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Recursive Contracts, Firm Longevity, and Rat Races: Theory and Experimental Evidence*

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Abstract

This paper investigates the relationship between firm longevity and rat races in an environment where long-lived firms are operated by overlapping generations of short-lived players. We first present a complete information model in which workers in the young generation are offered employment contracts designed by the firms’ owners who belong to the old generation. When old, employed workers are granted ownership rights as long as the firm continues to operate. We test the theoretical predictions of the model in a laboratory experiment. In line with our model’s predictions, as firm longevity increases, the recursive nature of the contracts leads to a rat race characterized by low wages, high effort levels, and rent dissipation.

JEL Classifications: C91, D02, D21, D86, D92

Keywords: Overlapping-generations models; Recursive contracts; Rat races; Experiments

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1 Introduction

Many firms are ongoing organizations made up of individuals who join when young and leave when old. In such firms, young workers often move on to executive roles at a later stage of their career. As executive roles typically entitle individuals to a share of the firm’s profits, a young worker’s decision to join a firm depends not only on the current rewards, but also the prospect of future rents that might arise through internal promotion (e.g., Baker, Gibbs and Holmstrom, 1994a, 1994b; Ferrall, 1996; Landers et al., 1996).

This paper examines, theoretically and experimentally, the structure of contracts that can emerge in overlapping-generations (OLG) firms when future ownership is a perquisite of employment. Contract design in such organizations is a recursive problem. The contracts that young workers are willing to accept depend on the value of owning the firm in the future, if promoted. The value of owning the firm in the future depends on the contract that they will be able to impose on the next generation of workers. Hence, firm longevity, which affects future ownership, plays a critical role in the design of contracts within these firms.

To study the impact of firm longevity on contract design, we consider a modified version of Bardsley and Sherstyuk (2006) and begin our analysis with a theoretical model investigating how the expected value of owning a firm affects contract design. We consider a simple environment with complete information and homogenous agents to focus on the recursive nature of the contracts. In the model, firm owners, members of the older generation, design the contracts offered to the young generation of workers. Workers who accept contracts will become owners of their firm as long as the firm continues operating. In this environment, the expected value of owning a firm depends on the firm’s longevity, which is determined by the probability that the firm continues operating in the future.

We show that when the expected value of owning the firm is high, the unique subgame-perfect equilibrium is a rat race characterized by inefficiently high levels of effort and the dissipation of firm rents. The reason for this is the inability of owners

\(^{1}\) As we explain in Section 3, an increase in firm longevity can also be interpreted as an increase in the probability that a worker is promoted or a decrease in the discounting of future rents. We relate the discounting of future rents to firm longevity as it is more amenable to experimental testing.
to write inter-generational contracts. Since the next (unborn) generation of workers is not privy to the original contract, the recursive structure implicitly includes a third party in the negotiation whose interests are not represented and whose actions are non-contractible. The additional rents offered from the next generation induce workers to accept lower wages or exert higher effort than they would in a single-period firm. Once wages are reduced to zero, owners offer contracts with inefficiently high levels of effort that workers are willing to accept. The model thus suggests a negative relationship between the expected longevity of a firm and its efficiency.

We test the predictions of the theoretical model using an OLG labor-market experiment. We consider two treatments that differ in the expected longevity of firms. In the first treatment (Efficient), firm longevity is expected to be short and our model predicts an efficient outcome. In the second treatment (Rat Race), firm longevity is expected to be longer and our model predicts a rat race. The results from our laboratory experiment are consistent with the predictions of our model. In the Efficient treatment, an equilibrium emerges with the efficient effort level, and wages act as the primary contracting instrument. In contrast, in the Rat-Race treatment, markets converge to a rat-race equilibrium with inefficiently high levels of effort, low wages, and high earnings inequality between owners and workers. The pronounced difference in the outcomes across the two treatments is remarkable given that the contracts offered by owners in the early periods of the experiment in both treatments are efficient and fair.

The negative relationship we establish between firm longevity and efficiency raises questions about when we would expect to observe rat races in real life. The key features of our model are that owners are residual claimants who have been internally promoted; they are given full contracting authority but cannot sell the firm to an outsider. These features appear to be common in partnerships in which ownership of non-fungible assets (such as reputation or corporate skills) is eventually passed vertically within the firm from the older to the younger generation, and in which workers’ effort is closely monitored. Partnerships with such features are common in many fields, including law, accounting, investment banking, management consulting, advertising, and medicine. Partners in these firms have full, if implicit, contracting authority vis-a-vis the promotion of associates. The reputation of the firm is embodied
in its people and culture. Since selling the firm to outsiders would incur a high reputational cost, the promise of future promotion is credible\textsuperscript{2}. Indeed, the fact that most empirical studies on rat races use data from partnerships (e.g., Ferrall, 1996; Landers, et al., 1996) suggests that recursive contracting is an important determinant of rat races. More generally, we would expect to encounter forces similar to those in our model in organizations where the ability to exercise authority in the future is an important motivation for joining the organization in the first place\textsuperscript{3}.

The paper proceeds as follows. After reviewing the related literature in Section 2, we present the theoretical model in Section 3. We discuss our experimental design in Section 4 and present the experimental results in Section 5. Section 6 concludes.

2 Related literature

Our paper contributes to the literature on the emergence of rat races in organizations. In economics, the term rat race is typically used to describe situations in which an individual is enticed to overwork, and where rents from this work are dissipated. Previous studies have shown that rat races may emerge due to adverse selection and moral hazard\textsuperscript{4}. In this literature, our theoretical framework is closest to that of Bardsley and Sherstyuk (2006). Although they mainly focus on a model with heterogeneous agents and adverse selection, their analysis reveals that rat races can also emerge in environments with complete information if contracts are recursive. We simplify and extend their framework to focus on the relationship between firm longevity and rat races. By varying firm longevity, we can explore the effect of the

\textsuperscript{2}Tadelis (1999) and Mailath and Samuelson (2001) discuss models in which reputation can be traded in the external market, not just within the firm.

