A SIMPLE MODEL OF INSURANCE MARKET DYNAMICS

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Summary. The purpose of the paper is to construct and study a simple but realistic model of an insurance market. The model has a minimalist construction in the sense that the number of parameters defining it is strictly limited and the elimination of any one of them would destroy its realism. There are, in fact, 11 essential parameters.

Each of the parameters has a physical interpretation. Some determine competitive effects within the market, some barriers to entry, and so on. The effect of each on various aspects of the market is examined in the presence of simulated loss experience. The aspects of the market considered include stability of premium rates, profitability, market concentration, and others.

Some of the parameters are capable of use as regulatory controls. Two parameters, in addition to the original 11, are explicit price controls.

Despite its simplicity, the model displays considerably complex behaviour. Some results are intuitive but some are not. For this reason, regulatory controls need to be applied with great caution lest they induce perverse effects, possibly even the reverse of those intended.

The effect of the parameters on market behaviour is first studied in the absence of catastrophic events from the loss experience. Subsequently, the effect of a single such event is studied.

Keywords: competition, insurance market, market concentration, market cycles, price control, regulation, stability.

1. Introduction

This paper constructs a model of a competitive insurance market. The intention is that the model be as simple as possible, i.e. contain as few parameters as possible, while capturing all the essential dynamics of a real market. This enables the effects of individual components of the model, e.g. barriers to industry entry, to be studied.

Some of these effects may be relevant to public policy. For example, high insurance profit margins, or highly volatile insurance premium rates, are viewed unfavourably by consumer advocates. Higher levels of insurer failure may be viewed more unfavourably than lower by insurer regulators or legislatures.

The paper simulates the behaviour of a market according to the defined model, and examines the effects of variation of individual parameters, or small groups of parameters, that define specific economic effects on the market. It may be possible to influence these parameters, and hence the behaviour of the market, if this is seen as desirable public policy.

This approach to insurance market modelling is somewhat different from previous examples in the literature. Of course, the literature contains numerous studies of particular isolated aspects of the market. The emphasis here, however, is on the detail of any single aspect and more on the integration of all market dynamics into a single model.

In a relatively little known paper that might reasonably be considered a seminal contribution to the subject now known as **dynamic financial analysis (DFA)**, Coutts & Devitt (1989), introduced stochastic modelling of a single insurance operation but did not link this to competitor behaviour through competitive dynamics. A related paper by Daykin et al (1987) covered similar ground.

Daykin & Hey (1990) and Daykin, Pentikäinen & Pesonen (1994) added some simple competitive dynamics, but retained the main focus on the behaviour of a single insurer in a competitive market. Market cycles were regarded as exogenous.

Subsequent years have seen a great deal of DFA development but, by its nature, this work too is usually concerned with the future financials of a single insurer, with or without recognition of external competitive influences.

2. Model specification

2.1 Outline

As mentioned in Section 1, the intention here is to construct a simple model that recognises the major dynamics of an insurance market. It is useful to begin by listing these. It is supposed that the insurance market, consisting of a number of insurers, operates in discrete time. The unit of time is arbitrary but may be thought of as the minimum period during which an insurer is able to re-assess and respond to its market circumstances.

It is assumed that the market underwrites a single line of business. An equivalent assumption is that each insurer underwrites the same composition of business by line, and that, over time, its premium rates in all lines vary by the same percentage factors. There is clear scope for generalisation here.

In each period each insurer behaves as follows:

2.1.1 It has certain target premium rates which, if achieved in its underwriting, would achieve certain corporate goals, e.g. return on equity.

2.1.2 These target premium rates are tempered by competitive forces that are manifest in the form of the rates charged by competitors in the immediate past.

2.1.3 The resulting premium rates brought to market by the insurer will generate underwriting results for the quarter. These will be influenced by the loss experience, which is a stochastic quantity and not fully under the insurer's control. It may include catastrophe (CAT) events.

2.1.4 These results update the insurer's balance sheet, including solvency position.

2.1.5 At the end of the period, the competitive position of the market is re-balanced. The insurer will exit the market if its solvency position has fallen below a threshold of viability.

Further, at the end of the period:

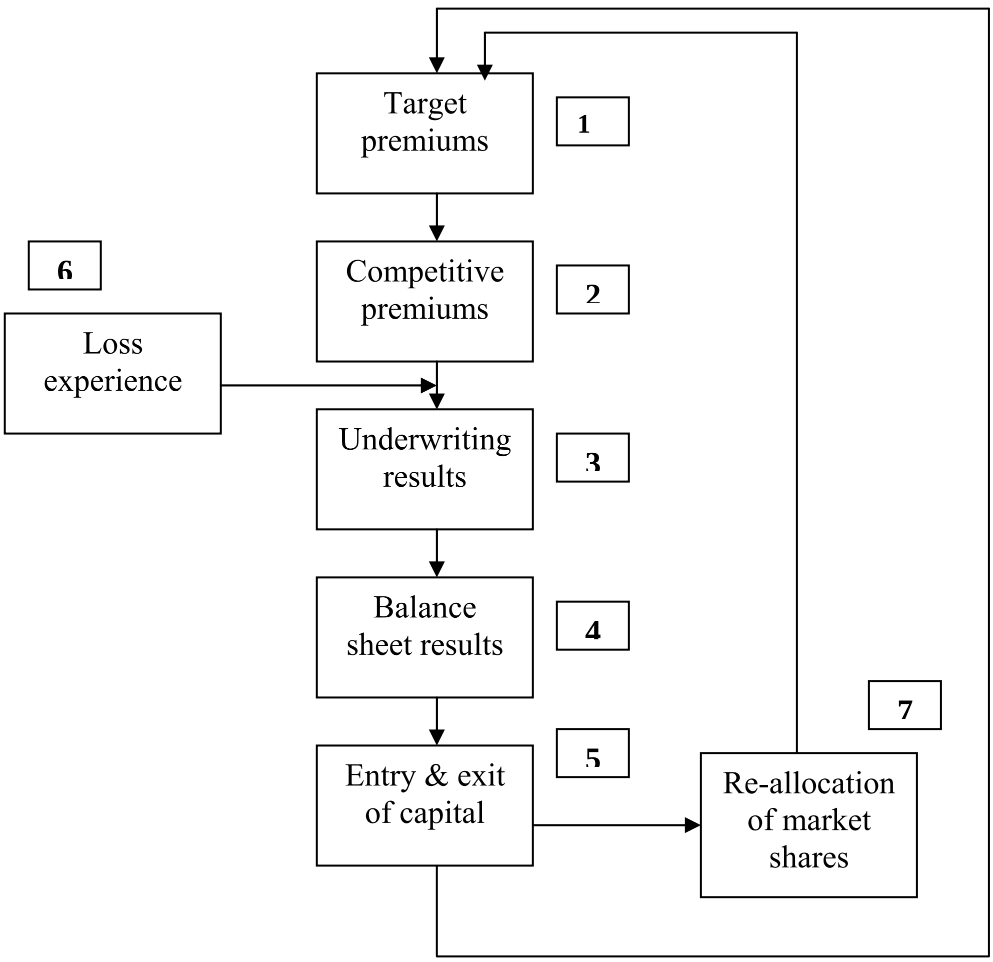
2.1.6 New insurers may enter the market. They will be lured to do so if the profit margins emerging in 2.1.3 are sufficiently lucrative.

2.1.7 Market shares of all insurers now participating in the market are re-balanced. Business will be attracted from insurers offering higher premium rates to those offering lower.

The process then re-commences at 2.1.1.

This process is represented in diagrammatic form in Figure 2.1. The mathematical details of the dynamics appear in Section 2.2. A boxed number n in the figure means that the detailed description appears in Section 2.2.n.

Figure 2.1
Market process over a single period



2.2 Detail

This sub-section gives the mathematical details of the market dynamics. It is reiterated that emphasis is on simplicity, on capturing the essential market detail with as few parameters as possible.

First some notation. A quantity represented in bold italics is an input to the process. The system parameters are also listed in bold italics.

Notation

Let

t = iteration number (suppressed within an iteration, but explicit when the relation between iterations is at issue)

I_t = number of insurers in n-th iteration

i = label for i-th insurer (in order of volume of exposure units)

E_{it} = number of exposure units underwritten

$E_t = \Sigma_i E_{it}$ = total exposure for whole market (supposed constant over time)

K_{it} = net assets

$S_{it} = K_{it}/(P_{it}E_{it})$ = solvency ratio

r_F = risk free rate of return

r_M = share market expected rate of return

λ_C = expected CAT claim frequency (for whole market)

μ_C = expected CAT claim size (for whole market)

λ_N = expected non-CAT claim frequency per exposure unit (common to all insurers)

μ_N = expected non-CAT claim size (common to all insurers)

T_{it} = target premium per unit exposure

P_{it} = competitive premium per unit exposure

L_{itC} = CAT incurred losses

L_{itN} = non-CAT incurred losses

π_{it} = insurance profit

D_{it} = dividend paid

For simplicity, it is assumed that all premiums are received at the start of a year and all associated claims paid at the end of that year. It is further assumed that each insurer invests its premiums received P in risk free assets, and its net assets K in equities. Then let

r_{Kit} = insurer's return on equity = $r_F + (r_M - r_F)[K_{it}/(K_{it} + E_{it}P_{it})]$

Steady state

The system will have a steady state in which, if losses and investment return occurring in each period are equal to their expected values, premiums will be sufficient to maintain a constant solvency ratio after returning an economic profit. In the competitive market this steady state will be viewed by participants as some sort of normative, or medium term average, state.

Define

K_0 = steady state capital per exposure unit

P_0 = steady state premium per exposure unit

$S_0 = K_0/P_0$ = steady state solvency ratio

r_{K0} = steady state expected return on equity.

The undiscounted risk premium per unit exposure is $\lambda_N\mu_N + \lambda_C\mu_C/E$, and so, on the assumption of no correlation between claim costs (payable at period end) and the share market,

$$P_0 = (\lambda_N\mu_N + \lambda_C\mu_C/E)/(1+r_F) \tag{2.1}$$

This premium will be sufficient to pay claims. Steady state capital K_0 generates an economic return r_M, which equates to period profit. Note that tax, and particularly the issue of tax deadweight, is not considered here.

It follows that K_0 is a free variable, and there is a different steady state, with a different P_0, for each choice of K_0. The value of K_0 is determined by forces external to the market, such as regulatory constraints and consumers' concerns about security.

By the definition of r_K given above,

$$r_{K0} = r_F + (r_M - r_F)[K_0/(K_0 + P_0)] \tag{2.2}$$

System dynamics

System parameters are of two types:

- those that describe the environment within which the market exists (call these **environmental parameters**); and
- those that describe the market dynamics within that environment (call these **dynamical parameters**).

Those listed above in bold italics are of the former type, and are summarised for convenience in Table 2.1.

Table 2.1
Environmental parameters in market model

K_0	Steady state capital per unit exposure
E	Total exposure for whole market
r_F	Risk free rate of return
r_M	Share market expected rate of return
λ_C	Expected CAT claim frequency (for whole market)
μ_C	Expected CAT claim size (for whole market)
λ_N	Expected non-CAT claim frequency per exposure unit (common to all insurers)
μ_N	Expected non-CAT claim size (common to all insurers)

The dynamics of the model are as set out below, where the numbering follows that in Figure 2.1.

(1) Target premiums

$$T_{it} = P_0 \exp [- k_1 (S_{it} - S_0)] \tag{2.3}$$

Target premiums increase (decrease) as solvency falls below (rises above) steady state solvency. Thus, k_1 may be viewed as **premium-to-solvency sensitivity** parameter.

