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# Empirical Evidence on Inflation and Unemployment in the Long Run*

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**Abstract**

We examine the relationship between inflation and unemployment in the long run, using quarterly US data from 1952 to 2010. Using a band-pass filter approach, we find strong evidence that a positive relationship exists, where inflation leads unemployment by some 3 to $3\frac{1}{2}$ years, in cycles that last from 8 to 25 or 50 years. Our statistical approach is atheoretical in nature, but provides evidence in accordance with the predictions of Friedman (1977) and the recent New Monetarist model of Berentsen, Menzio, and Wright (2011): the relationship between inflation and unemployment is positive in the long run.

**JEL Codes**: E24, E31

**Key words**: Inflation, Unemployment, Long-Run Phillips Curve

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# 1 Introduction

Since the publication of Bill Phillips' (1958) seminal paper, the relationship between inflation and unemployment has been a central focus for macroeconomists and policymakers. Most of this focus has been directed at establishing whether or not a negative relationship exists in the short run and what this may imply for policy. In his Presidential Address to the American Economic Association, Milton Friedman (1968) introduced the concept of the "natural" rate of unemployment, which led to the view that, in the long run, the Phillips curve is perfectly vertical – that is, the unemployment rate is independent of inflation and monetary policy. Thus, the nature of the short-run relationship and whether or not it represents a usable policy trade-off (for example, Lucas, 1972)) has, for the most part, dominated the discussion. However, in his Nobel lecture, Friedman (1977) argued that, in the long run (beyond the business cycle frequency), a positive relationship may exist between these two variables, due to the distortionary effects of the inflation tax. Moreover, he found evidence of this positive relationship in US data, observing average values for successive quinquennia. This long-run relationship has received considerably less attention but is, arguably, at least as important to consider.

Theoretically, the existence of a positive relationship in the long run has been suggested by a variety of modelling frameworks. In a real business cycle model with a cash-in-advance (CIA) constraint and employment lotteries (as in Rogerson, 1988)), Cooley and Hansen (1989) quantified this relationship, where the inflation tax induces substitution into untaxed leisure. Shi (1998) introduced a CIA constraint in a monetary search model based on Shi (1997), but with labour search as in Diamond (1982), Mortensen (1982) and Pissarides (1985) (hereafter, DMP), and found that, in the long run, two forces push the effect of money growth on unemployment in opposite directions. The first is a search-inducing effect: higher inflation induces buyers to search more intensively, which increases firm sales, and reduces unemployment. The second is the inflation effect: higher inflation reduces the shadow value of sales, thus reducing profitability, and hence hiring and employment. In the calibrated version of the model, the second effect dominates the first, leading to a positive relationship between inflation and unemployment.

The theoretical ambiguity of the sign of the relationship has been echoed in other studies. For example, Rocheteau, Rupert, and Wright (2007) and Dong (2010) studied monetary search models based on Lagos and Wright (2005) but with employment lotteries. In these papers, the sign of the slope of the long-run Phillips curve depends on complementarities in the utility function: high inflation induces substitution out of cash-intensive goods, and if labour-intensive goods are complements, or leisure is a substitute, for these cash-intensive goods, then the slope of the Phillips curve will be negative. Otherwise, the relationship will be positive. Liu (2008) considered a related model, but with labour market search replacing employment lotteries as the source of unemployment, and where employed workers are taxed differently from unemployed ones. She found that this asymmetry in the tax treatment can be a further source of ambiguity.

Two recent studies, however, argued for an unambiguously positive theoretical relationship in the long run. Kumar (2010) used a model based on Shi (1998), but without endogenous search intensity and with four alternative wage determination mechanisms: individual Nash bargaining, union bargaining, efficiency wages, and wage posting. In all cases, he found that higher inflation rates reduce real money balances for any given consumption level – thus increasing real wage settlements, reducing firm profitability, entry, and increasing unemployment. Berentsen, Menzio, and Wright (2011) – hereafter, BMW – considered a model based on Lagos and Wright (2005), but with labour search as in DMP. They demonstrated that the monetary steady state equilibrium unemployment rate is strictly increasing in the inflation rate, with the following intuitive reasoning: higher inflation increases the cost of holding money, which leads households to economize on real balances, which reduces trade, profitability, entry, and increases unemployment. In the empirical section of their paper BMW also argue that, in the US over the period 1955-2005, a positive relationship between inflation and unemployment has existed over the long run – once high frequency fluctuations have been removed.

In this paper we pursue this empirical issue further by formally testing the proposition that a positive relationship exists between these two variables in the long run. In order to examine the data for long-run patterns, or stylized facts, we employ an analytic method that is as free as possible from economic and statistical models – we apply band-pass filters that require only the specification of the cycle length. This contrasts with the methods used in BMW, who used the Hodrick-Prescott filter to extract the stochastic long-run trend of the U.S. inflation rate and unemployment rate, (i.e., the component of a time series with cycles that last longer than the business cycle). The long-run stochastic trend of the Hodrick-Prescott filter is likely not covariance-stationary for inflation and unemployment, and therefore may potentially cause spurious correlations among filtered components.[1] We use, instead, the nearly optimal band-pass filter developed by Christiano and Fitzgerald (2003) and extract covariance-stationary cycles of 8 to 50 years, i.e., cycles in the long run below the business cycle frequency. We then study dynamic cross-correlations of inflation with unemployment in order to establish whether or not there is a relationship in the long run. Our econometric methodology is in the spirit of Engle (1974), Lucas (1980), Geweke (1986), Backus and Kehoe (1992), King and Watson (1997) and Müller and Watson (2008), who used methods in the frequency domain to extract movements in data at a specific frequency. It is also in the spirit of Friedman (1977) using data averages, and Friedman and Schwartz (1963, 1982) using phase averages because the band-pass filter is a moving average in the time domain with optimal weights chosen in the frequency domain.

Empirical testing of the relationship between inflation and unemployment in the long run is not a trivial task. A commonly used econometric method for testing long-run neutrality in

[1] See Granger and Newbold (1974).

macroeconomic relationships has been the approach developed by Fisher and Seater (1993). King and Watson (1994, 1997)) used this approach to test for long-run neutrality, and illustrated how the results for the slope of the long-run Phillips curve depend on the identifying assumptions that need to be imposed. They found that a positively sloped Phillips curve is not inconsistent with their empirical results, depending on specific assumptions made about short and long-run trade-offs.[2] The econometric method of Fisher and Seater crucially relies on variables not being covariance stationary because a covariance stationary series (say around a deterministic time trend or around a constant mean), by definition, does not have shocks with permanent effects – so that there can be only a vertical long-run Phillips curve. On the other hand, if variables exhibit unit root behavior then shocks do have long-run permanent effects, and one can test whether a permanent shock to one variable affects the other in the long run. The second crucial assumption for the Fisher and Seater long-run neutrality tests is that the variables involved are not cointegrated because cointegration implies co-movement among variables in the long run – so long-run neutrality cannot hold. This literature typically finds little evidence for cointegration between inflation and unemployment. The approach of Fisher and Seater to long-run neutrality testing requires a careful assessment of the time-series properties of the variables involved before applying the neutrality tests. In particular, unit root behavior with no cointegration needs to be established. Furthermore, there are some econometric problems with the neutrality tests. The tests can have relatively low power and may therefore lead to unreliable empirical inference, as shown in Monte Carlo simulations by Coe and Nason (2004).

Alternative empirical approaches, not based on Fisher and Seater's (1993) method, have found support for a low frequency positive relationship between inflation and unemployment in the form of cointegration. Ireland (1999), for example, found evidence of cointegration, and observed that a ten-year centered moving average for US inflation and unemployment over the period from 1960 to 1997 follows an upward trend until the early 1980s (the "great inflation") and afterwards a downward trend (the "great moderation"). Beyer and Farmer (2007) also found evidence for cointegration among quarterly US inflation and unemployment with a positive relationship but in a model that includes an interest rate. Their explanation of the great inflation and moderation is based on a positively sloped long-run Phillips curve and non-superneutrality of money. They found that inflation and unemployment follow a stable cointegrating relation, whereas the cointegration of inflation and the nominal interest rate revealed a break (in 1979:3) and the Fisher hypothesis does not hold over the full sample (1970:1 to 1999:3). However, Doyle and Falk (2008) presented empirical evidence from other OECD countries, allowing for possible structural change in the cointegrating relationship between inflation and unemployment. In contrast, they mostly found evidence against cointegration.

Here, we apply an altogether different approach that does not rely on the presence of unit roots with or without cointegration. We isolate the low-frequency variability of the data with

[2]We applied their method to our data and found that this result also holds for our sample period.

band-pass filters and disregard other aspects of the original data. We focus on the long-run cycles that last from 8 to 50 years. Our method is therefore robust to high-frequency misspecifications of short-run relationships. Our empirical approach follows the monetarist tradition that generally emphasizes simple correlations and lead- and lag relationships, instead of estimating structural economic models. Our results are therefore independent of the assumptions of any particular economic model.

First, we apply the low-frequency-based tests for unit roots and roots local-to-unity, recently proposed by Müller and Watson (2008). This allows us to specify the time series process that any variable follows as an I(0) or I(1) process for the filtering, and also to assess the degree of persistence of shocks for each variable. It is meaningful to talk about a long-run Phillips curve only if an inflation or unemployment shock has effects on inflation or unemployment, respectively, that last for several periods, going beyond the business cycle frequency. If such persistent effects are present, then it is possible to explore whether shocks to inflation affect unemployment in the long run, and vice versa. An I(0) process is a covariance-stationary process and shocks do not have permanent effects. An I(1) process is a process integrated of order one, or, equivalently, a unit root process that is characterized by shocks that have permanent effects. On the other hand, a process may be formally I(0) but have large autoregressive roots close to unity, in which case the effects of shocks decay very slowly over time and shocks can have very persistent effects that last longer than the business cycle frequency.

