrate different bargaining orders as in Lemma 5 and Lemma 6, they are not presented for the consideration of space.

[21] See, for instance, Herrero (1984) and Osborne and Rubinstein (1990).

Proposition 3. *In the N-seller game with commitment, if the mall is profitable as in*

$$\sum_{i=1}^{N}\left(\frac{\delta}{1+\delta}\right)^{N-i}v_i<\left(\frac{\delta}{1+\delta}\right)^{N-1}, \tag{16}$$

there is a unique equilibrium outcome and a unique equilibrium bargaining order where the buyer bargains with the smallest remaining seller until an agreement is reached.

Consider the N-seller game without commitment, Proposition 4 extends the analysis in Section 3.1 to the N-seller case.

Proposition 4. *In the N-seller game without commitment, if the mall is profitable as in (16) and the sellers' sizes are significantly different as in*

$$\begin{aligned}&\left(1-\frac{\delta}{\delta+1}\right)(v_{n-1}-v_n)+\sum_{i=1}^{n-1}\left((v_i-v_{i+1})\sum_{j=n-i}^{n-1}\left(\frac{\delta}{1+\delta}\right)^{j}\right)\\ > \;&\left(\frac{\delta}{\delta+1}\right)^{n-1}-v_n\sum_{i=1}^{n}\left(\frac{\delta}{1+\delta}\right)^{i-1}\end{aligned} \tag{17}$$

for $n=2,...,N$, there is a unique equilibrium outcome and a unique bargaining order where the buyer bargains with the smallest remaining seller until an agreement is reached.

Condition (17) ensures that seller i sells first when the remaining sellers are $1,2,...,i$. If it is violated, there could multiple equilibria with different outcomes and different bargaining orders as in Case 2 of the two-seller game. Other equilibria can be constructed similarly as in Lemma 5 and 6. Besides the difficulties explained in Section 3, it is even harder to find a full characterization of the equilibria in the N-seller game. The larger number of sellers allows a much larger number of possible bargaining orders, and there is a large number of possible selling orders given each bargaining order. Moreover, when (17) is violated for some but not all n smaller than N, the full characterization is even more challenging.

5 Discussion and Applications

5.1 Finite vs. Infinite Horizons

If the bargaining game without commitment has a finite horizon, it has a unique equilibrium outcome even when the sellers' sizes are similar. In particular, suppose the game without commitment has a finite horizon of $2T+N-1$ periods where $T>N$.[22] Backward induction finds the unique equilibrium outcome of this game, where the buyer always bargains with the smallest remaining seller. Moreover, when T goes to infinity, the equilibrium outcome of this game converges to an equilibrium outcome in the game with no commitment and infinite horizon, and the limit outcome is the one associated with the bargaining order of increasing size. In this sense, we say that although the game without commitment could have multiple equilibrium outcomes when the sellers are very similar,

[22] For the consideration of space, the formal discussion is put in the supplementary notes for this paper.

only one outcome is "consistent" with the equilibrium outcome in game of finite horizon, and this outcome is associated with the bargaining order of increasing size.

5.2 Inside vs. Outside Options

It is natural to use inside options (ongoing profits from farming) to represent the sizes of sellers. However, outside options can also be used to represent the different sizes of sellers. In particular, suppose a seller can also sell his land in an outside market besides accepting and rejecting the offer when he receives an offer. If any seller chooses the outside option, the bargaining ends[23]. In this setting, our qualitative results would not be affected. Let us demonstrate this in an example of bargaining with commitment. Suppose that there are two sellers with outside options 0 and $1/2$ and the discount factor is close to 1. First, suppose that the buyer bargains with the small seller (with outside option 0) until he agrees. After the small seller agrees, the buyer bargains with the big seller (with outside option $1/2$). The surplus of this bargaining is the difference between the mall's value and the outside option of the big seller, $1 - 1/2$. Since the players are patient, they split this surplus evenly, and the big seller receives surplus $(1 - 1/2)/2$. Therefore, the buyer and the small seller split the remaining surplus $(1 - 1/2)/2$ evenly, and the surplus to the buyer is $(1 - 1/2)/4$. Second, suppose that the buyer bargains with the big seller until he agrees. Similarly, the surplus after the big seller agrees is $1 - 0$, and the small seller receives half of it, $1/2$. Since the total surplus is $1 - 1/2 - 0$, there is no surplus left for the buyer and the big seller. As a result, the buyer prefers to bargain with the small seller until he agrees. The above analysis only illustrates the comparison of two bargaining orders in the bargaining game with commitment, but we can similarly derive the counter parts of other results with inside options.

5.3 Coordination Among Sellers

It is an interesting question that whether the sellers gain by merging into a single agent who bargains on behalf of the sellers. The answer is no. Consider the same example in Section 5.2. Suppose that the two players merge into one agent. Then, this agent has outside option $1/2$, and the bargaining is a standard two-person bargaining game with an outside option. As a result, the agent receives $(1 - 1/2)/2$, which is half of the total surplus. However, the sellers receive a total surplus of $3/8$ if they do not merge. The main reason is that the buyer needs the agreement from each seller, therefore each seller has "veto" power. If the sellers merge and there are less players with "veto" power and their bargaining power is also reduced. Moreover, it may be beneficial for a seller to split his land and let some agents represent the different pieces.