(2) Competitive premiums

$P_{it} = \max(k_{11} P_0, \min(T_{it} [T_{it} / \mathrm{avg}(P'_{i,t-1})]^{-k2}, k_{12} P_0))$, $0<k_2<1$, for new companies in their first period of existence (2.4)

$= \max(k_{11} P_0, \min(k_{13} (T_{it} [T_{it} / \mathrm{avg}(P'_{i,t-1})]^{-k2}) + (1 - k_{13}) P_{i(t-1)}, k_{12} P_0))$ afterwards (2.5)

where $\mathrm{avg}(P'_{i,t-1})$ denotes the average of $P_{i-2,t-1}$, $P_{i-1,t-1}$, $P_{i,t-1}$, $P_{i+1,t-1}$, $P_{i+2,t-1}$, with non-existent values (e.g. $P_{-2,t-1}$) deleted from the average.

The main parameter in conceptual terms here is k_2. The member involving it shows that insurer i is competitively influenced by the two next larger and two next smaller insurers (recall that i indexes insurers by volume of exposure).

As the average premium of the market segment consisting of insurer i and these four competitors, as observed in the previous period, falls below the target premium of insurer i, the latter is influenced to bring to market premiums below its target. Conversely when competitor's premiums rise above insurer i's target.

The parameter k_2 is restricted to the interval (0,1). The extreme cases k_2=0 or 1 would be those in which respectively:

- market premium rates were totally insensitive to competition; and
- market premium rates were so sensitive that in any period that each insurer in the market simply set its premium rate to equal that observed in its own 5-insurer market segment in the preceding period.

Thus, k_2 may be viewed as **competition intensity** parameter.

Parameter k_{13} distinguishes separate cases for extant and start-up insurers. It exists only in the former case, and there 1-k_{13} represents competitive inertia in the sense that the larger is 1-k_{13}, the less the competitive effect just described. In fact, the insurer prices by giving weight 1-k_{13} to its premium rates of the preceding period and weight k_{13} to the competitive premium rates described above. In this sense k_{13} denotes **lack of competitive inertia**.

The start-up insurer, on the other hand, has no prior period's premium rates to create this inertia, and in effect is taken to adopt k_{13}=1.

The parameters k_{11} and k_{12} simply set **competitive premiums upper and lower bounds**, where the bounds are expressed as multiples of steady state premiums.

It is noteworthy that the structure of competitive premiums in (2.4) and (2.5) creates positive feedback in the system. As one insurer lowers premium rates, its peers will be induced to do likewise, creating increased market pressure for the first insurer to lower rates further.

(3) Underwriting results

$\pi_{it} = (K_{it} + E_{it}\ P_{it})\ (1+r_{Kit}) - L_{it} - K_{it}$ (2.6)

$D_{it} = \max\ (0,\min[K_{it}+ \pi_{it} - k_3P_{it}E_{it},\ k_{10}(K_{it}+ \pi_{it} - S_0P_{it}E_{it})]$ (2.7)

The definition of r_{Kit} assumes that each insurer invests net assets K in a cross section of the equity market, and the technical reserves $E_{it}\ P_{it}$ in risk free assets. Equation (2.6) assumes that the expected rate of investment return is realized in each period.

This has the effect of factoring investment risk out of the market dynamics. However, as investment risk is supposed to be random, this should not change the fundamental properties of those dynamics, but simply remove an element of volatility.

At the end of each period, the net assets of insurer i amount to $K_{it}+ \pi_{it}$, opening net assets for the period having been augmented by period profit. This is the amount available to pay a dividend.

It is assumed that dividend will be limited so that the remaining net assets do not fall short of a multiple k_3 of period premium. Thus k_3 is a **floor solvency ratio** (for dividend purposes).

The second argument of the min function in (2.7) conditions each dividend payment on the steady state solvency ratio. In effect, it sets this as a target solvency ratio, and no dividend will be paid if the closing solvency ratio is less than the target. If the closing solvency ratio exceeds the target, then k_{10} functions as a **dividend payout ratio** (relative to the excess over target solvency).

(4) Balance sheet results

$K_{i,t+1} = K_{it} + \pi_{it} - D_{it}$ (2.8)

This relation simply states that closing net assets are equal to opening net assets increased by period profit and decreased by any dividend payment.

(5) Entry and exit of capital

Insurer i exits if

$K_{it} < k_3\ E_{it}\ P_{it}$ (2.9)

This relation states that any insurer will exit the market if its solvency ratio falls below a threshold value k_3. Thus, k_3 is a **minimum viable solvency ratio**.

Let

$\pi_t = \Sigma_i\ \pi_{it}$ = market-wide insurance profit (2.10)

$P_t = \Sigma_i\ E_{it}\ P_{it}/\ E_t$ = market average premium per exposure (2.11)

If

$\pi_t\ /\ E_t\ P_t > k_4$ and $\pi_{t-1}\ /\ E_t\ P_{t-1} > k_4$ (2.12) then m_{t+1} new insurers will be introduced into the market, where

$$m_{t+1} = \text{int}\,[k_5\, \Sigma^1_{j=0}\, \{\Sigma_i\, (\pi_{t-j} / E_t\, P_{t-j} - k_4)\}] \tag{2.13}$$

with int[x] defined as the integral part of x.

Each of these new insurers is assumed to have initial capital

$$K_{i,t+1} = k_6\, \Sigma_j\, K_{jt} \tag{2.14}$$

where j runs over all insurers in the market in period t. Each of these insurers commences with premium per exposure P_t.

By (2.12), capital is attracted into the market if the market-wide profit margin exceeds a threshold k_4 in each of two consecutive periods. In this sense, k_4 denotes the **threshold capital attraction profit margin**.

By (2.13), the number of new insurers attracted into the market is (subject to its integral nature) proportional to the sum, over the prior two years, and over all insurers then in the market, of the excess profit margins over the threshold. The constant of proportionality k_5 may be regarded as a **new capital attraction per unit market profitability** parameter.

The capitalisation of new entrants to the insurance market must bear some relation to the strength capitalisation of the market generally if they are to be able to compete. Relation (2.14) postulates that each new insurer is capitalised at $100k_6$% of the preceding period's total market capitalisation. Thus k_6 is a **new entrant capitalisation** parameter.

(6) Loss experience

Let

n_C = realised CAT claim count (for whole market)
~ Poisson (λ_C)

m_C = realised CAT claim size (for whole market)
~ Pareto (mean=μ_C, minimum= $100M)

n_{Nit} = realised non-CAT claim count (**not** per exposure unit) (independent drawings for different insurers)
~ Poisson ($E_i\, \lambda_N$)

m_{Nit} = realised non-CAT claim size (independent drawings for different insurers)
~ Gamma (mean=μ_N, c.v.=1000%)

Then

$$L_{it} = n_C\, m_C\, (E_{it}/E_t) + n_{Nit}\, m_{Nit} \tag{2.15}$$

(7) Re-allocation of market shares

Let

$\rho_{it} = E_{it}/ E_t$ = market share.

Define the (unscaled) transfer function, from insurer r to s as

$$\alpha_{rst} = \max(0, P_{rt} - P_{st}).\max(k_8, \rho_{st}) \tag{2.16}$$

Then define

$$\alpha_{rt} = \Sigma_s\, \alpha_{rst} \tag{2.17}$$

$$\tau_{rst} = [1 - \exp(-k_7\, \alpha_{rt})]\, \alpha_{rst}/\alpha_{rt} \tag{2.18}$$

Define the (unscaled) re-allocated exposures as

$$E^*_{i,t+1} = E_{it}\, [1 - \Sigma_s\, \tau_{ist}] + \Sigma_r\, E_{rt}\, \tau_{rit} \tag{2.19}$$

Then suppose that the resulting market shares are re-scaled as follows:

$$\rho_{i,t+1} = E^*_{i,t+1} / \Sigma_r\, E^*_{r,t+1} \tag{2.20}$$

The sequence (2.16) to (2.20) is explained as follows. The transfer function α_{rst} is non-zero only if $P_{rt} > P_{st}$. In this case some business is expected to flow from insurer r to insurer s, and the transfer function is intended to control the strength of the flow.

Note the factor of $\max(k_8, \rho_{st})$ in the transfer function. This recognises an assumption that the volume of business transferring to insurer s will depend not only on the differential in premium rates but also the market presence of insurer s, represented by its market share ρ_{st}. For example, if s and s* are large and small insurers respectively with $P_{rt} > P_{st} = P_{s*t}$, then the volume of business attracted from r to s will exceed that attracted from r to s*.

It is assumed, however, that there is a lower limit on this effect of market presence. Otherwise insurers with small market shares would never attract any significant amounts of business. Below a certain market share, k_8, the transfer function is just as if market share were k_8. Thus, k_8 is **market presence limit** parameter.

The strength of the flow of business from insurer r to insurer s is described by (2.18). Here, the square bracketed member represents the total transfer of business **away** from insurer r. It is zero when $P_{rt} = P_{st}$, and increases steadily as P_{st} decreases relative to P_{rt}. The ratio following the square bracket apportions this loss of business to other insurers in proportion with the transfer functions α_{rst}.

Exposure volumes for the next period are determined accordingly in (2.19). Volumes calculated this way, will not necessarily sum ti the total market volume, and so are re-scaled to do so by (2.20).

In this system of re-balancing of market shares, the strength of flows between insurers is governed by the parameter k_7, which may be viewed as a **market price-sensitivity** parameter.

Note that, if both max(.) terms of (2.16) are ignored, the square bracketed member of (2.18) becomes $[1 - \exp[-k_7\, \Sigma_s\, (P_{rt} - P_{st})]]$. A single term of the summation gives $[1 - \exp[-k_7\, (P_{rt} - P_{st})]]$ which is just the rate of transfer of business from insurer r to insurer s (for $P_{rt} > P_{st}$) when price-elasticity is k_7. The summation within the square bracket makes allowance for all possible destinations of lapsed business of insurer r.

If $\rho_i < k_9$, then insurer i exits and the market shares are rescaled.

$$E_{i,t+1} = E\ \rho_{i,t+1} \tag{2.21}$$

It is assumed that unsuccessful insurers do not continue to dwindle to infinitesimal sizes, but exit the market after falling below some critical market share. The parameter k_9 represents the **minimum viable market share**.

The dynamical parameters k_1 to k_{13} are summarised for convenience in Table 2.2.

Table 2.2
Dynamical parameters in market model

k_1	Premium-to-solvency sensitivity
k_2	Competition intensity
k_3	Floor solvency ratio (for dividend purposes)
k_4	Threshold capital attraction profit margin
k_5	New capital attraction per unit market profitability
k_6	New entrant capitalisation
k_7	Market price-sensitivity
k_8	Market presence limit
k_9	Minimum viable market share
k_{10}	Dividend payout ratio (relative to the excess over target solvency)
k_{11}	Competitive premiums lower bound
k_{12}	Competitive premiums upper bound
k_{13}	Lack of competitive inertia

Policy variables
Certain of the variables in Table 2.2 are subject to regulation, and so may be regarded as policy variables. These are listed in Table 2.3.

Table 2.3
Policy variables

k_3	Floor solvency ratio (for dividend purposes)
k_4	Threshold capital attraction profit margin
k_6	New entrant capitalisation
k_9	Minimum viable market share
k_{10}	Dividend payout ratio (relative to the excess over target solvency)
k_{11}	Competitive premiums lower bound
k_{12}	Competitive premiums upper bound

A variable has been included in Table 2.3 only if capable of **direct regulation**. The variables omitted are essentially the controlling the competitive dynamics of the market. Some of the variables omitted might be influenced indirectly by policy. For example, premium-to-solvency sensitivity (k_1) could be influenced by the imposition of solvency-dependent risk capital charges.

3. Simulation set-up

Market behaviour has been simulated over the periods t=1,2,…,60 by means of the model described in Section 2. Loss experience has been generated as random drawings of the quantities n_C, m_C, n_{Nit} and m_{Nit}. A **base case**, involving a base set of parameters, has been simulated, and then other cases obtained by varying the parameters singly or in small related groups. The same drawing of loss experience is used for all simulations.