Second, we apply band-pass filters in order to isolate the components of inflation and unemployment with frequencies lower than the business cycle. We apply the band-pass filter of Christiano and Fitzgerald (2003), guided by the outcome of the Müller-Watson tests as to whether or not the time series involve a unit root. Third, we compute dynamic cross-correlations for the low-frequency filtered components of inflation and unemployment in order to possibly uncover stylized facts. Fourth and last, we apply tests for structural stability of the relationship among the filtered components, using structural change tests proposed by Bai and Perron (1998, 2003). These tests are for breaks at unknown dates. Hansen (2001) emphasized the importance of treating break dates as unknown when testing for structural change.

We find, first, that the inflation rate has a unit root, with breaks, and that the unemployment rate is stable with a long memory process. Therefore both series have persistence – and a long-run analysis of the relationship between the two is meaningful. When considering this relationship, we find that the peaks and troughs of the two filtered series are not aligned in time – with the misalignment involving several years. We therefore consider both leads and lags. When considering cycles with a frequency of 8-50 years, we find that the only correlations that are significant at the 10% level are those at leads 4-24 (1-6 years), where inflation peaks ahead of unemployment. At these leads, all correlations are positive. The correlations at leads 9-18 are significant at the 1% level. The maximum correlation occurs at lead 13 and takes a relatively large value of 0.8338. We also consider other cycle lengths, such as 8-25 years, and find similar results. Thus, a higher inflation rate (at time t) is associated with a higher unemployment rate $3\frac{1}{4}$ years later (at time t+13, i.e., at lead i=13).

Based on our empirical findings, we argue that the Phillips curve has a positive slope in the long run. After formally testing for breaks, we conclude that the long-run association of unemployment with inflation is very stable over our sample period. Although we used different and more formal methods, our findings support the position in Berentsen, Menzio, and Wright (2011), and those of Friedman (1977).

The remainder of this paper is structured as follows. Section 2 provides a detailed description of the methodologies we use. Section 3 describes the data. The main results are presented in Section 4, and Section 5 concludes. All tables and figures are at the end of the paper.

# 2 The Empirical Methodology

## 2.1 Band-Pass Filtering

### 2.1.1 A Short Explanation of Some Basic Concepts for Spectral Analysis

To start, we briefly explain a few basic spectral concepts needed for the discussion of the band-pass filter.[3] A covariance stationary time series $y_t$, with $t = -\infty, \ldots, \infty$, can be represented in the spectral or frequency domain by applying a so-called Fourier transformation to the sequence of the auto-covariances of $y_t$.[4] The spectrum $s_y(\omega)$ is given by

$$s_y(\omega) = (2\pi)^{-1} \sum_{j=-\infty}^{\infty} \gamma_y^j \exp(-i\,\omega j), \tag{1}$$

where $\omega$ is the frequency of oscillation measured in radians and $\gamma_y^j$ represents the auto-covariances, $cov(y_t,\ y_{t-j})$, which are absolutely summable. The spectrum is a periodic function with period $2\pi$ in the interval $[-\pi,\ \pi]$ that is symmetric around $\omega = 0$ so that analysis in the interval $[0,\ \pi]$ is sufficient. The spectrum decomposes the variance of $y_t$ by frequency with the components at different frequencies orthogonal to each other. The integral of the spectrum over the frequency band, say from $\omega_1$ to $\omega_2$, represents the contribution that cycles in this band make to the overall variance of $y_t$. The spectrum of inflation and the unemployment rate can be estimated with, for example, the Bartlett-smoothed periodogram as long as the series are I(0).[5]

A coherence is a squared correlation, measured at a particular frequency, between one covariance-stationary series, $y_t$, and another covariance-stationary series, $x_t$. The coherence is given by

$$coh(\omega) = \frac{|s_{yx}(\omega)|^2}{s_y(\omega)\ s_x(\omega)}\ , \tag{2}$$

[3] A more detailed introductory treatment is given, for example, in Hamilton (1994).

[4] For an I(1) series in levels, the spectrum is not defined at frequency zero.

[5] If a time series is I(1), the series in first-differenced form has a well defined spectrum.

where $s_{yx}(\omega)$ is the cross-spectrum of $y_t$ and $x_t$, defined in the same way as $s_y(\omega)$ in equation (1), except that $\gamma_y^j$ is replaced with the covariance $\gamma_{yx}^j$. For the estimation of coherences, the Bartlett lag-window, among others, can be used.

### 2.1.2 Filtering out Bands From the Spectrum

Filtering can be done in the time domain or frequency domain and can take various forms. A simple filter is a one-sided moving average, say over 5 or 10 years. Another commonly used filter is the Hodrick and Prescott (1997) filter, which filters out frequencies below and above a certain $\omega$, where $\omega$ depends on the smoothing parameter chosen (see Ravn and Uhlig, 2002). The Hodrick-Prescott filter is not a band-pass filter whereas the Christiano and Fitzgerald (2003) filter is.[6] A band-pass filter picks the spectrum in a given band of frequencies that is passed through, say, the spectrum from $\omega_1$ to $\omega_2$, and eliminates or filters out the spectra at other frequencies. It extracts cycles of a specified duration. The actual filtering for the Christiano and Fitzgerald filter is carried out in the time domain but the optimal filter weights are derived in the frequency domain.

The ideal linear filter in the time domain is given by an infinite two-sided moving average, producing the filtered series $y_t^*$:

$$y_t^* = \sum_{j=-\infty}^{\infty} a_j \, y_{t-j}.$$

The filter weights $a_j$ are chosen in order to pick out or pass-through specific frequency bands, such as cycles that last from $1\frac{1}{2}$ years to 8 years (business cycles) or from 8 to 50 years (long-run cycles). The spectrum of the ideally filtered series is given by

$$s_{y^*}(\omega) = |A(\omega)|^2 \, s_y(\omega),$$

with the power transfer function $|A(\omega)|^2$ determined by the filter gain $|A(\omega)|$ that reflects the weight attached to the spectrum at a given frequency:

$$A(\omega) = \sum_{j=-\infty}^{\infty} a_j \exp(-i\,\omega j),$$

which can also be expressed as

$$A(\omega) = |A(\omega)| \sum_{j=-\infty}^{\infty} \exp(-i\,\phi(\omega)),$$

where $\phi(\omega)$ is the phase shift that a filter may introduce. Phase shift means that the peaks and troughs in the filtered series are not consistent with those in the unfiltered series. For a

[6] There has been a debate in the literature over whether a filter, such as the Hodrick-Prescott filter, produces spurious cycles. However, this issue has been laid to rest by Pedersen (2001). See also the reply by Cogley (2001).

symmetric filter, the phase shift can be shown to be always zero.

The problem with the ideal filter is that it has an infinite number of weights and therefore requires an infinite number of observations so that it is generally not feasible. An approximation to the ideal filter is required. We will use the approximation suggested by Christiano and Fitzgerald (2003). Their approximate filter is optimal in a mean-squared-error sense and in addition leads to a filtered series without missing filtered observations at the beginning and end of the sample. These are advantages of the Christiano-Fitzgerald filter over the alternative band-pass filter of Baxter and King (1999). In addition, Christiano and Fitzgerald demonstrated how their filter outperforms the Baxter-King filter at low frequency filtering with US data.

### 2.1.3 The Christiano-Fitzgerald Filter

The filter of Christiano and Fitzgerald (2003) employs a two-sided moving average filter with different weights for every time period t in a sample of size $T$, with t=1, …, $T$. The optimal weights are chosen so that the mean squared error between the ideally filtered series $y_t^*$ and the finite-sample approximation $\widehat{y}_t^*$ is minimized:

$$\min E\left[(y_t^* - \widehat{y}_t^*)^2 \mid y\right]$$

for $y = [y_1, \ldots, y_T]$. In the frequency domain, the problem is stated as

$$\min \int_{-\pi}^{\pi} \left|A(\omega) - B^{p,f}(\omega)\right|^2 s_y(\omega)\, d\omega. \tag{3}$$

The optimal filter weights derived by Christiano and Fitzgerald are

$$B^{p,f}(\omega) = \sum_{j=-f}^{p} b_j^{p,f} \exp(-i\,\omega j)$$

for $f = T - t$ and $p = t - 1$. The filter weights are adjusted according to the importance of the spectrum at a given frequency. This filter is optimal, in a mean-squared-error sense, for every observation in the sample. Also, it can be applied to I(0) and I(1) series.

In the case of a random walk without drift, the optimal filter is Christiano and Fitzgerald's "random walk filter":

$$\widehat{y}_t^* = b_0 y_t + b_1 y_{t+1} + \cdots + b_{T-1-t} y_{T-1} + \tilde{b}_{T-t} y_T + b_1 y_{t-1} + \cdots + b_{t-2} y_2 + \tilde{b}_{t-1} y_1,$$

with

$$b_j = (j\pi)^{-1} \left[\sin(\omega_2\, j) - \sin(\omega_1\, j)\right]$$

for $j = 3, 4, \ldots, T-2$. The formulae for $t = 1, 2, T-1$ and $T$ are straightforward (see Christiano and Fitzgerald, 2003, p. 438) and $\tilde{b}_{T-t}$ and $\tilde{b}_{t-1}$ follow from summation constraints. Before the random walk filter is applied, the raw series should have a non-zero drift or a deterministic time trend removed, if appropriate.

The Christiano-Fitzgerald filter is an asymmetric filter that uses all sample observation for filtering at every point in time, t. Asymmetry may introduce phase shifts. However, the sensitivity analysis in Christiano and Fitzgerald showed that phase shift is generally negligible. Furthermore, their sensitivity analysis also showed that the random walk filter works very well, even when the true time series process of a variable is quite different. It is therefore unnecessary to estimate the time series process for a specific variable in most instances because the random walk approximation is optimal or nearly optimal. In terms of the optimality criterion, the random walk filter dominates the Baxter-King filter. However, as a sensitivity check we apply the alternative symmetric filter (with no phase shift) suggested by Baxter and King (1999).

The Baxter-King filter minimizes instead of equation (3) the following expression:

$$\min \int_{-\pi}^{\pi} \left| A(\omega) - B^{bk}(\omega) \right|^2 \, d\omega. \tag{4}$$

The distance between the ideal and proposed filter is not weighted by the spectrum. The weighted moving average used for filtering uses $k$ leads and $k$ lags at every $t$. Hence, the Baxter-King filter does not produce a filtered series for the $k$ observations at the beginning and at the end of the sample. Also, the filter weights are fixed across observations. It requires the choice of $k$, using some rule of thumb. In contrast, the Christiano-Fitzgerald filter uses all observations for filtering every time and produces filtered series for all endpoints.