\textsuperscript{3}The apprenticeship system of medieval and pre-modern Europe is another example. Apprentices actually paid substantial amounts to be apprenticed, and were bound to work for a subsistence wage for a number of years. See Wallis (2008).

\textsuperscript{4}Akerlof (1976) was the first to show that rat races may emerge if workers’ abilities are unobservable to the employer. See also Stiglitz (1975), Miyazaki (1975), Landers, et al. (1996), and Andersson (2002) for models with adverse selection. In these models, workers may work inefficiently long hours to signal their higher ability. Models with moral hazard emphasize that promotion tournaments, used to provide workers with incentives not to shirk, may result in inefficiently high effort levels. See, for example, Gibbons and Murphy (1992), Ferrall (1996), Holmstrom (1999), and Baker, Choi and Gulati (2006).
probability that the firm continues operating in the future (i.e., the discount rate) on the equilibrium contract and firm efficiency. This allows us to specify a tractable model that can be tested in the laboratory.

Focusing on the discount rate also allows us to clearly demonstrate our contribution to the literature on OLG organizations. The question of how cooperation can be sustained across overlapping generations was first raised by Cremer (1986). He argued that young workers may exert efficient levels of effort as an equilibrium of a repeated game played between overlapping generations of individuals even if workers have an incentive to shirk and are finitely lived. This insight was confirmed in a Folk theorem proved by Kandori (1992), and refined by Salant (1991) and Smith (1992). The behavior of the young is disciplined by the fear that the next generation might punish them if they deviate from the cooperative norm. As in other Folk theorems, the young generation can be motivated to exert efficient levels of effort, provided that lifetimes are not too short, there is sufficient overlap between generations, and the rate at which future payoffs are discounted are low.

However, in many cases, decision making authority accrues naturally to the older members of an organization. This can lead to outcomes of a different nature. In our model, the owners have full control rights and can impose their decisions on the workers, whose actions are fully contractible. In a single-period firm, the ability of the owners to extract rents is limited only by the fact that the workers have an outside option. Increasing the expected lifetime of the firm relaxes the incentive compatibility constraint, allowing the owners to impose harsher and increasingly more inefficient contracts. The lower discount rate increases the ability of the older generation to tempt workers today with the rents of the future generation. Therefore, in sharp contrast to models where the Folk Theorem applies, reducing the discount rate lowers efficiency in our model.

On the experimental side, we contribute to the literature in two ways. First, our paper is the first to study rat races in the laboratory. Second, methodologically, our experiment is the first to use an OLG structure to study contract design in a

5A similar insight applies to Hammond’s pension game (Hammond, 1975) and Binmore’s Mother-Daughter game (Binmore, 2007). In these models, it is the young generation who makes the decisions and who imposes an externality on the old generation. See also Ando and Kobayashi (2008), Morrison and Wilhelm (2004), and Bar-Isaac (2007).
principle-agent framework. Previously, an OLG structure has mainly been used to test macroeconomic theories\footnote{A few exceptions are Aliprantis and Plott (1992), Offerman, Potters and Verbon (2001), and Chermak and Krause (2002), who study competitive equilibria, cooperation, and intra-generational environmental issues, respectively, using OLG experiments.}

3 Theoretical framework

3.1 The model

We consider a discrete-time model with short-lived agents and a single long-lived firm. At each time $t$, there are $n$ individuals born into the economy. Each individual lives for two periods. Utility is separable between periods and there is no discounting. All individuals are homogeneous, risk neutral, rational, and act to maximize their monetary payoffs.

The single firm in the economy is composed of a worker from the young generation and an owner from the old generation. At the beginning of each period, the owner designs a contract specifying a non-negative wage $w$ for the worker and an effort level $e$. She makes a ‘take it or leave it’ offer of this contract to a single individual in the young generation\footnote{See Duffy (2008) for a survey of this literature.} The effort of the worker yields deterministic output to the owner which can be sold at gross profit $\kappa e$. There is complete information, with no moral hazard or adverse selection, and the contract can be enforced costlessly.

If the contract is turned down, both the owner and the worker receive their reservation utility, which is normalized to zero in both periods. If the contract is accepted, the owner exerts no effort and receives the surplus $V = \kappa e - w$. The agent exerts effort $e$ and receives utility $w - c(e)$, where $c(e)$ is the cost of effort. We assume that $e$ is deterministic.\footnote{Note that the equilibrium prediction of the model is identical to a case where the owner posts the offer publicly and has $n > 1$ homogenous individuals in the young generation compete for the contract. Our model is also isomorphic to a model with one agent per generation and where the owner has full bargaining rights. We model with $n$ agents to make the relation between the theory and experiments clear.} The assumption that there is a single worker is without loss of generality. If there are $k$ identical workers, equally likely to inherit the firm, then the probability of inheritance is $\frac{1}{k}$ while the firm value increases $k$ times. Thus, the expected value of future ownership is invariant to $k$.\footnote{The assumption that there is a single worker is without loss of generality. If there are $k$ identical workers, equally likely to inherit the firm, then the probability of inheritance is $\frac{1}{k}$ while the firm value increases $k$ times. Thus, the expected value of future ownership is invariant to $k$.}
c (e) is smooth and convex, that \( c (0) = 0 \), and that \( 0 \leq c' (0) < \kappa \). These assumptions imply that there exists an effort level \( e > 0 \) such that \( e > c (e) \), which ensures that first-best effort is positive. We also assume that effort is bounded above: there is an effort level \( \bar{e} > 0 \) such that \( c (e) \to \infty \) as \( e \to \bar{e} \). As is usual in the Principal-Agent literature, we assume that if a worker is at the participation margin, then they will accept the contract.