Initially, this is done for the case n_C=0, i.e. the effect of catastrophes is excluded. The whole procedure is then repeated with catastrophes included.

The base case is chosen as representing a reasonably stable and uneventful market.

Table 3.1 sets out the values adopted for the environmental parameters, and Table 3.2 the dynamical parameter values for the base case.

Table 3.1
Environmental parameter values

	Parameter	Value
K_0	Steady state capital per unit exposure	\$145
E	Total exposure for whole market	1.5M
r_F	Risk free rate of return	4%
r_M	Share market expected rate of return	12%
λ_C	Expected CAT claim frequency (for whole market)	2%
μ_C	Expected CAT claim size (for whole market)	\$120M
λ_N	Expected non-CAT claim frequency per exposure unit (common to all insurers)	12%
μ_N	Expected non-CAT claim size (common to all insurers)	\$1000

Table 3.2
Base case dynamical parameter values

k_1	0.667
k_2	0.25
k_3	0.1
k_4	0.2
k_5	30
k_6	0.000333
k_7	0.1
k_8	0.01
k_9	0.0006
k_{10}	0.7
k_{11}	0
k_{12}	1000
k_{13}	0.75

Table 3.3 lists the initial values of the free variables in the market. It indicates that each of the 100 initial market participants underwrites 75,000 exposure units. The premium of $160 never appears in the market, but functions as $P_{i(0)}$ in the application of (2.3) for t=1. It is chosen to differ significantly from the steady state premiums in the examples below, in order to illustrate the reversion to steady state over time in the case of a stable system.

Table 3.3
Initial values of market

Number of market participants	20
Market share of each participant	5%
Premium per exposure unit	$160
Premium per exposure unit	$160
Opening capital (total market)	$270M

Note that the price controls k_{11} and k_{12} are, in effect, switched off in the base case (compare the steady state premium of $115.4 calculated in accordance with (2.1)).

Different variations in the dynamical parameter values appearing in Table 3.2 affect the following aspects of the market:

- Length of market cycles
- Depth of market cycles
- Market concentration (as represented by the Herfindahl-Hirschman index)
- Longevity of initial participants in the market
- Longevity of new entrants to the market
- Diversity of premium rates on offer

- Market profitability
- Number of market participants
- Market solvency
- Insolvency rate
- Expected policy holder deficit (EPD).

4. Simulation results

4.1 Base case

Base case parameters are set out in Table 3.2 and it is assumed for the time being that n_C=0. Figure 4.1 to Figure 4.5 illustrate the major dynamics of this market. The general picture is one of a typical market, with:

- Generally stable premium rates and solvency (Figure 4.1);
- Largely stable number of market participants but with the occasional entrant or exit (Figure 4.2);
- A marked diversity of premium rates available in the market (Figure 4.3);
- An average profit margin that is variable but generally positive (Figure 4.4);
- Some market concentration over time but only of a moderate sort (Figure 4.5).

Figure 4.1

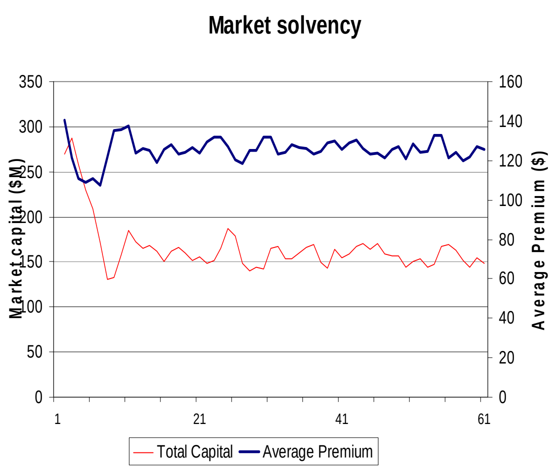


Figure 4.2

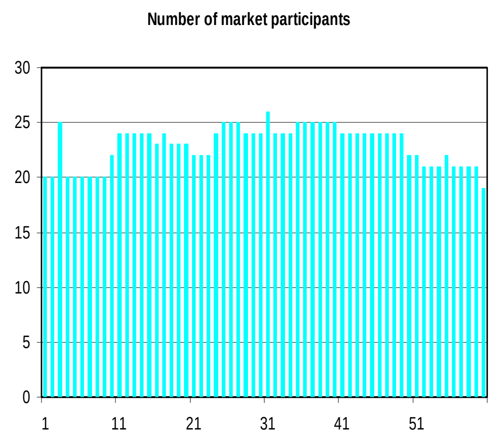


Figure 4.3

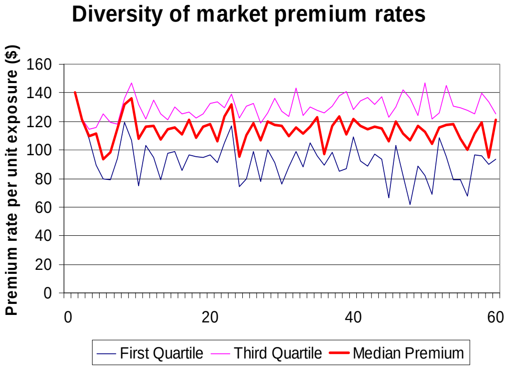


Figure 4.4

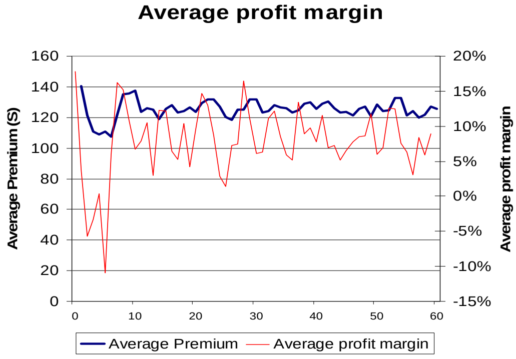
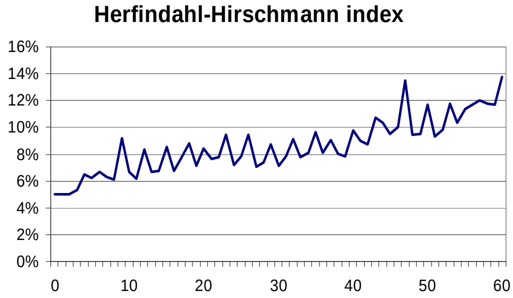


Figure 4.5



4.2 Cyclic market behaviour

Certain parameter values in the model cause market premium rates to exhibit cyclic behaviour. This is illustrated in Figure 4.6 where decreasing values of k_2 are associated with premium cycles of increasing amplitude. the plot for the base value of k_2 is shown as a heavy line.

The empirical autocorrelation function (ACF) for average premium is shown for the same values of k_2 in Figure 4.7. The horizontal axis displays the autocorrelation lag. Again increasing amplitude is associated with decreasing k_2.

Figure 4.6

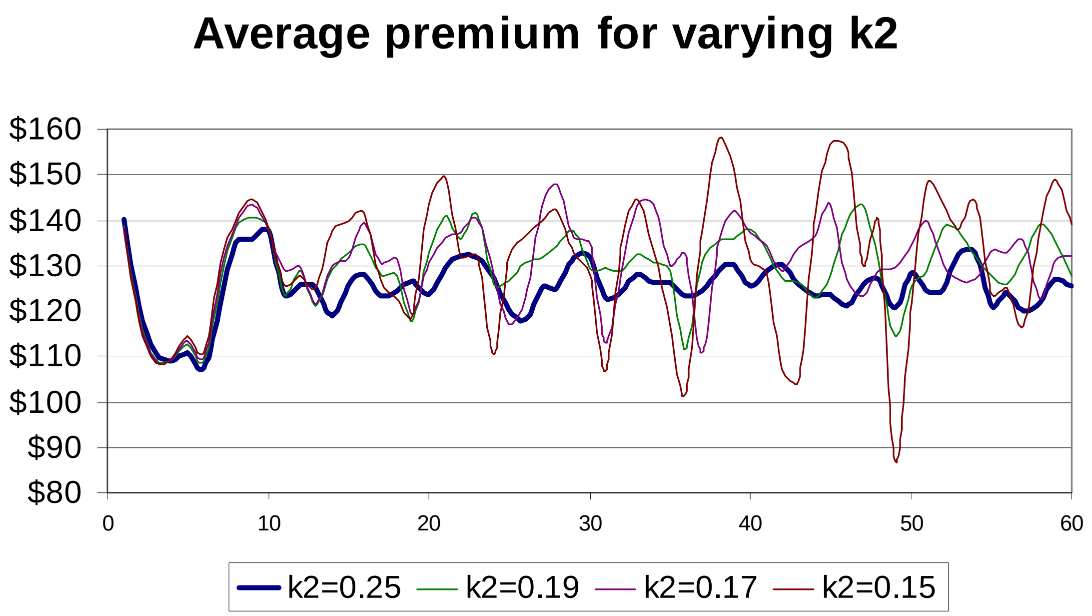
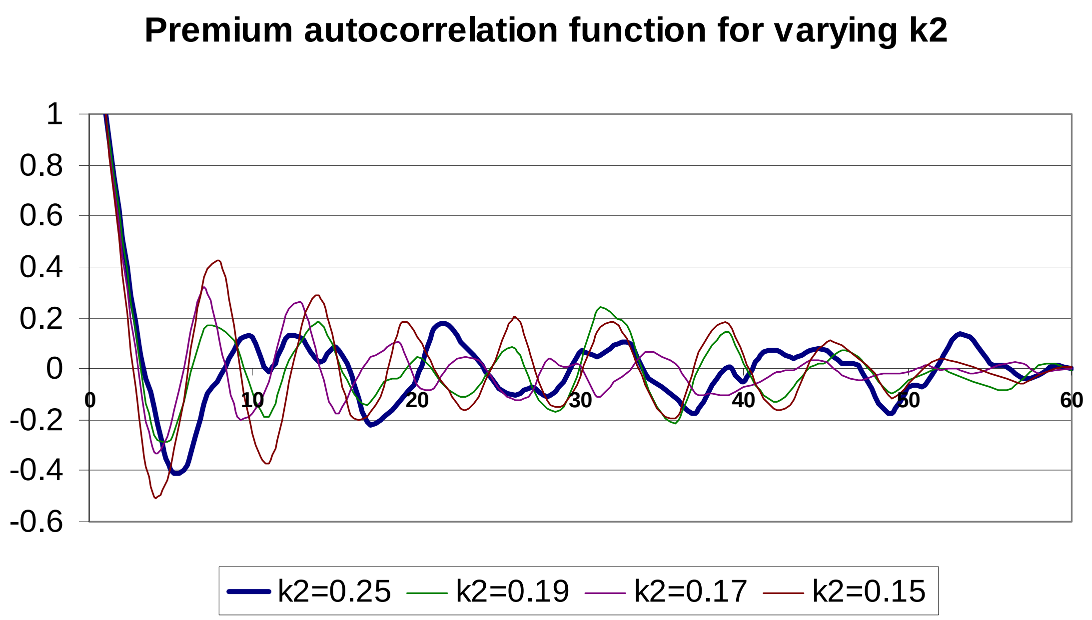


Figure 4.7



The discussion immediately following (2.4) indicates that k_2 is a competition intensity parameter and that reduction in its value represents a weakening of competition in the market. The numerical results above therefore imply that weakened competition (from the base case) will induce cyclic behaviour in market premium rates.

It is interesting to note that Figure 4.7 implies a periodicity of 6 or 7 time intervals. This value is not unique to variation of k_2. As will be seen, when the market exhibits cyclic behaviour, induced by whichever parameter, the cycles are usually of about this periodicity.

A "time interval" here is the length of time required for each re-balancing of the market under the competitive dynamics. The value of this interval is unknown. It is interesting to note, however, that, if it were a year, the above occurrence of 6- or 7-year cycle would agree with the empirical results of Cummins & Outreville (1987), who found market cycles of about this period in a number of countries.