The Christiano-Fitzgerald filter produces covariance-stationary filtered components. This is essential in order to avoid spurious correlations due to unit roots in variables analyzed (see Granger and Newbold, 1977). The same holds true for the band-pass filter of Baxter and King (1999) and the business cycle component of the Hodrick-Prescott filter. However, the long-run filtered component (at frequencies below the business cycle) of the Hodrick-Prescott filter is non-stationary when an I(1) series is filtered and could lead therefore to spurious correlations. Furthermore, Corbae, Ouliaris and Phillips (2002) pointed out a potential problem of spectral analysis when non-stationary time series are involved. There is leakage from the zero frequency component into all other frequencies for discrete Fourier transforms of an I(1) process. Corbae and Ouliaris (2006) presented a new leakage-corrected frequency-domain filter. We applied this filter to our data to extract 8 to 50 year bands and found that the correlations for the longer-run components are very similar to the ones we found with the Christiano-Fitzgerald filter.[7]

[7]However, we noticed some marked differences for the filtered components at the end-points of the sample. As argued by Corbae and Ouliaris, their new filter may be particularly useful in getting better output gap measures for empirical macroeconomic models.

### 2.2 Testing for Structural Change

When testing for breaks, a test based on unknown break dates should be applied. Hansen (2001) showed how imposing candidate breaks for the Chow test for structural change can lead to finding breaks incorrectly,when there are none. Also, when a break test designed for a known break date is used to search for (unknown) breaks in a given sample, the limiting distribution of a break test generally changes and is different from the case of known break dates (Carrion-i-Silvestre, Kim and Perron, 2009). Inference based on distributions for break tests assuming known dates when breaks are unknown is therefore not valid. Further, when a time series possibly involves breaks, unit root tests should allow for breaks under the null hypothesis and under the alternative hypothesis when testing for a unit root. The unknown true data generating process may be an I(0) process with breaks or an I(1) process with breaks.[8]

All break tests that we apply test for structural change at unknown points in time. Furthermore, we estimate all break dates consistently from the data. Our strategy is to first scrutinize each individual time series separately, in unfiltered or raw form. Then, we subject the relationship between the band-pass filtered series to break tests in order to establish whether the long-run relationship between inflation and the unemployment rate is stable. Individual unfiltered time series may be subject to breaks. However, these breaks may be due to short-run instabilities and therefore may not be present in the longer-run filtered data. In other words, the question is whether breaks in individual time series leave the long-run relationship between inflation and unemployment unaffected.

Lastly, it is important to allow for multiple breaks when testing for structural change. A test for one structural break only may incorrectly lead to the finding of no structural change when there are instead multiple structural breaks in the true data generating process. Simulations in Bai and Perron (2006) demonstrated that multiple breaks can lead to very low power of tests for a single break.

## 3 The Data

The quarterly U.S. data cover the period from 1952Q1 to 2010Q1. The last observation was the most recent available one when the data were collected in June and July of 2010. All data were seasonally adjusted at the source and downloaded from the Federal Reserve Economic Data (FRED) base at the Federal Reserve Bank of St. Louis at http://research.stlouisfed.org/fred2, except for the sweep-adjusted monetary aggregate.

The consumer price index (series CPIAUCS2) is for all items based on an index setting prices in the period 1982-1984 to 100. The CPI-based inflation rate is calculated as the quarterly year-on-year percentage change: $\{ln(P_t) - ln(P_{t-4})\}\,100$, where $P$ is the CPI-index.[9] As

[8] See, among others, Lee and Strazicich, (2001).

[9] Using instead the annualized quarter-on-quarter changes, $\{ln(P_t) - ln(P_{t-1})\}\,400$, leads to very similar

an alternative measure of inflation, we calculated the percentage changes for the GDP-deflator. The implied GDP-deflator was calculated based on the ratio of the nominal GDP (series GDP) and the real GDP (series GDPC1) in chained 2005 dollars. The civilian unemployment rate (series UNRATE) covers persons 16 years of age and older. The CPI-based inflation rate and the unemployment rate are depicted in Figures 1 and 2.

The treasury bill rate is from the secondary market for 3-month bills (TB3MS). The monetary aggregate is taken from two sources. The M1 data (series M1SA) for the period from 1952Q1 to 1958Q4 are from the historical monetary data web site of the Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/aggreg/; retrieved in June 2011). There were no sweep programs operating in that time period and the series definition is consistent with the one that we use for subsequent years. The data for the period from 1959Q1 to 2009Q4 are form a web site that reports M1 adjusted for balances in retail and demand deposit sweep accounts.[10] M1 is defined as the sum of currency and coins held by the public, travellers checks, demand deposits and other checkable deposits. We use the sweep adjusted measure M1S. Sweep programs started as early as the 1970s and are automated programs that move funds between accounts included in M1 and other instruments with zero reserve requirements, such as money market deposit accounts, money market mutual funds, and overnight and offshore instruments (see Cynamon, Dutkowsky and Jones, 2006). Sweep balances increased sharply in the mid-1990s and by 2003 underreporting of M1 balances due to sweeps reached nearly 70%.

# 4 The Results

## 4.1 The Persistence of Shocks to Inflation and Shocks to Unemployment

In order for inflation and the unemployment rate to be related in the long run, shocks need to have persistent effects. This means that inflation and the unemployment rate should both follow a process with persistence such as a near unit root, long memory or unit root process. One measure of persistence is the size of the largest autoregressive (AR) root of a time series. We calculated the 95% confidence band for the largest AR root of inflation and the unemployment rate, following Stock (1991), by inverting the augmented Dickey-Fuller test statistic.[11] Table 1 reports results for the case of a unit root test regression with a constant term only and for the case with a constant terms and a deterministic time trend. The confidence bands include a unit root, for both variables, whether or not a time trend is

results that are not reported. All qualitative results remain the same.

[10]http://sweepmeasures.com by Cynamon, Barry Z., Donald H. Dutkowsky and Barry E. Jones, "Sweep-Adjusted Monetary Aggregates for the United States", retrieved in June 2011.

[11]See Stock and Watson (2007) for a related application to U.S. GDP price index inflation. Also, lag augmentations for the augmented Dickey-Fuller test regression were chosen with Akaike's information criterion. A sequential $t$-test produced an identical lag structure.

present. More importantly, the confidence bands indicate very high persistence, with roots larger than 0.95 for inflation and larger than 0.91 for unemployment, reaching values slightly above one in both cases.

Müller and Watson (2008) developed several new statistical tests for assessing persistence and low-frequency variability of a time series. Tests for unit roots, long memory (or fractional integration) and local-to-unity autoregressive parameters generally rely on controlling for high frequency variation in the data when assessing persistence and low-frequency variability, as is the case, for example, when adding the lag augmentations for the above augmented Dickey-Fuller test. The approach of Müller and Watson avoids potential misspecification of high frequency variability by exclusively focusing on the low frequency of the spectrum of a time series, below the business cycle frequency, that is for cycles that last longer than 32 quarters or 8 years. The low-frequency component of a series is filtered out using a weighted average of trigonometric cosine expansions, instead of Fourier expansions and associated band-pass filters.[12] The cosine expansions produce diagonal long-run covariance matrices for the models of the persistent processes considered and also cause less leakage of higher frequencies in Müller and Watson's applications than Fourier expansions.

The LFST-test of Müller and Watson (2008) is a low-frequency version of a point-optimal test with a null hypothesis of an I(0) process that maximizes power against a point alternative hypothesis of a local-level model. A local-level model is a simple form of an unobserved components or state-space model with a stochastic level that differs at each (local) point in time. The local-level model is non-stationary and also referred to as a random walk plus noise model. The random walk is the I(1) component of the model with permanent effects and the noise is the I(0) component with only temporary effects on the time series. Their LFUR-test is also a point-optimal test at the low frequency but for the null hypothesis of a unit root model that maximizes power against a point alternative hypothesis of a local-to-unity (or near-unit-root) model. The weight for the I(1) component ($g$) for the LFST-test and the local-to-unity parameter ($c$) for the LFUR-test are chosen so that a 5% level test has approximately 50% power at the alternative for which it is optimal.[13]

The $S$- and $H$-tests of Müller and Watson (2008) are tests for misspecified persistence and low-frequency heteroscedasticity. We apply these two tests to the I(0) and I(1) specifications for inflation and unemployment. The $S$-test assesses whether the low-frequency variance of a time series is consistent with either an I(0) or an I(1) specification. Persistence of the shocks to a time series translates into heteroscedasticity of the weighted average of the cosine expansions. Hence, misspecification of the persistence of a time series produces heteroscedasticity that is more or less persistent than under the null hypothesis of either an I(0) or I(1) statistical model.

The $H$-test is also a test for low-frequency heteroscedasticity but in the shocks instead

[12] Müller and Watson found that 13 weighted averages adequately capture the below business cycle variability in post-WWII U.S. macroeconomic time series for the demeaned case. For the detrended case, 14 weighted averages suffice.

[13] Uniformly most powerful tests do not exist in this context. Following Müller and Watson (2008), we set $c$=14 for the demeaned case and $c$=28 for the detrended case, and $g$=10 and 20, respectively.

of in the trigonometrically transformed time series. Heteroscedasticity in the shocks implies autocorrelation in the cosine-transformed series. The $H$-test checks whether there is too much low-frequency variability in a time series in order to be consistent with the null model. The $H$- and $S$-tests are also designed to maximize power for a 5% level test at 50% at the alternative for which it is optimal.