5.4 Heterogeneous Discounting

It is a natural extension to consider players with different discount factors. For the purpose of illustration, we only consider the two-seller game with commitment. Suppose the players have discount factors, $\delta_1, \delta_2, \delta_B$ for sellers 1, 2 and the buyer. Equation (5) becomes

$$p_i^1 = v_i + r_i (1 - v_i)$$

[23] Since the buyer still needs the land, he would start a new bargaining with the new owners, which makes the situation more complicated.

where $r_i = (1 - \delta_B)\, \delta_i / (1 - \delta_i \delta_B)$ and the and equations (6) and (7) become

$$\begin{aligned} p_2 &= v_2 (1 - \delta_2) + \delta_2 q_2 \\ \delta_B \left(1 - p_1^1\right) - q_2 &= \delta_B \left(\delta_B \left(1 - p_1^1\right) - p_2\right) \end{aligned}$$

Then, similar analysis implies that, if the buyer bargains with the small seller until he agrees, the equilibrium prices are $p_1^1 = v_1 + r_1 (1 - v_1)$, $p_2^2 = v_2 + r_2 (\delta_B (1 - p_1) - v_2)$, and the buyer's payoff is $\pi_B = (1 - r_2)(r_1 (1 - v_1) - v_2)$. Similarly, if the buyer bargains with the big seller until he agrees, the buyer's payoff is $\pi'_B = (1 - r_1)(r_2 (1 - v_2) - v_1)$. If $\delta_2 \leq \delta_1$, the buyer's payoff is higher if she bargains with seller 2 (the small seller) first, and our qualitative results would not be affected. However, if seller 2 is much more patient than seller 1, it may be efficient for seller 1 to sell first. This effect could dominate the effect from "sizes", and the buyer may bargain with seller 1 until he agrees in an equilibrium.

5.5 Order of Making Offers

This paper assumes that the buyer makes the first offer in each round of bargaining, however, our result is not hinged with this assumption. For example, consider the same example in Section 5.2, and suppose the sellers make the first offer in each round of bargaining. It is easy to see that the equilibrium prices and payoff are the same. What is important to our results is the presence of inside/outside options. Without inside/outside options, there could be multiple equilibria with different bargaining orders (see, for instance, Cai, 2003 and Li, 2010).

5.6 Cash-Offer vs. Contingent Contracts

This paper considers cash-offer contracts, which are prevalent in real estate business. However, there are other bargaining situations where contingent contracts could be used. Under a contingent contract, the payments are not made until all the sellers have agreed. In contrast to our paper, if the buyer uses contingent contracts in our model, the surplus remains the same after the agreements. Therefore, multiple equilibria with different bargaining orders could arise[24], and our results on unique equilibrium bargaining order would not hold.

5.7 Simultaneous vs. Sequential Offers

In many situations, it is hard or impossible for the buyer to make simultaneous offers to the sellers. However, if the buyer can make simultaneous offers to all the sellers, there is an equilibrium that the sellers agree in the first period because this game is of complete information. Therefore, we cannot discuss bargaining orders in such setup, and our results do not apply.

5.8 Applications and Other Extensions

Besides land purchasing and the two other examples in the introduction, the model is also applicable to voting scenarios. For example, when a country wants to join a trade organization, it has to get permission from all the respective existing members. The

[24] See also Li (2010)

members have different attitudes toward the entry, and the member that likes the entry least corresponds to the seller with largest size in the model. As a result, the applicant should start with the member who favors the entry most.

We may wonder what would happen if the buyer could choose to commit to a bargaining order by paying a fixed positive cost. As in Section 3.2, if the sizes of the sellers are very similar and the buyer does not commit to a bargaining order, there could be multiple equilibrium outcomes including the outcome if the buyer can commit. Therefore, the buyer may not benefit from choosing commitment unless she wants to ensure a minimal payoff.

One of the most interesting extensions is to allow some players to hide some information, for example, the sellers sizes, past offers or deal prices[25]. It would be interesting to examine how the bargaining order affects the players' incentive to reveal their private information. Moreover, the model has the potential for more applications under some other modifications. For example, it can be extended to the case where the buyer needs not to purchase from all the sellers.[26] This modification accommodates the situation when the developer can change[27] the shape of the mall. Besides, it also fits the voting situations where winning requires not only a minimum number of votes but also all the votes from voters with veto rights. Moreover, it would be interesting to explore the situation where the bargaining order or the offers could be confidential,[28] as this paper only presents a game of complete information.

6 Conclusion

This paper studies a non-cooperative, complete-information and infinite-horizon bargaining game with one buyer and multiple sellers heterogeneous in their sizes. The bargaining order in this game is endogenously determined, and three different cases are considered. First, when the buyer can commit to a bargaining order, there is a unique equilibrium outcome where the buyer bargains with the smallest remaining seller until an agreement is reached. Second, when the buyer cannot commit to a bargaining order and the sellers are significantly different, there is also a unique equilibrium outcome associated with the same bargaining order. Third, when the sellers are similar to each other, the game without commitment still has an equilibrium with the same bargaining order, but there could be other equilibrium outcomes with different bargaining orders. However, only the equilibrium outcome, associated with the order of increasing size, is consistent with the equilibrium outcome in the finite-horizon bargaining game with no commitment.

[25] Noe and Wang (2004) examine a finite-horizon bargaining game in which the negotiation history could be secrect.

[26] Chowdhury and Sengupta (2008) also consider this feature.

[27] For example, Yardley (2008) reports a boutique supermarket was altered from its original design and built around a tiny house whose owner refused to sell.

[28] See also Noe and Wang (2004) and Chowdhury and Sengupta (2008).

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Appendices

A Proof of Proposition 1

Let $\bar{\pi}_B$ be the supremum of the buyer's equilibrium payoffs. The proof consists of five claims.

Claim A.1. *If the buyer's payoff is π_B in an equilibrium, the buyer's payoff is at least π_B in another equilibrium with an agreement in the first period.*

Proof: Suppose the buyer's payoff is π_B in the equilibrium E, where the bargaining order is $i_1, i_2, ...$, and the first agreement is reached in period $t > 1$. Then, E induces an equilibrium, E_t for the subgame $\Gamma(B, (i_t, i_{t+1}, ...))$. Note that the buyer does not choose another bargaining order in the subgame $\Gamma(B, (i_t, i_{t+1}, ...))$, but the order chosen in the first period also specifies the sequence of sellers, $i_t, i_{t+1}, ...$, in this subgame.