4.3 Diversity of premium rates

Figure 4.8 to Figure 4.10 illustrate the fact that diversity of premiums available in the market increases as the market price-sensitivity parameter k_7 is increased. Note that Figure 4.9 reproduces Figure 4.3.

Figure 4.10 appears to have become cyclic. This is indeed the case, as is illustrated by Figure 4.11 which plots the ACF of average premium in the market. It may be checked that cyclic behaviour steadily intensifies as k_7 increases above the base value of 0.10.

Figure 4.8

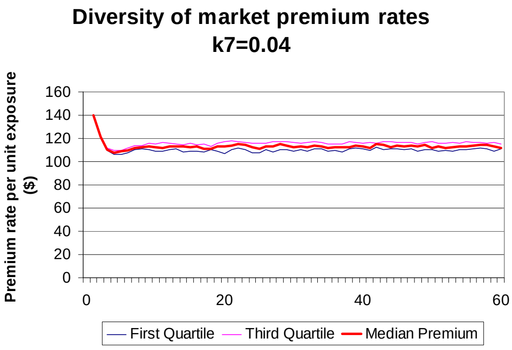


Figure 4.9

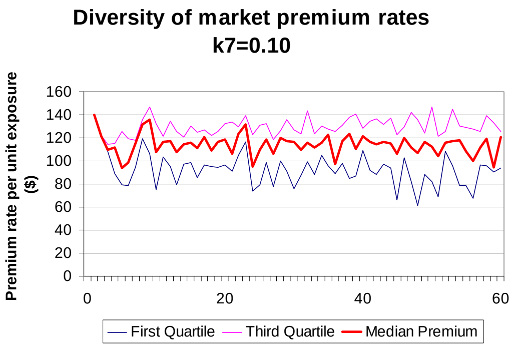


Figure 4.10

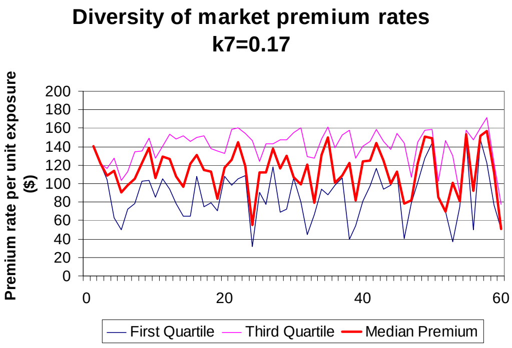
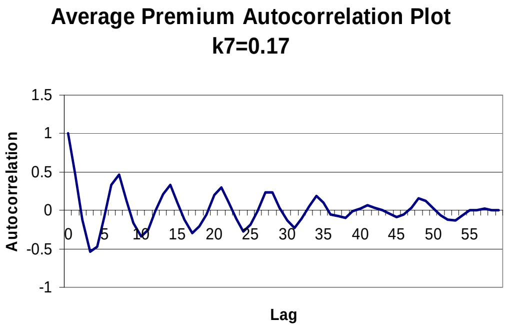


Figure 4.11



4.4 Number of market participants and market concentration

4.4.1 Dividend payout ratio

The number of market participants and market concentration are strongly affected by the dividend payout ratio k_{10}. Figure 4.12 to Figure 4.17 illustrate the effect of increasing k_{10} from its base value of 70% to 90%. Note that Figure 4.12 and Figure 4.13 reproduce Figure 4.2 and Figure 4.5.

Figure 4.12

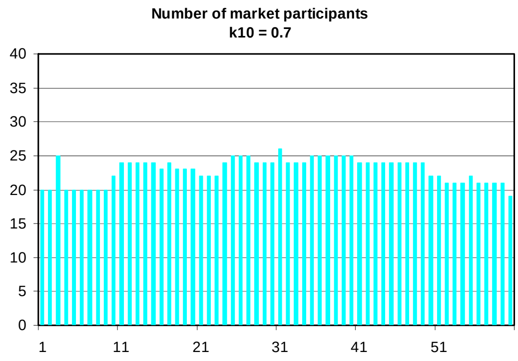


Figure 4.13

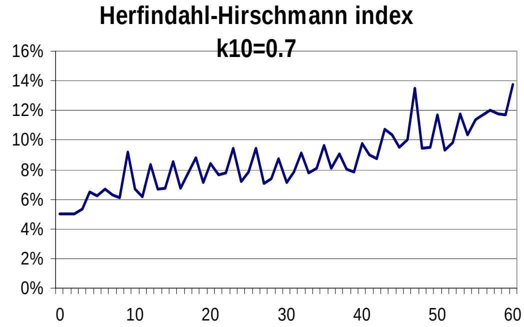


Figure 4.14

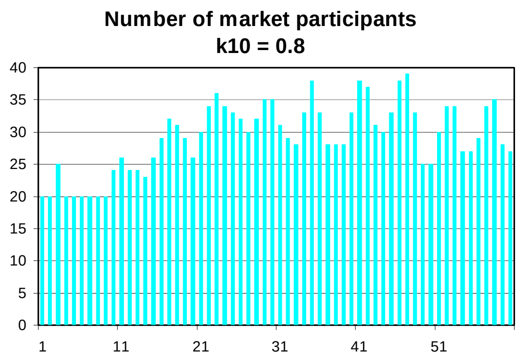


Figure 4.15

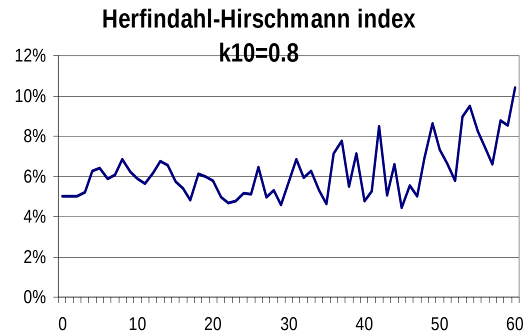


Figure 4.16

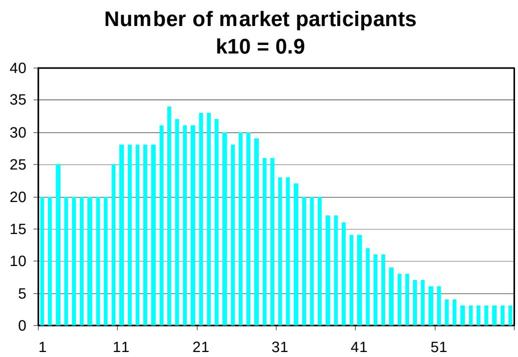
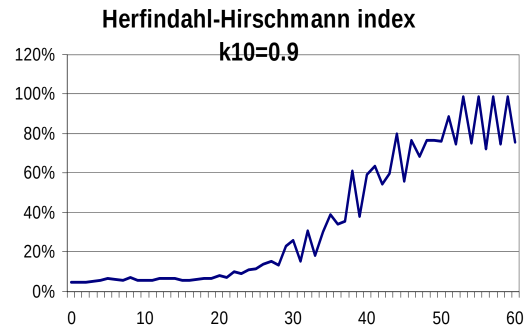


Figure 4.17



It can be seen that the increase in k_{10} from 0.7 to 0.8 causes an appreciable increase in the number of market participants, though without a reduction in the concentration index and at the expense of the introduction of cyclic behaviour. The latter is visible in Figure 4.14, and is confirmed by Figure 4.18.

Figure 4.18

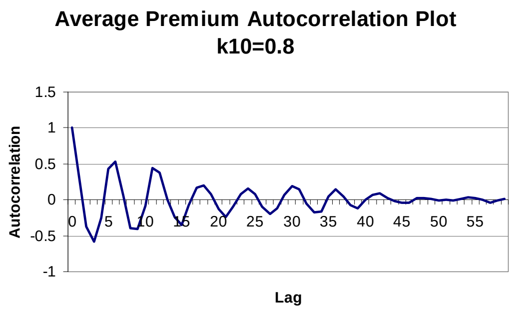
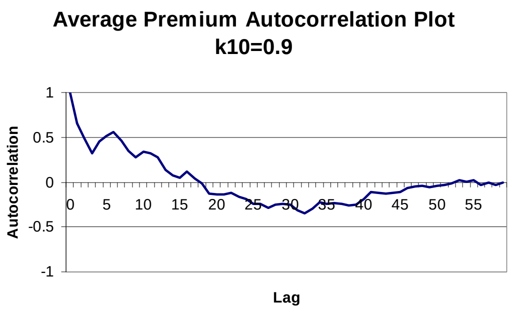


Figure 4.19



Interestingly, further increase of k_{10} from 0.8 to 0.9 causes an initial increase followed by a crash in the number of market participants (Figure 4.16). The cyclic market behaviour is eliminated and average premium in the market follows a process more resembling AR(1) (Figure 4.19).

4.4.2 New capital attraction per unit market profitability

It is interesting that the number of participants in the market is not permanently increased by increasing the parameter k_5 governing new capital attraction per unit market profitability. New participants are certainly attracted when this parameter is increased from its base value of 30 to 45, but intense competition sets in, creating violent market cycles (Figure 4.20), leading to the demise of many.

The average insolvency rate, defined as the annual proportion of insurers (by number) going insolvent, averaged over all 60 years, is 10%, compared with 0.15% for the base case. Ultimately, the number of participants is low (Figure 4.21).

Figure 4.20

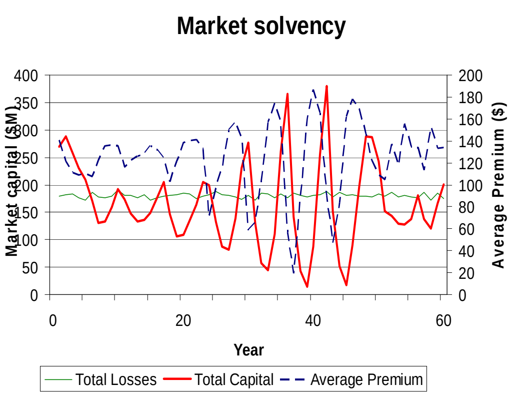
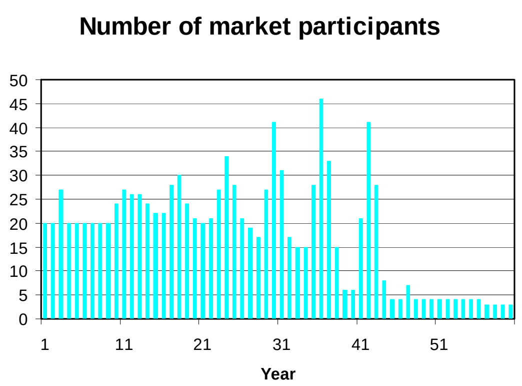


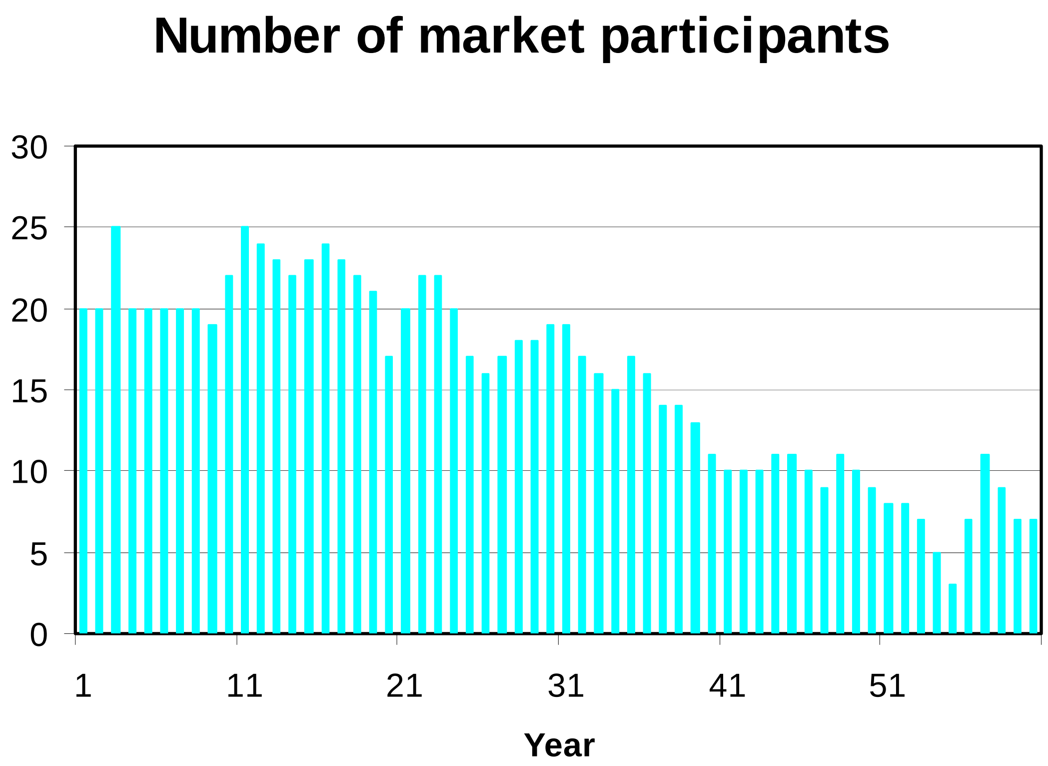
Figure 4.21



4.4.3 Minimum viable market share

The effect of the minimum viable market share parameter k_9 is as expected. Increasing it reduces the number of market participants, as seen by a comparison of Figure 4.22 (k_9=0.0025) with Figure 4.2 (k_9=0.0006).