Table 2 report results for the test of Müller and Watson (2008) for the case with a constant only. The null hypothesis that inflation follows an I(0) process is clearly rejected by the LFST-test with a $p$-value of 0.01 in favor of the alternative hypothesis with a process of permanent effects of shocks. Furthermore, the $S$-test and the $H$-test indicate that the persistence and low-frequency heteroscedasticity of inflation in levels are not consistent with a covariance stationary or I(0) specification. Moreover, the LFUR-test cannot reject the null hypothesis of a unit root or I(1) process against the alternative hypothesis of a near unit root. The $S$-test and $H$-test both show that the persistence and heteroskedasticity of inflation are adequately captured at low frequencies by the I(1) specification. Once we allow for a deterministic time trend, we get the same result in favor of an I(1) specification. Therefore, inflation is best modelled as a unit root process.[14]

The low-frequency based tests for the unemployment rate suggest that an I(0) specification fits the data best. The LFST-test for the specification with a constant term and no time trend cannot reject the null hypothesis of an I(0) model with a $p$-value of 0.11. In addition, no misspecification of persistence and heteroscedasticity are detected by the $S$-test and $H$-test. The I(1) specification is rejected by the LFUR-test ($p$=0.02). Once we include a deterministic time trend in the model, the result is inconclusive. At the 5% level, the LFST-test rejects the I(0) model and the LFUR-test rejects the I(1) model, with the first test rejecting at the 6% level and the second test rejecting at the 7% level. The $S$- and $H$-test do not reject the I(0) and I(1) models. A specification without a deterministic time trend seems reasonable for the unemployment rate and we will therefore treat it as I(0) in our analysis. Furthermore, we estimated the autoregressive parameter for a first-order autoregressive model for the unemployment rate to be 0.98. This value indicates very high persistence of shocks for the unemployment rate, i.e., a long memory process.

In summary, there is considerable persistence in the unemployment and inflation rates, which makes it meaningful to analyze the co-variability of these two time series in the long run, i.e., at low frequencies. Hence, after first testing for breaks in the next section, we filter out low-frequency components and study their dynamic cross-correlations.

## 4.2 Testing for Structural Change in the Raw Data

We test for breaks in each individual time series in order to establish whether the underlying data generating process is better approximated by an I(0) or I(1) behavior once breaks are

[14] Müller and Watson (2008) found the same results for quarterly U.S. GDP-deflator based inflation from 1952Q1 to 2005Q3.

allowed for. This is motivated by the seminal paper of Perron (1989) that demonstrated that unit root tests may give misleading results in favor of I(1) when the true process involves breaks. In other words, we check whether the persistence of shocks in each series is possibly spurious and due to breaks. We apply the testing methodology for structural breaks of Carrion-i-Silvestre, Kim and Perron (2009). The first step of this methodology is to apply a Wald-type test of Perron and Yabu (2009), Exp-$W_{RQF}$, as a pre-test in order to establish whether there is any break at all present in a univariate series. A time series can be thought of as involving a deterministic part (a constant and time trend) and a noise component. For the Exp-$W_{RQF}$ test, the noise component can be I(0) or I(1). In other words, this break test can be applied when it is not known a priori whether the series has a unit root or not. The test allows for an unknown break in the intercept, deterministic time trend or in both. The test is based on robust quasi-flexible generalized least squares. As recommended by Perron and Yabu (2009), we apply the Bayesian information criterion (BIC) to select the order of the autoregressive model for the noise component. The spectral density at frequency zero is estimated either as an autoregressive spectral regression using again BIC or with a non-parametric spectral kernel with automatic band-width selection, depending on the value of the estimate of the truncated sum of the autoregressive coefficients of the noise component. We choose, as is common, a value of 0.15 for the sample truncation parameter $\varepsilon$ when testing for breaks.

When the Exp-$W_{RQF}$ test indicates that there is at least one break in a time series, we apply next the $MZ_{\alpha}^{GLS}(\widehat{\lambda})$ test of Carrion-i-Silvestre et al. (2009).[15] This is a test with good power and size properties in the Monte Carlo simulations carried out by those authors. The test is applied sequentially for up to five breaks in order to determine the total number of breaks in a univariate time series. The break dates are again assumed unknown a priori and are estimated consistently from the data with the algorithm in Carrion-i-Silvestre et al., which is a modified version of the algorithm in Bai and Perron (2003). The test is based on a modified or M-class unit root test with quasi-generalized least squares detrending, as studied in Ng and Perron (2001). The null hypothesis of a unit root is tested against the alternative of stationarity or trend-stationarity. However, it was extended by Carrion-i-Silvestre et al. to allow for multiple breaks under both the null and the alternative hypotheses, which is a novel feature. Previous tests generally allowed for multiple breaks only under the alternative hypothesis. The test therefore allows to assess whether a time series is a trend-stationary process with breaks or an I(1) process with breaks. The test can be applied to models with or without deterministic time trends. Also, the break is allowed to be in the intercept, trend or both. In our applications, we set again the trimming parameter $\varepsilon = 0.15$.[16] Carrion-i-Silvestre et al. showed that their testing methodology involving the pretest has good size and power

[15]The estimated sample fractions where the breaks occur are denoted by the vector $\widehat{\lambda}$. The $MZ_{\alpha}^{GLS}(\widehat{\lambda})$ test is recommended in the conference version of the paper, Carrion-i-Silvestre et al. (2007), which discussed individual test performances in more detail than the published version.

[16]The Gauss codes for the Exp-$W_{RQF}$ and $MZ_{\alpha}^{GLS}(\widehat{\lambda})$ tests were downloaded from Pierre Perron's web site at http://people.bu.edu/perron/.

and that it is superior to that of alternative available methods.

Table 3 reports the values for the Exp-$W_{\mathrm{RQF}}$ test for a single break, which tests for a break regardless of whether the noise component of the series has a unit root or is covariance stationary. The null hypothesis that there is no break in the inflation rate is rejected at the 5% level once we allow for potential breaks in the intercept and in the slope coefficient of the deterministic time trend. On the other hand, the null hypothesis of no break cannot be rejected for a specification with potential breaks in the intercept only. Based on Figure 1, a trend specification seems a reasonable specification and we therefore conclude that there is at least one break in the inflation series. For the unemployment rate, the null hypothesis of no break cannot be rejected by the Exp-$W_{\mathrm{RQF}}$ test at the 5% level, regardless of how the deterministic part is specified. The unemployment rate is therefore stable over our sample period, regardless of whether it is I(0) or I(1). The application of the $\mathrm{MZ}_{\alpha}^{\mathrm{GLS}}(\widehat{\lambda})$ tests in Table 3 allow us to determine the number of breaks for the inflation rate and whether the series is I(1) with breaks or is I(0) around a deterministic part that has breaks. For the inflation rate, we find evidence for a unit root specification with five breaks (in 1963Q3, 1972Q1, 1980Q3, 1989Q1 and in 1999Q2).[17]

Our main purpose, here, is simply to establish that the time series show persistence for there to be long-run effects of shocks. Our tests confirm that a unit root process with breaks provides a good approximation to the underlying unknown data generating process for inflation and that the unemployment rate is a covariance-stationary process with very long memory and no breaks. Hence, there is indeed persistence in both time series so that a longer-run analysis of the relationship between the inflation rate and the unemployment rate is meaningful.

### 4.3 Inflation and Unemployment in the Long Run: Band-Pass Filtered Data

To consider this potential relationship, we start by calculating the spectra of inflation and the unemployment rate. Because the inflation rate is behaving as an I(1) process in our sample, we use the first difference of inflation instead of levels. Figure 3 shows the coherences between unemployment and first-differenced inflation. We plot cycles per period against estimated coherences based on the Bartlett kernel because cycles per period make the interpretation of the graph easier than using frequencies. Cycles per period, denoted by $\frac{1}{p}$, relate to frequency $\omega$ as $\frac{1}{p} = \frac{\omega}{2\pi}$. Therefore, $\frac{1}{p} = 0.0313$ implies a periodicity of 32 quarter, which is 8 years or 32 periods per cycle and $\omega = 0.1963$. A cycle of 200 quarters duration (50 years), i.e., $p = 200$, means that cycles per period $\frac{1}{p} = .005$ and $\omega = 0.0314$. The graph shows that the contribution of the variance of one series to the variance of the other series at a given frequency is similarly large at the low frequencies (in the long run) as it is at higher frequencies, like the business

[17] It is interesting to consider possible reasons for the breaks, like L.B. Johnson's Great Society programs and the associated change in fiscal regime, the 1973 oil shock, the change in monetary policy in the period 1979-1982, the end of the Cold War in the late 1980s, and the dot-com bubble at the end of the 1990s.

cycle where usually $0.0313 \leq \frac{1}{p} \leq 0.1667$ and $6 \leq p \leq 32$ (or $1\,\frac{1}{2}$ to 8 years).

We apply the band-pass filter of Christiano and Fitzgerald (2003) in order to pick out bands in the frequency domain from each time series that last from 8 to 50 years. We treat inflation as an I(1) series and the unemployment rate as an I(0) series for the filtering. The time period chosen for the filtering band is the long run in the sense that we look beyond the business cycle frequency, although we exclude the very long run beyond 50 years that is likely shaped by slow moving factors such a demographics.

Figure 4 depicts the filtered series. Visually, it is very evident that the peaks and troughs of the two transformed data series are very similar, but not aligned in time: there is a misalignment in peaks and troughs involving several years. It is also evident that inflation leads unemployment. We study leads and lags formally in the next section.

### 4.4 Dynamic Cross-Correlations

Table 4 reports the dynamic cross-correlations of the band-pass filtered series for cycles from 8 to 50 years. We consider correlations of the filtered inflation rate at time $t$ with the filtered unemployment rate at time $t+i$ for leads up to i=100 quarters and lags (negative i) back to i=-100 quarters. The time span between the maximum lead and lag involves therefore 200 quarters, which is one full maximum cycle in our chosen band. Following the approach in Stock and Watson (1998) for business cycles, we look for the peaks in the absolute values of the dynamic cross-correlations. In order to assess whether the reported cross-correlations are significantly different from zero, we calculate critical values with the bootstrap method. The filtered components are generated series so that standard critical values and standard confidence bands are not applicable.[18] We follow Christiano and Fitzgerald (2003) and fit to each unfiltered time series an MA process.[19] The best fitting MA that assures white-noise residuals is of order 7 for inflation, modelled as I(1), and of order 14 for the unemployment rate, modelled as I(0). Next, we parametrically bootstrap each MA process for the actual sample size (after deleting 200 observations to mitigate start-up effects). We use 20,000 replications with Gaussian errors generated under the null hypothesis of zero cross-correlations in the data generating process. We start every data generating process at the historical level. In each replication, we apply the band-pass filter to the artificially generated sample and calculate the dynamic cross-correlations for the filtered components for all leads and lags. Next, we calculate the 1%, 2.5%, 5% and 10% critical values from the 20,000 replications generated for each correlation coefficient. We use the computer software package GAUSS version 9 for all simulations. We use the 5% level of significance for deciding whether or not to reject the null hypothesis of zero cross-correlation.