The key benefit of employment in our model is the possibility of becoming the owner in the future. With probability \( \alpha \) the firm continues to operate and the worker in period \( t \) inherits the firm in period \( t + 1 \). As owners have both the right to choose the contract for the next generation and the right to all residual claims, the model is recursive, with the promise of future residual rights being used to entice workers to participate today.

We interpret \( \alpha \) as the firm’s survival probability so that \( \frac{1}{1-\alpha} \) is the firm’s expected lifetime, but there are other interpretations which fit our environment. In the context of a long-lived partnership or firm, a useful interpretation of \( \alpha \) is the probability of internal promotion. In thinking about the transfer of skills as in apprenticeships, \( 1 - \alpha \) may be the probability of external discovery, which eliminates the monopoly rents enjoyed by the industry. It may also, of course, represent pure time discounting.

We assume there is no lending or borrowing and that individuals are born with no assets other than their endowment of labor. As workers cannot borrow, these assumptions impose a non-negativity constraint on wages.\(^9\) Individuals have rational expectations, and anticipate the future equilibrium when choosing their actions. Since all individuals are assumed to act to maximize their monetary payoff, the owner will choose \( w \geq 0 \) and \( e \geq 0 \) to maximize her rent

\[
V = \kappa e - w, \tag{1}
\]

\(^9\) We make this assumption to exclude the possibility of Ponzi equilibria with ever increasing inter-generational transfers and arbitrarily large effort levels. The upper bound \( \bar{e} \) must be finite but can be large.

\(^{10}\) This can be relaxed to allow workers to borrow up to some bounded credit limit. Such borrowing relaxes the participation constraint and increases the range of \( \alpha \) for which contracting is efficient, but does not significantly change the analysis. Without a limit on borrowing and for \( \alpha \) close to 1, no equilibrium exists.
subject to the individual rationality constraint and the non-negativity constraint on wage. Individual rationality requires that

\[ w + \alpha V^+ \geq c(e), \]  

(2)

where we write \( V^+ \) for the next period’s rent. Noting that the individual rationality constraint will always bind and eliminating \( w \) from (1), it follows that

\[ V = \alpha V^+ + \kappa e - c(e). \]  

(3)

Non-negativity requires \( w \geq 0 \) which, when combined with (1), implies that

\[ V \leq \kappa e. \]  

(4)

Combining equations (3) and (4), the rent obeys the Bellman equation

\[ V = \max_{e \geq 0} \min \left[ \kappa e, \alpha V^+ + \kappa e - c(e) \right]. \]  

(5)

This recursion determines the contract dynamics and reflects the basic dilemma of the worker, who may be prepared to accept a harsh employment contract today if they expect that they can impose a similar contract on the next generation tomorrow.

### 3.2 Equilibrium contract

We show in the Appendix that there is a unique subgame-perfect equilibrium, that this equilibrium is stationary, and that the value function is given explicitly by

\[ V = V(\alpha) = \max_{e \geq 0} \min \left[ \kappa e, \frac{\kappa e - c(e)}{1 - \alpha} \right]. \]  

(6)

In order to understand this equation, notice that it builds in two constraints. The first is the wage non-negativity constraint \( V \leq \kappa e \), which requires that the rent that the owner can extract from the firm is bounded by current period production. The second is the value constraint \( V \leq \frac{\kappa e - c(e)}{1 - \alpha} \), which requires that the rent is bounded by the total value of the firm: the per-period surplus \( \kappa e - c(e) \) multiplied by the
expected lifetime $\frac{1}{1-\alpha}$ \footnote{This constraint aggregates the individual rationality constraints of the current and all future generations.} Equation (6) states that these are the only constraints on rent extraction, and that jointly they determine the entire dynamic equilibrium.

The interaction of these constraints in $(e, V)$ space is shown in Figure 1. The wage constraint requires that the contract point $(e, V)$ lie below the diagonal line $V = \kappa e$. This constraint does not depend on $\alpha$. The value constraint requires that the contract point lie below the total value curve $V = \frac{\kappa e - c(e)}{1-\alpha}$, which bows upwards as $\alpha$ increases. We plot these curves and the equilibrium contract $(e, V)$ for a range of values from $\alpha = 0$ (the single period firm) to $\alpha = 1$ (the infinitely long-lived firm). Since $w = \kappa e - V$, the wage $w$ can also be read directly off this diagram as the vertical distance from the contract point $(e, V)$ to the diagonal line $V = \kappa e$.

![Figure 1: Equilibrium effort $e$ and rent $V$ for various continuation probabilities between $\alpha = 0$ and $\alpha = 1$](image)

When $\alpha = 0$, we have a single period firm that will not survive into any future
period. The owner chooses the first-best effort level $e^{FB}$, defined by $c'(e^{FB}) = \kappa$, and a wage $w$ that is equal to the effort cost $c(e^{FB})$. This is the smallest wage that will induce the worker to participate. Thus, the wage constraint does not bind. As can be seen from Figure 1, the contract point lies on the vertical line $e = e^{FB}$ at the highest point that satisfies the participation constraint.

As $\alpha$ increases, the expected lifetime of the firm increases and the value curve shifts up to include the benefits of future ownership. The owner adjusts the contract to extract this surplus. Since $c(e)$ is convex and $c'(e^{FB}) = \kappa$, the owner at first finds it in her best interest to lower wages rather than to increase effort above the first-best level. The contract point thus shifts vertically up the efficient effort line $e = e^{FB}$. This shift continues, as $\alpha$ increases further, until we reach the threshold value $\alpha^* = \frac{c(e^{FB})}{\kappa e^{FB}}$ where the wage constraint begins to bind.