Figure 4.22

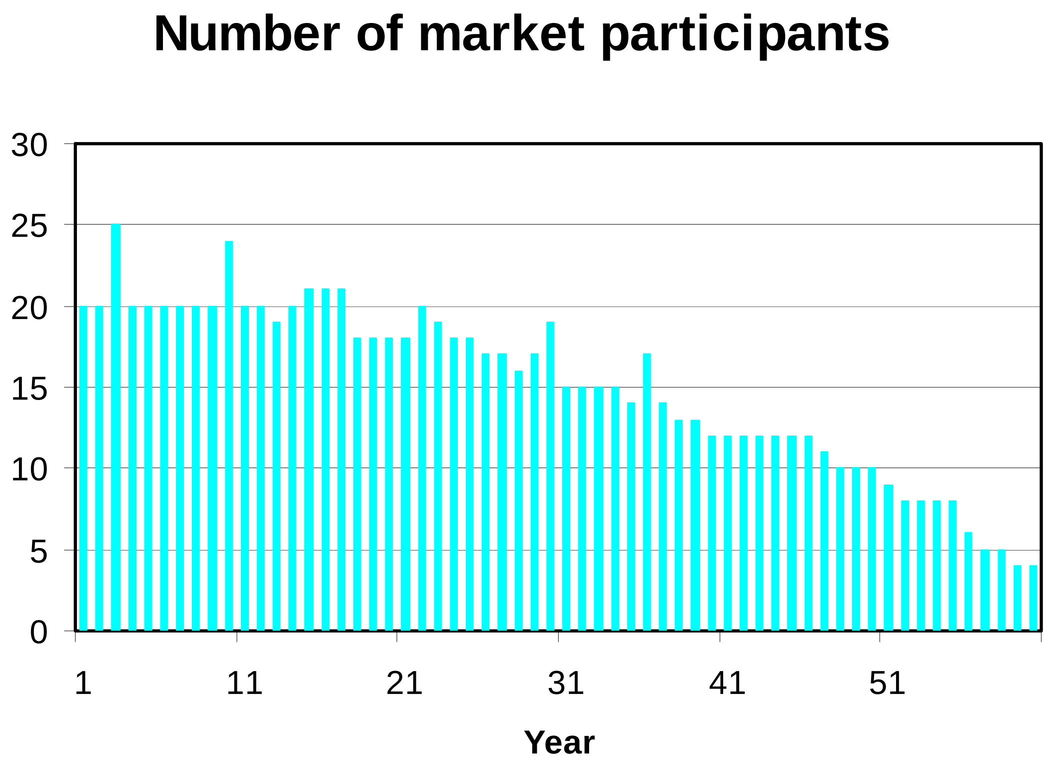


4.4.4 Market presence limit

The effect of the market presence limit parameter k_8 is also largely as expected. Decreasing it reduces the number of market participants, as seen by a comparison of Figure 4.23 (k_8=0.001) with Figure 4.2 (k_8=0.01).

Increasing k8 above 0.01 appears to have little effect. This suggests that there may be a limit to the value of advertising simply to achieve brand prominence. Beyond that limit advertising might need to switch to some form of product differentiation to achieve effectiveness.

Figure 4.23



4.5 Effects of competition

The effects of competition are reflected in the premium-to-solvency and competition intensity parameters k_1 and k_2. One of the major effects of these parameters is the generation of complex patterns of cyclic behaviour. Figure 4.24 displays, for various values of k_1, the intervals of k_2 in which cyclic behaviour is observed.

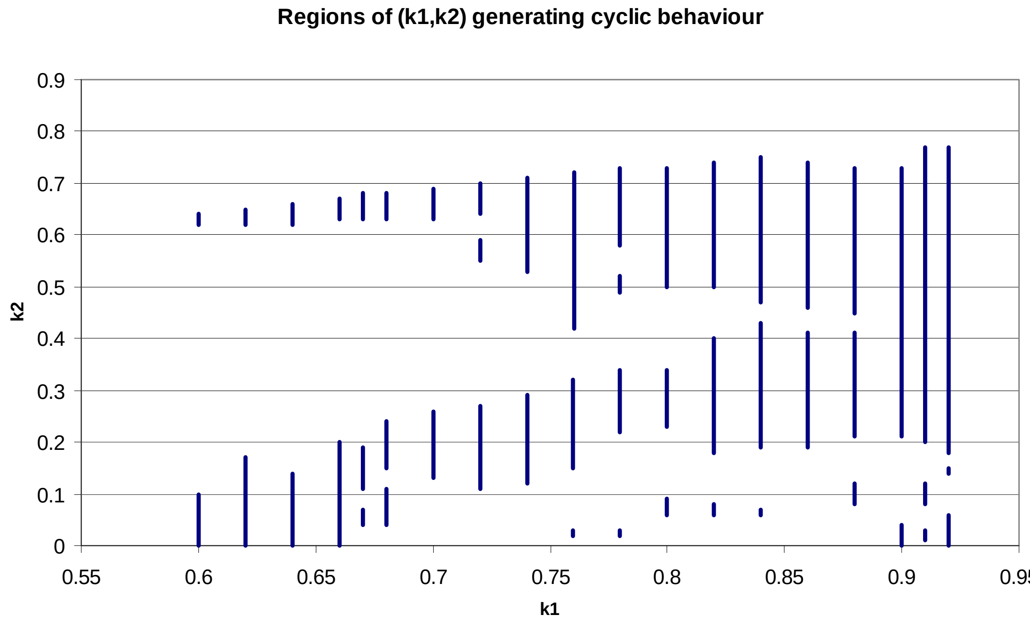
It should be said in connection with Figure 4.24 that there is some imprecision in the differentiation between cyclic and non-cyclic cases. Generally, however, a case has been classified as cyclic if the ACF of average premium displays either:

- a clear cyclic pattern (as in Figure 4.11 and Figure 4.18); or
- a well-defined peak well in excess of 0.2.

The regions of cyclicity exhibit catastrophe-like behaviour (Poston & Stewart, 1976) with bifurcation into sub-regions as k_1 increases, and merger of other sub-regions. It may be observed in the most general terms, however, that cyclic behaviour becomes steadily more difficult to avoid as k_1 increases.

This means that a high preoccupation with solvency in the pricing of business by market participants is likely to lead to cyclic behaviour of prices. The parameter k_1 is not one that can be regulated directly. It may, however, subject to indirect influence by the application of some form of penalty for low solvency. Figure 4.24 indicates that this might have some unwelcome effects.

Figure 4.24



The same figure shows that cyclic behaviour is liable to emerge if competition, as represented by k_2, is either too strong or too weak. Competition of moderate strength is more conducive to market stability.

It is not surprising that increasing the competition intensity k_2 tends to reduce the average profit of the market. Figure 4.25 plots the average market profit margin for the base case (k_2=0.25) and also for k_2=0.45 and 0.75. It can be seen that, once the initial transients have passed, the plots clearly show the said ordering with respect to k_2.

Figure 4.25

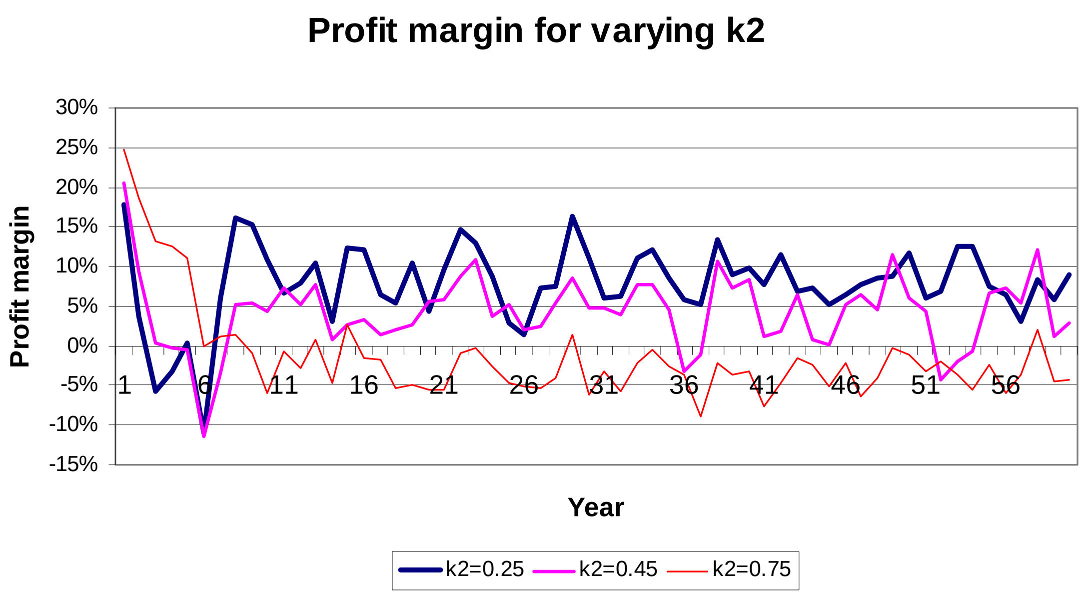
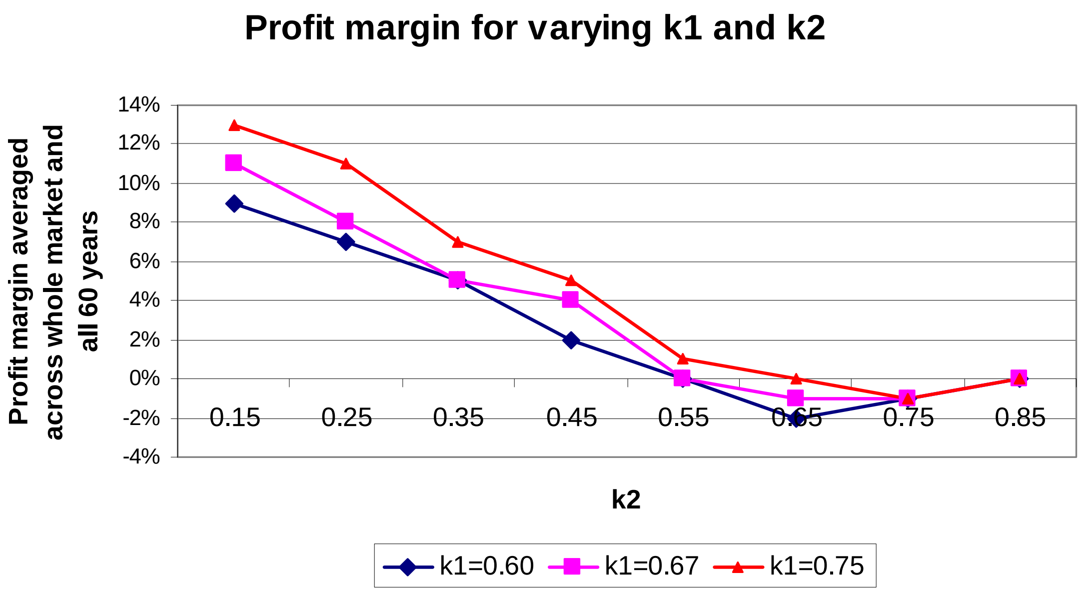


Figure 4.26 gives more detail on the variation of profit margin with k_2. the profit margin displayed here is averaged across the whole market, as in Figure 4.25,

but also averaged across all60 years of simulated experience. This average profit margin is plotted against k_2. this is done for several values of k_1.