The dynamic cross-correlations between filtered inflation and lags and leads in the filtered unemployment rate vary from a minimum of -0.4995 at lag 43 to a maximum of 0.8338 at lead

[18] A so-called generated regressors problem arises.

[19] See also Haug and Dewald (2011), using annual data and different filtered series.

13. Table 4 reports results only up to 53 leads and lags in order to conserve space. The cross-correlations for leads and lags from 54 to 100 are all between -0.4733 (at lead 80) and 0.1626 (at lag 75) and are not significantly different from zero at the 10% level, based on bootstrap critical values. The contemporaneous correlation is 0.2647. The maximum cross-correlation, in absolute terms, between inflation and unemployment components is therefore positive, and the inflation rate at time t moves ahead of the unemployment rate by 13 quarters or $3\frac{1}{4}$ years, i.e., the maximum reaction of the unemployment rate occurs 13 periods later at time t+13, which we refer to in the tables as a lead of i quarters (here, i=13) for the unemployment rate. In other words, an increase in the inflation rate is followed by an increase in the unemployment rate with a maximum effect that occurs 13 quarters later, and vice versa. This relationship is highly significant at the 1% level and takes on a quite large value of 0.8338 for the cross-correlation. On the other hand, all correlations between inflation and lags of the unemployment rate are insignificantly different from zero at the 10% level. The only correlations that are significant are those for leads 4 to 24, and of those leads 6 to 21 are significant at the 5% level. At leads 9 to 18, all correlations are significant at the 1% level. More importantly, all significant cross-correlations take on positive values.

Our analysis does not allow us to assign causation, however, our finding is consistent with the hypothesis that inflation and the unemployment rate are positively related in the long run.[20] We find that a higher inflation rate is followed by a higher unemployment rate in cycles of durations longer than the business cycle that last 8 to 50 years, and vice versa. This effect is not contemporaneous but occurs with a delay for the unemployment rate of $3\frac{1}{4}$ years. The correlation of inflation with unemployment (at lead i=13) is highly significant and takes on a relatively large value of 0.8338. Our findings are consistent with the proposition of Friedman (1977) and the theoretical model in Berentsen, Menzio, and Wright (2011) that inflation and the unemployment rate are positively related in the long run. Moreover, we do find support for a relationship in which inflation is associated positively with unemployment in later periods.

## 4.5 Sensitivity Analysis

There is general agreement among economists that the business cycle lasts from about 6 to 32 quarters (Stock and Watson, 1998), excluding high frequency noise. However, long-run cycles are not a well defined period, except that they are cycles that last longer than the business cycle. In order to assess how the dynamic cross-correlations evolve once we change the cycle length to shorter cycles, we considered cycles in the frequency band from 8 to 25 years, or 32 to 100 quarters. Results are reported in Table 5. The correlations are somewhat smaller in magnitude compared to those in Table 4. The maximum correlation correlation is 0.7592 at lead 14 in Table 5 compared to 0.9338 at lead 13 in Table 4. The lead time of the maximum correlation stays about the same with only a one quarter difference. In absolute

[20] Comin and Gertler (2006) and Müller and Watson (2008) also found pronounced dynamics below the business cycle frequency for other time series.

terms, negative correlations increase slightly, with a minimum value of -0.5640 at lead 35 in Table 5 compared to a value of -0.4995 at lag 43 in Table 4. None of the correlations in Table 5 is significant at the 5% level, except for correlations for leads 9 to 18, where they all take on positive values. The peak correlation at lead 14 is again significantly different from zero at the 1% level. Furthermore, we explored bands from 8 to 20, 8 to 30 and 8 to 40 years and got the same qualitative results with very similar magnitudes and lag and lead structures. Our result in the previous section is hence robust to alternate definitions of the length of the long-run cycle.

Berentsen at al. (2011) used a sample that ended in 2005Q4 for filtering with the Hodrick-Prescott filter. The recent global recession may possibly have affected our results and break tests cannot pick up breaks close to the endpoints of a sample due to the necessary truncations of the sample during testing. Therefore, we repeated our analysis in Section 4 for the period from 1952Q1 to 2005Q4. The evidence seems to favor an I(1) specification for the unemployment rate over this period so that we apply a unit root specification for filtering the unemployment rate, however, the results are not much different for an I(0) specification instead. The peak correlation occurs at lead 14 and takes on a value of 0.6945. The smallest cross-correlation occurs at lag 42 with a value of -0.5264. We find again that inflation moves ahead of the unemployment rate by $3\frac{1}{2}$ years in this case, and the relationship is positive with an only slightly smaller magnitude of the cross-correlation.

In order to examine how the results are affected by using a different filtering method, we apply the filter of Baxter and King (1999). This is a symmetric filter that should not produce phase shift. The Christiano and Fitzgerald filter, on the other hand, is an asymmetric filter that may introduce some phase shift. We apply the Baxter-King filter and filter out again cycles in the band from 8 to 50 years. At first, we used $k = 12$ for the lead and lag lengths as recommended by Baxter and King (1999) for filtering at the business cycle frequency. However, the empirical power transfer function showed a poor fit and we increased $k$ to 28 in order to improve the approximation to the ideal filter and get a reasonable fit. The results are reported in Table 6. The most significant correlations occur again at leads 7 to 18, all being significant at the 5% level and positive. The maximum absolute value of the dynamic cross-correlations occurs at lead 12 and is positive, with a slightly lower value of 0.7624 compared to the value of 0.8338 for the Christiano-Fitzgerald filter in Table 4. It is again significant at the 1% level. The phase shift involves only one period and is therefore minimal. We note that the Baxter-King filter produces no filtered series for 28 periods at each sample end. Our results demonstrate that phase shift is not a problem in the application of the Christiano-Fitzgerald filter to our data set. The peak cross-correlation occurs at about the same lead-time across the two filters.

Finally, we study whether the results are sensitive to the way inflation is measured and whether money has a similar role as inflation in the long-run relationship with unemployment. We replace the CPI-based inflation rate used so far with an inflation rate based on the implicit GDP price deflator. The results (not reported) with the GDP price index inflation rate show the same pattern across leads and lags for the dynamic cross-correlations as in Table 4. The maximum absolute correlation is at lead 14 with a positive value of 0.7948. Based on Monte Carlo bootstrap critical values, it is significantly different from zero at the 1% level.[21] Therefore, the qualitative results are unchanged. The theoretical model of Berentsen et al. (2011) suggests that money should be closely related in the long run to inflation and therefore also have a positive relationship with the unemployment rate. Furthermore, if the Fisher hypothesis holds, the nominal interest rate should move one-for-one with the inflation rate in the long run, unless long-run real interest rates change. We follow the long-run studies on money of Lucas (1980, 2000) and Ball (2001), among others, and select a monetary aggregate based on M1. The monetary aggregate that we use is M1S, which is M1 adjusted for sweep balances. The tests of Müller and Watson (2008) support an I(1) specification for the M1S growth rate (results are available from the authors).[22]

The dynamic cross-correlations between filtered M1S growth and the filtered unemployment rate, for cycles of 8 to 50 years, follow a pattern that is similar to the one reported in Table 4 for the filtered components of inflation and unemployment but with different timing for peaks and troughs. They reach the largest absolute value when M1S growth peaks ahead of unemployment by 35 quarters or $8\frac{3}{4}$ years. This correlation is statistically significantly different from zero at the 5% level, based on bootstrap critical values, and takes on a value of 0.5842. Other significant correlations are positive and occur when M1S growth leads by 29 to 41 quarters (at the 10% level) and by 31 to 38 quarters (at the 5% level). All other dynamic cross-correlations are insignificantly different from zero at the 10% level. The results for M1S growth are consistent with our findings for inflation and unemployment in the long run. We found that inflation leads unemployment by $3\frac{1}{4}$ years in cycles of 8 to 50 years. This implies that money growth takes some $5\frac{1}{2}$ years for its impact on the inflation component in these long cycles.[23]

## 4.6 Testing for Structural Change in the Filtered Data

Testing for breaks in the medium- or longer-run relationship is necessary in order to check whether the above uncovered regularities or stylized facts in the data are structural features of the economy that are independent of monetary and fiscal policies. We apply the structural change tests of Bai and Perron (1998, 2003) to the dynamic cross-correlations that we

[21] We fitted an MA(10) to the unfiltered GDP price index inflation series for the data generating process in the simulations.

[22] See Haug and Dewald (2011) for further discussion of the time series properties of monetary aggregates.

[23] We also considered the correlations between the filtered 3-month Treasury bill rate and the filtered unemployment rate in 8 to 50 year cycles. The filter extracts the long-run movements of interest rates embedded in the term structure. The correlations turned out to be all not statistically significantly different from zero at the 5% level. The same result holds for cycles of 8 to 25 years. We therefore conjecture that the relationship between interest rates and the inflation rate in the long run does not support the long-run Fisher hypothesis over our sample period. If the Fisher hypothesis held, we would expect a significant one-for-one movement of nominal interest rates and inflation rates in the long run so that interest rates should behave very similarly to inflation in relation to the unemployment rate as far as long-run cycles are concerned.

calculated for the long-run filtered components of 8 to 50 years and of 8 to 25 years. By construction, the filtered components are covariance stationary so that the break tests of Bai and Perron (1998, 2003) are appropriate. Over our sample period, different fiscal and monetary policies have been in place. The methodology of Bai and Perron allows for estimating the number of breaks and the break dates consistently. It also provides tests for testing for the significance of the breaks. Bai and Perron (2003) developed an efficient algorithm to obtain global minima for the sum of squared residuals. The econometric framework accounts for possible heteroscedasticity and autocorrelation in the residuals. As recommended by the authors, we first apply the double maximum test UDmax that is based on sequential F-type tests of the null hypothesis of no break against the alternative hypothesis of one break at an unknown date. This test allows us to establish whether there are any breaks at all in the sample. In the presence of breaks, we would apply Bai and Perron's sequential sup-F test of the null hypothesis of $l$ breaks against the alternative hypothesis of $l+1$ breaks in order to determine the number of breaks and the unknown break dates.