As $\alpha$ increases beyond $\alpha^*$ we move from the efficient region into the rat race region. The value of future ownership continues to increase, making participation more attractive to the worker. However, the owner cannot extract this surplus by further lowering the wage, which is constrained below at 0. In fact, the owner has no efficient instrument that can be used to extract this additional surplus. The best that she can do is to increase the effort level, even though this is beyond the efficient, first-best level. As can be seen from the Figure 1, the contract point moves out along the diagonal line into the rat race region. As $\alpha$ approaches 1, effort approaches the fully dissipative rat race effort $e^{RR}$ defined by $\kappa e^{RR} = c(e^{RR})$.

Figure 2 summarizes how the observable contract $(w(\alpha), e(\alpha))$ evolves as $\alpha$ increases and the firm’s longevity increases. In the efficient region, when $\alpha \leq \alpha^*$, the effort is first best, but the wage declines as $\alpha$ increases. This is because the value of future ownership is increasing, and this future reward is a substitute for current compensation through the wage. When $\alpha$ increases beyond $\alpha^*$, we move into the rat race region where the wage constraint is binding, and rents generated by an increase in $\alpha$ can only be captured by requiring higher effort. This results in an increasingly severe rat race, with an inefficiently high effort level determined by the worker’s individual rationality constraint. We summarize this characterization in the following proposition:

**Proposition 1** Let $\alpha$ be the firm’s survival probability.
1. If $\alpha \leq \frac{c(e^{FB})}{\kappa e^{FB}}$, then the effort level $e(\alpha) = e^{FB}$ is efficient. The wage $w(\alpha)$ is non-negative and is decreasing in $\alpha$.

2. If $\frac{c(e^{FB})}{\kappa e^{FB}} < \alpha < 1$, then the wage is zero and the effort level is inefficiently high with $e(\alpha) = \frac{c(e(\alpha))}{\alpha \kappa}$. Effort is increasing in $\alpha$.

The value $V(\alpha)$ of owning the firm can be decomposed into the sum of the production surplus $\kappa e(\alpha) - c(e(\alpha))$ and a transfer $c(e(\alpha)) - w(\alpha)$ from the young generation to the old. As $\alpha$ increases and the firm becomes more long lived, the production surplus decreases (due to the inefficient effort level) while the transfer increases, eventually becoming the main contributor to owner profit. The value of owning the firm comes increasingly from the ability to extract a transfer from the young and intergenerational inequality increases as firm longevity increases.

As can be seen from Figure 1, although the value $V(\alpha)$ of the firm increases it remains bounded, even though the expected lifetime $\frac{1}{1-\alpha}$ becomes infinite and there is no discounting. In fact, $V(\alpha) \to \kappa e^{RR} = c(e^{RR}) < \infty$ as $\alpha \to 1$. This is because as firm longevity increases, the firm is becoming less efficient. The per-period surplus $\kappa e - c(e)$ converges to zero more rapidly than the expected lifetime increases. The next proposition summarizes these results.

Figure 2: Wage $w$ and effort $e$ as functions of $\alpha$
Proposition 2 Let $\alpha$ be the firm’s survival probability. As $\alpha$ increases, the value of owning the firm increases and the inequality between the old and the young generation increases. Moreover, as $\alpha \to 1$, the per-period surplus, $\kappa e - c(e)$, converges to zero and complete rent dissipation takes place.

Note that this inefficiency would not occur if it were possible to write complete intergenerational contracts, which would guarantee efficient effort in each generation. It is precisely the inability to constrain rent seeking in the next generation which creates inefficiency today.

4 Experiment

To test the predictions of the model, we designed an OLG labor-market experiment. The reason for conducting the experiment is that while the dynamic program is tractable to analyze, the acceptance of rat-race contracts depends on three assumptions which may be violated in practice.

First, in order to accept a rat-race contract, a worker must expect future workers to be willing to accept a similar contract when s/he becomes an owner. Of course, the willingness of future workers to accept such contracts will depend on their expectations about the behavior of future generations, and so on. Previous experiments (e.g., Smith, Suchanek and Williams, 1988; Lei, Noussair and Plott, 2001; Hussam, Porter and Smith, 2008; Chaudhuri, Schotter, and Sopher, 2009). In our experiment, failure to establish common expectations about future contracts is likely to prevent convergence to a rat-race equilibrium.

Second, owners and workers must be able to calculate the workers’ expected payoff from accepting a contract. This requires them to consider the expected payoff from becoming an owner and the short-run payoff from the contract being offered. Previous experiments (e.g., Johnson, Camerer, Sankar, and Rymon, 2002) have provided evidence that a significant fraction of participants in laboratory experiments does not look into the future when making decisions. Note that the uncertainty this creates about individual rationality may also hamper the formation of common expectations.
Third, rat-race contracts imply a substantial inequality in earnings between owners and workers. Moreover, even if workers do not care about the short-term inequality in earnings, they may still decline rat-race contracts if they dislike the risk they are being exposed to with the uncertainty regarding firm longevity. Evidence from laboratory experiments suggests that many individuals care about the distribution of earnings and are risk averse (see, e.g., Holt and Laury, 2002; Sobel, 2005).

4.1 Experimental design

The experiment consists of a sequence of labor markets which we refer to as “periods.” In each period, three firms operate in the market. Every firm is owned by an individual (the owner) who needs to hire one of six workers to operate the firm. Hiring is conducted through a continuous-time one-sided market in which owners offer employment contracts. Workers can accept a single employment contract at any point in time, while employers can revise the terms in their contracts as many times as they wish until they hire a worker or the period ends. A period ends when all owners have employed a worker or two minutes have passed.

Employment contracts consist of a wage $w \in \{0, 1, \ldots, 150\}$ and a binding effort level $e \in \{0, 20, \ldots, 100\}$. We chose to have a discrete choice set for effort in order to simplify the subjects’ decision problem. Workers and owners who do not agree on a contract receive zero earnings. The owner’s earnings from the period are given by $1.5e - w$ while the worker’s earnings are given by $w - c(e)$, where $c(e)$ is given in Table 1. The efficient level of effort which maximizes the difference between $1.5e - c(e)$ is 40.