The steady decline of average profit margin with increasing competition intensity up to about k_2=0.65 is evident for each value of k_1 considered. The decline appears to flatten for higher values of k_2.

Figure 4.26



4.6 Regulatory controls

4.6.1 Barriers to entry

Setting a floor under market profitability for the introduction of new participants in the market (parameter k_4) can affect the number of participants. This is illustrated by It can be seen that, as the barrier to entry is raised, the number of market participants steadily reduces. In fact, for the case k4=0.24 and all higher values, there are no new entrants to the market, only the occasional exit.

Figure 4.27 to Figure 4.30, which track market behaviour as k_{11} increases from 0.15 to 0.24. The case k_4=0.2 replicates Figure 4.2.

It can be seen that, as the barrier to entry is raised, the number of market participants steadily reduces. In fact, for the case k_4=0.24 and all higher values, there are no new entrants to the market, only the occasional exit.

Figure 4.27

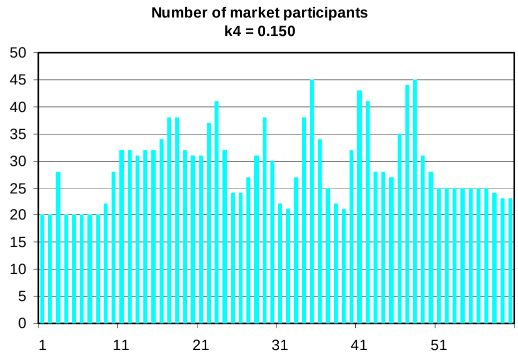


Figure 4.28

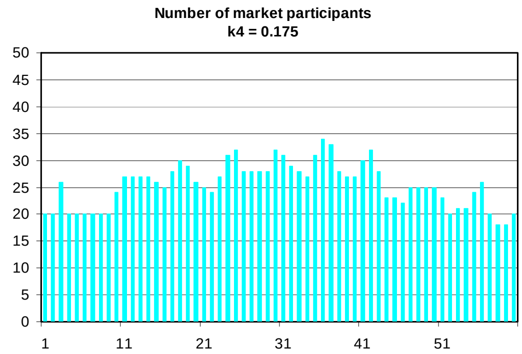


Figure 4.29

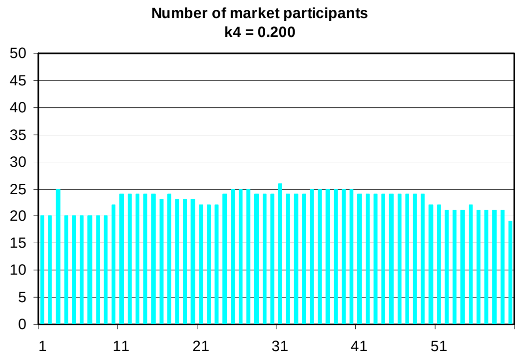
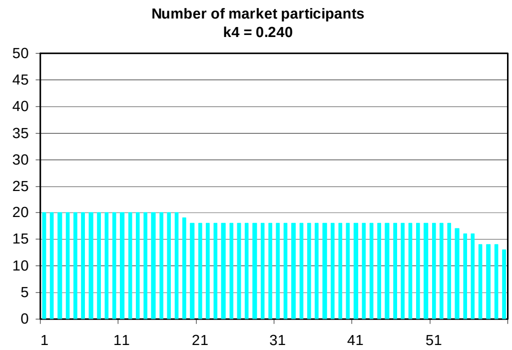


Figure 4.30



It is also noteworthy that, taken over the whole 60 intervals, the effect of variation in k_4 is largely transient; the final number of market participants never differs much from the initial number. This is because a lower barrier to entry admits more new entrants, but with a lower average viability. This is illustrated by Figure 4.31 and Figure 4.32.

Figure 4.31

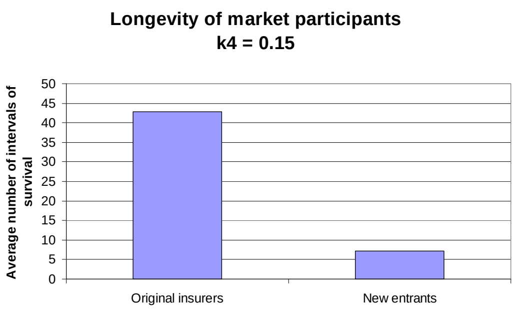
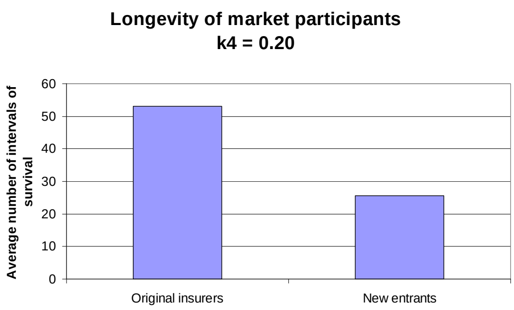


Figure 4.32



A different barrier to entry is discussed at the end of Section 4.6.4.

4.6.2 Price regulation

Premium floor

Setting a floor under market premium rates (parameter k_{11}) can result in drastic changes to the number of participants in the market. This is illustrated by Figure 4.33 to Figure 4.38, which track market behaviour as k_{11} increases from 0.65 to 1. The case k_{11}=0 is found in Figure 4.2.

Increasing required market profitability from a low level attracts new entrants. However, the behaviour of the market becomes acutely sensitive to changes in this parameter as it approaches unity (full funding).

Moreover, increasing k_{11} beyond a certain level affects the market perversely. While new entrants are attracted, their survival is short and ultimately the number of market participants declines. When this occurs, average market profitability declines in sympathy. This is illustrated by Figure 4.39, which displays the evolution of market profitability over the 60-year period for various values of k_{11} close to unity.

Figure 4.33

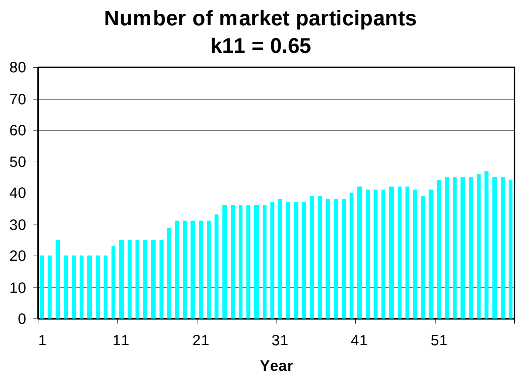


Figure 4.34

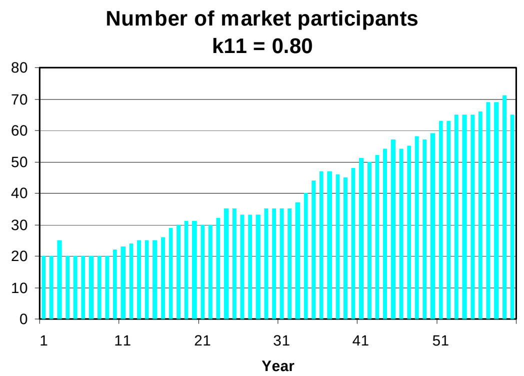


Figure 4.35

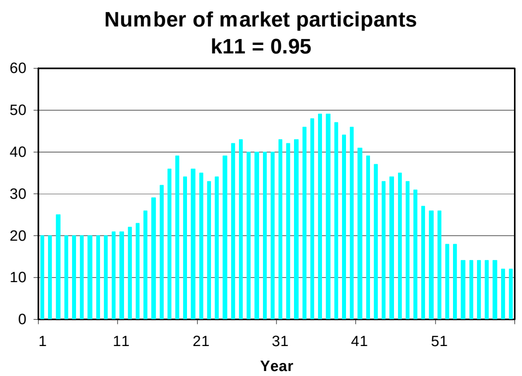


Figure 4.36

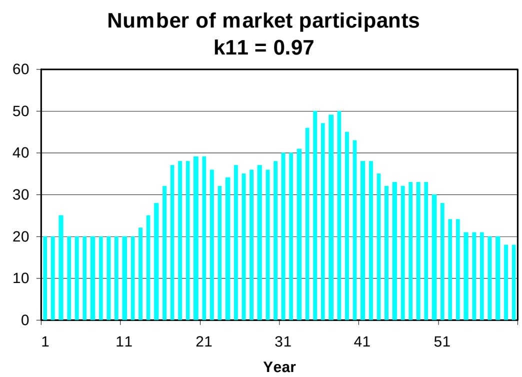


Figure 4.37

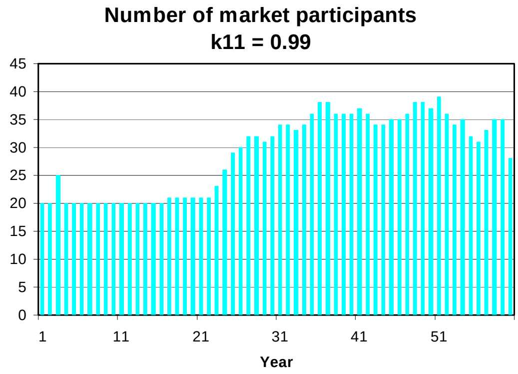


Figure 4.38

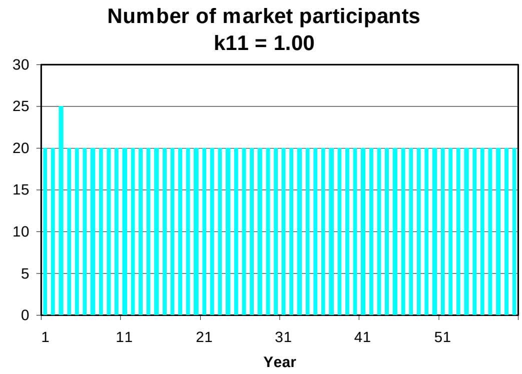
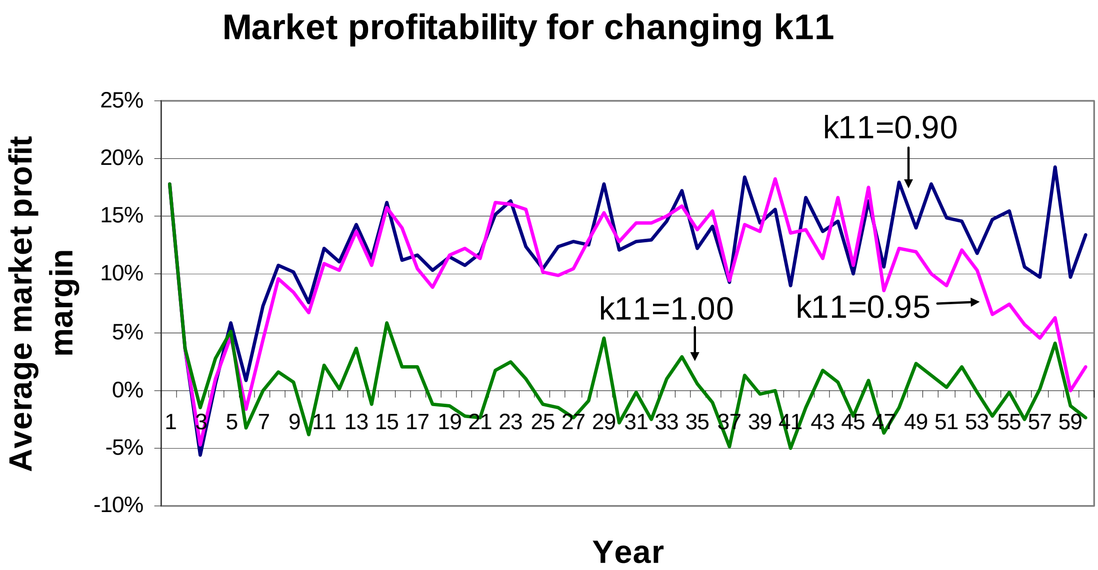