We choose again the trimming parameter $\varepsilon = 0.15$. We regress the filtered inflation rate series on a constant and the filtered unemployment series by least squares.[24] We test for structural change in the peak cross-correlations. For the UDmax test, we calculate bootstrap critical values with the estimated MA processes fitted before to the unfiltered raw data. We impose the variance-covariance structure found in the data onto the data generating process used for the simulations. We filter each artificial sample and apply Bai and Perron's UDmax test to the filtered data. We use 10,000 replications and calculate the 1%, 5% and 10% critical values for the UDmax test.

We calculated a test statistic value of 0.05 for the UDmax test for the stability of the relationship between the filtered components of inflation and the unemployment rate for cycles of 8 to 50 years at lead 13. This value is well below the 10% bootstrap-based simulated critical value from our Monte Carlo simulations.[25] The same result holds for other significant dynamic cross-correlations in Table 4. Therefore, the null hypothesis of no structural change cannot be rejected at conventional significance levels. For cycles of 8 to 25 years in Table 5, the same conclusion holds.[26]

We applied the same stability analysis to the significant correlations between the components of money growth and unemployment for cycles of 8 to 50 years. The UDmax test statistic does not indicate breaks. It takes on a value of 0.02 at lead 35 with bootstrap critical values of 0.91, 2.07 and 8.91 for the 10%, 5% and 1% levels respectively.[27]

We conclude, therefore, that the relationship between the filtered inflation and unemploy-

[24] Regressing instead the filtered unemployment series on the filtered inflation series and a constant has no effect on the results.

[25] The 10%, 5% and 1% bootstrap critical values are 0.51, 1.16 and 9.14, respectively.

[26] The UDmax test has a value of 0.03 for the dynamic cross-correlation at lead 14, with associated 10%, 5% and 1% bootstrap critical values of 0.82, 2.15 and 13.99.

[27] Similarly, the peak correlation for the filtered Treasury bill rate with filtered unemployment, though not significant itself, seems to be stable according to the UDmax test, taking a value of 0.06 with a 10% critical bootstrap value of 0.75.

ment rates is stable over time. In the long run for cycles of 8 to 25 or 8 to 50 years duration, inflation is positively related to the unemployment rate with a lead of 14 or 13 quarters. Our data with quarterly observations stretching from 1952 to 2010 suggests that an increase in the inflation rate is associated with a higher unemployment rate about 3 to $3\frac{1}{2}$ years later. This long-run relationship is stable and unaffected by different monetary and fiscal regimes in operation during this time period. Furthermore, a long-run positive relationship also exits between money growth and the unemployment rate.

# 5 Conclusion

Using US data from 1952Q1-2010Q1 we found a positive relationship between inflation and unemployment in the medium to long run. This relationship is not contemporaneous, and its precise magnitude depends on the assumed frequency of cycles. The highest level of cross-correlation occurs when cycles are 8-50 years in length, and where unemployment responds to inflation after 13 quarters ($3\frac{1}{4}$ years). The only correlations that are significant at or better than the 10% level are those where inflation leads unemployment by 1 to 6 years, and the only correlations that are significant at the 5% level are those where inflation leads unemployment by $1\frac{1}{2}$ to $5\frac{1}{4}$ years. At the 1% level of significance, the only correlations that are significant are those where inflation leads unemployment by $2\frac{1}{4}$ to $4\frac{1}{2}$ years. All significant correlations take on positive values.

We also found that these results are quite robust. Similar results hold for cycles of shorter lengths, for different time periods, different filters, and different measures of inflation. Finally, we found that this long-run relationship is stable, and not affected by different fiscal and monetary policy regimes.

We consider this to be strong evidence that inflation and unemployment are linked, in a positive way, in the long run – where inflation leads unemployment by approximately 3 years. This fact comes out quite graphically, also, in Figure 4 of the paper, which plots the filtered data. The time paths of the filtered data are strikingly similar, with inflation clearly leading by a few years.

The methods we used were as theory-neutral as possible and so we believe that they are compatible with several different theoretical interpretations, including those of Friedman (1977), and Berentsen, Menzio, and Wright (2011). Of course, this study does not provide a test of these theories, but it does provide some evidence in support of their predictions.

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Figure 1. CPI-Based Inflation Rate, Percentages, Year-on-Year, 1953Q1 to 2010Q1

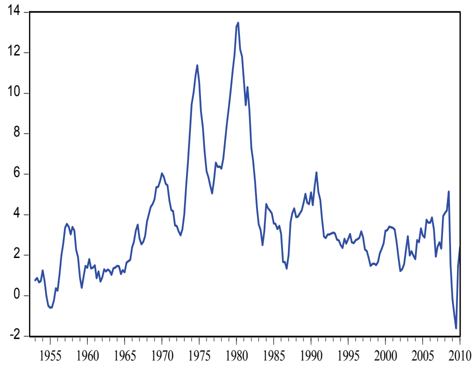


Figure 2. Unemployment Rate, Quarterly, 1952:Q1 to 2010:Q1

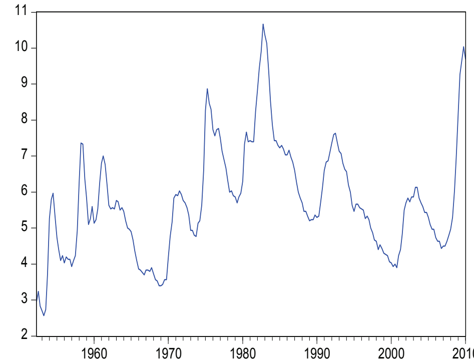


Figure 3. Coherences for Unemployment and Differenced Inflation

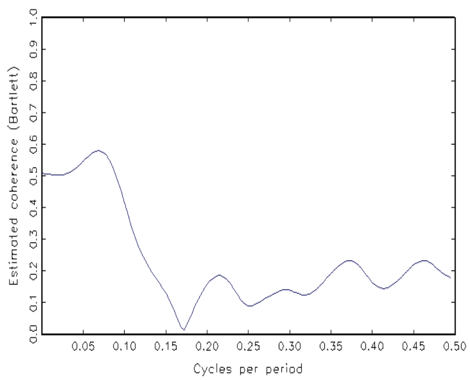



Figure 4. Filtered Inflation and Unemployment, 8 to 50 Year Cycles

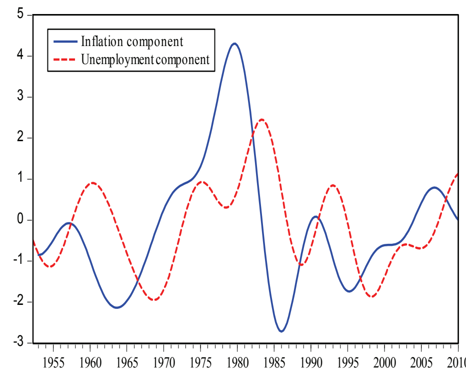



**Table 1**. Confidence Bands for the Largest Autoregressive Root

| | Model with | | | |
|---|---|---|---|---|
| | constant only: | | constant and deterministic trend: | |
| | inflation | unemployment | inflation | unemployment |
| 95% confidence band[a] | 0.942 – 1.017 | 0.909 – 1.011 | 0.952 – 1.022 | 0.913 – 1.018 |

*Note:* [a] A maximum of 14 lags was considered for the augmented Dickey-Fuller unit root test regression used for the calculation of the confidence bands. Akaike's information criterion and a sequential *t*-test chose 12 lags for inflation and 13 lags for unemployment for the lag augmentations in both cases.

**Table 2**. Low-Frequency Tests of Müller and Watson (2008)[a]

| | Model with | | | |
|---|---|---|---|---|
| | constant only: | | constant and deterministic trend: | |
| | inflation | unemployment | inflation | unemployment |
| LFST-test | 10.16 (0.01) | 1.43 (0.11) | 79.51 (0.0004) | 2.45 (0.06) |
| *S*-test for the I(0) model | 5.44 (0.02) | 0.38 (0.62) | 9.52 (0.01) | 0.38 (0.64) |
| *H*-test for the I(0) model | 6.10 (0.01) | 0.17 (0.89) | 7.09 (0.01) | 0.25 (0.73) |
| LFUR-test | 1.77 (0.15) | 9.03 (0.02) | 0.14 (0.48) | 3.80 (0.07) |
| *S*-test for the I(1) model | 0.51 (0.43) | 1.59 (0.10) | 0.39 (0.62) | 1.00 (0.19) |
| *H*-test for the I(1) model | 1.15 (0.14) | 0.27 (0.65) | 1.23 (0.13) | 0.41 (0.49) |