<table>
<thead>
<tr>
<th>Units of effort</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of effort</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>60</td>
<td>100</td>
<td>140</td>
</tr>
</tbody>
</table>

Table 1: Units and cost of effort

Experimental sessions consist of 21 subjects. At any given point in the experiment,

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12In the first three periods of the experiment, the labor market operated for 4 minutes in order to give time to participants to familiarize themselves with the experiment and the computer interface.
13In order to facilitate learning, owners were also provided with a profit calculator which allowed them to calculate their earnings and that of the worker for any given contract.
there are 3 owners, 6 workers, and 12 observers. Observers are included in our design to reduce incentives for strategic behavior which might arise from subjects being reborn, and to create variation in the set of potential workers and firms. As their name suggests, observers are able to observe the decisions made in the labor market, but they do not directly participate in it. All players have access to the complete history of trades including information about the earnings of owners and workers.\footnote{This information may facilitate learning as it allows individuals to observe strategies that are associated with higher payoffs (e.g., Apesteguia, Huck and Oechssler, 2007; Apesteguia, Huck, Oechssler, and Weidenholzer, in press).}

As with the theoretical model, our interest is how the prospect of future ownership impacts the efficiency of traded contracts. To link current ownership to future ownership, a (virtual) dice is rolled at the end of each period to determine whether the three firms continue to operate. If firms continue their operation, employed workers become owners in the next period. Six of the twelve observers are randomly chosen to become workers and all remaining participants become observers.\footnote{In the case that a firm did not hire a worker, a random observer is assigned the role of the owner. This rarely happened in practice with less than 0.2 percent of owners remaining unmatched (31 out of 2382 owners across sessions).} If the firms do not continue their operation, roles in the following period are randomly assigned.

We deliberately chose to have an excess supply of workers in the labor market. As explained in footnote\footnote{See, for example, Roth, Prasnikar, Okuno-Fujiwara, and Zamir (1991) where fairness is eliminated in an environment with competition.} the equilibrium prediction of a model with an excess supply of workers and competition is the same as having the owner make a ‘take it or leave it’ offer to a single member of the young generation. An excess supply of workers, which exists in most naturally occurring labor markets, also allowed us to reduce the impact of risk attitudes and social preferences on behavior. The existence of workers with heterogeneous preferences may play less of a role in real labor markets where individuals can select into firms. Self-selection implies that rat races may emerge even if many workers are risk averse and care about inequality in earnings.\footnote{See, for example, Roth, Prasnikar, Okuno-Fujiwara, and Zamir (1991) where fairness is eliminated in an environment with competition.}

4.2 Experimental treatments

The experiment consists of two treatments that differ only with respect to the probability that a firm will continue to operate in the following period ($\alpha$). The values for
α were chosen such that in one treatment (Efficient), the efficient level of effort would be exerted in equilibrium, whereas in the other treatment (Rat Race), the equilibrium level of effort would be inefficiently high. We ran 6 sessions per treatment and 126 subjects participated in each treatment.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>α</th>
<th>Wage</th>
<th>Effort</th>
<th>Owner’s Payoff</th>
<th>Worker’s Payoff</th>
<th>Per-Period Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rat Race</td>
<td>5/6</td>
<td>0</td>
<td>80</td>
<td>120</td>
<td>−100</td>
<td>20</td>
</tr>
<tr>
<td>Efficient</td>
<td>2/6</td>
<td>0</td>
<td>40</td>
<td>60</td>
<td>−20</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 2: Experimental treatments and predictions

Table 2 presents the treatments we ran and the corresponding predictions of our model based on Propositions 1 and 2. In the Rat-Race treatment, firms have a higher survival probability (α = 5/6). In both treatments, wages are predicted to be 0 in equilibrium. Hence, the participation constraint is binding. This implies that in the Rat-Race treatment, the employer can demand a higher effort level from the worker, since the worker’s probability of promotion is higher. While this increases the payoff of the employer, it reduces the per-period surplus created, $1.5e - c(e)$.

4.3 Experimental procedures

All of the experiments were run in the Experimental Economics Laboratory at the University of Melbourne in September and October of 2009. The experiments were conducted using z-Tree (Fischbacher, 2007). All of the 252 participants were undergraduate students at the University, who were randomly invited from a pool of more than 2000 volunteers using ORSEE (Greiner, 2004).

Upon arrival to the laboratory, participants were asked to read the instructions and answer some practice questions\textsuperscript{17} Their answers to the practice questions were checked by the experimenter. Once the answers of all participants were checked, the experimenter read aloud a summary of the instructions. The purpose of the summary

\textsuperscript{17} A copy of the instructions and the practice questions can be downloaded from http://www.economics.unimelb.edu.au/staff/nisvan/instructions-rat-race.pdf.
was to ensure that the main features of the experiment were common knowledge amongst the participants.

Each session lasted at least 75 minutes. After the 75 minutes passed, the experiment ended when the existing set of firms ended. Hence, each session had a random ending time.

Payments to the subjects were made in cash at the end of the experiment based on the earnings they accumulated throughout the experiment. Participants were paid at the rate of E$10 = 1 AUD. In addition, each subject received a show-up fee of $15. Since payoffs during the experiment could be negative, the subjects could use the show-up fee to prevent bankruptcy during the experiment. The average payment at the end of the experiment was 47.30 AUD. At the time of the experiment, 1 AUD = 0.80 USD.

5 Results

Our analysis of the experimental data focuses on the theoretical predictions stated in Propositions 1 and 2. For each result, we provide support based on descriptive statistics, figures, and statistical tests.

Result 1 In line with Proposition 1, an increase in firm longevity leads to a rat race characterized by inefficiently high effort levels and low wages.