If there is no agreement in E, the buyer gets zero in both E and E_t. If there is an agreement in E, the sellers sell at the same prices in E_t as in E but $t-1$ periods earlier, therefore the buyer receives a higher payoff than in E. Hence the buyer's payoff in E_t is at least π_B. As a result, it is another equilibrium where the buyer chooses order $i_t, i_{t+1}, ...$ and every player follows the strategies in E_t. In this equilibrium, the buyer's payoff is at least π_B and an agreement is reached in period 1. ■

Claim A.2. *$\bar{\pi}_B$ can be approached by buyer's equilibrium payoffs in the subgames with either order* $1, 1, ...$ *or order* $2, 2, ...$

Proof: Since $\bar{\pi}_B$ can be approached by a sequence of identical payoffs, this claim does not exclude the possibility of unique equilibrium in the subgames with either order $1, 1, ...$ or order $2, 2, ...$ From now on, we are going to say "the equilibrium (payoff) associated with order $1, 1, ...$" instead of "the equilibrium (payoff) in the subgame with order $1, 1, ...$". By the definition of supremum $\bar{\pi}_B$, there exists a sequence of buyer's equilibrium payoffs $\{\pi_B^k\}_{k=1}^{\infty}$ that converges to $\bar{\pi}_B$. Pick any equilibrium payoff π_B^k from this sequence, and denote an associated equilibrium as E^k (there could be other equilibria yielding the same payoff π_B^k for the buyer) and the associated bargaining order as $i_1, i_2, i_3, ...$.

Given equilibrium E^k, denote $\hat{\pi}_B^k$ as the buyer's payoff in the subgame $\Gamma(B, (i_2, i_3, ...))$. Since $\{\pi_B^k\}_{k=1}^{\infty}$ converges to the supremum payoff $\bar{\pi}_B$, it must have a subsequence $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ such that $\pi_B^{k_m} \geq \hat{\pi}_B^k$ for all m. There could be two cases: there is a payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ that is associated with an order starting with i_1, or there is no payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ that is associated with an order starting with i_1.

In the first case, we are going to construct an equilibrium, denoted as E^{*k} below, in an subgame with an order whose first *two* sellers are i_1 such that the buyer's payoff in E^{*k} is no less than that in E^k. Since the constuction is relatively involved, it is helpful to explain its outline first. We first consider the buyer's payoff $\hat{\pi}_B^k$ in the subgame with order $i_2, i_3, ...$ in equilibrium E^k, then we find an equilibrium E^{k_j} in an subgame with order $i_1, i_2', i_3'...$ such that the buyer's payoff is no less than $\hat{\pi}_B^k$. Using the strategies in equilibrium E^{k_j}, we construct the equilibrium E^{*k} in the subgame with order $i_1, i_1, i_2', i_3'....$ The construction in the second case is similar.

In the first case, suppose there is a payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$, $\pi_B^{k_j}$, which is associated with an order starting with i_1. Given $\pi_B^{k_j}$, let E^{k_j} be an equilibrium associated with payoff $\pi_B^{k_j}$, and denote the order associated with this equilibrium as $i_1, i'_2, i'_3, \ldots$.

Because of Claim A.1, we can assume that the first agreement is reached in the first period in both E^k and E^{k_j} without loss of generality. Let us first examine the strategies in equilibrium E^k associated with order $i_1, i_2, i_3, \ldots$. In the first period of subgame $\Gamma(i_1, (i_1, i_2, i_3, \ldots))$, seller i_1 suggests a price $q_{i_1}^k$ such that

$$\delta\left(1 - p^1(v_{\tilde{i}_1})\right) - q_{i_1}^k = \delta \hat{\pi}_B^k, \tag{A.1}$$

where $p^1(v_{\tilde{i}_1})$ is the price for the player other than i_1 and $\hat{\pi}_B^k$ is the buyer's payoff in the subgame $\Gamma(B, (i_2, i_3, \ldots))$ according to equilibrium E^k. The price $q_{i_1}^k$ is accepted by the buyer. In the first period of subgame $\Gamma(B, (i_1, i_2, i_3, \ldots))$, the buyer suggests a price $p_{i_1}^k$ such that seller i_1 is indifferent between accepting and rejecting,

$$p_{i_1}^k = H_{i_1,1} + \delta q_{i_1}^k, \tag{A.2}$$

and $p_{i_1}^k$ is also accepted by the seller.

An equilibrium, E^{*k}, with the order $i_1, i_1, i'_2, i'_3, \ldots$ is described as follows. The strategies in the subgame $\Gamma(B, (i_1, i'_2, i'_3, \ldots))$ are the same as those in equilibrium E^{k_j}. In the subgame $\Gamma(i_1, (i_1, i_1, i'_2, i'_3, \ldots))$, seller i_1 suggests a price $q_{i_1}^{*k}$ and it is accepted. In the subgame $\Gamma(B, (i_1, i_1, i'_2, i'_3, \ldots))$, the buyer suggests a price $p_{i_1}^{*k}$ and it is also accepted. In particular, in the first period of $\Gamma(i_1, (i_1, i_1, i'_2, i'_3, \ldots))$, seller i_1 suggests a price $q_{i_1}^{*k}$ such that

$$\delta\left(1 - p^1(v_{\tilde{i}_1})\right) - q_{i_1}^{*k} = \delta \pi_B^{k_j}. \tag{A.3}$$

In the first period of $\Gamma(B, (i_1, i_1, i'_2, i'_3, \ldots))$, the buyer offers such that seller i_1 is indifferent between accepting and rejecting and the seller accepts it, so

$$p_{i_1}^{*k} = H_{i_1,1} + \delta q_{i_1}^{*k} \tag{A.4}$$

Now let us compare equilibria E^{k_j} and E^{*k}. By definition of $\pi_B^{k_j}$, we have $\pi_B^{k_j} \geq \hat{\pi}_B^k$, therefore (A.1) and (A.3) imply $q_{i_1}^{*k} \leq q_{i_1}^k$, which, combined with (A.2) and (A.4), gives $p_{i_1}^{*k} \leq p_{i_1}^k$. Notice that $p_{i_1}^{*k}$ and $p_{i_1}^k$ are accepted in E^k and E^{*k} respectively, so the buyer's payoff in E^{*k} is no less than that in E^k.

In the second case, each payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ has equilibrium order starts with $\tilde{i}_1 \neq i_1$. Since $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ converges to the supremum by definition, there also exists a payoff $\pi_B'^{k_j}$ in this sequence that is no less than $\hat{\pi}_B^k$. Denote the order associated with $\pi_B'^{k_j}$ as $\tilde{i}_1, i''_2, i''_3\ldots$ Similarly as in the first case, there is an equilibrium with order $\tilde{i}_1, \tilde{i}_1, i''_2, i''_3\ldots$ giving the buyer at least π_B^k.