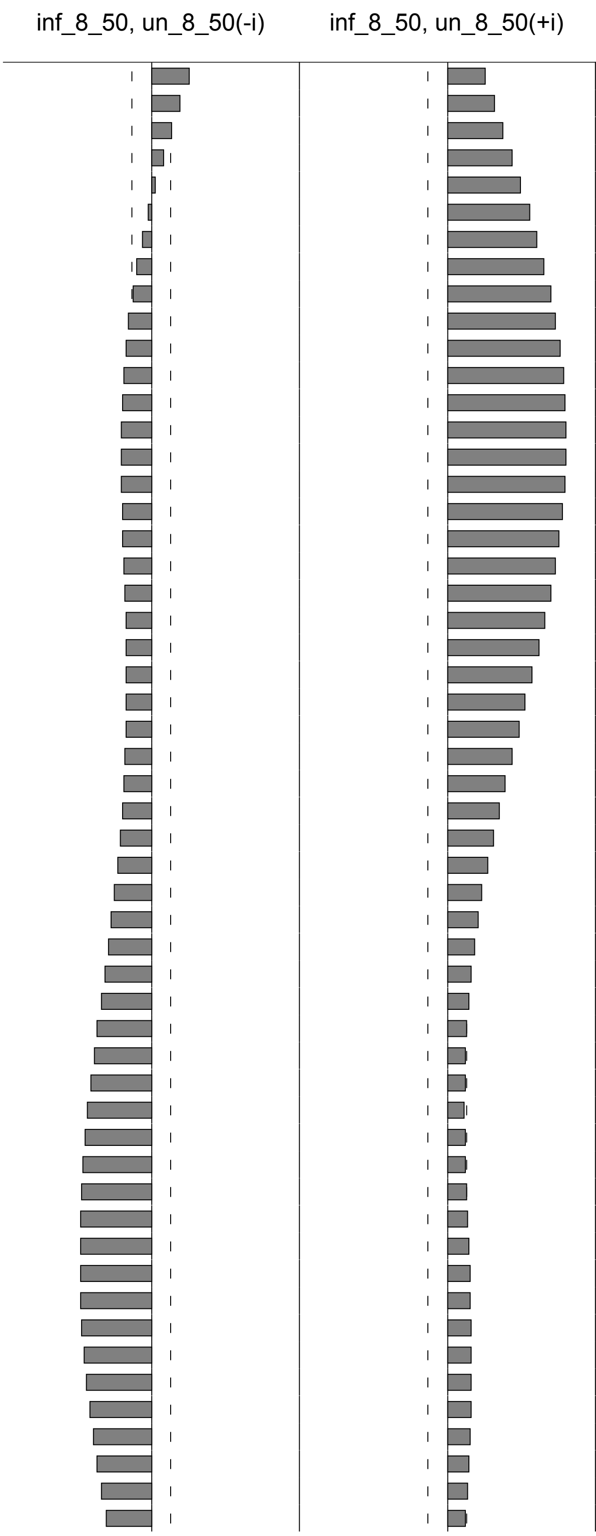
*Note:* [a] The values in parentheses are *p*-values.

**Table 3**. Tests for Breaks at Unknown Dates[a]

| Test | Model with | | | |
|---|---|---|---|---|
| | break in constant only: | | break in constant and deterministic time trend: | |
| | inflation | unemployment | inflation | unemployment |
| Exp-$W_{RQF}$ $H_0$: no break | 0.40 | 1.65 | **3.24**** | 1.78 |
| $MZ_\alpha^{GLS}(\lambda)$ $H_0$: 1 break | n/a | n/a | **-26.02** (-23.70) | n/a |
| $MZ_\alpha^{GLS}(\lambda)$ $H_0$: 2 breaks | n/a | n/a | **-34.18** (-29.47) | n/a |
| $MZ_\alpha^{GLS}(\lambda)$ $H_0$: 3 breaks | n/a | n/a | **-38.40** (-35.40) | n/a |
| $MZ_\alpha^{GLS}(\lambda)$ $H_0$: 4 breaks | n/a | n/a | **-46.59** (-41.75) | n/a |
| $MZ_\alpha^{GLS}(\lambda)$ $H_0$: 5 breaks | n/a | n/a | -21.99 (-47.82) | n/a |

*Note:* [a] Significance at the 5% level is denoted by **. The asymptotic critical values for the pre-test Exp-$W_{RQF}$ test are 3.12, 1.74 and 1.26 for a break in the constant only, and 4.47, 3.12 and 2.48 for a break in the constant and time trend slope (Perron and Yabu, 2009, Table 7, p. 373). The 5% critical values for the $MZ_\alpha^{GLS}(\lambda)$ test are given in parentheses below the test statistic values. They are based on estimated response surfaces. Bold entries indicate rejection at the 5% level.

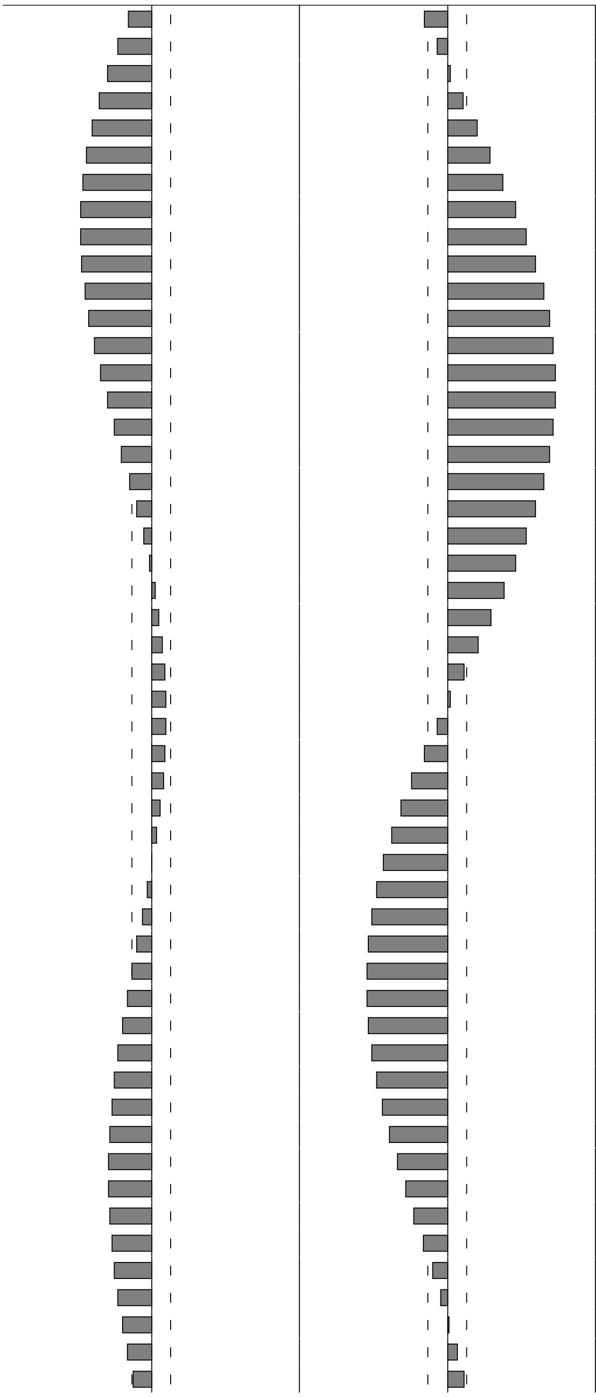
Table 4. Dynamic Cross-Correlations for Christiano-Fitzgerald Filtered Components, 8 to 50 Year Bands, Inflation (inf_8_50) and Unemployment (un_8_50) at Lag (-i) or Lead (+i)




| i | lag | MC signif. | lead | MC signif. |
|---|---|---|---|---|
| 0 | 0.2647 | | 0.2647 | |
| 1 | 0.2013 | | 0.3281 | |
| 2 | 0.1404 | | 0.3918 | |
| 3 | 0.0831 | | 0.4547 | |
| 4 | 0.0300 | | 0.5157 | * |
| 5 | -0.0183 | | 0.5739 | * |
| 6 | -0.0613 | | 0.6282 | ** |
| 7 | -0.0986 | | 0.6776 | *** |
| 8 | -0.1301 | | 0.7213 | *** |
| 9 | -0.1559 | | 0.7586 | **** |
| 10 | -0.1760 | | 0.7889 | **** |
| 11 | -0.1908 | | 0.8117 | **** |
| 12 | -0.2007 | | 0.8267 | **** |
| 13 | -0.2061 | | 0.8338 | **** |
| 14 | -0.2078 | | 0.8331 | **** |
| 15 | -0.2065 | | 0.8246 | **** |
| 16 | -0.2028 | | 0.8088 | **** |
| 17 | -0.1975 | | 0.7862 | **** |
| 18 | -0.1915 | | 0.7574 | **** |
| 19 | -0.1854 | | 0.7231 | *** |
| 20 | -0.1800 | | 0.6842 | ** |
| 21 | -0.1759 | | 0.6416 | ** |
| 22 | -0.1737 | | 0.5962 | * |
| 23 | -0.1739 | | 0.5491 | * |
| 24 | -0.1768 | | 0.5011 | * |
| 25 | -0.1826 | | 0.4534 | |
| 26 | -0.1915 | | 0.4067 | |
| 27 | -0.2035 | | 0.3619 | |
| 28 | -0.2185 | | 0.3197 | |
| 29 | -0.2363 | | 0.2808 | |
| 30 | -0.2565 | | 0.2458 | |
| 31 | -0.2787 | | 0.2149 | |
| 32 | -0.3024 | | 0.1885 | |
| 33 | -0.3272 | | 0.1668 | |
| 34 | -0.3523 | | 0.1496 | |
| 35 | -0.3772 | | 0.1369 | |
| 36 | -0.4012 | | 0.1285 | |
| 37 | -0.4238 | | 0.1239 | |
| 38 | -0.4443 | | 0.1228 | |
| 39 | -0.4622 | | 0.1246 | |
| 40 | -0.4771 | | 0.1287 | |
| 41 | -0.4884 | | 0.1346 | |
| 42 | -0.4960 | | 0.1415 | |
| 43 | -0.4995 | | 0.1487 | |
| 44 | -0.4987 | | 0.1558 | |
| 45 | -0.4937 | | 0.1620 | |
| 46 | -0.4844 | | 0.1668 | |
| 47 | -0.4710 | | 0.1697 | |
| 48 | -0.4536 | | 0.1702 | |
| 49 | -0.4326 | | 0.1680 | |
| 50 | -0.4082 | | 0.1628 | |
| 51 | -0.3808 | | 0.1545 | |
| 52 | -0.3510 | | 0.1429 | |
| 53 | -0.3191 | | 0.1280 | |

Note: Dashed lines indicate conventional 95% confidence bands. MC signif. refers to the significance level based on 20,000 Monte Carlo simulations: ****, ***, ** and * indicate significance at the 1%, 2.5%, 5% and 10% levels respectively.