Figure 3 presents the evolution of the mean effort level for the accepted contracts in each treatment. As can be seen in Panel A, effort converges to the efficient level \( e = 40 \) in the Efficient treatment and remains at this level for the reminder of the experiment. Effort in the Efficient treatment appears thus to be remarkably well predicted by our model. The same applies for the Rat-Race treatment. As can be seen in Panel B, effort starts at a similar level as in the Efficient treatment, but increases rapidly towards the theoretical prediction \( e = 80 \). This suggests that, while efficiency may be a salient consideration in the determination of the initial contracts,

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18 We had four cases of bankruptcy across all experimental sessions. All four cases were in the Rat-Race treatment. Subjects were given additional money and allowed to continue in the experiment. It was explained that the additional money would be subtracted from their final payment.
the recursive structure of the firm leads effort to inefficient levels, as predicted by the model.

Statistical support for these observations can be found in Table 3. Columns 1 to 4 present the mean effort, mean wage, and mean firm value, along with the theoretical predictions. Columns 5 and 6 present the p-values from tests examining whether there are significant deviations from the theoretical predictions in the Efficient and Rat-Race treatments, respectively. Column 7 presents the p-values from tests examining whether there are significant differences across treatments. These tests are from a regression analysis using session-level random effects. To account for the fact that experimental sessions varied in length due to their stochastic ending, we use data from the last 30 periods in each session.¹⁹

As can be seen in the first row of Table 3 (Columns 1-4), effort in both treatments is notably close to the theoretical prediction. Average effort in the Efficient treatment is 40.8 which is not significantly different from the predicted level of 40 (p-value = 0.82). Similarly, mean effort in the Rat-Race treatment is 74.6 which is not significantly different from the theoretical prediction of 80 (p-value = 0.11). Finally, in line with the theoretical predictions, the effort level in the Rat-Race treatment is approximately double that of the Efficient treatment, a treatment effect which is highly significant (p-value < 0.01).

Figure 4 presents the evolution of the mean wage for the accepted contracts in each treatment. As can be seen, wages start at similar levels in both treatments (around E$35) and decline over time. In the Efficient treatment, wages stabilize after approximately 30 periods at roughly E$15, while wages in the Rat-Race treatment continue to decrease over the course of the experiment. The statistical results in Table 3 reveal that the deviation from the theoretical prediction (E$0) is significant in both treatments (p-value < 0.01). Average wages are significantly higher in the Efficient treatment than in the Rat-Race treatment (p-value < 0.01), even though the actual difference is small (E$5.7).

¹⁹Regression results include only the longevity treatment variable as the explanatory variable and session level random effects. Differences across treatment are qualitatively unaffected if we include all periods in the statistical analysis or if we include time and demographic controls. Differences across treatments are also highly significant if we use non-parametric Mann-Whitney tests with session means as observations.
Figure 3: Average effort by period for the Efficient and Rat-Race Treatments.

<table>
<thead>
<tr>
<th></th>
<th>Efficient Treatment</th>
<th>Rat Race Treatment</th>
<th>Statistical tests (entries are p-values)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prediction</td>
<td>Mean</td>
<td>Prediction</td>
</tr>
<tr>
<td>Effort</td>
<td>40</td>
<td>40.8</td>
<td>80</td>
</tr>
<tr>
<td>Wage</td>
<td>0</td>
<td>15.8</td>
<td>0</td>
</tr>
<tr>
<td>Value of firm</td>
<td>40</td>
<td>39.5</td>
<td>20</td>
</tr>
<tr>
<td>Employer’s profit</td>
<td>60</td>
<td>45.4</td>
<td>120</td>
</tr>
<tr>
<td>Worker’s profit</td>
<td>-20</td>
<td>-5.9</td>
<td>-100</td>
</tr>
</tbody>
</table>

The table presents the mean values from the experiment. The means are calculated using all observations from the last 30 periods of each experimental session. Columns (5) and (6) present the p-values of tests examining whether there are significant deviations in the experiment from our theoretical predictions in the Efficient and Rat Race treatment, respectively. Column (7) presents the p-values of tests examining whether there are significant differences across the two treatments.

Table 3: Summary statistics, theoretical predictions, and statistical tests
Figure 4: Average wage by period for the Efficient and Rat-Race Treatments.

One possible reason for the higher-than-predicted wages is the high degree of risk and inequality implied by the equilibrium contracts in both treatments. As discussed in Section 4, while we chose to have an excess supply of workers in order to mitigate the impact of these factors, it is not entirely surprising that workers require premia to accept a contract. In the Rat-Race treatment, the positive wages observed are consistent with workers exhibiting low degrees of risk (or loss) aversion. Rationalizing the wages in the Efficient treatment in the same way, by contrast, requires an unreasonably high level of risk or loss aversion. This suggests that other forces may be at play in this environment. For example, subjects may underweight the probability of the firm surviving in the Efficient treatment or may view zero profit as a salient reference point in accepting contracts.

Having established the formation of rat races, we next study some of their consequences.

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\footnote{Using a CRRA utility function of the form $\frac{x^{1-\sigma}}{1-\sigma}$, the data is rationalized with a $\sigma = .091$. Using a Koszegi-Rabin (2006) loss aversion function, the data can be rationalized with a loss aversion parameter of $\lambda = 1.11$.}

\footnote{Using a CRRA utility function, the data is rationalized with a $\sigma = 4.5$. Using a Koszegi-Rabin (2006) loss aversion function, the data can be rationalized with a loss aversion parameter of $\lambda = 4$.}
Result 2 In line with Proposition 2, the formation of rat races leads to a dissipation of rents. Inequality in payoffs between owners and workers increases in $\alpha$.