Figure 4.39



Premium ceiling

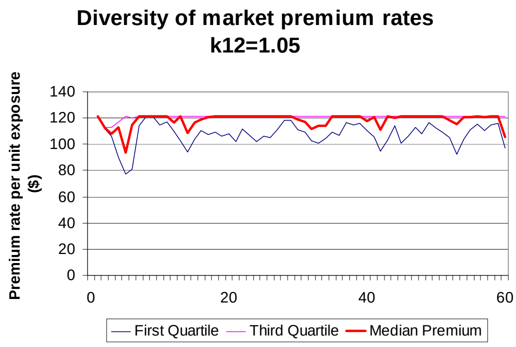
Setting a ceiling on premium (parameter k_{12}) does not affect the number of market participants so dramatically but it still affects the diversity of premiums available in the market if set sufficiently low.

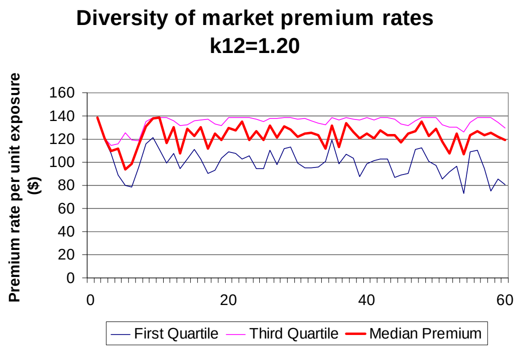
A comparison of Figure 4.40 and Figure 4.41 with Figure 4.3 illustrates. In the former the premium ceiling allows a maximum margin of 5% in excess of the economic premium rate. The restricted range of premium rates is such that the upper quartile often coincides with the median.

In Figure 4.41 a margin of up to 20% is permitted but the loss of premium diversity is still appreciable.

Figure 4.40

Figure 4.41





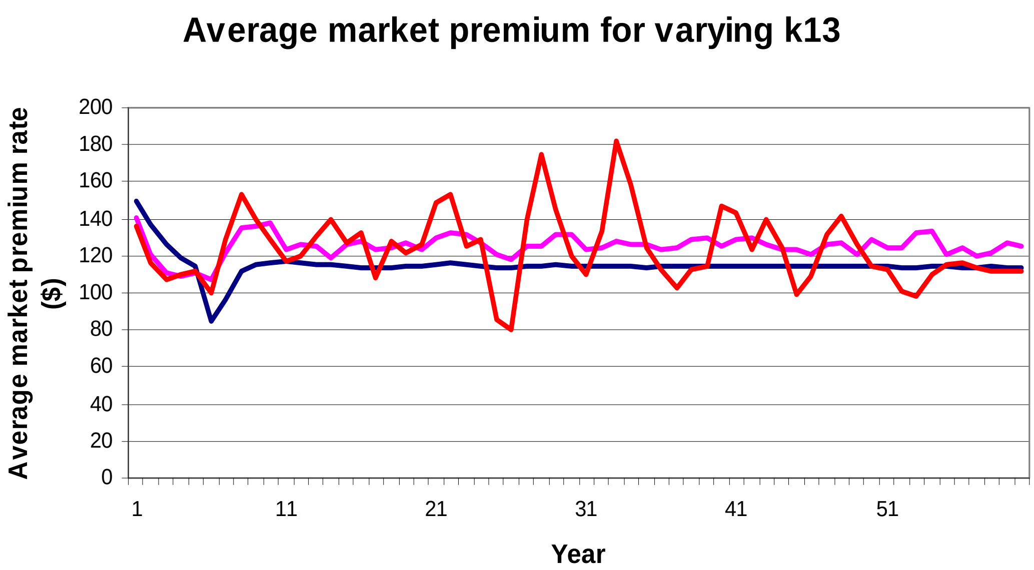
4.6.3 Regulation of price stability

The parameter k_{13} regulates price stability. The closer it is to zero, the less the movement in premium rates from year to year. Direct insurers often apply some pricing mechanism of this sort voluntarily in order to protect market share.

It would be difficult to apply the parameter at the regulatory level in exactly the form specified in (2.5). However, some variant of it, e.g. premium rates do not vary from one period to the next by more than p%, could be so applied.

Figure 4.42 tracks average market premium through the 60 years of simulation for three values of k_{13}, viz. 0.4, 0.75 and 0.9. All other parameters remain set at their base case values.

Figure 4.42

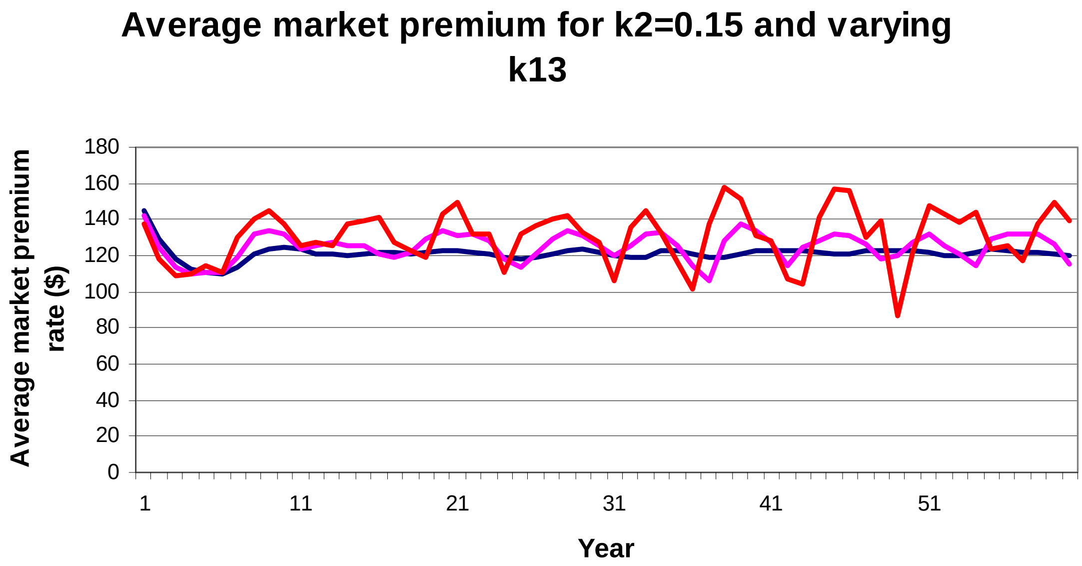


The plot shows increasing instability of premium rates with increasing k_{13}. Reducing this parameter to increase price stability causes a modest reduction in the number of market participants.

The same parameter can also be used to control market cycles that would otherwise exist. For example, Figure 4.6 shows pronounced cycles in the case

k_2=0.15 and all other parameters at their base values including k_{13}=0.75. Figure 4.47 illustrates how these cycles are steadily reduced as k_{13} is reduced to 0.6 and 0.5.

Figure 4.43



4.6.4 Solvency maintenance

The parameter k_3 plays two roles. It sets a floor solvency ratio that must not be breached by dividend payout (see (2.7)). It also sets the same floor for continued underwriting. Breach of the floor leads to enforced exit from the industry.

Comparison of Figure 4.44 with the base case in Figure 4.1, and of Figure 4.45 with Figure 4.2, illustrates the effect of trebling the solvency floor, raising this parameter from its base value of 0.1 to 0.3. It is seen that a regulatory action that might have been considered responsible in fact has the effect of driving out a large proportion of market participants and creating violent market cycles in the process.

Figure 4.44

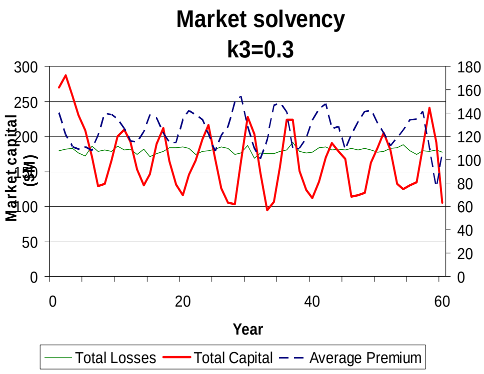
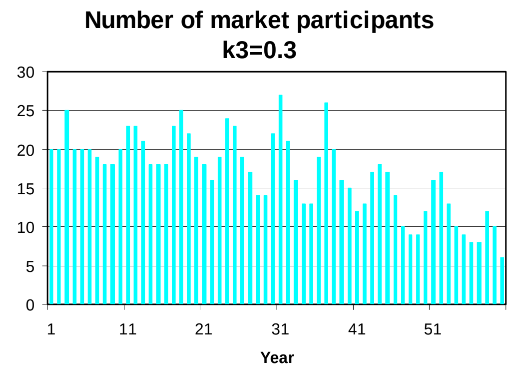
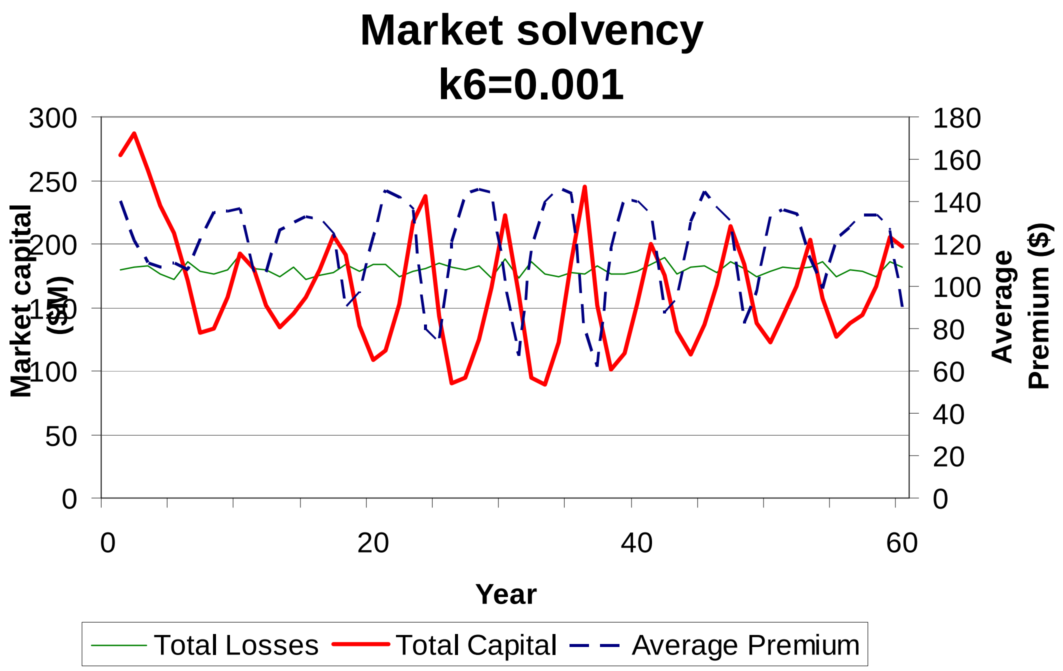


Figure 4.45



A similar effect is observed when the capital requirement for market entry is raised. Comparison of Figure 4.46 with Figure 4.1 shows the effect of trebling the market entry parameter k_6.