In sum, we have shown that for any sequence of equilibrium payoffs converging to $\bar{\pi}_B$, there is another sequence also converge to $\bar{\pi}_B$, and each equilibrium payoff in this sequence has the same first two elements in its bargaining order.

By induction, if there exists a sequence of equilibria with first t elements identical whose buyer's payoffs converge to $\bar{\pi}_B$, there is another sequence of equilibria with identical first $t+1$ elements, where the buyer's payoffs converge to $\bar{\pi}_B$. Hence, $\bar{\pi}_B$ can be approached by equilibrium payoffs in the subgames with either order $1, 1, \ldots$ or order $2, 2, \ldots$. ■

Claim A.3. $\Gamma(B,(2,2,...))$ *has a unique equilibrium with an agreement in the first period,* $\Gamma(B,(1,1,...))$ *either has no agreement or has a unique equilibrium with an agreement in the first period.*

Proof: The proof is similar to the analysis for the Rubinstein bargaining game.[29] In particular, the supremum of p_2 and the supremum of q_2 in the equilibria of $\Gamma(B,(2,2,...))$ satisfy (6) and (7), so do the infimum of p_2 and the infimum of q_2. Therefore the supremum and infimum of p_2 coincide, which implies that the selling price for seller 2 is unique. As a result, the unique outcome is $\left(p_1^1, p_2^2, 2, 1\right)$ where p_2^2 is given in (9).

Given the bargaining order, seller 1 sells first or there is no agreement in $\Gamma(B,(1,1,...))$. Suppose seller 1 sells first, his price is at least v_1, then the buyer's payoff is at most $-v_1 - \delta p_2^1 + \delta$, which is negative when $\delta v_2 + (1+\delta) v_1 > \delta$. As a result, $\Gamma(B,(1,1,...))$ has no agreement if $\delta v_2 + (1+\delta) v_1 > \delta$. On the other hand, if $\delta v_2 + (1+\delta) v_1 \leq \delta$, similarly as for $\Gamma(B,(2,2,...))$, $\Gamma(B,(1,1,...))$ also has a unique equilibrium with an agreement in the first period. ■

Claim A.4. *The buyer's equilibrium payoff in* $\Gamma(B,(1,1,...))$ *is less than* $\bar{\pi}_B$.

Proof: $\Gamma(B,(2,2,...))$ has a unique equilibrium according to Claim A.3, and the equilibrium is given in Lemma 2. Therefore, the buyer's payoff is given in (10), which is positive because of (8).

Claim A.3 implies that $\Gamma(B,(1,1,...))$ has no agreement or a unique equilibrium with an agreement in the first period. If $\Gamma(B,(1,1,...))$ has no agreement, the buyer's payoff is zero. If $\Gamma(B,(1,1,...))$ has a unique equilibrium with an agreement in the first period, an analogue of (6) and (7) gives the unique selling price for seller 1, so the buyer's payoff is

$$\pi'_B = \frac{1}{1+\delta}\left[\delta \frac{1}{1+\delta}(1-v_2) - v_1\right]$$

which is also less than π_B^*. Hence, the buyer's equilibrium payoff in $\Gamma(B,(1,1,...))$ is less than π_B^*, so it is also less than $\bar{\pi}_B$. ■

Claim A.5. *The buyer's equilibrium payoff is* $\bar{\pi}_B$ *only when the bargaining order is* $2, 2, ...$

Proof: Claim A.2 to A.4 imply that the buyer's equilibrium payoff in $\Gamma(B,(2,2,...))$ is $\bar{\pi}_B$, so it is sufficient to show that the buyer's equilibrium payoff given any other order is less than $\bar{\pi}_B$. Suppose there is an equilibrium, E_2 with another order $i_1, i_2, i_3, ...$ where the buyer's payoff is also $\bar{\pi}_B$, we are going to show that this assumption leads to a contradiction.

First consider the case with $i_1 = 2$. Since the order $i_1, i_2, ...$ is different from $2, 2, ...$, assume $i_2 = 1$ without loss of generality.[30] By the same reason in Claim A.1, E_2 has an

[29] See Fudenberg and Tirole (1991), pp 115-116.

[30] To see why it is without loss of generality to assume i_2 is the first 1 in the order, consider, for example, the case where i_3 is the first 1 in the bargaining order. If the buyer's payoff is less than $\bar{\pi}_B$ in the subgame with order $i_2, i_3, ...$, then by backward induction her payoff is also less than $\bar{\pi}_B$ in the game with order $i_1, i_2, ...$

agreement in period 1. Since seller 2's price is no less than v_2, we have

$$\delta\left(1-p_1^1\right)-v_2 \geq \bar{\pi}_B \tag{A.5}$$

In the first period of $\Gamma(2,(2,1,i_3,...))$, seller 2 offers q_2'' such that

$$\delta\left(1-p_1^1\right)-q_2''=\delta\pi_B'' \tag{A.6}$$

where π_B'' is the payoff of $\Gamma(B,(1,i_3,...))$ in E_2, and $q_2'' \geq v_2$ is guaranteed by

$$\delta\left(1-p_1^1\right)-v_2 \geq \bar{\pi}_B \geq \pi_B''$$

where the first inequality is given by (A.5) and the second inequality comes from the definition of $\bar{\pi}_B$. Similarly, the buyer offers p_2'' such that seller 2 is indifferent between accepting and rejecting in the first period of $\Gamma(B,(2,i_2,i_3,...))$, so

$$p_2''=H_{2,1}+\delta q_2'' \tag{A.7}$$

In the unique equilibrium given the order $2,2,...$, the buyer offers p_2^2 and seller 2 offers q_2^2 according to (6) and (7) that can be rewritten as

$$\delta\left(1-p_1^1\right)-q_2^2=\delta\bar{\pi}_B \tag{A.8}$$

Claim A.4 implies that $\pi_B''<\bar{\pi}_B$, so we have $q_2^2<q_2''$ by comparing (A.8) and (A.8). Similarly, by comparing (6) and (A.7), we have $p_2^2<p_2''$, hence the buyer's payoff in E_2 is less than $\bar{\pi}_B$. This is a contradiction to the definition of E_2.