Recall that the per-period value of the firm is $1.5e^{-c}(e)$. This value is not affected by wages which represent a net transfer. Given the high degree of accuracy in the effort predictions, it is unsurprising that the value of the firm is also well-predicted. As can be seen in Column 2 in Table 3, the mean per-period value of the firm in the Efficient treatment is E$39.5 which is remarkably close to the predicted E$40, and not statistically different from it ($p$-value = 0.77). Similarly, the mean per-period value of the firm in the Rat-Race treatment is E$22.7 which is close to the predicted E$20, and not statistically different from it ($p$-value = 0.12). In contrast, the per-period value of the firm is significantly different across the two treatments ($p$-value $< 0.01$). Thus, the main prediction of the theoretical model — that an increase in firm longevity can lead to a dissipation of rents — is borne out in our data.

With regards to the inequality in owner-worker payoffs mentioned in Result 2, it is useful to examine the data from the twelve individual sessions. This will also allow us to better understand how the two treatments differ in their convergence patterns. In the top panel of Figure 5, the profit of workers and owners is shown for each of the six sessions of the Efficient treatment. As can be seen, there is remarkably little difference in behavior across sessions with workers having a mean loss of E$6 and owners a mean profit of roughly E$45. As shown in Table 3, profits do not fully converge to the theoretical predictions ($p$-value $\leq 0.05$). This is because the wages are higher than those predicted by the theoretical model, as described above.

As predicted by the model, the inequality between the payoffs of owners and workers is even greater in the Rat-Race treatment, with owners enjoying a mean profit of E$102 and workers having a mean loss of E$79 ($p$-value $< 0.01$). As shown in the bottom panel of Figure 5, behavior in all sessions seems to have followed a similar pattern with payoff inequality increasing over time. However, there are some differences in the convergence speeds across sessions. Specifically, Sessions 2, 5 and 6 in the Rat-Race treatment display a slower rate of convergence. This can be explained by the empirical probability of continuation which differed across sessions. In Table 4, we present the survival rate of firms based on the number of times established firms were replaced by new firms by the 10th, 20th, 40th and last period. As can be
seen in the first column, the probability that old firms continue operating is markedly low in the first 10 periods for Sessions 2, 5, and 6 in the Rat-Race Treatment (56%, 56% and 67%, respectively, compared to 100%, 89% and 89% for the other sessions). These are the sessions where convergence was the slowest.

<table>
<thead>
<tr>
<th></th>
<th>Period 10</th>
<th>Period 20</th>
<th>Period 40</th>
<th>Last Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>1.00</td>
<td>.84</td>
<td>.92</td>
<td>.88</td>
</tr>
<tr>
<td>Session 2</td>
<td>.56</td>
<td>.79</td>
<td>.79</td>
<td>.80</td>
</tr>
<tr>
<td>Session 3</td>
<td>.89</td>
<td>.95</td>
<td>.87</td>
<td>.86</td>
</tr>
<tr>
<td>Session 4</td>
<td>.89</td>
<td>.79</td>
<td>.79</td>
<td>.78</td>
</tr>
<tr>
<td>Session 5</td>
<td>.56</td>
<td>.74</td>
<td>.85</td>
<td>.83</td>
</tr>
<tr>
<td>Session 6</td>
<td>.67</td>
<td>.74</td>
<td>.79</td>
<td>.85</td>
</tr>
</tbody>
</table>

Table 4: Empirical Probability of Continuation in Rat-Race Treatments

6 Conclusion

In many firms, workers’ decisions are motivated by the prospect of being promoted to a senior role that entitles them to a share of the firm’s future profits. Since the profits depend on the contracts that future workers will be willing to accept, the contract design in these firms is recursive.

We have shown that recursive contracts can give rise to a rat race with high effort levels, low wages, and rent dissipation if the expected value of owning the firm is sufficiently high. Our model illustrates that rat races may be more common in firms that are expected to continue operating for longer periods of time. This finding suggests that firm longevity can have adverse effects and reduce efficiency. This is in contrast to findings in previous studies (reviewed in Section 2) that have associated firm longevity with higher efficiency arising from the ability of individuals to enforce cooperation across generations of workers. We are not aware of any empirical evidence linking rat races with firm longevity.

To obtain empirical support for our model, we designed a laboratory experiment. The experimental results lend strong support to the predictive power of our model. Despite the computational complexity of the decision problem, the exposure to risk,
Figure 5: Profits of the owner and worker for each of the twelve treatments.
the presence of payoff inequalities, and the well-documented difficulty of establishing common expectations between individuals, we find that increases in firm longevity lead to rat rates characterized by inefficiently high levels of effort and substantial inequality in earnings between owners and workers. To our knowledge, this is the first experiment to document the emergence of rat races in a laboratory environment.

Given the inefficiency of the rat race equilibrium, it is interesting to consider what institutions or policy interventions might prevent rat races from emerging. Restricting wages to a minimum will not prevent rat races as owners, in response, will simply demand higher levels of effort from workers. However, imposing a cap on the maximum number of hours could be effective. Reducing the rents available to owners and increasing the number of senior hires from outside the firm will also weaken the rat race. These remedies are a topic for future studies.

7 Appendix: Uniqueness of the Subgame Perfect Equilibrium

We consider the version of the model where one individual is born at each date, and the owner makes a take it or leave it offer to the worker. To reduce notation we set $\kappa = 1$.

The contract space is $S = \{(e, \pi) : 0 \leq e < \bar{e}, \, \pi \leq e\}$, where we write $\pi = e - w$ for the current period profit to the owner. A history is a finite sequence $h = (c_0, c_1, c_2, \ldots)$ where $c_t = (e_t, \pi_t) \in S$ is a contract offered at date $t$. At history $h$ the worker chooses a set $A \subset S$ of contracts that will be accepted, and the owner chooses a contract $c \in S$.

We consider subgame perfect equilibria. In such an equilibrium the owner and the worker solve the interlinked Bellman equations

$$V_h = \max \left[ 0, \sup_{(e, \pi) \in S} \chi_A(e, \pi) \pi \right],$$

$$U_h = \max \left[ 0, \sup_{A \subset S} \chi_{A^+}(e, \pi) \left( e - \pi - c(e) + \alpha V_h(c) \right) \right].$$
The supremum over an empty set is, by convention, \(-\infty\). The set \(A_+ = \{(e, \pi) \in A : \pi \geq 0\}\) is the set of contracts in \(A\) that are not dominated by the owner’s outside option.