Figure 4.46



4.7 Catastrophe events

As noted at the start of Section 4.1, all results hitherto are based on the assumption of no CAT claims (n_C=0). In the present sub-section, the value n_C=2% is included in the simulation of loss experience. Figure 4.47 plots the simulated experience, consisting of the CAT experience added to that underlying the results of Sections 4.1 to 4.6.

The simulated CAT experience, which is conspicuous in the plot, comprises a single event in year 36 with a total cost of $107M to the market. This increase the total cost of losses for the year by more than 50%, and the single CAT event accounts for 83% of steady state market capital.

Figure 4.47

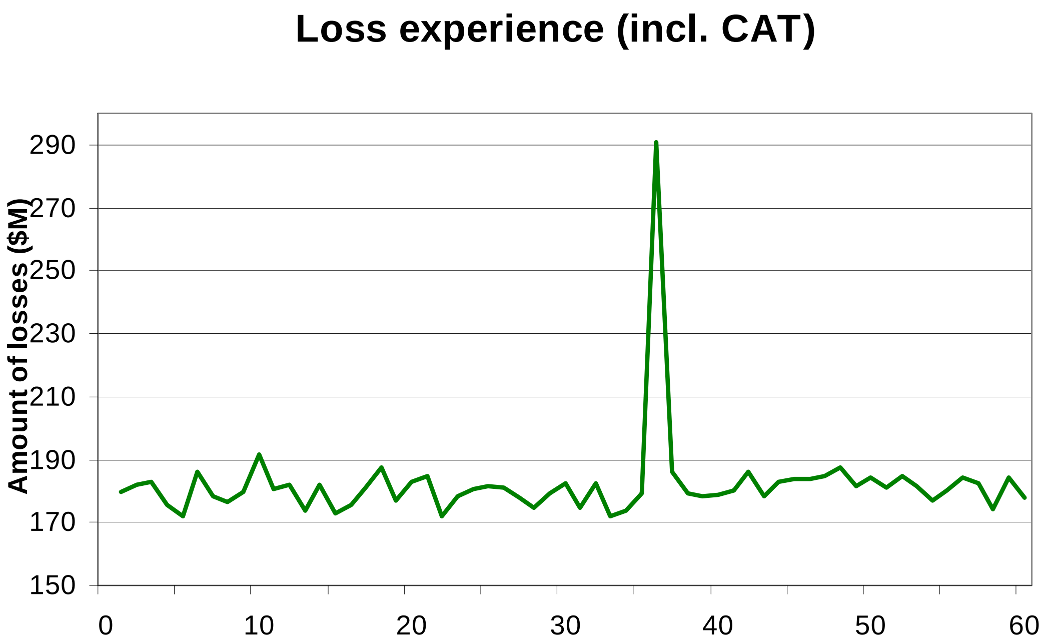


Figure 4.48 and Figure 4.49 illustrate the effect of the loss on the market. Initially, it extinguishes nearly half of the market participants and drives market solvency down to about one-third of its steady state value. However, this drives prices up to record levels and there is a rapid influx of new participants. This accords entirely with the model of Winter (1991).

Cummins & Danzon (1991) also find empirical evidence that an insurance price crisis, such as appears in Figure 4.48 can arise from unexpected losses to insurers in the form of reserve adjustments. Presumably, this result can be extrapolated to unexpected losses from catastrophic events.

Figure 4.48

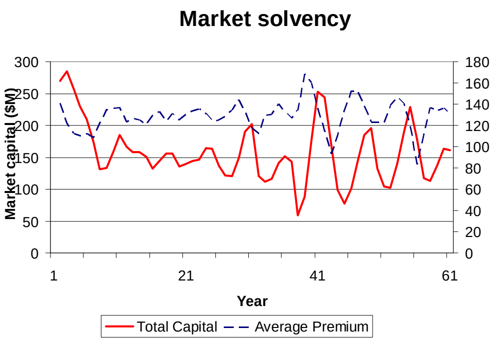
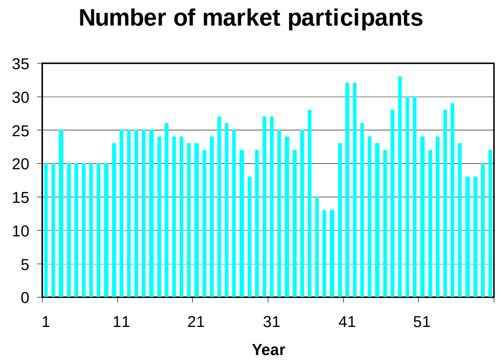


Figure 4.49



A series of cycles in premium rates, solvency and number of market participants then sets in. Over the course of a cycle, premium rates vary by 25% or more of their steady state value. Market solvency varies by 70% or so of its steady state value. These results are consistent with those of Balzer (1982) and Balzer &

Benjamin (1980), who show that feedback of claim costs into premiums can create cycles if the feedback lag becomes too great.

Whether these are features of a healthy market is perhaps an ideological matter, but it may be useful to consider whether they can be mitigated by some form of market intervention. The most obvious form is price control, particularly the prescription of a ceiling.

This is investigated in Figure 4.50 and Figure 4.51, where the premium ceiling parameter k_{12} is set to 1.1, and in Figure 4.52 and Figure 4.53 where k_{12}=1.2. It seen that this form of price regulation can in fact reduce the depth of the premium cycles considerably. The price for this is a modest reduction in the number of market participants. The cycles in market solvency are not much effected.

Figure 4.50

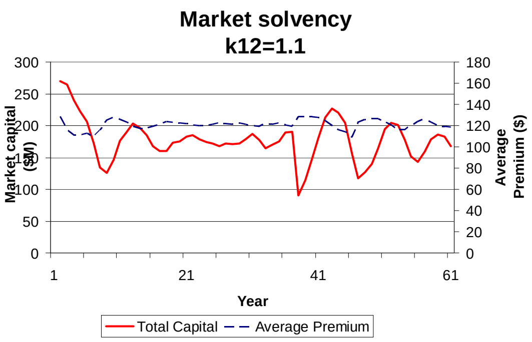


Figure 4.51

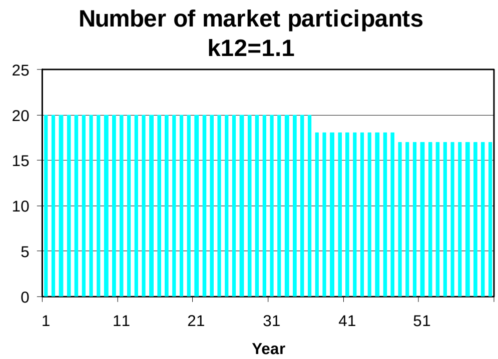


Figure 4.52

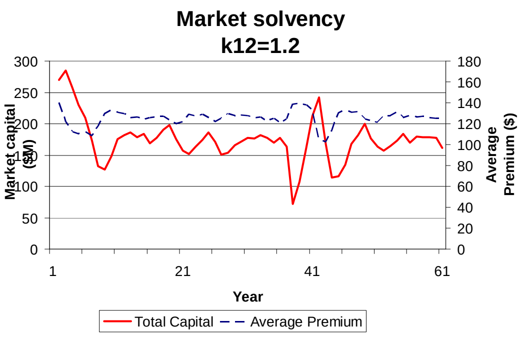
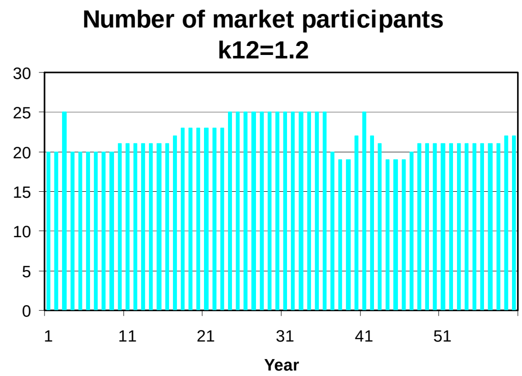


Figure 4.53



5. Conclusion

5.1 General commentary

The model of an insurance market constructed here is minimalist in the sense that it attempts to capture all of the main features of a market using as few parameters as possible. It does so with 13 dynamical parameters. Of these two are upper and lower limits on price which are inessential to the functioning of the market. So it is fair to regard the market as comprising 11 parameters. It is suggested that the model is minimalist in that the elimination of any one of these eleven would be fatal to its realism.

Despite the relatively sparse parameterisation, the model displays considerable complexity. Some of its results are reminiscent of catastrophe theory (Figure 4.24) and a number are counter-intuitive. Regulation of the market solvency provides an example. Excessive increase in the strictness of the required solvency ratio is seen in Section 4.6.4 to induce violent cyclic behaviour in the average market premium rate and solvency. This is in addition to the more predictable effect of driving out a certain proportion of market participants.

5.2 Competition

Competition has occasionally been seen as a de-stabiliser of insurance markets. In Winter (1991), for example, competition slowly erodes market profitability until solvency becomes overly stressed. A sudden premium rate crunch then ensues, attracting new capital into the market, and the cycle re-commences.

This viewpoint is valid to a limited extent. For example, Section 4.3 shows that strong competitive effects, resulting from high price-sensitivity of consumers, can indeed induce cyclic behaviour in average market premiums rates.

However, the effects of competition extend well beyond this. For example, Section 4.5 shows that the dependency on certain combinations of competition parameters of whether or not the market displays cyclic behaviour is extremely complex.

Some of the major competitive effects are intuitive. For example, the average profit margin in the market declines with increasing competition, and it increases as insurers become more concerned with solvency (Section 4.5).

Some competitive effects are beneficial to the market and some are not. For example, Section 4.2 shows that too low an intensity of competition leads to cyclic behaviour in premium rates, and that this is eliminated in a more competitive market. On the other hand, high price-sensitivity of consumers, a different aspect of competition, also leads to cycles (Section 4.3).

5.3 Use of policy variables

Policy variables need to be used with considerable care because their effects may be counter-intuitive and even the reverse of those intended.

Perhaps the most controversial use of policy variables are price controls, in the form of upper or lower limits on premium rates. It is seen in Section 4.6.2 that setting a floor under premium rates does not have the expected effect if the floor is set toot high.

In fact, raising the floor towards a requirement of full funding premium rates acts as a barrier to entry. The market becomes bland, with virtually no new entrants, very stable premium rates but no diversity between them. Moreover, average profit margin is zero.

This last effect is counter to the intention. The fact is that setting a lower floor under premium rates leads to **higher**, not lower, average rates.

Setting a ceiling on premium rates does not have unexpected consequences, but a low ceiling produces a similarly bland market.

Another quantity with major potential as a policy variable is dividend payout ratio. Section 4.4.1 shows that increasing this has counter-intuitive effects. High values of the ratio induce market cycles. Even higher values eliminate these but decimate the market participants.

5.4 Catastrophe events

Catastrophes have sometimes been viewed in the literature as responsible for market cycles, or at least for erratic behaviour of premium rates. This is certainly confirmed by the model investigated here. Indeed, while the effect on the market of a single major event may be transient, it can have a long persistency (Section 4.7).

The result of this, even with a relatively low catastrophe frequency, would be that each new event would occur before the cycles induced by the previous one had died out. In this situation the market would exhibit continuous cyclic behaviour.

A price control, in the form of a ceiling on premium rates, can mitigate this behaviour.

6. Acknowledgement

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