Similarly, there is also a contradiction when $i_1=1$. Hence, if the bargaining order is different from $2,2,...$, the buyer's equilibrium payoff is less than $\bar{\pi}_B$. ■

Claim A.1 to A.5 prove Proposition 1. ■

B Proof of Proposition 2

Lemma B.1. *Perpetual disagreement cannot be an equilibrium outcome.*

Proof: It is easy to see $\pi_B=0$ under perpetual disagreement.

If the buyer first purchases from seller 1, she would have negative payoff, so seller 2 is the first seller and sells in period t_2 for a price of p_2^2, then seller 1 sells in the next period at price p_1^1. In order to have a balanced budget, the total payments to all the players should equal the total value of the mall,

$$p_2^2+\delta p_1^1+\delta\pi_B=\delta,$$

which implies

$$p_2^2=\delta-\delta p_1^1-\delta\pi_B. \tag{B.1}$$

So seller 2's payoff is

$$\begin{aligned}\pi_2 &= H_{2,t_2-1}+\delta^{t_2-1}p_2^2\\ &= H_{2,t_2-1}+\delta^{t_2-1}\left(\delta-\delta p_1^1-\delta\pi_B\right)\end{aligned}$$

where the second inequality comes from (B.1). π_2 is a function of t_2 and π_B, and it has an upper bound:

$$\begin{aligned}\overline{\pi}_2 &\equiv \max_{t_2\geq 2,\pi_B\geq 0}\left[H_{2,t_2-1}+\delta^{t_2-1}\left(\delta-\delta p_1^1-\delta\pi_B\right)\right]\\ &= \delta-\delta p_1^1\end{aligned}$$

When seller 2 faces an offer, his payoff also has the same upper bound. As a result, if seller 2 rejects the buyer's offer, he gets one period of harvest in the current period and at most $\overline{\pi}_2$ in the next period. Because $H_{2,1}+\delta\overline{\pi}_2<\overline{\pi}_2$, there exits $\varepsilon>0$ such that

$$H_{2,1}+\delta\overline{\pi}_2+\varepsilon<\overline{\pi}_2.$$

If the buyer offers to seller 2 the price $H_{2,1}+\delta\overline{\pi}_2+\varepsilon$ in period 1, seller 2 would accept it because he receives at most $H_{2,1}+\delta\overline{\pi}_2$ otherwise. Then the buyer's payoff π_B satisfies the following budget balance condition:

$$\left(H_{2,1}+\delta\overline{\pi}_2+\varepsilon\right)+\delta p_1^1+\delta\pi_B=\delta$$

so

$$\begin{aligned}\pi_B &= -\left(H_{2,1}+\delta\overline{\pi}_2+\varepsilon\right)-p_1^1+1\\ &> -\left(H_{2,1}+\delta\overline{\pi}_2\right)/\delta-p_1^1+1\\ &= 0\end{aligned}$$

Hence the buyer could always guarantee himself positive payoff by offering $H_{2,1}+\delta\overline{\pi}_2+\varepsilon$ to seller 2 in the first period. So permanent disagreement cannot be an equilibrium outcome.
■

Proof of Lemma 4: Lemma B.1 implies that there is always a first seller. Suppose the first seller sells in period t of $\hat{\Gamma}(B,2)$ and $t>1$. As discussed after Lemma 3, the buyer would have a negative payoff if seller 1 sells first, so the first seller must be seller 2. Let m_B and M_B be the infimum and supremum of equilibrium payoffs of the buyer, and we have $m_B, M_B>0$.

Let (π_1,π_2,π_B) be a payoff vector, and $U(j,j')$ denotes the set of all equilibrium payoff vectors of subgame $\hat{\Gamma}(j,j')$.

Suppose seller 2 sells immediately at price p_2 in subgame $\hat{\Gamma}(B,2)$, then seller 2 and the buyer's payoffs are

$$\begin{aligned}\pi_B &= \delta-\delta p_1^1-p_2\\ \pi_2 &= p_2\end{aligned}$$

which imply

$$\pi_2+\pi_B=\delta-\delta p_1^1\equiv\pi_{-1}(B,2)$$

Similarly, if seller 2 sells in the first period of subgame $\hat{\Gamma}(2,B)$, we have

$$\pi_2+\pi_B=\pi_{-1}(B,2)$$

If seller 2 sells at price p_2 in the third period of $\hat{\Gamma}(B,1)$, the payoffs for seller 2 and the buyer are

$$\begin{aligned}\pi_B &= \delta^3 - \delta^3 p_1^1 - \delta^2 p_2 \\ \pi_2 &= H_{2,2} + \delta^2 p_2\end{aligned}$$

which imply

$$\pi_2 + \pi_B = \delta^3 - \delta^3 p_1^1 + H_{2,2} \equiv \pi_{-1}(B, 1)$$

Similarly, if seller 2 sells in the second period of $\hat{\Gamma}(1, B)$, we have

$$\pi_2 + \pi_B = \delta^2 - \delta^2 p_1^1 + H_{2,1} \equiv \pi_{-1}(1, B)$$

Let m_{B2} and M_{B2} be the infimum and supremum of π_2 in $U(B, 2)$ and m_{2B} and M_{2B} be the infimum and supremum of π_2 in $U(2, B)$. In $\hat{\Gamma}(B, 2)$, we have

$$M_{B2} = H_{2,1} + \delta M_{2B} \tag{B.2}$$

$$m_{B2} = H_{2,1} + \delta m_{2B} \tag{B.3}$$

In $\hat{\Gamma}(2, B)$, we have

$$\pi_{-1}(2, B) - M_{2B} = \delta M_B \tag{B.5}$$

$$\pi_{-1}(2, B) - m_{2B} = \delta m_B \tag{B.6}$$

Since there could not be agreement in the first two periods of subgame $\hat{\Gamma}(B, 1)$, the infimum and supremum of π_B in the subgame is $\delta^2 M_B$ and $\delta^2 m_B$.