The indicator functions are defined by \(\chi_A (e, \pi) = 1\) if \((e, \pi) \in A\) and \(\chi_A (e, \pi) = 0\) if \((e, \pi) \not\in A\); and similarly for \(\chi_{A_+}\). The history \(h|c\) is the history \(h\) with the contract \(c = (e, \pi)\) appended at the end. Since we are at an equilibrium, the suprema are actually attained, and we could write \(\text{max}\) rather than \(\text{sup}\).

By subgame perfection, the worker will always accept any individually rational contract. Thus

\[
A (h) = \{(e, \pi) \in S : e - \pi - c(e) + \alpha V_{h|c} \geq 0\}.
\]

Consider the owner’s problem. The set \(A\) is bounded above by the functions \(\pi = e\) and \(\pi = e - \pi - c(e) + \alpha V_{h|c}\). Since \(c(e) \to \infty\) as \(e \to \bar{e}\), \(A\) has compact upper sections and there exists a point in \(A\) that maximizes \(\pi\). It is clear that this maximum must occur at a value of \(e\) in the interval where the constraint \(e - \pi - c(e) + \alpha V_{h|c} \geq 0\) binds, since otherwise a small increase in \(e\) would increase \(\pi\). But \(e - \pi - c(e) + \alpha V_{h|c}\) is strictly concave on this interval, so the maximum must be unique. It may, however, be below the owner’s reservation value of 0. But the owner’s optimal choice is unique in this case as well: she chooses the outside option with payoff 0.

We thus conclude that the strategies of both the owner and the worker at history \(h\), and in particular the value \(V_h\), are entirely determined by the number \(\alpha V_{h|c}\). Thus the equilibrium history is completely determined by the sequence of values \(V_t = V_{h_t}\), where \(h_t\) is the history truncated at date \(t\), and these values satisfy the recursion

\[
V_t = \max \left[ 0, \max_e \min [e, \alpha V_{t+1} + e - c(e)] \right].
\]

Since \(V_{t+1} \geq 0\), and the owner can always choose the efficient contract yielding a strictly positive surplus, it follows that \(V_t > 0\) and we can write the simpler recursion

\[
V_t = \max_e \min [e, \alpha V_{t+1} + e - c(e)].
\]

It is convenient to write \(Z_t = \alpha V_t\) for the expected valuation prior to knowing whether the firm continues. To simplify notation, we write \(Z\) for \(Z_t\) and \(Z_+\) for \(Z_{t+1}\).
We will consider how \( V \) is determined by \( Z_+ \). We consider separately the cases where the non-negativity constraint \( \pi \leq e \) does and does not bind.

If the non-negativity constraint does not bind then \( V = v_0 (Z_+) \), where the function \( v_0 (z) \) is determined by

\[
\begin{align*}
  c' (e) &= 1, \\
  v_0 (z) &= z + e - c (e)
\end{align*}
\]

so

\[
v_0 (z) = e^{FB} - c (e^{FB}) + z.
\]

Note that \( v_0 (0) = e^{FB} - c (e^{FB}) > 0 \), and \( v_0' (z) = 1 \).

If the non-negativity constraint does bind then \( V = v_1 (Z_+) \), where the function \( v_1 (z) \) is determined by

\[
\begin{align*}
  e &= z + e - c (e), \\
  v_1 (z) &= e
\end{align*}
\]

so

\[
v_1 (z) = c^{-1} (z)
\]

is the inverse to the cost of effort function.

It follows that the valuation is determined by \( V = v (Z_+) \), where

\[
v (z) = \begin{cases} 
  v_0 (z) & \text{if } z \leq z^{FB} \\
  v_1 (z) & \text{if } z \geq z^{FB}
\end{cases}
\]

where \( z^{FB} = c (e^{FB}) \). To see this, notice that \( v_0 (z) \geq v_1 (z) \), \( v_0 (z^{FB}) = v_1 (z^{FB}) \), and \( v_0' (z^{FB}) = v_1' (z^{FB}) \). Thus \( v_0 (z) \) is the tangent line that supports \( v_1 (z) \) at \( z^{FB} \).

For \( z \geq z^{FB} \), \( v (z) \) is the inverse cost function, while for \( z \leq z^{FB} \) it is the tangent supporting the inverse cost function at \( z^{FB} \). Thus it is continuously differentiable, monotonic increasing with \( 0 \leq v' (z) \leq 1 \), and concave. It is strictly non-negative.

\( ^{22} \)We use upper case letters \( V, Z \) for equilibrium values, and lower case variables \( \pi, z, e \) for other variables.
and bounded, with \( v(0) = e^{FB} - c(e^{FB}) > 0 \) and \( \lim_{z \to \infty} v(z) = \bar{e} \).

Contract dynamics are determined by the relations

\[
V = v(Z), \\
V = \frac{Z}{\alpha}
\]

and can be determined in the usual way from the loci of these two relations in \((Z, V)\) space. Since the loci cross once, it is clear that for \(0 < \alpha \leq 1\) there exists a unique stationary equilibrium. But this is the only equilibrium. Any other equilibrium can only be supported by a value trajectory that either increases without bound, violating the assumption that \(e \leq \bar{e}\), or decreases without bound, violating the constraint that \(e \geq 0\).

We note, in conclusion, that the steady state condition \(V = v(\alpha V)\) can be written equivalently as

\[
V = \max_{\bar{e}} \min \left[ e, \frac{e - c(e)}{1 - \alpha} \right].
\]

This can be verified by considering separately the cases where the non-negativity constraint \(\pi \leq e\) does or does not bind.
References