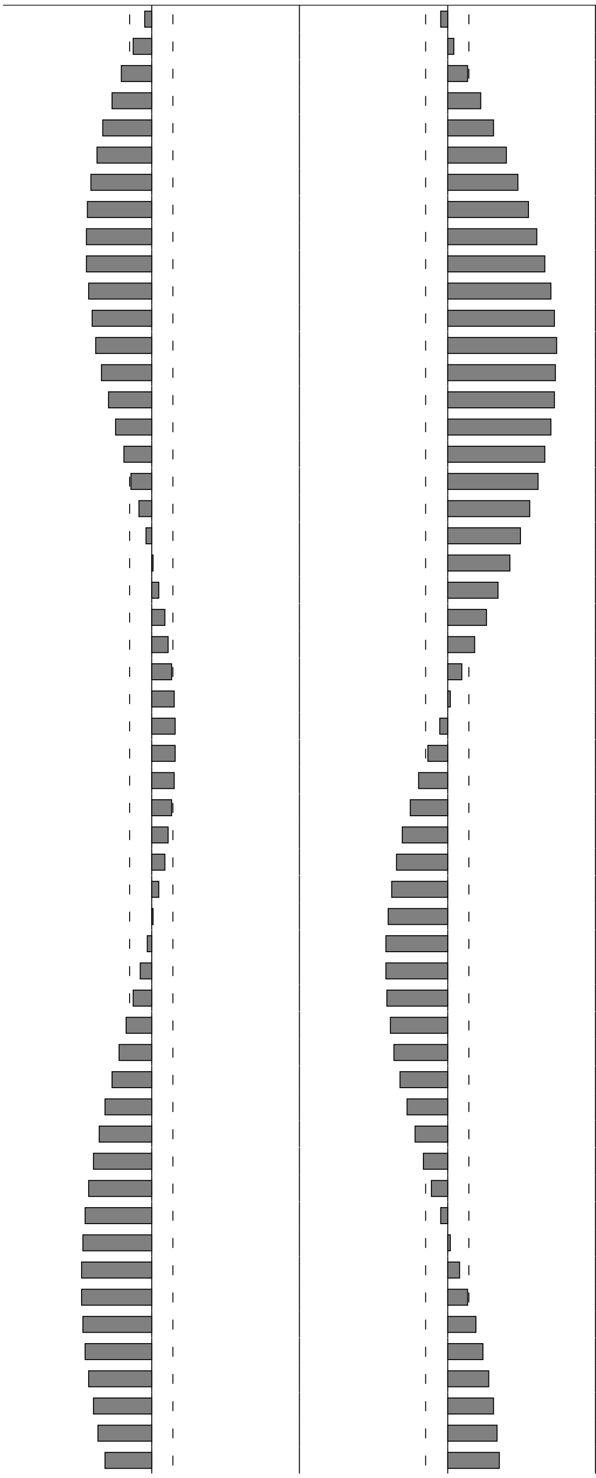
Table 5. Dynamic Cross-Correlations for Christiano-Fitzgerald Filtered Components, 8 to 25 Year Bands, Inflation (inf_8_25) and Unemployment (un_8_25) at Lag (-i) or Lead (+i)



| inf_8_25, un_8_25(-i) | inf_8_25, un_8_25(+i) | i | lag | MC signif. | lead | MC signif. |
|---|---|---|---|---|---|---|
| | | 0 | -0.1585 | | -0.1585 | |
| | | 1 | -0.2368 | | -0.0713 | |
| | | 2 | -0.3065 | | 0.0204 | |
| | | 3 | -0.3663 | | 0.1147 | |
| | | 4 | -0.4153 | | 0.2095 | |
| | | 5 | -0.4528 | | 0.3026 | |
| | | 6 | -0.4783 | | 0.3920 | |
| | | 7 | -0.4919 | * | 0.4754 | |
| | | 8 | -0.4938 | * | 0.5511 | * |
| | | 9 | -0.4847 | | 0.6172 | ** |
| | | 10 | -0.4652 | | 0.6721 | *** |
| | | 11 | -0.4366 | | 0.7147 | *** |
| | | 12 | -0.4001 | | 0.7438 | **** |
| | | 13 | -0.3571 | | 0.7587 | **** |
| | | 14 | -0.3093 | | 0.7592 | **** |
| | | 15 | -0.2583 | | 0.7452 | **** |
| | | 16 | -0.2057 | | 0.7169 | *** |
| | | 17 | -0.1533 | | 0.6752 | ** |
| | | 18 | -0.1025 | | 0.6209 | ** |
| | | 19 | -0.0550 | | 0.5554 | * |
| | | 20 | -0.0118 | | 0.4800 | |
| | | 21 | 0.0257 | | 0.3966 | |
| | | 22 | 0.0567 | | 0.3070 | |
| | | 23 | 0.0804 | | 0.2132 | |
| | | 24 | 0.0965 | | 0.1173 | |
| | | 25 | 0.1047 | | 0.0214 | |
| | | 26 | 0.1051 | | -0.0724 | |
| | | 27 | 0.0979 | | -0.1623 | |
| | | 28 | 0.0837 | | -0.2464 | |
| | | 29 | 0.0631 | | -0.3231 | |
| | | 30 | 0.0371 | | -0.3908 | |
| | | 31 | 0.0066 | | -0.4485 | |
| | | 32 | -0.0273 | | -0.4951 | |
| | | 33 | -0.0633 | | -0.5301 | * |
| | | 34 | -0.1003 | | -0.5531 | * |
| | | 35 | -0.1369 | | -0.5640 | * |
| | | 36 | -0.1721 | | -0.5631 | * |
| | | 37 | -0.2047 | | -0.5508 | * |
| | | 38 | -0.2336 | | -0.5278 | * |
| | | 39 | -0.2579 | | -0.4952 | |
| | | 40 | -0.2770 | | -0.4539 | |
| | | 41 | -0.2902 | | -0.4054 | |
| | | 42 | -0.2970 | | -0.3511 | |
| | | 43 | -0.2972 | | -0.2923 | |
| | | 44 | -0.2908 | | -0.2307 | |
| | | 45 | -0.2777 | | -0.1677 | |
| | | 46 | -0.2583 | | -0.1048 | |
| | | 47 | -0.2330 | | -0.0435 | |
| | | 48 | -0.2023 | | 0.0151 | |
| | | 49 | -0.1667 | | 0.0697 | |
| | | 50 | -0.1271 | | 0.1193 | |

Note: See Table 4.

Table 6. Dynamic Cross-Correlations for Baxter-King Filtered Components, k=28, 8 to 50 Year Bands, Inflation (inf_8_50) and Unemployment (un_8_50) at Lag (-i) or Lead (+i)



| inf_8_50, un_8_50(-i) | inf_8_50, un_8_50(+i) | i | lag | MC signif. | lead | MC signif. |
|---|---|---|---|---|---|---|
| | | 0 | -0.0426 | | -0.0426 | |
| | | 1 | -0.1288 | | 0.0472 | |
| | | 2 | -0.2075 | | 0.1399 | |
| | | 3 | -0.2771 | | 0.2332 | |
| | | 4 | -0.3364 | | 0.3247 | |
| | | 5 | -0.3842 | | 0.4121 | |
| | | 6 | -0.4201 | | 0.4932 | * |
| | | 7 | -0.4435 | | 0.5663 | ** |
| | | 8 | -0.4545 | * | 0.6296 | *** |
| | | 9 | -0.4534 | * | 0.6818 | *** |
| | | 10 | -0.4408 | | 0.7218 | **** |
| | | 11 | -0.4178 | | 0.7489 | **** |
| | | 12 | -0.3856 | | 0.7624 | **** |
| | | 13 | -0.3457 | | 0.7623 | **** |
| | | 14 | -0.2994 | | 0.7488 | **** |
| | | 15 | -0.2487 | | 0.7224 | **** |
| | | 16 | -0.1953 | | 0.6841 | *** |
| | | 17 | -0.1410 | | 0.6350 | *** |
| | | 18 | -0.0874 | | 0.5764 | ** |
| | | 19 | -0.0360 | | 0.5095 | * |
| | | 20 | 0.0117 | | 0.4360 | * |
| | | 21 | 0.0547 | | 0.3573 | |
| | | 22 | 0.0918 | | 0.2750 | |
| | | 23 | 0.1222 | | 0.1908 | |
| | | 24 | 0.1453 | | 0.1065 | |
| | | 25 | 0.1606 | | 0.0236 | |
| | | 26 | 0.1680 | | -0.0562 | |
| | | 27 | 0.1675 | | -0.1314 | |
| | | 28 | 0.1591 | | -0.2005 | |
| | | 29 | 0.1431 | | -0.2622 | |
| | | 30 | 0.1200 | | -0.3154 | |
| | | 31 | 0.0903 | | -0.3591 | |
| | | 32 | 0.0549 | | -0.3927 | |
| | | 33 | 0.0145 | | -0.4156 | |
| | | 34 | -0.0301 | | -0.4276 | * |
| | | 35 | -0.0778 | | -0.4288 | * |
| | | 36 | -0.1277 | | -0.4191 | * |
| | | 37 | -0.1785 | | -0.3990 | |
| | | 38 | -0.2292 | | -0.3690 | |
| | | 39 | -0.2784 | | -0.3299 | |
| | | 40 | -0.3251 | | -0.2828 | |
| | | 41 | -0.3679 | | -0.2290 | |
| | | 42 | -0.4057 | | -0.1699 | |
| | | 43 | -0.4375 | * | -0.1073 | |
| | | 44 | -0.4623 | * | -0.0427 | |
| | | 45 | -0.4797 | * | 0.0220 | |
| | | 46 | -0.4889 | * | 0.0852 | |
| | | 47 | -0.4897 | * | 0.1451 | |
| | | 48 | -0.4819 | * | 0.2003 | |
| | | 49 | -0.4657 | * | 0.2493 | |
| | | 50 | -0.4413 | * | 0.2909 | |
| | | 51 | -0.4092 | | 0.3245 | |
| | | 52 | -0.3699 | | 0.3493 | |
| | | 53 | -0.3240 | | 0.3649 | |

Note: See Table 4. The Monte Carlo simulations were applied to the Baxter-King filtered series.