Since $\delta < 1$ and $m_B, M_B > 0$, infimum and supremum of π_B of the whole game must satisfies

$$M_B = \pi_{-1}(B, 2) - m_{B2} \tag{B.7}$$

$$m_B = \pi_{-1}(B, 2) - M_{B2} \tag{B.8}$$

Since there is a unique solution to (B.2)-(B.8), we have

$$\begin{aligned}M_{B2} &= m_{B2} \\ M_{2B} &= m_{2B} \\ M_B &= m_B\end{aligned}$$

Suppose that there is an equilibrium with delay, then the equilibrium payoff of the buyer must be lower than that in an equilibrium without delay. Thus we have a contradiction because we would have $m_B < M_B$. Hence, there is unique element in $U(B, 2)$, $U(2, B)$, $U(B, 1)$ and $U(1, B)$. ■

C Proof of Proposition 3

Proof: The arguments for Lemma 2 and Proposition 1 also apply to the N-seller game with commitment. As a result, only a sketch of the proof is provided below.

Let π_B^{N-1*} be the unique equilibrium payoff in $(N-1)$-seller game. As in Lemma 2, let $\left(p_N^N, q_N^N\right)$ be the solution to the equations

$$\begin{aligned} p_N &= H_{N,1} + \delta q_N && \text{(C.1)} \\ \delta \pi_B^{N-1*} - q_N &= \delta\left(\delta \pi_B^{N-1*} - p_N\right) && \text{(C.2)} \end{aligned}$$

There is an equilibrium where the buyer suggests price p_N^N to seller N and accepts price no more than q_N^N from seller N; and seller N suggests price q_N^N and accepts price no less than p_N^N. The corresponding equilibrium payoff for the buyer is

$$\pi_B^{N*} = \delta \pi_B^{N-1*} - p_N^N$$

and it is easy to show by induction that

$$\pi_B^{N*} = \frac{1}{1+\delta}\left(\left(\frac{\delta}{\delta+1}\right)^{N-1} - \sum_{i=1}^{N}\left(\frac{\delta}{1+\delta}\right)^{N-i} v_i\right) \tag{C.3}$$

Equilibrium price for seller N can be solved from (C.1), (C.2) and (C.3),

$$p_N^N = \frac{\delta}{1+\delta}\left(\left(\frac{\delta}{\delta+1}\right)^{N-1} - \sum_{i=1}^{N}\left(\frac{\delta}{1+\delta}\right)^{N-i} v_i\right) \tag{C.4}$$

As a result, (16) ensures that the mall is profitable for the buyer and seller N. Moreover, it is easy to verify that (16) also implies that any other seller's equilibrium price is also higher than the value of his land. (C.4) gives the equilibrium price for seller N in the N-seller game, and it also gives any seller n's equilibrium price p_n^n in the N-seller game if N is replaced with n in the equation. Therefore, the unique equilibrium outcome is $\left(p_1^1, p_2^2, ..., p_N^N, N, N-1, ..., 1\right)$.

Following the same reasoning in the proof of Proposition 1, we can show that there is a unique equilibrium outcome with the unique equilibrium bargaining order $N, N, ...$ ■

D Proof of Proposition 4

Consider the N-seller game without commitment. Lemma D.1 and Proposition 4 extends the analysis in Section 3.1 to the N-seller case. (C.3) also defines a function $\pi_B^{N*}(v_1, ..., v_N)$ for any N, so $\pi_B^{N-1*}(\mathbf{v}_{-i})$ denotes the equilibrium payoff in the $(N-1)$-seller game without seller i where $\mathbf{v}_{-i} = (v_1, ..., v_{i-1}, v_{i+1}, ..., v_N)$. It is easy to see from (C.3) that π_B^{n*} is a linear function of $v_1, ..., v_n$, and v_i's coefficient is smaller than v_{i+1}.

Lemma D.1. *For every $i \in \{1, ..., N-1\}$, let $\left(p_i^N, q_{Bi}^N\right)$ be the solution to the equations*

$$\begin{aligned} p_i &= H_{i,N+1} + \delta^{N+1} p_i^i && \text{(D.1)} \\ \delta \pi_B^{N-1^*}(\mathbf{v}_{-i}) - q_{Bi} &= \delta \pi_B^{N*} && \text{(D.2)} \end{aligned}$$

If (17) for $n = 2, ..., N$ and (16) are satisfied, given any $\left(q_i^N, p_{Bi}^N\right)$ such that $q_i^N > q_{Bi}^N$ and $p_{Bi}^N < q_i^N$, the following strategies constitute an equilibrium for the game $\hat{\Gamma}(B, N)$:

i) seller N suggests price q_N^N and accepts price no less than p_N^N,

ii) seller i suggests price q_i^N and accepts price no less than p_i^N for $i = 1, \cdots, N-1$, *iii) the buyer bargains with seller N before the first agreement; suggests price p_N^N to seller N and price p_{Bi}^i to seller $i = 1, \cdots, N-1$; and accepts price no more than q_N^N from seller N and price no more than q_{Bi}^N from seller $i = 1, \cdots, N-1$.*

Proof: Induction on the number of sellers is used to prove. Suppose the lemma is true for $N = k$. By the same backward induction analysis in Lemma 3, the above lemma is also true for $N = k + 1$. Therefore, only the arguments related with the interpretations are given below.

The conditions in this lemma also have similar interpretations as in Lemma 3. For any $i < N$, (D.1) means seller i is indifferent between accepting and rejecting the buyer's offer p_i^N, and (D.2) ensures that the buyer is indifferent between accepting and rejecting seller i's offer q_{Bi}^N.

The sellers sell in the order of increasing size in the first N periods. If seller $N-1$ and seller N exchange their selling time, the buyer's payoff is $\pi_B^{N*}(v_1, ..., v_{N-2}, v_N, v_{N-1})$, and (17) for $n = N$ is equivalent to

$$\pi_B^{N*}(v_1, ..., v_{N-2}, v_N, v_{N-1}) < 0.$$

Since v_i's coefficient is smaller than v_{i+1}'s in π_B^{N*} for all i, (17) also implies that the buyer receives a negative payoff if any seller other than N sells first. Moreover, (17) holds for $n = 2, ..., N$, so the smallest remaining seller has to be the first to sell otherwise the buyer receives a negative payoff. ■

Proof of Proposition 4: Similarly as in Lemma 4, the equilibrium outcome implied by Lemma D.1 is also unique in $\hat{\Gamma}(B, N)$. As in Section 3, if the buyer chooses a seller other than N to bargain with first, the resulting subgame is a proper subgame of $\hat{\Gamma}(B, N)$, so there is no agreement in the first two periods and the buyer chooses seller N to bargain with in the third period and an agreement is reached immediately. As a result, if the buyer bargains with any other seller first, all the selling prices remain the same but there would be two periods of delay, hence the buyer bargains with the smallest remaining seller. ■

Supplementary Notes for "Bargaining Order in a Multi-Person Bargaining Game"

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Abstract

This note discusses the two-seller game with a finite horizon of $2T+1$ periods and no commitment. This game has a unique equilibrium outcome where the buyer bargains with the smaller seller until an agreement is reached. As T goes to infinity, the outcome converges to an equilibrium outcome in the two-seller game with infinite horizon. Moreover, the limit outcome in the game with infinite horizon is again associated with the bargaining order in which the buyer bargains with the smaller seller until an agreement is reached.

1 Two-seller game with $2T+1$ periods

This note studies the two-seller game with a finite horizon of $2T+1$ periods and no commitment.[1] "No commitment" is omitted hereafter since the game with commitment is not discussed here. The two-seller game with $2T+1$ periods is the same as the two-seller game defined in Section 3 of "Bargaining Order in a Multi-Person Bargaining Game" except that the game ends after $2T+1$ periods.

$\bar{G}(j, j', t)$ denotes the two-seller game that has a horizon of t periods, and player j offering to j' in the first period. $\bar{G}(i, t)$ denotes the one-seller game that has a horizon of t periods and B making the first offer. Recall that $G(j, j')$ denotes the two-seller game with infinite horizon where player j makes the first offer to j', and $G(i)$ denotes the one-seller game with infinite horizon where i is the only seller and the buyer makes the first offer. The buyer offers $p^n_{i,t}$ to i in the first period of the n-seller game with t periods, and seller i offers $q^n_{i,t}$ in the second period of the n-seller game with t periods.

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[1] The argument in this note can also be extended to the N-seller game with a horizon of $2T+N-1$ periods and no commitment.

Lemma S1: *In the two-seller game with 3 periods, if the mall is profitable as in*

$$(1-\delta)\,v_2+\delta\,(1-\delta)\,v_1\leq\delta\,(1-2\delta)\,, \tag{S1}$$

the buyer bargains with seller 2 first until an agreement is reached.

Proof: Backward induction is used. Let us first examine the last period. If neither seller has agreed in the last period, every player gets 0. Suppose that only seller i has not agreed in the last period. If the buyer offers in the last period, she suggests

$$p_{i,1}^{1}=0.$$

If seller i offers in the last period, he suggests

$$q_{i,1}^{1}=1.$$

where the second subscript indicates in which period the prices are considered.

Let us move to the second period. On the one hand, suppose that neither seller has agreed in period 2. If seller i offers in period 2, he suggests $q_{i,2}^{2}$ such that the buyer is indifferent between accepting and rejecting:

$$\delta\left(1-p_{j,1}^{1}\right)-q_{i,2}^{2}=0$$

On the other hand, suppose only seller i has not agreed. If the buyer offers in period 2, she offers $p_{i,2}^{1}$ such that the seller is indifferent between accepting and rejecting:

$$p_{i,2}^{1}=H_{i,1}+\delta q_{i,1}^{1}$$

where the right land side is the harvest of period 2 and seller i's price in period 3.

Finally, consider the first period. If the buyer bargains with seller i in the first period, she offers $p_{i,3}^{2}$ such that seller i is indifferent between accepting and rejecting:

$$p_{i,3}^{2}=H_{i,1}+\delta q_{i,2}^{2}$$

As a result, if the buyer bargains with seller 2 in the first period, her payoff is

$$\pi_{B,3}=\delta\left(1-p_{1,2}^{1}\right)-p_{2,3}^{2}$$

In contrast, if the buyer bargains with seller 1 in the first period, her payoff is

$$\pi_{B,3}'=\delta\left(1-p_{2,2}^{1}\right)-p_{1,3}^{2}$$

(S1) ensures that $\pi_{B,3}$ is non-negative. If $\pi'_{B,3}$ is negative, then there is no agreement in the first two periods and the mall is not built, so the buyer prefers bargaining with seller 2 in the first period. If $\pi'_{B,3}$ is positive, it can be verified that $\pi_{B,3} > \pi'_{B,3}$, therefore the buyer chooses seller 2 in the first period. ■

Since the buyer offers first in a one-seller game with a finite horizon, seller 1 offers in the last period of $\bar{G}(1, 2t+2)$, therefore we have

$$\begin{aligned} p^1_{1,2t+2} &= H_{2,1} + \delta q^1_{1,2t+1} \\ 1 - q^1_{1,2t+1} &= \delta\left(1 - p^1_{1,2t}\right) \end{aligned}$$

for all t. Hence, we have

$$p^1_{1,2t+2} = v_2 \frac{1-\delta^{2t+2}}{1+\delta} + \delta \frac{1-\delta^{2t}}{1+\delta} + \delta^{2t+1} \tag{S2}$$

Similarly, the buyer offers in the last period of $\bar{G}(1, 2t+1)$, therefore

$$\begin{aligned} p^1_{1,2t+1} &= H_{2,1} + \delta q^1_{1,2t} \\ 1 - q^1_{1,2t} &= \delta\left(1 - p^1_{1,2t-1}\right) \end{aligned}$$

for all t and

$$p^1_{1,2t+1} = v_2 \frac{1-\delta^{2t}}{1+\delta} + \delta \frac{1-\delta^{2t}}{1+\delta} + \delta^{2t} \tag{S3}$$

Consider the two-seller game with $2t+1$ periods. Suppose that the buyer always bargains with the smallest seller until an agreement is reached, then $p^2_{2,2t+1}$ for $t = 1, 2, ...$ can be solved recursively from the equations below

$$p^2_{2,2t+1} = H_{2,1} + \delta q^2_{2,2t} \tag{S4}$$

$$\delta\left(1 - p^1_{1,2t-1}\right) - q^2_{2,2t} = \delta\left[\delta\left(1 - p^1_{1,2t-2}\right) - p^2_{2,2t-1}\right] \tag{S5}$$

Lemma S2: *Suppose that the buyer chooses to bargain with seller 2 until an agreement is reached in the two-seller game with $2k-1$ periods if the mall is profitable given the horizon $2k-1$ as in*

$$\delta\left(1 - p^1_{1,2k-2}\right) - p^2_{2,2k-1} \geq 0, \tag{S6}$$

then the buyer also chooses to bargain with seller 2 until an agreement is reached in the two-seller game with a horizon of $2k+1$ periods if the mall is profitable given the horizon $2k+1$ as in

$$\delta\left(1 - p^1_{1,2k+2}\right) - p^2_{2,2k+1} \geq 0. \tag{S7}$$

Proof: First, I claim that (S7) implies (S6), which means that if the mall is profitable given a horizon of $2k+1$ periods, it is also profitable given a shorter horizon of $2k-1$ periods. (S4) and (S5) imply that

$$p^2_{2,2k+1} = H_{2,1} + \delta^2\left(1 - p^1_{1,2k-1}\right) - \delta^3\left(1 - p^1_{1,2k-2}\right) + \delta^2 p^2_{2,2k-1}$$

Therefore

$$\begin{aligned} & \pi_{B,2k+1} - \pi_{B,2k-1} \\ = \quad & \delta\left(1 - p^1_{1,2k}\right) - \delta^2\left(1 - p^1_{1,2k-1}\right) - H_{2,1} - \left(1 - \delta^2\right)\pi_{B,2k-1} \\ \leq \quad & \delta\left(1 - p^1_{1,2k}\right) - \delta^2\left(1 - p^1_{1,2k-1}\right) - H_{2,1} \end{aligned}$$

where $\pi_{B,2k+1} \equiv \delta\left(1 - p^1_{1,2k+2}\right) - p^2_{2,2k+1}$ and $\pi_{B,2k-1} \equiv \delta\left(1 - p^1_{1,2k-2}\right) - p^2_{2,2k-1}$. Substituting (S2) and (S3) into the equation above, we can verify that the right hand side of the above equation is negative. Hence, $\pi_{B,2k+1} < \pi_{B,2k-1}$, therefore (S7) implies (S6).

Suppose the buyer chooses seller 1 in the first period. If no agreement is reached in the first two periods, then in period 3 the buyer choose seller 2 to bargain with by assumption. It is easy to see that the buyer gets higher payoff by choosing seller 2 in the first period.

If an agreement is reached in the first two periods, seller 1 is indifferent between accepting and rejecting in period $t = 1$ and the buyer is indifferent between accepting and rejecting in period $t = 2$. It is easy to see that the buyer gets a higher payoff by choosing seller 2 in the first period. ■

Lemma S1 and S2 prove the proposition below.

Proposition S1: *In the two-seller game with* $2T+1$ periods, *if the mall is profitable as in*

$$\delta\left(1 - p^1_{1,2T}\right) - p^2_{2,2T+1} \geq 0,$$

there is a unique equilibrium outcome where the buyer bargains with the smaller seller until an agreement is reached.

2 Finite horizon vs. Infinite horizon

Lemma S3: *The equilibrium price in* $\bar{G}(i,t)$ *converges to the equilibrium price in* $G(i)$ *as* t *approaches infinity.*[2]

Lemma S4: *If the mall is profitable as in (8), seller 2's equilibrium price in* $G(B,2,2T+1)$ *converges to the equilibrium price* p^2_2 *in* $G(B,2)$ *as* T *approaches infinity.*

[2] See in Osborne and Rubinstein (1990), pp 54.

Proof: According to Lemma S2, the mall is profitable given any finite horizon if (8) is satisfied. Therefore, Proposition 1 implies that the buyer bargaining with the smaller seller until an agreement is reached and the equilibrium strategies can be solved from (S4) and (S5).

If T approaches infinity, (S4) and (S5) becomes

$$\begin{aligned}\lim_{t\to\infty} p^2_{2,2t+1} &= H_{2,1} + \delta \lim_{t\to\infty} q^2_{2,2t} \\ \delta\left(1 - \lim_{t\to\infty} p^1_{1,2t-1}\right) - \lim_{t\to\infty} q^2_{2,2t} &= \delta\left[\delta\left(1 - \lim_{t\to\infty} p^1_{1,2t-2}\right) - \lim_{t\to\infty} p^2_{2,2t-1}\right]\end{aligned}$$

Lemma S3 implies that $\lim_{t\to\infty} p^1_{1,2t-1} = \lim_{t\to\infty} p^1_{1,2t-2} = p^1_1$, so

$$\begin{aligned}\lim_{t\to\infty} p^2_{2,2t+1} &= H_{2,1} + \delta \lim_{t\to\infty} q^2_{2,2t} \\ \delta\left(1 - p^1_1\right) - \lim_{t\to\infty} q^2_{2,2t} &= \delta\left[\delta\left(1 - p^1_1\right) - \lim_{t\to\infty} p^2_{2,2t-1}\right]\end{aligned}$$

Therefore, it can be verified that the equations above imply $\lim_{t\to\infty} p^2_{2,2t+1} = \lim_{t\to\infty} p^2_{2,2t-1} = p^2_2$ and $\lim_{t\to\infty} q^2_{2,2t} = q^2_2$, where p^2_2 and q^2_2 are equilibrium offers in the two-seller game with infinite horizon. As a result, the lemma above is proved. ■

Lemma S3 and S4 prove the proposition below.

Proposition S2: *If the mall is profitable as in (8), the equilibrium outcome of $\bar{G}(B, 2, 2T+1)$ converges to the equilibrium outcome $\left(2, 1, p^1_1, p^2_2\right)$ of $G(B, 2)$ as T goes to infinity.*